Capacitated Human Migration Networks and Subsidization

Anna Nagurney
Department of Operations and Information Management
Isenberg School of Management
University of Massachusetts
Amherst, Massachusetts 01003
and
Patrizia Daniele and Giorgia Cappello
Department of Mathematics and Computer Science
University of Catania, Italy

February 2020; revised June 2020

In: *Dynamics of Disasters - Impact, Risk, Resilience, and Solutions,*
I.S. Kotsireas, A. Nagurney, P.M. Pardalos, and Arsenios Tsokas, Editors
Springer Nature Switzerland AG, 2021, pp 195-217.

Abstract: Large-scale migration flows are posing immense challenges for governments around the globe, with drivers ranging from climate change and disasters to wars, violence, and poverty. In this paper, we introduce multiclass human migration models under user-optimizing and system-optimizing behavior in which the locations associated with migration are subject to capacities. We construct alternative variational inequality formulations of the governing equilibrium/optimality conditions that utilize Lagrange multipliers and then derive formulae for subsidies that, when applied, guarantee that migrants will locate themselves, acting independently and selfishly, in a manner that is also optimal from a societal perspective. An algorithm is proposed, implemented, and utilized to compute solutions to numerical examples. Our framework can be applied by governmental authorities to manage migration flows and population distributions for enhanced societal welfare.

Keywords: human migration networks, variational inequalities, system-optimization, user-optimization, capacities, subsidies, societal welfare

1. Introduction

Governments of many nations are increasingly being faced with large-scale human migration flows not only within their national borders but also across their borders. The drivers of migratory flows are many, including: wars, conflicts, violence and strife, and poverty, as well as challenges and disruptions posed by climate change and disasters, both sudden (earthquakes, wildfires, hurricanes, tornadoes, floods, tsunamis, landslides, etc.) and slow-onset (malnutrition and hunger, drought, disease epidemics, insect infestations, etc.). Migrants from time immemorial have sought a better quality of life for themselves, moving to locations to better their situations. The UNHCR (2020) reports that 70.8 million humans have fled their homes worldwide, the highest level of displacement ever recorded. According to the United Nations (2017), since the new millennium, the number of refugees and asylum seekers has increased from 16 to 26 million, comprising about 10% of total of the international migrants. The International Organization for Migration (2019) reports that there have been significant migration and displacement events during the last two years with such events resulting in hardship, trauma, and loss of life.

Many recent crises associated with migration have brought enhanced emphasis by both practitioners as well as academics on how to better address the associated challenges of migratory flows and the ultimate location of the migrants. Examples of epicenters of only a few of the migratory crises include: Venezuela (Kennedy (2019)), Central America (Bartenstein and McDonald (2019)), Libya (Sakuma (2020)), and Syria (United Nations Refugee Agency (2015)), with countries such as Mexico (Mattiace (2019)), Italy (Jones (2018)), Greece (Kitsantonis (2019)), and Cyprus (Stevis-Gridneff (2020)) serving as transit points for many refugees and asylum seekers in the dynamically evolving migration landscape (see also Papadaki et al. (2018)).

In particular, in many reports and studies, the capacity of nations to handle migrants, and we emphasize here that there are multiple classes of migrants (cf. Karagiannis (2016)), has risen to the fore as a critical characteristic. Examples of such studies have included even the United States in terms of migrants from Central America (O'Connor, Batalova, and Bolter (2019)); Colombia and other countries (Costa Rica and Ecuador) because of the issues in Venezuela and Nicaragua (Chinchilla et al. (2018)), as well as multiple countries in Europe as possible destination locations of migrants (Parkinson (2015) and European Commission (2019)).

In this paper, we develop user-optimized (U-O) and system-optimized (S-O) multiclass models of human migration under capacities associated with the migrant classes and loca-

tions. Our work builds on that of Nagurney, Daniele, and Cappello (2020), but with the generalization of the inclusion of capacities. Such a generalization is especially timely, as noted above. Moreover, to-date, the majority of research on human migration networks, from an operations research and mathematical modeling perspective, has focused on the modeling of migration flows assuming user-optimizing behavior, originating with the work of Nagurney (1989). In other words, it has been assumed that the migrants act selfishly and independently; see also Nagurney (1990), Nagurney, Pan, and Zhao (1992a, b), Pan and Nagurney (1994, 2006), Isac, Bulavsky, and Kalashnikov (2002), Kalashnikov et al. (2008), Causa, Jadamba, and Raciti (2017), Nagurney and Daniele (2020), Nagurney, Daniele, and Nagurney (2019), Capello and Daniele (2019), for a spectrum of U-O migration models. Davis et al. (2013), in turn, utilize a complex network approach for human migration and utilize an international dataset for their quantitative analysis.

System-optimization in multiclass human migration networks is also important since governments may wish to maximize societal welfare and hope that migrants locate accordingly. However, the latter may be extremely challenging unless proper policies/incentives are put into place. Indeed, Altemeyer-Bartscher et al. (2016) have argued for an effective cost-efficient mechanism for the distribution of refugees in the European Union, for example. Clearly, that would require some form of central control and cooperation/coordination.

Note that there are analogues to U-O and S-O network models, with a long history, in the transportation science literature (cf. Wardrop (1952), Beckmann, McGuire, and Winsten (1956), Dafermos and Sparrow (1969), and Boyce, Mahmassani, and Nagurney (2005)). Such concepts were made explicit, for the first time, in human migration networks, by Nagurney, Daniele, and Cappello (2020). We emphasize that in the transportation science literature the concern is total cost minimization in the case of system-optimization and individual cost minimization in the case of user-optimization, along with route selection, subject to the conservation of flow equations. In the human migration network context, in contrast, we are concerned with total utility maximization in the case of S-O and individual utility maximization in the case of U-O behavior and the selection of locations.

In addition, in this paper, we provide a quantitative mechanism, in the form of subsidies, that, when applied, guarantees that the system-optimized solution of our multiclass capacitated human migration network problem is also user-optimized. This is very important, since it enables governments, and policy-making bodies, to achieve optimal societal welfare in terms of the location of the migrants in the network economy, while the migrants locate independently in a U-O manner! Our work extends that of Nagurney, Daniele, and Capello (2020) to the capacitated network economy domain. Furthermore, we provide alternative

variational inequality formulations of both the new U-O and S-O models, which include Lagrange multipliers associated with the location capacity constraints as explicit variables. Their values at the equilibrium/optimal solutions provide valuable economic information for decision-makers.

This paper is organized as follows. In Section 2, we present the capacitated multiclass human migration network models, under S-O and under U-O behaviors. Associated with each location as perceived by a class, is an individual utility function, that, when multiplied by the population of that class at that location, yields the total utility function for that location and class. As in our earlier work (cf. Nagurney (1989), to start), the utility associated with a location and class can, in general, depend upon the vector of populations of all the classes at all the locations in the network economy. We assume a fixed population of each class in the network economy and are interested in determining the distributions of the populations among the locations under S-O and U-O behaviors. For each model, we provide alternative variational inequality formulations. We also illuminate the role that is played by the Lagrange multipliers associated with the class capacities on the locations in the network economy.

In Section 3, we outline the procedure for the calculation of the multiclass subsidies in order to guarantee, even in the capacitated case, that the system-optimized solution is, simultaneously, also user-optimized. Hence, once the subsidies are applied, the migrants will locate themselves individually in the network economy in a manner that is optimal from a societal perspective. As argued in Nagurney, Daniele, and Cappello (2020), there are analogues of our subsidies to tolls in transportation science. In the case of congested transportation networks, the imposition of tolls (see Dafermos and Sparrow (1969), Dafermos and Sparrow (1971), Dafermos (1973), Lawphongpanich, Hearn, and Smith (2006)), results in system-optimized flows also being user-optimized. In other words, once the tolls are imposed, travelers, acting independently, select routes of travel which result in a system optimum, that minimizes the total cost to the society. In this paper, we construct policies for human migration networks that maximize societal welfare but in the case of capacities.

