A Mean-Variance Disaster Relief Supply Chain Network Model for Risk Reduction with Stochastic Link Costs, Time Targets, and Demand Uncertainty

Anna Nagurney
Department of Operations and Information Management
Isenberg School of Management
University of Massachusetts
Amherst, Massachusetts 01003
Ladimer S. Nagurney
Department of Electrical and Computer Engineering
University of Hartford, West Hartford, CT 06117

In *Dynamics of Disasters: Key Concepts, Models, Algorithms, and Insights*, I.S. Kotsireas, A. Nagurney, and P.M. Pardalos, Eds., Springer International Publishing Switzerland, 2016, pp. 231-255.

Abstract: In this paper, we develop a mean-variance disaster relief supply chain network model with stochastic link costs and time targets for delivery of the relief supplies at the demand points, under demand uncertainty. The humanitarian organization seeks to minimize its expected total operational costs and the total risk in operations with an individual weight assigned to its valuation of the risk, as well as the minimization of expected costs of shortages and surpluses and tardiness penalties associated with the target time goals at the demand points. The risk is captured through the variance of the total operational costs, which is relevant to the reporting of the proper use of funds to stakeholders, including donors. The time goal targets associated with the demand points enable prioritization as to the timely delivery of relief supplies. The framework handles both the pre-positioning of relief supplies, whether local or nonlocal, as well as the procurement (local or nonlocal), transport, and distribution of supplies post-disaster. The time element is captured through link time completion functions as the relief supplies progress along paths in the supply chain network. Each path consists of a series of directed links, from the origin node, which represents the humanitarian organization, to the destination nodes, which are the demand points for the relief supplies. We propose an algorithm, which yields closed form expressions for the variables at each iteration, and demonstrate the efficacy of the framework through a series of illustrative numerical examples, in which trade-offs between local versus nonlocal procurement, post- and pre-disaster, are investigated. The numerical examples include a case study on hurricanes hitting Mexico.

Keywords: supply chains, disaster relief, humanitarian logistics, network optimization, risk reduction, undertainty, time constraints, variational inequalities.

1. Introduction

Natural disasters, such as earthquakes, hurricanes, tsunamis, floods, tornadoes, fires, and droughts, invoke all phases of the disaster management cycle from preparedness and mitigation to response and recovery. Notable recent examples of disasters include: Hurricane Katrina in 2005 and Superstorm Sandy in 2012, the two costliest disasters to strike the U.S., the earthquake in Haiti in 2010, the triple disaster in Fukushima, Japan in 2011, and the devastating earthquake in Nepal in 2015. As noted in Nagurney and Qiang (2009), the number of disasters is growing as well as the number of people affected by disasters. Hence, the development of appropriate analytical tools that can assist humanitarian organizations and nongovernmental organizations as well as governments in the various disaster management phases has become a challenge to both researchers and practitioners.

Recently, there has been growing interest in constructing integrated frameworks that can assist in multiple phases of disaster management. Network-based models and tools, which allow for graphical depiction of disaster relief supply chains and the flexibility of adding nodes and links, coupled with effective computational procedures, in particular, offer promise. Such models necessarily have to be optimization-based and must incorporate stochastic elements since in disaster situations there is uncertainty associated with the demand for relief supplies and also uncertainty associated with various link costs along with variances.

In addition, as noted in Nagurney, Masoumi, and Yu (2015), time plays a critical role in disaster relief supply chains and, therefore, time must be a fundamental element in disaster relief models. The U.S. Federal Emergency Management Agency (FEMA) has identified key benchmarks to response and recovery, which emphasize time and are: to meet the survivors' initial demands within 72 hours, to restore basic community functionality within 60 days, and to return to as normal of a situation within 5 years (Fugate (2012)). Walton, Mays, and Haselkorn (2011) further reinforce the importance of speed in emergency response guidelines for disaster relief operations (see also USAID (2005) and UNHCR (2007)). Timely and efficient delivery of relief supplies to the affected population not only decreases the fatality rate but may also prevent chaos. In the case of cyclone Haiyan, the strongest typhoon ever recorded in terms of wind speed, which devastated areas of Southeast Asia, especially, the Philippines, where 11 million people were affected, slow relief delivery efforts forced people to seek any possible means to survive. Several relief trucks were attacked and had food stolen, and some areas were reported to be on the brink of anarchy (Chicago Tribune (2013) and CBS News (2013)). In Nepal, post the April 2015 7.8 magnitude earthquake, there was near chaos at the Katmandu airport with relief airplanes not able to land, with numerous

Nepalese citizens seeking to leave while Nepalese expatriates attempted to return to help their families (Luke and McVicker (2015)). The BBC News (2015) reported that the slow distribution of aid led to clashes between protesters and riot police.

Furthermore, humanitarian relief organizations, for the most part, receive their primary funding and support from donors. Hence, they are responsible to these and other stakeholders in terms of accountability of the use of their financial funds (see Toyasaki and Wakolbinger (2014)). It has been estimated that logistics accounts for about 80% of the total costs in disaster relief (Van Wassenhove (2006)). Thus, humanitarian organizations must utilize their resources in the most effective and efficient way while delivering relief supplies in a timely manner. As noted by Tzeng, Cheng, and Huang (2007), once a disaster strikes, effective disaster relief efforts can mitigate the damage, reduce the number of fatalities, and bring relief to the survivors. For additional background, see the recent edited volume on disaster management and emergencies by Vitoriano, Montero, and Ruan (2013), which includes a survey on decision aid models for humanitarian logistics by Ortuño et al. (2013).

In this paper, we develop a mean-variance disaster relief supply chain network model with stochastic link costs and time targets for delivery of the relief supplies at the demand points, under demand uncertainty. The model is inspired by the supply chain network integration model for risk reduction in the case of mergers and acquisitions developed by Liu and Nagurney (2011), coupled with the integrated disaster relief framework of Nagurney, Masoumi, and Yu (2015). Liu and Nagurney (2011) used a mean-variance (MV) approach for the measurement of risk associated with link supply chain network costs, but in a corporate, not a humanitarian, setting. That work also assessed synergies associated with mergers and acquisitions.

The MV approach to risk reduction dates to the work of the Nobel laureate Harry Markowitz (1952, 1959) and is still relevant in finance (Schneeweis, Crowder, and Kazemi (2010)), in supply chains (Chen and Federgruen (2000) and Kim, Cohen, and Netessine (2007)), as well as in disaster relief and humanitarian operations, where the focus, to-date, has been on inventory management (Ozbay and Ozguven (2007) and Das (2014)). However, the model constructed here is the first to integrate preparedness and response in a supply chain network framework with a mean-variance approach for risk reduction under demand and cost uncertainty and time targets plus penalties for shortages and surpluses. Bozorgi-Amiri et al. (2013) developed a model with uncertainty on the demand side and also in procurement and transportation using expected costs and variability with associated weights but did not consider the critical time elements as well as the possibility of local versus nonlocal procurement post- or pre-disaster.

In addition, Boyles and Waller (2010) developed a MV model for the minimum cost network flow problem with stochastic link costs and emphasized that an MV approach is especially relevant in logistics and distribution problems with critical implications for supply chains. They noted that a solution that only minimizes expected cost and not variances may not be as reliable and robust as one that does.

In our model, the humanitarian organization seeks to minimize its expected total operational costs and the total risk in operations with an individual weight assigned to its valuation of the risk, as well as the minimization of expected costs of shortages and surpluses and tardiness penalties associated with the target time goals at the demand points. The risk is captured through the variance of the total operational costs, which is of relevance also to the reporting of the proper use of funds to stakeholders, including donors. The time goal targets associated with the demand points enable prioritization of demand points as to the timely delivery of relief supplies. This framework handles both the pre-positioning of relief supplies, whether local or nonlocal, as well as the procurement (local or nonlocal), transport, and distribution of supplies post-disaster. There is growing empirical evidence showing that the use of local resources in humanitarian supply chains can have positive impacts (see Matopoulos, Kovacs, and Hayes (2014)). Earlier work on procurement with stochastic components did not distinguish between local or nonlocal procurement (see Falasca and Zobel (2011)).

The time element in our model is captured through link time completion functions as the relief supplies progress along paths in the supply chain network. Each path consists of a series of directed links, from the origin node, which represents the humanitarian organization, to the destination nodes, which are the demand points for the relief supplies.

The literature on humanitarian operations and disaster relief has been growing. Below we highlight publications that are relevant to aspects of supply chain network activities, such as procurement, transportation, storage, and distribution. Hale and Moberg (2005) proposed a set covering location model to identify secure sites for the storage of emergency supplies. Balcik and Beamon (2005) studied facility location in humanitarian relief. Beamon and Kotleba (2006) developed a stochastic inventory control model determining optimal order quantities and reorder points for a long-term emergency relief response. Barbarosoglu and Arda (2004) and Falasca and Zobel (2011) proposed two-stage stochastic models for the procurement and transportation of disaster relief items. Also, Mete and Zabinsky (2010) introduced a two-stage stochastic model for the storage and distribution of medical supplies to be used in case of emergencies. Huang, Smilowitz, and Balcik (2012) presented performance measures for the efficiency, efficacy, and equity of relief distribution.

Nagurney and Qiang (2012) proposed network robustness and performance measures in addition to synergy assessment of supply chain network integration in the case of humanitarian partnerships (see also Nagurney and Qiang (2009) and Nagurney, Yu, and Qiang (2012)). The synergy measure can be used to determine the potential benefits of horizontal cooperation and coordination between humanitarian organizations. Qiang and Nagurney (2012) introduced a bi-criteria indicator for performance evaluation of supply chains of critical needs products under capacity and demand disruptions. Rottkemper, Fischer, and Blecken (2012) presented a bi-criteria mixed-integer programming model for the inventory relocation of relief items. Ortuño, Tirado, and Vitoriano (2011) and Vitoriano et al. (2011) developed goal programming frameworks for the distribution of relief goods while considering targets for attributes such as the cost and travel time.

