Consumer Learning of Product Quality with Time Delay: Insights from Spatial Price Equilibrium Models with Differentiated Products

Dong “Michelle” Li[1], Anna Nagurney[2], and Min Yu[3]

[1]Department of Management & Marketing
College of Business
Arkansas State University, State University, Arkansas 72467

[2]Department of Operations and Information Management
Isenberg School of Management
University of Massachusetts, Amherst, Massachusetts 01003

[3]Pamplin School of Business Administration
University of Portland
Portland, Oregon 97203

October 2016; revised April 2017, August 2017, and October 2017

Omega (2018), **18**, pp 150-168.

Abstract:

In this paper, we present spatial price equilibrium network models, both static and adaptive, with differentiated products under perfect quality information for producers and consumers and under quality information asymmetry with consumer learning of product quality with a time delay. The adaptive model with information asymmetry is able to adapt to the uncertainty in consumer learning as well as in supply, demand, transportation cost, and product quality over time. In addition, we provide measures of consumer welfare under perfect quality information and under information asymmetry as well as the value of perfect quality information for consumers. The models are formulated and qualitatively analyzed using variational inequality theory. We establish theoretically and illustrate computationally that, under appropriate assumptions, the equilibrium solution, consisting of supply and demand markets prices, quality levels, and product flows, of the adaptive spatial price equilibrium model with information asymmetry converges to that of the corresponding static model with perfect quality information. The models are especially relevant to agricultural products where spatial price equilibrium models have found wide application. We also present several numerical examples with practical insights provided.

Keywords: spatial price equilibrium, consumer learning, networks, information asymmetry, product quality, value of perfect information, adaptive model

1. Introduction

Spatial price equilibrium models have served as the foundation for the study of numerous perfectly competitive markets, including agricultural and energy markets, since the pioneering work of Samuelson (1952) and Takayama and Judge (1971). Notably, the models recognize that supply markets are generally spatially dispersed and the same holds for the demand markets for products, with transportation costs playing a crucial role in the differences between the associated supply market prices and demand market prices. Such models, which often also have a network structure (see Dafermos and Nagurney (1984)), have captured the interest of researchers as well as practitioners in regional science, economics, and operations research/management science and also form the foundations for a variety of supply chain network equilibrium models (cf. Nagurney (2006) and Zhang (2006)).

Spatial price equilibrium models can be quantity-based or price-based, static or dynamic (see, e.g., Nagurney (1999) and the references therein), and their formulation, analysis, and solution has utilized optimization theory, variational inequality theory, as well as complementarity theory, often coupled with network theory. For a survey of spatial price equilibrium models, with a focus on transportation, see van den Bergh, Nijkamp, and Rietveld (1996). For a general survey of spatial price equilibrium models, see Labys and Yang (1997) and for a related survey focusing on spatial economic location, see Kilkenny and Thisse (1999).

As emphasized in Nagurney, Li, and Nagurney (2014), specific examples of product-based applications of spatial price equilibrium models have included multiple ones in agricultural industries (see, e.g., Thompson (1989)), such as beef (Sohn (1970)), potatoes (Howard (1984)), dairy (Bishop, Pratt, and Novakovic (1994)), cereal grains (Ruijs et al. (2001)), soybeans (Barraza De La Cruz, Pizzolato, and Barraza De La Cruz (2010)), with an early application being to eggs (cf. Judge (1956)). Furthermore, spatial price equilibrium models are also pertinent to the mineral ore and energy industries (see Hwang et al. (1994), Labys and Yang (1997), and Labys (1999)), especially to the coal (Newcomb and Fan (1980)), aluminum (Newcomb, Reynolds, and Masbruch (1990)), and natural gas (Irwin and Yang (1996)) sectors.

Interestingly, although many of the applications of spatial price equilibrium models arise in sectors in which products are used regularly and repetitively by consumers, notably, agricultural products, there has been only limited research, to-date, surrounding the integration of product quality into such models. Furthermore, especially in the case of agricultural products there have been numerous cases of serious shortcoming in terms of quality of food (Strom (2013) and McDonald (2014)), which have resulted in illnesses and even death. For example, in 2009, over 400 peanut butter products were recalled after 8 people died and more than 500 people were sickened by salmonella poisoning, the source of which was a peanut butter plant in Georgia (Harris (2009)). Moreover, in early 2016, CNN reported that packaged salads contaminated with listeria virus, which were sold under a variety of names and came from a processing facility in Springfield, OH, had killed one and left eleven hospitalized (Christensen (2016)). The only research that we are aware of that includes product quality in spatial price equilibrium models is that of Nagurney, Li, and Nagurney (2014), who explored information asymmetry in a spatial price equilibrium model with homogeneous products in which producers at supply markets are aware of the quality of their products but consumers at the demand markets are unaware of the origin of the products and respond to the *average* quality of the product at the demand markets. The impact of minimum quality standards was also investigated therein.

In this paper, in contrast, we present two distinct, novel models. We, nevertheless, retain the definition of quality used by Nagurney, Li, and Nagurney (2014), which has also been utilized by numerous authors (see, e.g., Nagurney and Li (2016)); that is, we define quality as "the degree to which a specific product conforms to a design or specification." The first model is a static model, as is the model of Nagurney, Li, and Nagurney (2014), but it is a spatial price equilibrium model with perfect information on product quality and with differentiated products in which consumers at the demand markets recognize that the products come from different supply markets. Moreover, supply and demand functions are utilized, rather than supply price and demand price functions. The second model is an adaptive spatial price equilibrium model under quality information asymmetry, in which consumers at the demand markets learn the quality of the product with a time delay. In response to consumer learning of quality, producers at the supply markets adapt their decisions over time. In addition, this model also allows them to adapt to the unpredictable changes in supply, demand, transportation cost, and product quality over time.

We also provide measures to quantify the consumer welfare under the scenario of perfect information and that under quality information asymmetry, for each pair of supply and demand markets, along with the value of perfect information for consumers. The impacts of quality information asymmetry and consumer learning of quality on the dynamics of equilibrium supply price, demand price, shipment, and quality pattern and consumer welfare are studied by comparing the two models. Both models are formulated and qualitatively analyzed using variational inequality theory. We theoretically establish that, as time approaches infinity, the equilibrium supply prices, demand prices, product quality, and product shipments between supply and demand markets under information asymmetry converge to the equilibrium of the corresponding static model under perfect information. In addition, we demonstrate this result computationally through several numerical examples with practical insights provided.

Most of the learning models of quality in the field of operations research, such as those in Yelle (1979), Fine (1986), Koulamas (1992), Teng and Thompson (1996), Wang and Lee (2001), Vörös (2006), and Khan, Jaber, and Ahmad (2014), provided insights on the statics or dynamics of firms'/suppliers' knowledge and the learning of quality in production and in product design. The results in this paper, in contrast, capture the statics and dynamics of the equilibrium quality knowledge of both suppliers and consumers with consumer learning of product quality.

The necessity and importance of incorporating consumer learning of quality in models for food and agricultural industries, where the spatial price equilibrium models are widely applied and quality information asymmetry present, are revealed and emphasized in various empirical studies, such as in Hayes et al. (1995), Johnston et al. (2001), and Huffman et al. (2007). Furthermore, one of the pioneering studies in consumer learning of product quality, Shapiro (1982), argued that consumer learning should involve adjusting towards true quality. Erdem and Keane (1996) assumed that consumers' past quality perception and choice affected current quality and choice. Mehta, Rajiv, and Srinivasan (2004) and Zhao, Zhao, and Helsen (2011) extended Erdem and Keane (1996) and stated that consumers' behavior depended on the timing of signals. Erdem, Keane, and Sun (2008) then modeled the signaling role of price in frequently purchased consumer goods.

Some additional empirical work on consumer learning and quality is as follows. Israel (2005) applied a model with consumer learning, departure, and purchasing in the case of automobile insurance. Iyengar, Ansari, Gupta (2007) developed a model with consumer learning of quality and quantity with pricing schemes. Ching (2010) investigated aggregated consumer learning with consumer heterogeneity due to price sensitivity to study the dynamics of the demand for prescription drugs. Archak, Ghose, and Ipeirotis (2011) estimated a model based on sales and consumer data from Amazon to study consumer learning of reviews and quality. Zhao et al. (2013) proposed a model with consumer learning and quality to study the effect of online product reviews on consumer purchases of experiential products.

In addition, in a literature review of consumer learning by Ching, Erdem, and Keane (2013), it is pointed out that integrating learning models of demand with supply side models remains under-explored and should be another important area for future research. Indeed, this gap is filled by our results, since, as mentioned above, both the supply side and the demand side and the evolution and adaption of product prices and flows over space and time are captured in this paper along with consumer learning of product quality. It also contributes to the existing literature on consumer learning in terms of generality and scope. The underlying network of the models consists of multiple differentiated supply markets, multiple spatially distributed demand markets, and the flows on links joining each pair of them.

The paper is organized as follows. In Section 2, we present both spatial price equilibrium models, state the equilibrium conditions, and derive the variational inequality formulations. We also present the measures of consumer welfare for both models, along with the value of perfect information. In Section 3, we present qualitative properties of the equilibrium patterns, specifically, existence and uniqueness results. We establish that, the equilibrium pattern of the adaptive spatial price equilibrium model with quality information asymmetry, over time, approaches that of the corresponding static spatial price equilibrium model with perfect quality information. The algorithm is presented in Appendix C and explicit formulae derived for the computation of the underlying variables of the models at each iteration. Section 4 then presents numerical examples with practical insights that further support the modeling framework, theoretical contributions, and the applications of the paper. We summarize our results and present our conclusions in Section 5.

2. Spatial Price Equilibrium with Quality Information and Product Differentiation

In this Section, two spatial price equilibrium models with quality information and product differentiation are presented with associated variational inequality formulations. These two models are developed under the scenarios of, respectively, perfect quality information and information asymmetry in quality. We also measure the consumer welfare at equilibrium under these two scenarios and construct the value of perfect quality information for consumers.

Please refer to Figure 1 for the underlying network structure of the spatial price equilibrium problem with quality information and product differentiation.

We assume that there are m supply markets and n demand markets that are spatially separated. A typical supply market is denoted by i; $i = 1\ldots,m$, and a typical demand market is denoted by j; $j = 1,\ldots,n$. There is a distinct but substitutable product produced by each of the m supply markets and is consumed at the n demand markets. The consumers at the demand markets differentiate the products by their supply markets, that is, their

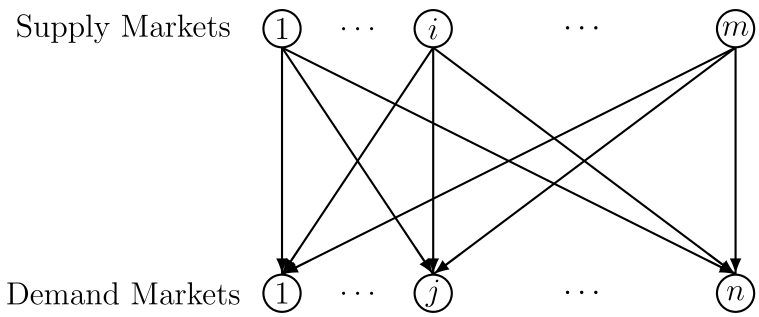

Figure 1: The Bipartite Network Structure of the Spatial Price Equilibrium Problems with Quality Information and Product Differentiation

points of origins.

2.1 Spatial Price Equilibrium with Product Differentiation Under Perfect Quality Information

We first develop the spatial price equilibrium model with product differentiation under the scenario of perfect quality information. Let π_i denote the nonnegative supply price at supply market i, and let ρ_{ij} denote the nonnegative demand price of product i (i.e., the product produced at supply market i) at demand market j. Q_{ij} is the nonnegative shipment of product i to demand market j. We group supply prices into the vector $\pi \in R^m_+$, demand prices into the vector $\rho \in R^{mn}_+$, and product shipments into the vector $Q \in R^{mn}_+$. In this paper, all vectors are assumed to be column vectors.

The supply price at supply market i is the price charged at the supply market for one unit of product i. It identifies the total cost associated with the supply market in the case of perfect competition, where spatial price equilibrium models are often applied to markets of agricultural products. A great number of studies, models, and applications with supply prices can be found in the economics and regional science literature (e.g., Harberger (1971), Florian and Los (1982), Krichene (2002), and Ball, Meen, and Nygaard (2010)) and in the operations research and management science literature (e.g., McCarl, Moskowitz, and Furtan (1977), Dafermos and Nagurney (1984), Dong et al. (2005), and Zhang, Shang, and Li (2011)). In this paper, we assume that the quantities of the product to provide depend on their supply prices. This assumption is reasonable and practical and has been used in many applications of spatial price equilibrium models (e.g., Dafermos (1986), Ruijs et al. (2001), and Barraza De La Cruz, Pizzolato, and Barraza De La Cruz (2010)).

The product supply/output produced at supply market i is denoted by s_i. We allow for the general situation where the supply at supply market i may depend upon the entire supply price pattern, that is,

$$s_i = s_i(\pi), \quad i = 1, \ldots, m. \tag{1}$$

The function s_i is assumed to be monotonically increasing in its own supply price π_i, but monotonically decreasing in the supply prices of the other supply markets.

Let c_{ij} denote the unit transportation cost associated with shipping the product from supply market i to demand market j, where c_{ij} is given by

$$c_{ij} = c_{ij}(Q), \quad i = 1, \ldots, m; j = 1, \ldots, n. \tag{2}$$

The unit transportation cost also includes, as appropriate, any unit transaction cost. We let the unit transportation costs depend, in general, upon the entire product shipment pattern, and they are assumed to be monotonically increasing in product shipments.

Let q_i denote the quality level of product i, which depends on the supply price of product i (Akerlof (1970) and Leland (1979)), that is,

$$q_i = q_i(\pi_i), \quad i = 1, \ldots, m. \tag{3}$$

The function q_i is assumed to be monotonically increasing in π_i. This is reasonable since we expect that enhanced quality comes at a higher supply price. We group all $q_i(\pi_i)$ elements into the vector $q(\pi)$.

In practice, the quality of a product is determined by the effort and amount that the supplier spends on the product, which, more specifically, are the effort/costs in making, ensuring, and assuring the quality of the product. Such effort/costs and the relationships between them and the product quality (Juran and Gryna (1993), Dale and Wan (2002), and Kannan and Tan (2005)) can be quantified, measured, and implemented by firms via a variety of schemes in practice, some of which have been described in Juran and Gryna (1988), Feigenbaum (1991), and Schiffauerova and Thomson (2006).

The supply markets in our models, which produce the products, measure, quantify, and know the relationships between the effort/costs associated with their products and the actual product quality and determine their product quality based on the effort/costs they can afford. Such effort/costs of each supply market are identified by its supply price (the price charged at the supply market). Thus, in our paper, the quality of a product depends on the associated supply price. The higher the price charged by a supplier, the more effort/costs the supplier will be able to spend in providing a higher quality product. The evidence that quality is determined by cost/payment can be found in the literature (e.g., Keller and Noori (1988), Kaya and Özer (2009), and Liu, Zhang, and Tang (2015)).

