Centralized Supply Chain Network Optimization with UAV-based Last Mile Deliveries

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Accepted for Publication in *Transportation Research C*

Abstract

This paper proposes a centralized supply chain network optimization model that maximizes the total profit obtained by a company that produces and/or outsources production, stores, ships and sells products to customers using a fleet made up of trucks and, in the last mile, also of drones. The model includes realistic features of unmanned aerial vehicles (UAVs) in the form of drones with fundamental limitations such as low battery capacities and short delivery ranges. The constrained nonlinear optimization problem is formulated as a variational inequality. Existence and uniqueness results for the solution of the variational inequality are provided along with the results of detailed numerical simulations that emphasize the advantages of the use of a hybrid fleet from enhanced profits to reduction in air pollution. Our quantitative results reveal great promise and insights for the logistics industry in the use of emerging UAV technologies for last mile parcel deliveries as a practical solution within a holistic supply chain network context.

Key words: Parcel delivery, Unmanned Aerial Vehicle, Supply chains, Networks, Nonlinear optimization, Variational formulation

1 Introduction

Unmanned aerial vehicles (UAVs), also known as drones, were originally used for military purposes. Following the rapid technological progress recorded during the 2000s and, thanks to their versatility, UAVs have also been used in the civilian sector, where they are employed in aerial surveillance, in aerial cinematographic shots, in operations search and rescue missions, in the monitoring of power lines and oil pipelines, and in the monitoring of flora and fauna. In 2013, Amazon first announced 30-minute drone deliveries and the company successfully piloted its so-called 'Prime Air' drone delivery service in Cambridge, UK in December 2016 and in the USA in March 2017. Later, many other companies, such as Walmart, DHL, UPS, FedEx, Uber Eats, and others, also chose delivery-by-drone in order to provide faster-than-ever service [21]. Parallel to advances in UAV technologies, customer expectations of delivery options have risen with the real-time tracking of orders, same-day and free delivery options, growing service requirements and even greater supply chain complexity. The use of drones for delivery services, nevertheless, reveals some difficulties. First of all, drones can often only carry light and low-volume parcels, and usually only one package at a time. Furthermore, they have a limited battery life, which allows them to go only for short distances. Hence, a hybrid delivery system combining traditional trucks and drones may reduce traffic issues, costs, including environmental ones, and, at the same time, address flight range limitations, as the transportation is initially done with trucks and, in the last mile, the delivery can be done also with drones. Drones can also be launched from trucks in a hybrid system (see [4], [8]).

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Last mile delivery can be defined as the process of delivering a shipment from a transportation hub to the final delivery destination. This means that the last phase of the supply chain network has been reached and the product is almost at the end of its journey. In our model in this paper, we construct a supply chain network with five sets of nodes consisting of a company at the top, warehouses of third-party sellers and of the company in the second tier of nodes, followed by fulfillment centers, and then delivery stations, with customer locations at the bottom tier. Trucks are involved in the transportation of the products between all the tiers, but, in the final stretch, which is the last mile, also drones can be used. The transportation takes place on links of the supply chain network that join the nodes. Obviously, the advantage of using trucks consists mainly in their large load capacity and in their ability to make long journeys, while the use of drones can reduce traffic congestion and pollutant emissions in urban areas, reduce fuel and manpower consumption, eliminate the need for parking, and improve the timeliness of deliveries.

The last leg or link in the supply chain network may be less than a mile in length or it may be hundreds of miles long: what's important is that the last mile takes deliveries from the delivery stations to the end customers. This transportation component is, typically, a complex, expensive part of the supply chain network. Imagine the hundreds, if not thousands, of deliveries made by a company each day. The use of drones for last mile delivery has gained significant attention in recent years due to its potential to improve delivery efficiency and to reduce transportation costs as well as environmental costs. Many scientists have conducted research on optimizing the last mile with drones (see [18] for a literature review in which authors selected a collection of recent papers and classified them as routing, cargo distribution optimization, battery management, data communication and environmental protection). See [23] for a survey of routing problems with drones, primarily in the context of parcel delivery.

In [31], the authors studied humanitarian logistics; specifically, they analyzed drone applications in last mile distribution and proposed an optimization model for the delivery of multiple packages of light-weight relief items via drones to remote locations within a disaster prone area (see also [13] and [12] for optimization models for the provision of services with UAVs in disaster management phases, and of the 5G-network edge in rural areas, respectively). In [10], the authors proved that using UAVs for last mile logistics is not only cost-effective, but also reduces carbon emissions. They proposed a mixed-integer linear routing model for UAV last mile parcel deliveries and developed a genetic algorithm. Some researchers have also studied the optimal location of drone-beehives based on an economic viability criterion (see, for example, [3] where four different scenarios are analyzed to estimate the potential benefit obtained by citizens from last mile drone delivery services, through a modeling framework using high-resolution data on the EU-wide population and land use). Borghetti et al. in [6] carried out a stated preferences analysis in a real case study in the city of Milan in order to assess the propensity of users to use drones or not. The authors also performed a financial feasibility analysis to evaluate the costs and the revenues for a logistics operator in charge of the UAV-based last mile logistics service.

Many researchers have investigated delivery networks with trucks and drones simultaneously. The first study considering drone-truck collaboration dates back to 2015. In [26], the authors presented a Mixed Integer Linear Programming Problem with the aim of minimizing the time at which both vehicles complete service and return to the depot such that every customer is either served by the truck, or by the drone that operates in synchronization with the truck. Subsequently, many other authors have used mixed integer linear programming models and heuristic or exact algorithms to investigate several variants of the problem of finding the optimal routing for a set of trucks and a set of drones operating jointly (see, for instance, [1], [7], [9], [15], [34]). In [4], the authors introduce a vehicle routing problem with flexible drones aiming at minimizing the return time of the very last vehicle (drone or truck) to the depot after completing its service. In [11], the authors consider how an online food delivery platform can improve last mile delivery services' performance using multi-source data. They propose a data-driven optimization approach that combines machine learning techniques with capacitated vehicle routing optimization. In [29], the authors study the Vehicle Routing and Scheduling Problem with Time-Dependent Costs in which the probability of a successful delivery is taken into account when planning the vehicle routes. In [32], the authors optimize the partitioning of delivery locations into small clusters, so that from each focal point per cluster (that is, drone launch location), where a truck is parked, a drone fleet is launched towards the nearby customer locations. They also analyzed the route of the truck (through all the launch locations) such that the customer demands are met by a drone or truck. Authors in [5] proposed a compact Integer Linear Programming formulation for a variant of the Traveling Salesman Problem where routing decisions are integrated with customer-to-drone and customer-to-truck assignment decisions and truck-and-drone synchronization constraints. Another problem where the movements of a single truck and multiple UAVs (launched from the truck) are synchronized is proposed in [25], where the mathematical model aims to allocate customers to UAVs and the truck. In the latter, the authors formulated a Mixed-Integer Linear Programming Problem and developed a routing algorithm with the objective of minimizing customer waiting times for deliveries.

In this paper, we study a five-tier supply chain network for a company which can buy from third-party sellers and/or can produce in-house and sells different products which are stored in different warehouses. Such warehouses are geographically distributed and are connected to fulfillment centers, which handle order processing, picking, packing and shipping. The products are then transported to the delivery stations which are physical locations out of which the company conducts its logistics delivery services. Finally, the products are delivered to the customers at their locations by trucks and/or drones.

We analyze the entire supply chain network, including the last mile delivery, which can be done by trucks and/or by using drones. We study the problem from a system point of view; that is, from that of the company, which coordinates and manages the whole process. We obtain a nonlinear optimization problem with the aim of maximizing the total profit of the company, given by the difference between the total revenues and the total costs. The model proposed in this paper seeks to determine the optimal quantities of products to buy from third-party sellers and/or to self-produce and the optimal flows for each link of the network. The proposed model is able to establish which nodes of the network are best to use from an optimization perspective and the same for the links, and whether to make last mile delivery with drones and/or with trucks. Our supply chain network optimization model also takes into account the maximum capacity of trucks and drones and the maximum distance that a drone can reach, due to its battery duration. Furthermore, the model includes an environmental component in the form of incentives for using drones to reduce congestion and pollution. The numerical simulations we provide, firstly, suggest that using UAVs at the last mile can significantly improve the objective function of profit maximization, by reducing the delivery costs. Another consequence of the use of drones is the reduction of environmental emissions. Some additional supply chain network configurations also reveal how the proposed model takes into account real drone limitations, such as their limited capacity and battery life. Finally, a sensitivity analysis demonstrates that the choice of incentives (by National Institutions) can affect the environmental impacts, since companies alter their decision-making strategies accordingly.

The paper is organized as follows. In Section 2, we describe the supply chain network and in Section 3, we introduce the variables, the parameters and the functions that allow us to present the constrained nonlinear optimization problem. In Section 4, we deduce the associated variational inequality formulation and provide the related existence and uniqueness results. In Section 5, we present detailed results of numerical simulations in order to illustrate key aspects of the centralized supply chain network optimization model and to validate its effectiveness. Finally, Section 6 is devoted to our conclusions and further research.

2 The Supply Chain Network Description

We consider a supply chain network as the one depicted in Figure 1, where five different tiers are present. At the top level node, there is a company (such as Amazon) that can produce the products and/or store the self-produced products at its warehouse W_{I+1} located in the second tier of nodes. In the second tier, there are also I third-party sellers' warehouses, where the typical one is denoted by i. We consider L different kinds of products, with the typical one denoted by l. In the third level of nodes of the supply chain network, there are J fulfillment centers, with the typical one denoted by j; in the fourth tier, there are S delivery stations, with the typical one denoted by s. Finally, in the bottom tier, there are K customer locations, with the typical one denoted by k. The different nodes of the supply chain network are connected by links. Specifically, the links between the company and the third-party sellers' warehouses and the company's warehouse are virtual links (denoted with dashed arrows in Figure 1), since actually there is no transportation of commodities between the first and the second tiers of the network. These links could, nevertheless, be associated with some economic activities. Specifically, we assume that links between the company and the third-party sellers' warehouses are associated with the purchasing of products, while the link between the company and its warehouse is associated with the manufacturing activity. Moreover, we assume that the products produced by the third-party sellers or the company are sold and sent from the warehouses to the fulfillment centers. From fulfillment centers the products can be sent both to delivery stations or directly to customers. Finally, delivery stations ship the products to the customers. Furthermore, trucks are used for long journeys and heavy goods and, therefore, in the transportation of products between: warehouses and fulfillment centers, the fulfillment centers and the delivery stations or the fulfillment centers and the customers. In the "last mile"; that is, the final stretch that connects delivery stations and customer locations, both trucks or drones can be used. We denote by black arrows the links associated with trucks and by blue dashed arrows the links associated with drones. As noted in the Introduction, the use of drones or other types of Unmanned Aerial Vehicles in the "last mile" is justified by the need to reduce pollution and congestion in urban areas. For this reason, we also introduce an incentive to use this means of transport.