The paper is organized as follows. In Section 2, we construct the mean-variance supply chain network model for disaster relief and provide its variational inequality formulation, with nice features for computations. In Section 3, we present the Euler method, which yields closed form expressions for the variables at each iteration, and then apply it to solve two sets of numerical examples. The first set consists of a small example with 5 variants whereas the second set consists of a larger example focusing on Mexico, and a variant. We have identified Mexico as an appropriate setting for the larger set of examples due to its natural disaster risk profile in terms of hurricanes, storms, floods, earthquakes, and droughts. Specifically, we focus on multiple hurricanes hitting Mexico, as happened in 2013, with two hurricanes, Manuel and Ingrid, making landfall within 24 hours of each other and affecting Acapulco and the Mexico City area, respectively. In Section 4, we summarize the results and present our conclusions.

2. The Mean-Variance Disaster Relief Supply Chain Network Model for Risk Reduction

In this section, we construct the mean-variance disaster relief supply chain network model in which the humanitarian organization seeks to minimize its expected total operational costs and the total risk in operations with an individual weight assigned to its valuation of the risk, as well as the expected costs of shortages and surpluses and tardiness penalties associated with the target time goals at the demand points. The risk is captured through the variance of the total operational costs. The time goal targets associated with the demand points enable prioritization of demand points as to the timely delivery of relief supplies. The framework handles both the pre-positioning of relief supplies as well as the procurement, transport, and distribution of supplies post-disaster, whether local or nonlocal. The time element is captured through link time completion functions as the relief supplies progress via paths in the supply chain network. The paths consist of a series of directed links, from the origin node to the destination nodes, which are the demand points for the relief supplies.

2.1 Model Foundations and Notation

The network topology of the mean-variance disaster relief supply chain network is given in Figure 1 and is denoted by $G = [N, L]$, where N denotes the set of nodes and L the set of links. The organization is associated with node 1, which also serves as the (abstract) origin node. The demand points, which receive the disaster relief supplies, are denoted by nodes $R_1, \ldots, R_{n_R}$. We emphasize that the supply chain network topology may be modified/adapted for specific instances and situations. It, nevertheless, reflects the essential elements of a disaster relief supply chain and the associated activities of procurement, transportation, storage, processing, and, finally, the ultimate distribution to the demand points, as reflected by the links in Figure 1. Also, the progression of time in Figure 1 is reflected in the link directions from left to right.

Specifically, links joining node 1 to nodes $C_1, \ldots, C_{n_C}$ are procurement links. Procurement, depending upon the scenario, may be done locally or not, as depicted in Figure 1. Transportation links connect the procurement nodes to storage nodes denoted by $S_1, \ldots, S_{n_S,1}$. Storage is reflected by the links joining the latter nodes to nodes: $S_{1,2}, \ldots, S_{n_S,2}$. Also, the links connecting node 1 to nodes $S_{1,2}, \ldots, S_{n_S,2}$ represent nonlocal procurement post-disaster and, hence, obviate the need for storage on links: $S_{1,1}$ to $S_{1,2}$, through $S_{n_S,1}$ to $S_{n_S,2}$. Joining the storage nodes are transportation links with individual links corresponding to a specific mode of transportation. In humanitarian operations it is important to distinguish among modes of transportation since relief supplies might be airlifted, arrive

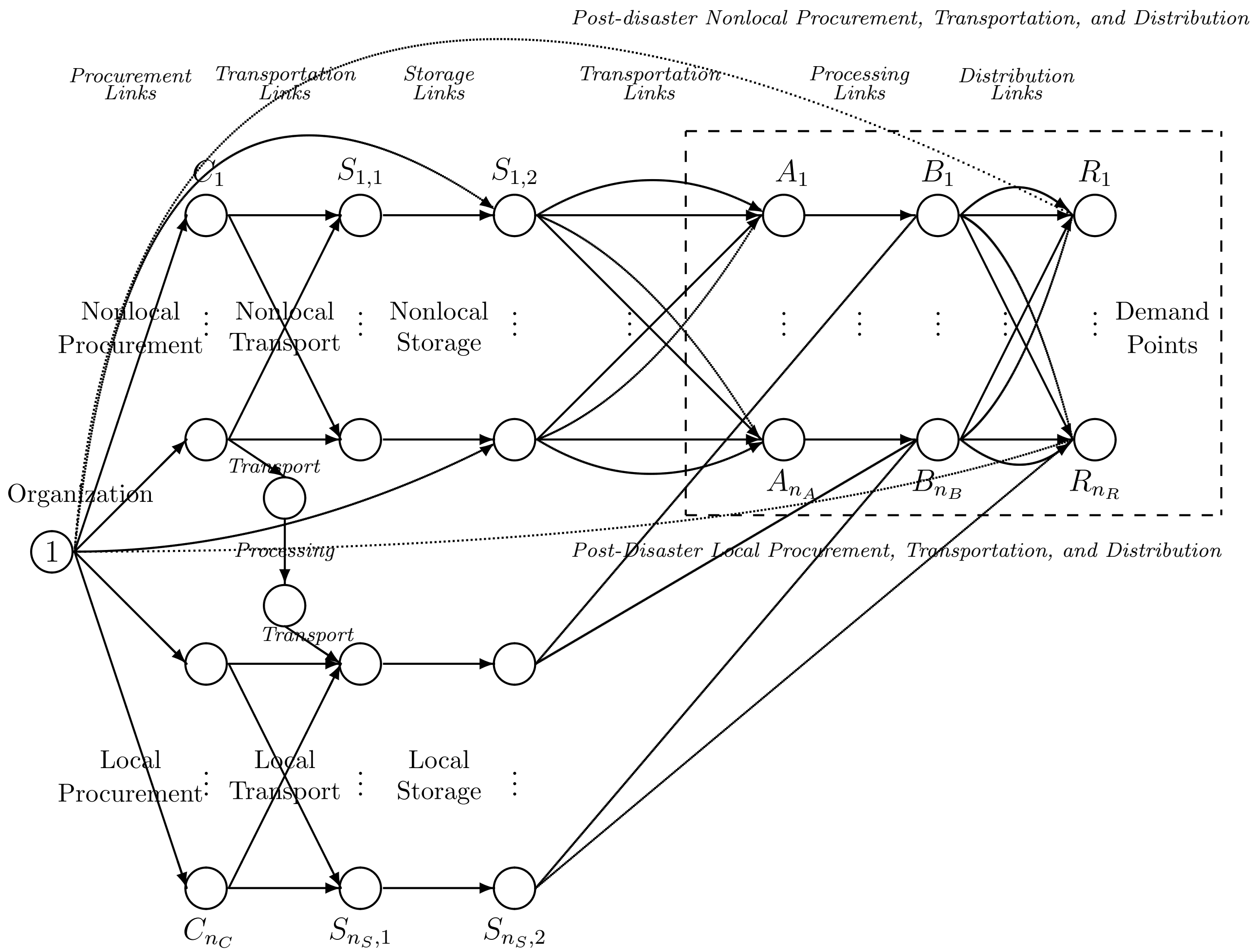

Figure 1: Network Topology of the Mean-Variance Disaster Relief Supply Chain

via ground transportation or even maritime transport, depending on the geography and the status of the critical infrastructure. The nodes: $A_1, \ldots, A_{n_A}$ are the arrival portals with the links emanating from such nodes reflecting processing links. In the case of imports across national boundaries there might be customs inspections, import duties and fees, and other processing prior to the ultimate consolidation for final distribution of supplies (see,e.g., Lorch (2015) and Harris (2015)). The processing facilities are denoted by nodes: $B_1, \ldots, B_{n_B}$. The links joining the nodes $B_1, \ldots, B_{n_B}$ in Figure 1 with the demand point nodes $R_1, \ldots, R_{n_R}$ are the distribution links, which include the last mile distribution operations. The supply chain network topology revealed in Figure 1 is a substantive generalization of the one in Nagurney, Masoumi, and Yu (2015) to include the options of local procurement, transportation, and distribution post-disaster as reflected by the links joining node 1 to the demand point nodes

Table 1: Notation for the Mean-Variance Disaster Relief Model

Notation	**Definition**
x_p	the nonnegative flow of the relief item on path p. We group the flows on all paths into the vector $x \in R_+^{n_\mathcal{P}}$.
f_a	the flow of the relief item on link a; $a \in L$.
v_k	the projected demand for the disaster relief item at point k; $k = 1, \ldots, R_{n_R}$.
d_k	the actual (uncertain) demand at point k; $k = 1, \ldots, R_{n_R}$.
Δ_k^-	the amount of shortage of the relief item at demand point k; $k = 1, \ldots, R_{n_R}$.
Δ_k^+	the amount of surplus of the relief item at demand point k; $k = 1, \ldots, R_{n_R}$.
λ_k^-	the unit penalty corresponding to a shortage of the relief item at demand point k; $k = 1, \ldots, R_{n_R}$.
λ_k^+	the unit penalty corresponding to a surplus of the relief item at demand point k; $k = 1, \ldots, R_{n_R}$.
$\tau_a(f_a)$	the completion time of the activity on link a; $a \in L$, with $\tau_a(f_a) = \hat{t}_a f_a + t_a$, where $\hat{t}_a$ and t_a are ≥ 0, $\forall a \in L$.
T_k	target for the completion time of the activities on paths corresponding to demand point k determined by the organization's decision-maker where $k = 1, \ldots, n_R$.
T_{kp}	the target time for demand point k with respect to path $p \in \mathcal{P}_k$. $T_{kp} = T_k - t_p$, where $t_p = \sum_{a \in L} t_a \delta_{ap}$, where $\delta_{ap} = 1$, if link a is contained in path p, and is equal to 0, otherwise.
z_p	the amount of deviation with respect to target time T_{kp} associated with late delivery of the relief item to k on path p, $\forall p \in \mathcal{P}$. We group the z_ps into the vector $z \in R_+^{n_\mathcal{P}}$.
$\gamma_k(z)$	the tardiness penalty function corresponding to demand point k; $k = 1, \ldots, n_R$.
ω_a	an exogenous random variable affecting the total operational cost on link a; $a \in L$.
$\hat{c}_a(f_a, \omega_a)$	the total operational cost on link a; $a \in L$.

as well as the partitioning of pre-disaster choices according to whether they are local or not.

We assume that there exists at least one path in the disaster relief supply chain network connecting the origin (node 1) with each demand point: $R_1, \ldots, R_{n_R}$.

The links in the supply chain network are denoted by a, b, c, etc. The paths are denoted by p, q, etc., with the set of paths joining origin node 1 with demand point k denoted by $\mathcal{P}_k$, and the set of paths joining the node 1 with all demand points denoted by $\mathcal{P}$ with this set having $n_\mathcal{P}$ elements.