In Section 4, we outline the computational algorithm, which we then apply to compute solutions to numerical examples that illustrate the theoretical results in this paper in a practical format. We summarize our results and present our conclusions in Section 5.

2. The Capacitated Multiclass Human Migration Network Models

In this Section, we construct the capacitated multiclass network models of human migration. We first present the system-optimized model and then the user-optimized one. The notation follows that in Nagurney, Daniele, and Cappello (2020), where, as mentioned in the Introduction, no capacities on the populations at the locations were imposed.

We assume that the human migrants have no movement costs associated with migrating from location to location since we are concerned with the long-term population distribution behaviors under both principles of system-optimization and user-optimization. The network representation of the models is given in Figure 1.

There are J classes of migrants, with a typical class denoted by k, and n locations corresponding to locations that the multiclass populations can migrate to, with a typical location denoted by i. There are assumed to be no births and no deaths in the network economy.

In the network representation, locations are associated with links. A link can correspond to a country or a region within a country and the network economy can capture multiple countries. If a government is interested in within country migration, exclusively, then the network economy (network) would correspond to that country.

Table 1 contains the notation for the models. All vectors here are assumed to be column vectors.

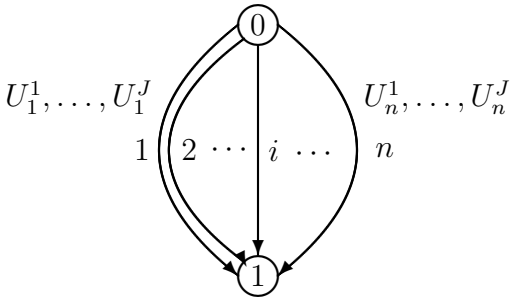

Figure 1: The Network Structure of the Multiclass Human Migration Models

According to Table 1, there is a utility function U_i^k associated with each class k; $k = 1, \ldots, J$, and location i; $i = 1, \ldots, n$, which captures how attractive location i is for that class k. Observe that (see Table 1), the utility, and, hence, the total utility, $\hat{U}_i^k$, associated with location i and class k, may, in general, depend upon the population distribution of all the classes at all the locations. The OECD (2019), for example, recognizes that different locations may be more or less attractive to distinct classes of migrants.

Table 1: Notation for the Multiclass Human Migration Models

Notation	Definition
p_i^k	the population of class k at location i. We group the $\{p_i^k\}$ elements into the vector $p^k \in R_+^n$. We then further group the p^k vectors; $k = 1, \ldots, J$, into the vector $p \in R_+^{Jn}$.
cap_i^k	the nonnegative capacity at location i for class k; $k = 1, \ldots, J; i = 1, \ldots, n$.
β_i^k	the Lagrange multiplier associated with capacity constraint for k at i; $k = 1, \ldots, J; i = 1, \ldots, n$. We group all these Lagrange multipliers into the vector $\beta \in R_+^{Jn}$.
P^k	the fixed population of class k in the network economy; $k = 1, \ldots, J$.
$U_i^k(p)$	the utility of individuals of class k at location i; $i = 1, \ldots, n$. We group the utility functions for each k into the vector $U^k \in R^n$ and then group all such vectors for all k into the vector $U \in R^{Jn}$.
$\hat{U}_i^k(p)$	the total utility of class k at location i; $i = 1, \ldots, n$. The total utility $\hat{U}_i^k(p) = U_i^k(p) \times p_i^k$; $k = 1, \ldots, J; i = 1, \ldots, n$.

We now present the constraints. The population distribution of each class among the various locations must sum up to the population of that class in the network economy, that is, for each class k; $k = 1, \ldots, J$:

$$\sum_{i=1}^{n} p_i^k = P^k. \tag{1}$$

Furthermore, the population of each class at each location must be nonnegative, that is,

$$p_i^k \geq 0, \quad \forall i; \forall k, \tag{2}$$

and not exceed the capacity:

$$p_i^k \leq cap_i^k, \quad \forall i; \forall k. \tag{3}$$

The feasible set $K^1 \equiv \{p |$ (1), (2), (3) hold$\}$.

We assume here that

$$\sum_{i=1}^{n} cap_i^k \geq P^k, \tag{4}$$

for all classes k. In other words, we assume that the network economy has sufficient capacity to accommodate the population of each class. Hence, the feasible set K^1 is nonempty. Moreover, it is compact.

2.1 The Capacitated System-Optimization (S-O) Problem

The government (or governments), in the case of system optimization, wishes to maximize the total utility in the network economy, which reflects the societal welfare, subject to the constraints. The capacitated system-optimization (S-O) problem is:

$$\text{Maximize} \quad \sum_{k=1}^{J}\sum_{i=1}^{n} \hat{U}_i^k(p) = \sum_{k=1}^{J}\sum_{i=1}^{n} U_i^k(p) \times p_i^k \tag{5}$$

subject to constraints (1) through (3).

We assume that the total utility functions for all the classes at all the locations are concave and continuously differentiable. Then, from classical results (cf. Kinderlehrer and Stampacchia (1980) and Nagurney (1999)), we know that the optimal solution, denoted by p', satisfies the variational inequality (VI) problem: determine $p' \in K^1$, such that

$$-\sum_{k=1}^{J}\sum_{i=1}^{n}\left[\sum_{l=1}^{J}\sum_{j=1}^{n}\frac{\partial \hat{U}_j^l(p')}{\partial p_i^k}\right] \times (p_i^k - p_i^{k'}) \geq 0, \quad \forall p \in K^1. \tag{6}$$

A solution p' to VI (6) is guaranteed to exist under our imposed assumptions since the feasible set K^1 is compact and the total utility functions are continuously differentiable. Uniqueness of the solution p' then follows under the assumption that all the utility functions are strictly concave.

We now present an alternative variational inequality to the one in (6), which we utilize to compute the S-O solution in numerical examples. Furthermore, the solution of the alternative VI allows us to determine the optimal Lagrange multipliers associated with the location class capacities in the S-O context. The Lagrange multipliers at the optimal solution provide valuable economic information. We define the feasible set $K^2 \equiv \{(p,\beta)|(1),(2)\,\text{hold and}\,\beta \in R_+^{Jn}\}$.

Alternative Variational Inequality Formulation of the Capacitated S-O Problem

A solution to the S-O problem also satisfies the VI: determine $(p',\beta') \in K^2$ such that

$$-\sum_{k=1}^{J}\sum_{i=1}^{n}\left[\sum_{l=1}^{J}\sum_{j=1}^{n}\frac{\partial \hat{U}_j^l(p')}{\partial p_i^k} - \beta_i^{k'}\right] \times (p_i^k - p_i^{k'}) + \sum_{k=1}^{J}\sum_{i=1}^{n}\left[cap_i^k - p_i^{k'}\right] \times (\beta_i^k - \beta_i^{k'}) \geq 0,$$
$$\forall (p,\beta) \in K^2. \tag{7}$$

The above result follows from Bertsekas and Tsitsiklis (1989), page 287. Capacities have also been applied to links in various supply chain system-optimization problems and varia-

tional inequality formulations constructed; see, for example, Nagurney (2010) and Nagurney, Yu, and Qiang (2011).

