

Multicommodity Trade Network Equilibrium with Mixed Demand and Food Security

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Abstract: Rising food insecurity is a pressing global concern. Many developing countries rely on a combination of international trade and food aid to satisfy the nutritional and caloric demands of their populations. Trade openness has been shown to have positive long-term effects on food security; however, it can cause short-term crises in food availability and affordability. Strategic grain reserves are among the tools governments use to control food prices and ensure availability while maintaining trade openness. This paper constructs a multicommodity trade network equilibrium model with mixed demand to examine the implications of strategic grain reserves for food security. The model incorporates fixed and elastic demand components, representing commodities held in reserves and those entering markets through trade, respectively. The equilibrium conditions are stated, and alternative variational inequality formulations are presented. Several simple examples are solved analytically for illustration, and an algorithmic framework for computing equilibrium solutions to larger cases is provided. A set of numerical examples is presented, inspired by wheat exports from the Black Sea region to North and Sub-Saharan Africa, as well as purchases by the World Food Program. The impacts of the Russo-Ukrainian war on trade patterns and the effects of varying strategic reserve levels are assessed. We observe that each additional one million metric tons of fixed demand raises the corresponding market's equilibrium demand price by \$0.830–\$0.886 per metric ton, with the cumulative increase reaching approximately \$11 per metric ton over the reserve levels considered and substantial cross-market spillovers.

Key words: multicommodity trade, network equilibrium, food security, mixed demand, variational inequalities

1. Introduction

Food insecurity is growing globally with estimates of as many as 343 million people in 74 countries being acutely food insecure in 2024. Chronic food insecurity also continues to be high, with as many as 757 million being undernourished in 2023 (see World Bank, FAO, and WFP (2025)). The reasons are manifold from the COVID-19 pandemic to geopolitical crises around the world plus climate change and related disasters and even certain economic policies, including restrictions on trade (cf. Nagurney et al. (2024)).

Grains, which are commodities, and include wheat, maize, and rice, represent more than 50% of the calories consumed by humans (see Nikose-Rose (2022)) and comprise 37% of their consumed protein (Nagurney (2024)). Hence, grain commodities are essential to food security. Since not all grain commodity needs may be met within a nation, due, for example, to the inability to produce a sufficient amount, trade of commodities is critical to food security. According to Yin et al. (2024), in the two decades since 1990, global grain exports increased to 124.11 billion USD from 36.48 billion USD, with more than 25% of the demand for grain being met by international trade. Having sufficient, nutritious food is also fundamental to the security of a country. For example, in a recent report (see Sova and Zembilci (2023)), it is recognized that “hunger emergencies must be addressed before they metastasize into large-scale security threats.” Even in the United States, officials are becoming increasingly aware that food security affects national security.

Sova and Zembilci (2023) document that, in 2022 alone, more than 12,500 protests took place in countries facing rising food and fuel prices. Chronic food insecurity increases both the likelihood as well as the intensity of armed conflicts (see Martin-Shields and Stojetz (2019)), with a brutal loop between food insecurity and conflicts, including wars. For example, in 2011, the rising cost of bread, made from the commodity of wheat, and a staple of the Egyptian diet, with Egypt being the biggest importer of wheat, helped to topple its President Hosni Mubarak in uprisings referred to as the Arab Spring. Subsequent challenges occurred in Egypt after Russia’s full-scale invasion of Ukraine in February 2022 (see Deng and El-Fekki (2023)).

The governments of multiple countries, in addition to Egypt, due to concerns associated with possible food shortages and exacerbating emergencies, have been purchasing grain for their citizenry and also storing grain in what are known as strategic grain reserves. Not only is this happening in certain MENA (Middle Eastern and North African) countries, such as Egypt, but also in poorer ones, including Bangladesh, Pakistan, and Mali, and, interestingly, even in highly developed countries such as Norway, Switzerland, Japan, South Korea, and

Saudi Arabia (see World Bank, FAO, and WFP (2025)). Of course, certain NGOs, such as the World Food Program, are also purchasers of grains for distribution to the needy in food-challenged areas. Prior to Russia’s full-scale invasion of Ukraine, for example, the WFP bought about 50% of its wheat from Ukraine (see Nagurney (2022a)). The WFP has been the largest international organization fighting hunger, with reports that it helped to nourish more than 100 million people in 2024 (Spitzer (2025)) and spent \$9.8 billion on aid in 2024, with the US contributing nearly half of the support. USAID, according to Better World Campaign (2025), had, until recently, been one of the largest buyers of US-grown crops, spending around \$2.1 billion a year to provide food aid globally. The WFP expected a 40% reduction in donations in 2025, as compared to 2024 (Spitzer (2025)).

With countries, as well as humanitarian organizations, purchasing grains, it is important to quantitatively assess the impacts of such purchases on commodity outputs, commodity supply market prices as well as demand market prices, and on commodity trade volumes. Such purchasing is essentially price inelastic, although both nations and organizations wish to acquire commodities at the lowest cost, which needs to also include the transportation cost.

In this paper, we construct a multicommodity trade network equilibrium model with mixed demand and a focus on food security. The model builds upon the spatial price equilibrium framework and incorporates a fixed demand and an elastic demand for each commodity at each demand market. The fixed demand can represent purchases for strategic grain reserves or by humanitarian organizations, whereas the elastic demand represents the demand satisfied through regular trade. The concept of mixed demand was previously introduced by Pinyagina (2017) in the context of traffic and telecommunications networks. Here, we adapt this concept to a multicommodity trade network equilibrium model. We provide two equivalent statements of the governing equilibrium conditions and derive a finite-dimensional variational inequality formulation. The model enables the assessment of the impacts of fixed demands on commodity supply, trade flows, supply prices, and demand prices. We present illustrative examples and larger algorithmically solved numerical examples.

2. Literature Review and Organization of the Paper

We now highlight the literature that is relevant to this paper. We first focus on the impact of trade on food security, then on perfectly competitive spatial price equilibrium models with the integration of food security issues, and, finally, turn to mixed demand models, a very recent concept with few papers in the literature.

2.1 Trade and Food Security

The tutorial by Johnson and Smilowitz (2014) highlights food security as an important application of community-based operations research. Much of the public sector OR literature on food security has focused on food bank operations or the problem of food waste. The tutorial by Davis et al. (2024) provides an overview of food bank supply chains. Food banks are the first line of response to the issue of food insecurity in developed countries and, hence, are essential nonprofit organizations (Tarasuk, Fafard St-Germain, and Loopstra (2020)). However, on a larger scale, developing countries rely on international agricultural trade to satisfy the nutritional and caloric demands of their populations. Furthermore, vulnerable communities in those countries are primarily served through international food aid (e.g., by WFP) rather than local food banks. Accordingly, there is a need for more research in the public sector OR literature on food security in the context of international agricultural trade. Relatedly, Alam et al. (2024) analyze the mechanisms that enable food grain supply chains to mitigate the ripple effects of disruptions and identify sourcing diversification as an important means of improving supply chain resilience and protecting food security.

There is a vast body of papers on the impact of trade, trade openness, and trade policies on food security in the economics literature. For a review of various perspectives on the role of international trade in food security, one can refer to Clapp (2015). Wu and Guclu (2013) analyze the international trade patterns of maize and suggest that overreliance on a specific supplier could pose food security risks in the event of disruptions. Martin (2017) highlights that international agricultural trade raises production efficiency in supplying countries and reduces food price volatilities by diversifying supply sources. He further argues that price-insulation policies redistribute volatilities rather than reducing them. Dithmer and Abdulai (2017) conduct a cross-country data analysis to assess the food security impacts of trade openness and show a positive and significant impact in terms of the availability and quality of food. Smith and Glauber (2020) argue that domestic agricultural production in low-income countries is vulnerable to the impacts of climate change, pushing these countries towards relying on international agricultural trade. They also highlight the policy challenge arising from the trade-off between the long-term positive effects of trade openness and short-term food availability crises for vulnerable households.

More recent studies have examined the resilience of international agricultural trade in the face of global disruptions and the food security impacts of trade policies. Anderson (2022) reviews trade-related food policies under increasing climate, price, and policy volatility and emphasizes the importance of maintaining open international food markets. Akter (2022) reviews the effects of food export restrictions and argues that such restrictions can

distort production incentives and intensify price and availability pressures in international markets despite their temporary domestic benefits. Arita et al. (2022) use a monthly gravity-based model to assess the effects of the COVID-19 pandemic on global agricultural trade and find that agricultural trade was relatively resilient, although the effects varied across commodity groups. Jafari, Engemann, and Zimmermann (2023) examine international food trade and regional trade agreements from a network perspective and show that regional trade agreements are positively associated with agricultural trade connectivity. Ji et al. (2024) investigate the structure and vulnerability of global trade networks for rice, maize, wheat, and soybeans and demonstrate that the concentration and interdependence of international food trade can expose countries to the transmission of disruptions. Emediegwu and Rogna (2024) analyze the transmission of international commodity prices to local markets in developing and fragile economies and show that international price shocks can be transmitted rapidly to domestic food markets.

The Russo-Ukrainian war has further highlighted the dependence of global food security on international agricultural trade. Hellegers (2022) assesses the disrupting effect of Russia’s full-scale invasion of Ukraine on agricultural commodity markets and food prices. She argues that the food security of vulnerable countries depends on other countries’ policies, highlighting the need for a global approach towards international agricultural trade. Motaleb, Kruseman, and Snapp (2022) quantitatively examine how disruptions to Russian and Ukrainian wheat exports may affect wheat prices, consumption, and nutritional intake. They show that reductions in global wheat trade can increase prices and reduce wheat-derived calorie and protein intake, particularly in import-dependent developing countries. Abay et al. (2023) examine the implications of the Russo-Ukrainian war for global and regional food security and emphasize the importance of protecting vulnerable importing countries in the short run and diversifying food, fertilizer, energy, and trade sources in the longer run.

One of the policy tools available to governments in the face of disruptions to international agricultural trade is the use of strategic grain reserves. Strategic grain reserves have been used by vulnerable countries to address food shortages and recurrent global grain price volatilities. Wright and Cafiero (2011) argue that the expense and negative incentive effects of these reserves can be controlled by tying reserve volumes to the needs of vulnerable populations and distributing the reserves among them in severe emergencies. Fraser, Legwegoh, and KC (2015) provide a historical review of the use of grain reserves and demonstrate that reserve levels are directly linked to price volatilities. The World Bank (2021) assesses the use of strategic grain reserves in Zambia and Zimbabwe and provides an overview of lessons learned and policy recommendations. Nóia Júnior et al. (2022) assess the production and

stock management responses required to offset disruptions to Ukrainian wheat exports and highlight the role of coordinated releases of existing wheat stocks in mitigating immediate shortages. Chellai (2024) empirically analyzes the objectives and obstacles of strategic grain reserves in Arab nations and suggests that there is a negative relationship between wheat reserves and wheat prices in these countries.

The existing literature has studied strategic grain reserves, international agricultural trade, and food security from various perspectives. However, to the best of our knowledge, government purchases for strategic grain reserves have not been modeled as a fixed demand component alongside elastic demand in a multicommodity international trade network equilibrium model. This paper addresses this gap.

2.2 Spatial Price Equilibrium Models with a Focus on Food Security

As emphasized in Nagurney, Li, and Nagurney (2014), spatial price equilibrium (SPE) models, originally developed by Samuelson (1952) and Takayama and Judge (1964, 1971), have been widely applied to the trade of various commodities, including agricultural commodities such as grains. Spatial and trade network equilibrium models have also been examined from several perspectives. Fajgelbaum and Schaal (2020) develop a spatial equilibrium model in which locations are connected through a transportation network and transportation costs depend on congestion and infrastructure investment. They examine how transportation network design affects trade flows and spatial economic outcomes. Birge et al. (2022) examine spatial price integration in commodity markets with capacitated transportation networks and characterize the effects of supply and demand conditions, transportation costs, and network capacity constraints on equilibrium prices. They apply their framework to gasoline markets in the southeastern United States to identify the price effects of transportation bottlenecks and supply disruptions. Candogan, Epitropou, and Vohra (2021) study competitive equilibrium in trading networks through a generalized network flow formulation and provide a polynomial-time method for computing equilibrium outcomes.

SPE and other network equilibrium models have been closely connected with advances in variational inequality theory (cf. Dafermos and Nagurney (1984), Nagurney, Thore, and Pan (1996), Daniele (2004), Nagurney and Besik (2022)). Variational inequality formulations enable the incorporation of additional features such as multiple routes connecting supply and demand markets; nonseparable, asymmetric, and nonlinear functions (see Florian and Los (1982), Nagurney (1999)); and perishability of commodities (Nagurney, Pour, and Samadi (2024)). They also provide a framework for investigating a variety of trade measures (Nagurney, Nicholson, and Bishop (1996), Nagurney and Samadi (2026)). Recent studies have also

applied variational inequality theory to a range of supply chain and trade network equilibrium problems. Hamdouch and Ghouli (2020) construct a supply chain network equilibrium model with general stochastic price-dependent demand and formulate the governing equilibrium conditions as a finite-dimensional variational inequality. Ma et al. (2020) develop a supply chain network equilibrium model with time-based competition and heterogeneous consumer preferences and represent the equilibrium as a multinomial logit-based variational inequality. Feng et al. (2022) propose a global supply chain network equilibrium model to analyze the effects of tariffs, quotas, and subsidies under competition among product-market chains and derive a variational inequality formulation of the equilibrium conditions.

Geopolitical conflicts, the negative impacts of climate change on agriculture and trade routes (see, e.g., Besik and Nagurney (2025) and the references therein), the continuing effects of the COVID-19 pandemic, and rising food insecurity have motivated recent agricultural trade network models. Nagurney et al. (2023) introduced a multicommodity international trade spatial price equilibrium model with a focus on agricultural commodities. The model includes exchange rates and policy instruments such as tariffs, subsidies, and quotas. The case study therein examined the impacts of Russia’s full-scale invasion of Ukraine. Ukraine was known as the breadbasket of Europe, if not the world. MENA countries relied heavily on wheat imports from Ukraine, as did the World Food Program. Nagurney et al. (2024a, 2024b) further extended their findings by investigating various disruptions to production capacities and trade routes because of Russia’s war on Ukraine, with insights also applicable to disaster scenarios under uncertainty (see Nagurney et al. (2024b) and the references therein). Hassani et al. (2025), in turn, proposed a multiperiod, multicommodity trade network equilibrium model incorporating storage facilities and capacity constraints. Their wartime Ukraine case study included commodities such as wheat and corn. Nagurney (2024) constructed a multicommodity international trade network equilibrium model that incorporated nutritional minimum standards to support food security, along with consumer subsidies, and derived explicit formulae for them. As in the previously noted studies, the theoretical and algorithmic framework was that of variational inequalities. The numerical examples focused on Ukraine, MENA countries, and the staple commodity of wheat.

2.3 Mixed Demand Models

By mixed demand here, as noted earlier, is meant that, at a given demand market, there is a fixed demand, which needs to be satisfied for each commodity, which, for example, we assume is to be purchased by the government and/or NGOs, in the case of agricultural commodities for food security, as well as an elastic demand, which is sensitive to price,

and corresponds to the free market. It is important to recognize that, in the case of traffic network equilibrium models, historically, one distinguished between fixed demand (cf. Dafermos and Sparrow (1969) for a classical paper) and elastic demand (see the seminal work of Beckmann, McGuire, and Winsten (1956)) with numerous extensions of each of these developed over the decades using, for example, variational inequality theory (cf. Dafermos (1980, 1982), Nagurney (1999), Patriksson (2015), and the references therein). Pinyagina (2017) introduced a network equilibrium model with mixed demand focusing on traffic, relevant to both transportation and telecommunications. The model was a single modal one with origin/destination pairs (O/D) and the fixed and elastic demand associated with the O/D pairs, with the latter identified with travel disutilities. In this paper, in contrast, the mixed demand concept is adapted to a multicommodity trade network equilibrium model with a focus on food security, with the mixed demand associated with each commodity at each demand market. The model builds on the established spatial price equilibrium and variational inequality frameworks. The underlying functions consist of the multicommodity supply price functions and demand price functions as well as link cost functions, from which the path costs are constructed. To the best of our knowledge, mixed demand has not previously been incorporated into a multicommodity spatial price equilibrium model for international trade and food security.