The notation for the model is summarized in Table 1.

The notation is similar to that in Nagurney, Masoumi, and Yu (2015) but with appropriate additions to capture link total cost uncertainty.

2.2 Formulation of the Mean-Variance Disaster Relief Supply Chain Network Model with Risk Reduction

Before constructing the objective function, we recall some preliminaries.

In the model, the demand is uncertain due to the unpredictability of the actual demand at the demand points. The literature contains examples of supply chain network models with uncertain demand and associated shortage and surplus penalties (see, e.g., Dong, Zhang, and Nagurney (2004), Nagurney, Yu, and Qiang (2011), Nagurney and Masoumi (2012), and Nagurney, Masoumi, and Yu (2015)). For example, the probability distribution of demand might be derived using census data and/or information gathered during the disaster preparedness phase. Since d_k denotes the actual (uncertain) demand at destination point k, we have:

$$P_k(D_k) = P_k(d_k \leq D_k) = \int_0^{D_k} \mathcal{F}_k(u)du, \qquad k = 1, \ldots, n_R, \tag{1}$$

where P_k and $\mathcal{F}_k$ denote the probability distribution function, and the probability density function of demand at point k, respectively.

Recall from Table 1 that v_k is the "projected demand" for the disaster relief item at demand point $k; k = 1, \ldots, n_R$. The amounts of shortage and surplus at destination node k are calculated, respectively, according to:

$$\Delta_k^- \equiv \max\{0, d_k - v_k\}, \qquad k = 1, \ldots, n_R, \tag{2a}$$

$$\Delta_k^+ \equiv \max\{0, v_k - d_k\}, \qquad k = 1, \ldots, n_R. \tag{2b}$$

The expected values of shortage and surplus at each demand point are, hence:

$$E(\Delta_k^-) = \int_{v_k}^{\infty} (u - v_k)\mathcal{F}_k(u)du, \qquad k = 1, \ldots, n_R, \tag{3a}$$

$$E(\Delta_k^+) = \int_0^{v_k} (v_k - u)\mathcal{F}_k(u)du, \qquad k = 1, \ldots, n_R. \tag{3b}$$

The expected penalty incurred by the humanitarian organization due to the shortage and surplus of the relief item at each demand point is equal to:

$$E(\lambda_k^- \Delta_k^- \ + \ \lambda_k^+ \Delta_k^+) = \lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+), \qquad k = 1, \ldots, n_R. \tag{4}$$

We have the following two sets of conservation of flow equations. The projected demand at destination node k, v_k, is equal to the sum of flows on all paths in the set $\mathcal{P}_k$, that is:

$$v_k \equiv \sum_{p \in \mathcal{P}_k} x_p, \qquad k = 1, \ldots, n_R. \tag{5}$$

The flow on link a, f_a, is equal to the sum of flows on paths that contain that link:

$$f_a = \sum_{p \in \mathcal{P}} x_p \; \delta_{ap}, \qquad \forall a \in L, \tag{6}$$

where δ_{ap} is equal to 1 if link a is contained in path p and is 0, otherwise.

The objective function faced by the organization's decision-maker, which he seeks to minimize, is the following:

$$E\left[\sum_{a \in L} \hat{c}_a(f_a, \omega_a)\right] + \alpha Var\left[\sum_{a \in L} \hat{c}_a(f_a, \omega_a)\right] + \sum_{k=1}^{n_R} (\lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+)) + \sum_{k=1}^{n_R} \gamma_k(z)$$

$$= \sum_{a \in L} E\left[\hat{c}_a(f_a, \omega_a)\right] + \alpha Var\left[\sum_{a \in L} \hat{c}_a(f_a, \omega_a)\right] + \sum_{k=1}^{n_R} (\lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+)) + \sum_{k=1}^{n_R} \gamma_k(z), \tag{7}$$

where E denotes the expected value, Var denotes the variance, and α represents the risk aversion factor (weight) for the organization that the organization's decision-maker places on the risk as represented by the variance of the total operational costs. The objective function (7) includes the expected total operational costs on all the links, the weighted variance of those costs, the expected costs due to shortages or surpluses at the demand points, and the sum of tardiness penalties at the demand points in the disaster relief supply chain network.

Here we consider total operational link cost functions of the form:

$$\hat{c}_a = \hat{c}_a(f_a, \omega_a) = \omega_a \hat{g}_a f_a + g_a f_a, \quad \forall a \in L, \tag{8}$$

where $\hat{g}_a$ and g_a are positive-valued for all links $a \in L$. We permit ω_a to follow any probability distribution and the ωs of different supply chain links can be correlated with one another. As noted in Liu and Nagurney (2011), the term $\hat{g}_a f_a$ in (8) represents the part of the total link operational cost that is subject to variation of ω_a with $g_a f_a$ denoting that part of the total cost that is independent of ω_a. The random variables ω_a, $a \in L$ can capture various elements of uncertainty, due, for example, to disruptions because of the disaster, and price uncertainty for storage, procurements, transport, processing, and distribution services.

The goal of the decision-maker is, thus, to minimize the following problem, with the objective function in (7), in lieu of (8), taking the form in (9) below:

$$\text{Minimize} \sum_{a\in L} E(\omega_a)\hat{g}_a f_a + \sum_{a\in L} g_a f_a + \alpha Var(\sum_{a\in L} \omega_a \hat{g}_a f_a) + \sum_{k=1}^{n_R} (\lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+)) + \sum_{k=1}^{n_R} \gamma_k(z) \tag{9}$$

subject to constraint (6) and the following constraints:

$$x_p \geq 0, \quad \forall p \in \mathcal{P}, \tag{10}$$

$$z_p \geq 0, \quad \forall p \in \mathcal{P}, \tag{11}$$

$$\sum_{q\in\mathcal{P}} \sum_{a\in L} \hat{t}_a x_q \delta_{aq} \delta_{ap} - z_p \leq T_{kp}, \quad \forall p \in \mathcal{P}_k; k = 1, \ldots, n_R, \tag{12}$$

with the T_ks defined in Table 1. Constraint (10) guarantees that the relief item path flows are nonnegative. Constraint (10) guarantees that the path deviations with respect to target times on the respective paths are nonnegative, and (12) captures the goal target information for the paths.

In view of constraint (6) we can reexpress the objective function in (9) in path flows (rather than in link flows and path flows) to obtain the following optimization problem:

$$\text{Minimize} \sum_{a\in L} \left[E(\omega_a)\hat{g}_a \sum_{q\in\mathcal{P}} x_q \delta_{aq} + g_a \sum_{q\in\mathcal{P}} x_q \delta_{aq} \right] + \alpha Var(\sum_{a\in L} \omega_a \hat{g}_a \sum_{q\in\mathcal{P}} x_q \delta_{aq})$$

$$+ \sum_{k=1}^{n_R} (\lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+)) + \sum_{k=1}^{n_R} \gamma_k(z) \tag{13}$$

subject to constraints: (10) – (12).

Let K denote the feasible set:

$$K \equiv \{(x, z, \mu) | x \in R_+^{n_\mathcal{P}}, z \in R_+^{n_\mathcal{P}}, \text{ and } \mu \in R_+^{n_\mathcal{P}}\}, \tag{14}$$

where recall that x is the vector of path flows of the relief item, z is the vector of time deviations on paths, and μ is the vector of Lagrange multipliers corresponding to the constraints in (12) with an individual element corresponding to path p denoted by μ_p.

Before presenting the variational inequality formulation of the optimization problem immediately above, we review the respective partial derivatives of the expected values of shortage and surplus of the disaster relief item at each demand point with respect to the path flows, derived in Dong, Zhang, and Nagurney (2004), Nagurney, Yu, and Qiang (2011), and Nagurney, Masoumi, and Yu (2012). In particular, they are given by:

$$\frac{\partial E(\Delta_k^-)}{\partial x_p} = P_k\left(\sum_{q\in\mathcal{P}_k} x_q\right) - 1, \quad \forall p \in \mathcal{P}_k; \quad k = 1,\ldots,n_R, \tag{15a}$$

and,

$$\frac{\partial E(\Delta_k^+)}{\partial x_p} = P_k\left(\sum_{q\in\mathcal{P}_k} x_q\right), \quad \forall p \in \mathcal{P}_k; \quad k = 1,\ldots,n_R. \tag{15b}$$

We now present the variational inequality formulation of the mean-variance disaster relief supply chain network problem for risk reduction. We assume that the underlying functions in the model are convex and continuously differentiable The proof is immediate following the proof of Theorem 1 in Nagurney, Masoumi, and Yu (2015).

Theorem 1

The optimization problem (13), subject to its constraints (10) – (12), is equivalent to the variational inequality problem: determine the vector of optimal path flows, the vector of optimal path time deviations, and the vector of optimal Lagrange multipliers $(x^*, z^*, \mu^*) \in K$*, such that:*

$$\sum_{k=1}^{n_R}\sum_{p\in\mathcal{P}_k}\Bigg[\sum_{a\in L}(E(\omega_a)\hat{g}_a + g_a)\delta_{ap} + \alpha\frac{\partial Var(\sum_{a\in L}\omega_a\hat{g}_a\sum_{q\in\mathcal{P}}x_q^*\delta_{aq})}{\partial x_p}$$

$$+\lambda_k^+ P_k(\sum_{q\in\mathcal{P}_k} x_q^*) \; - \; \lambda_k^-(1 - P_k(\sum_{q\in\mathcal{P}_k} x_q^*)) + \sum_{q\in\mathcal{P}}\sum_{a\in L}\mu_q^* g_a\delta_{aq}\delta_{ap}\Bigg] \times [x_p - x_p^*]$$

$$+ \sum_{k=1}^{n_R}\sum_{p\in\mathcal{P}_k}\left[\frac{\partial\gamma_k(z^*)}{\partial z_p} - \mu_p^*\right] \times [z_p - z_p^*]$$

$$+ \sum_{k=1}^{n_R}\sum_{p\in\mathcal{P}_k}\left[T_{kp} + z_p^* - \sum_{q\in\mathcal{P}}\sum_{a\in L} g_a x_q^*\delta_{aq}\delta_{ap}\right] \times [\mu_p - \mu_p^*] \geq 0, \qquad \forall (x, z, \mu) \in K. \tag{16}$$