Furthermore, as related to Amazon as the company, there exist two types of fulfillment. According to Fulfillment By Amazon (FBA), factories sell their products to Amazon and, when a customer makes a purchase, Amazon picks, packs and ships the order. On the other hand, Fulfillment By Merchant (FBM) is when the factory handles the entire shipping process. It uses its own resources and sends the products directly to the customers. In this paper, we analyze the FBA type of fulfillment. Therefore, the aim of the company is to maximize the total profit of its supply chain network.

In this paper, we are assuming that, for both the battery life and the drone capacity, each drone starts at D_s, reaches C_k, and comes back (i.e., doesn't route). Indeed, there could be more trucks or drones making the same journey when there is a need to increase the total maximum capacity. Therefore, we are not focusing on the Vehicle Routing Problem (VRP). Hence, in our framework, it does not matter if it's a single truck that, for example, starts from G_j and goes to two or more D_s or if it is the case of multiple trucks that go to a single D_s.

3 The Optimization Problem

In this section, we focus on the mathematical formulation of the problem. As previously described, the supply chain network consists of $I+1$ warehouses $(W_1, \dots, W_i, \dots, W_I, W_{I+1})$, J fulfillment centers $(G_1, \dots, G_j, \dots, G_J)$, S delivery stations $(D_1, \dots, D_s, \dots, D_S)$ and K customer locations $(C_1, \dots, C_k, \dots, C_K)$. Moreover, we take into account L different types of products. In Table 1, we report the symbols and definition of all the sets.

The aim of the supply chain network optimization problem is to determine:

- the optimal quantities of each product $l \in L$ that the company has to buy from third-party

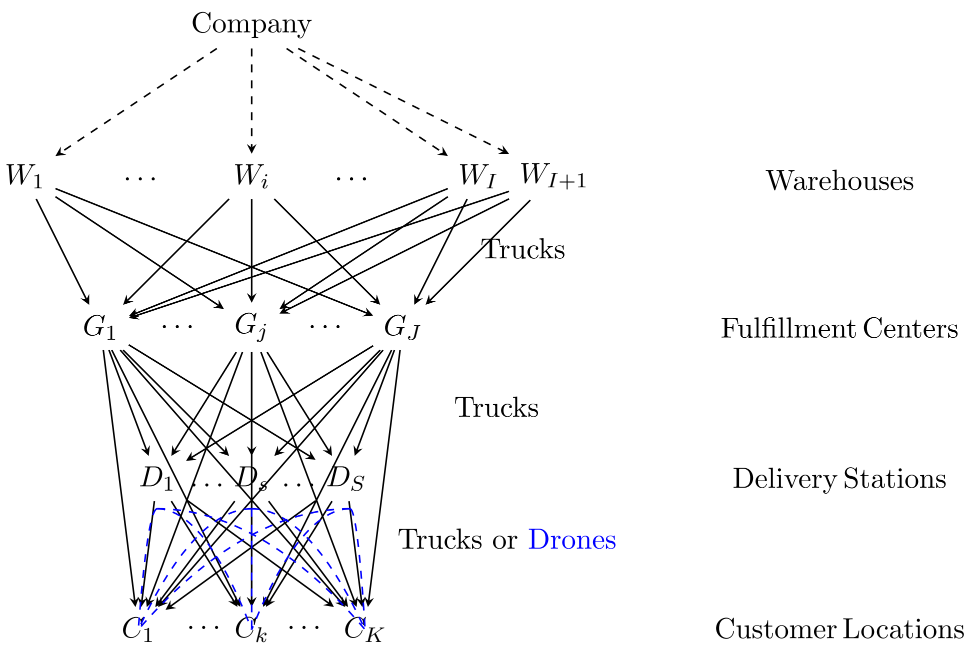

Figure 1: Supply chain network topology

Sets	**Definitions**
$W = \{W_i : i = 1, \dots, I+1\}$	set of warehouses
$G = \{G_j : j = 1, \dots, J\}$	set of fulfillment centers
$D = \{D_s : s = 1, \dots, S\}$	set of delivery stations
$C = \{C_k : k = 1, \dots, K\}$	set of customer locations
$\mathcal{L} = \{l : l = 1, \dots, L\}$	index set of products

Table 1: Symbols and definitions of sets

sellers (α_i^l, $\forall i = 1, \dots, I$) and to self-produce (α_{I+1}^l);

- the optimal quantities of each product $l \in L$ that the company has to transport from each warehouse W_i, $\forall i = 1, \dots, I+1$ to each fulfillment center G_j, $\forall j = 1, \dots, J$ (x_{ij}^l, $\forall l = 1, \dots, L$, $\forall i = 1, \dots, I+1$, $\forall j = 1, \dots, J$);
- the optimal quantities of each product $l \in L$ that the company has to transport from each fulfillment center G_j, $\forall j = 1, \dots, J$ to each delivery station D_s, $\forall s = 1, \dots, S$ (y_{js}^l, $\forall l = 1, \dots, L$, $\forall j = 1, \dots, J$, $\forall s = 1, \dots, s$) and to each customer location C_k, $\forall k = 1, \dots, K$ (z_{jk}^l, $\forall l = 1, \dots, L$, $\forall j = 1, \dots, J$, $\forall k = 1, \dots, K$);
- the optimal quantities of each product $l \in L$ that the company has to transport from each delivery station D_s, $\forall s = 1, \dots, S$ to each customer location C_k, $\forall k = 1, \dots, K$ via trucks (w_{sk}^{1l}, $\forall l = 1, \dots, L$, $\forall s = 1, \dots, S$, $\forall k = 1, \dots, K$) and via drones (w_{sk}^{2l}, $\forall l = 1, \dots, L$, $\forall s = 1, \dots, S$, $\forall k = 1, \dots, K$).

Table 2 summarizes all the variables and their definitions.

Table 3 and Table 4 show all the parameters and the cost functions of the model, respectively.

We are interested in optimizing the supply chain network of the company, whose aim is to maximize its profits. Therefore, the company seeks to maximize the difference between the revenue and the costs. Hence, the objective function consists of:

Variables	Definitions
α_i^l	quantity (in kg) of product l produced by a third-party seller, bought by the company (for the customers) and stored in the third-party seller's warehouse W_i, $i = 1, \dots, I$. While α_{I+1}^l represents the quantity (in kg) of product l produced by the company. We group such quantities into the vector $\alpha \in \mathbb{R}_+^{(I+1)L}$.
x_{ij}^l	quantity of product l sent by W_i to G_j, $i = 1, \dots, I+1$, $j = 1, \dots, J$ (in kg). We group such quantities into the vectors $x_{ij} \in \mathbb{R}_+^L$ and $x \in \mathbb{R}_+^{(I+1)JL}$.
y_{js}^l	quantity of product l sent by G_j to D_s, $j = 1, \dots, J$, $s = 1, \dots, S$ (in kg). We group such quantities into the vectors $y_{js} \in \mathbb{R}_+^L$, $y_j \in \mathbb{R}_+^{LS}$ and $y \in \mathbb{R}_+^{JSL}$.
z_{jk}^l	quantity of product l sent by G_j to C_k, $j = 1, \dots, J$, $k = 1, \dots, K$ (in kg). We group such quantities into the vectors $z_{jk} \in \mathbb{R}_+^L$, $z_j \in \mathbb{R}_+^{LK}$ and $z \in \mathbb{R}_+^{JKL}$.
w_{sk}^{1l}	quantity of product l sent by D_s, to C_k $s = 1, \dots, S$, $k = 1, \dots, K$ (in kg) via trucks. We group such quantities into the vectors $w_{sk}^1 \in \mathbb{R}_+^L$ and $w^1 \in \mathbb{R}_+^{SKL}$.
w_{sk}^{2l}	quantity of product l sent by D_s, to C_k $s = 1, \dots, S$, $k = 1, \dots, K$ (in kg) via drones. We group such quantities into the vectors $w_{sk}^2 \in \mathbb{R}_+^L$ and $w^2 \in \mathbb{R}_+^{SKL}$.

Table 2: Symbols and definitions of variables

- the handling costs of the company ($C^{(hand)} = c(\alpha, x, y, z, w^1, w^2)$);
- the total expense for the purchase of products, C^{PurPr}, from all W_i, $i = 1, \dots, I$ $\left(\sum_{l=1}^{L}\sum_{i=1}^{I} \gamma_i^l \alpha_i^l\right)$ and the production cost for products, when they are produced by the company itself ($\gamma_{I+1}^l \alpha_{I+1}^l$);
- the total transportation cost from each warehouse to each fulfillment center $\left(C^{(WG)} = \sum_{i=1}^{I+1}\sum_{j=1}^{J} c_{ij}(x_{ij})\right)$; from each fulfillment center to each delivery station $\left(C^{(GD)} = \sum_{j=1}^{J}\sum_{s=1}^{S} \hat{c}_{js}(y_{js}, z_j)\right)$; from each fulfillment center to each customer location $\left(C^{(GC)} = \sum_{j=1}^{J}\sum_{k=1}^{K} \bar{c}_{jk}(y_j, z_{jk})\right)$; from each delivery station to each customer location via truck $\left(C_{tr}^{(DC)} = \sum_{s=1}^{S}\sum_{k=1}^{K} \tilde{c}_{sk}^1(w_{sk}^1)\right)$; and from each delivery station to each customer location via drone $\left(C_{dr}^{(DC)} = \sum_{s=1}^{S}\sum_{k=1}^{K} \tilde{c}_{sk}^2(w_{sk}^2)\right)$;
- the incentive for sustainable mobility (in order to reduce pollution and congestion) that the company receives from the National Institution if it uses drones (instead of trucks), in the links between the delivery stations and the customer locations $\left(I^{SM} = \sum_{l=1}^{L}\sum_{s=1}^{S}\sum_{k=1}^{K} I_{sk} w_{sk}^{2l}\right)$;