Under perfect quality information, the actual quality levels of the products are known to consumers as soon as they observe the products. Therefore, consumers' perception of the quality of product i at demand market j, denoted by $\hat{q}_{ij}$, is the same as the actual quality of the product, that is,

$$\hat{q}_{ij} = q_i = q_i(\pi_i), \quad i = 1, \dots, m; j = 1, \dots, n. \tag{4}$$

We group all $\hat{q}_{ij}$s into the vector $\hat{q}$. Here, it is assumed that there is no deterioration in product quality during the transportation process; hence, the actual product quality at the supply markets and that perceived by consumers at the demand markets are exactly the same in this model with perfect quality information.

Furthermore, let d_{ij} denote the demand for product i at demand market j. The function d_{ij} may depend, in general, upon the entire demand price pattern, as well as on consumers' perception of the quality levels of all products, since consumers located at the demand markets respond not only to the prices of the products but also to their perception of product quality. Thus, we have

$$d_{ij} = d_{ij}(\rho, \hat{q}), \quad i = 1, \dots, m; j = 1, \dots, n. \tag{5a}$$

The function d_{ij} is assumed to be monotonically increasing in the perception of the quality of product i, but monotonically decreasing in the perceived quality levels of the other products. It is also assumed to be monotonically decreasing in product i's demand prices, but monotonically increasing in the demand prices of the other products (Banker, Khosla, and Sinha (1998)).

Since, under perfect quality information, consumers' knowledge and perception of product quality is the same as the actual product quality (cf. (4)), we can re-express the demand functions d_{ij}; $i = 1, \dots, m$; $j = 1, \dots, n$, in demand prices and the actual product quality, that is,

$$d_{ij} = d_{ij}(\rho, q(\pi)), \quad i = 1, \dots, m; j = 1, \dots, n. \tag{5b}$$

Shipment quantities, supply prices, and demand prices must be nonnegative. Therefore, we have

$$Q_{ij} \geq 0, \quad i = 1, \dots, m; j = 1, \dots, n, \tag{6}$$

$$\rho_{ij} \geq 0, \quad i = 1, \dots, m; j = 1, \dots, n, \tag{7}$$

$$\pi_i \geq 0, \quad i = 1, \ldots, m, \tag{8}$$

and we define the feasible set $K^1 \equiv \{(Q, \rho, \pi) \in R_+^{2mn+m}\}$.

We assume that all the supply, demand, quality, and unit transportation cost functions are continuous.

We now state the spatial price equilibrium conditions with product differentiation under perfect quality information. The equilibrium conditions are based on the well-known spatial price equilibrium conditions of Samuelson (1952) and Takayama and Judge (1971) (see also Nagurney (1999)) but with product quality included.

Definition 1: Spatial Price Equilibrium Conditions with Product Differentiation Under Perfect Quality Information

A product shipment, demand price, and supply price pattern $(Q^*, \rho^*, \pi^*) \in K^1$ is a spatial equilibrium with product differentiation under perfect quality information if it satisfies the following conditions: for each pair of supply and demand markets (i, j); $i = 1, \ldots, m$; $j = 1, \ldots, n$:

$$\pi_i^* + c_{ij}(Q^*) \begin{cases} = \rho_{ij}^*, & \text{if } Q_{ij}^* > 0, \\ \geq \rho_{ij}^*, & \text{if } Q_{ij}^* = 0, \end{cases} \tag{9}$$

and

$$d_{ij}(\rho^*, q(\pi^*)) \begin{cases} = Q_{ij}^*, & \text{if } \rho_{ij}^* > 0, \\ \leq Q_{ij}^*, & \text{if } \rho_{ij}^* = 0, \end{cases} \tag{10}$$

and for each supply market i; $i = 1, \ldots, m$:

$$s_i(\pi^*) \begin{cases} = \sum_{j=1}^n Q_{ij}^*, & \text{if } \pi_i^* > 0, \\ \geq \sum_{j=1}^n Q_{ij}^*, & \text{if } \pi_i^* = 0. \end{cases} \tag{11}$$

According to (9), there is a positive quantity of the product shipped from a supply market to a demand market in equilibrium, if the supply price at the originating supply market plus the associated unit transportation cost is equal to the demand price at the demand market. If the supply price plus the unit transportation cost exceeds that demand price, then there will be no trade of that product between the pair of supply and demand markets. According to (10) and (11), if the demand price at a demand market is zero, then the product shipments into that demand market can exceed the demand; similarly, if a supply price at a supply market is zero, then the supply can exceed the product shipments out of that supply market.

We now establish the variational inequality formulation of the above spatial price equilibrium conditions under perfect quality information.

Theorem 1: Variational Inequality Formulation of Spatial Price Equilibrium with Product Differentiation Under Perfect Quality Information

A product shipment, demand price, and supply price pattern $(Q^*, \rho^*, \pi^*) \in K^1$ is a spatial price equilibrium with product differentiation under perfect quality information according to Definition 1 if and only if it satisfies the variational inequality problem:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}(\pi_i^* + c_{ij}(Q^*) - \rho_{ij}^*) \times (Q_{ij} - Q_{ij}^*) + \sum_{i=1}^{m}\sum_{j=1}^{n}(Q_{ij}^* - d_{ij}(\rho^*, q(\pi^*))) \times (\rho_{ij} - \rho_{ij}^*)$$

$$+ \sum_{i=1}^{m}(s_i(\pi^*) - \sum_{j=1}^{n} Q_{ij}^*) \times (\pi_i - \pi_i^*) \geq 0, \quad \forall (Q, \rho, \pi) \in K^1. \tag{12}$$

Proof: Please see Appendix A.

We now put variational inequality (12) into standard form (cf. Nagurney (1999); see also Liu and Nagurney (2011) and Qiang et al. (2013)): determine $X^* \in \mathcal{K} \subset R^N$, such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{13}$$

where $\mathcal{K}$ is the feasible set, which must be closed and convex. The vector X is an N-dimensional vector, and $F(X)$ is a given continuous function from $\mathcal{K}$ to R^N. $\langle \cdot, \cdot \rangle$ denotes the inner product in N-dimensional Euclidean space. We define the vector $X \equiv (Q, \rho, \pi)$ and the vector $F(X) \equiv (F^1(X), F^2(X), F^3(X))$ with $F^1(X)$ consisting of components $F^1_{ij}(X) = \pi_i + c_{ij}(Q) - \rho_{ij}$; $i = 1, \ldots, m$; $j = 1, \ldots, n$, $F^2(X)$ consisting of components $F^2_{ij}(X) = Q_{ij} - d_{ij}(\rho, q(\pi))$; $i = 1, \ldots, m$; $j = 1, \ldots, n$, and $F^3(X)$ with components $F^3_i(X) = s_i(\pi) - \sum_{j=1}^{n} Q_{ij}$; $i = 1, \ldots, m$. Also, we define the feasible set $\mathcal{K} \equiv K^1$, and let $N = 2mn + m$. Then, variational inequality (12) can be put into standard form (13).

For additional background on the variational inequality problem, we refer the reader to the book by Nagurney (1999).

Consumer Welfare Under Perfect Quality Information

We now provide a measure of the consumer welfare at equilibrium under perfect quality information. We assume that the demand functions (5b) (and (5a)) are invertible, which allows for a simpler calculation; thus, we first construct the demand price functions

$$\rho_{ij} = \rho_{ij}(d, q(\pi)), \quad i = 1, \ldots, m; j = 1, \ldots, n, \tag{14}$$

where d is the vector with all d_{ij}s.

The consumer welfare associated with product i at demand market j at equilibrium under perfect quality information, CW_{ij}^P, is then given by

$$CW_{ij}^P = \int_0^{d_{ij}(\rho^*,q(\pi^*))} \rho_{ij}(\hat{d}_{ij}^*, d_{ij}, q(\pi^*))\,\mathrm{d}(d_{ij}) - \rho_{ij}^* d_{ij}(\rho^*, q(\pi^*)), \quad i = 1, \ldots, m; j = 1, \ldots, n, \tag{15}$$

where $\hat{d}_{ij}^* \equiv (d_{11}(\rho^*, q(\pi^*)), \ldots, d_{i,j-1}(\rho^*, q(\pi^*)), d_{i,j+1}(\rho^*, q(\pi^*)), \ldots, d_{mn}(\rho^*, q(\pi^*)))$ (Spence (1975) and Wildman (1984)).

Please note that, once the equilibrium solution X^* of this model is achieved, it will remain in equilibrium. In the event that changes in the supply, demand, transportation cost, and/or quality functions occur, the equilibrium solution X^* and the consumer welfare will be updated by re-applying the model with the new functions.

2.2 Spatial Price Equilibrium with Product Differentiation Under Information Asymmetry in Quality

We now develop the spatial price equilibrium model with product differentiation under information asymmetry in quality. In this paper, the information asymmetry in quality that we consider is that caused by the time delay of consumer learning of product quality. The producers at each supply market determine and, hence, know the actual quality level of the product that they supply, based on their current knowledge of supply, demand, transportation cost, and quality. However, at the demand markets, it takes time for consumers to learn. After the actual product quality is learned by consumers, their perception of quality will change and their demand will change as well. The market equilibrium is then updated to adapt to this change, in addition to other changes that might occur in supply, demand, transportation cost, and quality from time to time. Afterwards, based on the new equilibrium, producers at the supply markets will re-determine the actual quality of their products. Therefore, the spatial price equilibrium problem with product differentiation under information asymmetry in quality is an *adaptive spatial price equilibrium problem.* Learning problems are modeled as adaptive problems in Holmes (1991), Van Zandt and Lettai (2003), Ho, Camerer, and Chong (2007), and in Durlauf and Blume (2010).

In this paper, in the scenario with quality information asymmetry, a time period represents the necessary time duration for consumers to learn/observe the actual quality of the products. We denote the nonnegative shipment of product i to demand market j in period t by Q_{ij}^t, and group all the Q_{ij}^t elements in period t into the vector $Q^t \in R_+^{mn}$. The supply price of product i in period t is π_i^t, and all the π_i^t elements in period t are grouped into the vector $\pi^t \in R_+^m$. ρ_{ij}^t is the demand price of product i at demand market j in period t. We group all ρ_{ij}^t elements in period t into the vector $\rho^t \in R_+^{mn}$.

In period t, where t is a positive integer and $t = 1, 2, 3, \ldots$, the supply at supply market i, s_i^t, the unit transportation cost associated with shipping the product from supply market i to demand market j, c_{ij}^t, and the quality of product i, q_i^t, are given by the corresponding functions:

$$s_i^t = s_i^t(\pi^t), \quad i = 1, \ldots, m, \tag{16}$$

$$c_{ij}^t = c_{ij}^t(Q^t), \quad i = 1, \ldots, m; j = 1, \ldots, n, \tag{17}$$

$$q_i^t = q_i^t(\pi_i^t), \quad i = 1, \ldots, m. \tag{18}$$

We group the $q_i^t(\pi_i^t)$ elements in period t into the vector $q^t(\pi^t)$.

Here, we define the beginning of each time period as the time point that consumers learn/observe the actual product quality of the product. We express consumers' perception of the quality of product i; $i = 1, \ldots, m$, at demand market j; $j = 1, \ldots, n$, at the beginning of period t, as

$$\hat{q}_{ij}^t = \begin{cases} \hat{q}_{ij}^1, & \text{if } \ t = 1, \\ \hat{q}_{ij}^t(q_i^{t-1}, \hat{q}_{ij}^{t-1}), & \text{if } \ t \geq 2. \end{cases} \tag{19a}$$

In the first period, that is, when $t = 1$, product i just enters the demand markets as a new product. For consumers, their perception of the quality of a new product would not be dependent upon the intrinsic attributes of it, but rather the extrinsic attributes such as warranty, brand name, and packaging (Zeithaml (1988)). Such extrinsic attributes are determined and, hence, known by the producer. Therefore, we assume that, consumers' perception of the quality of product i at the beginning of the first period, $\hat{q}_{ij}^1$; $i = 1, \ldots, m$, $j = 1, \ldots, n$, is a constant known by the supply markets. However, thereafter, at the beginning of each period t; $t \geq 2$, the actual quality of product i is observed/learned by consumers, which is q_i^{t-1}, the latest equilibrium quality of the supply market. As a result, consumers' perception of quality will change. It changes to a value depending on the actual product quality that consumers just learned, q_i^{t-1}, and consumers' previous knowledge of quality which can be measured by their latest perception of quality, $\hat{q}_{ij}^{t-1}$ (cf. Shapiro (1982), Erdem and Keane (1996), Mehta, Rajiv, and Srinivasan (2004), and Archak, Ghose, and Ipeirotis (2011)).

In other words, the perception of quality in a period depends on the actual quality levels in all previous periods, since $\hat{q}_{ij}^t = \hat{q}_{ij}^t(q_i^{t-1}, \hat{q}_{ij}^{t-1})$ applies to every t; $t \geq 2$. Furthermore, for every t, the function $\hat{q}_{ij}^t$ is continuous and assumed to be monotonically increasing in the actual product quality, q_i^{t-1}, and in the perception of quality, $\hat{q}_{ij}^{t-1}$, of the preceding period. We group all the $\hat{q}_{ij}^t$ elements in period t into the vector $\hat{q}^t$.

In light of (18), $\hat{q}_{ij}^t$ can be re-expressed as

$$\hat{q}_{ij}^t = \begin{cases} \hat{q}_{ij}^1, & \text{if } t=1, \\ \hat{q}_{ij}^t(q_i^{t-1}(\pi_i^{t-1}), \hat{q}_{ij}^{t-1}), & \text{if } t \geq 2, \end{cases} \tag{19b}$$

for all $i=1,\ldots,m; j=1,\ldots,n$. We group all the $\hat{q}_{ij}^t(q_i^{t-1}(\pi_i^{t-1}), \hat{q}_{ij}^{t-1})$ elements in each period into the vector $\hat{q}^t(q^{t-1}(\pi^{t-1}), \hat{q}^{t-1})$.

Consequently, the demand for product i at demand market j in period t, d_{ij}^t, can be written as follows:

$$d_{ij}^t = \begin{cases} d_{ij}^1(\rho^1, \hat{q}^1), & \text{if } t=1, \\ d_{ij}^t(\rho^t, \hat{q}^t) = d_{ij}^t(\rho^t, \hat{q}^t(q^{t-1}(\pi^{t-1}), \hat{q}^{t-1})), & \text{if } t \geq 2. \end{cases} \tag{20}$$

We group all the demands in period t into the vector $d^t \in R_+^{mn}$.

The assumptions (i.e., continuity and monotonicity) on the supply, demand, transportation cost, and quality functions, respectively, are the same as those under perfect information.