Furthermore, in this paper, we provide two equivalent statements of the governing equilibrium conditions, along with a variational inequality formulation suitable for computational solution. The first statement of the equilibrium conditions utilizes a lower bound on the demand for each commodity at each demand market, which is equal to its fixed demand, and a Lagrange multiplier associated with the lower bound constraint. Interestingly, Lebow, Spelter, and Ince (2003), in a US Department of Agriculture report, introduced an SPE model with lower bounds on the commodity demands, but the model had separable supply price and demand price functions and fixed transportation costs. The model allowed for the transformation of raw materials, and the methodology was that of linear programming.

It is important to mention that upper bounds have been imposed on supplies (production quantities), commodity shipments, and demands in spatial price equilibrium models; see, for example, the models of Besik and Nagurney (2025) and Hassani et al. (2025). However, the present paper incorporates lower bounds on demands, outside of nonnegativity constraints, into a general multicommodity SPE framework and links these lower bounds to fixed purchases for strategic grain reserves of relevance to food security.

2.4 Organization of the Paper

In addition to the Introduction in Section 1 and the Literature Review in this Section 2, the paper includes the multicommodity trade network equilibrium model in Section 3 with the equilibrium conditions in Subsection 3.1, accompanied by the variational inequality formulation, and illustrative examples in Subsection 3.2. The algorithmic scheme is given in Section 4 along with the closed-form expressions for the multicommodity path flows and the Lagrange multipliers in each step of the algorithm. The algorithm is then applied in Section 5 to compute solutions to a set of numerical examples drawn from the real world. We summarize our results, present the conclusions, and also give suggestions for future research in Section 6.

3. The Multicommodity Trade Network Equilibrium Model with Mixed Demand

We now present the multicommodity trade network equilibrium model with mixed demand. There are m supply markets, that can be in the same or in different countries, and n demand markets, that also can be in the same or in different countries. Hence, they are spatially separated. A typical supply market is denoted by i and a typical demand market is denoted by j . The model is a perfectly competitive one with the supply markets involved in the production of K commodities and with the demand markets being their consumption sites. We denote a typical commodity by k and emphasize that our focus here is on agricultural commodities since they are the staples for food security. The supply and demand markets are connected by a network with its topology consisting of a graph $G = [N, L]$. The set N consists of the supply market origin nodes, the demand market destination nodes, intermediate nodes for transshipment, etc. L , in turn, is the set of n_L directed links with a link representing a mode of transportation and, if need be, a storage link. In Figure 1 we depict a possible network topology for the multicommodity trade. In the model, there may be multiple paths/routes connecting a supply market with a demand market. The set of paths is denoted by P with n_P being the number of paths in the network. We let P^i denote the set of paths from supply market i to all the demand markets and we let P_j denote the set of paths from all the supply markets to demand market j . P_j^i denotes the set of paths joining supply market i with demand market j . A typical link is denoted by a and a typical path by p . A link can correspond to a specific mode of transportation (truck, rail, maritime, air, etc.) or even a distribution or storage link.

The fundamental notation for the model is given in Table 1. All vectors are column vectors. All costs and prices are in a common currency.

Table 1: Notation for the Multicommodity Trade Network Equilibrium Model with Mixed Demand

Variables	Definition
s_i^k	the supply of commodity k at supply market i . We group all the commodity supplies into the vector $s \in R^{Km}$.
Q_p^k	the flow of commodity k on path p . We group the commodity path flows into the vector $Q \in R_+^{Kn_P}$.
f_a^k	the flow of commodity k on link a . We group the commodity link flows into the vector $f \in R_+^{Kn_L}$.
d_j^k	the total demand for commodity k at demand market j . We group all the commodity total demands into the vector $d \in R^{Kn}$.
$d_j^{k,e}$	the elastic component of the total demand for commodity k at demand market j . We group all the elastic commodity demands into the vector $d^e \in R^{Kn}$.
μ_j^k	the Lagrange multiplier associated with the minimum demand constraint for commodity k at demand market j . We group all the Lagrange multipliers into the vector $\mu \in R_+^{Kn}$.
Functions	Definition
$\pi_i^k(s)$	the supply price function for commodity k at supply market i . We group all the supply price functions into the vector $\pi(s) \in R^{Km}$.
$c_a^k(f)$	the unit cost of transportation of commodity k on link a , $\forall k, \forall a \in L$.
$C_p^k(Q)$	the unit cost of transportation of commodity k on path p . We group all the path costs into the vector $C \in R^{Kn_P}$.
$\rho_j^k(d^e)$	the demand price function for commodity k at demand market j that depends on the volume of commodity elastic demands. We group all the demand price functions into the vector $\rho(d^e) \in R^{Kn}$.
Parameters	Definition
$d_j^{k,f}$	the fixed demand for the purchase of commodity k at demand market j , $\forall k, \forall j$.

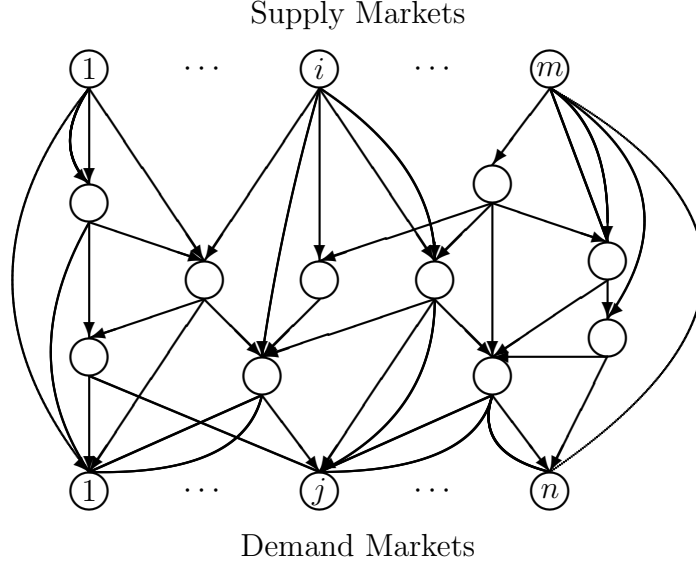


Figure 1: A Multicommodity Trade Network Topology

Conservation of Flow Equations

We now introduce the conservation of flow equations including the mixed demand constraints that allow us to capture the fixed demands that correspond to purchases of commodities by countries for their citizenry, as well as purchases by humanitarian organizations and NGOs such as the World Food Program.

The commodity path flows must be nonnegative, that is:

$$Q_p^k \geq 0, \quad \forall k, \forall p \in P. \quad (1)$$

Since links may be shared by paths, the commodity link flows are related to the commodity path flows as follows:

$$f_a^k = \sum_{p \in P} Q_p^k \delta_{ap}, \quad \forall k, \forall a \in L. \quad (2)$$

Hence, the commodity flow on a link is equal to the sum of flows of that commodity on paths that contain that link.

The supply constraints are:

$$s_i^k = \sum_{p \in P^i} Q_p^k, \quad \forall k, \forall i. \quad (3)$$

According to (3), the supply (amount produced) of commodity k at each supply market i is equal to the shipments of the commodity from the supply market to all the demand markets.

The mixed demand constraints are:

$$d_j^k = d_j^{k,f} + d_j^{k,e}, \quad \forall k, \forall j. \quad (4)$$

The constraints in (4) state that the total demand for commodity k at demand market j is equal to the fixed demand for the commodity at the demand market plus the elastic demand for the commodity at the demand market. It is important to distinguish between the fixed demand and the elastic demand at each demand market for a commodity since the demand commodity prices will only depend on the elastic demand for the commodities. The fixed demand for a commodity at a demand market is price inelastic since the governments and humanitarian organizations are focused on saving lives.

Furthermore, we must have that the sum of the shipments of each commodity to each demand market is equal to its total demand, that is:

$$\sum_{p \in P_j} Q_p^k = d_j^k, \quad \forall k, \forall j. \quad (5)$$

Here, we assume that, for each commodity k and demand market j , there is at least one path connecting a supply market to the demand market. Since no explicit upper bounds are imposed on the supplies or commodity shipments, this assumption ensures that the fixed demands can be satisfied, but at the associated supply price and transportation costs. Specifically, a feasible commodity path flow pattern can be constructed by assigning $d_j^{k,f}$ to one such path for each commodity k and demand market j . If explicit supply or transportation capacities were imposed and the fixed demands exceeded the quantities that could be delivered through the network, the feasible set would be empty and the equilibrium problem in its current form would have no solution. Such a setting would require the incorporation of unmet fixed demand and a rationing scheme.

Path and Link Cost Relationships

The unit transportation cost of a commodity on a path is equal to the sum of the unit transportation costs for that commodity on the links that comprise the path:

$$C_p^k(Q) = \sum_{a \in L} c_a^k(f) \delta_{ap}, \quad \forall k, \forall p \in P, \quad (6)$$

where $\delta_{ap} = 1$, if link a is contained in path p , and 0, otherwise.

Some Reformulations

In view of (3), we can redefine the commodity supply price functions in terms of commodity shipments, that is:

$$\tilde{\pi}_i^k(Q) \equiv \pi_i^k(s), \quad \forall k, \forall i \quad (7)$$

and, in view of (4) and (5), we can redefine the commodity demand price functions in terms of commodity shipments thus:

$$\tilde{\rho}_j^k(Q) = \rho_j^k(d^e), \quad \forall k, \forall j. \quad (8)$$

We assume that all the functions are continuous.

We do not impose explicit capacities on the supplies (production outputs) at the supply markets nor on the link flows. However, capacities can be captured by making the associated functions more highly nonlinear.

Also, in order to provide a statement of the governing equilibrium conditions which yields a variational inequality formulation amenable to effective algorithmic solution, we reexpress the fixed demand constraints (4), with use of (5), as:

$$\sum_{p \in P_j} Q_p^k \geq d_j^{k,f}, \quad \forall k, \forall j. \quad (9)$$

With each commodity k and demand market j we associate a Lagrange multiplier μ_j^k for its constraint in (9).

3.1 The Equilibrium Conditions

We now present the equilibrium conditions with mixed demand and derive the variational inequality formulation. We also provide an alternative, but equivalent, statement of the former.

The Multicommodity Trade Network Equilibrium Conditions

We define the feasible set $\mathcal{K}$ thus:

$$\mathcal{K} \equiv \{(Q, \mu) | (Q, \mu) \in R_+^{K(n_P+n)}\},$$

which is the nonnegative orthant, and we state the governing multicommodity trade network equilibrium conditions below.

Definition 1: Multicommodity Trade Network Equilibrium Conditions with Mixed Demand

A multicommodity path flow and Lagrange multiplier pattern $(Q^*, \mu^*) \in \mathcal{K}$ is a multicommodity trade network equilibrium with mixed demand, if the following conditions hold: For all commodities k ; $k = 1, \dots, K$, and for all paths $p \in P$ connecting pairs of supply and demand markets (i, j) ; $i = 1, \dots, m$ and $j = 1, \dots, n$:

$$\tilde{\pi}_i^k(Q^*) + C_p^k(Q^*) - \mu_j^{k*} \begin{cases} = \tilde{\rho}_j^k(Q^*), & \text{if } Q_p^{k*} > 0, \\ \geq \tilde{\rho}_j^k(Q^*), & \text{if } Q_p^{k*} = 0; \end{cases} \quad (10)$$

plus, for all commodities k and for all paths p terminating at the demand markets:

$$\sum_{i=1}^m \sum_{p \in P_j^i} Q_p^{k*} \begin{cases} = d_j^{k,f}, & \text{if } \mu_j^{k*} > 0, \\ \geq d_j^{k,f}, & \text{if } \mu_j^{k*} = 0. \end{cases} \quad (11)$$

According to equilibrium conditions (10) and (11), if there is a positive elastic demand for a commodity k at a demand market j ; that is, $d_j^{k,e*} > 0$, which occurs when (cf. (11)) $\sum_{i=1}^m \sum_{p \in P_j^i} Q_p^{k*} > d_j^{k,f}$, then the elastic demand price for the commodity is exactly equal to its supply price plus the unit transportation cost on each path with a positive flow of the commodity. However, if $d_j^{k,e*} = 0$, in which case $\sum_{i=1}^m \sum_{p \in P_j^i} Q_p^{k*} = d_j^{k,f}$, then the price that the humanitarian organization or government pays for the commodity, which is equal to its supply price plus unit transportation path cost, can exceed the demand market price, $\tilde{\rho}_j^k(Q^*)$, which is the price the consumers are willing to pay. Of course, there will be zero flow of a commodity on a path if the supply price plus the unit transportation cost on the path minus the associated Lagrange multiplier for the lower bound demand constraint at the demand market exceeds the demand price.

An Equivalent Set of Equilibrium Conditions with Emphasis on the Mixed Demand

We now provide an equivalent set of equilibrium conditions to (10) and (11), further emphasizing the mixed demand aspect and the prices.

A multicommodity path flow pattern Q^* can be associated with a Lagrange multiplier pattern μ^* such that $(Q^*, \mu^*) \in \mathcal{K}$ is a multicommodity trade network equilibrium according to Definition 1 if and only if Q^* satisfies the following conditions:

1. If $Q_p^{k*} > 0$, for $p \in P_j^i$, then

$$\tilde{\pi}_i^k(Q^*) + C_p^k(Q^*) = \min_{\substack{h=1,\dots,m \\ q \in P_j^h}} \{ \tilde{\pi}_h^k(Q^*) + C_q^k(Q^*) \}; \quad (12)$$

that is, there are positive commodity shipments only on paths with minimal supply price plus unit path transportation cost among all paths serving demand market j .

This condition follows from the first case in (10), noting that the same μ_j^{k*} is associated with all paths serving demand market j . Specifically, all paths in P_j with positive commodity shipment for k have the same minimal “cost” equal to $\tilde{\rho}_j^k(Q^*) + \mu_j^{k*}$, while paths in P_j with zero commodity shipment have a cost no less than this value.

2. If $d_j^{k,e*} > 0$ and $Q_p^{k*} > 0$, for $p \in P_j^i$, then

$$\tilde{\pi}_i^k(Q^*) + C_p^k(Q^*) = \tilde{\rho}_j^k(Q^*). \quad (13)$$

Therefore, for positive commodity shipments on paths and positive elastic demand for the associated demand market and commodity, the supply price of the commodity plus the unit transportation cost on the path is exactly equal to the demand price for the commodity at the demand market.

Note that this condition corresponds to the case where $\mu_j^{k*} = 0$, and, according to (10), then (13) holds.

3. If $Q_p^{k*} = 0$ or $d_j^{k,e*} = 0$, for $p \in P_j^i$, then

$$\tilde{\pi}_i^k(Q^*) + C_p^k(Q^*) \geq \tilde{\rho}_j^k(Q^*); \quad (14)$$

that is, the demand market price for commodity k cannot exceed the supply price at the origin supply market plus the unit transportation path cost.

Observe that if $Q_p^{k*} = 0$, then, according to (10), (14) will hold. Also, if $d_j^{k,e*} = 0$, then, since $\mu_j^{k*} \geq 0$, it follows from (10) that (14) is satisfied.

Theorem 1: Variational Inequality Formulation of the Multicommodity Trade Network Equilibrium Conditions with Mixed Demand

A multicommodity path flow and Lagrange multiplier pattern $(Q^*, \mu^*) \in \mathcal{K}$ is a multicommodity trade network equilibrium pattern under mixed demand, according to Definition 1, if and only if it satisfies the variational inequality:

$$\sum_{k=1}^K \sum_{i=1}^m \sum_{j=1}^n \sum_{p \in P_j^i} [\tilde{\pi}_i^k(Q^*) + C_p^k(Q^*) - \mu_j^{k*} - \tilde{\rho}_j^k(Q^*)] \times [Q_p^k - Q_p^{k*}]$$

$$+ \sum_{k=1}^K \sum_{j=1}^n \left[\sum_{i=1}^m \sum_{p \in P_j^i} Q_p^{k*} - d_j^{k,f} \right] \times [\mu_j^k - \mu_j^{k*}] \geq 0, \quad \forall (Q, \mu) \in \mathcal{K}. \quad (15)$$

Proof:

First, we establish necessity; that is, we show that if $(Q^*, \mu^*) \in \mathcal{K}$ satisfies equilibrium conditions (10) and (11) then it also satisfies variational inequality (15).