Parameters	Definitions
γ_i^l	purchase price of product l from $W_i,\ i=1,\dots,I$; while γ_{I+1}^l is the production cost for the good l, when it is produced by the company (in €)
I_{sk}	incentive for using drones (to reduce pollution and congestion) on the link between D_s and C_k
γ_{jk}^l	selling price of product l from G_j to C_k, established by the company (in €)
γ_{sk}^l	selling price of product l from D_s to C_k, established by the company (in €)
Q_i^l	maximum quantity (in kg) of product l available at W_i
r_k^l	demand of product l from customer location C_k (in kg)
n_i^{tr}	number of trucks available at W_i
W	maximum capacity of each truck (in kg)
n_s^{dr}	number of drones available at D_s
p_d	maximum weight that each drone is able to carry (in kg)
p_l	weight of a unit of product l (in kg)
P_l^{dr}	a parameter that is equal to 0 if $p_l > p_d$, otherwise (if $p_l \leq p_d$) it is equal to $\sum_{k=1}^{K} r_k^l$
$\tilde{d}_{sk}$	distance between delivery station D_s and customer location C_k (in km)
b_d	drone battery life, that is, the distance allowed taking into account both outward and return trips (in km)
B_{sk}^{dr}	a parameter that is equal to 0 if $\tilde{d}_{sk} > b_d$, while it is equal to $\sum_{k=1}^{K} r_k^l$ if $\tilde{d}_{sk} \leq b_d$

Table 3: Symbols and definitions of parameters

Functions	Definitions
$c(\alpha, x, y, z, w^1, w^2)$	handling costs of the company, which include managing costs
$c_{ij}(x_{ij})$	transportation cost from W_i to G_j
$\hat{c}_{js}(y_{js}, z_j)$	transportation cost from G_j to D_s
$\bar{c}_{jk}(y_j, z_{jk})$	transportation cost from G_j to C_k
$\tilde{c}_{sk}^1(w_{sk}^1)$	transportation cost from D_s to C_k through trucks
$\tilde{c}_{sk}^2(w_{sk}^2)$	transportation cost from D_s to C_k through drones
$c_i^{tr}(\alpha, w^2)$	handling costs of W_i for the trucks; they depend on α and w^2 because the size of the truck chosen depends on the quantity of goods to be transported by trucks
$c_s^{dr}(w^2)$	handling costs of D_s for the drones, including drone battery disposal costs

Table 4: Symbols and definitions of functions

- the handling costs of all trucks $\left(C^{(tr)} = \sum_{i=1}^{I+1} c_i^{tr}(\alpha, w^2) \right)$ and of all drones $\left(C^{(dr)} = \sum_{s=1}^{S} c_s^{dr}(w^2) \right)$;

- the total revenues, TR, obtained from the selling of products from each fulfillment center to each customer $\left(\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{k=1}^{K}\gamma_{jk}^{l}z_{jk}^{l}\right)$; and those obtained from the selling of products from each delivery station to each customer via trucks and drones $\left(\sum_{l=1}^{L}\sum_{s=1}^{S}\sum_{k=1}^{K}\gamma_{sk}^{l}w_{sk}^{1l} \text{ and } \sum_{l=1}^{L}\sum_{s=1}^{S}\sum_{k=1}^{K}\gamma_{sk}^{l}w_{sk}^{2l}\right)$.

Observe that the revenues (γ_{jk}^{l} and γ_{sk}^{l}) take into account the only gains obtained by the company.

We remark that, in reality, not all the third-party sellers' warehouses need to provide products to all the fulfillment centers, and not all the fulfillment centers need to be connected to all the delivery stations or all the customers locations, and not all the delivery stations need to be associated with all the customer locations. In this paper, since the transportation costs differ according to the origin and destination of the link, we also take into account this important and real aspect of different distances that may be involved for transportation between nodes. Of course, one could also associate a very high cost with a link, which would work essentially like the link being unavailable.

The optimization problem is as follows:

$$\max_{\alpha,x,y,z,w^1,w^2}\Bigg\{-c(\alpha,x,y,z,w^1,w^2)-\sum_{l=1}^{L}\sum_{i=1}^{I+1}\gamma_i^l\alpha_i^l-\sum_{i=1}^{I+1}\sum_{j=1}^{J}c_{ij}(x_{ij})-\sum_{j=1}^{J}\sum_{s=1}^{S}\hat{c}_{js}(y_{js},z_j)$$
$$-\sum_{j=1}^{J}\sum_{k=1}^{K}\bar{c}_{jk}(y_j,z_{jk})-\sum_{s=1}^{S}\sum_{k=1}^{K}\tilde{c}_{sk}^{1}(w_{sk}^{1})-\sum_{s=1}^{S}\sum_{k=1}^{K}\tilde{c}_{sk}^{2}(w_{sk}^{2})+\sum_{l=1}^{L}\sum_{s=1}^{S}\sum_{k=1}^{K}I_{sk}w_{sk}^{2l} \tag{1}$$
$$-\sum_{i=1}^{I+1}c_i^{tr}(\alpha,w^2)-\sum_{s=1}^{S}c_s^{dr}(w^2)+\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{k=1}^{K}\gamma_{jk}^{l}z_{jk}^{l}+\sum_{l=1}^{L}\sum_{s=1}^{S}\sum_{k=1}^{K}\gamma_{sk}^{l}w_{sk}^{1l}+\sum_{l=1}^{L}\sum_{s=1}^{S}\sum_{k=1}^{K}\gamma_{sk}^{l}w_{sk}^{2l}\Bigg\}$$

$$\alpha_i^l \le Q_i^l, \quad \forall l=1,\dots,L,\ \forall i=1,\dots,I+1 \tag{2}$$
$$\sum_{j=1}^{J}x_{ij}^{l} \le \alpha_i^l, \quad \forall l=1,\dots,L,\ \forall i=1,\dots,I+1 \tag{3}$$
$$\sum_{i=1}^{I+1}x_{ij}^{l} \ge \sum_{s=1}^{S}y_{js}^{l}+\sum_{k=1}^{K}z_{jk}^{l}, \quad \forall j=1,\dots,J,\ \forall l=1,\dots,I \tag{4}$$
$$\sum_{j=1}^{J}y_{js}^{l} \ge \sum_{k=1}^{K}w_{sk}^{1l}+\sum_{k=1}^{K}w_{sk}^{2l}, \quad \forall s=1,\dots,S,\ \forall l=1,\dots,L \tag{5}$$
$$\sum_{j=1}^{J}z_{jk}^{l}+\sum_{s=1}^{S}w_{sk}^{1l}+\sum_{s=1}^{S}w_{sk}^{2l}=r_k^l, \forall k=1,\dots,K,\ \forall l=1,\dots,L \tag{6}$$
$$\sum_{l=1}^{L}\sum_{j=1}^{J}x_{ij}^{l} \le W\cdot n_i^{tr}, \quad \forall i=1,\dots,I+1 \tag{7}$$
$$\sum_{l=1}^{L}\sum_{k=1}^{K}w_{sk}^{2l} \le p_d n_s^{dr}, \quad \forall s=1,\dots,S \tag{8}$$
$$w_{sk}^{2l} \le P_l^{dr}, \quad \forall l=1,\dots,L,\ \forall s=1,\dots,S,\ \forall k=1,\dots,K \tag{9}$$
$$w_{sk}^{2l} \le B_{sk}^{dr}, \quad \forall s=1,\dots,L,\ \forall k=1,\dots,K,\ \forall l=1,\dots,L \tag{10}$$
$$\alpha_i^l,\ x_{ij}^l,\ y_{js}^l,\ z_{jk}^l,\ w_{sk}^{1l},\ w_{sk}^{2l} \ge 0$$
$$\forall l=1,\dots,L, \forall i=1,\dots,I+1, \forall j=1,\dots,J, \forall s=1,\dots,S, \forall k=1,\dots,K. \tag{11}$$

Constraint (2) ensures that the quantity of each type of product sold by a third-party seller to the company (or self-produced by the organization) does not exceed the maximum quantity that can be produced. Constraint (3) establishes that the total amount of product sent from a warehouse to all the fulfillment centers must be less than or equal to that bought by the correspondent third-party seller or that self-produced. Constraint (4) affirms that the quantity of product l sent from G_j to all the delivery stations and the customers must be less than or equal to that received by all the warehouses. Constraint (5) states that the quantity of product l that D_s receives from all the fulfillment centers must be greater than or equal to that which it sells to all customers in both ways (trucks and drones). Constraint (6) establishes that the request from each C_k must be satisfied by all fulfillment centers and all delivery stations. Constraint (7) states that the sum of weights of all the products that are shipped from each W_i to all fulfillment centers must not exceed the maximum capacity of the trucks available at node W_i. Obviously, such a constraint is also guaranteed in the lower levels of the supply chain network, where smaller quantities of products are transported (since constraint (4) and constraint (5) hold). Constraint (8) states that the total weight of all the products transported from each D_s to the customer locations must be less than or equal to the total capacity of the drones available at node D_s. Moreover, for the drone to be able to carry any product (allocated on it), constraint (9) requires that the weight of the transported product l (from D_s to C_k) must be less than the capacity of the drone; otherwise, the drone cannot be used (and the truck is the only allowed transportation mode). Note that such a constraint forbids that a single heavy product is carried by more than one drone. Indeed, if a unit of product l weighs more than the maximum capacity allowed by the drone (that is, if $p_l > p_d$), it cannot be transported by drone. Per definition, in this case, the parameter $P_l^{dr} = 0$ and constraint (9) becomes $w_{sk}^{2l} \leq 0$, which is equivalent to $w_{sk}^{2l} = 0$ (since the domain of the variable, constraint (11), holds). On the contrary, the parameter P_l^{dr} assumes such a large numerical value that the constraint (9) is always satisfied. Indeed, the variable w_{sk}^{2l} of the quantity of product l sent from D_s to C_k must necessarily be less than the sum of the requests for product l from all the customer locations. Constraint (10) guarantees that, if the distance between a node of the fourth level and a node of the last level exceeds the duration (in meters) of the drone battery (that is, if $\tilde{d}_{sk} > b_d$), the drone cannot be used ($B_{sk}^{dr} = 0 \Rightarrow w_{sk}^{2l} \leq 0 \Leftrightarrow w_{sk}^{2l} = 0$).

4 The Variational Inequality Formulation

We now provide a variational formulation of problem (1)-(11). Variational inequalities, indeed, are a very powerful mathematical tool both from a computational point of view as well as from a qualitative perspective since a solid theory of existence and uniqueness of the solutions has been developed. The following well-known theorem holds (see [27]).

Theorem 4.1 *Let X^* be a solution to the optimization problem:*

$$\min f(X)$$

$$\textit{subject to: } X \in \mathcal{K}, \tag{12}$$

where f is a continuously differentiable function and $\mathcal{K}$ is a closed (not empty) convex set. Then, X^* is a solution to the variational inequality problem $VI(F, \mathcal{K})$:

$$\textit{Find } X^* \in \mathcal{K} \textit{ such that:} \quad \langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{13}$$

where $F(X) \equiv \nabla f(X)$ is the gradient of $f(\cdot)$, $\mathcal{K} \subset \mathbb{R}^N$ is the feasible set, and $\langle \cdot, \cdot \rangle$ denotes the inner product in the Euclidean space $\mathbb{R}^N_+$. Moreover, if $f(X)$ is a convex function and X^* is a solution to $VI(F, \mathcal{K})$, then X^* is a solution to the optimization problem (12).