2.2 The Capacitated User-Optimization (U-O) Problem

We now introduce the capacitated user-optimized version of the above S-O model. The new model extends the classical one introduced in Nagurney (1989) to include capacities.

The Capacitated Equilibrium Conditions

Mathematically, a multiclass population vector $p^* \in K^1$ is said to be U-O or, equivalently, a capacitated equilibrium, if for each class k; $k = 1, \ldots, J$; and all locations $i; i = 1, \ldots, n$:

$$U_i^k(p^*) \begin{cases} \geq \lambda^k, & \text{if} \quad p_i^{k*} = cap_i^k, \\ = \lambda^k, & \text{if} \quad 0 < p_i^{k*} < cap_i^k, \\ \leq \lambda^k, & \text{if} \quad p_i^{k*} = 0. \end{cases} \tag{8}$$

From (8) one can see that locations with no population of a class are those with the lowest utilities; those locations with a positive population of a class, with the population not at the capacity for the location and class will have equalized utility for that class and higher than the unpopulated locations of that class. Moreover, the equalized utility will be equal to an indicator λ^k. The indicator λ^k is, actually, the Lagrange multiplier associated with constraint (1) for k with the value at the equilibrium. Those locations with a class k at its capacity have a utility greater than or equal to λ^k.

A capacitated U-O solution p^* satisfies the VI: determine $p^* \in K^1$ such that

$$\sum_{k=1}^{J} \sum_{i=1}^{n} -U_i^k(p^*) \times (p_i^k - p_i^{k*}) \geq 0, \quad \forall p \in K^1. \tag{9}$$

We now prove the equivalence of the solution to the Capacitated Equilibrium Conditions (8) and the VI (9).

Indeed, it is easy to see that, according to (8), for a fixed k and i, the equilibrium conditions imply that

$$\left[\lambda^k - U_i^k(p^*)\right] \times \left[p_i^k - p_i^{k*}\right] \geq 0, \quad \forall p_i^k : 0 \leq p_i^k \leq cap_i^k. \tag{10}$$

Observe that, if $p_i^{k*} = 0$, (10) holds true; if $p_i^{k*} = cap_i^k$, then (10) also holds, and (10) also holds if $0 < p_i^{k*} < cap_i^k$.

Summing now (10) over all k and all i, yields:

$$\sum_{k=1}^{J}\sum_{i=1}^{n}\left[\lambda^k - U_i^k(p^*)\right] \times \left[p_i^k - p_i^{k*}\right] \geq 0, \quad \forall p \in K^1. \tag{11}$$

But, because of (1), (11) simplifies to precisely (9).

Furthermore, we now show that if p^* satisfies VI (9), then the p^* also satisfies the Capacitated Equilibrium Conditions (8).

In (9), we set $p_i^l = p_i^{l*}$ for all $l \neq k$, which yields:

$$\sum_{i=1}^{n} -U_i^k(p^*) \times (p_i^k - p_i^{k*}) \geq 0, \quad \forall p_i^k : 0 \leq p_i^k \leq cap_i^k; \quad \sum_{i=1}^{n} p_i^k = P^k. \tag{12}$$

If there are two locations, say, r and s with positive populations not at their capacities, set for a sufficiently small $\epsilon > 0$:

$$p_r^k = p_s^{k*} - \epsilon; \quad p_s^k = p_r^{k*} + \epsilon$$

and all other p_i^ks equal to p_i^{k*}. Clearly, such a population distribution is also feasible. Substitution into (12) yields, after algebraic simplification:

$$(-U_r^k(p^*) + U_s^k(p^*)) \times (p_s^{k*} - p_r^{k*} - \epsilon) \geq 0. \tag{13}$$

Similarly, by constructing another feasible population pattern:

$$p_r^k = p_s^{k*} + \epsilon, \quad p_s^k = p_r^{k*} - \epsilon,$$

with all other $p_i^k = p_i^{k*}$, and substitution into (12) yields

$$(U_r^k(p^*) - U_s^k(p^*)) \times (p_s^{k*} - p_r^{k*} - \epsilon) \geq 0. \tag{14}$$

(13) and (14) can only hold true if

$$U_r^k(p^*) = U_s^k(p^*)$$

which we call λ^k. Hence, the second condition in (8) has been established.

On the other hand, suppose that $p_i^{k*} \geq 0$ for all i, but $p_r^{k*} > 0$ and $p_s^{k*} = 0$. For a sufficiently small $\epsilon > 0$, construct $p_r^k = p_r^{k*} - \epsilon$ and $p_s^k = p_s^{k*} + \epsilon$, with all other p_i^ks equal to p_i^{k*} and substitute these values into (12). After, algebraic simplification, we obtain:

$$(U_r^k(p^*) - U_s^k(p^*))\epsilon \geq 0,$$

hence,

$$U_r^k(p^*) \geq U_s^k(p^*)$$

and the third condition in (8) is verified.

Now, in order to verify that a solution to VI (9) also satisfies the top condition in (8), if for some location r: $p_r^{k*} = cap_r^k$, then we construct a feasible distribution pattern such that:

$$p_r^k = p_r^{k*} - \epsilon, \quad p_s^k = p_s^{k*} + \epsilon,$$

with $\epsilon > 0$ sufficiently small and all other $p_i^k = p_i^{k*}$. Substitution into (12), after algebraic simplification yields:

$$U_r^k(p^*) \geq U_s^k(p^*)$$

and the conclusion follows. With the above arguments, we have shown that a capacitated equilibrium p^* is equivalent to the solution of the VI (9).

We now provide an alternative VI formulation of the capacitated equilibrium conditions. This result is immediate by making note of Nagurney (1989) demonstrating that the U-O human migration model (without capacities) is isomorphic to a traffic network equilibrium problem (cf. Dafermos and Sparrow (1969) and Dafermos (1980)) and, hence, in the case of capacities, also isomorphic to a traffic network equilibrium problem with side constraints (see Larsson and Patriksson (1999)) and with special structure.

Alternative Variational Inequality Formulation of the U-O Problem

The U-O solution satisfies the variational inequality problem: determine $(p^*, \beta^*) \in K^2$ such that

$$\sum_{k=1}^{J} \sum_{i=1}^{n} \left[-U_i^k(p^*) + \beta_i^{k*}\right] \times (p_i^k - p_i^{k*}\rangle + \sum_{k=1}^{J} \sum_{i=1}^{n} \left[cap_i^k - p_i^{k*}\right] \times (\beta_i^k - \beta_i^{k*}) \geq 0, \quad \forall (p, \beta) \in K^2. \tag{15}$$

2.3 Illustrative Examples

We first present an uncapacitated example for which we provide U-O and S-O solutions. We then add capacities to the locations and report the new U-O and S-O solutions. There is a single class in the network economy and three locations. The total population is: $P^1 = 120$ and the utility functions at the three locations are:

$$U_1^1(p) = -p_1^1 + 190, \quad U_2^1(p) = -p_2^1 + 200, \quad U_3^1(p) = -p_3^1 + 210.$$

The user-optimized solution is:

$$p_1^{1*} = 30.00, \quad p_2^{1*} = 40.00, \quad p_3^{1*} = 50.00,$$

yielding $\lambda^1 = 160$, since

$$U_1^1(p^*) = U_2^1(p^*) = U_3^1(p^*) = 160.00.$$

The S-O solution, on the other hand, is:

$$p_1^{1'} = 35.00, \quad p_2^{1'} = 40.00, \quad p_3^{1'} = 45.00.$$

We now impose capacities as follows:

$$cap_1^1 = 60.00, \quad cap_2^1 = 60.00, \quad cap_3^1 = 30.00,$$

and solve for the U-O and S-O solutions.