Note that, according to equilibrium conditions (10), for a fixed commodity k , a fixed pair of supply and demand markets (i, j) , and path $p \in P_j^i$, one must have that:

$$[\tilde{\pi}_i^k(Q^*) + C_p^k(Q^*) - \mu_j^{k*} - \tilde{\rho}_j^k(Q^*)] \times [Q_p^k - Q_p^{k*}] \geq 0, \quad (16)$$

for any Q_p^k such that $Q_p^k \geq 0$.

Indeed, if $Q_p^{k*} = 0$, then, according to (10),

$$[\tilde{\pi}_i^k(Q^*) + C_p^k(Q^*) - \mu_j^{k*} - \tilde{\rho}_j^k(Q^*)] \geq 0. \quad (17)$$

Since $Q_p^k \geq 0$, we also know that $(Q_p^k - Q_p^{k*}) \geq 0$ and, therefore, (16) holds.

On the other hand, if $Q_p^{k*} > 0$, then, according to (10):

$$[\tilde{\pi}_i^k(Q^*) + C_p^k(Q^*) - \mu_j^{k*} - \tilde{\rho}_j^k(Q^*)] = 0, \quad (18)$$

and, consequently, (16) also holds.

Also, from (11) it follows that for a fixed commodity k and demand market j :

$$\left[\sum_{i=1}^m \sum_{p \in P_j^i} Q_p^{k*} - d_j^{k,f} \right] \times [\mu_j^k - \mu_j^{k*}] \geq 0, \quad \forall \mu_j^k \geq 0 \quad (19)$$

since if $\mu_j^{k*} = 0$ then the left-hand side of the expression in (19) and the right-hand side are each nonnegative, and, if $\mu_j^{k*} > 0$, then the left-hand side of the expression in (19) is equal to 0 and, thus, (19) also holds.

Summing up the inequality in (16) over all commodities and all supply markets and all demand markets and the paths connecting them, and summing up the inequality in (19) over all commodities and all demand markets yields variational inequality (15). Necessity has, thus, been established.

We now turn to establishing sufficiency. We show that, if $(Q^*, \mu^*) \in \mathcal{K}$ satisfies variational inequality (15), then it also satisfies equilibrium conditions (10) and (11).

Let $\mu_j^k = \mu_j^{k*}$ for all k and all j and also let $Q_q^g = Q_q^{g*}$, for all commodities g , $g \neq k$, and for all paths $q \in P_l^h$, for all hl except for path $p \in P_j^i$ and substitute the resultants into (15). Then, (15) reduces to:

$$[\tilde{\pi}_i^k(Q^*) + C_p^k(Q^*) - \mu_j^{k*} - \tilde{\rho}_j^k(Q^*)] \times [Q_p^k - Q_p^{k*}] \geq 0, \quad \forall Q_p^k \geq 0. \quad (20)$$

The equilibrium conditions (10) then follow for commodity k , this path $p \in P_j^i$, and, hence, for all commodities k , and for all paths in P_j^i plus for all paths in all other supply and demand market pairs.

Now let $Q_p^k = Q_p^{k*}$ for all p and all k and let $\mu_l^g = \mu_l^{g*}$ for all commodities $g \neq k$ and for all demand markets $l \neq j$ and substitute these into (15). The result obtained is:

$$\left[\sum_{i=1}^m \sum_{p \in P_j^i} Q_p^{k*} - d_j^{k,f} \right] \times [\mu_j^k - \mu_j^{k*}] \geq 0, \quad \forall \mu_j^k \geq 0, \quad (21)$$

from which equilibrium conditions (11) follow. The proof is complete. $\square$

We now provide an alternative variational inequality (VI) formulation to that in (15).

Theorem 2

A multicommodity path flow pattern $Q^* \in \mathcal{K}^2$, where $\mathcal{K}^2 \equiv \{Q | Q \geq 0, \sum_{i=1}^m \sum_{p \in P_j^i} Q_p^k \geq d_j^{k,f}, \forall k, \forall j\}$, satisfying

$$\sum_{k=1}^K \sum_{i=1}^m \sum_{j=1}^n \sum_{p \in P_j^i} [\tilde{\pi}_i^k(Q^*) + C_p^k(Q^*) - \tilde{\rho}_j^k(Q^*)] \times [Q_p^k - Q_p^{k*}] \geq 0, \quad \forall Q \in \mathcal{K}^2, \quad (22)$$

admits a nonnegative Lagrange multiplier pattern μ^* such that $(Q^*, \mu^*) \in \mathcal{K}$ satisfies VI (15).

Proof: The result follows by using similar arguments as in the proof of the Theorem in Nagurney (2022b), which makes use of Lagrange analysis. Since Q^* satisfies VI (22), it is also a solution to the following linear optimization problem:

$$\min_{Q \in \mathcal{K}^2} \sum_{k=1}^K \sum_{i=1}^m \sum_{j=1}^n \sum_{p \in P_j^i} [\tilde{\pi}_i^k(Q^*) + C_p^k(Q^*) - \tilde{\rho}_j^k(Q^*)] \times Q_p^k. \quad (23)$$

Indeed, VI (22) states that the value of the objective function at any $Q \in \mathcal{K}^2$ is no smaller than its value at Q^* .

The lower bound constraints defining $\mathcal{K}^2$ can be written as:

$$d_j^{k,f} - \sum_{i=1}^m \sum_{p \in P_j^i} Q_p^k \leq 0, \quad \forall k, \forall j. \quad (24)$$

With each such constraint, we associate a Lagrange multiplier $\mu_j^k \geq 0$. The Lagrange function is:

$$\begin{aligned} \mathcal{L}(Q, \mu) \equiv & \sum_{k=1}^K \sum_{i=1}^m \sum_{j=1}^n \sum_{p \in P_j^i} [\tilde{\pi}_i^k(Q^*) + C_p^k(Q^*) - \tilde{\rho}_j^k(Q^*)] \times Q_p^k \\ & + \sum_{k=1}^K \sum_{j=1}^n \left[d_j^{k,f} - \sum_{i=1}^m \sum_{p \in P_j^i} Q_p^k \right] \times \mu_j^k. \end{aligned} \quad (25)$$

Since (23) is a finite-dimensional linear program that attains a finite optimal value at Q^* , strong duality for linear programming guarantees the existence of an optimal dual solution, represented by a nonnegative Lagrange multiplier vector μ^* , such that (Q^*, μ^*) is a saddle point of the Lagrange function. Therefore, we have that:

$$\mathcal{L}(Q^*, \mu) \leq \mathcal{L}(Q^*, \mu^*) \leq \mathcal{L}(Q, \mu^*), \quad \forall Q \geq 0, \quad \forall \mu \geq 0. \quad (26)$$

The right-hand inequality in (26) yields:

$$\sum_{k=1}^K \sum_{i=1}^m \sum_{j=1}^n \sum_{p \in P_j^i} [\tilde{\pi}_i^k(Q^*) + C_p^k(Q^*) - \mu_j^{k*} - \tilde{\rho}_j^k(Q^*)] \times [Q_p^k - Q_p^{k*}] \geq 0, \quad \forall Q \geq 0. \quad (27)$$

The left-hand inequality in (26) yields:

$$\sum_{k=1}^K \sum_{j=1}^n \left[\sum_{i=1}^m \sum_{p \in P_j^i} Q_p^{k*} - d_j^{k,f} \right] \times [\mu_j^k - \mu_j^{k*}] \geq 0, \quad \forall \mu \geq 0. \quad (28)$$

Adding (27) and (28) yields:

$$\begin{aligned} & \sum_{k=1}^K \sum_{i=1}^m \sum_{j=1}^n \sum_{p \in P_j^i} [\tilde{\pi}_i^k(Q^*) + C_p^k(Q^*) - \mu_j^{k*} - \tilde{\rho}_j^k(Q^*)] \times [Q_p^k - Q_p^{k*}] \\ & + \sum_{k=1}^K \sum_{j=1}^n \left[\sum_{i=1}^m \sum_{p \in P_j^i} Q_p^{k*} - d_j^{k,f} \right] \times [\mu_j^k - \mu_j^{k*}] \geq 0, \quad \forall (Q, \mu) \in \mathcal{K}. \end{aligned} \quad (29)$$

This is exactly VI (15). The proof is complete. $\square$

We use VI (15) as the formulation for algorithm implementation and the solution of larger-scale examples in Section 5. We now put it into standard form (cf. Nagurney (1999)), VI($F, \mathcal{K}$), where one seeks to determine a vector $X^* \in \mathcal{K} \subset R^{\mathcal{N}}$, such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \quad (30)$$

where F is a given continuous function from $\mathcal{K}$ to $R^{\mathcal{N}}$, $\mathcal{K}$ is a given closed, convex set, and $\langle \cdot, \cdot \rangle$ denotes the inner product in $\mathcal{N}$ -dimensional Euclidean space.

We define $X \equiv (Q, \mu)$, with $\mathcal{K}$ as previously defined, and $\mathcal{N} = K(n_P + n)$. Also, $F(X)$ consists of the elements $F_p^k(X) \equiv [\tilde{\pi}_i^k(Q) + C_p^k(Q) - \mu_j^k - \tilde{\rho}_j^k(Q)]$, $\forall k, \forall i, \forall j, \forall p \in P_j^i$ and the elements $F_j^k(X) = \left[\sum_{i=1}^m \sum_{p \in P_j^i} Q_p^k - d_j^{k,f} \right]$, $\forall k, \forall j$. It follows that variational inequality (15) can be put into standard form (30).

Similarly, VI (22) can also be put into standard form (30) if we define $\mathcal{K} \equiv \mathcal{K}^2$, $X \equiv Q$, $\mathcal{N} = Kn_P$, and the components of $F(X)$ consisting of the elements:

$$F_p^k(X) \equiv [\tilde{\pi}_i^k(Q) + C_p^k(Q) - \tilde{\rho}_j^k(Q)], \quad \forall k, \forall i, \forall j, \forall p \in P_j^i.$$

Since neither feasible set underlying the two variational inequalities (15) and (22) is compact, existence is not guaranteed solely under the assumption of the underlying functions being continuous from the classical theory of variational inequalities (cf. Kinderlehrer and Stampacchia (1980) and Nagurney (1999)). However, if $F(X)$ that enters the VI (30) is coercive, that is,

$$\frac{\langle F(X) - F(X_0), X - X_0 \rangle}{\|X - X_0\|} \rightarrow \infty \quad (31)$$

as $\|X\| \rightarrow \infty$ for $X \in \mathcal{K}$ and for some $X_0 \in \mathcal{K}$. then VI($F, \mathcal{K}$) always has a solution.

Pinyagina (2017) imposed a type of coercivity condition on the path cost and travel disutility functions for the mixed demand traffic network equilibrium problem.

Theorem 3: Existence

Assume that $\mathcal{K}^2$ is nonempty and that the function $F(Q)$ that enters VI (22) is affine and can be expressed as:

$$F(Q) = AQ + b, \quad (32)$$

where all the entries of the matrix A are nonnegative and all its diagonal entries are strictly positive. Then VI (22) has a solution. Consequently, there exists a nonnegative Lagrange multiplier pattern μ^ such that (Q^*, μ^*) satisfies VI (15).*

Proof:

Fix a vector $Q_0 \in \mathcal{K}^2$ and let $z = Q - Q_0$. Denote the positive and negative parts of z by z^+ and z^- , respectively, so that $z = z^+ - z^-$. Since $Q \geq 0$, the vector z^- is bounded componentwise by Q_0 .

Let $A = [A_{lr}]_{l,r=1}^{K_{n_P}}$. Since all the diagonal entries of A are strictly positive, let:

$$\alpha = \min_l A_{ll} > 0. \quad (33)$$

Since all the entries of A are nonnegative, we have that:

$$(z^+)^T A z^+ \geq \alpha \|z^+\|^2. \quad (34)$$

Since z^- is bounded and $z = z^+ - z^-$, it follows that $\|z^+\| \rightarrow \infty$ as $\|Q\| \rightarrow \infty$. Also, since z^- is bounded, there exist nonnegative constants c_1 and c_2 such that

$$z^T A z \geq \alpha \|z^+\|^2 - c_1 \|z^+\| - c_2. \quad (35)$$

Therefore, it follows that:

$$\frac{\langle F(Q) - F(Q_0), Q - Q_0 \rangle}{\|Q - Q_0\|} = \frac{z^T A z}{\|z\|} \rightarrow \infty, \quad (36)$$

as $\|Q\| \rightarrow \infty$ for $Q \in \mathcal{K}^2$. Hence, the coercivity condition is satisfied, and VI (22) has a solution. The result for VI (15) then follows from Theorem 2. The proof is complete. $\square$

The conditions of Theorem 3 are satisfied by the functions used in all the numerical examples. In the illustrative examples, the supply price and link cost functions are increasing linear functions and the demand price functions are decreasing linear functions. In Examples 5 through 10, all the coefficients in the supply price functions are nonnegative, all the link cost functions have strictly positive slopes, and all the demand price functions have strictly negative slopes. Therefore, the induced path flow coefficient matrices have nonnegative entries and strictly positive diagonal entries. Hence, VI (22), and, consequently, VI (15), has a solution for each of the numerical examples.

Regarding uniqueness, it is well-known from the standard theory of variational inequalities that, if the function $F(Q)$ that enters VI (22) is strictly monotone on $\mathcal{K}^2$, then the equilibrium path flow pattern Q^* is unique (cf. Kinderlehrer and Stampacchia (1980) and Nagurney (1999)). A weaker, and often more relevant, uniqueness result can be stated in terms of the aggregate network variables. In particular, if the mapping consisting of the supply price

functions $\pi(s)$, the link cost functions $c(f)$, and the negatives of the demand price functions $-\rho(d^e)$ is strictly monotone over the set of feasible induced supply, link flow, and elastic demand patterns, then the equilibrium supply pattern s^* , commodity link flow pattern f^* , and elastic and total demand patterns d^{e*} and d^* are unique (cf. Dafermos (1980) and Nagurney (1999)). However, the equilibrium path flow pattern Q^* need not be unique, since distinct path flow patterns may induce the same supply, link flow, and demand patterns.

3.2 Illustrative Examples

We now present several examples to illustrate the concepts. For simplicity, the unit for the commodity shipments, supplies, and demands is one million metric tons. Henceforth, when we refer to a “ton” we mean a “metric ton.” The prices and costs are per ton.

Illustrative Example 1

Consider the network topology in Figure 2 where supply market node 1 corresponds to Egypt and demand market node 1 also corresponds to Egypt. As noted in the Introduction, Egypt is a big purchaser of grains for its citizenry. We consider only a single commodity such as wheat and, hence, we suppress the superscript of 1.

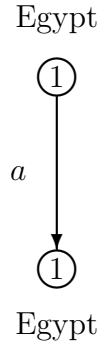


Figure 2: Network Topology for Illustrative Example 1

The supply price is: $\pi_1(s) = s_1 + 300$. The unit transportation cost on link a is $c_a(f) = f_a + 20$, and the demand price is $\rho_1(d^e) = -d_1^e + 330$. There is a single path $p_1 = (a)$ and, therefore, $f_a = Q_{p_1}$ and $C_{p_1}(Q) = c_a(f)$ in this example.

Let $d_1^f = 5$, which is close to how many million tons of wheat Egypt expects to purchase from its farmers (see Millingmea.com (2025)).

We explore what the commodity shipment would be if the supply price plus the unit cost of transportation were equal to the demand price. Because of the conservation of flow equations, $s_1^* = Q_{p_1}^*$ and $d_1^e = Q_{p_1}^* - 5$. Using these substitutions, the single variable equation

becomes:

$$Q_{p_1}^* + 300 + Q_{p_1}^* + 20 = -(Q_{p_1}^* - 5) + 330, \quad (37)$$

which simplifies to:

$$3Q_{p_1}^* = 15,$$

with solution: $Q_{p_1}^* = 5$, which is precisely the fixed demand and, therefore, $d_1^{e*} = 0$.