Please note that this model captures the learning behavior of the consumers and how the supply markets respond to it over time, period by period. Furthermore, the functions (16) - (20) allow for unpredictable changes in the supply, demand, transportation cost, quality, and/or quality perception functions that may occur in any time period. Therefore, the producers at the supply markets must make decisions period by period in response to such changes over time.

Since the product shipments, supply prices, and the demand prices must be nonnegative, we define the feasible set $K^{2^t} \equiv \{(Q^t, \rho^t, \pi^t) \in R_+^{2mn+m}\}$ for each period t. We now state the adaptive spatial price equilibrium conditions with product differentiation under information asymmetry in quality.

Definition 2: Adaptive Spatial Price Equilibrium Conditions with Product Differentiation Under Information Asymmetry in Quality

A product shipment, demand price, and supply price pattern $(Q^{t^*}, \rho^{t^*}, \pi^{t^*}) \in K^{2^t}$, is a spatial equilibrium with product differentiation under quality information asymmetry in period t; $t=1,2,\ldots,$ if it satisfies the following conditions: for each pair of supply and demand markets (i,j); $i=1,\ldots,m$; $j=1,\ldots,n$:

$$\pi_i^{t^*} + c_{ij}^t(Q^{t^*}) \begin{cases} = \rho_{ij}^{t*}, & \textit{if } Q_{ij}^{t^*} > 0, \\ \geq \rho_{ij}^{t*}, & \textit{if } Q_{ij}^{t^*} = 0, \end{cases} \tag{21}$$

and

if $t=1$,

$$d_{ij}^1(\rho^{1^*}, \hat{q}^1) \begin{cases} = Q_{ij}^{1^*}, & \textit{if } \rho_{ij}^{1^*} > 0, \\ \leq Q_{ij}^{1^*}, & \textit{if } \rho_{ij}^{1^*} = 0; \end{cases} \tag{22a}$$

if $t \geq 2$,

$$d_{ij}^t(\rho^{t^*}, \hat{q}^t(q^{t-1}(\pi^{t-1^*}), \hat{q}^{t-1})) \begin{cases} = Q_{ij}^{t^*}, & \text{if } \rho_{ij}^{t^*} > 0, \\ \leq Q_{ij}^{t^*}, & \text{if } \rho_{ij}^{t^*} = 0, \end{cases} \tag{22b}$$

and for each supply market i*;* $i = 1, \ldots, m$*:*

$$s_i^t(\pi^{t^*}) \begin{cases} = \sum_{j=1}^n Q_{ij}^{t^*}, & \text{if } \pi_i^{t^*} > 0, \\ \geq \sum_{j=1}^n Q_{ij}^{t^*}, & \text{if } \pi_i^{t^*} = 0. \end{cases} \tag{23}$$

We now provide the variational inequality formulation of the above adaptive spatial price equilibrium conditions under information asymmetry in quality. The proof of Theorem 2 is similar to that of Theorem 1.

Theorem 2: Variational Inequality Formulation of the Adaptive Spatial Price Equilibrium with Product Differentiation Under Information Asymmetry in Quality

A product shipment, demand price, and supply price pattern $(Q^{t^*}, \rho^{t^*}, \pi^{t^*}) \in K^{2^t}$ *is a spatial price equilibrium with product differentiation under information asymmetry in quality in period* t*;* $t = 1, 2, \ldots,$ *according to Definition 2 if and only if it satisfies the variational inequality problem:*

if $t = 1$*:*

$$\sum_{i=1}^m \sum_{j=1}^n (\pi_i^{1^*} + c_{ij}^1(Q^{1^*}) - \rho_{ij}^{1^*}) \times (Q_{ij}^1 - Q_{ij}^{1^*}) + \sum_{i=1}^m \sum_{j=1}^n (Q_{ij}^{1^*} - d_{ij}^1(\rho^{1^*}, \hat{q}^1)) \times (\rho_{ij}^1 - \rho_{ij}^{1^*})$$

$$+ \sum_{i=1}^m (s_i^1(\pi^{1^*}) - \sum_{j=1}^n Q_{ij}^{1^*}) \times (\pi_i^1 - \pi_i^{1^*}) \geq 0, \quad \forall (Q^1, \rho^1, \pi^1) \in K^{2^1}, \tag{24a}$$

and if $t \geq 2$*:*

$$\sum_{i=1}^m \sum_{j=1}^n (\pi_i^{t^*} + c_{ij}^t(Q^{t^*}) - \rho_{ij}^{t^*}) \times (Q_{ij}^t - Q_{ij}^{t^*}) + \sum_{i=1}^m \sum_{j=1}^n (Q_{ij}^{t^*} - d_{ij}^t(\rho^{t^*}, \hat{q}^t(q^{t-1}(\pi^{t-1^*}), \hat{q}^{t-1})) \times (\rho_{ij}^t - \rho_{ij}^{t^*})$$

$$+ \sum_{i=1}^m (s_i^t(\pi^{t^*}) - \sum_{j=1}^n Q_{ij}^{t^*}) \times (\pi_i^t - \pi_i^{t^*}) \geq 0, \quad \forall (Q^t, \rho^t, \pi^t) \in K^{2^t}. \tag{24b}$$

We now put variational inequalities (24a) and (24b) into standard form: determine $X^{t^*} \in \mathcal{L}^t \subset R^N$ for time period t, such that

$$\langle G^t(X^{t^*}), X^t - X^{t^*} \rangle \geq 0, \quad \forall X^t \in \mathcal{L}^t, \tag{25}$$

where $\mathcal{L}^t$ is the closed and convex feasible set. The vector X^t is an N-dimensional vector, and $G^t(X^t)$ is a given continuous function from $\mathcal{L}^t$ to R^N. We define the vector $X^t \equiv (Q^t, \rho^t, \pi^t)$ and the vector $G^t(X^t) \equiv (G^{t^1}(X^t), G^{t^2}(X^t), G^{t^3}(X^t))$ for time period t, with $G^{t^1}(X^t)$ consisting of components $G^{t^1}_{ij}(X^t) = \pi^t_i + c^t_{ij}(Q^t) - \rho^t_{ij}$; $i = 1, \ldots, m$; $j = 1, \ldots, n$, $G^{t^2}(X^t)$ consisting of components $G^{t^2}_{ij}(X^t) = Q^1_{ij} - d^1_{ij}(\rho^1, \hat{q}^1))$ if $t = 1$ and $G^{t^2}_{ij}(X^t) = Q^t_{ij} - d^t_{ij}(\rho^t, \hat{q}^t(q^{t-1}(\pi^{t-1}), \hat{q}^{t-1}))$ if $t \geq 2$; $i = 1, \ldots, m$; $j = 1, \ldots, n$, and $G^{t^3}(X^t)$ with components $G^{t^3}_i(X^t) = s^t_i(\pi^t) - \sum_{j=1}^n Q^t_{ij}$; $i = 1, \ldots, m$. Also, we define the feasible set $\mathcal{L}^t \equiv K^{2^t}$. Then, variational inequalities (24a) and (24b) can be put into standard form (25).

Consumer Welfare Under Information Asymmetry in Quality

The measure of consumer welfare under quality information asymmetry at equilibrium is now constructed. We assume that the demand functions (20) are invertible and we first derive the corresponding demand price functions in each period, that is,

$$\rho^t_{ij} = \begin{cases} \rho^1_{ij}(d^1, \hat{q}^1), & \text{if } \ t = 1, \\ \rho^t_{ij}(d^t, \hat{q}^t(q^{t-1}(\pi^{t-1}), \hat{q}^{t-1})), & \text{if } \ t \geq 2, \end{cases} \tag{26}$$

for all $i = 1, \ldots, m$; $j = 1, \ldots, n$. This is done, as was done for the static model with perfect quality information, in order to simplify the calculation of the measure.

In period t, the consumer welfare of product $i; i = 1, \ldots, m$, at demand market $j; j = 1, \ldots, n$, at equilibrium is given by

$$CW^{I^t}_{ij} = \begin{cases} \int_0^{d^1_{ij}(\rho^{1^*}, \hat{q}^1)} \rho^1_{ij}(\tilde{d}^{1^*}_{ij}, d^1_{ij}, \hat{q}^1)\, \mathrm{d}(d^1_{ij}) - \rho^{1^*}_{ij} d^1_{ij}(\rho^{1^*}, \hat{q}^1), & \text{if } \ t = 1, \\ \int_0^{d^t_{ij}(\rho^{t^*}, \hat{q}^t(q^{t-1}(\pi^{t-1^*}), \hat{q}^{t-1}))} \rho^t_{ij}(\tilde{d}^{t^*}_{ij}, d^t_{ij}, \hat{q}^t(q^{t-1}(\pi^{t-1^*}), \hat{q}^{t-1}))\, \mathrm{d}(d^t_{ij}) & \\ -\rho^{t^*}_{ij} d^t_{ij}(\rho^{t^*}, \hat{q}^t(q^{t-1}(\pi^{t-1^*}), \hat{q}^{t-1})), & \text{if } \ t \geq 2, \end{cases} \tag{27}$$

where $\tilde{d}^{1^*}_{ij} \equiv (d^1_{11}(\rho^{1^*}, \hat{q}^1), \ldots, d^1_{i,j-1}(\rho^{1^*}, \hat{q}^1), d^1_{i,j+1}(\rho^{1^*}, \hat{q}^1), \ldots, d^1_{mn}(\rho^{1^*}, \hat{q}^1))$, and $\hat{d}^{t^*}_{ij} \equiv (d^t_{11}(\rho^{t^*}, \hat{q}^t(q^{t-1}(\pi^{t-1^*}), \hat{q}^{t-1})), \ldots, d^t_{i,j-1}(\rho^{t^*}, \hat{q}^t(q^{t-1}(\pi^{t-1^*}), \hat{q}^{t-1})), d^t_{i,j+1}(\rho^{t^*}, \hat{q}^t(q^{t-1}(\pi^{t-1^*}), \hat{q}^{t-1})), \ldots, d^t_{mn}(\rho^{t^*}, \hat{q}^t(q^{t-1}(\pi^{t-1^*}), \hat{q}^{t-1}))$.

2.3 Value of Perfect Quality Information for Consumers

For a spatial price equilibrium problem with quality information and product differentiation, the value of perfect quality information for consumers is estimated by comparing the consumer welfare under perfect quality information and the corresponding welfare under information asymmetry in quality. The maximum amounts that consumers are willing to pay for gaining access to perfect quality information is then the difference between these two welfare values, which indicates how much more welfare consumers can obtain with perfect quality information.

Hence, in period t, the value of perfect quality information of product i for consumers at demand market j is:

$$CVPI_{ij}^{t} = CW_{ij}^{P} - CW_{ij}^{I^t}, \quad i = 1, \ldots, m; j = 1, \ldots, n. \tag{28}$$

3. Qualitative Properties

In this Section, we provide some qualitative properties of the solution to variational inequalities (22) and (25). In particular, we derive existence and uniqueness results.

Recall (cf. Nagurney (1999)) that $F(X)$ in (13) is *monotone* if

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle \geq 0, \quad \forall X^1, X^2 \in \mathcal{K}. \tag{29}$$

$F(X)$ is *strictly monotone* if

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle > 0, \quad \forall X^1, X^2 \in \mathcal{K},\ X^1 \neq X^2. \tag{30}$$

$F(X)$ is *strongly monotone* X^*, if there is an $\eta > 0$, such that

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle \geq \eta \|X^1 - X^2\|^2, \quad \forall X^1, X^2 \in \mathcal{K}. \tag{31}$$

The monotonicity of a function F is closely related to the positive-definiteness of its Jacobian matrix ∇F (cf. Nagurney (1999)). Specifically, if ∇F is positive-semidefinite, then F is monotone; if ∇F is positive-definite, then F is strictly monotone; and, if ∇F is strongly positive-definite, then F is strongly monotone.

The above results for monotonicity, strict monotonicity, and strong monotonicity also hold for $G^t(X^t)$ in (25).

Assumption 1

Suppose that for our spatial price equilibrium problems with quality information and product differentiation, there exists a sufficiently large B and a sufficiently large $\bar{B}$, such that, for any supply and demand market pair (i, j):

$$F_{ij}^1(X) = \pi_i + c_{ij}(Q) - \rho_{ij} > 0, \tag{32}$$

$$F_{ij}^2(X) = Q_{ij} - d_{ij}(\rho, q(\pi)) > 0, \tag{33}$$

$$G_{ij}^{t^1}(X^t) = \pi_i^t + c_{ij}^t(Q^t) - \rho_{ij}^t > 0, \qquad \forall t, \tag{34}$$

$$G_{ij}^{t^2}(X^t) = Q_{ij}^1 - d_{ij}^1(\rho^1, \hat{q}^1)) > 0, \qquad t = 1, \tag{35}$$

$$G_{ij}^{t^2}(X^t) = Q_{ij}^t - d_{ij}^t(\rho^t, \hat{q}^t(q^{t-1}(\pi^{t-1}), \hat{q}^{t-1})) > 0, \quad t \geq 2, \tag{36}$$

for all shipment patterns Q with $Q_{ij} \geq B$ and Q^t with $Q_{ij}^t \geq B$ and for all demand price patterns ρ with $\rho_{ij} \geq \bar{B}$ and ρ^t with $\rho_{ij}^t \geq \bar{B}$. In addition, suppose that there exists a sufficiently large $\hat{B}$, such that, for any supply market i:

$$F_i^3(X) = s_i(\pi) - \sum_{j=1}^{n} Q_{ij} > 0, \tag{37}$$

$$G_i^{t^3}(X^t) = s_i^t(\pi^t) - \sum_{j=1}^{n} Q_{ij}^t > 0, \quad \forall t, \tag{38}$$

for all supply price patterns π with $\pi_i \geq \hat{B}$ and π^t with $\pi_i^t \geq \hat{B}$.

We now provide an existence result, whose proof can be established using similar arguments as the proof of Proposition 6.1 in Nagurney and Zhang (1996) for the spatial price equilibrium problem without quality information.

Theorem 3: Existence

Any spatial price equilibrium problem with quality information and product differentiation, as described in Section 2, that satisfies Assumption 1 possesses at least one equilibrium shipment, demand price, and supply price pattern.

We now present the uniqueness result, the proof of which follows from the basic theory of variational inequalities (cf. Nagurney (1999)).

Theorem 4: Uniqueness

Suppose that $F(X)$ in (13) is strictly monotone on $\mathcal{K}$. Then the solution X^* to variational inequality (13) is unique, if one exists.

Similarly, suppose that G^t in (25) is strictly monotone on $\mathcal{L}^t$. Then the solution X^{t^*} to variational inequality (25) is unique, if one exists.

In addition, an existence and uniqueness result is presented in the following, the proof of which follows from the basic theory of variational inequalities (cf. Nagurney (1999)).

Theorem 5: Existence and Uniqueness

Suppose that F is strongly monotone. Then there exists a unique solution to variational inequality (13).