The new U-O solution, satisfying VI (15), is:

$$p_1^{1*} = 40.00, \quad p_2^{1*} = 50.00, \quad p_3^{1*} = 30.00,$$

with Lagrange multipliers associated with the capacities of:

$$\beta_1^{1*} = 0.00, \quad \beta_2^{1*} = 0.00, \quad \beta_3^{1*} = 30.00.$$

The new S-O solution, satisfying VI (7), is:

$$p_1^{1'} = 42.50, \quad p_2^{1'} = 47.50, \quad p_3^{1'} = 30.00,$$

with Lagrange multipliers associated with the capacities of:

$$\beta_1^{1'} = 0.00, \quad \beta_2^{1'} = 0.00, \quad \beta_3^{1'} = 45.00.$$

Observe that the S-O solution is distinct from the U-O solution in both the uncapacitated and the capacitated versions.

Remark

We now show how the optimal Lagrange multipliers can be utilized. For example, if one modifies the utility functions by reducing each of them by the value of the optimal Lagrange multiplier associated with the location and the class then the same user-optimizing solution

is obtained as the one for the problem with the corresponding capacities. Indeed, proceeding as above, we modify the utility functions as:

$$\tilde{U}_1^1(p) = -p_1^1 + 190 - 0 = -p_1^1 + 190,$$

$$\tilde{U}_2^1(p) = -p_2^1 + 200 - 0 = -p_2^1 + 200,$$

$$\tilde{U}_3^1(p) = -p_3^1 + 210 - 30 = -p_3^1 + 180,$$

and observe that the capacitated U-O solution: $p_1^{1*} = 40.00$, $p_2^{1*} = 50.00$, $p_3^{1*} = 30.00$ remains optimal.

Similarly, one can modify the utility functions in the same manner, but by using the optimal Lagrange multipliers for the S-O problem, to obtain the same S-O solution as for the problem with the capacities.

Hence, government decision-makers, in order to limit the population of certain (or all) classes at certain (or all) locations can accomplish this through regulations corresponding to the capacities or by modifying the utility functions accordingly to yield the same result.

Now, we describe how subsidies (which may be viewed as a positive intervention) can, once imposed, make the capacitated S-O solution also a capacitated U-O one.

3. Subsidies to Guarantee the Capacitated S-O Solution is Also a Capacitated U-O Solution

In Nagurney, Daniele, and Cappello (2020) a procedure was introduced for the calculation of subsidies that, once applied to the locations with a positive population of a class under S-O, guaranteed that migrants operating under the U-O behavioral principle would select locations that were also optimal from a societal standpoint; that is, they were system-optimized.

Here we show that the same general construct is also applicable to capacitated problems of human migration.

The procedure is as follows. We first solve for the capacitated system-optimized solution p' satisfying VI (7), or, equivalently, VI (6). For each class k, we denote those locations with a positive population by $k_1, \ldots, k_{n_k}$, where n_k is the number of locations in the network economy with a positive population of class k. We also introduce notation for subsidies associated with the different locations for each class denoted by class k by: $(subsidy)_{k_1}$, $(subsidy)_{k_2}$, ..., $(subsidy)_{k_{n_k}}$. We then enumerate those location in a list as follows:

$$U_{k_1}^k(p') + subsidy_{k_1}^k = \mu^k,$$

$$U^k_{k_2}(p') + subsidy^k_{k_2} = \mu^k, \tag{16}$$

and so on until

$$U^k_{k_{n_k}}(p') + subsidy^k_{k_{n_k}} = \mu^k.$$

Note that μ^k is the incurred utility for class k after the subsidies are distributed for the class at the locations with positive populations of that class. Also, we can number those locations for that class with zero populations of that class (if there are any) as follows:

$$U^k_{k_{n_{k+1}}}(p') + subsidy^k_{k_{n_k+1}} \leq \mu^k,$$

and so on until

$$U^k_{k_n}(p') + subsidy^k_{k_n} \leq \mu^k. \tag{17}$$

Expressions (16) and (17) reveal that the appropriate governmental authority chooses the μ^k for each class k, and then the subsidy for each location for that class is determined by straightforward subtraction.

In order to select an appropriate μ^k, as noted in Nagurney, Daniele, and Cappello (2020) for the uncapacitated case, the μ^ks can be set as: $\max_{k_l;l=1,\ldots,n_k} U^k_{k_l}(p')$. All thus calculated are nonnegative and, furthermore, all migrants enjoy the maximal utility for each class at all the populated locations. Also, for the subsidies associated with locations with no populations of a class k (see (17)), we set those subsidies zero.

Returning to the above simple example, we note that $\mu^1 = 180.00$, and the above subsidy formulae simplify to:

$$U^1_1(p') + subsidy^1_1 = \mu^1,$$
$$U^1_2(p') + subsidy^1_2 = \mu^1,$$
$$U^1_3(p') + subsidy^1_3 = \mu^1,$$

or

$$147.50 + subsidy^1_1 = 180.00,$$
$$152.50 + subsidy^1_2 = 180.00,$$
$$180.00 + subsidy^1_3 = 180.00,$$

which yields:

$$subsidy^1_1 = 32.50, \quad subsidy^1_2 = 27.50, \quad subsidy^1_3 = 0.00.$$

Observe that an application of the above subsidies modifies the utility functions as follows:

$$\tilde{U}^1_1(p) = -p^1_1 + 190 + 32.50, \quad \tilde{U}^1_2(p) = -p^1_2 + 200 + 27.50, \quad \tilde{U}^1_3(p) = -p^1_3 + 210 + 0.$$

Clearly, the S-O solution

$$p_1^{1'} = 42.50, \quad p_2^{1'} = 47.50, \quad p_3^{1'} = 30.00,$$

is at the same time U-O, since the utilities are equalized (and maximal) under this S-O pattern and, hence, migrants will select locations, although acting selfishly and individually, accordingly, because of the subsidies.

The above subsidies may be viewed as investments by government(s). As for the budgets, if an individual government experiences a budgetary shortfall, additional financing may be provided by a supra authority such as the World Bank, the United Nations, or if in Europe, the European Union. Altemeyer-Bartscher et al. (2016) have argued for closer cooperation among countries regarding migration crises and also advocated for an economic approach as to distribution of the migrants. Here, we provide a quantitative approach with explicit formulae for implementation.

As noted earlier, climate change as well as disasters may act as drivers of human migrations. Robinson, Dilkina, and Moreno-Cruz (2020), for example, provide a machine learning approach to migration in the United States under sea level rise but emphasize that their approach is not yet ready for policy making. They, as Bier, Zhou, and Du (2019), consider sea level rise due to climate change, and migration within a country - the United States. The latter authors observe that offering a subsidy (e.g., a partial buyout) can be effective if the government has a significantly lower discount rate than residents. However, they assume homogeneous residents, whereas we consider multiclass ones and we also allow for multiple countries and not just regions within a country. For edited volumes on dynamics of disasters, see Kotsireas, Nagurney, and Pardalos (2016, 2018). Once a disaster or disasters strike, one would modify the fixed populations of the various classes in the economy, as need be, along with the utility functions and rerun the model(s), along with the subsidies. In the case of disasters, we can expect that populations will decrease and so would utility functions associated with locations that have been negatively impacted.