Variational inequality (16) can be put into standard form (Nagurney (1999)) as follows: determine $X^* \in \mathcal{K}$ such that:

$$\left\langle F(X^*), X - X^*\right\rangle \geq 0, \qquad \forall X \in \mathcal{K}, \tag{17}$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in n-dimensional Euclidean space. If the feasible set is defined as $\mathcal{K} \equiv K$, and the column vectors $X \equiv (x, z, \mu)$ and $F(X) \equiv (F_1(X), F_2(X), F_3(X))$, where:

$$F_1(X) = \left[\sum_{a \in L} (E(\omega_a)\hat{g}_a + g_a)\delta_{ap} + \alpha \frac{\partial Var(\sum_{a \in L} \omega_a \hat{g}_a \sum_{q \in \mathcal{P}} x_q \delta_{aq})}{\partial x_p} \right.$$

$$\left. + \lambda_k^+ P_k(\sum_{q \in \mathcal{P}_k} x_q) - \lambda_k^- (1 - P_k(\sum_{q \in \mathcal{P}_k} x_q)) + \sum_{q \in \mathcal{P}} \sum_{a \in L} \mu_q g_a \delta_{aq} \delta_{ap}, \quad p \in \mathcal{P}_k; \quad k = 1, \ldots, n_R \right],$$

$$F_2(X) = \left[\frac{\partial \gamma_k(z)}{\partial z_p} - \mu_p, \quad p \in \mathcal{P}_k; \quad k = 1, \ldots, n_R \right],$$

and

$$F_3(X) = \left[T_{kp} + z_p - \sum_{q \in \mathcal{P}} \sum_{a \in L} g_a x_q \delta_{aq} \delta_{ap}, \quad p \in \mathcal{P}_k; \quad k = 1, \ldots, n_R, \right], \tag{18}$$

then variational inequality (16) can be re-expressed as standard form (17).

We utilize variational inequality (16) for our computations to obtain the optimal path flows and the optimal path time deviations. Then we use (6) to calculate the optimal link flows of disaster relief items in the supply chain network.

3. The Algorithm and Numerical Examples

In this section, we present the Euler method, which is induced by the general iterative scheme of Dupuis and Nagurney (1993) and then apply it to compute solutions to several numerical examples to illustrate the modeling framework. The realization of the Euler method for the solution of mean-variance disaster relief supply chain network problem governed by variational inequality (16) results in subproblems that can be solved explicitly and in closed form. Specifically, recall that at an iteration τ of the Euler method (see also Nagurney and Zhang (1996)) one computes:

$$X^{\tau+1} = P_{\mathcal{K}}(X^\tau - a_\tau F(X^\tau)), \tag{19}$$

where $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$ and F is the function that enters the variational inequality problem: determine $X^* \in \mathcal{K}$ such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{20}$$

where $\langle \cdot, \cdot \rangle$ is the inner product in n-dimensional Euclidean space, $X \in R^n$, and $F(X)$ is an n-dimensional function from $\mathcal{K}$ to R^n, with $F(X)$ being continuous.

As shown in Dupuis and Nagurney (1993); see also Nagurney and Zhang (1996), for convergence of the general iterative scheme, which induces the Euler method, among other methods, the sequence $\{a_\tau\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \to 0$, as $\tau \to \infty$. Specific conditions for convergence of this scheme can be found for a variety of network-based problems, similar to those constructed here, in Nagurney and Zhang (1996) and the references therein.

Explicit Formulae for the Euler Method Applied to the Disaster Relief Supply Chain Network Variational Inequality (16)

The elegance of this procedure for the computation of solutions to the disaster relief supply chain network problem modeled in Section 2 can be seen in the following explicit formulae. Specifically, (19) for the supply chain network problem governed by variational inequality problem (16) yields the following closed form expressions for the product path flows, the time deviations, and the Lagrange multipliers, respectively:

$$x_p^{\tau+1} = \max\{0, x_p^\tau + a_\tau(\lambda_k^-(1 - P_k(\sum_{q\in\mathcal{P}_k} x_q^\tau)) - \lambda_k^+ P_k(\sum_{q\in\mathcal{P}_k} x_q^\tau) - \sum_{a\in L}(E(\omega_a)\hat{g}_a + g_a)\delta_{ap}$$

$$-\alpha\frac{\partial Var(\sum_{a\in L}\omega_a\hat{g}_a\sum_{q\in\mathcal{P}} x_q^\tau\delta_{aq})}{\partial x_p} - \sum_{q\in\mathcal{P}}\sum_{a\in L}\mu_q^\tau g_a\delta_{aq}\delta_{ap})\}, \quad \forall p \in \mathcal{P}_k; \quad k = 1, \ldots, n_R, \quad (21)$$

$$z_p^{\tau+1} = \max\{0, z_p^\tau + a_\tau(\mu_p^\tau - \frac{\partial\gamma_k(z^\tau)}{\partial z_p})\}, \qquad \forall p \in \mathcal{P}_k; \quad k = 1, \ldots, n_R, \text{ and} \qquad (22)$$

$$\mu_p^{\tau+1} = \max\{0, \mu_p^\tau + a_\tau(\sum_{q\in\mathcal{P}}\sum_{a\in L} g_a x_q^\tau\delta_{aq}\delta_{ap} - T_{kp} - z_p^\tau\}, \qquad \forall p \in \mathcal{P}_k; \qquad k = 1, \ldots, n_R. \quad (23)$$

In view of (21), we can define a generalized marginal total cost on path p; $p \in \mathcal{P}$, denoted by $G\hat{C}'_p$, where

$$G\hat{C}'_p \equiv \sum_{a\in L}(E(\omega_a)\hat{g}_a + g_a)\delta_{ap} + \alpha\frac{\partial Var(\sum_{a\in L}\omega_a\hat{g}_a\sum_{q\in\mathcal{P}} x_q\delta_{aq})}{\partial x_p}. \qquad (24)$$

In our numerical examples, we provide explicit formulae for the link generalized marginal total costs, from which the general marginal total cost on each path, as in (24), can be constricted by summing up the former on links that comprise each given path.

3.1 Numerical Examples

In order to fix ideas and concepts, we first present a smaller example for clarity purposes, along with variants, and then construct a larger example, also with a variant. We implemented the Euler method, as described above, in FORTRAN, using a Linux system at the

University of Massachusetts Amherst. The convergence criterion was $\epsilon = 10^{-6}$; that is, the Euler method was considered to have converged if, at a given iteration, the absolute value of the difference of each variable (see (21), (22), and (23)) differed from its respective value at the preceding iteration by no more than ϵ. The sequence $\{a_\tau\}$ was: $.1(1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}\ldots)$. We initialized the algorithm by setting each variable equal to 0.00.

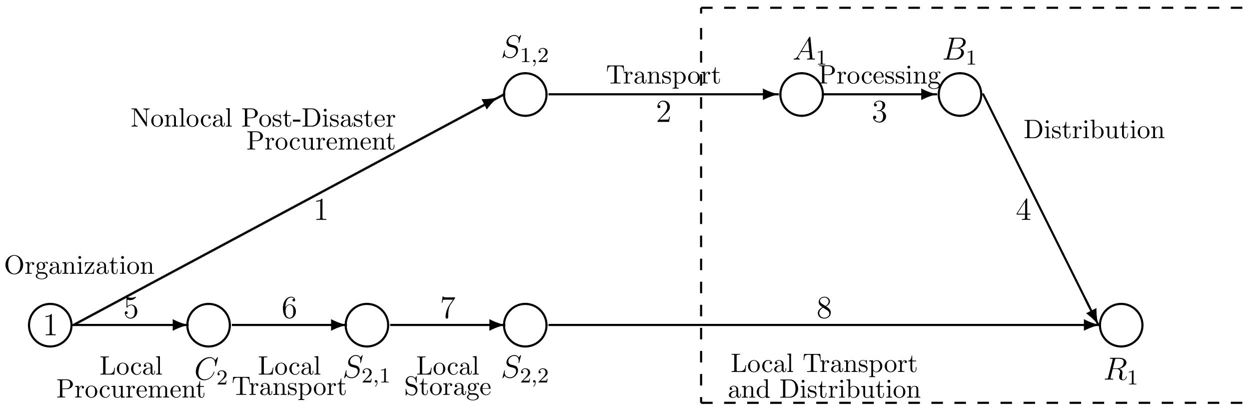

Figure 2: Disaster Relief Supply Chain Network Topology For Example 1 and its Variants

Example 1 and Variants

The disaster relief supply chain network topology for Example 1 and its variants is given in Figure 2. This might correspond to an island location that is subject to major storms. The humanitarian relief organization is depicted by node 1 and there is a single demand point for the relief supplies denoted by R_1, which is located on the island. The organization is considering two options, that is, strategies, reflected by the two paths connecting node 1 with node R_1 with path p_1 consisting of the links: 1, 2, 3, and 4, and path p_2 consisting of the links: 5, 6, 7, and 8. Path p_1 consists of nonlocal post-disaster procurement, transport, processing, and ultimate distribution, whereas path p_2 consists of the activities: local procurement, local transport and local storage, pre-disaster, followed by local transport and distribution. The local transport and distribution are done by ground transport. However, the transport on link 2 is done by air.

The covariance matrix associated with the link total cost functions $\hat{c}_a(f_a, \omega_a)$, $a \in L$, is the 8×8 matrix $\sigma^2 I$. In the variants of Example 1 we explore different values for σ^2 and also different values for α, the risk aversion factor (see (13)). The organization's risk aversion factor $\alpha = 1$ in Example 1 and its Variants 1, 2, and 3.

The demand for the relief item at the demand point R_1 (in thousands of units) is assumed to follow a uniform probability distribution on the interval $[10, 20]$. The path flows and the link flows are also in thousands of units. Therefore,

$$P_{R_1}(\sum_{p\in\mathcal{P}_1} x_p) = \frac{\sum_{p\in\mathcal{P}_1} x_p - 10}{20-10} = \frac{x_{p_1}+x_{p_2}-10}{10}.$$

We now describe how we construct the marginalized total link costs for the numerical examples from which the marginalized total path costs as in (24) are then constructed.