Similarly, suppose that G^t is strongly monotone. Then there exists a unique solution to variational inequality (25).

In the adaptive spatial price equilibrium problem under information asymmetry in quality discussed in Section 2.2, consumers' perceived quality in a period depends on the actual product quality and the perceived quality in the preceding period. Therefore, the equilibrium solution in period t does not only depend on the inputs in period t but also on the results from period $t-1$. In other words, the function $G^t(X^t)$ can be rewritten as $G^t(X^t) = G^t(X^t, X^{t-1})$. In addition, the feasible sets K^1 and K^{2t} are identical, that is, $K^1 = K^{2t} \subset R_+^{2mn+m}$, $\forall t$. For purpose of discussion, we define the following notation:

$$g(x, y) \equiv G^t(X^t, X^{t-1}), \quad \forall t. \tag{39}$$

$$K \equiv K^1 = K^{2t}, \quad \forall t, \tag{40}$$

Theorem 6: Convergence of Variational Inequality (24b) Under Information Asymmetry in Quality

Assume that, in the adaptive spatial price equilibrium model under information asymmetry, the supply, demand, transportation cost, quality, and quality perception functions remain unchanged over time, and there is a constant $\theta > 0$ such that

$$|||\nabla_x g^{-\frac{1}{2}}(x^1, y^1)\nabla_x g(x^2, y^2)\nabla_x g^{-\frac{1}{2}}(x^3, y^3)||| \leq \theta < 1, \tag{41}$$

for all $(x^1, y^1), (x^2, y^2), (x^3, y^3) \in K$, where $||| \cdot |||$ denotes the standard norm of a matrix; and that infimum over $K \times K$ of the minimum eigenvalue of $\nabla_x g(x, y)$ is positive. Then as $t \to \infty$, the solution X^{t^*} to variational inequality (24b) of the problem under information asymmetry converges to the solution X^* to the corresponding variational inequality (12) under perfect quality information.

Proof: Please see Appendix B.

Moreover, a necessary condition for (41) to hold is that the G^t are strictly monotone for all t.

In the next Section, we apply the Euler method (please see Appendix C) to compute solutions to numerical spatial price equilibrium problems with quality information and product differentiation.

4. Numerical Examples

In this Section, we present several numerical examples, which we solve via the Euler method. We implemented the Euler method using Matlab on an OS X 10.10.5 system. The convergence tolerance is 10^{-6}, so that the algorithm is deemed to have converged when the absolute value of the difference between each successive product shipment, demand price, and supply price is less than or equal to 10^{-6}. The sequence $\{a_\tau\}$ is set to: $\{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \ldots\}$. We initialize the algorithm by setting the demand of each product at 10 and equally distributed the demand among the demand markets; the demand and supply prices are set to 0 initially.

In the numerical examples, we utilize linear demand functions (see, e.g., Kaya and Özer (2009), Xie et al. (2011), and Liu, Zhang, and Tang (2015)) and linear supply functions (e.g., Takayama and Judge (1971), Barraza De La Cruz, Pizzolato, and Barraza De La Cruz (2010), and Koizumi (2015) in applications to agricultural products).

Moreover, in the numerical examples, the perceived quality in time period t; $t \geq 2$, is measured as

$$\hat{q}^t_{ij} = \alpha^t_j q^{t-1}_i(\pi^{t-1}_i) + (1 - \alpha^t_j)\hat{q}^{t-1}_{ij}, \quad i = 1, \ldots, m; j = 1, \ldots, n, \tag{42}$$

where $0 \leq \alpha^t_j \leq 1$; $j = 1, \ldots, n$. The higher the α^t_j, in period t, the greater the impact of actual product quality on consumers' quality perception, and the less the impact of consumers' previous knowledge of quality. Similar weighted sum functions are used to measure quality perception in Shapiro (1982) and in Ching, Erdem, and Keane (2013).

Example 1

In this example, there are two supply markets consisting of farms, and each supply market produces a milk product of its brand name. The operations and technology of the production and quality management at the two supply markets are similar. Each supply market provides its product to city 1, city 2, and city 3 (i.e., demand markets), which are spatially dispersed. Supply market 1 is closer to city 1 than to the other cities, while supply market 2 is closer to city 2, and, among all three cities, city 3 is the farthest to both supply markets. Consumers' sensitivity towards the price and quality of the two milk products is similar in these three cities. The network topology for this example is given in Figure 2.

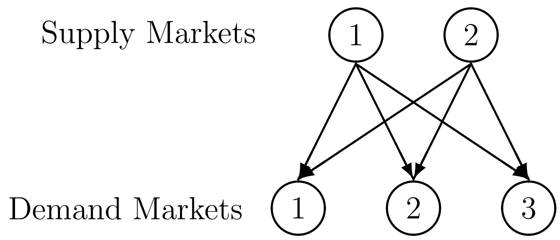

Figure 2: Example 1 Network Topology

Under perfect quality information, the data are as follows.

The supply functions are:

$$s_1(\pi_1, \pi_2) = 2\pi_1 - 0.5\pi_2 - 2, \quad s_2(\pi_1, \pi_2) = 2\pi_2 - 0.5\pi_1 - 2.$$

The unit transportation cost functions are:

$$c_{11}(Q_{11}) = Q_{11} + 6, \quad c_{12}(Q_{12}) = 2Q_{12} + 7, \quad c_{13}(Q_{13}) = 4Q_{13} + 5,$$

$$c_{21}(Q_{21}) = 2Q_{21} + 7, \quad c_{22}(Q_{22}) = Q_{22} + 5, \quad c_{23}(Q_{23}) = 4Q_{23} + 6.$$

The quality functions are:

$$q_1(\pi_1) = 2\pi_1 - 3, \quad q_2(\pi_2) = 2\pi_2 - 3,$$

and the demand functions are:

$$d_{11}(\rho_{11}, \rho_{21}, \hat{q}_{11}, \hat{q}_{21}) = -\rho_{11} + 0.4\hat{q}_{11} + 0.1\rho_{21} - 0.05\hat{q}_{21} + 35,$$

$$d_{12}(\rho_{12}, \rho_{22}, \hat{q}_{12}, \hat{q}_{22}) = -\rho_{12} + 0.4\hat{q}_{12} + 0.1\rho_{22} - 0.05\hat{q}_{22} + 35,$$

$$d_{13}(\rho_{13}, \rho_{23}, \hat{q}_{13}, \hat{q}_{23}) = -\rho_{13} + 0.4\hat{q}_{13} + 0.1\rho_{23} - 0.05\hat{q}_{23} + 35,$$

$$d_{21}(\rho_{11}, \rho_{21}, \hat{q}_{11}, \hat{q}_{21}) = -\rho_{21} + 0.4\hat{q}_{21} + 0.1\rho_{11} - 0.05\hat{q}_{11} + 35,$$

$$d_{22}(\rho_{12}, \rho_{22}, \hat{q}_{12}, \hat{q}_{22}) = -\rho_{22} + 0.4\hat{q}_{22} + 0.1\rho_{12} - 0.05\hat{q}_{12} + 35,$$

$$d_{23}(\rho_{13}, \rho_{23}, \hat{q}_{13}, \hat{q}_{23}) = -\rho_{23} + 0.4\hat{q}_{23} + 0.1\rho_{13} - 0.05\hat{q}_{13} + 35.$$

The equilibrium solution is reported in Table 1, which is a summary of the results for Examples 1, 2, 3, 4, and 5.

Due to the similarity between the two supply markets in production, quality management, transportation, and in demand, the differences in their quality, supply amounts, and supply prices are insignificant.

Since supply market 1 is closer to city 1 than to the other cities, the shipment from supply market 1 to city 1 is the largest among all shipments from supply market 1. In addition, city 3 is the farthest from supply market 1; thus, the shipment to city 3 is the smallest due to a high transportation cost. This explains why, among the three demand prices of product 1, the demand price charged in city 1 is the lowest and that in city 3 is the highest. Since consumers in city 3 pay a higher price for product 1 than consumers in the other two cities, their welfare associated with product 1 is the smallest. Similar results are observed for supply market 2.

We now model this example again under quality information asymmetry. The problem remains the same, so the functional forms of the supply, demand, transportation cost, and quality functions in each period are the same as those under perfect information, but with different variables from period to period.

As discussed in Section 2.2, extrinsic attributes of the products, such as warranty, brand name, and packaging, may result in consumers' initial perceived quality levels being positive values. Here, we assume consumers' perceived quality levels in period 1 are:

$$\hat{q}^1_{11} = 18, \quad \hat{q}^1_{12} = 22, \quad \hat{q}^1_{13} = 25,$$

$$\hat{q}^1_{21} = 18, \quad \hat{q}^1_{22} = 22, \quad \hat{q}^1_{23} = 25.$$

Consumers in different cities perceive the initial quality of the same product distinctively based on their preferences. Consumers in city 3 are most attracted by the initial extrinsic attributes of the two milk products when they just enter the market, and consumers in city 1 are least attracted. In addition, from period 2 onwards, (42) is used to measure the perception of quality with α^t_1, α^t_2, α^t_3=0.7, $\forall t \geq 2$.

We consider a finite number of periods, 20 periods, in this example. During these 20 periods, no changes occur in the supply, demand, transportation cost, actual quality, or quality perception functions. The evolution of the equilibrium product shipments, equilibrium supply prices, equilibrium demand prices, actual product quality and the perception of quality, consumer welfare, and the values of perfect quality information under quality information asymmetry from period 1 to period 20 is presented in Figures 3 and 4, which indicates the impacts of quality information asymmetry and consumer learning of quality over time. The associated results under perfect quality information are also shown in the same figures.

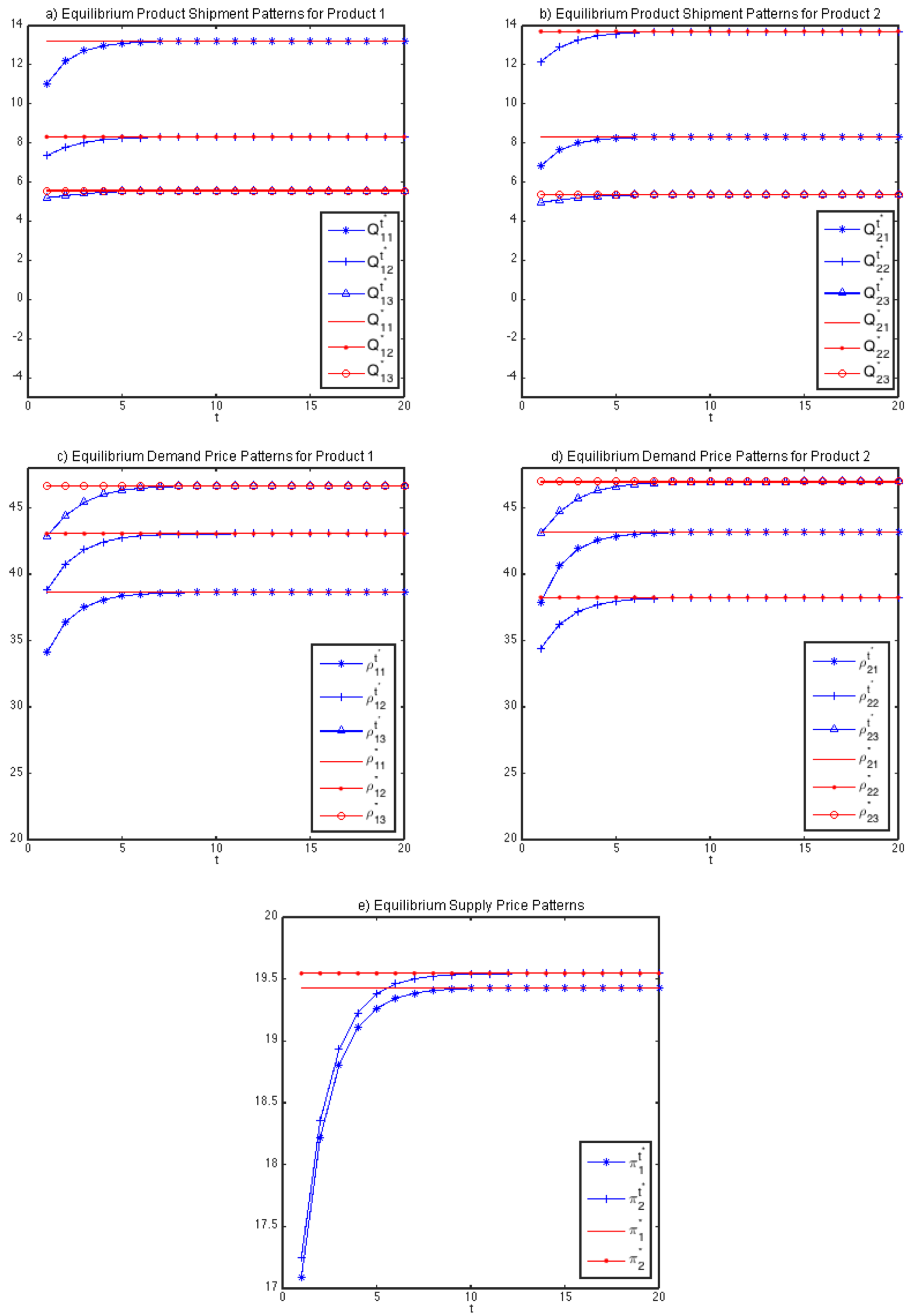

Figure 3: Evolution of the Equilibrium Product Shipment Pattern, Equilibrium Demand Price Pattern, and Equilibrium Supply Price Pattern Under Quality Information Asymmetry from Period 1 to Period 20 for Example 1, Along with Associated Results Under Perfect Quality Information