4. The Algorithm and Numerical Examples

We apply the Euler method of Dupuis and Nagurney (1993) for the solution of the capacitated network models of human migration. As discussed therein (see also Nagurney and Zhang (1996)), the Euler method is induced by a general iterative scheme, and was inspired by the theory of projected dynamical systems, whose set of stationary points coincides with the set of solutions to an appropriate variational inequality problem. The Euler method, in fact, can be viewed as a time-discretization of the underlying continuous time trajectories of

the projected dynamical system until a solution is achieved. It has been applied to numerous network problems, including supply chain ones (see Nagurney (2006)).

4.1 The Algorithm

For purposes of standardizing the mechanism, we utilize similar notation to that in Nagurney, Daniele, and Cappello (2020) and put variational inequality (7) into standard form (Nagurney (1999)): determine $X^{**} \in \mathcal{K} \subset R^N$ such that:

$$\langle F(X^{**}), X - X^{**} \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{18}$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in N-dimensional Euclidean space. $F(X)$ is a given continuous function such that $F(X) : X \rightarrow \mathcal{K} \subset R^N$. $\mathcal{K}$ is a closed and convex set.

We define the vector $X \equiv (p, \beta)$ and the vector $F(X)$ with elements: $F^1_{k,i}(p, \beta) \equiv \sum_{l=1}^{J} \sum_{j=1}^{n} - \frac{\partial \hat{U}_j^l(p)}{\partial p_i^k}$ and $F^2_{k,i}(p, \beta) \equiv cap_i^k - p_i^k$; $k = 1, \ldots, J$; $i = 1, \ldots, n$. We define the feasible set as $\mathcal{K} \equiv K^2$ and $N = 2Jn$. Thus, VI (7) can be put into the standard form (18) with $X^{**} = (p', \beta')$. Similarly, VI (15) can also be put into standard form with X and $\mathcal{K}$ as above and with the components of its $F(X)$ given by $-U_i^k(p, \beta)$, $cap_i^k - \beta_i^k$; $\forall k$, $\forall i$, and with $X^{**} = (p^*, \beta^*)$.

At iteration τ, the statement of the Euler method is:

$$X^{\tau+1} = P_{\mathcal{K}}(X^\tau - a_\tau F(X^\tau)), \tag{19}$$

where $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$ and F is the function that enters the variational inequality problem (18).

Dupuis and Nagurney (1993) proved that, for convergence of the general iterative scheme, which induces the Euler method, the sequence $\{a_\tau\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \rightarrow 0$, as $\tau \rightarrow \infty$. Specific conditions for convergence of the Euler method within many network-based models can be found in Nagurney and Zhang (1996) and in Nagurney (2006) and the references therein.

The Euler method nicely exploits the special network structure of the models as depicted in Figure 1 and allows for closed form expressions at each iteration for the computation of the Lagrange multipliers associated with the capacity constraints. We solve the network subproblems of special structure, which are separable quadratic programming problems, using the exact equilibration algorithm (cf. Dafermos and Sparrow (1969) and Nagurney (1999)). This algorithm yields the exact solution at each iteration for the populations.

4.2 Numerical Examples

The algorithm was implemented in FORTRAN and a Unix system at the University of Massachusetts Amherst used for the computations. The series $\{a_\tau\}$ in the algorithm was set to: $1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \ldots$ with the convergence tolerance ϵ equal to 10^{-5}. In other words, the algorithm was considered to have converged when the absolute value of each of the computed population values for each class at two successive iterations was less than or equal to .00001.

For continuity, and cross comparison purposes, the data for the uncapacitated examples was taken from Nagurney, Daniele, and Cappello (2020) and to these we added capacities. For completeness, we report both the uncapacitated (solved in the paper above) and the capacitated versions, reported for the first time here.

In our numerical examples, the network economy consists of two classes of migrants and five locations.

Utility Function and Fixed Population Data

The fixed populations in the network economy of the two classes are, respectively:

$$P^1 = 1,000.00 \quad P^2 = 2,000.00.$$

The utility functions and the total utility functions for class 1 are:

$$U_1^1(p) = -2p_1^1 - .2p_1^2 + 2,000, \quad \hat{U}_1^1(p) = -2(p_1^1)^2 - .2p_1^2p_1^1 + 2,000p_1^1,$$

$$U_2^1(p) = -3p_2^1 - .1p_2^2 + 1,500, \quad \hat{U}_2^1(p) = -3(p_2^1)^2 - .1p_2^2p_2^1 + 1,500p_2^1,$$

$$U_3^1(p) = -p_3^1 - .3p_3^2 + 3,000, \quad \hat{U}_3^1(p) = -(p_3^1)^2 - .3p_3^2p_3^1 + 3,000p_3^1,$$

$$U_4^1(p) = -p_4^1 - .2p_4^2 + 2,500, \quad \hat{U}_4^1(p) = -(p_4^1)^2 - .2p_4^2p_4^1 + 2,500p_4^1,$$

$$U_5^1(p) = -2p_5^1 - .3p_5^2 + 4,000, \quad \hat{U}_5^1(p) = -2(p_5^1)^2 - .3p_5^2p_5^1 + 4,000p_5^1.$$

The utility functions and the total utility functions for class 2 are:

$$U_1^2(p) = -p_1^2 - .4p_1^1 + 4,000, \quad \hat{U}_1^2(p) = -(p_1^2)^2 - .4p_1^1p_1^2 + 4,000p_1^2,$$

$$U_2^2(p) = -2p_2^2 - .6p_2^1 + 3,000, \quad \hat{U}_2^2(p) = -2(p_2^2)^2 - .6p_2^1p_2^2 + 3,000p_2^2,$$

$$U_3^2(p) = -p_3^2 - .2p_3^1 + 5,000, \quad \hat{U}_3^2(p) = -(p_3^2)^2 - .2p_3^1p_3^2 + 5,000p_3^2,$$

$$U_4^2(p) = -2p_4^2 - .3p_4^1 + 4,000, \quad \hat{U}_4^2(p) = -2(p_4^2)^2 - .3p_4^1p_4^2 + 4,000p_4^2,$$

$$U_5^2(p) = -p_5^2 - .4p_5^1 + 3,000, \quad \hat{U}_5^2(p) = -(p_5^2)^2 - .4p_5^1p_5^2 + 3,000p_5^2.$$

We first recall the uncapacitated U-O and S-O solutions obtained in Nagurney, Daniele, and Cappello (2020) and then report the capacitated solutions based on the new models constructed here. We also report the calculated subsidies in the more general capacitated case introduced in this paper. We provide two sets of examples.

4.2.1 Numerical Example Set 1

The uncapacitated U-O solution for the numerical example with the above data is:

Class 1 Uncapacitated U-O Population Distribution

$$p_1^{1*} = 0.00, \quad p_2^{1*} = 0.00, \quad p_3^{1*} = 167.31, \quad p_4^{1*} = 41.68, \quad p_5^{1*} = 791.01.$$

Class 2 Uncapacitated U-O Population Distribution

$$p_1^{2*} = 415.89, \quad p_2^{2*} = 0.00, \quad p_3^{2*} = 1,382.41, \quad p_4^{2*} = 201.69, \quad p_5^{2*} = 0.00.$$

The uncapacitated S-O solution is:

Class 1 Uncapacitated S-O Population Distribution

$$p_1^{1'} = 0.00, \quad p_2^{1'} = 0.00, \quad p_3^{1'} = 120.43, \quad p_4^{1'} = 314.39, \quad p_5^{1'} = 565.19.$$

Class 2 Uncapacitated S-O Population Distribution

$$p_1^{2'} = 606.48, \quad p_2^{2'} = 53.23, \quad p_3^{2'} = 1,076.35, \quad p_4^{2'} = 263.94, \quad p_5^{2'} = 0.00.$$