For our numerical examples, we have that:

$$\sum_{a\in L}\sigma^2\hat{g}_a^2 f_a^2 = Var(\sum_{a\in L}\omega_a\hat{g}_a f_a) = Var(\sum_{a\in L}\omega\hat{g}_a\sum_{q\in\mathcal{P}} x_q\delta_{aq}), \tag{25}$$

so that:

$$\frac{\partial Var(\sum_{a\in L}\omega_a\hat{g}_a\sum_{q\in\mathcal{P}} x_q\delta_{aq})}{\partial x_p} = 2\sigma^2\sum_{a\in L}\hat{g}_a^2 f_a\delta_{ap}. \tag{26}$$

In view of (26) and (24) we may define the generalized marginal total cost on a link a, $\hat{g}c'_a$, as:

$$g\hat{c}'_a \equiv E(\omega_a)\hat{g}_a + g_a + \alpha 2\sigma^2\hat{g}_a^2 f_a, \tag{27}$$

so that

$$G\hat{C}'_p = \sum_{a\in L} g\hat{c}'_a\delta_{ap}, \quad \forall p\in\mathcal{P}. \tag{28}$$

Table 2 contains the link total operational cost functions, the expected value of the random variable associated with the total operational cost on each link, and the marginal generalized total link cost, as well as the link time completion functions, and the optimal link flows for Example 1 with $\sigma^2 = .1$ and for Variant 1 with $\sigma^2 = 1$. The time target at demand point R_1, $T_1 = 48$ (in hours). The link time completion functions for links: 5, 6, and 7 are 0.00 since these are completed prior to the disaster and the supplies on the path with these links are, hence, immediately available for local transport and distribution. Also, we set $\lambda_1^- = 1000$ and $\lambda_1^+ = 100$. The organization is significantly more concerned with a shortage of the relief item than with a surplus. The tardiness penalty function $\gamma_{R_1}(z) = 3(\sum_{p\in\mathcal{P}_{R_1}} z_p^2)$.

The optimal flow on path p_1, $x^*_{p_1}$, in Example 1 with $\sigma^2 = .1$ is 4.70. and that for path p_2, $x^*_{p_2}$, is 14.18, with the projected demand $v_{R_1} = x^*_{p_1} + x^*_{p_2} = 18.88$. In Variant 1 of Example 1 with $\sigma^2 = 1$, the new optimal path flow on path p_1, $x^*_{p_1} = 4.90$, and on path p_2, $x^*_{p_2} = 12.84$, with $v_{R_1} = x^*_{p_1} + x^*_{p_2} = 17.74$. The values of $z^*_{p_1}$ and $z^*_{p_2}$ are both 0.00 for both these examples and the Lagrange multipliers $\mu^*_{p_1}$ and $\mu^*_{p_2}$ are also both 0.00 since the time target for delivery at R_1, post-disaster, is met by both paths for R_1.

Table 2: Link Total Cost, Expected Value of Random Link Cost, Marginal Generalized Link Total Cost, and Time Completion Functions for Example 1 and Variant 1 and Optimal Link Flows

Link a	$\hat{c}_a(f_a, \omega_a)$	$E(\omega_a)$	Marginal Generalized Total Link Cost $g\hat{c}_a'$	$\tau_a(f_a)$	f_a^*;$\alpha = 1$; $\sigma^2 = .1$	f_a^*; $\alpha = 1$; $\sigma^2 = 1$
1	$\omega_1 3f_1 + f_1$	1	$\alpha 18\sigma^2 f_1 + 4$	$f_1 + 1$	4.70	4.90
2	$\omega_2 2f_2 + f_2$	1	$\alpha 8\sigma^2 f_2 + 3$	$f_2 + 2$	4.70	4.90
3	$\omega_3 .5f_3 + f_3$	1	$\alpha .5\sigma^2 f_3^2 + 1.5$	$f_3 + .5$	4.70	4.90
4	$\omega_4 .4f_4 + f_4$	1	$\alpha .32\sigma^2 f_4 + 1.4$	$f_4 + 1$	4.70	4.90
5	$\omega_5 2f_5 + f_5$	1	$\alpha 8\sigma^2 f_5 + 3$	0.00	14.18	12.84
6	$\omega_6 .1f_6 + f_6$	1	$\alpha .02\sigma^2 f_6 + 1.1$	0.00	14.18	12.84
7	$\omega_7 f_7 + f_7$	1	$\alpha 2\sigma^2 f_7 + 2$	0.00	14.18	12.84
8	$\omega_8 .5f_8 + f_8$	1	$\alpha .5\sigma^2 f_8 + 1.5$	$.2f_8 + 2$	14.18	12.84

One can see from the optimal solution to Example 1 and Variant 1 that, as the variance-covariance term σ^2 increases from .1 to 1, the amount of optimal flow on path p_2, which corresponds to local procurement, transport, and storage, decreases whereas the amount procured nonlocally post-disaster, increases. Given increased uncertainty as to the operational costs locally since the disaster may impact the storage location(s), for example, and local transport routes as well, it is better to preposition less of the relief item locally. Also, interestingly, when $\sigma^2 = 1$, less of the relief item is provided (17.74) than when $\sigma^2 = .1$ (18.88). The humanitarian relief organization must report to its stakeholders, including donors, and, hence, it must adhere to the minimization of its objective function and with greater variability, there are greater associated costs.

Variants 2 and 3 of Example 1 are constructed as follows and the data are reported in Table 3. For Variant 2, we retain the data for Example 1 with $\sigma^2 = .1$ but now assume that air transport, due to the expected storm damage of the island airport, is no longer possible. Maritime transport is, nevertheless, available, so link 2 in Figure 2 now corresponds to maritime transport rather than air transport. All the data, hence, for Variant 2 are as for Example 1 except that the total operational cost data and the time completion data for link 2 change as reported in Table 3.

Variant 3 is constructed from Variant 2 but with $\sigma^2 = 1$ (as in Variant 1 of Example 1). The optimal solutions for Variants 2 and 3 are reported in Table 3. In Variant 2, only the prepositioning of relief items locally with local procurement as a strategy is optimal since $x^*_{p_1} = 0.00$ and $x^*_{p_2} = 18.84$. The maritime transport is simply too costly. The time target is met with the prepositioning strategy and, hence, the time deviations on the paths, $z^*_{p_1}$ and

$z^*_{p_2}$, are equal to 0.00 as are the path Lagrange multipliers: $\mu^*_{p_1}$ and $\mu^*_{p_2}$. In Variant 3, on the other hand, as the covariance σ^2 term increases from .1 to 1, there is diversification of risk, with both strategies now being applied, that is, maritime transport, post-disaster, and the prepositioning of supplies locally. The time target is met in Variant 3 as well. In Variant 2, $v_{R_1} = 18.84$, whereas in Variant 3, $v_{R_1} = 17.41$. We see, as we did in Table 2, that an increase in σ^2 results in fewer relief supplies being delivered in total according to the optimal solution. Hence, relief organizations should try, if at all possible, to reduce the uncertainty associated with their total operational costs in their disaster relief supply chain networks.

Table 3: Link Total Cost, Expected Value of Random Link Cost, Marginal Generalized Link Total Cost, and Time Completion Functions for Example 1 Variants 2 and 3 and Optimal Link Flows

Link a	$\hat{c}_a(f_a, \omega_a)$	$E(\omega_a)$	Marginal Generalized Total Link Cost $g\hat{c}'_a$	$\tau_a(f_a)$	f^*_a; $\alpha = 1$; $\sigma^2 = .1$	f^*_a; $\alpha = 1$; $\sigma^2 = 1$
1	$\omega_1 3f_1 + f_1$	1	$\alpha 18\sigma^2 f_1 + 4$	$f_1 + 1$	0.00	0.51
2	$\omega_2 12f_2 + 10f_2$	1	$\alpha 288\sigma^2 f_2 + 3$	$3f_2 + 10$	0.00	0.51
3	$\omega_3 .5f_3 + f_3$	1	$\alpha .5\sigma^2 f_3 + 1.5$	$f_3 + .5$	0.00	0.51
4	$\omega_4 .4f_4 + f_4$	1	$\alpha .32\sigma^2 f_4 + 1.4$	$f_4 + 1$	0.00	0.51
5	$\omega_5 2f_5 + f_5$	1	$\alpha 8\sigma^2 f_5 + 3$	0.00	18.84	16.90
6	$\omega_6 .1f_6 + f_6$	1	$\alpha .02\sigma^2 f_6 + 1.1$	0.00	18.84	16.90
7	$\omega_7 f_7 + f_7$	1	$\alpha 2\sigma^2 f_7 + 2$	0.00	18.84	16.90
8	$\omega_8 .5f_8 + f_8$	1	$\alpha .5\sigma^2 f_8 + 1.5$	$.2f_8 + 2$	18.84	16.90

In Variants 4 and 5 we explore the impact on the strategies and on the optimal link flows of increasing the risk aversion factor α. Specifically, in Variant 4 we utilize the Variant 1 data in Table 2 but we increase α to 10 and in Variant 5 we increase α even more to 100. We report the input data and results for $\alpha = 10$ and for $\alpha = 100$ in Table 4.

In Variant 4, the optimal path flow pattern is: $x^*_{p_1} = 3.17$ and $x^*_{p_2} = 8.10$, with $v_{R_1} = 11.27$. In Variant 5, the optimal path flow pattern is: $x^*_{p_1} = .68$ and $x^*_{p_2} = 1.74$, with $v_{R_1} = 2.46$. As the risk-aversion factor α increases, the flows on the paths decrease and, hence, also the total relief supply deliveries at the demand point R_1 decrease. In Variants 4 and 5 the time target is, again, met and, hence, the values of $z^*_{p_1}$, $z^*_{p_2}$ and $\mu^*_{p_1}$ and $\mu^*_{p_2}$ are again all 0.00.