The supply price at the equilibrium is $\pi_1(s^*) = 305$, the unit path transportation cost is $C_{p_1}(Q^*) = 25$, and the demand price $\rho_1(d^{e*}) = 330$. Hence, the equilibrium conditions (10) and (11) clearly hold with $\mu_1^* = 0$. According to Tridge (2025), Egypt is expecting to pay its farmers about \$309-\$323 per ton of wheat.

Illustrative Example 2

In Illustrative Example 2, we introduce a new demand market, neighboring countries, since Egypt does export some flour, which we use as a proxy for wheat. The new demand market is referred to as demand market 2. The estimate of its fixed demand is $d_2^f = 1$; that is, 1 million tons (see Miller Magazine (2025)). The network topology is as given in Figure 3.

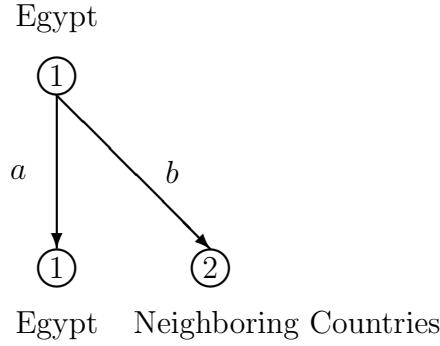


Figure 3: Network Topology for Illustrative Examples 2, 3, and 4

In Illustrative Example 2, we keep the data as in Illustrative Example 1 but expand the data for the new demand market 2 and the new transportation route $p_2 = (b)$ so that $f_b = Q_{p_2}$. We have $c_b(f) = f_b + 25$. Hence, $C_{p_2}(Q) = c_b(f) = Q_{p_2} + 25$. Also, the demand price at the added demand market 2 is: $\rho_2(d^e) = -d_2^e + 310$.

By applying the equilibrium conditions (10) and (11) for this example, we obtain the following equilibrium commodity flow pattern: $Q_{p_1}^* = 5$ and $Q_{p_2}^* = 1$. Hence, $s_1^* = 6$ and $d_1^{e*} = 0$ and $d_2^{e*} = 0$ so the total demand at each demand market corresponds only to the respective fixed demand at the market. The Lagrange multipliers at the equilibrium are: $\mu_1^* = 1$ and $\mu_2^* = 22$. The supply price at the equilibrium $\pi_1(s^*) = 306$. The unit

path transportation costs are: $C_{p_1}(Q^*) = 25$ and $C_{p_2}(Q^*) = 26$. The demand prices are: $\rho_1(d^{e*}) = 330$ and $\rho_2(d^{e*}) = 310$. In this example, both Lagrange multipliers are positive at the equilibrium, and Egypt only produces the fixed amount that is purchased by its domestic market and its neighboring countries. Note that, in this example, the supply price plus the unit transportation cost to each demand market is higher than the price the consumers are willing to pay, but the government does pay that amount for their fixed demands.

Illustrative Example 3

According to Millingmea.com (2025), Egypt was expected to purchase 4-5 million tons from local farmers in 2024-2025. This example, hence, reduces the fixed demand in Egypt from 5 million tons to 4 million tons, with the rest of the data as in Illustrative Example 2. The new equilibrium commodity trade pattern is now: $Q_{p_1}^* = 4\frac{1}{3}$, $Q_{p_2}^* = 1$, with $d_1^{e*} = \frac{1}{3}$, $d_2^{e*} = 0$, and $s_1^* = 5\frac{1}{3}$. The supply market price is now: $\pi_1(s^*) = 305\frac{1}{3}$; the unit path transportation costs are: $C_{p_1}(Q^*) = 24\frac{1}{3}$ and $C_{p_2}(Q^*) = 26$, and the demand prices are: $\rho_1(d^{e*}) = 329\frac{2}{3}$ and $\rho_2(d^{e*}) = 310$ with the Lagrange multipliers being: $\mu_1^* = 0$ and $\mu_2^* = 21\frac{1}{3}$. The equilibrium supply now decreases from 6 to $5\frac{1}{3}$. The government in Egypt pays a lower price for the wheat in Egypt ($329\frac{2}{3}$ rather than 331) and the consumers in Egypt also acquire the wheat in the free market at a price of $329\frac{2}{3}$, whereas in the previous example all the wheat went to the government.

Illustrative Example 4

In Illustrative Example 4, we explore what would be the equilibrium solution if the governments did not purchase a fixed amount of wheat for their citizens, and, therefore, the total demand at each of the two demand markets is equal to the elastic demand at the respective market. Hence, in this example, we have that $d_1^f = 0$ and $d_2^f = 0$.

We can immediately see, from the fixed term in the supply price function and the fixed cost term in $C_{p_2} = c_b$, with the sum of both being equal to 325, that this value exceeds the fixed term in the demand price function at the second demand market, which is 310. Hence, according to the equilibrium conditions, $Q_{p_2}^* = 0$. This leaves us with a single variable equation to solve for $Q_{p_1}^*$ using the linear equation: $\pi_1 + C_{p_1} = \rho_1(d^e)$, or $2Q_{p_1}^* + 320 = -Q_{p_1}^* + 330$, with the solution: $Q_{p_1}^* = 3\frac{1}{3}$. The supply price $\pi_1(s^*) = 303\frac{1}{3}$. The path cost $C_{p_1}(Q^*) = 23\frac{1}{3}$ and the demand price $\rho_1(d^{e*}) = 326\frac{2}{3}$, whereas $\rho_2(d^{e*}) = 310$.

Since the demand price in neighboring countries that consumers are willing to pay for the wheat, $\rho_2(d^{e*}) = 310$, is lower than the supply price of wheat in Egypt and the unit transportation cost to neighboring countries, there is no shipment of wheat from Egypt to

them, as there was in Illustrative Examples 2 and 3 when there was a fixed demand associated with procurement in neighboring countries. In Illustrative Example 4, the farmers in Egypt, under a pure market mechanism, receive the lowest price for their wheat, as compared to the supply prices in the preceding examples, and produce the least amount of wheat. However, the demand price for wheat in Egypt for consumers, in Illustrative Example 4, is the lowest of all the examples at $\rho_1(d^{e*}) = 326\frac{2}{3}$.

Of course, the same wheat production output (supply) and commodity shipment of wheat as above from Egyptian farmers to the Egyptian demand market, and associated supply price, unit transportation path cost, and demand price in Egypt would prevail, if, in Illustrative Example 1, the fixed demand in Egypt was set to zero.

4. The Algorithm

We now, for the reader's convenience and easy accessibility, recall the modified projection method of Korpelevich (1977), that we implement and apply in Section 5 to compute solutions to larger-scale numerical trade network equilibrium examples with mixed demand. We, also, provide the explicit formulae induced by the algorithm, at each iteration, for the computation of the multicommodity path flows and the Lagrange multipliers. The modified projection method has been used to solve a wide range of spatial price and related supply chain network equilibrium problems formulated as variational inequality problems (see: Nagurney (1999, 2006, 2023) and Hassani et al. (2025)). It converges under reasonable assumptions on the function $F(X)$ that enters the variational inequality (30) and it does not require strong monotonicity of $F(X)$ for convergence as does the projection method of Dafermos (1980). Specifically, the modified projection method is guaranteed to converge if the function $F(X)$ that enters the variational inequality problem (30) is monotone and Lipschitz continuous and a solution exists.

We recall that F is monotone if

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle \geq 0, \quad \forall X^1, X^2 \in \mathcal{K}. \quad (38)$$

It is Lipschitz continuous if there exists an $\eta > 0$, known as the Lipschitz constant, such that

$$\|F(X^1) - F(X^2)\| \leq \eta \|X^1 - X^2\|, \quad \forall X^1, X^2 \in \mathcal{K}. \quad (39)$$

It is not unreasonable to assume that the supply price functions $\tilde{\pi}(Q)$ will be monotone increasing in Q ; the demand price functions $\tilde{\rho}(Q)$ will be monotone decreasing in Q , and that the path cost functions $C(Q)$ will be monotone increasing in Q ; in addition, that these

functions are Lipschitz continuous. Further discussion of this algorithm, when applied to path-based network equilibrium problems and spatial price equilibrium problems can be found in Nagurney et al. (2023). In particular, as noted therein, the modified projection method, as recognized by Solodov and Tseng (1996) is a “very practical method.” As is emphasized below, the modified projection method is amenable to implementation for the solution of the multicommodity trade model with mixed demand.

In the statement of the algorithm, t denotes an iteration counter.

The Modified Projection Method

Step 0: Initialization

Initialize with $X^0 \in \mathcal{K}$. Set the iteration counter $t = 1$ and let β be a scalar such that $0 < \beta \leq \frac{1}{\eta}$, where η is the Lipschitz constant.

Step 1: Computation

Compute $\bar{X}^t$ by solving the variational inequality subproblem:

$$\langle \bar{X}^t + \beta F(X^{t-1}) - X^{t-1}, X - \bar{X}^t \rangle \geq 0, \quad \forall X \in \mathcal{K}. \quad (40)$$

Step 2: Adaptation

Compute X^t by solving the variational inequality subproblem:

$$\langle X^t + \beta F(\bar{X}^t) - X^{t-1}, X - X^t \rangle \geq 0, \quad \forall X \in \mathcal{K}. \quad (41)$$

Step 3: Convergence Verification

If $|X^t - X^{t-1}| \leq \epsilon$, with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $t := t + 1$ and go to Step 1.

Since the feasible set $\mathcal{K}$ underlying the multicommodity trade network equilibrium model as formulated in (15) (and (30)) is the nonnegative orthant, the solution of each of the subproblems in (40) and (41) can be obtained via closed-form expressions for the multicommodity path flows and the Lagrange multipliers, respectively, as follows:

Explicit Formulae at Iteration t for the Multicommodity Path Flows in Step 1

The modified projection method results in the following closed-form expressions for (40) for the multicommodity flows for the solution of variational inequality (15):

$$\bar{Q}_p^{kt} = \max\{0, Q_p^{kt-1} + \beta(\tilde{\rho}_j^k(Q^{t-1}) - \tilde{\pi}_i^k(Q^{t-1}) - C_p^k(Q^{t-1}) + \mu_j^{kt-1})\}, \quad \forall k, \forall p \in P. \quad (42)$$

Explicit Formulae at Iteration t for the Multicommodity Lagrange Multipliers in Step 1

The modified projection method results in the following closed-form expressions for (40) for the multicommodity Lagrange multipliers for the solution of variational inequality (15):

$$\bar{\mu}_j^{kt} = \max\{0, \mu_j^{kt-1} + \beta(d_j^{k,f} - \sum_{i=1}^m \sum_{p \in P_j^i} Q_p^{kt-1})\}, \quad \forall k, \forall j. \quad (43)$$

Explicit Formulae at Iteration t for the Multicommodity Path Flows in Step 2

The modified projection method results in the following closed-form expressions for (41) for the multicommodity flows for the solution of variational inequality (15):

$$Q_p^{kt} = \max\{0, Q_p^{kt-1} + \beta(\tilde{\rho}_j^k(\bar{Q}^t) - \tilde{\pi}_i^k(\bar{Q}^t) - C_p^k(\bar{Q}^t) + \bar{\mu}_j^{kt})\}, \quad \forall k, \forall p \in P. \quad (44)$$

Explicit Formulae at Iteration t for the Multicommodity Lagrange Multipliers in Step 2

The modified projection method results in the following closed-form expressions for (41) for the multicommodity Lagrange multipliers for the solution of variational inequality (15):

$$\mu_j^{kt} = \max\{0, \mu_j^{kt-1} + \beta(d_j^{k,f} - \sum_{i=1}^m \sum_{p \in P_j^i} \bar{Q}_p^{kt})\}, \quad \forall k, \forall j. \quad (45)$$

The multicommodity equilibrium supplies, elastic and total demands, and the link flows can be easily determined from the computed equilibrium multicommodity path flows.

5. Algorithmically Solved Numerical Examples

In this section, we conduct a case study inspired by the export of grains from the Black Sea region to African countries before and after Russia's full-scale invasion of Ukraine. On the supply side, we have Bulgaria, Romania, Ukraine, and Russia. On the demand side, we consider WFP's strategic hub in Djibouti, which supports its humanitarian operations across North and Sub-Saharan Africa, as well as Egypt, Tunisia, and Ethiopia. Egypt, Tunisia, and Ethiopia are among the African countries that carry strategic grain reserves (World Bank, FAO, and WFP (2025)). WFP procures both in advance through its Global Commodity Management Facility (GCMF), with commodities stored in its inventory hubs, and in response to disasters (Renoux (2025), WFP (2025a)). WFP's pre-disaster procurement can be

treated as a mix of fixed and elastic demand, since it is price-sensitive and adjusts the timing and sourcing of its purchases, but it also needs to procure and store a minimum quantity to ensure rapid response capability. In Examples 5 through 9, where we are focusing on the single commodity of wheat, the superscript k is suppressed. In Example 10, which considers two commodities, wheat is denoted by $k = 1$ and corn by $k = 2$. The shipment volumes represent yearly commodity trade flows in metric tons, with all instances of “tons” referring to metric tons, and all prices expressed in U.S. dollars per metric ton of commodity.

Data for the examples are derived from publicly available online reports. The yearly imports for demand countries are derived from IndexMundi, and the share of supply countries in the imports of each demand country is based on percentages reported on the TrendEconomy Open Data Portal and country-specific reports of the U.S. Department of Agriculture (USDA). Furthermore, the data on WFP are based on WFP’s yearly procurement reports. The transportation costs are based on reports from Tridge, Fastmarkets, USDA Agricultural Marketing Service, Reuters, and data used in Hassani et al. (2025) (see, for example, Belikova (2022), USDA Agricultural Marketing Service (2024), USDA Agricultural Marketing Service (2025), Tridge (2024)). Supply prices are derived from the values used in Hassani et al. (2025) and from FOB prices reported by Fastmarkets and APK-Inform (e.g., APK-Inform (2024a, 2024b); Belikova (2025)) after excluding domestic handling and transportation costs. Demand prices are based on average global commodity prices and consumer prices in the specific demand countries where online reports are available. Note that the total annual volume of wheat exports from the supply countries in the network has historically far exceeded the total annual wheat imports of the network’s demand countries (see, for example, IndexMundi (2026a, 2026b, 2026c, 2026d)).