We now impose the following capacities on the locations for the classes in the above problem.

$$cap_1^1 = 500.00, \quad cap_2^1 = 500.00, \quad cap_3^1 = 500.00, \quad cap_4^1 = 500.00, \quad cap_5^1 = 200.00,$$

$$cap_1^1 = 500.00, \quad cap_2^1 = 500.00, \quad cap_3^1 = 400.00, \quad cap_4^1 = 500.00, \quad cap_5^1 = 500.00.$$

The capacitated U-O solution is:

Class 1 Capacitated U-O Population Distribution

$$p_1^{1*} = 0.00, \quad p_2^{1*} = 0.00, \quad p_3^{1*} = 500.00, \quad p_4^{1*} = 300.00, \quad p_5^{1*} = 200.00.$$

Class 2 Capacitated U-O Population Distribution

$$p_1^{2*} = 500.00, \quad p_2^{2*} = 226.67, \quad p_3^{2*} = 400.00, \quad p_4^{2*} = 500.00, \quad p_5^{2*} = 373.33.$$

The optimal Lagrange multipliers are:

Class 1 Capacitated U-O Lagrange Multipliers

$$\beta_1^{1*} = 0.00, \quad \beta_2^{1*} = 0.00, \quad \beta_3^{1*} = 280.00, \quad \beta_4^{1*} = 0.00, \quad \beta_5^{1*} = 1,388.01.$$

Class 2 Capacitated U-O Lagrange Multipliers

$$\beta_1^{2*} = 953.33, \quad \beta_2^{2*} = 0.00, \quad \beta_3^{2*} = 1,953.33, \quad \beta_4^{2*} = 363.33, \quad \beta_5^{2*} = 0.00.$$

One can see, from this example, that at all the locations with populations of a class at the capacity, there is an associated positive Lagrange multiplier. Also, it is clear that the capacitated U-O solution is quite distinct from the uncapacitated one. For example, all the locations have a positive population of class 2 under the capacitated solution. Moreover, in the uncapacitated case, location 5 is most attractive for class 1, whereas location 3 is most attractive for class 2. In contrast, in the capacitated case, location 3 is now most popular for class 1, whereas locations 1 and 4 are most popular (and at the capacities) for class 2.

The capacitated S-O solution is:

Class 1 Capacitated S-O Population Distribution

$$p_1^{1'} = 88.82, \quad p_2^{1'} = 0.00, \quad p_3^{1'} = 242.55, \quad p_4^{1'} = 468.63, \quad p_5^{1'} = 200.00.$$

Class 2 Capacitated S-O Population Distribution

$$p_1^{2'} = 500.00, \quad p_2^{2'} = 244.65, \quad p_3^{2'} = 400.00, \quad p_4^{2'} = 436.07, \quad p_5^{2'} = 419.29.$$

The optimal Lagrange multipliers are:

Class 1 Capacitated S-O Lagrange Multipliers

$$\beta_1^{1'} = 0.00, \quad \beta_2^{1'} = 0.00, \quad \beta_3^{1'} = 0.00, \quad \beta_4^{1'} = 0.00, \quad \beta_5^{1'} = 1,561.77.$$

Class 2 Capacitated S-O Lagrange Multipliers

$$\beta_1^{2'} = 925.30, \quad \beta_2^{2'} = 0.00, \quad \beta_3^{2'} = 2,057.33, \quad \beta_4^{2'} = 0.00, \quad \beta_5^{2'} = 0.00.$$

Under the uncapacitated S-O, location 5 is most attractive for class 1 and location 3 is for class 2. However, in the capacitated case, location 4 is best for class 1 and location 1 for class 2, with locations 3 through 5 also quite competitive.

We now report the calculated subsidies, which are obtained using the described procedure in Section 3. We note that $\mu^1 = 3,474.21$ and $\mu^2 = 4,551.50$ - these values represent the highest utility of each class at a location evaluated at the S-O solution, which are obtained for class 1 at location 5 and for class 2 at location 3. The calculated subsidies are:

Class 1 **Subsidies**

$$subsidy_1^1 = 1,751.85,\ subsidy_2^1 = 1,998.67,\ subsidy_3^1 = 836.76,\ subsidy_4^1 = 1,530.05,$$

$$subsidy_5^1 = 0.00.$$

Class 2 **Subsidies**

$$subsidy_1^2 = 1,087.03,\ subsidy_2^2 = 2,040.79,\ subsidy_3^2 = 0.00,\ subsidy_4^2 = 1,564.22,$$

$$subsidy_5^2 = 2,050.79.$$

4.2.2 Numerical Example Set 2

The data were as in the first numerical example set except now we considered a sizable decrease in the populations of each of the two classes due to a disaster. As argued in Nagurney, Daniele, and Cappello (2020), this could occur in the form of a pandemic, that is, a healthcare disaster hitting the network economy. We note that the novel coronavirus outbreak that originated in Wuhan, China (Shih, Denyer, and Taylor (2020)) was officially declared a pandemic by the World Health Organization on March 11, 2020 (cf. Branswell and Joseph (2020)). This coronavirus causes the disease known as Covid-19. The data in this example was as in Numerical Example 1, except that now we assumed that 50% of the population of each class has perished, that is,

$$P^1 = 500.00 \quad P^2 = 1,000.00.$$

The uncapacitated U-O solution for the numerical example with the above data is:

Class 1 **Uncapacitated U-O Population Distribution**

$$p_1^{1*} = 0.00, \quad p_2^{1*} = 0.00, \quad p_3^{1*} = 0.00, \quad p_4^{1*} = 0.00, \quad p_5^{1*} = 500.00.$$

Class 2 **Uncapacitated U-O Population Distribution**

$$p_1^{2*} = 0.00, \quad p_2^{2*} = 0.00, \quad p_3^{2*} = 1,000.00, \quad p_4^{2*} = 0.00, \quad p_5^{2*} = 0.00.$$

The uncapacitated computed S-O solution is:

Class 1 **S-O Uncapacitated Population Distribution**

$$p_1^{1'} = 0.00, \quad p_2^{1'} = 0.00, \quad p_3^{1'} = 47.98, \quad p_4^{1'} = 43.17, \quad p_5^{1'} = 408.85.$$

Class 2 **S-O Uncapacitated Population Distribution**

$$p_1^{2'} = 206.96, \quad p_2^{2'} = 0.00, \quad p_3^{2'} = 694.96, \quad p_4^{2'} = 98.08, \quad p_5^{2'} = 0.00.$$

As noted in Nagurney, Daniele, and Cappello (2020), in the S-O solution one sees a greater "spreading out" of the classes among the locations than in the U-O solution.