Table 4: Link Total Cost, Expected Value of Random Link Cost, Marginal Generalized Link Total Cost, and Time Completion Functions for Example 1 Variants 4 and 5 and Optimal Link Flows

Link a	$\hat{c}_a(f_a, \omega_a)$	$E(\omega_a)$	Marginal Generalized Total Link Cost $g\hat{c}'_a$	$\tau_a(f_a)$	f_a^*;$\alpha = 10$; $\sigma^2 = 1$	f_a^*; $\alpha = 100$; $\sigma^2 = 1$
1	$\omega_1 3f_1 + f_1$	1	$\alpha 18\sigma^2 f_1 + 4$	$f_1 + 1$	3.17	.68
2	$\omega_2 2f_2 + f_2$	1	$\alpha 8\sigma^2 f_2 + 3$	$f_2 + 2$	3.17	.68
3	$\omega_3 .5f_3 + f_3$	1	$\alpha .5\sigma^2 f_3 + 1.5$	$f_3 + .5$	3.17	.68
4	$\omega_4 .4f_4 + f_4$	1	$\alpha .32\sigma^2 f_4 + 1.4$	$f_4 + 1$	3.17	.68
5	$\omega_5 2f_5 + f_5$	1	$\alpha 8\sigma^2 f_5 + 3$	0.00	8.10	1.74
6	$\omega_6 .1f_6 + f_6$	1	$\alpha .02\sigma^2 f_6 + 1.1$	0.00	8.10	1.74
7	$\omega_7 f_7 + f_7$	1	$\alpha 2\sigma^2 f_7 + 2$	0.00	8.10	1.74
8	$\omega_8 .5f_8 + f_8$	1	$\alpha .5\sigma^2 f_8 + 1.5$	$.2f_8 + 2$	8.10	1.74

Example 2 and Variant

Example 2, and its variant, consider a realistic, larger scenario setting. The supply chain network topology is as given in Figure 3. Specifically, with the larger Example 2, and its variant, we focus on Mexico.

According to the United Nations (2011), Mexico is ranked as one of the world's thirty most exposed countries to three or more types of natural disasters, notably, storms, hurricanes, floods, as well as earthquakes, and droughts. For example, as reported by The International Bank for Reconstruction and Development/The World Bank (2012), 41% of Mexico's national territory is exposed to storms, hurricanes, and floods; 27% to earthquakes, and 29% to droughts. The hurricanes can come from the Atlantic or Pacific oceans or the Caribbean. As noted by de la Fuente (2011), the single most costly disaster in Mexico were the 1985 earthquakes, followed by the floods in the southern state of Tabasco in 2007, with damages of more than 3.1 billion U.S. dollars.

We consider a humanitarian organization such as the Mexican Red Cross, which is interested in preparing for another possible hurricane, and recalls the devastation wrought by Hurricane Manuel and Hurricane Ingrid, which struck Mexico within a 24 hour period in September 2013. Ingrid caused 32 deaths, primarily, in eastern Mexico, whereas Manuel resulted in at least 123 deaths, primarily in western Mexico (NOAA (2014)). According to Pasch and Zelinsky (2014), the total economic impact of Manuel alone was estimated to be approximately $4.2 billion (US), with the biggest losses occurring in Guerrero. In particular,

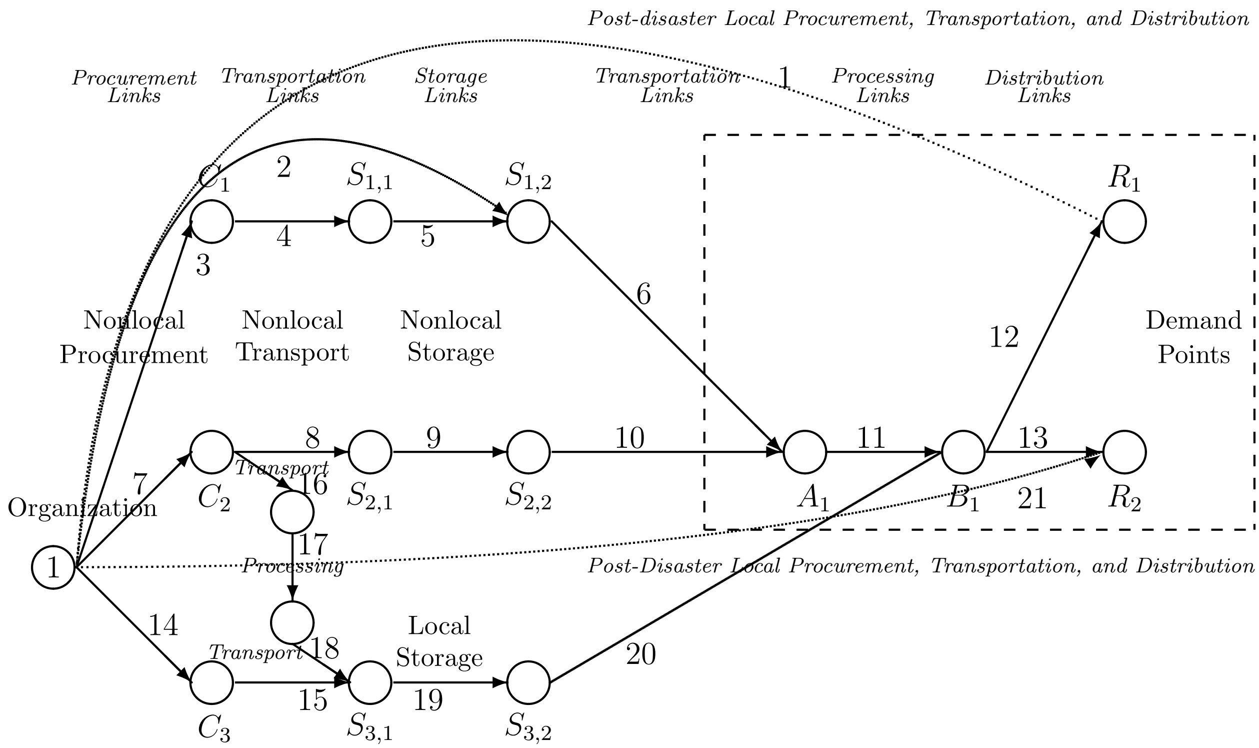

Figure 3: Disaster Relief Supply Chain Network Topology for Example 2 and its Variant

in Example 2, we assume that the Mexican Red Cross is mainly concerned about the delivery of relief supplies to the Mexico City area and the Acapulco area. Ingrid affected Mexico City and Manuel affected the Acapulco area and also points northwest.

The Mexican Red Cross represents the organization in Figure 3 and is denoted by node 1. There are two demand points, R_1 and R_2, for the ultimate delivery of the relief supplies. R_1 is situated closer to Mexico City and R_2 is closer to Acapulco. Nonlocal procurement is done through two locations in Texas, C_1 and C_2. Because of good relationships with the U.S. and the American Red Cross, there are two nonlocal storage facilities that the Mexican Red Cross can utilize, both located in Texas, and represented by links 5 and 9 emanating from $S_{1,1}$ and $S_{2,1}$, respectively. Local storage, on the other hand, is depicted by the link emanating from node $S_{3,1}$, link 19. The Mexican Red Cross can also procure locally (see C_3). Nonlocal procurement, post-disaster, is depicted by link 2, whereas procurement locally, post-disaster, and direct delivery to R_1 and R_2 are depicted by links 1 and 21, respectively. Link 11 is a processing link to reflect processing of the arriving relief supplies from the U.S. and we assume one portal A_1, which is in southcentral Mexico. Link 17 is also a processing link but that processing is done prior to storage locally and pre-disaster. Such a link is needed if the goods are procured nonlocally (link 7). The transport is done via road in the disaster relief supply chain network in Figure 3.

The demand for the relief items at the demand point R_1 (in thousands of units) is assumed to follow a uniform probability distribution on the interval $[20, 40]$. The path flows and the link flows are also in thousands of units. Therefore,

$$P_{R_1}(\sum_{p\in\mathcal{P}_1} x_p) = \frac{\sum_{p\in\mathcal{P}_1} x_p - 20}{40-20} = \frac{\sum_{i=1}^{6} x_{p_i} - 20}{20}.$$

Also, the demand for the relief item at R_2 (in thousands of units) is assumed to follow a uniform probability distribution on the interval $[20, 40]$. Hence,

$$P_{R_2}(\sum_{p\in\mathcal{P}_2} x_p) = \frac{\sum_{p\in\mathcal{P}_1} x_p - 20}{40-20} = \frac{\sum_{i=7}^{12} x_{p_i} - 20}{20}.$$

The time targets for the delivery of supplies at R_1 and R_2, respectively, in hours, are: $T_1 = 48$ and $T_2 = 48$. The penalties at the two demand points for shortages are: $\lambda_1^- = 10,000$ and $\lambda_2^- = 10,000$ and for surpluses: $\lambda_1^+ = 100$ and $\lambda_2^+ = 100$. The tardiness penalty function $\gamma_{R_1}(z) = 3(\sum_{p\in\mathcal{P}_{R_1}} z_p^2)$ and the tardiness penalty function $\gamma_{R_2}(z) = 3(\sum_{p\in\mathcal{P}_{R_2}} z_p^2)$.

As in Example 1 and its variants, we assume that, for Example 2, the covariance matrix associated with the link total cost functions $\hat{c}_a(f_a, \omega_a)$, $a \in L$, is a 21×21 matrix $\sigma^2 I$. In Example 2, $\sigma^2 = 1$ and the risk aversion factor $\alpha = 10$ since the humanitarian organization is risk-averse with respect to its costs associated with its operations.

The additional data for Example 2 are given in Table 5, where we also report the computed optimal link flows via the Euler method, which are calculated from the computed path flows reported in Table 6. Note that the time completion functions in Table 5, $\tau_a(f_a)$, $\forall a \in L$, are 0.00 if the links correspond to procurement, transport, and storage, pre-disaster, since such supplies are immediately available for shipment once a disaster strikes.

The definitions of the paths joining node 1 with R_1 and node 1 with R_2, the optimal path flows, optimal path deviations, and the optimal Lagrange multipliers for Example 2 are reported in Table 6. Note that there are 6 paths joining node 1, representing the organization with R_1, and 6 paths joining node 1 with R_2. The paths represent sequences of decisions and activities that must be executed for the relief supplies to reach the destinations.

The largest volumes of relief supplies flow on paths p_1 and p_6 for R_1 and on paths p_{11} and p_{12} for R_2. All these paths correspond to local procurement. Paths p_6 and p_{11} correspond also to local storage. The projected demands are: $v_{R_1} = 26.84$ and $v_{R_2} = 26.76$.