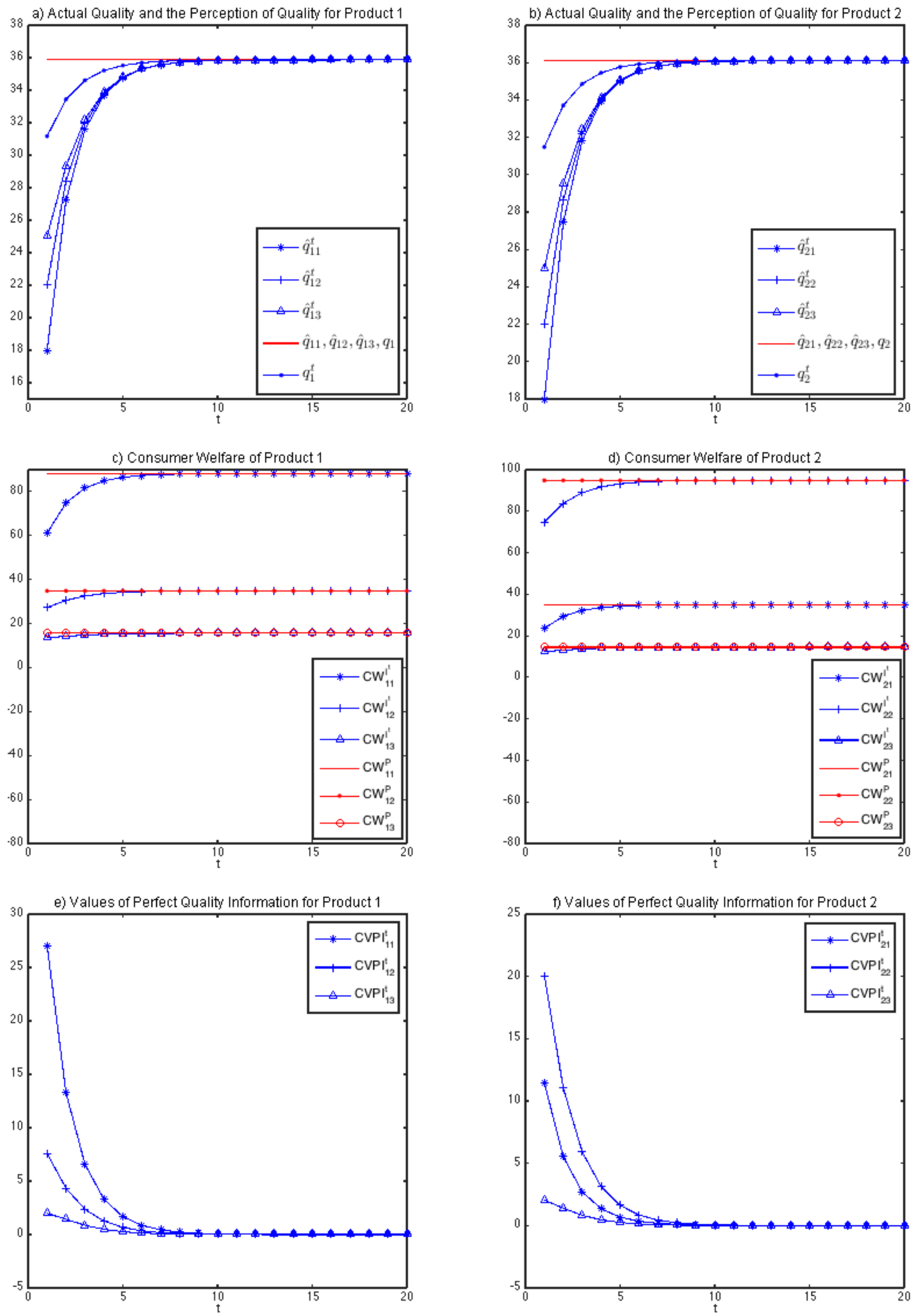

Figure 4: Evolution of Actual Quality and the Perception of Quality, Consumer Welfare, and the Values of Perfect Quality Information Under Quality Information Asymmetry from Period 1 to Period 20 for Example 1, Along with Associated Results Under Perfect Quality Information

The values of perfect quality information, which are the maximum amounts consumers are willing to pay to obtain perfect quality information, approach to 0, as more information is revealed to consumers over time. As shown in these figures, the evolution of the equilibrium under quality information asymmetry approaches the equilibrium under perfect quality information over time.

Example 2

Example 2 is the same as in Example 1, but in this example, supply market 1 applies a new technology that is able to improve product more efficiently from time period 8 onwards. With this new technology, a higher supply price needs to be charged for product 1.

From time period 8, the supply function and the quality function of supply market 1 under information asymmetry become:

$$s_1^t(\pi_1^t, \pi_2^t) = 1.75\pi_1^t - 0.5\pi_2^t - 2, \quad q_1^t(\pi_1^t) = 0.1{\pi_1^t}^2, \quad \forall t \geq 8.$$

The model under perfect information is applied again with the above new functional forms, with the new equilibrium solution reported in Table 1. The evolution of the results under quality information asymmetry from period 1 to period 20 is shown in Figures 5 and 6, along with the associated equilibrium with the new supply and quality functions under perfect quality information. As revealed in these figures, prior to time period 8, all results approach the equilibrium solution under perfect information of Example 1. However, after the changes in supply and quality functions take place in period 8, all results approach the new equilibrium solution under perfect information.

In this example, as supply market 1 becomes much more efficient in improving quality than before, from time period 8 onwards, its quality increases and almost doubles at the new equilibrium. However, a higher supply price is charged at the same time; therefore, its supply price increases as well, resulting in increases in the demand prices of product 1. Due to the higher quality of product 1, its demands rise in the three cities. Nonetheless, the demands of product 2 decrease due to competition, which leads to slight increases in its demand prices and supply price. As a result of the slightly increased supply price of product 2, its quality increases slightly as well. It is worth noting that, although only supply market 1 adopts a new technology that efficiently improves its quality level, because of competition, the product quality of its competitor, supply market 2, increases as well.

Moreover, after the changes occur, the consumer welfare of product 2 slightly decreases because of increases in its demand prices. For product 1, due to its much higher quality, its

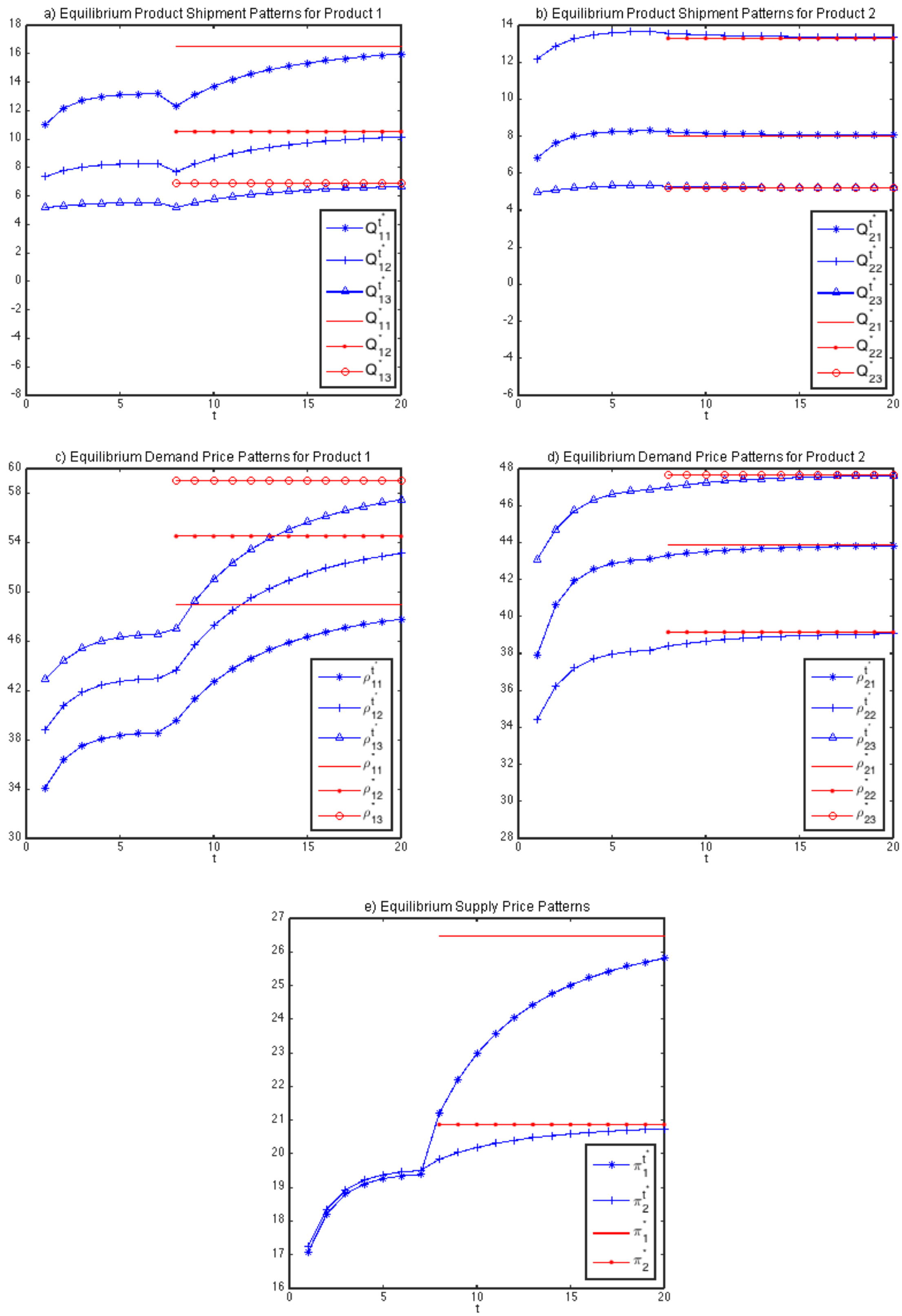

Figure 5: Evolution of the Equilibrium Product Shipment Pattern, Equilibrium Demand Price Pattern, and Equilibrium Supply Price Pattern Under Quality Information Asymmetry from Period 1 to Period 20 for Example 2, Along with Associated Results Under Perfect Quality Information with the New Functions for Example 2

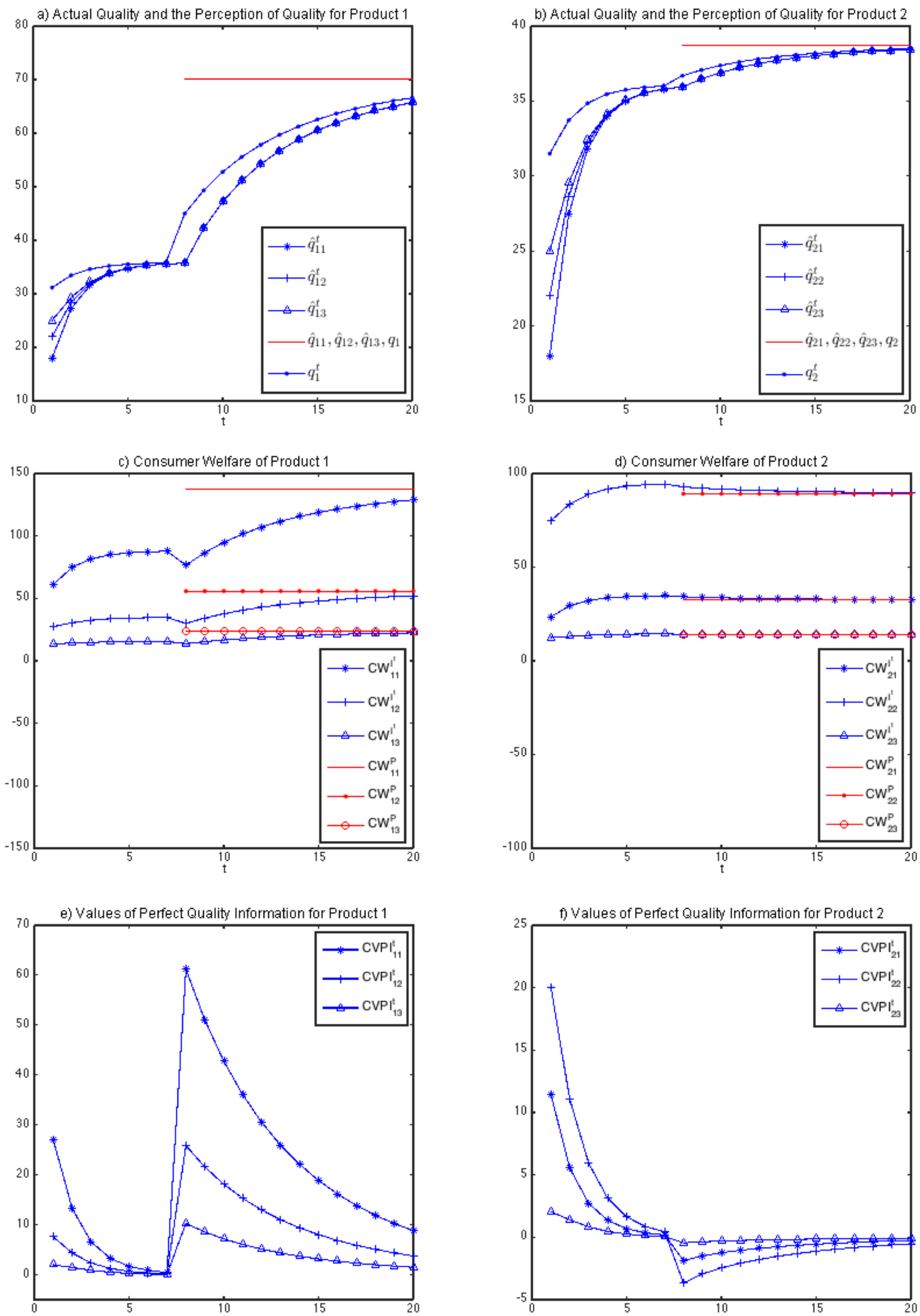

Figure 6: Evolution of Actual Quality and the Perception of Quality, Consumer Welfare, and the Values of Perfect Quality Information Under Quality Information Asymmetry from Period 1 to Period 20 for Example 2, Along with Associated Results Under Perfect Quality Information with the New Functions for Example 2

consumer welfare increases to a much higher value. The values of perfect information of the two products evolve to 0 over time.

Example 3

In the Introduction, we define quality as the "the degree to which a specific product conforms to a design or specification." The design and specification that the quality of a product conforms to are the quality standards and requirements of the supply market, which can include those of the industry, both international and domestic standards and requirements, and consumers' expectations and needs for quality. However, such designs and specifications may evolve over time and lead to changes in the measurement of quality.

This example is the same as Example 1, except for the following. In time period 5, due to technology development in milk production, sterilization, and in quality management, a new process, and higher expectation from consumers, stricter quality requirements and higher standards for milk products are adopted. As a result, the measurement of quality changes. The two supply markets then re-evaluate the relationship between their quality and supply prices and determine new quality functions.

The new quality functions under information asymmetry are:

$$q_1^t(\pi_1^t) = \pi_1^t - 1.5, \quad q_2^t(\pi_2^t) = \pi_2^t - 1.5, \quad \forall t \geq 5.$$

As in these functions, the same supply price leads to half of the quality value as before.

The new equilibrium solution under perfect information with the new functional forms is reported in Table 1. It is interesting to note that, the supply prices and the demand are lower than those before the change (i.e., those reported in Table 1 for Example 1). As a result, the new equilibrium quality level is actually less than half of the equilibrium quality before the change. Consumer welfare is reduced significantly as well.

Therefore, in this case, it is very important for the supply markets to improve current operations and quality management in order to improve quality more efficiently. As shown in the results for Example 2, this may improve product quality significantly. If no improvement is done to the current situation, quality and consumer welfare will both suffer.

For the evolution of the results under information asymmetry, initially, all results approach the equilibrium solution under perfect information of Example 1. After the changes in quality functions occur in time period 5, the results approach the new equilibrium solution under perfect information.