We kept the same capacities as in the first set. The Euler Method now yielded the following solution:

The capacitated U-O solution for the numerical example with the above data is:

Class 1 **Capacitated U-O Population Distribution**

$$p_1^{1*} = 0.00, \quad p_2^{1*} = 0.00, \quad p_3^{1*} = 300.00, \quad p_4^{1*} = 0.00, \quad p_5^{1*} = 200.00.$$

Class 2 **Capacitated U-O Population Distribution**

$$p_1^{2*} = 0.00, \quad p_2^{2*} = 400.00, \quad p_3^{2*} = 0.00, \quad p_4^{2*} = 400.00, \quad p_5^{2*} = 200.00.$$

The optimal Lagrange multipliers are:

Class 1 **Capacitated U-O Lagrange Multipliers**

$$\beta_1^{1*} = 0.00, \quad \beta_2^{1*} = 0.00, \quad \beta_3^{1*} = 0.00, \quad \beta_4^{1*} = 0.00, \quad \beta_5^{1*} = 1,020.00.$$

Class 2 **Capacitated U-O Lagrange Multipliers**

$$\beta_1^{2*} = 0.00, \quad \beta_2^{2*} = 0.00, \quad \beta_3^{2*} = 940.00, \quad \beta_4^{2*} = 0.00, \quad \beta_5^{2*} = 0.00.$$

The capacitated computed S-O solution is:

Class 1 **S-O Capacitated Population Distribution**

$$p_1^{1'} = 0.00, \quad p_2^{1'} = 0.00, \quad p_3^{1'} = 124.08, \quad p_4^{1'} = 175.91, \quad p_5^{1'} = 200.00.$$

Class 2 S-O Capacitated Population Distribution

$$p_1^{2'} = 414.66, \quad p_2^{2'} = 0.00, \quad p_3^{2'} = 400.00, \quad p_4^{2'} = 185.34, \quad p_5^{2'} = 0.00.$$

The optimal Lagrange multipliers are:

Class 1 Capacitated S-O Lagrange Multipliers

$$\beta_1^{1'} = 0.00, \quad \beta_2^{1'} = 0.00, \quad \beta_3^{1'} = 0.00, \quad \beta_4^{1'} = 0.00, \quad \beta_5^{1'} = 1,144.49.$$

Class 2 Capacitated S-O Lagrange Multipliers

$$\beta_1^{2'} = 0.00, \quad \beta_2^{2'} = 0.00, \quad \beta_3^{2'} = 967.27, \quad \beta_4^{2'} = 0.00, \quad \beta_5^{2'} = 0.00.$$

We now report the subsidies that, when imposed, guarantee that the capacitated S-O solution obtained above for the second numerical example is also U-O. Here we had that $\mu^1 = 3,599.99$ and $\mu^2 = 4,575.18$.

Class 1 Subsidies

$$subsidy_1^1 = 1,682.92, \; subsidy_2^1 = 2,099.99, \; subsidy_3^1 = 844.07, \; subsidy_4^1 = 1,312.97,$$

$$subsidy_5^1 = 0.00.$$

Class 2 Subsidies

$$subsidy_1^2 = 989.84, \; subsidy_2^2 = 1,575.18, \; subsidy_3^2 = 0.00, \; subsidy_4^2 = 998.63,$$

$$subsidy_5^2 = 1,655.18.$$

5. Summary and Conclusions

Problems of human migration are issues of global concern and are presenting immense challenges to governments around the world. Multiple countries are dealing with different classes of migratory flows and the ensuing difficulties when faced with capacities at locations under their jurisdictions. Rigorous, appropriate policies may help to better reallocate migrants across suitable locations.

Historically, many of the mathematical models of human migration have utilized a network formalism and have assumed user-optimizing behavior, that is, that migrants select locations, which are best for themselves, as revealed through utility functions that depend on the

population distributions among the locations of the different classes of migrants. However, such behavior may lead to costs to society and even reduced societal welfare.

Hence, in this paper, we build upon the recent work of Nagurney, Daniele, and Cappello (2020), who proposed both system-optimized and user-optimized multiclass migration network models, and demonstrated how incentives, in the form of subsidies, when applied, guarantee that the system-optimized solution, which maximizes the total utility in the network economy, becomes, at the same time, user-optimizing. Migrants, thus, under such subsidies, and acting selfishly and independently, would select locations to migrate to and locate at that are optimal from the system perspective.

In this paper, we propose a novel extension of that work, in the form of capacities at different locations associated with the classes of migrants. This brings a greater realism in capturing challenges faced by various governments who are dealing with refugees, asylum seekers, etc. For each U-O and S-O model we provide alternative variational inequality formulations of the governing equilibrium/optimality conditions. We then utilize the variational inequality formulations with Lagrange multipliers associated with the multiclass capacity constraints to gain deeper insights into appropriate policies. We show that the Lagrange multipliers can be utilized to modify the utility functions so that the capacities are made implicit. Moreover, we show how, through the use of appropriately constructed formulae for subsidies, once applied, the system-optimized solution becomes, at the same time, user-optimized. This provides a more positive approach to the redistribution of human migrants and enhances societal welfare.

In addition, in this paper, we provide an effective computational procedure, which exploits the underlying special network structure of our models. The algorithm is implemented, and the solutions to a series of numerical examples computed. We report the user-optimized and the system-optimized solutions, both uncapacitated and capacitated, along with the subsidies for the latter. Our theoretical framework can be applied in practice under different scenarios, along with sensitivity analysis, as, for example, in the case of disasters, when there are population changes and/or modifications to utility functions because of impacted infrastructure.

Acknowledgments

The first two authors are grateful for the insights gained through participation in the Dynamics of Disasters conferences in Kalamata, Greece and the very useful discussions.

The authors acknowledge the very helpful comments and suggestions provided by two

anonymous reviewers on an earlier version of this paper.

References

Altemeyer-Bartscher, M., Holtemöller, O., Lindner, A., Schmalzbauer, A., Zeddies, G., 2016. On the distribution of refugees in the EU. Intereconomics 51(4), 220-228.

Bartenstein, B., McDonald, M.D., 2019. Exporting people: How Central America encourages mass migration. Bloomberg, July 19; available at: https://www.bloomberg.com/news/articles/2019-06-19/migrant-crisis-at-border-how-central-america-encourages-exodus

Beckmann, M., McGuire, C.B., Winsten, C.B., 1956. Studies in the Economics of Transportation. Yale University Press, New Haven, Connecticut.

Bertsekas, D.P., Tsitsiklis, J.N., 1989. Parallel and Distributed Computation: Numerical Methods. Prentice-Hall, Englewood Cliffs, NJ.

Bier, V.M., Zhou, Y., Du, H., 2019. Game-theoretic modeling of pre-disaster relocation. The Engineering Economist. DOI: 10.1080/0013791X.2019.1677837

Boyce, D.E., Mahmassani, H.S., Nagurney, A., 2005. A retrospective on Beckmann, McGuire and Winsten's Studies in the Economics of Transportation. Papers in Regional Science 84(1), 85-103.

Branswell, H., Joseph, A., 2020. WHO declares the coronavirus outbreak a pandemic. STAT. March 11; available at: https://www.statnews.com/2020/03/11/who-declares-the-coronavirus-outbreak-a-pandemic/

Cappello, G., Daniele, P., 2019. A variational formulation for a human migration model. In: Paolucci M., Sciomachen A., Uberti P., Editors, Advances in Optimization and Decision Science for Society, Services and Enterprises. AIRO Springer Series, vol 3. Springer Nature Switzerland, pp. 185-195.

Causa, A., Jadamba, B., Raciti, F., 2017. A migration equilibrium model with uncertain data and movement costs. Decisions in Economics and Finance 40, 159-175.

Chinchilla, L., Orozco, M., Pérez Bravo, T., Rodriguez, R., 2018. Do countries have enough capacity to handle migrants? Latin American Advisor, August 24. Available at: https://www.thedialogue.org/analysis/do-countries-have-enough-capacity-to-handle-migrants/

Dafermos, S., 1980. Traffic equilibrium and variational inequalities. Transportation Science 14(1), 42-54.

Dafermos, S.C., 1973. Toll patterns for multiclass-user transportation networks. Transportation Science 7(3), 211-223.

Dafermos, S., Sparrow, F.T., 1971. Optimal resource allocation and toll patterns in user-optimized transport networks. Journal of Transport Economics and Policy 5(2), 184-200.