Let all the involved cost functions be continuously differentiable and convex. The following result allows us to obtain the variational formulation of the proposed model (see, for instance, [28]).

Theorem 4.2 *A vector $(\alpha^*, x^*, y^*, z^*, w^{1*}, w^{2*}) \in \mathbb{K}$ is an optimal solution to the problem* (1)-(11) *if and only if such a vector is a solution to the variational inequality:*
Find $(\alpha^*, x^*, y^*, z^*, w^{1*}, w^{2*}) \in \mathbb{K}$ such that:

$$\sum_{i=1}^{I+1}\sum_{l=1}^{L}\left[\frac{\partial c(\alpha^*, x^*, y^*, z^*, w^{1*}, w^{2*})}{\partial \alpha_i^l} + \gamma_i^l + \frac{\partial c_i^{tr}(\alpha^*, w^{2*})}{\partial \alpha_i^l}\right] \times (\alpha_i^l - \alpha_i^{l*})$$
$$+\sum_{i=1}^{I+1}\sum_{j=1}^{J}\sum_{l=1}^{L}\left[\frac{\partial c(\alpha^*, x^*, y^*, z^*, w^{1*}, w^{2*})}{\partial x_{ij}^l} + \frac{\partial c_{ij}(x_{ij}^*)}{\partial x_{ij}^l}\right] \times (x_{ij}^l - x_{ij}^{l*})$$
$$+\sum_{j=1}^{J}\sum_{s=1}^{S}\sum_{l=1}^{L}\left[\frac{\partial c(\alpha^*, x^*, y^*, z^*, w^{1*}, w^{2*})}{\partial y_{js}^l} + \frac{\partial \hat{c}_{js}(y_{js}^*, z_j^*)}{\partial y_{js}^l} + \frac{\partial \overline{c}_{jk}(y_j^*, z_{jk}^*)}{\partial y_{js}^l}\right] \times (y_{js}^l - y_{js}^{l*})$$
$$+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{l=1}^{L}\left[\frac{\partial c(\alpha^*, x^*, y^*, z^*, w^{1*}, w^{2*})}{\partial z_{jk}^l} + \frac{\partial \hat{c}_{js}(y_{js}^*, z_j^*)}{\partial z_{jk}^l} + \frac{\partial \overline{c}_{jk}(y_j^*, z_{jk}^*)}{\partial z_{jk}^l} - \gamma_{jk}^l\right] \times (z_{jk}^l - z_{jk}^{l*})$$
$$+\sum_{s=1}^{S}\sum_{k=1}^{K}\sum_{l=1}^{L}\left[\frac{\partial c(\alpha^*, x^*, y^*, z^*, w^{1*}, w^{2*})}{\partial w_{sk}^{1l}} + \frac{\partial \tilde{c}_{sk}^1(w_{sk}^{1l*})}{\partial w_{sk}^{1l}} - \gamma_{sk}^l\right] \times (w_{sk}^{1l} - w_{sk}^{1l*})$$
$$+\sum_{s=1}^{S}\sum_{k=1}^{K}\sum_{l=1}^{L}\left[\frac{\partial c(\alpha^*, x^*, y^*, z^*, w^{1*}, w^{2*})}{\partial w_{sk}^{2l}} + \frac{\partial \tilde{c}_{sk}^2(w_{sk}^{2l*})}{\partial w_{sk}^{2l}} - I_{sk}\right.$$
$$\left. + \frac{\partial c_i^{tr}(\alpha^*, w^{2*})}{\partial w_{sk}^{2l}} + \frac{\partial \tilde{c}_s^{dr}(w^{2*})}{\partial w_{sk}^{2l}} - \gamma_{sk}^l\right] \times (w_{sk}^{2l} - w_{sk}^{2l*}) \geq 0$$
$$\forall (\alpha, x, y, z, w^1, w^2) \in \mathbb{K}, \qquad (14)$$

where

$$\mathbb{K} := \left\{(\alpha, x, y, z, w^1, w^2) \in \mathbb{R}_+^{L[(I+1)+J(I+1)+JS+JK+2SK]} : \text{ (2)-(10) } hold\right\}. \qquad (15)$$

Proof Since the cost functions are continuously differentiable and convex, the function f in (1) is continuously differentiable and convex.
Let $N = L[(I+1) + J(I+1) + JS + JK + 2SK]$ be the dimension of the vectors belonging to the feasible set. We define the N-dimensional vector $X = (\alpha, x, y, z, w^1, w^2)$ and the N-dimensional vector $F(X) \equiv \nabla f(X) = (F^1(X), F^2(X), F^3(X), F^4(X), F^5(X), F^6(X))$, where the (i,l)-th component, F_{il}^1, of $F^1(X)$ is given by

$$F_{il}^1 = \frac{\partial c(\alpha, x, y, z, w^1, w^2)}{\partial \alpha_i^l} + \gamma_i^l + \frac{\partial c_i^{tr}(\alpha, w^2)}{\partial \alpha_i^l},$$

the (i,j,l)-th component, F_{ijl}^2, of $F^2(X)$ is given by

$$F_{ijl}^2 = \frac{\partial c(\alpha, x, y, z, w^1, w^2)}{\partial x_{ij}^l} + \frac{\partial c_{ij}(x_{ij})}{\partial x_{ij}^l},$$

the (j,s,l)-th component, F_{jsl}^3, of $F^3(X)$ is given by

$$F_{jsl}^3 = \frac{\partial c(\alpha, x, y, z, w^1, w^2)}{\partial y_{js}^l} + \frac{\partial \hat{c}_{js}(y_{js}, z_j)}{\partial y_{js}^l} + \frac{\partial \overline{c}_{jk}(y_j, z_{jk})}{\partial y_{js}^l},$$

the (j,k,l)-th component, F_{jkl}^4, of $F^4(X)$ is given by

$$F_{jkl}^4 = \frac{\partial c(\alpha, x, y, z, w^1, w^2)}{\partial z_{jk}^l} + \frac{\partial \hat{c}_{js}(y_{js}, z_j)}{\partial z_{jk}^l} + \frac{\partial \overline{c}_{jk}(y_j, z_{jk})}{\partial z_{jk}^l} - \gamma_{jk}^l,$$

the (s,k,l)-th component, F^5_{skl}, of $F^5(X)$ is given by

$$F^5_{skl} = \frac{\partial c(\alpha, x, y, z, w^1, w^2)}{\partial w^{1l}_{sk}} + \frac{\partial \tilde{c}^1_{sk}(w^{1l}_{sk})}{\partial w^{1l}_{sk}} - \gamma^l_{sk},$$

and, finally, the (s,k,l)-th component, F^6_{skl}, of $F^6(X)$ is given by

$$F^6_{skl} = \frac{\partial c(\alpha, x, y, z, w^1, w^2)}{\partial w^{2l}_{sk}} + \frac{\partial \tilde{c}^2_{sk}(w^{2l}_{sk})}{\partial w^{2l}_{sk}} - I_{sk} + \frac{\partial c^{tr}_i(\alpha, w^2)}{\partial w^{2l}_{sk}} + \frac{\partial \tilde{c}^{dr}_s(w^2)}{\partial w^{2l}_{sk}} - \gamma^l_{sk}.$$

The feasible set $\mathcal{K}$ is defined as $\mathbb{K}$ and variational inequality problem (14) is put into standard form (13).
Moreover, the feasible set $\mathbb{K}$, given the constraints (2)-(11), is clearly convex and closed. Therefore, Theorem 4.1 has been established. □

Following the classical theory of variational inequalities (see, for instance, [27]), we have the following existence result:

Theorem 4.3 (Existence) *If $\mathcal{K}$ is a compact and convex and F is a continuous function on $\mathcal{K}$, then variational inequality problem $VI(F,\mathcal{K})$ (13) admits at least a solution X^*.*

Moreover, we have the following uniqueness result:

Theorem 4.4 (Uniqueness) *Under the assumptions of Theorem 4.3, if the function $F(X)$ in (13) is strictly monotone on $\mathcal{K}$, that is:*

$$\langle (F(X_1) - F(X_2))^T, X_1 - X_2 \rangle > 0, \quad \forall X_1,\ X_2 \in \mathcal{K}, X_1 \neq X_2,$$

then variational inequality (13) or, equivalently, variational inequality (14), admits a unique solution.

Theorem 4.5 *Variational inequality problem (14) admits at least one solution.*

Proof As already discussed in the proof of Theorem 4.2, the function f is continuously differentiable and convex. Therefore, the function $F \equiv \nabla f(X)$ is continuous on $\mathbb{K}$. Moreover, we observed that the feasible set $\mathbb{K}$ is convex and closed. Compactness follows by observing that:

- all the variables are non-negative (see constraint (11));
- from constraint (2) we obtain that $\alpha^l_i < +\infty$, $\forall i = 1, \ldots, I+1$, $\forall l = 1, \ldots, L$;
- from constraint (3), the previous deduction, and constraint (7) we obtain that $x^l_{ij} < +\infty$, $\forall i = 1, \ldots, I+1$, $\forall j = 1, \ldots, J$, $\forall l = 1, \ldots, L$;
- from constraint (4) and the previous deduction we have that $y^l_{js} < +\infty$, $\forall j = 1, \ldots, J$, $\forall s = 1, \ldots, S$, $\forall l = 1, \ldots, L$;
- from constraint (4), the previous deductions, and constraint (6) it follows that $z^l_{jk} < +\infty$, $\forall j = 1, \ldots, J$, $\forall k = 1, \ldots, K$, $\forall l = 1, \ldots, L$;
- from constraint (5) and the previous deductions, constraint (6), and constraints (8)-(10) we have that both $w^{1l}_{sk}, w^{2l}_{sk} < +\infty$, $\forall s = 1, \ldots, S$, $\forall k = 1, \ldots, K$, $\forall l = 1, \ldots, L$.

Therefore, the feasible set $\mathbb{K}$ is a closed and bounded set and, hence, we can affirm that the feasible set $\mathbb{K}$ is also a compact set. The hypotheses of Theorem 4.3 are all satisfied and the thesis follows. □

5 Numerical Simulations

In this section, we first illustrate the supply chain network topology and the parameters used for the numerical simulations. Then, we detail the performed simulations and emphasize the advantages of using UAVs as revealed through the results. Furthermore, we propose some additional configurations to show the potential of the mathematical model, which is able to capture some important real-world aspects such as the limited drone capacity and battery duration. Finally, we provide results from a sensitivity analysis to further emphasize the environmental impacts of drone usage.