Table 5: Link Total Cost, Expected Value of Random Link Cost, Marginal Generalized Link Total Cost, and Time Completion Functions for Example 2 and Optimal Link Flows

Link a	$\hat{c}_a(f_a, \omega_a)$	$E(\omega_a)$	Marginal Generalized Total Link Cost $g\hat{c}'_a$	$\tau_a(f_a)$	f^*_a; $\alpha = 10$; $\sigma^2 = 1$
1	$\omega_1 6f_1 + f_1$	2	$\alpha 72\sigma^2 f_1 + 13$	$f_1 + 15$	9.07
2	$\omega_2 3f_2 + f_2$	2	$\alpha 18\sigma^2 f_2 + 7$	$f_2 + 7$	2.54
3	$\omega_3 2f_3 + f_3$	1	$\alpha 8\sigma^2 f_3 + 3$	0.00	2.57
4	$\omega_4 3f_4 + f_4$	1	$\alpha 18\sigma^2 f_4 + 4$	0.00	2.57
5	$\omega_5 2f_5 + f_5$	1	$\alpha 8\sigma^2 f_5 + 3$	0.00	2.57
6	$\omega_6 2f_6 + f_6$	2	$\alpha 8\sigma^2 f_6 + 5$	$2f_6 + 10$	5.11
7	$\omega_7 2f_7 + f_7$	1	$\alpha 8\sigma^2 f_7 + 3$	0.00	8.51
8	$\omega_8 3f_8 + f_8$	1	$\alpha 18\sigma^2 f_8 + 4$	0.00	4.36
9	$\omega_9 2f_9 + f_9$	1	$\alpha 8\sigma^2 f_9 + 3$	0.00	4.36
10	$\omega_{10} 2f_{10} + f_{10}$	1	$\alpha 8\sigma^2 f_{10} + 3$	$2f_{10} + 10$	4.36
11	$\omega_{11} f_{11} + f_{11}$	2	$\alpha 2\sigma^2 f_{11} + 3$	$f_{11} + 2$	9.47
12	$\omega_{12} f_{12} + f_{12}$	2	$\alpha 2\sigma^2 f_{12} + 3$	$f_{12} + 6$	17.78
13	$\omega_{13} f_{13} + f_{13}$	2	$\alpha 2\sigma^2 f_{13} + 3$	$f_{13} + 7$	17.64
14	$\omega_{14} f_{14} + f_{14}$	1	$\alpha 2\sigma^2 f_{14} + 2$	0.00	21.79
15	$\omega_{15} f_{15} + f_{15}$	1	$\alpha 2\sigma^2 f_{15} + 2$	0.00	21.79
16	$\omega_{16} f_{16} + f_{16}$	1	$\alpha 2\sigma^2 f_{16} + 2$	0.00	4.15
17	$\omega_{17} .5f_{17} + f_{17}$	1	$\alpha\sigma^2 .5f_{17} + 1.5$	0.00	4.15
18	$\omega_{18} f_{18} + f_{18}$	1	$\alpha 2\sigma^2 f_{18} + 2$	0.00	4.15
19	$\omega_{19} .5f_{19} + f_{19}$	2	$\alpha\sigma^2 .5f_{19} + 1.5$	0.00	25.94
20	$\omega_{20} f_{20} + f_{20}$	2	$\alpha 2\sigma^2 f_{20} + 2$	$2f_{20} + 5$	25.94
21	$\omega_{21} 6f_{21} + f_{21}$	2	$\alpha 72\sigma^2 f_{21} + 13$	$f_{21} + 14$	9.13

Both pre-positioning and procurement post-disaster strategies are optimal and, hence, used. This makes sense since the organization is interested in risk reduction and, therefore, utilizes a portfolio of strategies. In fact, in Example 2 all paths have positive flow.

Table 6: Path Definitions, Target Times, Optimal Path Flows, Optimal Path Time Deviations, and Optimal Lagrange Multipliers for Example 2

	Path Definition (Links)	x_p^*	z_p^*	μ_p^*
$\mathcal{P}_{R_1}$: Set of Paths Corresponding to Demand Point R_1	$p_1 = (1)$	9.07	0.00	0.00
	$p_2 = (2, 6, 11, 12)$	1.27	34.75	208.53
	$p_3 = (3, 4, 5, 6, 11, 12)$	1.29	25.26	151.56
	$p_4 = (7, 8, 9, 10, 11, 12)$	2.18	23.78	142.69
	$p_5 = (7, 16, 17, 18, 19, 20, 12)$	2.98	50.48	302.85
	$p_6 = (14, 15, 19, 20, 12)$	10.06	50.48	302.85
$\mathcal{P}_{R_2}$: Set of Paths Corresponding to Demand Point R_2	$p_7 = (2, 6, 11, 13)$	1.27	35.48	212.88
	$p_8 = (3, 4, 5, 6, 11, 13)$	1.29	25.99	155.91
	$p_9 = (7, 8, 9, 10, 11, 13)$	2.18	24.51	147.04
	$p_{10} = (7, 16, 17, 18, 19, 20, 13)$	1.17	51.20	307.19
	$p_{11} = (14, 15, 19, 20, 13)$	11.74	51.20	307.19
	$p_{12} = (21)$	9.13	0.00	0.00

Example 2 - Variant 1

In Variant 1 of Example 2, we kept the data as in Example 2, but now we assumed that the humanitarian organization has a better forecast for the demand at the two demand points. The demand for the relief items at the demand point R_1 again follows a uniform probability distribution but on the interval $[30, 40]$ so that:

$$P_{R_1}(\sum_{p\in\mathcal{P}_1} x_p) = \frac{\sum_{p\in\mathcal{P}_1} x_p - 30}{40 - 30} = \frac{\sum_{i=1}^{6} x_{p_i} - 30}{10}.$$

Also, the demand for the relief item at R_2 follows a uniform probability distribution on the interval $[30, 40]$ so that:

$$P_{R_2}(\sum_{p\in\mathcal{P}_2} x_p) = \frac{\sum_{p\in\mathcal{P}_2} x_p - 30}{40 - 30} = \frac{\sum_{i=7}^{12} x_{p_i} - 30}{10}.$$

The computed path flows are reported in Table 7.

The projected demands are: $v_{R_1} = 31.84$ and $v_{R_2} = 31.79$. The greatest percentage increase in path flow volumes occurs on paths p_1 and p_6 for demand point R_1 and on paths p_{11} and p_{12} for demand point R_2, reinforcing the results obtained for Example 2.

For both Example 2 and its variant the time targets are met for paths p_1 and p_2 since $\mu_{p_1}^*$ and $\mu_{p_2}^* = 0.00$ for both examples. Hence, direct local procurement post-disaster is effective time-wise, and cost-wise. Mexico is a large country and this result is quite reasonable.

Table 7: Path Definitions, Target Times, Optimal Path Flows, Optimal Path Time Deviations, and Optimal Lagrange Multipliers for Variant 1 of Example 2

	Path Definition (Links)	x_p^*	z_p^*	μ_p^*
$\mathcal{P}_{R_1}$: Set of Paths Corresponding to Demand Point R_1	$p_1 = (1)$	11.30	0.00	0.00
	$p_2 = (2, 6, 11, 12)$	1.37	43.13	258.78
	$p_3 = (3, 4, 5, 6, 11, 12)$	1.49	33.42	200.49
	$p_4 = (7, 8, 9, 10, 11, 12)$	2.58	32.28	193.69
	$p_5 = (7, 16, 17, 18, 19, 20, 12)$	2.81	64.37	386.19
	$p_6 = (14, 15, 19, 20, 12)$	12.29	64.37	386.19
$\mathcal{P}_{R_2}$: Set of Paths Corresponding to Demand Point R_2	$p_7 = (2, 6, 11, 13)$	1.37	43.92	263.49
	$p_8 = (3, 4, 5, 6, 11, 13)$	1.49	34.20	205.20
	$p_9 = (7, 8, 9, 10, 11, 13)$	2.57	33.07	198.40
	$p_{10} = (7, 16, 17, 18, 19, 20, 13)$	1.96	65.15	390.90
	$p_{11} = (14, 15, 19, 20, 13)$	13.04	65.15	390.90
	$p_{12} = (21)$	11.36	0.00	0.00

4. Summary and Conclusions

In this paper, we developed a mean-variance disaster relief supply chain network model for risk reduction with stochastic link costs, uncertain demands for the relief supplies and time targets associated with the demand points. The humanitarian organization seeks to minimize the expected value of the total operational costs and the weighted variance of these costs in the supply chain network plus the penalized expected shortages and surpluses as well as the deviations from the time targets. Each link has an associated time completion function and the decision-maker determines his risk-aversion. This framework handles, in an integrated manner, both the pre-positioning of supplies, which can be local or nonlocal, as well as the procurement of supplies, both local and nonlocal, post-disaster. The model allows for the investigation of the optimal strategies associated with the paths which are composed of links comprising the necessary activities from procurement to ultimate delivery of the relief supplies to the victims at the demand points.

We presented the optimization model, along with its variational inequality formulation, which enables computation via the Euler method, which, in turn, yields closed form expressions for the path variables at each iteration. Through a series of numerical examples, we illustrated the concepts and computational procedure. Specifically, we presented a series of smaller examples and then construct a set of larger examples, based on a study focusing on Mexico, in the case of hurricanes. We find that, in the case of the Mexico study, although all strategies are used, in that all the path flows are positive, the most highly recommended strategies, in terms of path flow volumes, are those with local procurement, whether with storage pre-disaster, or direct procurement, post-disaster.

The model extends the model of Nagurney, Masoumi, and Yu (2015) in several dimensions:

1. It considers stochastic link costs, which are relevant given uncertainty in disaster relief supply chain network operations.

2. The objective function includes the minimization of the expected costs as well as the variance with an associated weight for the latter to denote the humanitarian organization's value of risk reduction.

3. The supply chain network topology allows for the procurement and pre-positioning of supplies locally and is more general than that in earlier literature.

4. The generality of the framework allows for numerous sensitivity analysis exercises to evaluate risk-aversion, the assessment of the impacts of the size of penalties on shortages and supplies, as well as modifications to the cost and time completion functions.

The framework consolidates decision-making associated with two phases of disaster management: preparedness and response, incorporates uncertainty in costs and demands and includes the critical time element. Future research may include extending this framework to assess synergies associated with horizontal cooperation among humanitarian organizations in relief operations. In addition, it would be interesting to consider multiple supplies with different associated priorities as well as to include transportation time uncertainty in future work.