Dafermos S.C., Sparrow, F.T., 1969. The traffic assignment problem for a general network. Journal of Research of the National Bureau of Standards 73B, 91-118.

Davis, K.F., D;Odorico, P., Laio, F., Ridolfi, L., 2013. Global spatio-temporal patterns in human migration: A complex network perspective. PLOS ONE 8(1), e53723.

Dupuis, P., Nagurney, A., 1993. Dynamical systems and variational inequalities. Annals of Operations Research 44, 9-42.

European Commission, 2019. Communication from the Commission on the European Parliament, the European Council and the Council. Progress report on the Implementation of the European Agenda on Migration. Brussels, Belgium, 6.3.2019 COM(2019) 126 final.

International Organization for Migration, 2019. World migration report for 2020. Geneva, Switzerland.

Isac, G., Bulavsky, V.A., Kalashnikov, V.V., 2002. Complementarity, Equilibrium, Efficiency and Economics. Kluwer Academic Publishers, Dordrecht, The Netherlands.

Jones, G., 2018. Italy will no longer be 'Europe's refugee camp', vows new government. World News June 4.

Kalashnikov, V., Kalashnykova, N., Rojas, R.L. Munos, M.M., Uranga, C., Rojas, A.L., 2008. Numerical experimentation with a human migration model. European Journal of Operational Research 189, 208-229.

Karagiannis, G.M., 2016. Emergency management perspective of the European migrant crisis. Newsletter - special edition, The International Emergency Management Society, issue 4, October, 12-24, Brussels, Belgium.

Kennedy, M., 2019. U.N. says more than 4 million people have left Venezuela. NPR; available at: https://www.npr.org/2019/06/07/730687807/u-n-says-more-than-4-million-people-

have-left-venezuela

Kinderlehrer, D., Stampacchia, G., 1980. An Introduction to Variational Inequalities and Their Applications. Academic Press, New York.

Kitsantonis, N., 2019. Greek refugee camps are near catastrophe, Rights Chief warns. The New York Times, October 31.

Kotsireas, I.S., Nagurney, A., Pardalos, P.M., Editors, 2016. Dynamics of Disasters: Key Concepts, Models, Algorithms, and Insights. Springer International Publishing Switzerland.

Kotsireas, I.S., Nagurney, A., Pardalos, P.M., Editors, 2018. Dynamics of Disasters: Algorithmic Approaches and Applications. Springer International Publishing Switzerland.

Larsson, T., Patriksson, M., 1999. Side constrained traffic equilibrium models - analysis, computation and applications. Transportation Research B 33(4), 233-264.

Lawphongpanich, S., Hearn, D.W., Smith, M.J., Editors, 2006. Mathematical and Computational Models for Congestion Charging. Springer Science + Business Media, New York.

Mattiace, T., 2019. Mexico: Onset of migration crisis. Global Risk Insights, November 24. Available at:
https://globalriskinsights.com/2019/11/mexico-onset-of-migration-crisis/

Nagurney, A., 1989. Migration equilibrium and variational inequalities. Economics Letters 31, 109-112.

Nagurney, A., 1990. A network model of migration equilibrium with movement costs. Mathematical and Computer Modelling 13, 79-88.

Nagurney, A., 1999. Network Economics: A Variational Inequality Approach, second and revised edition. Kluwer Academic Publishers, Dordrecht, The Netherlands.

Nagurney, A., 2006. Supply Chain Network Economics: Dynamics of Prices, Flows and Profits. Edward Elgar Publishing, Cheltenham, England.

Nagurney, A., 2010. Optimal supply chain network design and redesign at minimal total cost and with demand satisfaction. International Journal of Production Economics 128, 200-208.

Nagurney, A., Daniele, P., 2020. International human migration networks under regulations. European Journal of Operational Research, in press.

Nagurney, A., Daniele, P., Cappello, G., 2020. Human migration networks and policy interventions: Bringing population distributions in line with system-optimization. International Transactions in Operational Research, in press.

Nagurney, A., Daniele, P., Nagurney, L.S., 2019. Refugee migration networks and regulations: A multiclass, multipath variational inequality framework. Isenberg School of Management, University of Massachusetts Amherst.

Nagurney, A., Pan, J., Zhao, L., 1992a. Human migration networks. European Journal of Operational Research 59, 262-274.

Nagurney, A., Pan, J., Zhao, L., 1992b. Human migration networks with class transformations. In: Structure and Change in the Space Economy. Lakshmanan, T.R., Nijkamp, P., Editors, Springer-Verlag, Berlin, Germany, pp. 239-258.

Nagurney, A., Yu, M., Qiang, Q., 2011. Supply chain network design for critical needs with outsourcing. Papers in Regional Science 90, 123-142.

Nagurney, A., Zhang, D., 1996. Projected Dynamical Systems and Variational Inequalities with Applications. Kluwer Academic Publishers, Boston, Massachusetts.

O'Connor, A., Batalova, J., Bolter, J., 2019. Central American immigrants in the United States. Migration Information Source, August 15. Available at:
https://www.migrationpolicy.org/article/central-american-immigrants-united-states

OECD, 2019. How do OECD countries compare in their attractiveness for talented migrants? Migration Policy Debates, No. 19, May.

Pan, J., Nagurney, A., 1994. Using Markov chains to model human migration in a network equilibrium framework. Mathematical and Computer Modelling 19, 31-39.

Pan, J., Nagurney, A., 2006. Evolution variational inequalities and projected dynamical systems with application to human migration. Mathematical and Computer Modelling 43(5-6), 646-657.

Papadaki, S., Banias, G., Achillas, C., Aidonis, D., Folinas, D., Bochtis, D., Papangelou, S., 2018. A humanitarian logistics case study for the intermediary phase accommodation center for refugees and other humanitarian disaster victims. In: Dynamics of Disasters: Algorithmic Approaches and Applications, Kotsireas, I.S., Nagurney, A., Pardalos, P.M., Editors, Springer International Publishing Switzerland, pp. 157-202.

Parkinson, J., 2015. Which countries are in the best position to take migrants? BBC News Magazine, September 9. Available at: https://www.bbc.com/news/magazine-34138831

Robinson, C., Dilkina, B., Moreno-Cruz, J., 2020. Modeling migration patterns in the USA under sea level rise. PLoS ONE 15(1): e0227436. https://doi.org/10.1371/journal.pone.0227436

Sakuma, A., 2020. Damned for trying. Available at: http://www.msnbc.com/specials/migrant-crisis/libya

Shih, G., Denyer, S., Taylor, A., 2020. Worries grow that quarantine in China not enough to stem increasingly virulent coronavirus. Washington Post, January 27.

Stevis-Gridneff, M., 2020. Asylum seekers find a new route to Europe, flowing into a divided Cyprus. The New York Times, January 28.

UNHCR, 2020. Figures at a glance. Available at: https://www.unhcr.org/en-us/figures-at-a-glance.html

United Nations, 2017. Population facts. Department of Economic and Social Affairs, Population Division, New York, December; available at: https://www.un.org/en/development/desa/population/publications/pdf/popfacts/PopFacts_2017-5.pdf

United Nations Refugee Agency, 2015. The sea route to Europe: The Mediterranean passage in the age of refugees. July 1; available at: https://www.unhcr.org/en-us/protection/operations/5592bd059/sea-route-europe-mediterranean-passage-age-refugees.html

Wardrop, J.G., 1952. Some theoretical aspects of road traffic research. Proceedings of the Institution of Civil Engineers, Part II 1: pp. 325-378.