5.1 Simulation Setting

The network topology of the considered supply chain is depicted in Figure 2. The supply chain network consists of a company, two warehouses (a third-party seller's warehouse and the company's warehouse), one fulfillment center, two delivery stations and three customer locations. We take into account such

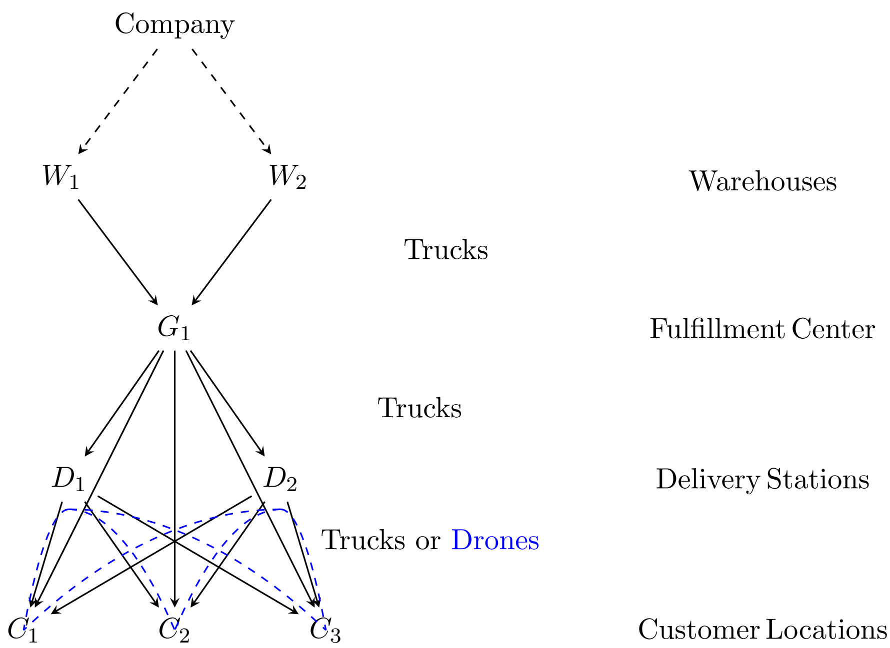

Figure 2: Supply chain network topology for the numerical simulations

a supply chain network, and consider a single product, in order to gain a better understanding of the results obtained. Moreover, large geographical areas can be divided into smaller areas and associated supply chain networks.

In the previous section, we assumed that all the cost functions are continuously differentiable and convex. Hence, we assume that the typical cost function c, depending on the variable (or variables vector) x, has the following general quadratic expression:

$$c(x) = \beta_1 \cdot x^2 + \beta_2 \cdot x, \tag{16}$$

where $\beta_1 > 0$ and $\beta_2 \geq 0$. The choice of such an expression and parameters is motivated by observing that all the assumptions are verified and these functions are well-suited to reality. Specifically, it is clear that, since all the parameters are greater than or equal to zero, the cost functions are always non-negative and are increasing functions. Moreover, by noting that $\beta_1 > 0$, we can affirm that the costs are convex. Finally, since there is not a constant coefficient, when $x = 0$, we obtain that $c(0) = 0$, in accordance with reality.

Therefore, the handling cost of the company has the following expression:

$$c(\alpha, x, y, w^1, w^2) = \beta_1^{(han)} \cdot (2\alpha, x, y, w^1, w^2)^2 + \beta_2^{(han)} \cdot (2\alpha, x, y, w^1, w^2),$$

where $(2\alpha, x, y, w^1, w^2)$ represents the sum of all variables, with a double weight for the variables α_1 and α_2 which we are assuming to have a greater contribution to handling costs than the other variables.

In these simulations, we assume that the transportation costs from the fulfillment center to the delivery stations only depend on the transported amount of the product (that is, y_{js}): $\hat{c}_{js}(y_{js}, z_j) = \hat{c}_{js}(y_{js})$. Analogously, we assume that $\bar{c}_{jk}(y_j, z_{jk}) = \bar{c}_{jk}(z_{jk})$. On the other hand, the handling costs of W_i for the trucks depend on α and w^2 as follows:

$$c_i^{tr}(\alpha, w^2) = \beta_1^{(tr)} \cdot \left(\alpha_1 + \alpha_2 - \omega\left(\sum_{s,k} w_{sk}^2\right)\right)^2 + \beta_2^{(tr)} \cdot \left(\alpha_1 + \alpha_2 - \omega\left(\sum_{s,k} w_{sk}^2\right)\right).$$

Indeed, the handling costs for the trucks take into account the transported quantities via trucks (through all the links of the network, from the first level to the last one), while the amount of products transported via UAVs must be subtracted (with a weight of $\omega = 0.5$, because it refers only to the last mile).

All other cost functions follow the generic expression described in (16). We emphasize that any type of expression that satisfies the assumptions that the functions are continuously differentiable and convex could be used and it need not be of a quadratic form.

Cost functions	**Description**	β_1	β_2
$c(\alpha, x, y, z, w^1, w^2)$	Handling	0.5	0.5
$c_{ij}(x_{ij})$	Transp. from W_1 to G_1	0.2	0.2
	Transp. from W_2 to G_1	0.1	0.1
$\hat{c}_{js}(y_{js}, z_j)$	Transp. from G_1 to D_1	0.1	0.1
	Transp. from G_1 to D_2	0.1	0.1
$\bar{c}_{jk}(y_j, z_{jk})$	Transp. from G_1 to C_1	50	50
	Transp. from G_1 to C_2	1	1
	Transp. from G_1 to C_3	50	50
$\tilde{c}_{sk}^1(w_{sk}^1)$	Transp. from D_1 to C_1 (trucks)	2	2
	Transp. from D_1 to C_2 (trucks)	8	8
	Transp. from D_1 to C_3 (trucks)	8	8
	Transp. from D_2 to C_1 (trucks)	8	8
	Transp. from D_2 to C_2 (trucks)	9	9
	Transp. from D_2 to C_3 (trucks)	2	2
$\tilde{c}_{sk}^2(w_{sk}^2)$	Transp. from D_1 to C_1 (UAVs)	1	1
	Transp. from D_1 to C_2 (UAVs)	4	4
	Transp. from D_1 to C_3 (UAVs)	4	4
	Transp. from D_2 to C_1 (UAVs)	4	4
	Transp. from D_2 to C_2 (UAVs)	4.5	4.5
	Transp. from D_2 to C_3 (UAVs)	1	1
$c_i^{tr}(\alpha, w^2)$	Trucks handling ($\forall i = 1, 2$)	0.3	0.3
$c_s^{dr}(w^2)$	UAVs handling ($\forall s = 1, 2$)	0.1	0.1

Table 5: Parameters of the cost functions numerical simulations

In Table 5, we report all the used β_1 and β_2 parameters, for each cost function. Note that we are assuming that there are no links between the fulfillment center G_1 to the customer locations C_1 and

C_3. Indeed, at the costs associated with the links from G_1 to C_1 and C_3, we set some very high values. We suppose, in contrast, that the second customer location, C_2, is very close to the fulfillment center. Furthermore, observe that the more expensive links are those of the last mile, between the delivery stations and the customer locations. More specifically, we assume that the drones are cheaper than the trucks and that the trucks in the last mile are the most expensive. These choices are motivated by reality. Moreover, we are assuming that the cheapest shipping method is chosen; for example, Amazon could use its own trucks or those of third parties.

Parameters	**Values**
Purchase price	$\gamma_1 = 3$
Production cost	$\gamma_2 = 4$
Incentive for UAVs	$I_{sk} = 2,\ \forall s = 1, 2,\ \forall k = 1, 2, 3$
Selling price from G_j	$\gamma_{jk} = 149,\ \forall j = 1,\ \forall k = 1, 2, 3$
Selling price from D_s	$\gamma_{sk} = 150,\ \forall s = 1, 2,\ \forall k = 1, 2, 3$
Maximum quantity at W_1	$Q_1 = 20$
Maximum quantity at W_2	$Q_2 = 10$
Demand from C_k	$r_k = 10,\ \forall k = 1, 2, 3$
Number of trucks at W_1	$n_1^{tr} = 4$
Number of trucks at W_2	$n_2^{tr} = 5$
Maximum capacity of trucks	$W = 6$
Number of UAVs at D_1	$n_1^{dr} = 5$
Number of UAVs at D_2	$n_2^{dr} = 5$
Maximum weight (UAV)	$p_d = 4$
Parameter on the weights	$P^{dr} = 30$
Parameter on distances	$B_{sk}^{dr} = 30,\ \forall s = 1, 2,\ \forall k = 1, 2, 3$

Table 6: Parameter values for numerical simulations

Table 6 displays all the values of the parameters used in the simulations. Observe that, as in reality, the maximum capacity (weight) of each truck is greater than that of each drone. We have that the selling price proposed by the fulfillment center is slightly lower than the prices proposed by the delivery stations (but the fulfillment centers are usually more difficult to reach, because they are further away). In these simulations, we are assuming that there are neither weight nor distance limitations; namely, the weight of the product is less than the maximum weight that each drone is able to carry ($P^{dr} = 30$) and the distance between each delivery station and each customer location is less than the distance allowed by the drone battery life ($B_{sk}^{dr} = 30,\ \forall s = 1, 2,\ \forall k = 1, 2, 3$). In the additional configurations (see Section 5.3) we modify these parameters.

5.2 Performed Simulations and Results

We perform two main numerical simulations, which focus on:

S1: a UAV-based last mile network; that is, a supply chain using UAVs for the last mile;

S2: a supply chain without using UAVs.

Therefore, we execute the simulations having the same supply chain network topology, functions and parameters as previously described, and which differ only in the presence (S1) or absence (S2) of drones in the last mile.

The optimal results for both the simulations S1 and S2 are computed by solving the variational inequality given in the previous section via the Euler Method (see [17]). We implemented the algorithm in Matlab on an LG laptop with a 12th Gen Intel(R) Core(TM) i7-1260P, 16 GB RAM. The optimal solutions are obtained in less than one second. Simulation S1 consists of 21 variables, while S2 consists of 15 variables.

The optimal solutions are reported in Table 7.

Variables		Optimal solutions S1	S2
α_i^{l*}	$i=1$	20	20
	$i=2$	10	10
x_{ij}^{l*}	$i=1, j=1$	20	20
	$i=2, j=1$	10	10
y_{js}^{l*}	$j=1, s=1$	11.38	10
	$j=1, s=2$	11.21	10
z_{jk}^{l*}	$j=1, k=1$	0	0
	$j=1, k=2$	7.41	10
	$j=1, k=3$	0	0
w_{sk}^{1l*}	$s=1, k=1$	0	0
	$s=1, k=2$	0	0
	$s=1, k=3$	0	10
	$s=2, k=1$	0	10
	$s=2, k=2$	0	0
	$s=2, k=3$	0	0
w_{sk}^{2l*}	$s=1, k=1$	7.98	–
	$s=1, k=2$	1.39	–
	$s=1, k=3$	1.92	–
	$s=2, k=1$	1.93	–
	$s=2, k=2$	1.20	–
	$s=2, k=3$	7.99	–

Table 7: Optimal solutions of the simulations

The total objective function values for each simulation are:

$$f^{(S1)}(\alpha^*, x^*, y^*, z^*, w^{1*}, w^{2*}) = 2850.6;$$

$$f^{(S2)}(\alpha^*, x^*, y^*, z^*, w^{1*}, w^{2*}) = 1137.$$

In Table 8, we report the partial contribution of the terms of the objective functions.