The function parameters are calibrated using the above data sources. More specifically, the intercepts are selected based on the reported supply prices, demand prices, and transportation costs, and the coefficients are adjusted so that the equilibrium path flows and prices in Examples 5 and 6 are close to the real-world values discussed in those examples. The off-diagonal terms in the supply price functions capture the interdependence among the supply countries in the Black Sea region. Note that these terms are positive but smaller than the corresponding diagonal terms, meaning that the supply in each country has the strongest impact on its own supply price. The supply price functions used in the numerical examples are, therefore, asymmetric and nonseparable. In other words, changes in the supply of one country affect the equilibrium supply prices and trade flows throughout the network, with larger off-diagonal coefficients resulting in stronger cross-market effects. After the functions are calibrated using Examples 5 and 6, they are used in Examples 7, 8, and 9 to examine

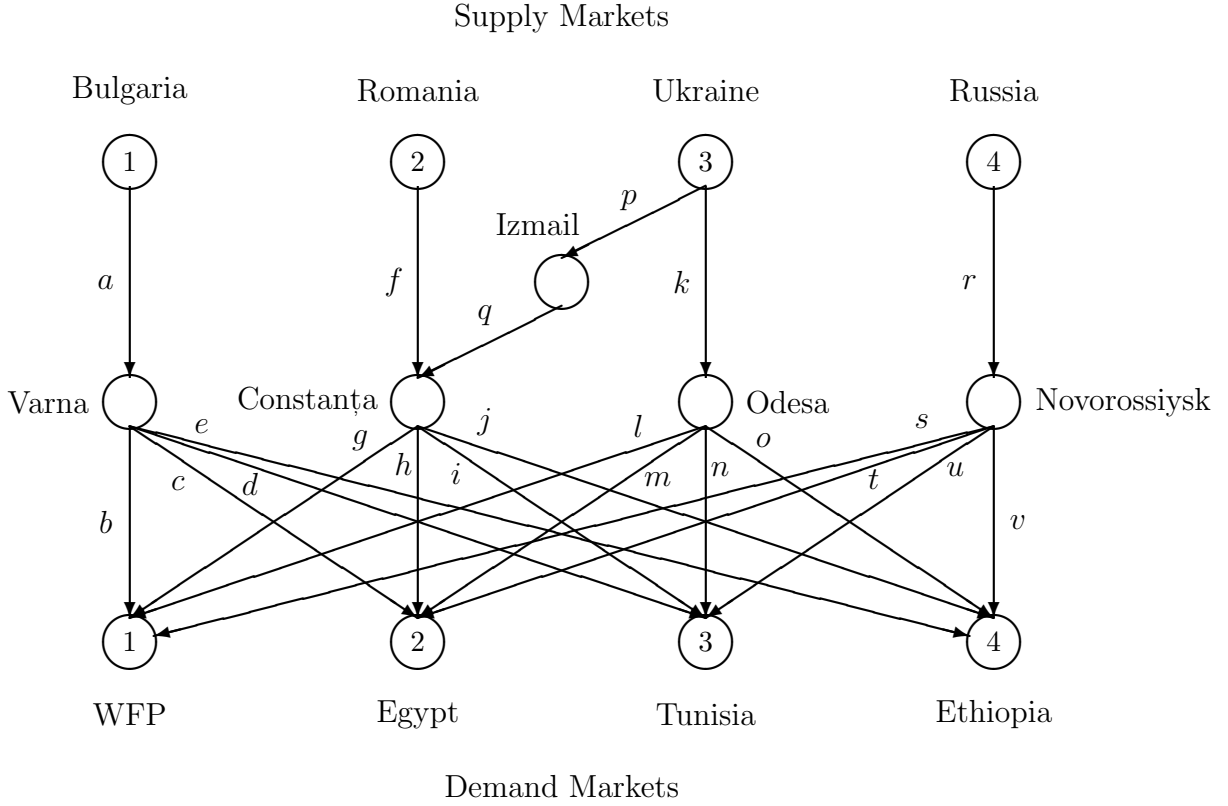


Figure 4: The Trade Network Topology for the Algorithmically Solved Numerical Examples

different scenarios. Example 10 examines the effects of adding a second commodity to the trade network.

The trade network for the examples is depicted in Figure 4. There are 22 links and 20 paths in the network. Paths p_1 through p_4 represent rail transport in Bulgaria to the Port of Varna (link a), and from there maritime shipment to WFP (link b), Egypt (link c), Tunisia (link d), and Ethiopia (link e), respectively. Paths p_5 to p_8 represent rail transport in Romania to the Port of Constanța (link f), and from there by ship to WFP (link g), Egypt (link h), Tunisia (link i), and Ethiopia (link j), respectively. Paths p_9 to p_{12} contain rail transport inside Ukraine to the Port of Odesa (link k), and then by ship to WFP (link l), Egypt (link m), Tunisia (link n), and Ethiopia (link o), respectively. Paths p_{13} to p_{16} include rail transport in Ukraine to the Port of Izmail (link p), then by barge to the Port of Constanța in Romania (link q), and from there maritime shipment to WFP (link g), Egypt (link h), Tunisia (link i), and Ethiopia (link j), respectively. Finally, paths p_{17} to p_{20} represent rail transport inside Russia to the Port of Novorossiysk (link r), and from there maritime transport to WFP (link s), Egypt (link t), Tunisia (link u), and Ethiopia (link v), respectively.

The modified projection method is implemented in MATLAB on a Microsoft Windows 10 system equipped with an 11th Gen Intel Core i7-1185G7 processor and 16 GB of RAM at the University of Massachusetts Amherst. We use $\beta = 0.44$ for all examples except Example 8, which is solved using $\beta = 0.03$ to reduce oscillations around the solution and improve the practical convergence rate. In Example 8, some of the fixed demand constraints are binding, and the corresponding Lagrange multipliers are positive; with larger values of β , the iterates tend to overshoot the solution and oscillate around it, resulting in substantially longer convergence times. This reduction in β is a practical numerical choice and does not reflect a change in the monotonicity or Lipschitz continuity of the mapping. We use $\eta = 2.24$ as a common Lipschitz constant for all the numerical examples. The convergence criterion is set at 10^{-2} ; that is, the algorithm is considered to have converged if the absolute difference between successive iterates of each path flow and each Lagrange multiplier is less than or equal to this value. Although the number of iterations required for convergence of the algorithm is large, the wall-clock runtimes for Examples 5, 6, 7, and 9 are just a few seconds. Example 10, which is the larger multicommodity example, takes around 22.47 seconds to converge, and Example 8, for which a smaller β is used to reduce oscillations around the solution, still requires less than a minute to converge. In other words, because of the closed-form expressions provided earlier, the algorithm is computationally inexpensive for the numerical examples considered here. All wall-clock runtimes were measured using the `tic` and `toc` functions in MATLAB and are reported in the discussions of the respective examples. The results for Examples 5 to 9 are reported in Tables 2, 3, and 4. The results of Example 10 are reported in Tables 5, 6, and 7. The computed solutions satisfy the equilibrium conditions with a high degree of numerical accuracy. Across all the numerical examples, the maximum absolute deviation from the equality condition on a positive-flow path is approximately \$0.167 per metric ton on path p_{18} in Example 8, which is less than 0.1% of the corresponding demand price. All zero-flow inequalities are satisfied, and the fixed-demand equilibrium conditions hold to numerical precision.

To assess the sensitivity of the computed equilibrium solutions to the convergence tolerance, we solved Example 5 using tolerance levels of 10^{-1} , 10^{-2} , 10^{-3} , 10^{-4} , and 10^{-5} . Using the solution obtained under the strictest tolerance as the benchmark, the normalized maximum differences in the equilibrium path flows were approximately 2.30%, 0.24%, 0.033%, and 0.003% under tolerances of 10^{-1} , 10^{-2} , 10^{-3} , and 10^{-4} , respectively. Here, the normalized maximum difference is defined as the maximum absolute difference between the path flows divided by the largest path flow under the 10^{-5} tolerance. The Lagrange multipliers were identical under all five tolerance levels. These results indicate that the computed equilibrium is stable with respect to reasonable changes in the convergence threshold.

To assess the sensitivity of the computed equilibrium solutions to the choice of initial conditions, we solved Example 5 using four starting points: zero path flows and zero Lagrange multipliers; a balanced feasible path flow vector in which each demand market’s fixed demand was divided equally among its available paths; a concentrated feasible path flow vector in which each demand market’s fixed demand was assigned to a single path; and zero path flows with all initial Lagrange multipliers set equal to 100. Using the solution obtained from the zero initialization as the benchmark, the maximum normalized difference in the equilibrium path flows across the alternative initializations was approximately 0.024%, and the maximum absolute difference in the Lagrange multipliers was zero. The normalized maximum difference is defined as the maximum absolute difference between the path flows divided by the largest path flow under the zero initialization. The same set of paths carried positive equilibrium flows under all four initializations.

Finally, regarding uniqueness in the numerical examples, the symmetric parts of the coefficient matrices associated with the aggregate supply price and link cost mappings and the negatives of the demand price mappings are positive definite. Hence, the equilibrium supply, commodity link flow, and demand patterns are unique under the strict monotonicity conditions discussed earlier. Although the associated path-based mapping is monotone but not strictly monotone, its nonzero null directions do not yield alternative feasible equilibrium solutions. In particular, the equilibrium flows on paths p_{13} through p_{16} are equal to zero in all the numerical examples, and any nonzero movement from an equilibrium path flow pattern along a null direction would result in at least one of these path flows becoming negative. Hence, no distinct feasible equilibrium path flow pattern exists, and the equilibrium path flow pattern Q^* is also unique for each of the numerical examples.

Example 5: Prior to Full-Scale Invasion - 2021

In Example 5, we focus on the trade of wheat from the Black Sea region to WFP, Egypt, Tunisia, and Ethiopia in 2021, i.e., the year before Russia’s full-scale invasion of Ukraine on February 24, 2022. The fixed demands are: $d_1^f = 750,000$, $d_2^f = 4,000,000$, $d_3^f = 500,000$, $d_4^f = 500,000$. According to Reidy (2025), Tunisia’s nationwide grain storage infrastructure has a capacity of around 0.5 million tons, and Ethiopia’s grain reserve capacity has been reported at around the same amount (FDRE Government Communication Service (2017)). Furthermore, USDA (2022) reports Egypt’s maximum wheat storage capability at around 5 million tons. We assume 4,000,000 tons in 2021 and 5,000,000 tons in 2024 since the Egyptian government has been consistently increasing its grain storage capacity over the course of recent years. WFP’s fixed demand is estimated based on the fact that, in the first

Table 2: Equilibrium Wheat Path Flows for Examples 5 Through 9

	Example 5	Example 6	Example 7	Example 8	Example 9
$Q_{p_1}^*$	80,353.74	32,546.49	125,909.85	283,842.73	0
$Q_{p_2}^*$	0	0	0	0	0
$Q_{p_3}^*$	288,964.41	160,805.74	453,162.88	1,338,888.66	0
$Q_{p_4}^*$	44,834.32	2,192.49	79,842.31	16,628.77	0
$Q_{p_5}^*$	229,528.35	0	0	990,344.74	0
$Q_{p_6}^*$	1,512,458.84	635,035.94	999,874.40	1,007,538.59	0
$Q_{p_7}^*$	141,068.16	0	179,218.95	147,893.92	0
$Q_{p_8}^*$	159,256.40	180,796.71	172,621.29	4,160,370.50	0
$Q_{p_9}^*$	784,422.69	305,781.21	138,953.19	1,979,413.18	19,477.21
$Q_{p_{10}}^*$	2,783,147.85	2,151,987.96	3,213,208.94	3,112,223.00	1,448,727.89
$Q_{p_{11}}^*$	581,543.54	731,549.98	741,337.96	2,318,577.00	467,010.83
$Q_{p_{12}}^*$	798,728.47	94,122.07	946,945.90	97,161.58	66,232.96
$Q_{p_{13}}^*$	0	0	0	0	0
$Q_{p_{14}}^*$	0	0	0	0	0
$Q_{p_{15}}^*$	0	0	0	0	0
$Q_{p_{16}}^*$	0	0	0	0	0
$Q_{p_{17}}^*$	190,673.65	30,196.73	119,554.24	2,746,399.35	5,331.95
$Q_{p_{18}}^*$	4,894,840.19	8,713,507.99	9,750,213.54	11,838,552.76	6,031,023.71
$Q_{p_{19}}^*$	114,928.86	415,083.64	158,414.97	2,532,212.84	480,545.38
$Q_{p_{20}}^*$	209,428.90	799,001.19	351,120.62	1,725,839.16	708,355.95

half of 2023, it shipped more than 725,000 tons of wheat from Ukraine (European Council (2025)), and that 2021 recorded the highest level of food procurement by WFP in twelve years (WFP (2022)). We assume that approximately half of WFP's procurement is carried out in anticipation of disasters.

The supply price functions are as follows:

$$\pi_1(s) = 0.0000075s_1 + 0.0000005s_2 + 0.0000014s_3 + 0.0000015s_4 + 230,$$

$$\pi_2(s) = 0.0000004s_1 + 0.0000050s_2 + 0.0000005s_3 + 0.0000006s_4 + 230,$$

$$\pi_3(s) = 0.0000001s_1 + 0.0000003s_2 + 0.0000045s_3 + 0.0000009s_4 + 220,$$

$$\pi_4(s) = 0.0000002s_1 + 0.0000003s_2 + 0.0000008s_3 + 0.0000042s_4 + 220.$$

The demand price functions are:

$$\rho_1(d^e) = -0.000040d_1^e + 330, \quad \rho_2(d^e) = -0.000009d_2^e + 350,$$

$$\rho_3(d^e) = -0.000045d_3^e + 335, \quad \rho_4(d^e) = -0.000023d_4^e + 325.$$

Table 3: Equilibrium Wheat Supplies, Total Demands, and Elastic Demands for Examples 5 Through 9

	Example 5	Example 6	Example 7	Example 8	Example 9
s_1^*	414, 152.47	195, 544.73	658, 915.05	1, 639, 360.16	0
s_2^*	2, 042, 311.75	815, 832.64	1, 351, 714.63	6, 306, 147.74	0
s_3^*	4, 947, 842.56	3, 283, 441.21	5, 040, 445.98	7, 507, 374.75	2, 001, 448.90
s_4^*	5, 409, 871.60	9, 957, 789.55	10, 379, 303.37	18, 843, 004.10	7, 225, 256.98
d_1^*	1, 284, 978.43	368, 524.43	384, 417.28	6, 000, 000.00	24, 809.16
d_2^*	9, 190, 446.88	11, 500, 531.88	13, 963, 296.88	15, 958, 314.34	7, 479, 751.61
d_3^*	1, 126, 504.97	1, 307, 439.36	1, 532, 134.76	6, 337, 572.42	947, 556.21
d_4^*	1, 212, 248.10	1, 076, 112.46	1, 550, 530.12	6, 000, 000.00	774, 588.91
d_1^{e*}	534, 978.43	18, 524.43	34, 417.28	0	24, 809.16
d_2^{e*}	5, 190, 446.88	6, 500, 531.88	8, 963, 296.88	3, 458, 314.34	7, 479, 751.61
d_3^{e*}	626, 504.97	807, 439.36	1, 032, 134.76	337, 572.42	947, 556.21
d_4^{e*}	712, 248.10	576, 112.46	1, 050, 530.12	0	774, 588.91

Table 4: Equilibrium Supply and Demand Prices for Examples 5 Through 9

	Example 5	Example 6	Example 7	Example 8	Example 9
π_1	\$249.17	\$107.59	\$108.79	\$112.81	\$106.55
π_2	\$246.10	\$108.63	\$109.77	\$120.51	\$106.55
π_3	\$247.79	\$105.91	\$110.55	\$119.25	\$101.91
π_4	\$247.38	\$104.29	\$104.86	\$112.87	\$101.68
ρ_1	\$308.60	\$184.25	\$170.75	\$200.00	\$178.91
ρ_2	\$303.29	\$181.00	\$166.22	\$199.25	\$175.12
ρ_3	\$306.81	\$179.32	\$170.78	\$197.17	\$173.99
ρ_4	\$308.62	\$184.44	\$171.64	\$200.00	\$179.09

The link cost functions are:

$$\begin{aligned}
c_a(f) &= 0.0000010f_a + 20, & c_b(f) &= 0.0000500f_b + 35, & c_c(f) &= 0.0000010f_c + 40, \\
c_d(f) &= 0.0000250f_d + 30, & c_e(f) &= 0.0000900f_e + 35, & c_f(f) &= 0.0000010f_f + 25, \\
c_g(f) &= 0.0000020f_g + 35, & c_h(f) &= 0.0000001f_h + 30, & c_i(f) &= 0.0000260f_i + 30, \\
c_j(f) &= 0.0000030f_j + 35, & c_k(f) &= 0.0000010f_k + 20, & c_l(f) &= 0.0000011f_l + 35, \\
c_m(f) &= 0.0000002f_m + 30, & c_n(f) &= 0.0000070f_n + 30, & c_o(f) &= 0.0000011f_o + 35, \\
c_p(f) &= 0.0000100f_p + 20, & c_q(f) &= 0.0000100f_q + 10, & c_r(f) &= 0.0000010f_r + 20, \\
c_s(f) &= 0.0000043f_s + 35, & c_t(f) &= 0.0000001f_t + 30, & c_u(f) &= 0.0000350f_u + 30, \\
c_v(f) &= 0.0000040f_v + 35.
\end{aligned}$$

The algorithm converges in approximately 3.81 seconds and requires 5,432,231 iterations. Based on the equilibrium path flows provided in Table 2, we see that WFP procures 80,354 tons of wheat from Bulgaria, 229,528 tons from Romania, 784,423 tons from Ukraine, and 190,674 tons from Russia. Egypt imports 1,512,459 tons from Romania, 2,783,148 tons from Ukraine, and 4,894,840 tons from Russia. Tunisia purchases 288,964 tons, 141,068 tons, 581,544 tons, and 114,929 tons of wheat from Bulgaria, Romania, Ukraine, and Russia, respectively. Finally, based on the equilibrium solutions, Ethiopia sources most of its wheat from Ukraine, with a path flow of 798,728 tons of wheat, while there are also purchases of 44,834 tons from Bulgaria, 159,256 tons from Romania, and 209,429 tons from Russia. Note that alternative paths in Ukraine, i.e., paths p_{13} to p_{16} , are not used since, in normal times, Ukraine (and also Bulgaria, Romania, and Russia) exports most of its grains via its efficient Black Sea ports. All Lagrange multipliers are equal to 0.