Acknowledgments

This paper is dedicated to the students in Professor Anna Nagurney's Humanitarian Logistics and Healthcare class in 2015 at the Isenberg School of Management and to all the victims of natural disasters over the centuries as well as to humanitarian professionals.

Professor Anna Nagurney thanks Professor Panos M. Pardalos of the University of Florida and Professor Ilias Kootsireas of Wilfrid Laurier University for the great collaboration on the co-organization the 2nd International Conference on Dynamics of Disasters in Kalamata, Greece.

The authors also thank the speakers and participants in the conference for comments and stimulating discussions on themes of the conference.

The authors acknowledge helpful comments from two anonymous reviewers on an earlier version of this paper and acknowledge Professor Kotsireas for handling the reviewing process.

References

Balcik, B., Beamon, B.M. (2005) Facility location in humanitarian relief. *International Journal of Logistics, Research and Applications* 11(2), 101–121.

Barbarosoglu, G., Arda, Y. (2004) A two-stage stochastic programming framework for transportation planning in disaster response. *The Journal of the Operational Research Society* 55(1), 43–53.

BBC News (2015) Nepal quake: Towns near epicentre 'devastated' - Red Cross. May 1.

Beamon, B., Kotleba, S. (2006) Inventory management support systems for emergency humanitarian relief operations in South Sudan. *International Journal of Logistics Management* 17(2), 187–212.

Boyles, S.D., Waller, S.T. (2010) A mean-variance model for the minimum cost flow problem with stochastic arc costs. Networks 56(3), 215-227.

Bozorgi-Amiri, A., Jabalameli, M.S., Mirzapour Al-e-Hashem, S.M.J. (2013) A multi-objective robust stochastic programming model for disaster relief logistics under uncertainty. *OR Spektrum* 35, 905-933.

CBS News (2013) Typhoon Haiyan slams into northern Vietnam, November 10. Available online at: http://www.cbsnews.com/news/typhoon-haiyan-slams-into-northern-vietnam.

Chen, F., Federgruen, A. (2000) Mean-variance analysis of basic inventory models. Working Paper, Columbia Business School, Columbia University, New York.

Chicago Tribune (2013) Typhoon Haiyan: Desperate Philippine survivors turn to looting, November 13. Available online at: http://http://www.chicagotribune.com/news/chi-philippines-typhoon-haiyan-20131113,0,4099086,full.story.

Das, R. (2014) *Advancement on Uncertainty Modeling in Humanitarian Logistics for Earthquake Response*. PhD dissertation, The Graduate School of Science and Engineering, Department of International Development Engineering, Tokyo Institute of Technology, Japan.

de la Fuente, A. (2011) Government expenditures in pre and post-disaster risk management. The World Bank, Washington, DC.

Dong, J., Zhang, D., Nagurney, A. (2004) A supply chain network equilibrium model with random demands. *European Journal of Operational Research* 156, 194-212.

Dupuis, P., Nagurney, A. (1993) Dynamical systems and variational inequalities. *Annals of Operations Research* 44, 9-42.

Falasca, M., Zobel, C.W. (2011) A two-stage procurement model for humanitarian relief supply chains. *Journal of Humanitarian Logistics and Supply Chain Management* 1(2), 151-169.

Fugate, W. (2012) The state of FEMA – Leaning forward: Go big, go early, go fast, be smart. A report by the US Department of Homeland Security. Available online at: www.fema.gov/pdf/about/state_of_fema/state_of_fema.pdf.

Hale, T., Moberg, C. (2005) Improving supply chain disaster preparedness: A decision process for secure site location. *International Journal of Physical Distribution & Logistics Management* 35(3), 195-207.

Harris, G. (2015) Nepal's bureaucracy blamed as quake relief supplies pile up. *The New York Times*, May 4.

Huang, M., Smilowitz, K., Balcik, B. (2012) Models for relief routing: Equity, efficiency and efficacy. *Transportation Research E* 48, 2-18.

Kim, S.H., Cohen, M., Netessine, S. (2007) Performance contracting in after-sales service supply chains. *Management Science* 53, 1843-1858.

Liu, Z., Nagurney, A. (2011) Risk reduction and cost synergy in mergers and acquisitions via supply chain network integration. *Journal of Financial Decision-Making* 7(2), 1-18.

Lorch, D. (2015) Rural Nepal still waiting for relief after devastating earthquake. *USA Today*, My 5.

Luke, S., McVicker, L. (2015). Local Nrpal earthquake survivor: "it was utter chaos." San Diego News, April 29.

Markowitz, H. (1952) Portfolio selection. *Journal of Finance* 56(6), 2019-2065.

Markowitz, H. (1959) *Portfolio Selection: Efficient Diversification of Investment.* John Wiley & Sons, New York.

Matopoulos, A., Kovacs, G., Hayes, O. (2014) Local resources and procurement practices in humanitarian supply chains: An empirical examination of large-scale house reconstruction projects. *Decision Sciences* 25(4), 621-646.

Mete, H.O., Zabinsky, Z.B. (2010) Stochastic optimization of medical supply location and distribution in disaster management. *International Journal of Production Economics* 126, 76-84.

Nagurney, A. (1999) *Network Economics: A Variational Inequality Approach*, second and revised edition, Kluwer Academic Publishers, Dordrecht, The Netherlands.

Nagurney, A., Masoumi, A.H. (2012) Supply chain network design of a sustainable blood banking system. In *Sustainable Supply Chains: Models, Methods and Public Policy Implications*, T. Boone, V. Jayaraman, and R. Ganeshan, Editors, Springer, London, England, pp 49-72.

Nagurney, A., Masoumi, A.H., and Yu, M. (2012), Supply Chain Network Operations Management of a Blood Banking System with Cost and Risk Minimization, *Computational Management Science* 9(2): 205-231.

Nagurney, A., Masoumi, A.H., Yu, M. (2015) An integrated disaster relief supply chain network model with time targets and demand uncertainty. In *Regional Science Matters: Studies Dedicated to Walter Isard*, P. Nijkamp, A. Rose, and K. Kourtit, Editors, Springer International Publishing Switzerland, pp 287-318.

Nagurney, A., Qiang, Q. (2009) *Fragile Networks: Identifying Vulnerabilities and Synergies in an Uncertain World*. John Wiley & Sons, Hoboken, New Jersey.

Nagurney, A., Qiang, Q, (2012) Fragile networks: Identifying vulnerabilities and synergies in an uncertain age. *International Transactions in Operational Research* 19, 123-160.

Nagurney, A., Yu, M., Qiang, Q. (2011) Supply chain network design for critical needs with outsourcing. *Papers in Regional Science* 90(1), 123-143.

Nagurney, A., Yu, M., Qiang, Q. (2012) Multiproduct humanitarian healthcare supply chains: A network modeling and computational framework. In the *Proceedings of the 23rd Annual POMS Conference*, Chicago, Illinois.

Nagurney, A., Zhang, D. (1996) *Projected Dynamical Systems and Variational Inequalities with Applications*. Kluwer Academic Publishers, Boston, Massachusetts.

NOAA (2014) WMO retires Ingrid and Manuel for Atlantic and eastern North Pacific basins. April 10.

Ortuño, M. T., Cristóbal, P., Ferrer, J. M., Martín-Campo, F. J., Muñoz, S., Tirado, G., Vitoriano, B. (2013) Decision aid models and systems for humanitarian logistics: A survey. In *Decision Aid Models for Disaster Management and Emergencies*, Atlantis Computational Intelligence Systems, vol. 7, Vitoriano, B., Montero, J., Ruan, D., Editors, Springer Business + Science Media, New York, 17-44.

Ortuño, M.T., Tirado, G., Vitoriano, B. (2011) A lexicographical goal programming based decision support system for logistics of humanitarian aid. *TOP* 19(2), 464-479.

Ozbay, K., Ozguven, E.E. (2007) Stochastic humanitarian inventory control model for disaster planning. *Proceedings of the Transportation Research Boards 86th Annual Meeting*, Washington, DC.

Pasch, R.J., Zelinsky, D.A. (2014) Hurricane Manuel. National Hurricane Center, Tropical cyclone report. EP132013, revised April 14.

Qiang, Q., Nagurney, A. (2012) A bi-criteria indicator to assess supply chain network performance for critical needs under capacity and demand disruptions. *Transportation Research A* 46(5), 801-812.

Rottkemper, B., Fischer, K., Blecken, A. (2012) A transshipment model for distribution and inventory relocation under uncertainty in humanitarian operations. *Socio-Economic Planning Sciences* 46, 98-109.

Schneeweis, T., Crowder, G., Kazemi, H. (2010) *The New Science of Asset Allocation Risk Management in a Multi-Asset World*. John Wiley & Sons, Hoboken, New Jersey.

The International Bank for Reconstruction and Development/The World Bank (2012) FONDEN: Mexico's natural disaster fund - A review. Washington, DC.

Toyasaki, F., Wakolbinger, T. (2014) Impacts of earmarked private donations for disaster fundraising. *Annals of Operations Research* 221(1), 427-447.

Tzeng, G.-H., Cheng, H.-J., Huang, T. (2007) Multi-objective optimal planning for designing relief delivery systems. *Transportation Research E* 43(6), 673-686.

UNHCR (2007) United Nations High Commissioner for Refugees UNHCR Handbook for Emergencies, third edition, July.

United Nations (2011). Global assessment report on disaster reduction.

USAID (2005) United States Agency for International Development, Office of Foreign Disaster Assistance, Field operations guide for disaster assessment and response, Version 4.0.

Van Wassenhove, L.N. (2006). Blackett memorial lecture. Humanitarian aid logistics: Supply chain management in high gear. *Journal of the Operational Research Society* 57(5), 475489.

Vitoriano, B., Montero, J., Ruan, D., Editors, *Decision Aid Models for Disaster Management and Emergencies*, Atlantis Computational Intelligence Systems, vol. 7. Springer Business + Science Media, New York.

Vitoriano, B., Ortuño, M., Tirado, G., Montero, M. (2011) A multi-criteria optimization model for humanitarian aid distribution. *Journal of Global Optimization* 51, 189-208.

Walton, R., Mays, R., Haselkorn, M. (2011) Defining "fast": Factors affecting the experience of speed in humanitarian logistics. In *Proceedings of the 8th International ISCRAM Conference*, Lisbon, Portugal, May.