5.2.1 Analysis of results

The optimal solutions of simulation S1 clearly show that 20 units of product ($\alpha_1^{1*} = 20$) are bought by Amazon (or a general company) from a third-party seller, while 10 products are self-produced ($\alpha_2^{1*} = 10$). These quantities of products are sent from the warehouses to the fulfillment center ($x_{11}^{1*} = 20$, $x_{21}^{1*} = 10$), from which they are then distributed to both the delivery stations ($y_{11}^{1*} = 11.38$ and $y_{12}^{1*} = 11.21$). Moreover, a certain amount of product ($z_{12}^{1*} = 7.41$) is also directly sent from the fulfillment center to the second customer location, C_2 (the one near the fulfillment center). We highlight that from all the delivery stations no products are sent to any customer location via trucks; indeed, we obtain that $w_{sk}^{11*} = 0$, $\forall s = 1, 2$, $\forall k = 1, 2, 3$. Hence, all the products are sent via drones ($w_{sk}^{21*} \gneq 0$, $\forall s = 1, 2$, $\forall k = 1, 2, 3$). Therefore, we can easily observe that using drones is more optimal than using trucks.

The obtained objective function to maximize clearly shows that the company earns a profit because it assumes a positive value (consisting of the difference between the overall revenue and the costs).

Terms of the objective function	S1	S2
$-C^{(hand)}$	-1023.7	-987
$-C^{PurPr}$	-100	-100
$-C^{(WG)}$	-95	-95
$-C^{(GD)}$	-27.76	-22
$-C^{(GC)}$	-62.39	-110
$-C_{tr}^{(DC)}$	0	-1760
$-C_{dr}^{(DC)}$	-213.41	–
I^{SM}	44.79	–
$-C^{(tr)}$	-111.72	-279
$-C^{(dr)}$	-52.38	–
TR	4492.6	4490
$f(\alpha^*, x^*, y^*, z^*, w^{1*}, w^{2*})$	2850.6	1137

Table 8: Contributions of the terms of the objective functions

Indeed, the positive terms are the total revenue (obtained by the selling of products to the customers) and the incentive for sustainable mobility (that is, the use of UAVs, instead of trucks, in order to reduce pollution and congestion), and these represent the greater part of the objective function. This aspect guarantees that the company obtains a profit in the selling of products. The handling cost is the term with the highest value. Among the transportation costs, the one from the delivery stations to the customer locations via drones is the greatest one, while, obviously, the transportation cost via trucks (from each D_s to each C_k) is null, because no trucks are used for the last mile. Note that, despite no trucks being used in the last mile, there are still handling costs of trucks, since they are used in the higher levels of the supply chain network.

The optimal solutions of the second simulation S2 show that $\alpha_1^{1*} = 20$ units of product are bought by Amazon from a third-party seller, while $\alpha_2^{1*} = 10$ products are self-produced, and these quantities of products are sent from the warehouses to the fulfillment center ($x_{11}^{1*} = 20$, $x_{21}^{1*} = 10$), as in the first simulation, S1. Unlike the first simulation, however, $y_{11}^{1*} = 10$ products are sent from the fulfillment center G_1 to the first delivery station D_1; an equivalent amount of products ($y_{12}^{1*} = 10$) is sent to the second delivery station D_2 and directly to the second customer location C_2 ($z_{jk}^{l*} = 10$). The first and third customer locations, instead, receive the requested products from the second and first delivery stations, respectively ($w_{21}^{11*} = 10$ and $w_{13}^{11*} = 10$), using the allowed means of transport, which is by truck.

Also, in this simulation, the company obtains a profit; indeed, the objective function (the difference between the revenues and the costs) assumes a positive value (see Table 8). In contrast to the first simulation, in which drones can be used, in this second simulation, where only trucks could be used, the most relevant objective function term is the one related to the transportation costs of the last mile, from each delivery station to each customer location, via trucks. Obviously, there are no transportation costs via UAVs, nor incentives for sustainable mobility and handling costs of drones.

5.2.2 Comparison of results

We now compare the results of the two simulations. By observing the obtained optimal solutions (see Table 7), we can affirm that the solutions are very similar (see Figure 3), except for the last mile shipments (see Figure 4). Indeed, the means of transport allowed in the last mile essentially represent the diversification between the two simulations. Specifically, in the first simulation, when the model chooses the optimal means of transportation between trucks and drones, the drones are chosen, because they are more convenient. In contrast, in the second simulation, we assume that there

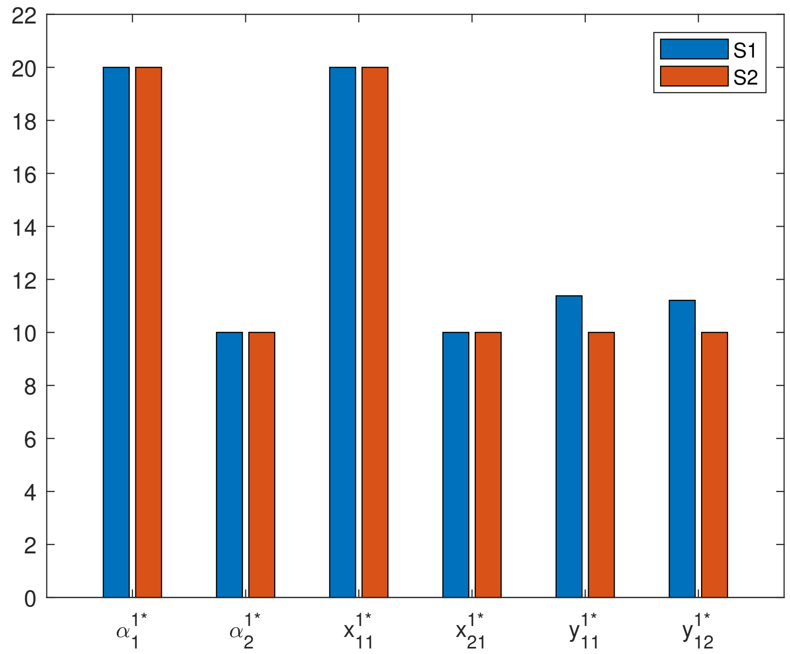

Figure 3: Optimal solutions: α_i^{l*}, x_{ij}^{l*} and y_{js}^{l*}.

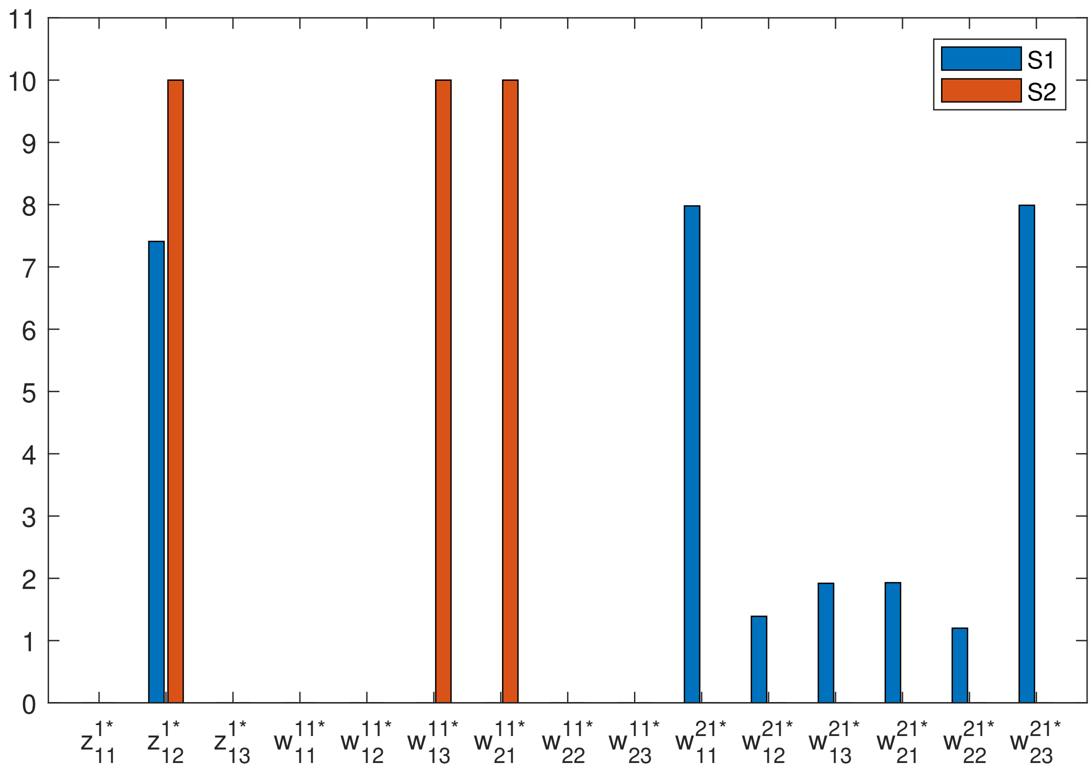

Figure 4: Optimal solutions: z_{jk}^{l*}, w_{sk}^{1l*} and w_{sk}^{2l*}

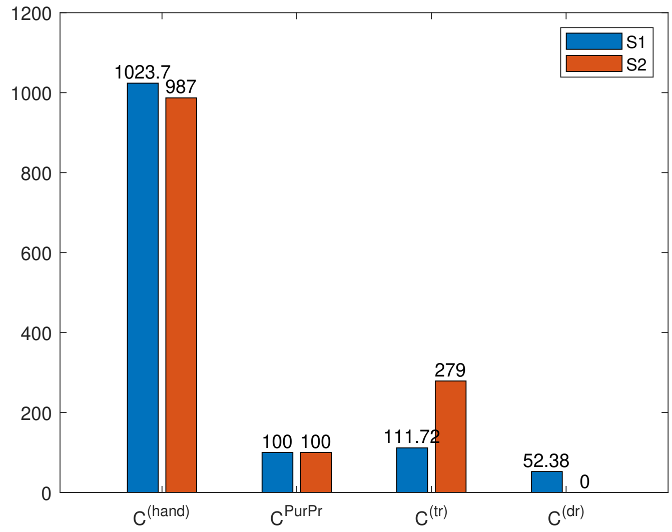

Figure 5: Objective function terms: handling costs of the company ($C^{(hand)}$), total expense for the purchase and the production cost (C^{PurPr}), handling costs of all trucks ($C^{(tr)}$) and of all UAVs ($C^{(dr)}$).

are no drones, and, hence, only trucks can be used for the last mile. Furthermore, in S1, the demand at the first customer location is satisfied by the delivery stations (more from the first one), as well as the demand at the third customer location (which is mostly satisfied by the second delivery station). On the contrary, the demand at the second customer location is mainly (and directly) satisfied by the fulfillment center. In contrast, in S2, the demand at the first customer location is satisfied by only the second delivery station; the demand at the second customer location by only the fulfillment center, directly; and the third customer location by only the first delivery station.