In reality, in 2021, WFP procured around 4.5 million tons of food (WFP (2022)). On average, around 70% of WFP’s food procurement consists of grains and cereals, and most of its wheat purchases are from the Black Sea region. For example, in 2022, WFP bought around 50% of its wheat from Ukraine, and the percentage was around 80% in 2023 (European Council (2025)). In 2021, WFP bought around 80,000, 270,000, 880,000, and 245,000 metric tons of grains from Bulgaria, Romania, Ukraine, and Russia, respectively (WFP (2022)). Furthermore, Egypt imported around 11.25 million tons of wheat in 2021, with Romania, Ukraine, and Russia providing 1.6, 2.8, and 4.95 million tons, respectively (IndexMundi (2025a), TrendEconomy (2025a)). Tunisia’s wheat imports in 2021 were around 1.8 million tons, with approximately 295,000, 131,000, 558,000, and 105,000 tons coming from Bulgaria, Romania, Ukraine, and Russia, respectively (IndexMundi (2025b), TrendEconomy (2025b)). Finally, Ethiopia imported around 1.7 million tons of wheat, with the shares of Bulgaria, Romania, Ukraine, and Russia being around 50,000, 155,000, 750,000, and 230,000 tons, respectively (IndexMundi (2025c), TrendEconomy (2025c)). The shipment volumes in Table 2 under Example 5 are close to the above real-world trade flows.

With regard to prices, based on the equilibrium prices presented in Table 4, we observe supply prices between \$246 and \$250 and demand prices in the range of \$303 to \$309. In actuality, in 2021, farmers in the Black Sea region were earning as high as \$270 per ton of their grains (Martyshev, Nivievskyi, and Bogonos (2023)), and the average global wheat prices were around \$320 per metric ton (Macrotrends (2025)). The equilibrium prices in Example 5 are comparable to the real-world prices at the time.

Example 6: After Full-Scale Invasion - 2024

In Example 6, we consider the trade of grains from the Black Sea region to North and Sub-Saharan Africa in 2024, i.e., the third year of the Russo-Ukrainian war. The full-scale invasion has changed the landscape of international agricultural trade in the Black Sea region. After the start of the full-scale invasion, Russia blockaded Ukraine's Black Sea ports, hampering the flow of grain exports out of Ukraine, and Ukraine sought alternative paths using its Danube River ports and a solidarity lane through Romania. Transportation costs in the Black Sea region soared due to the war-related risks, affecting all countries of the region, and as a result, wheat prices skyrocketed. From July 2022, the Black Sea Grain Initiative, brokered by Turkey and the United Nations, facilitated the export of grains from Ukrainian Black Sea ports, but transportation costs and, as a result, the demand prices, remained high. In July 2023, Russia suspended the Black Sea Grain Initiative, and since then, Ukraine has relied on its Black Sea export corridor and transport through NATO's territorial waters for higher safety.

The fixed demands in this example are: $d_1^f = 350,000$, $d_2^f = 5,000,000$, $d_3^f = 500,000$, $d_4^f = 500,000$.

The supply price functions are:

$$\pi_1(s) = 0.00000220s_1 + 0.00000001s_2 + 0.00000005s_3 + 0.00000020s_4 + 105,$$

$$\pi_2(s) = 0.00000001s_1 + 0.00000180s_2 + 0.00000005s_3 + 0.00000020s_4 + 105,$$

$$\pi_3(s) = 0.00000005s_1 + 0.00000005s_2 + 0.00000255s_3 + 0.00000025s_4 + 95,$$

$$\pi_4(s) = 0.00000003s_1 + 0.00000003s_2 + 0.00000009s_3 + 0.00000090s_4 + 95.$$

The demand price functions are:

$$\rho_1(d^e) = -0.000850d_1^e + 200, \quad \rho_2(d^e) = -0.000006d_2^e + 220,$$

$$\rho_3(d^e) = -0.000038d_3^e + 210, \quad \rho_4(d^e) = -0.000027d_4^e + 200.$$

The link cost functions are:

$$c_a(f) = 0.00000100f_a + 30, \quad c_b(f) = 0.00004500f_b + 45, \quad c_c(f) = 0.00000010f_c + 55,$$

$$c_d(f) = 0.00000950f_d + 40, \quad c_e(f) = 0.00075500f_e + 45, \quad c_f(f) = 0.00000100f_f + 30,$$

$$c_g(f) = 0.00000050f_g + 45, \quad c_h(f) = 0.00000245f_h + 40, \quad c_i(f) = 0.00000250f_i + 40,$$

$$\begin{aligned}
c_j(f) &= 0.00000001f_j + 45, & c_k(f) &= 0.00000100f_k + 30, & c_l(f) &= 0.00000020f_l + 45, \\
c_m(f) &= 0.000000084f_m + 40, & c_n(f) &= 0.000000015f_n + 40, & c_o(f) &= 0.000000265f_o + 45, \\
c_p(f) &= 0.00001000f_p + 30, & c_q(f) &= 0.00001000f_q + 20, & c_r(f) &= 0.00000100f_r + 25, \\
c_s(f) &= 0.000000018f_s + 45, & c_t(f) &= 0.000000020f_t + 40, & c_u(f) &= 0.000000020f_u + 40, \\
c_v(f) &= 0.000000025f_v + 45.
\end{aligned}$$

The algorithm converges in approximately 7.97 seconds and requires 13,152,826 iterations. All Lagrange multipliers are equal to 0. Based on the equilibrium path flows for Example 6 in Table 2, we observe that WFP sources the bulk of its wheat from Ukraine, with a path flow of 305,781 tons, while Bulgaria and Russia provide 32,546 and 30,197 tons, respectively. Egypt procures most of its wheat from Russia with an equilibrium path flow of 8,713,508 tons, with Ukraine and Romania following with 2,151,988 and 635,036 tons of wheat supply, respectively. Tunisia purchases 160,806 tons of wheat from Bulgaria, 731,550 tons from Ukraine, and 415,084 tons from Russia. Finally, Ethiopia procures 180,797 tons from Romania, 94,122 tons from Ukraine, 799,001 tons from Russia, and a negligible amount from Bulgaria. Note that paths p_{13} through p_{16} , which represent alternative transportation routes used by Ukraine during the initial months following the start of the full-scale invasion, were once again not used in 2024. This is to be expected, as Ukraine had been using its more efficient Black Sea export corridor. According to Melkozerova (2026), more than 90% of Ukraine's agricultural exports normally pass through its Odesa ports, and the alternative routes through the Danube River and European rail networks cannot match the scale and cost efficiency of maritime transport through Ukraine's Black Sea export corridor.

In reality, in 2024, WFP purchased 1.97 million tons of food, of which 68% were grains and cereals, and once again, most of the wheat procurement was from the Black Sea region. Ukraine was the top supply country with around 270,000 tons, and Russia provided around 44,000 tons. Bulgaria and Romania were not among the top ten source countries for WFP's procurement. The year 2024 saw the least amount of food purchased by WFP since 2012 due to funding cuts (WFP (2025b)). Furthermore, in 2024, Egypt imported 12.5 million tons of wheat, of which 0.66, 2.14, and 8.50 million tons came from Romania, Ukraine, and Russia, respectively (IndexMundi (2025a), USDA (2024a), USDA (2025a)). Tunisia, once again, imported around 1.8 million tons of wheat in 2024, with the shares of Bulgaria, Ukraine, and Russia at around 125,000, 760,000, and 410,000 tons, respectively (IndexMundi (2025b), USDA (2024b), USDA (2025b)). Finally, in 2024, Ethiopia imported around 1.4 million tons of wheat, with Romania, Ukraine, and Russia providing 200,000, 70,000, and 810,000 tons,

respectively (IndexMundi (2025c), USDA (2025c)). The results in Table 2 closely replicate the actual trade flows in 2024.

With regard to prices, compared to those in Example 5, supply prices decrease to between \$104 and \$109, and demand prices decrease to a range of \$179 to \$185. In fact, after the start of the full-scale invasion, farmers have had to shoulder the burden of higher transportation costs at the expense of their earnings and, at times, have earned even less than \$100 per ton of wheat (Balmforth and Polityuk (2022), Brower (2022)). On the demand side, after the start of the full-scale invasion, average global wheat prices rose as high as \$445, with prices in more vulnerable countries even reaching the \$500 level; however, markets have adapted as the war has dragged on, and demand prices decreased to as low as \$175 per ton of wheat in 2024 (Macrotrends (2025)). The supply and demand prices presented in Table 4 closely replicate the actual prices in 2024.

We can observe that Russia’s wheat exports increase by around 84% compared to those in Example 5, while Bulgaria, Romania, and Ukraine see decreases of around 53%, 60%, and 34%, respectively. In other words, Russia has by far become the leading supplier of wheat to the demand countries in the network. Given the huge volumes of wheat imported from Russia and Ukraine, any disruption to wheat supplies from these countries, at least in the short term, could hamper WFP’s food relief operations and decrease the availability of wheat in the demand countries, resulting in food insecurity issues and pushing the demand countries to use their strategic grain reserves. Furthermore, note that the total elastic demand in the network increases by around 12% compared to that in Example 5, and demand prices are also much lower, which, on an aggregate level, translates into higher wheat availability at more affordable consumer prices. In other words, markets have adapted to the impacts of the full-scale invasion, and the availability and affordability issues of the early months after the start of the war have been mitigated. However, observe that WFP’s total procurement decreases from around 1.285 million tons in 2021 to around 0.369 million tons in 2024, and its elastic demand decreases from around 0.535 million tons in 2021 to less than 20,000 tons in 2024. WFP serves the most vulnerable communities in North and Sub-Saharan Africa, and its decreased level of food assistance, whether due to funding cuts or geopolitical issues, puts already extremely food-insecure populations at even greater risk.

Example 7: The Full-Scale Invasion Stops

In Example 7, we consider the scenario in which the full-scale invasion stops and, therefore, transportation costs return to 2021 levels. Accordingly, this example uses the same supply and demand price functions as those in Example 6, but the link cost functions are as in Ex-

ample 5. All Lagrange multipliers are equal to 0. The algorithm converges in approximately 4.74 seconds and requires 7,430,664 iterations.

We can observe that with reduced transportation costs, trade flows naturally improve compared to those in Example 6. The supply from Bulgaria, Romania, Ukraine, and Russia increases by around 237%, 66%, 54%, and 4%, respectively. Russia's share of the total wheat supply in the network in this example is around 60%, which is lower than about 70% in Example 6 but much higher than roughly 42% in Example 5. In other words, even after the full-scale invasion has stopped and transportation costs have decreased, Russia maintains its leading position in the wheat markets of the demand countries, especially in Egypt. Compared to Example 6, Bulgaria's wheat exports to Tunisia and Ethiopia, as well as its wheat sales to WFP, increase significantly. Romania's wheat exports to Egypt and Tunisia increase, but its overall exports remain below the pre-war level in Example 5. Ukraine's share of wheat supply to WFP decreases, while its exports to Egypt and Ethiopia increase significantly compared to Example 6, and its shipments to Egypt, Tunisia, and Ethiopia remain higher than the pre-war levels observed in Example 5.

Based on the equilibrium path flows presented in Table 2, we observe that, among the demand countries, Egypt remains heavily reliant on the supply of wheat from Russia, meaning that any disruption to the wheat path flow originating from Russia could damage the availability of wheat in Egypt and result in food security concerns. In the same manner, Ethiopia relies heavily on wheat imports from Ukraine; however, the volume of Ethiopia's imports is around one-tenth that of Egypt, and, as such, it would be easier for Ethiopia to find alternative sources in the case of a disruption to wheat exports from Ukraine, and a much lower level of strategic grain reserves would help the country avoid a food availability crisis. Tunisia diversifies its wheat imports with relatively balanced volumes from the supply countries in the network, and, hence, disruptions to its wheat path flows originating from a specific supply country would not cause as severe a wheat availability crisis. Furthermore, similar to Ethiopia, Tunisia's demand is much lower than that of Egypt's, meaning that a much lower level of strategic grain reserves would suffice to shield the country against trade disruptions.

With transportation costs reduced, the supply prices increase only slightly, while the decreases in demand prices are more notable. In other words, the benefit of reduced transportation costs is appropriated by consumers rather than farmers, and farmers have to adapt to lower wheat prices. In fact, the decreasing trend in average global wheat prices has continued into 2025 and is projected to carry on into 2026 (Macrotrends (2025)). It is also worth noting that the total elastic demand in the network increases by around 40% compared to

that in Example 6. In other words, with the full-scale invasion stopped, the availability and affordability of wheat improve significantly, translating into higher food security in the demand countries. However, even at reduced transportation costs, WFP’s total and elastic demand remain close to those in Example 6; that is, one can associate WFP’s current operational hurdles more with its purchasing power (i.e., funding cuts) than with the prolonged full-scale invasion.

Example 8: Increased Fixed Demands

In this example, the demand countries aim to carry higher levels of strategic grain reserves in response to risks and global disruptions, and, hence, have higher fixed demands. The fixed demands in this example are: $d_1^f = 6,000,000$, $d_2^f = 12,500,000$, $d_3^f = 6,000,000$, $d_4^f = 6,000,000$. The rest of the data remains as in Example 6. We have that now $\mu_1^* = 2.23$ and $\mu_4^* = 2.01$, with the other Lagrange multipliers equal to 0. The algorithm converges in approximately 58.81 seconds and requires 101,353,591 iterations.

Note that the fixed demands in this example are intentionally selected at high levels and do not represent current reserve capacities or announced national reserve targets. Instead, Example 8 presents an extreme scenario in which some of the fixed demand constraints become binding, allowing us to examine the impacts of very high levels of strategic grain reserves and illustrate the implications of positive Lagrange multipliers. Such a scenario is motivated by increasing food security needs due to population growth and recurring international trade and supply disruptions. The positive Lagrange multipliers for WFP and Ethiopia indicate that their fixed demand constraints are binding. The values $\mu_1^* = 2.23$ and $\mu_4^* = 2.01$ represent the marginal costs of increasing their respective fixed demands by one additional metric ton. In other words, increasing the strategic reserve requirement would impose an additional cost of approximately \$2.23 per ton for WFP and \$2.01 per ton for Ethiopia. The zero Lagrange multipliers for Egypt and Tunisia indicate that their fixed demand constraints are not binding. Hence, the results show that, although higher levels of strategic grain reserves can improve long-term wheat availability, carrying excessively large reserves can increase costs and reduce current wheat affordability.

With increased fixed demands, Russia becomes the top source country of wheat for WFP, with flows from Ukraine and Romania, and to a lesser degree from Bulgaria, also significantly increasing. Providing around 74% of its wheat demand, Russia remains the major supplier of wheat to Egypt, while shipments from Romania and Ukraine also increase. Tunisia maintains a diverse portfolio of wheat imports from Russia, Ukraine, and Bulgaria. Interestingly, Romania becomes the major supplier of wheat to Ethiopia, with Russia following in second

place. Once again, Russia experiences the largest absolute increase in its total wheat supply, further highlighting the country’s strong share in the wheat markets of North and Sub-Saharan Africa, while Romania’s supply increases significantly as well. Given Ethiopia’s dependence on wheat imports from Romania in this example, major wheat availability risks in Ethiopia, and the corresponding food insecurity issues, would stem from disruptions to Romania’s wheat exports. Furthermore, once again, we observe that Egypt is heavily reliant on wheat imports from Russia to satisfy its population’s caloric demand and to fill its strategic grain reserve.