Example 4

This example considers the same problem as in Example 1, except that city 2 becomes much more congested than cities 1 and 3 from time period 10 onwards, as sections of major highways to city 2 are under construction/maintenance.

From period 10 onwards, the unit transportation cost functions to city 2 under information asymmetry are changed to:

$$c_{12}^t(Q_{12}^t, Q_{22}^t) = 2Q_{12}^{t^2} + Q_{12}^t Q_{22}^t, \quad c_{22}^t(Q_{22}^t, Q_{12}^t) = Q_{22}^{t^2} + Q_{12}^t Q_{22}^t, \quad \forall t \geq 10.$$

The new equilibrium solution under perfect information with the new functional forms is reported in Table 1. Compared to Example 1, it is now more expensive to ship to city 2 after the change in transportation conditions takes place, which leads to decreases in the product shipments to that city. The supply quantities from the two supply markets also decrease, and the supply prices and quality levels are reduced. The decreases in the product shipments to city 2 also lead to increases in demand prices there, and, hence, the consumer welfare in city 2 drops.

Under information asymmetry, before time period 10, all results approach the equilibrium solution under perfect information of Example 1. From time period 10, the results approach the new equilibrium solution under perfect information.

Example 5

This example is the same as Example 1, except that, from time period 12 onwards, consumers in city 2 become more sensitive to product quality than before. They are willing to purchase more of higher quality products and fewer of lower quality products.

The new demand functions at city 2 under information asymmetry, from time period 12 onwards, become the following:

$$d_{12}^t(\rho_{12}^t, \rho_{22}^t, \hat{q}_{12}^t, \hat{q}_{22}^t) = -\rho_{12}^t + 0.8\hat{q}_{12}^t + 0.1\rho_{22}^t - 0.05\hat{q}_{22}^t + 35,$$

$$d_{22}^t(\rho_{12}^t, \rho_{22}^t, \hat{q}_{12}^t, \hat{q}_{22}^t) = -\rho_{22}^t + 0.8\hat{q}_{22}^t + 0.1\rho_{12}^t - 0.05\hat{q}_{12}^t + 35.$$

The equilibrium solution under perfect information with the new demand functional forms is reported in Table 1. Since consumers in city 2 are willing to purchase more of higher quality products, from period 12 onwards, the quality of the two products improves in response to this change. Supply prices also increase, and demand prices rise as well, which decreases the

Table 1: Equilibrium and Induced Solutions to Examples 1, 2, 3, 4, and 5 Under Perfect Quality Information

	Example 1	Example 2	Example 3	Example 4	Example 5
Q_{11}^* (i.e., d_{11})	13.21	16.51	10.83	13.62	12.78
Q_{12}^* (i.e., d_{12})	8.31	10.51	6.74	3.02	14.21
Q_{13}^* (i.e., d_{13})	5.56	6.88	4.59	5.73	5.38
Q_{21}^* (i.e., d_{21})	8.32	8.01	6.74	8.69	7.95
Q_{22}^* (i.e., d_{22})	13.70	13.29	11.29	4.17	23.13
Q_{23}^* (i.e., d_{23})	5.35	5.20	4.37	5.58	5.12
π_1^*	19.43	26.48	16.14	15.72	23.42
π_2^*	19.55	20.87	16.24	14.16	24.96
ρ_{11}^*	38.64	48.99	32.97	35.34	42.20
ρ_{12}^*	43.05	54.51	36.63	46.51	58.83
ρ_{13}^*	46.67	58.99	39.50	43.63	49.95
ρ_{21}^*	43.19	43.88	36.72	38.54	47.85
ρ_{22}^*	38.25	39.15	32.53	44.18	53.09
ρ_{23}^*	46.96	47.68	39.74	42.48	51.45
s_1	27.09	33.91	22.16	22.37	32.37
s_2	27.38	26.50	22.41	18.45	36.20
q_1	35.86	70.11	14.63	28.44	43.84
q_2	36.09	38.74	14.74	25.31	46.91
CW_{11}	88.19	137.63	59.19	93.72	82.46
CW_{12}	34.89	55.84	22.97	4.60	101.92
CW_{13}	15.62	23.90	10.64	16.57	14.63
CW_{21}	34.97	32.38	22.95	38.17	31.88
CW_{22}	94.82	89.15	64.42	8.80	270.25
CW_{23}	14.47	13.68	9.67	15.74	13.25

demands in cities 1 and 3. In addition, since city 2 is more quality-sensitive than the other two, the demands in city 2 increase as quality increases. Therefore, the consumer welfare in city 2 increases, but that in cities 1 and 3 decreases.

Based on the results for this example, we conclude that, although only the consumers in one of the demand markets are willing to purchase more products of higher quality, product quality will increase in all demand markets. As a consequence, the demands and demand prices at the other demand markets will also be affected. Consumers who are willing to buy more higher quality products will gain more welfare, but consumers who do not may lose.

Furthermore, prior to period 12, all results under information asymmetry approach the equilibrium solution under perfect information of Example 1. From time period 12 onwards, they approach the new equilibrium solution under perfect information.

Example 6

Example 6 is the same as in Example 1. Nevertheless, under quality information asymmetry, in city 3, consumers' past knowledge/memory of quality plays a more significant role in their quality perception in this example with $\alpha_3^t = 0.5$, $\forall t \geq 2$, instead of 0.7. The impact of consumers' most recent observation of quality (i.e., the latest actual product quality) is, hence, less (cf. (42)).

The evolution of the results from period 1 to period 20 is shown in Figures 7 and 8. We now compare the results for Examples 1 and 6. In city 3, as consumers' past memory of quality is more dominant and their current quality observations are less, all results in city 3 evolve in a slower pace than those in the other cities compared to the results for Example 1, especially for the evolution of the perceived quality levels. The impacts of quality information asymmetry are now greater in city 3 with consumers' memory of quality playing a stronger role.

As stated in Shapiro (1982), consumers' perception of quality should always adjust towards the true quality, but, as shown in this example, if the consumers' past memory becomes more dominant, their adjustment towards the true quality would be slower.

In this Section, Examples 1 through 6 study the impacts of the changes in quality technology, quality measurement, transportation costs, consumers' sensitivity to quality, and consumers' past memory of quality on the results of the model with perfect quality information and the model with quality information asymmetry. The effects of information asymmetry and consumer learning are also studied via the comparisons of the two models and the evolution of the equilibrium, the consumer welfare, and the values of perfect information over time. Furthermore, the results for Examples 1 through 6 support the theoretical conclusion presented in Theorem 6 that, over time, the equilibrium pattern of the adaptive spatial price equilibrium model (i.e., the information asymmetry model) of a problem approaches the equilibrium for the corresponding static model (i.e., the perfect information model) of the problem.

In summary, the following practical insights can be drawn from Examples 1-6:

1) As shown in the results for Example 1, consumers at the demand markets that are closer to the supply markets will receive more product shipments, lower demand prices, and more consumer welfare than those in farther demand markets.

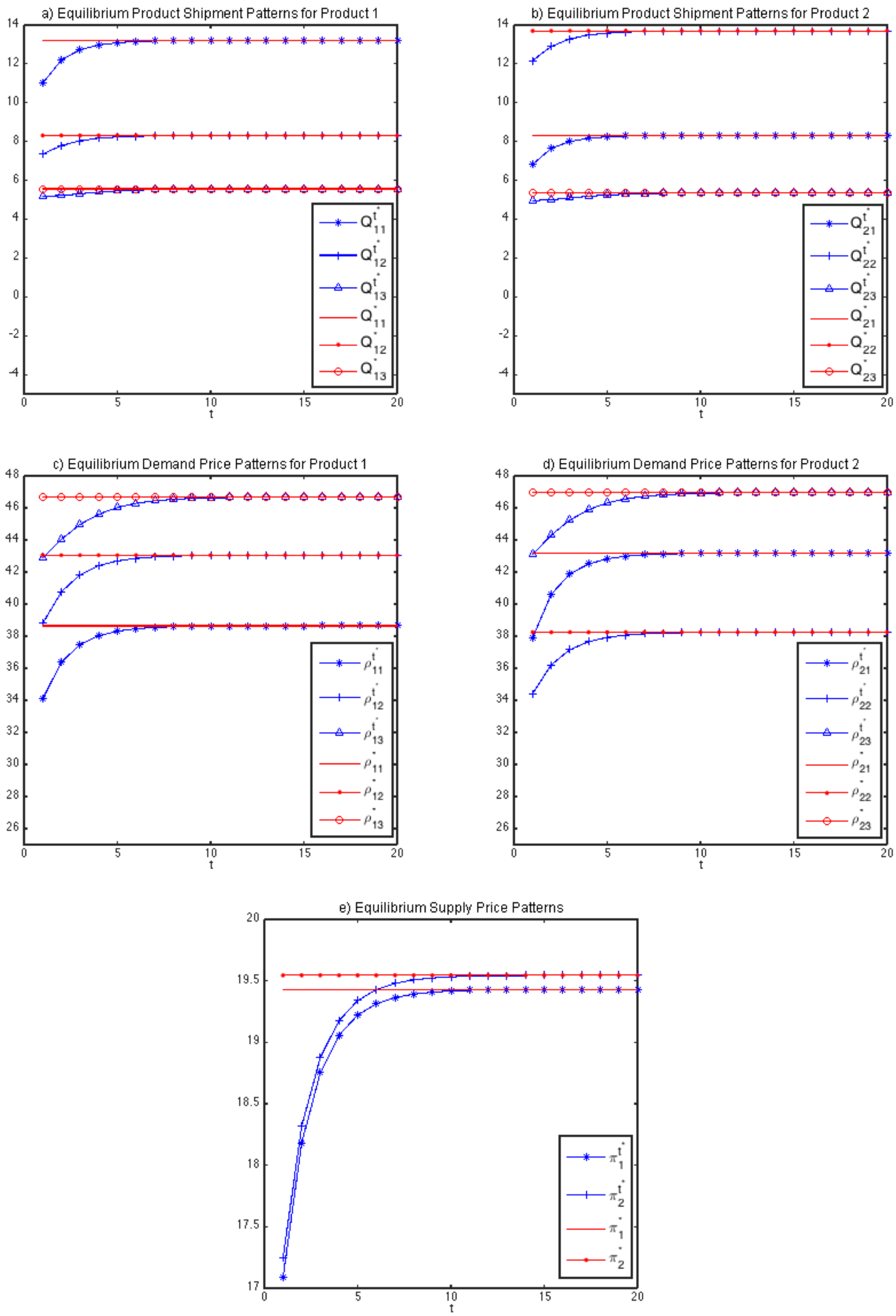

Figure 7: Evolution of the Equilibrium Product Shipment Pattern, Equilibrium Demand Price Pattern, and Equilibrium Supply Price Pattern Under Quality Information Asymmetry from Period 1 to Period 20 for Example 6, Along with Associated Results Under Perfect Quality Information with the New Functions for Example 6

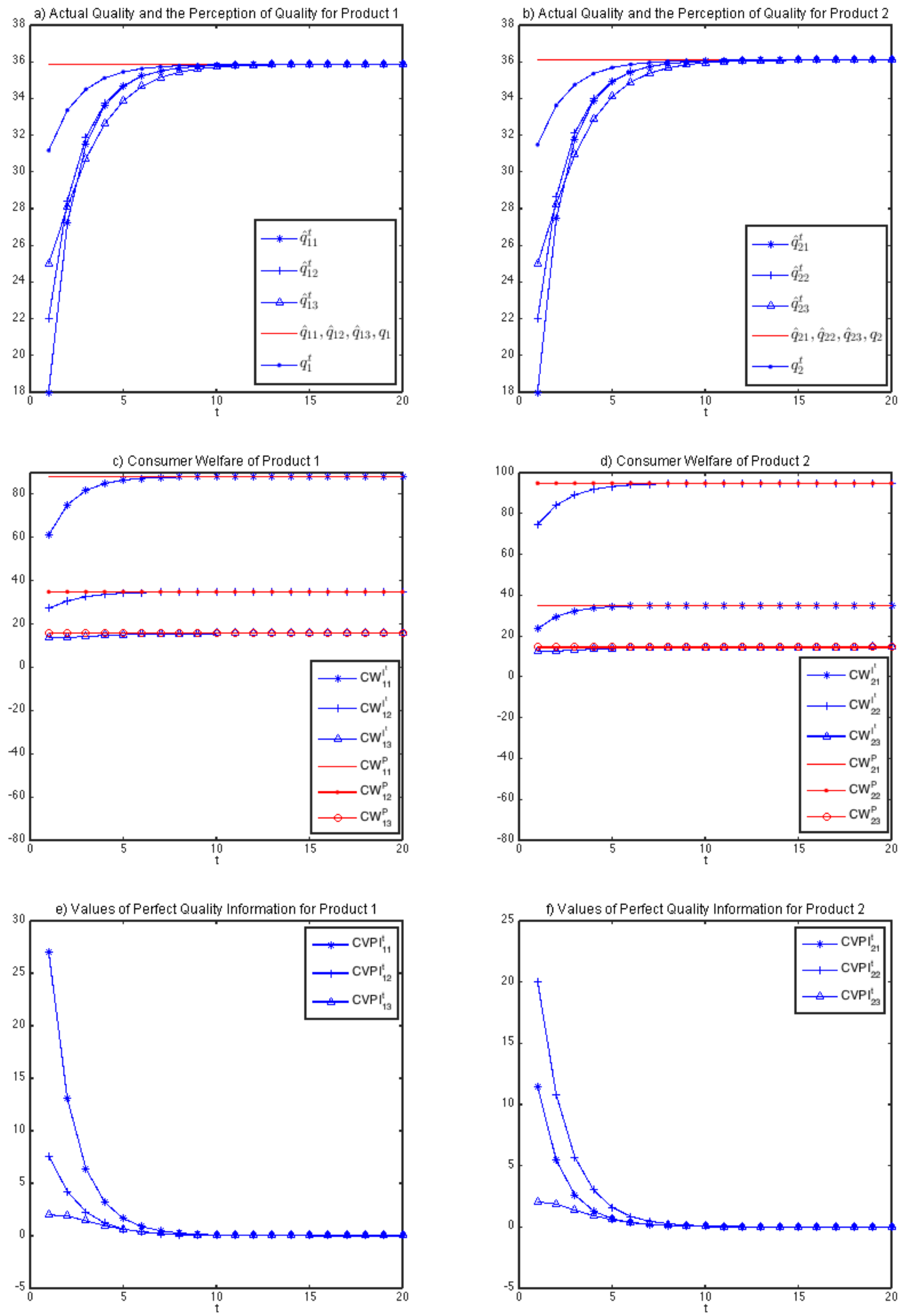

Figure 8: Evolution of Actual Quality and the Perception of Quality, Consumer Welfare, and the Values of Perfect Quality Information Under Quality Information Asymmetry from Period 1 to Period 20 for Example 6, Along with Associated Results Under Perfect Quality Information with the New Functions for Example 6

2) Comparing the results for Examples 1 and 2, a supply market's more efficient quality technology will enhance the consumer welfare of its own consumers but may hurt that of its competitors'.