From Figure 5 (and Table 8), we can observe that the handling costs of the company and the total expense for the purchase and the production of products in S1 and S2 are comparable. In contrast, the handling costs of all trucks for the second simulation are greater than those for the first one. The latter is motivated by the use of drones in the last mile for S1, which reduces the handling costs of trucks. Clearly, in simulation S2, there are no handling costs of drones since no drones can be used.

If we pay attention to the transportation costs (see Figure 6), we note that, in both the simulations, the highest ones are those related to the last mile, as expected (since we are assuming that the last mile links are the most expensive). In particular, we observe that the transportation cost from delivery stations to customer locations via trucks is null in S1 because the products are transported via drones, while they are very high in S2. Obviously, all the costs reported in Figure 5 and Figure 6 have a negative sign in the objective function.

Finally, from Figure 7, we can see that only in the first simulation does the company receive the incentive for sustainable mobility (since the drones are used); the total revenues (due to the products selling) of the two simulations are comparable; the objective function value of the first simulation is higher than that of the second simulation. Specifically, the difference between the two objective function values is:

$$\Delta f = f^{(S1)}(\alpha^*, x^*, y^*, z^*, w^{1*}, w^{2*}) - f^{(S2)}(\alpha^*, x^*, y^*, z^*, w^{1*}, w^{2*}) = 2850.6 - 1137 = 1713.6. \quad (17)$$

The positive value of the difference between the two objective functions, Δf, in (17), means that the first simulation yields a greater objective function value. Furthermore, if a company is evaluating whether to invest in UAVs or not, i.e., whether buying new drones and using them for the last mile is advisable or not, it can use the proposed model, according to which if the investment for drones is less than the value obtained by expression (17), then buying new drones is more suitable. Furthermore,

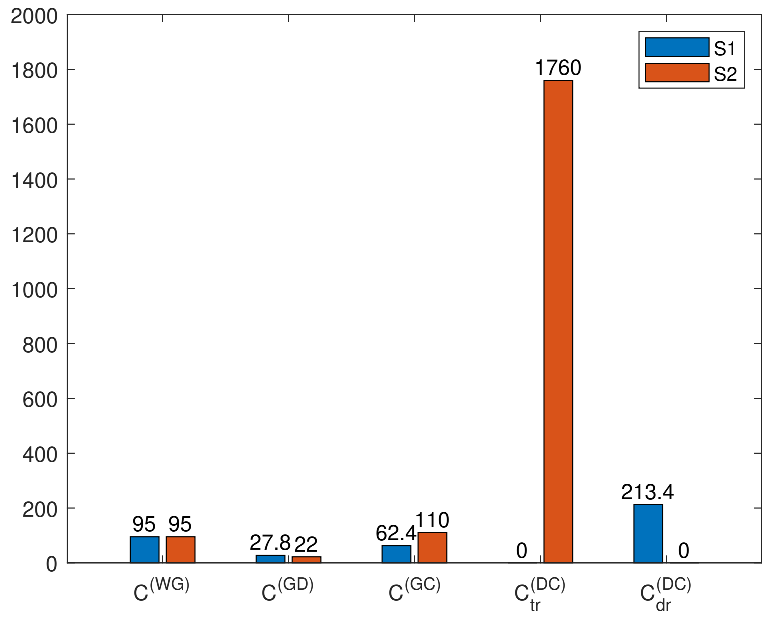

Figure 6: Objective function terms: transportation costs from warehouses to the fulfillment center (C^{WG}), from the fulfillment center to delivery stations (C^{GD}), from the fulfillment center to customer locations (C^{GC}), from delivery stations to customer locations via trucks (C_{tr}^{DC}) and via UAVs (C_{dr}^{DC}).

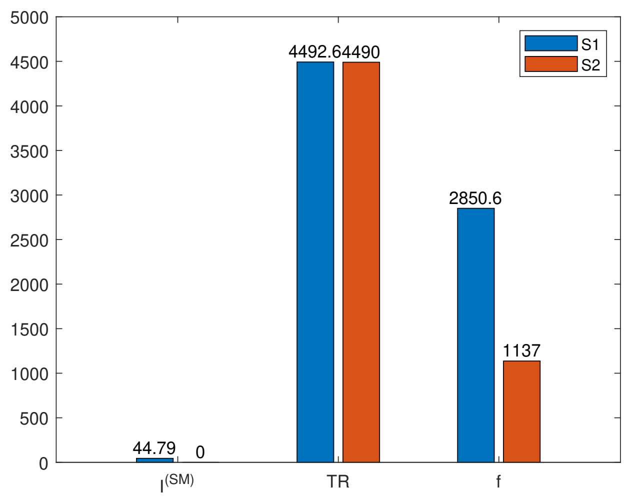

Figure 7: Objective function terms: incentive for sustainable mobility ($I^{(SM)}$), total revenue (TR) and total objective function (f).

we also calculate the percentage convenience of using UAVs as follows:

$$PC = \left| \frac{f^{(S1)}(\alpha^*, x^*, y^*, z^*, w^{1*}, w^{2*}) - f^{(S2)}(\alpha^*, x^*, y^*, z^*, w^{1*}, w^{2*})}{f^{(S1)}(\alpha^*, x^*, y^*, z^*, w^{1*}, w^{2*})} \right| \cdot 100\% =$$

$$= \left| \frac{\Delta f}{f^{(S1)}(\alpha^*, x^*, y^*, z^*, w^{1*}, w^{2*})} \right| \cdot 100\% = \left| \frac{1713.6}{2850.6} \right| \cdot 100\% = 60.11\%. \tag{18}$$

Expression (18) means that using drones improves the company's profit by more than 60%.

We now investigate on the environmental benefits obtained using drones. More specifically, we analyze the CO_2 emissions, or the more general GHG emissions for the delivery activities, in simulations S1 and S2 and compare them. It is known (see, for example, [22]) that the GHG emissions depend on the transported weight and volume of products (as well as the number of delivered products), on the type of used vehicles (that is, their sizes and their fuel factors) and on the travel distance. The two main equivalent approaches to the measure of GHG emissions from freight transport operations are based on energy consumption and on the level of transport activity, respectively (see [24], [16], [20], [33]). Therefore, we can estimate E, the total GHG emission CO_2 equivalent emissions (expressed as grams), as follows:

$$E = \sum_{k=1}^{K} \left[\sum_{j=1}^{J} F_{jk}^{(E)} \cdot \left(\sum_{l=1}^{L} p_l z_{jk}^{l*} \right) \cdot d_{jk} + \sum_{s=1}^{S} F_{sk}^{(E)} \cdot \left(\sum_{l=1}^{L} p_l w_{sk}^{1l*} \right) \cdot d_{sk} \right], \tag{19}$$

where, $F_{jk}^{(E)}$ (expressed as grams of CO_2e per kg-m: g/kgm) represents the emission factor for the link from node j to node k (which depends on the type of used truck) and d_{jk} (m) is the distance between node j and node k. Analogously, $F_{sk}^{(E)}$ and d_{sk} are the emission factor and the distance from s to k, respectively ([2]). Although the emission factor value is widely studied by many researchers ([19]), it is not the focus here. Hence, for this purpose, we use the EcoTransit World Model online environmental assessment tool (see [14] and [30] for a detailed description of the model and the calculator). By selecting the transport mode (truck) and vehicle type (size, fuel type, etc.), and inserting the distance (the origin/destination coordinates) and the total amount of products (in kilograms) transported for each link (that is, our optimal solutions), we obtain that the total GHG emission of simulation S1 is $E^{(S1)} = 3.2g$, while that of S2 is $E^{(S2)} = 27.3g$. Observe that in simulation S1 we have a little amount of emissions, since a percentage of product requested by $k = 2$ is directly satisfied from the fulfillment center, by trucks. In simulation S2, instead, we have a greater value of emissions, since all deliveries are made by trucks. Therefore, we calculate that 24.1g of GHG emissions might be eliminated through the use of drones yielding a reduction of 88.3The results show that the use of UAVs can significantly minimize the transportation costs and can also reduce the environmental emissions with incentives.

Therefore, the results show that the use of UAVs can significantly improve delivery efficiency and minimize the transportation costs. We also highlight that using drones the environmental emissions are significantly reduced.

5.3 Additional Configurations

We also examined other configurations of the UAV-based last mile supply chain network (S1), as follows:

- S1.1: with a large number of drones in each delivery station ($n_s^{dr} = 5$, $\forall s = 1, 2$) and without weight and distance limitations ($P^{dr} = 30$ and $B_{sk}^{dr} = 30$, $\forall s = 1, 2$, $\forall k = 1, 2, 3$);
- S1.2: with a large number of drones in each delivery station ($n_s^{dr} = 5$, $\forall s = 1, 2$) and without weight limitations ($P^{dr} = 30$) but with some distance limitations ($B_{12}^{dr} = B_{21}^{dr} = 30$, $B_{11}^{dr} = B_{13}^{dr} = B_{22}^{dr} = B_{23}^{dr} = 0$);

S1.3: without weight and distance limitations ($P^{dr} = 30$ and $B^{dr}_{sk} = 30,\ \forall s = 1, 2,\ \forall k = 1, 2, 3$) but with a limited number of drones in some delivery stations ($n^{dr}_1 = 5,\ n^{dr}_2 = 2$);

S1.4: with a limited number of drones in some delivery station ($n^{dr}_1 = 4,\ n^{dr}_2 = 2$), without weight limitations ($P^{dr} = 30$) and with some distance limitations ($B^{dr}_{12} = B^{dr}_{13} = B^{dr}_{21} = B^{dr}_{23} = 30$, while $B^{dr}_{11} = B^{dr}_{22} = 0$);

S1.5: with a large number of drones in each delivery station ($n^{dr}_s = 5,\ \forall s = 1, 2$), without distance limitations ($B^{dr}_{sk} = 30,\ \forall s = 1, 2,\ \forall k = 1, 2, 3$), but with weight limitations ($P^{dr} = 0$);

S1.6: with a large number of drones in each delivery station ($n^{dr}_s = 5,\ \forall s = 1, 2$), and with weight and distance limitations ($P^{dr} = 0$ and $B^{dr}_{sk} = 0,\ \forall s = 1, 2,\ \forall k = 1, 2, 3$).