The supply prices increase in all supply countries thanks to higher levels of trade and increased demand prices. With more of the wheat imports going towards satisfying the fixed demands, i.e., towards carrying higher volumes of wheat in strategic grain reserves, less elastic demand is satisfied, and as such, higher demand prices are observed. One can deduce that there is a trade-off between the amount carried in strategic grain reserves and current demand prices; that is, carrying unreasonably large volumes of strategic grain reserves for long-term safety could, in the short term, potentially damage the affordability of wheat in the markets.

Example 9: No Fixed Demands

In this example, the demand countries decide against having strategic grain reserves, i.e., there are now zero fixed demands. Therefore, we have that: $d_1^f = d_2^f = d_3^f = d_4^f = 0$. The rest of the data remains the same as in Example 6. All Lagrange multipliers are equal to zero. The algorithm converges in approximately 8.40 seconds and requires 11,471,929 iterations.

With no fixed demands, all the elastic wheat demands are provided by Ukraine and Russia, reducing Bulgaria’s and Romania’s share of the market to zero, which can hurt Bulgarian and Romanian farmers. Egypt and Ethiopia import most of their elastic wheat demand from Russia, while Tunisia maintains a balanced import portfolio. WFP’s demand drops to less than 25,000 tons, highlighting its low willingness to procure pre-disaster without needing to store grains in its strategic hubs. The wheat path flow from Russia to Egypt comprises over 80% of Egypt’s wheat demand, once again highlighting the country’s dependence on Russian wheat exports to meet its population’s caloric and nutritional needs. In the case of Ethiopia, the wheat path flow from Russia comprises over 91% of the country’s overall wheat imports. With no strategic grain reserves, all wheat path flows go towards satisfying elastic demands, and the total volume of elastic demand in the network increases by around 17% compared to that in Example 6, which translates into higher levels of wheat availability in the demand countries, as long as there is no disruption to wheat imports from Russia and Ukraine. Sup-

ply prices drop even below those in Example 6 due to a lower level of trade taking place in the network. The demand prices also decrease compared to those in Example 6, translating into higher levels of wheat affordability. However, one must note that strategic grain reserves are kept to build a long-term safety net to counter sudden commodity shortages and price hikes due to disruptions; hence, the short-term improvement in availability and affordability comes at an increased long-term risk.

Example 10: Two Commodities - Wheat and Corn

In Example 10, we extend Example 6 by considering the trade of two commodities, wheat and corn, from the Black Sea region to WFP, Egypt, Tunisia, and Ethiopia in 2024. Wheat is denoted by the superscript $k = 1$, while corn is denoted by $k = 2$. Wheat and corn are two of the most important grains traded through the Black Sea region. The two commodities share the same supply and transportation infrastructure. Therefore, changes in the supply and transportation of one commodity may affect the equilibrium supply prices, trade flows, and demand prices of the other commodity. For wheat, the fixed demands and demand price functions are the same as those in Example 6. We assume that the fixed demands for corn are equal to 0 across all demand markets since corn does not have a strategic role comparable to that of wheat in Egypt and Tunisia. Although corn is an important staple in Ethiopia, its demand is treated as fully elastic in this example.

The supply price functions for wheat are as follows:

$$\begin{aligned}
\pi_1^1(s) &= 0.000002200s_1^1 + 0.000000010s_2^1 + 0.000000050s_3^1 + 0.000000200s_4^1 \\
&+ 0.000000440s_1^2 + 0.000000002s_2^2 + 0.000000010s_3^2 + 0.000000040s_4^2 + 105, \\
\pi_2^1(s) &= 0.000000010s_1^1 + 0.000001800s_2^1 + 0.000000050s_3^1 + 0.000000200s_4^1 \\
&+ 0.000000002s_1^2 + 0.000000360s_2^2 + 0.000000010s_3^2 + 0.000000040s_4^2 + 105, \\
\pi_3^1(s) &= 0.000000050s_1^1 + 0.000000050s_2^1 + 0.000002550s_3^1 + 0.000000250s_4^1 \\
&+ 0.000000010s_1^2 + 0.000000010s_2^2 + 0.000000510s_3^2 + 0.000000050s_4^2 + 95, \\
\pi_4^1(s) &= 0.000000030s_1^1 + 0.000000030s_2^1 + 0.000000090s_3^1 + 0.000000900s_4^1 \\
&+ 0.000000006s_1^2 + 0.000000006s_2^2 + 0.000000018s_3^2 + 0.000000180s_4^2 + 95.
\end{aligned}$$

The supply price functions for corn are:

$$\pi_1^2(s) = 0.000000480s_1^1 + 0.000000002s_2^1 + 0.000000010s_3^1 + 0.000000030s_4^1$$

$$\begin{aligned}
& +0.0000002400s_1^2 + 0.000000010s_2^2 + 0.000000050s_3^2 + 0.000000150s_4^2 + 115, \\
\pi_2^2(s) &= 0.000000002s_1^1 + 0.000000420s_2^1 + 0.000000010s_3^1 + 0.000000030s_4^1 \\
& +0.000000010s_1^2 + 0.000002100s_2^2 + 0.000000050s_3^2 + 0.000000150s_4^2 + 109, \\
\pi_3^2(s) &= 0.000000010s_1^1 + 0.000000010s_2^1 + 0.000000640s_3^1 + 0.000000040s_4^1 \\
& +0.000000050s_1^2 + 0.000000050s_2^2 + 0.000003200s_3^2 + 0.000000200s_4^2 + 80, \\
\pi_4^2(s) &= 0.000000006s_1^1 + 0.000000006s_2^1 + 0.000000016s_3^1 + 0.000000220s_4^1 \\
& +0.000000030s_1^2 + 0.000000030s_2^2 + 0.000000080s_3^2 + 0.000001100s_4^2 + 120.
\end{aligned}$$

The link cost functions for wheat are as follows:

$$\begin{aligned}
c_a^1(f) &= 0.00000100f_a^1 + 0.00000050f_a^2 + 30, & c_b^1(f) &= 0.00004500f_b^1 + 0.00002250f_b^2 + 45, \\
c_c^1(f) &= 0.00000010f_c^1 + 0.00000005f_c^2 + 55, & c_d^1(f) &= 0.00000950f_d^1 + 0.00000475f_d^2 + 40, \\
c_e^1(f) &= 0.00075500f_e^1 + 0.00037750f_e^2 + 45, & c_f^1(f) &= 0.00000100f_f^1 + 0.00000050f_f^2 + 30, \\
c_g^1(f) &= 0.00000050f_g^1 + 0.00000025f_g^2 + 45, & c_h^1(f) &= 0.00000245f_h^1 + 0.000001225f_h^2 + 40, \\
c_i^1(f) &= 0.00000250f_i^1 + 0.00000125f_i^2 + 40, & c_j^1(f) &= 0.00000001f_j^1 + 0.000000005f_j^2 + 45, \\
c_k^1(f) &= 0.00000100f_k^1 + 0.00000050f_k^2 + 30, & c_l^1(f) &= 0.00000020f_l^1 + 0.00000010f_l^2 + 45, \\
c_m^1(f) &= 0.00000084f_m^1 + 0.00000042f_m^2 + 40, & c_n^1(f) &= 0.00000015f_n^1 + 0.000000075f_n^2 + 40, \\
c_o^1(f) &= 0.00000265f_o^1 + 0.000001325f_o^2 + 45, & c_p^1(f) &= 0.00001000f_p^1 + 0.00000500f_p^2 + 30, \\
c_q^1(f) &= 0.00001000f_q^1 + 0.00000500f_q^2 + 20, & c_r^1(f) &= 0.00000100f_r^1 + 0.00000050f_r^2 + 25, \\
c_s^1(f) &= 0.00000018f_s^1 + 0.00000009f_s^2 + 45, & c_t^1(f) &= 0.00000020f_t^1 + 0.00000010f_t^2 + 40, \\
c_u^1(f) &= 0.00000020f_u^1 + 0.00000010f_u^2 + 40, & c_v^1(f) &= 0.00000025f_v^1 + 0.000000125f_v^2 + 45.
\end{aligned}$$

The link cost functions for corn are:

$$\begin{aligned}
c_a^2(f) &= 0.00000100f_a^1 + 0.00000200f_a^2 + 30, & c_b^2(f) &= 0.00004500f_b^1 + 0.00009000f_b^2 + 45, \\
c_c^2(f) &= 0.00000010f_c^1 + 0.00000020f_c^2 + 55, & c_d^2(f) &= 0.00000950f_d^1 + 0.00001900f_d^2 + 40, \\
c_e^2(f) &= 0.00075500f_e^1 + 0.00151000f_e^2 + 45, & c_f^2(f) &= 0.00000100f_f^1 + 0.00000200f_f^2 + 30, \\
c_g^2(f) &= 0.00000050f_g^1 + 0.00000100f_g^2 + 45, & c_h^2(f) &= 0.00000245f_h^1 + 0.00000490f_h^2 + 40, \\
c_i^2(f) &= 0.00000250f_i^1 + 0.00000500f_i^2 + 40, & c_j^2(f) &= 0.00000001f_j^1 + 0.00000002f_j^2 + 45, \\
c_k^2(f) &= 0.00000100f_k^1 + 0.00000200f_k^2 + 30, & c_l^2(f) &= 0.00000020f_l^1 + 0.00000040f_l^2 + 45,
\end{aligned}$$

$$\begin{aligned}
c_m^2(f) &= 0.00000084f_m^1 + 0.00000168f_m^2 + 40, & c_n^2(f) &= 0.00000015f_n^1 + 0.00000030f_n^2 + 40, \\
c_o^2(f) &= 0.00000265f_o^1 + 0.00000530f_o^2 + 45, & c_p^2(f) &= 0.00001000f_p^1 + 0.00002000f_p^2 + 30, \\
c_q^2(f) &= 0.00001000f_q^1 + 0.00002000f_q^2 + 20, & c_r^2(f) &= 0.00000100f_r^1 + 0.00000200f_r^2 + 25, \\
c_s^2(f) &= 0.00000018f_s^1 + 0.00000036f_s^2 + 45, & c_t^2(f) &= 0.00000020f_t^1 + 0.00000040f_t^2 + 40, \\
c_u^2(f) &= 0.00000020f_u^1 + 0.00000040f_u^2 + 40, & c_v^2(f) &= 0.00000025f_v^1 + 0.00000050f_v^2 + 45.
\end{aligned}$$

The demand price functions for corn are as follows:

$$\begin{aligned}
\rho_1^2(d^{2,e}) &= -0.0009400d_1^{2,e} + 200, \\
\rho_2^2(d^{2,e}) &= -0.0000074d_2^{2,e} + 210, \\
\rho_3^2(d^{2,e}) &= -0.0000365d_3^{2,e} + 200, \\
\rho_4^2(d^{2,e}) &= -0.0000300d_4^{2,e} + 175.
\end{aligned}$$

Table 5: Equilibrium Wheat and Corn Path Flows for Example 10

	Wheat	Corn
$Q_{p_1}^*$	42,571.87	0
$Q_{p_2}^*$	0	0
$Q_{p_3}^*$	216,519.19	0
$Q_{p_4}^*$	2,908.22	0
$Q_{p_5}^*$	0	0
$Q_{p_6}^*$	736,685.83	21,201.88
$Q_{p_7}^*$	0	0
$Q_{p_8}^*$	341,806.85	0
$Q_{p_9}^*$	325,067.52	19,777.05
$Q_{p_{10}}^*$	788,967.98	3,617,079.60
$Q_{p_{11}}^*$	992,667.15	638,983.33
$Q_{p_{12}}^*$	131,378.71	0
$Q_{p_{13}}^*$	0	0
$Q_{p_{14}}^*$	0	0
$Q_{p_{15}}^*$	0	0
$Q_{p_{16}}^*$	0	0
$Q_{p_{17}}^*$	0	0
$Q_{p_{18}}^*$	9,788,989.47	0
$Q_{p_{19}}^*$	76,400.20	0
$Q_{p_{20}}^*$	568,852.77	0

The algorithm converges in approximately 22.47 seconds and requires 14,095,818 iterations. All Lagrange multipliers are equal to 0. Based on the equilibrium path flows for

Table 6: Equilibrium Supplies, Total Demands, and Elastic Demands for Example 10

	Wheat	Corn
s_1^*	261,999.28	0
s_2^*	1,078,492.69	21,201.88
s_3^*	2,238,081.35	4,275,839.97
s_4^*	10,434,242.45	0
d_1^*	367,639.39	19,777.05
d_2^*	11,314,643.28	3,638,281.48
d_3^*	1,285,586.54	638,983.33
d_4^*	1,044,946.56	0
d_1^{e*}	17,639.39	19,777.05
d_2^{e*}	6,314,643.28	3,638,281.48
d_3^{e*}	785,586.54	638,983.33
d_4^{e*}	544,946.56	0

Table 7: Equilibrium Supply and Demand Prices for Example 10

	Wheat	Corn
π_1	\$107.83	\$115.68
π_2	\$109.19	\$110.05
π_3	\$105.56	\$95.55
π_4	\$104.71	\$122.68
ρ_1	\$185.01	\$181.41
ρ_2	\$182.11	\$183.08
ρ_3	\$180.15	\$176.68
ρ_4	\$185.29	\$175.00

Example 10, we observe that the addition of corn affects the wheat trade flows throughout the network. Compared to Example 6, the wheat path flows from Bulgaria to WFP, Tunisia, and Ethiopia increase by around 31%, 35%, and 33%, respectively. Romania's wheat exports to Egypt and Ethiopia increase by around 16% and 89%, respectively. Ukraine's wheat supply to WFP, Tunisia, and Ethiopia increases by around 6%, 36%, and 40%, respectively, while its wheat exports to Egypt decrease by around 63%. Russia no longer supplies wheat to WFP, and its wheat exports to Tunisia and Ethiopia decrease by around 82% and 29%, respectively. However, its wheat exports to Egypt increase by around 12%, reaching approximately 9.8 million tons. With regard to corn, Ukraine supplies around 19,777 tons to WFP, 3.62 million tons to Egypt, and 638,983 tons to Tunisia, while Romania supplies around 21,202 tons to Egypt. In other words, Ukraine supplies more than 99% of the corn traded in the network.

In reality, in 2024, Egypt imported around 3.42 million tons of corn from Ukraine and

around 57,500 tons from Romania, and Tunisia imported around 615,000 tons of corn from Ukraine and around 6,400 tons from Romania in 2024. Overall, Ukraine and Romania supplied approximately 3.48 million tons of corn to Egypt and 622,000 tons to Tunisia in 2024. Furthermore, in actuality, Ukraine accounted for around 98% of the actual corn exports from the supply countries in the network to Egypt and Tunisia in 2024, which is close to its share of more than 99% of the corresponding equilibrium corn flows (World Bank WITS (2026a, 2026b)). Therefore, the corn path flows in Example 10 are close to the actual trade flows in 2024.

Based on the equilibrium supplies and demands, compared to Example 6, the wheat supplies from Bulgaria and Romania increase by around 34% and 32%, respectively, while Ukraine’s wheat supply decreases by around 32%. Russia’s wheat supply increases by around 5% and remains by far the largest source of wheat in the network. The total wheat demands of WFP, Egypt, Tunisia, and Ethiopia decrease by less than 1%, around 2%, around 2%, and around 3%, respectively. Furthermore, the total elastic wheat demand in the network decreases by around 3%. In other words, adding corn to the network has a larger impact on the allocation of wheat supplies among the supply countries than on the total wheat demands in the demand markets.

With regard to prices, the wheat supply prices remain close to those in Example 6, with changes of less than 1% across all supply countries. The wheat demand prices increase by less than 1% across all demand markets. Therefore, the addition of corn has a limited impact on the equilibrium wheat supply and demand prices and, accordingly, on the affordability of wheat in the demand countries. The corn supply prices range from approximately \$96 to \$123, while the corn demand prices range from \$175 to \$183. In actuality, Egypt purchased Ukrainian corn at a price of around \$185 per ton FOB in February 2024, while the international corn price was reported at around \$200 per ton later in the year (GrainTrade (2024), Reuters (2024)). Hence, the equilibrium corn demand prices are reasonably close to the corn prices observed in 2024, although one should note that the real-world prices are export or international market prices and are not directly comparable to the equilibrium demand prices.