3) From the results for Example 1 and Example 3, simply imposing stricter quality requirements will not improve consumer welfare, if no other effort is made.

4) Comparing Examples 1 and 4, traffic congestion will harm consumer welfare; thus, efficient and reliable transportation infrastructure is important for the benefit of consumers.

5) Based on the results for Examples 1 and 5, consumers who value quality more will benefit in terms of their welfare, but consumers who do not may obtain lower welfare.

6) From the results for Example 6, consumers who rely more on their past memory of quality will adjust to the actual quality in a slower manner than who do not.

5. Summary and Conclusions

Spatial price equilibrium models are foundational models for both agricultural as well as energy markets and their formulation and solution have been active areas of research in disciplines such as operations research, regional science, and economics because of their applicability in practice. In this paper, we advance the modeling, analysis, and understanding of spatial price equilibrium network models in which the products are differentiated and consumers respond to the quality of the products through the prices that they are willing to pay with consumers in our dynamic, adaptive spatial price equilibrium model, learning about the product quality over time.

The necessity and importance of incorporating consumer learning of quality in models for agricultural products, including food, where the spatial price equilibrium models are widely applied and quality information asymmetry exists, are emphasized in various empirical studies. Moreover, the literature, to-date, has noted that integrating learning models of demand with supply side models remains under-explored and should be another important area for future research. This gap is filled by our results, since both the supply side and the demand side and the evolution of product prices and flows over space and time are captured in this paper along with consumer learning of product quality. In addition, this paper also contributes to the existing literature on consumer learning in terms of generality and scope.

In particular, we develop both static and adaptive spatial price equilibrium models which capture product quality on both the production and the consumption sides. In the static model, consumers, located at the demand markets, have perfect information as to the quality of the products. In the adaptive spatial price equilibrium model, on the other hand, the producers have perfect information as to the quality of their products but the consumers are subject to information asymmetry in that, only after a time delay, do they obtain information about the actual product quality. The producers at the supply markets respond to consumer learning of quality and adapt accordingly over time. Responses and adaptions to the uncertainties in supply, demand, transportation cost, and product quality are also captured in the model. The latter is especially reasonable in the case of agricultural products, since consumers typically consume such products repetitively and will learn about the brand's product quality over time.

For both the static and the adaptive models we present the governing equilibrium conditions and formulate the problems as variational inequality problems. We also provide qualitative properties of the equilibrium supply price, demand price, product flow, and quality level patterns, in terms of existence and uniqueness results. We construct measures of consumer welfare under perfect quality information and under information asymmetry as well as the value of perfect quality information for consumers. The impacts of quality information asymmetry and consumer learning of quality on the dynamics of equilibrium supply price, demand price, shipment, and quality pattern and consumer welfare are studied by comparing the two models. We demonstrate theoretically and numerically that, under appropriate assumptions, the equilibrium pattern of the adaptive model with information asymmetry due to time delay, as the number of time periods approaches infinity, approaches the equilibrium pattern for the corresponding static model with perfect information on product quality.

The numerical examples presented in this paper further apply and compare the model with perfect quality information and the model with quality information asymmetry in cases with changes in quality technology, quality measurement, transportation costs, consumers' sensitivity to quality, and in the impact of consumers' past memory of quality, with practical insights provided. The impacts of information asymmetry are studied as well for these cases.

Acknowledgments

We sincerely thank the two anonymous referees and the Editor for their many valuable suggestions and comments.

Support from the summer research grant of the first author at the College of Business at Arkansas State University is acknowledged. The second author acknowledges support from the Radcliffe Institute for Advanced Study at Harvard University where she was a Summer Fellow in 2017. She also acknowledges support from the John F. Smith Memorial Fund at

the University of Massachusetts Amherst.

References

Akerlof, G.A., 1970. The market for ‘lemons’: Quality uncertainty and the market mechanism. Quarterly Journal of Economics, 84(3), 488-500.

Archak, N., Ghose, A., Ipeirotis, P.G., 2011. Deriving the pricing power of product features by mining consumer reviews. Management Science, 57(8), 1485-1509.

Ball, M., Meen, G., Nygaard, C., 2010. Housing supply price elasticities revisited: Evidence from international, national, local and company data. Journal of Housing Economics, 19(4), 255-268.

Banker, R.D., Khosla, I., Sinha, K.K., 1998. Quality and competition. Management Science, 44(9), 1179-1192.

Barraza De La Cruz, B.C., Pizzolato, N.D., Barraza De La Cruz, A., 2010. An application of the spatial equilibrium model to soybean production in Tocantins and neighboring states in Brazil. Pesquisa Operacional, 30(2), 443-464.

Bishop, P., Pratt, J., Novakovic, A., 1994. Using a joint-input, multi-product formulation to improve spatial price equilibrium models. Cornell University, Department of Agricultural Economics Staff Paper no. 94-06, Ithaca, New York.

Ching, A.T., 2010. Consumer learning and heterogeneity: Dynamics of demand for prescription drugs after patent expiration. International Journal of Industrial Organization, 28(6), 619-638.

Ching, A.T., Erdem, T., Keane, M.P., 2013. Learning models: An assessment of progress, challenges, and new developments. Marketing Science, 32(6), 913-938.

Christensen, J., 2016. One dead in listeria outbreak; salad suspected. CNN, January 22.

Dafermos, S., 1983. An iterative scheme for variational inequalities. Mathematical Programming, 26(1), 40-47.

Dafermos, S., 1986. Isomorphic multiclass spatial price and multimodal traffic network equilibrium models. Regional Science and Urban Economics, 16(2), 197-209.

Dafermos, S., Nagurney, A., 1984. Sensitivity analysis for the general spatial economic equilibrium problem. Operations Research, 32, 1069-1086.

Dale, B.G., Wan, G.M., 2002. Setting up a quality costing system: An evaluation of the key issues. Business Process Management Journal, 8(2),104-116.

Dong, J., Zhang, D., Yan, H., Nagurney, A., 2005. Multitiered supply chain networks: Multicriteria decision-making under uncertainty. Annals of Operations Research, 135(1), 155-178.

Dupuis, P., Nagurney, A, 1993. Dynamical systems and variational inequalities. Annals of Operations Research, 44, 9-42.

Durlauf, S.N., Blume, L.E., 2010. Learning and evolution in games: Adaptive heuristics. In: Durlauf, S.N., Blume, L.E. (Eds.). Game theory: The new Palgrave dictionary of economics (2nd ed.). London, England: Macmillan Publishers, pp. 178-183.

Erdem, T., Keane, M., 1996. Decision-making under uncertainty: Capturing dynamic brand choice processes in turbulent consumer goods markets. Marketing Science, 15(1), 1-20.

Erdem, T., Keane, M., Sun, B., 2008. A dynamic model of brand choice when price and advertising signal product quality. Marketing Science, 27(6), 1111-1125.

Feigenbaum, A.V., 1991. Total quality control (4th ed.). New York: McGraw-Hill.

Fine, C.H., 1986. Quality improvement and learning in productive systems. Management Science, 32(10), 1301-1315.

Florian, M., Los, M., 1982. A new look at static spatial price equilibrium models. Regional Science and Urban Economics, 12, 579-597.

Harberger, A.C., 1971. Three basic postulates for applied welfare economics: An interpretive essay. Journal of Economic literature, 9(3), 785-797.

Harris, G., 2009. Peanut plant broadens product list under recall. The New York Times, January 29.

Hayes, D.J., Shogren, J.F., Shin, S.Y., Kliebenstein, J.B., 1995. Valuing food safety in experimental auction markets. American Journal of Agricultural Economics, 77, 40-53.

Ho, T.H., Camerer, C.F., Chong, J.-K., 2007. Self-tuning weighted attraction learning in games. Journal of Economic Theory, 133, 177-198.

Holmes, C.H., 1991. Adaptive learning and roads to chaos: The case of the cobweb. Economics Letters, 36,127-132.

Howard, E.A., 1984. An analysis of interregional competition in the U.S. summer potato market, Master of Science dissertation, Department of Agricultural Economics, Texas A&M University, College Station, Texas.

Huffman, W.E., Rousu, M., Shogren, J.F., Tegene, A., 2007. The effects of prior beliefs and learning on consumers' acceptance of genetically modified foods. Journal of Economic Behavior & Organization, 63(1), 193-206.

Hwang, M.J., Yang, C.W., Kim, J., Irwin, C., 1994. Impact of environmental regulation on the optimal allocation of coal among regions in the United States. International Journal of Environment and Pollution, 4, 59-74.

Irwin, C.L., Yang, C.W., 1996. Impact analysis of carbon taxes on the steam coal and natural gas market in the eastern United States: A linear complementarity model. International Journal of Environment and Pollution, 6, 57-73.

Israel, M., 2005. Services as experience goods: An empirical examination of consumer learning in automobile insurance. The American Economic Review, 95(5), 1444-1463.

Iyengar, R., Ansari, A., Gupta, S., 2007. A model of consumer learning for service quality and usage. Journal of Marketing Research, 44(4), 529-544.

Johnston, R.J., Wessells, C.R., Donath, H., Asche, F., 2001. Measuring consumer preferences for ecolabeled seafood: an international comparison. Journal of Agricultural and Resource Economics, 26, 20-39.

Judge, G.G., 1956. A spatial equilibrium model for eggs. Connecticut Agricultural Experiment Station, Storrs, Connecticut.

Juran, J.M., Gryna, F.M., 1988. Quality control handbook (4th ed.). New York: McGraw-Hill.

Juran, J.M., Gryna, F.M., 1993. Quality planing and analysis. New York: McGraw-Hill.

Kannan, V.R., Tan, K.C., 2005. Just in time, total quality management, and supply chain management: Understanding their linkages and impact on business performance. Omega, 33(2), 153-162.

Kaya, M., Özer, Ö., 2009. Quality risk in outsourcing: Noncontractible product quality and private quality cost information. Naval Research Logistics, 56(7), 669-685.

Keller, G., Noori, H., 1988. Impact of investing in quality improvement on the lot size model. Omega, 16(6), 595-601.

Khan, M., Jaber, M.Y. Ahmad, A.R., 2014. An integrated supply chain model with errors in quality inspection and learning in production. Omega, 42(1),16-24.

Kilkenny, M., Thisse, J.F., 1999. Economics of location: A selective survey. Computers and Operations Research, 26(14), 1369-1394.

Koizumi, T., 2015. Biofuels and food security. Renewable and Sustainable Energy Reviews, 52, 829-841.

Koulamas, C., 1992. Quality improvement through product redesign and the learning curve. Omega, 20(2), 161-168.

Krichene, N., 2002. World crude oil and natural gas: A demand and supply model. Energy Economics, 24(6), 557-576.

Labys, W.C., 1999. Modeling mineral and energy markets. Boston, Massachusetts: Kluwer Academic Publishers.

Labys, W.C., Yang, C.W., 1997. Spatial price equilibrium as a foundation to unified spatial commodity modeling. Papers in Regional Science, 76(2), 199-228.

Leland, H.E., 1979. Quacks, lemons, and licensing: A theory of minimum quality standards. Journal of Political Economy, 87(6), 1328-1346.

Liu, Z., Nagurney, A., 2011. Supply chain outsourcing under exchange rate risk and competition. Omega, 39(5), 539-549.

Liu, G., Zhang, J., Tang, W., 2015. Joint dynamic pricing and investment strategy for perishable foods with price-quality dependent demand. Annals of Operations Research, 226(1), 397-416

McCarl, B.A., Moskowitz, H., Furtan, H., 1977. Quadratic programming applications. Omega, 5(1), 43-55.

McDonald, J., 2014. Meat scandal hits China Starbucks, Burger King. Masslive.com, July22; available at: http://www.masslive.com/news/index.ssf/2014/07/meatscandalhitschinastarbu.html

Mehta, N., Rajiv, S., Srinivasan, K., 2004. The role of forgetting in memory-based choice decisions. Quantitative Marketing and Economics, 2(2), 107-140.

Nagurney, A., 1999. Network economics: A variational inequality approach (2nd ed.). Boston, Massachusetts: Kluwer Academic Publishers.

Nagurney, A., 2006. Supply chain network economics: Dynamics of prices, flows, and Profits. Edward Elgar Publishing, Cheltenham, United Kingdom.

Nagurney, A., 2010. Formulation and analysis of horizontal mergers among oligopolistic firms with insights into the merger paradox: A supply chain network perspective. Computational Management Science, 7, 377-410.

Nagurney, A., Dupuis, P., Zhang, D., 1994. A dynamical systems approach for network oligopolies and variational inequalities. Annals of Regional Science, 28, 263-283.

Nagurney, A., Li, D., 2016. Competing on supply chain quality: A network economics perspective. Springer International Publishing Switzerland.

Nagurney, A., Li, D., Nagurney, L.S., 2014. Spatial price equilibrium with information asymmetry in quality and minimum quality standards. International Journal of Production Economics, 158, 300-313.

Nagurney, A., Yu, M., Qiang, Q., 2011. Supply chain network design for critical needs with outsourcing. Papers in Regional Science, 90, 123-142.

Nagurney, A., Zhang, D., 1996. Projected dynamical systems and variational inequalities with applications. Boston, Massachusetts: Kluwer Academic Publishers.

Newcomb, R.T., Fan, J., 1980. Coal market analysis issues. EPRI Report EA-1575. Palo Alto, California: Electric Power Research Institute.

Newcomb, R.T., Reynolds, S.S., Masbruch, T.A., 1990. Changing patterns of investment decision making in world aluminum. Resources and Energy, 11, 261-297.

Qiang, Q., Ke, K., Anderson, T., Dong, J., 2013. The closed-loop supply chain network with competition, distribution channel investment, and uncertainties. Omega, 41(2), 186-194.

Ruijs, A., Schweigman, C., Lutz, C., Sirpe, G., 2001. Cereal trade in developing countries: Stochastic spatial equilibrium models. Technical report, University of Groningen, The Netherlands.

Samuelson, P.A., 1952. Spatial price equilibrium and linear programming. American Economic Review, 42, 283-303.

Schiffauerova, A., Thomson, V., 2006. Managing cost of quality: Insight into industry practice. The TQM Magazine, 18(5), 542-550.

Shapiro, C., 1982. Consumer information, product quality, and seller reputation. The Bell Journal of Economics, 13(1), 20-35.

Sohn, H.K., 1970. A spatial equilibrium model of the beef industry in the United States, PhD Dissertation, Department of Agricultural Economics, University of Hawaii.

Spence, M., 1975. Monopoly, quality, and regulation. The Bell Journal of Economics, 6(2), 417-429.

Strom, S., 2013. Taylor farms, big food supplier, grapples with frequent recalls. The New York Times, August 19.

Takayama, T., Judge, G.G., 1971. Spatial and temporal price and allocation models. Amsterdam, The Netherlands: North-Holland.