5.1 Sensitivity Analysis on Fixed Demands

Following the discussions of Examples 8 and 9, we conduct sensitivity analysis on fixed demands in the four demand countries to better highlight the impact of various levels of strategic grain reserves on demand prices, that is, on the affordability element of food security. The results are presented in Figures 5 to 8. In each case, we keep the fixed component

of demand in three of the demand countries constant, similar to those in Example 6, and vary it in the remaining demand country between the values used in Examples 8 and 9. For example, in Figure 5, the fixed demand for WFP is varied between 0 and 6,000,000 metric tons, while the fixed demand in the other demand countries is kept at levels similar to those in Example 6 across all instances.

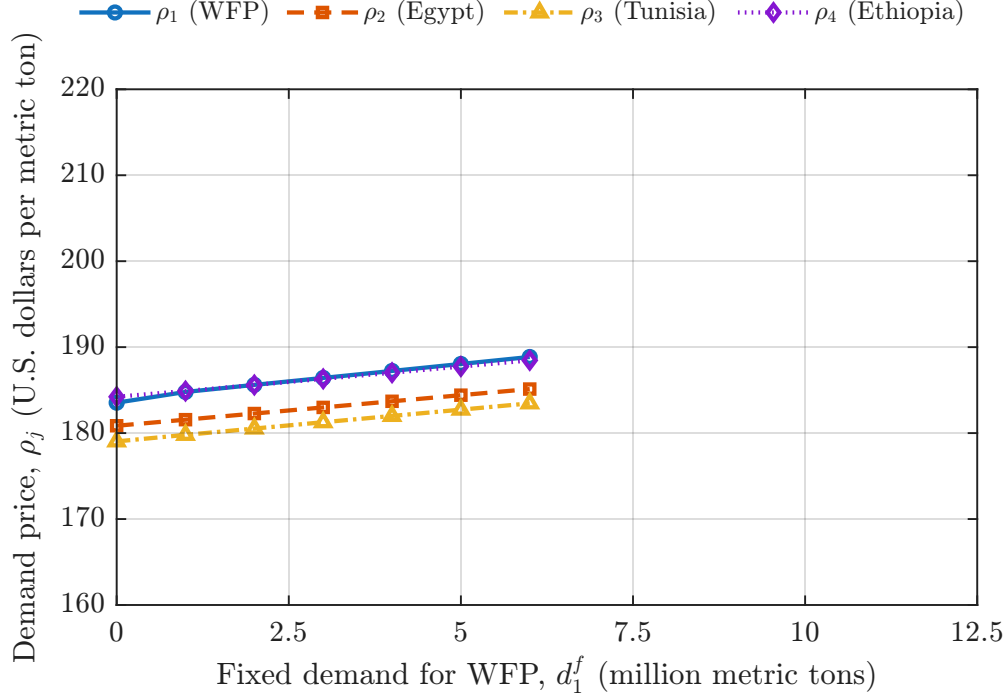


Figure 5: Sensitivity Analysis for Fixed demands with d_1^f Varying: Impact on Demand Prices

Note that the adverse effect of higher levels of strategic grain reserves on demand prices is evident across all instances. Also, one can observe that although the level of fixed demand, i.e., the level of strategic grain reserves carried in a country, clearly has the highest impact on the equilibrium demand price in that specific country, as seen by the slopes of the lines in Figures 5 to 8, it also increases the equilibrium prices in the other demand countries. In other words, one can deduce that markets are coupled, and the impact does not remain local but is rather regional. To reiterate the discussions of the numerical examples, there seems to be a trade-off at work. While strategic grain reserves address the issue of availability of essential agricultural commodities, there is, at least in the short term, an adverse effect on demand prices, which translates into affordability issues.

Table 8 reports the average increase in each equilibrium demand price for every additional one million metric tons of fixed demand over the ranges considered in Figures 5 through 8. The own-market rates of increase are approximately \$0.886 per metric ton for WFP,

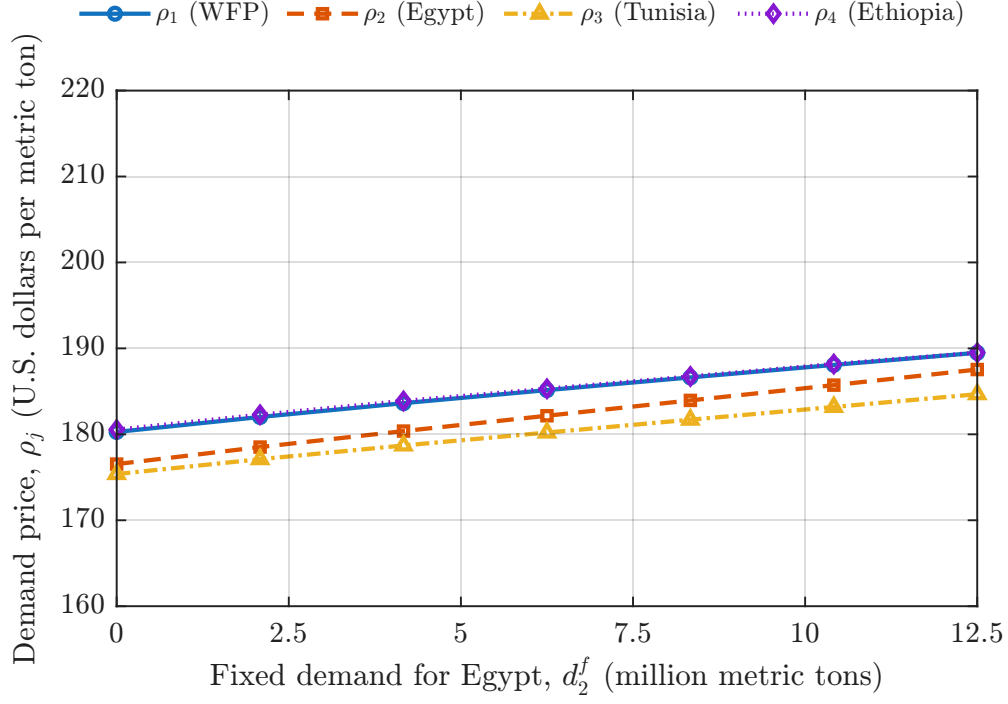


Figure 6: Sensitivity Analysis for Fixed demands with d_2^f Varying: Impact on Demand Prices

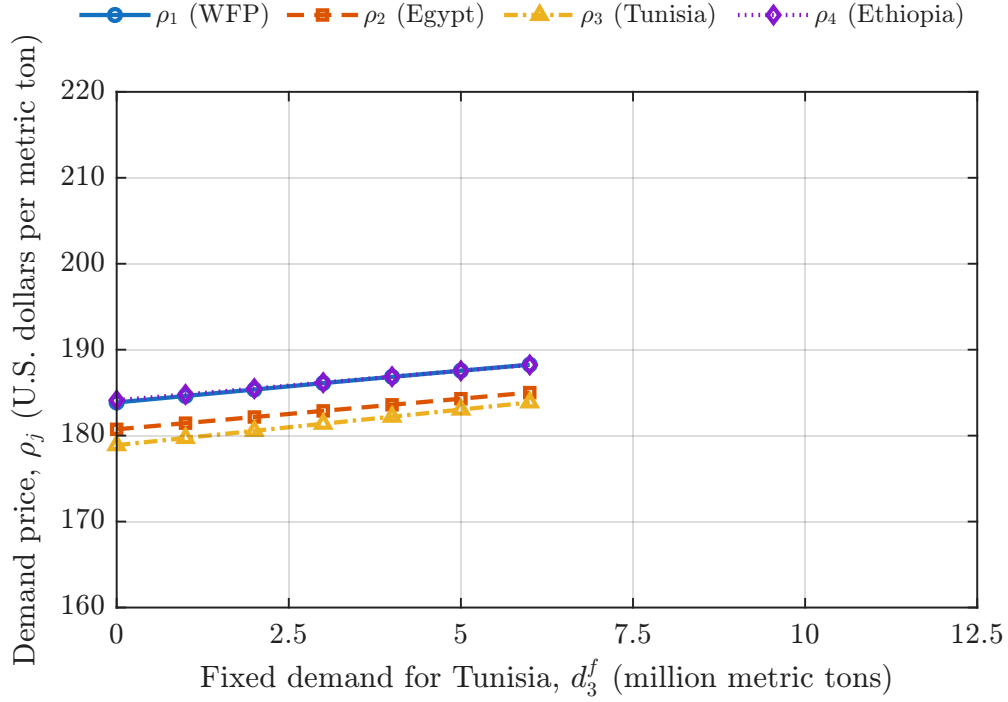


Figure 7: Sensitivity Analysis for Fixed demands with d_3^f Varying: Impact on Demand Prices

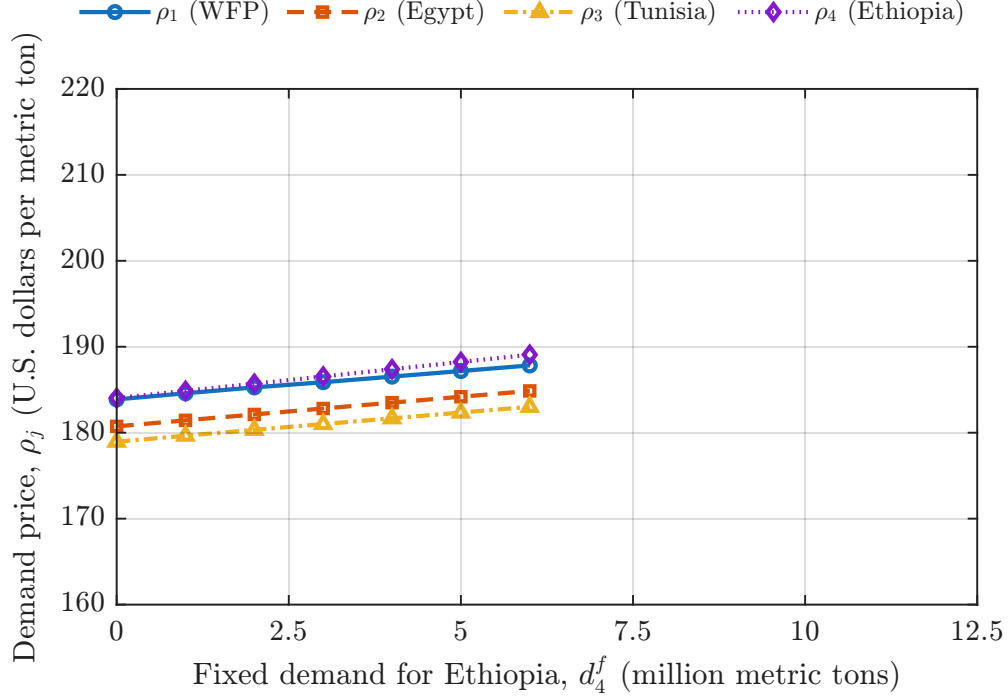


Figure 8: Sensitivity Analysis for Fixed demands with d_4^f Varying: Impact on Demand Prices

Table 8: Average Rates of Increase in Equilibrium Demand Prices with Fixed Demands Varying

	WFP (ρ_1)	Egypt (ρ_2)	Tunisia (ρ_3)	Ethiopia (ρ_4)
WFP (d_1^f)	0.886	0.712	0.738	0.705
Egypt (d_2^f)	0.737	0.880	0.743	0.716
Tunisia (d_3^f)	0.733	0.712	0.830	0.686
Ethiopia (d_4^f)	0.659	0.687	0.677	0.841

\$0.880 for Egypt, \$0.830 for Tunisia, and \$0.841 for Ethiopia. Therefore, the demand prices for WFP and Egypt increase the most in response to increases in their respective fixed demands, although the differences among the four demand markets are small. In all four sensitivity analyses, the own-market rate of increase is higher than the rates of increase in the other demand markets. The cross-market rates range from approximately \$0.659 to \$0.743 per metric ton for every additional one million metric tons of fixed demand. Increasing Egypt's fixed demand results in the largest average increase in the demand prices of the other markets, with an average rate of \$0.732, followed by increases in the fixed demands of WFP, Tunisia, and Ethiopia, with average rates of \$0.718, \$0.710, and \$0.674, respectively. The largest cross-market rate of increase is observed for Tunisia's demand price when Egypt's fixed demand increases, with an average rate of \$0.743. Except for the initially higher

rate of increase in WFP’s own demand price, the rates remain relatively stable in most cases. However, the rates of increase generally decline as Egypt’s fixed demand increases. In other words, increases in fixed demand have the largest impact on the demand price of the respective market, while also increasing the demand prices in the other markets.

6. Summary and Conclusions

Food insecurity has become a persistent issue across the globe due to disruptions caused by the COVID-19 pandemic, climate change, wars and conflicts, droughts, and other forms of disasters. While food banks are the first line of response to hunger issues in developed countries, many developing countries rely on a combination of international agricultural trade and international food aid to satisfy the nutritional and caloric demands of their populations. Trade openness has been shown to have positive long-term effects on food security; however, it can result in short-term food availability and affordability crises for more vulnerable households. Strategic grain reserves are one of the strategies governments use to control food prices and ensure availability while embracing trade openness. In this paper, we construct a multicommodity trade network equilibrium model with mixed demand and implications for food security. The mix of fixed (i.e., inelastic) and elastic demand components is interpreted as what demand countries carry in their strategic grain reserves and what directly reaches the national markets as a result of international agricultural trade.

The equilibrium conditions are provided, with the fixed component of demand captured through lower bounds on commodity path flows towards demand countries. Lagrange multipliers associated with the lower-bound constraints are introduced, and the equilibrium conditions are formulated as a variational inequality problem in commodity path flows and Lagrange multipliers. Several simple examples are solved analytically for illustrative purposes.

A set of larger numerical examples is then solved algorithmically. These examples are inspired by wheat exports from supply countries in the Black Sea region, namely Bulgaria, Romania, Ukraine, and Russia, to the demand countries of Tunisia, Egypt, and Ethiopia in North and Sub-Saharan Africa, plus purchases by the World Food Program for food aid operations in Sub-Saharan Africa. Two examples capture the supply, demand, trade, and price patterns of wheat prior to and after Russia’s full-scale invasion of Ukraine. One example then examines a scenario in which the full-scale invasion stops. Two examples assess the impacts of increased and decreased levels of fixed demand, i.e., higher and lower levels of strategic grain reserves carried. An additional multicommodity example examines the interactions between wheat and corn trade flows and prices. The results highlight that

Russia’s share of the wheat market in North and Sub-Saharan Africa has increased after the full-scale invasion, while damaging the agricultural sectors in other supply countries of the Black Sea region, particularly in the case of Ukraine, which has had to cope with the direct economic repercussions of Russia’s full-scale invasion for almost five years now. Specifically, Egypt is shown to be heavily reliant on imports of wheat from Russia, which poses food insecurity risks in the event of trade disruptions. The results also reveal that if the Russo-Ukrainian war ends, trade flows would improve and consumer prices would decrease, but Russia’s leading position in the market would remain intact, and the impacts in terms of increased supply prices would be negligible. It is further shown that WFP’s purchases remain below pre-war levels, indicating that the organization’s food aid operations are currently more affected by its purchasing power (that is, funding cuts). The solutions also highlight that carrying higher levels of strategic grain reserves can adversely impact demand prices in the demand countries, but the effect on supply prices would be positive due to increased trade. Conversely, decreased fixed demands would result in lower demand prices and higher wheat affordability; however, this may come at the cost of long-term food security if disruptions occur.

Many extensions are possible and interesting for future research. The model in this paper assumes a common currency, i.e., USD, and it would be worthwhile to incorporate exchange rates to assess the impacts of currency appreciation and depreciation on trade flows and supply and demand prices. Furthermore, accounting for the stochasticity of demand could allow for the examination of strategic grain reserve levels under uncertain market demand and humanitarian needs. Finally, empirical work involving substitutable agricultural commodities, such as wheat, corn, and rice, could incorporate cross-commodity effects in the demand price functions to examine changes in commodity demand following supply or trade disruptions.

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