Teng, J.T., Thompson, G.L., 1996. Optimal strategies for general price-quality decision models of new products with learning production costs. European Journal of Operational Research, 93(3), 476-489.

Thompson, R.L., 1989. Spatial and temporal price equilibrium agricultural models. In: Labys, W.C., Takayama, T., Uri, N. (Eds.). Quantitative methods for market oriented economic analysis over space and time. Brookfield, Vermont: Gower Publishing Co. Ltd., pp. 49-65.

van den Bergh, J.C.J.M., Nijkamp, P., Rietveld, P.,1996. Spatial equilibrium models: A survey with special emphasis on transportation. In: van den Bergh, J.C.J.M., Nijkamp, P., Rietveld, P. (Eds.). Recent advances in spatial equilibrium modelling. Heidelberg, Germany: Springer, pp. 48-76.

van Zandt, T., Lettai, M., 2003. Robustness of adaptive expectations as an equilibrium selection device. Macroeconomic Dynamics, 7, 89-118.

Vörös, J., 2006. The dynamics of price, quality and productivity improvement decisions. European Journal of Operational Research, 170(3), 809-823.

Wang, F.K., Lee, W., 2001. Learning curve analysis in total productive maintenance. Omega, 29(6), 491-499.

Wildman, S.S., 1984. A note on measuring surplus attributable to differentiated products. Journal of Industrial Economics, 33(1), 123-32.

Xie, G., Yue, W., Wang, S., Lai, K.K., 2011. Quality investment and price decision in a riskaverse supply chain. European Journal of Operational Research, 214(2), 403-410.

Yelle, L.E., 1979. The learning curve: Historical review and comprehensive survey. Decision Sciences, 10(2), 302-328.

Zeithaml, V.A., 1988. Consumer perceptions of price, quality, and value: A means-end model and synthesis of evidence. The Journal of Marketing, 52(3), 2-22.

Zhang, D., 2006. A network economic model for supply chain versus supply chain competition. Omega, 34(3), 283-295.

Zhang, G., Shang, J., Li, W., 2011. Collaborative production planning of supply chain under price and demand uncertainty. European Journal of Operational Research, 215(3), 590-603.

Zhao, Y., Yang, S., Narayan, V., Zhao, Y., 2013. Modeling consumer learning from online product reviews. Marketing Science, 32(1), 153-169.

Zhao, Y., Zhao, Y., Helsen, K., 2011. Consumer learning in a turbulent market environment: Modeling consumer choice dynamics after a product-harm crisis. Journal of Marketing Research, 48(2), 255-267.

Appendix A

Proof of Theorem 1:

We first establish necessity, that is, if $(Q^*, \rho^*, \pi^*) \in K^1$ satisfies the spatial price equilibrium conditions according to Definition 1, then it also satisfies variational inequality (12).

Note that, for a fixed pair of supply and demand markets (i, j), (9) implies that

$$(\pi_i^* + c_{ij}(Q^*) - \rho_{ij}^*) \times (Q_{ij} - Q_{ij}^*) \geq 0, \quad \forall Q_{ij} \geq 0. \tag{A1}$$

Indeed, since, if $Q_{ij}^* > 0$, we know, from the equilibrium conditions, that the expression to the left of the multiplication sign in (A1) will be zero, so (A1) holds true; also, if $Q_{ij}^* = 0$, then the expressions preceding and following the multiplication sign in (A1) will be nonnegative and, hence, the product is also nonnegative and (A1) holds true as well. Summing (A1) over all supply markets i and over all demand markets j, we obtain:

$$\sum_{i=1}^{m} \sum_{j=1}^{n} (\pi_i^* + c_{ij}(Q^*) - \rho_{ij}^*) \times (Q_{ij} - Q_{ij}^*) \geq 0, \quad \forall Q \in R_+^{mn}. \tag{A2}$$

Analogously, it follows that if each ρ_{ij}^* satisfies (10), then

$$(Q_{ij}^* - d_{ij}(\rho^*, q(\pi^*))) \times (\rho_{ij} - \rho_{ij}^*) \geq 0, \quad \forall \rho_{ij} \geq 0. \tag{A3}$$

Summing (A3) over all supply markets i and over all demand markets j, we obtain:

$$\sum_{i=1}^{m} \sum_{j=1}^{n} (Q_{ij}^* - d_{ij}(\rho^*, q(\pi^*))) \times (\rho_{ij} - \rho_{ij}^*) \geq 0, \quad \forall \rho \in R_+^{mn}. \tag{A4}$$

In the same way, if each π_i^* satisfies (11), then

$$(s_i(\pi^*) - \sum_{j=1}^{n} Q_{ij}^*) \times (\pi_i - \pi_i^*) \geq 0, \quad \forall \pi_i \geq 0. \tag{A5}$$

Summing (A5) over all supply markets i, yields

$$\sum_{i=1}^{m} (s_i(\pi^*) - \sum_{j=1}^{n} Q_{ij}^*) \times (\pi_i - \pi_i^*) \geq 0, \quad \forall \pi_i \in R_+^{m}. \tag{A6}$$

Combining now (A2), (A4), and (A6), we obtain variational inequality (12).

We now establish sufficiency, that is, if $(Q^*, \rho^*, \pi^*) \in K^1$ satisfies variational inequality (12) then it also satisfies the spatial price equilibrium conditions (9), (10), and (11).

Let $Q_{ij} = Q^*_{ij}$, $\forall (i,j) \neq (k,l)$, $\pi_i = \pi^*_i$, $\forall i$, and $\rho_{ij} = \rho^*_{ij}$, $\forall (i,j)$, and substitute into (12). The resultant is:

$$(\pi^*_k - c_{kl}(Q^*) - \rho^*_{kl}) \times (Q_{kl} - Q^*_{kl}) \geq 0, \quad \forall Q_{kl} \geq 0. \tag{A7}$$

But (A7) implies that, if $Q^*_{kl} = 0$, then $(\pi^*_k - c_{kl}(Q^*) - \rho^*_{kl}) \geq 0$, since $Q_{kl} \geq 0$; and, if $Q^*_{kl} > 0$, then $(\pi^*_k - c_{kl}(Q^*) - \rho^*_{kl}) = 0$, since $(Q_{kl} - Q^*_{kl})$ can be positive, negative, or zero. Because these results hold for any pair (k,l), we can conclude that the equilibrium conditions (9) are satisfied by the product shipment pattern satisfying (12).

Similarly, we now let $Q_{ij} = Q^*_{ij}$, $\forall (i,j)$, $\pi_i = \pi^*_i$, $\forall i$, and $\rho_{ij} = \rho^*_{ij}$, $\forall (i,j) \neq (k,l)$. Substitution of these into (12) yields

$$(Q^*_{kl} - d_{kl}(\rho^*, q(\pi^*))) \times (\rho_{kl} - \rho^*_{kl}) \geq 0, \quad \forall \rho_{kl} \geq 0. \tag{A8}$$

According to (A8), if $\rho^*_{kl} = 0$, then $(Q^*_{kl} - d_{kl}(\rho^*, q(\pi^*))) \geq 0$; and, if $\rho^*_{kl} > 0$, for (A8) to hold, $(Q^*_{kl} - d_{kl}(\rho^*, q(\pi^*))) = 0$. Since these results hold for any pair (k,l), we know that $\rho^* \in R^{mn}_+$ satisfying variational inequality (12) also satisfies equilibrium conditions (10).

Analogously, we now let $Q_{ij} = Q^*_{ij}$, $\forall (i,j)$, $\rho_{ij} = \rho^*_{ij}$, $\forall (i,j)$, and $\pi_i = \pi^*_i$, $\forall i \neq k$. Substitution into (12) yields:

$$(s_k(\pi^*) - \sum_{j=1}^{n} Q^*_{kj}) \times (\pi_k - \pi^*_k) \geq 0, \quad \forall \pi_k \geq 0. \tag{A9}$$

According to (A9), if $\pi^*_k = 0$, then $(s_k(\pi^*) - \sum_{j=1}^{n} Q^*_{kj}) \geq 0$; and, if $\pi^*_k > 0$, for (A9) to hold, $(s_k(\pi^*) - \sum_{j=1}^{n} Q^*_{kj}) = 0$. Since these results hold for any pair (k,l), we know that $\pi^*_k \in R^m_+$ satisfying variational inequality (12) also satisfies the equilibrium conditions in (11).

The proof is complete.□

Appendix B

Proof of Theorem 6:

Note that, if the adaptive spatial price equilibrium model under information asymmetry satisfies the following two assumptions: 1) the supply, demand, transportation cost, quality, and quality perception functions remain unchanged over time; 2) consumers' perceived quality in one period depend on the actual product quality of that period and their perceived quality in the preceding period, then the spatial price equilibrium problem in each period can be converted into the corresponding problem under perfect quality information, that is, the smooth function $G^t(X^t, X^{t-1})$ has the following property:

$$G^t(X^t, X^t) = F(X), \quad \forall t. \tag{A10}$$

In such case, in view of the definition (39), we have

$$g(x, x) = F(X). \tag{A11}$$

Following Theorem 2.2 in Nagurney (1999) (see also Dafermos (1983)), we conclude that the sequence $\{X^{t^*}\}$ converges, that is, $X^{t^*} \to X^*$, as $t \to \infty$. □

Appendix C

Algorithm:

The Euler method is employed for the computation of the solutions for the spatial price equilibrium models with quality information and product differentiation presented in Section 2. This method is induced by the general iterative scheme of Dupuis and Nagurney (1993).

Specifically, for the spatial price equilibrium model with product differentiation under perfect quality information, recall that at iteration τ of the Euler method (see also Nagurney and Zhang (1996)), one computes:

$$X^{\tau+1} = P_{\mathcal{K}}(X^\tau - a_\tau F(X^\tau)), \tag{A12}$$

where $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$ and F is the function that enters the variational inequality problem (13).

Various applications to the solutions of other network models can be found in Nagurney, Dupuis, and Zhang (1994), Nagurney (2010), Nagurney, Yu, and Qiang (2011), and in Nagurney, Li, and Nagurney (2014).

Explicit Formulae for the Euler Method Applied to the Spatial Price Equilibrium with Product Differentiation Under Perfect Quality Information

The Euler method yields, at each iteration, explicit formulae for the computation of the product shipment, demand price, and supply price pattern. In particular, we have the following closed form expression:

for the product shipments $i = 1, \ldots, m; j = 1, \ldots, n$:

$$Q_{ij}^{\tau+1} = \max\{0, Q_{ij}^\tau + a_\tau(-\pi_i^\tau - c_{ij}(Q^\tau) + \rho_{ij}^\tau)\};$$

for the demand prices $i = 1, \ldots, m; j = 1, \ldots, n$:

$$\rho_{ij}^{\tau+1} = \max\{0, \rho_{ij}^{\tau} + a_\tau(-Q_{ij}^{\tau} + d_{ij}(\rho^\tau, q(\pi^\tau)))\},$$

and for the supply prices $i = 1, \ldots, m$:

$$\pi_i^{\tau+1} = \max\{0, \pi_i^{\tau} + a_\tau(-s_i(\pi^\tau) + \sum_{j=1}^{n} Q_{ij}^{\tau})\}.$$

Similarly, for the spatial price equilibrium model with product differentiation under information asymmetry in quality, for time period t, at iteration τ of the Euler method, one computes:

$$X^{t^{\tau+1}} = P_{\mathcal{L}^t}(X^{t^\tau} - a_\tau G^t(X^{t^\tau})), \tag{A13}$$

where $P_{\mathcal{L}}^t$ is the projection on the feasible set $\mathcal{L}^t$ and G^t is the function that enters the variational inequality problem (25).

Explicit Formulae for the Euler Method Applied to the Spatial Price Equilibrium with Product Differentiation Under Information Asymmetry in Quality

For the spatial price equilibrium model with product differentiation under information asymmetry in quality, for time period t, the Euler method yields, at each iteration, the following closed form expression:

for the product shipments $i = 1, \ldots, m; j = 1, \ldots, n$:

$$Q_{ij}^{t^{\tau+1}} = \max\{0, Q_{ij}^{t^\tau} + a_\tau(-\pi_i^{t^\tau} - c_{ij}^t(Q^{t^\tau}) + \rho_{ij}^{t^\tau})\},$$

for the demand prices $i = 1, \ldots, m; j = 1, \ldots, n$:

$$\rho_{ij}^{t^{\tau+1}} = \begin{cases} \max\{0, \rho_{ij}^{1^\tau} + a_\tau(-Q_{ij}^{1^\tau} + d_{ij}^1(\rho^{1^\tau}, \hat{q}^1))\}, & \text{if } t = 1, \\ \max\{0, \rho_{ij}^{t^\tau} + a_\tau(-Q_{ij}^{t^\tau} + d_{ij}^t(\rho^{t^\tau}, \hat{q}^t(q^{t-1}(\pi^{t-1^*}), \hat{q}^{t-1})))\}, & \text{if } t \geq 2, \end{cases}$$

and for the supply prices $i = 1, \ldots, m$:

$$\pi_i^{t^{\tau+1}} = \max\{0, \pi_i^{t^\tau} + a_\tau(-s_i^t(\pi^{t^\tau}) + \sum_{j=1}^{n} Q_{ij}^{t^\tau})\}.$$

We now provide the convergence result. The proof is direct from Theorem 6.10 in Nagurney and Zhang (1996).

Theorem A1

In the spatial price equilibrium model with product differentiation under perfect quality information, let $F(X)$ be strictly monotone at any equilibrium pattern, and assume that Assumption 1 is satisfied. Also, assume that F is Lipschitz continuous, that is,

$$\|F(X^1) - F(X^2)\| \leq L\|X^1 - X^2\|, \quad \forall X^1, X^2 \in \mathcal{K},$$

where L is a positive number known as the Lipschitz constant. Then there exists a unique equilibrium product shipment, demand price, and supply price pattern $(Q^*, \rho^*, \pi^*) \in \mathcal{K}$ and any sequence generated by the Euler method as given by (A12), where $\{a_\tau\}$ satisfies $\sum_{\tau=0}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \to 0$, as $\tau \to \infty$ converges to (Q^*, ρ^*, π^*).

Similarly, in the spatial price equilibrium model with product differentiation under information asymmetry in quality, let $G^t(X^t)$ be strictly monotone at any equilibrium pattern for time period t, and assume that Assumption 1 is satisfied. Also, assume that $G^t(X^t)$ is Lipschitz continuous. Then there exists a unique equilibrium product shipment, demand price, and supply price pattern for time period t $(Q^{t^*}, \rho^{t^*}, \pi^{t^*}) \in \mathcal{L}^t$ and any sequence generated by the Euler method as given by (A13), where $\{a_\tau\}$ satisfies $\sum_{\tau=0}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \to 0$, as $\tau \to \infty$ converges to $(Q^{t^*}, \rho^{t^*}, \pi^{t^*})$.