We summarize the main parameters of these additional configurations in Table 9. For these new

Simulations	n^{dr}_1	n^{dr}_2	P^{dr}	B^{dr}_{11}	B^{dr}_{12}	B^{dr}_{13}	B^{dr}_{21}	B^{dr}_{22}	B^{dr}_{23k}
S1.1	5	5	30	30	30	30	30	30	30
S1.2	5	5	30	0	30	0	30	0	0
S1.3	5	2	30	30	30	30	30	30	30
S1.4	4	2	30	0	30	30	30	0	30
S1.5	5	5	0	30	30	30	30	30	30
S1.6	5	5	0	0	0	0	0	0	0

Table 9: Main parameters of the additional configurations

simulations, we assume that the transportation cost from the fulfillment center to the second customer location is the same as thhose to C_1 and C_3 (with $\beta_1 = \beta_2 = 50$; that is, C_2 is very far from G_1). We also choose the following parameters for the transportation costs from D_s to C_k, via trucks (see Table 10). As described in the previous simulations, we set the parameters of the transportation costs

Cost functions	**Description**	$\beta_1 = \beta_2$
$\tilde{c}^1_{sk}(w^1_{sk})$	Transp. from D_1 to C_1 (trucks)	2
	Transp. from D_1 to C_2 (trucks)	3
	Transp. from D_1 to C_3 (trucks)	4
	Transp. from D_2 to C_1 (trucks)	4
	Transp. from D_2 to C_2 (trucks)	3
	Transp. from D_2 to C_3 (trucks)	2

Table 10: Parameters for the cost functions for the additional configurations.

from D_s to C_k via drones as half of those shown in Table 10. All the other parameters are chosen as previously described in Section 5.1.

The optimal solutions are reported in Table 11. Comparing the results of the six configurations, we can observe that the quantities of products bought or produced by the company (α^*_i) remain unchanged, as well as the quantities sent by the warehouses to the fulfillment center (x^*_{ij}). For all the configurations, there is no product directly sent by the fulfillment center to the customer locations ($z^*_{jk} = 0$, for $j = 1,\ \forall k = 1, 2, 3$). This is motivated by the very large distance between G_1 and $C_k,\ \forall k = 1, 2, 3$. In configuration S1.1 no trucks are used for the last mile because trucks are more expensive than drones.

In configuration S1.2, we are assuming that, due to the limited battery duration, a drone starting from the delivery station D_1 cannot arrive (and come back) at the first and third customer locations and a drone starting from D_2 cannot reach either C_2 or C_3. Hence, UAVs are only used to transport products from D_1 to C_2 and from D_2 to C_1, while trucks are used for the other deliveries.

	S1.1	S1.2	S1.3	S1.4	S1.5	S1.6
α_1^*	20.00	20.00	20.00	20.00	20.00	20.00
α_2^*	10.00	10.00	10.00	10.00	10.00	10.00
x_{11}^*	20.00	20.00	20.00	20.00	20.00	20.00
x_{21}^*	10.00	10.00	10.00	10.00	10.00	10.00
y_{11}^*	15.00	14.38	20.05	18.26	15.00	15.00
y_{12}^*	15.00	15.62	9.95	11.74	15.00	15.00
z_{11}^*	0.00	0.00	0.00	0.00	0.00	0.00
z_{12}^*	0.00	0.00	0.00	0.00	0.00	0.00
z_{13}^*	0.00	0.00	0.00	0.00	0.00	0.00
w_{11}^{1*}	0.00	1.82	0.33	3.20	6.83	6.83
w_{12}^{1*}	0.00	0.66	0.28	0.71	5.00	5.00
w_{13}^{1*}	0.00	3.19	0.08	0.24	3.17	3.17
w_{21}^{1*}	0.00	0.63	0.17	1.51	3.17	3.17
w_{22}^{1*}	0.00	0.62	0.62	0.92	5.00	5.00
w_{23}^{1*}	0.00	6.81	1.16	1.30	6.83	6.83
w_{11}^{2*}	6.62	0.00	8.15	0.00	0.00	0.00
w_{12}^{2*}	5.00	8.72	6.41	8.37	0.00	0.00
w_{13}^{2*}	3.38	0.00	4.80	5.75	0.00	0.00
w_{21}^{2*}	3.38	7.56	1.35	5.29	0.00	0.00
w_{22}^{2*}	5.00	0.00	2.69	0.00	0.00	0.00
w_{23}^{2*}	6.62	0.00	3.96	2.71	0.00	0.00

Table 11: Optimal solutions of the additional configurations

Configuration S1.3 is characterized by a lower number of drones in delivery station D_2 ($n_2^{dr} = 2$, instead of 5 drones as used for the previous configurations. The maximum amount of product that can be delivered via drones from a delivery station, D_s, is given by $n_s^{dr} \cdot p_d$ (see constraint (8)). The maximum amount of product that can be delivered from D_2 is, thus, given by $n_2^{dr} \cdot p_d = 8$ and it is fully utilized; indeed, we have that $w_{21}^{2*} + w_{22}^{2*} + w_{23}^{2*} = 1.35 + 2.69 + 3.96 = 8.00$. Observe that the product delivered in configuration S1.3 by the first delivery station is greater than those in configurations S1.1 and S1.2 (see the optimal variables y_{11}^* and w_{1k}^* in Table 11).

In configuration S1.4, we suppose both a lower number of drones in delivery stations ($n_1^{dr} = 4$ and $n_2^{dr} = 2$) and the limitations due to the battery duration (a drone cannot reach C_1 from D_1 and C_2 from D_2). Therefore, we find out that the products are mainly delivered by the first delivery station, using drones, but also that trucks have to be used (because of the limited usage of UAVs).

Finally, observe that the optimal solutions of configurations S1.5 and S1.6 are the same. Indeed, both these configurations have a weight limitation ($P^{dr} = 0$); that is, the weight of a unit of product is greater than the maximum weight that each drone is able to carry. Therefore, since constraint (9) holds, whether there is a limitation due to battery or not, no drones can be used and only trucks are allowed.

These results show that the proposed model can effectively take into account very important aspects, such as the number of drones available, their capacity (the maximum weight they are able to carry, each drone individually and in total) and also the maximum distance that each drone can travel (which is limited by its battery life).

5.4 Sensitivity analysis on the incentive values

We now provide a sensitivity analysis on the incentive values, I_{sk}, for using UAVs to reduce pollution and congestion on the links between each delivery station, D_s, and each customer location, C_k, $\forall s =$

$1, \ldots, S$ and $\forall k = 1, \ldots, K$. Clearly, by varying the values of these parameters, the term of the objective function related to the incentive for sustainable mobility, I^{SM}, undergoes a modification and, therefore, the optimal solutions can change.

We test seven different new simulations (SA1, SA2, SA3, SA4, SA5, SA6 and SA7), varying the incentive values, I_{sk}, $\forall s = 1, 2$, $\forall k = 1, 2, 3$, in an increasing manner, as shown in Table 12.

	SA1	SA2	SA3	SA4	SA5	SA6	SA7
I_{sk}	0	1	2	4	8	16	32

Table 12: Incentive value for each sensitivity analysis configuration

All the other parameters are set as shown in Section 5.1, that is, as in simulation S1, and with no direct links between the fulfillment center and the customer locations.

	SA1	SA2	SA3	SA4	SA5	SA6	SA7
α_1^*	20.00	20.00	20.00	20.00	20.00	20.00	20.00
α_2^*	10.00	10.00	10.00	10.00	10.00	10.00	10.00
x_{11}^*	20.00	20.00	20.00	20.00	20.00	20.00	20.00
x_{21}^*	10.00	10.00	10.00	10.00	10.00	10.00	10.00
y_{11}^*	13.57	13.61	13.83	14.10	14.82	15.26	15.26
y_{12}^*	16.43	16.39	16.17	15.90	15.18	14.74	14.74
z_{11}^*	0.00	0.00	0.00	0.00	0.00	0.00	0.00
z_{12}^*	0.00	0.00	0.00	0.00	0.00	0.00	0.00
z_{13}^*	0.00	0.00	0.00	0.00	0.00	0.00	0.00
w_{11}^{1*}	1.95	1.76	1.55	0.00	0.00	0.00	0.00
w_{12}^{1*}	0.00	0.00	0.00	0.00	0.00	0.00	0.00
w_{13}^{1*}	0.00	0.00	0.00	0.00	0.00	0.00	0.00
w_{21}^{1*}	0.00	0.00	0.00	0.00	0.00	0.00	0.00
w_{22}^{1*}	10.00	9.94	9.61	8.66	7.09	0.00	0.00
w_{23}^{1*}	0.00	0.00	0.00	0.00	0.00	0.00	0.00
w_{11}^{2*}	8.05	8.24	8.45	9.31	8.97	8.04	8.04
w_{12}^{2*}	0.00	0.06	0.39	1.34	2.91	5.30	5.30
w_{13}^{2*}	3.57	3.54	3.44	3.45	2.94	1.93	1.93
w_{21}^{2*}	0.00	0.00	0.00	0.69	1.03	1.96	1.96
w_{22}^{2*}	0.00	0.00	0.00	0.00	0.00	4.70	4.70
w_{23}^{2*}	6.43	6.46	6.56	6.55	7.06	8.07	8.07

Table 13: Optimal solutions of the sensitivity analysis simulations

The optimal solutions for each sensitivity analysis configuration are reported in Table 13. We observe that the quantities of product bought and produced (α_i^*, $\forall i = 1, 2$) and the quantities of product sent from the warehouses to the fulfillment center (x_{ij}^*, $\forall i = 1, 2,\ j = 1$) are the same in all the simulations. The optimal quantities sent by the fulfillment center to the delivery stations (y_{js}^*, $\forall j = 1,\ \forall s = 1, 2$) are comparable. In terms of greater detail, we note that the quantity sent to the first delivery station varies increasingly; in contrast, the one sent to the second delivery station varies decreasingly. No products are directly sent from the fulfillment center to the customer locations ($z_{jk}^* = 0,\ \forall j = 1,\ \forall k = 1, 2, 3$), since we are assuming a very high cost for these links.

The main differences among the different sensitivity analysis simulations lie in the optimal quantities of product sent via trucks or drones in the last mile (that is, from the delivery stations to the customer locations). Specifically, trucks are used from the D_1 to C_1 only when the unit incentive for using drones is less than or equal to 2 ($I_{11} \leq 2$). While trucks are used from the D_2 to C_2 only when

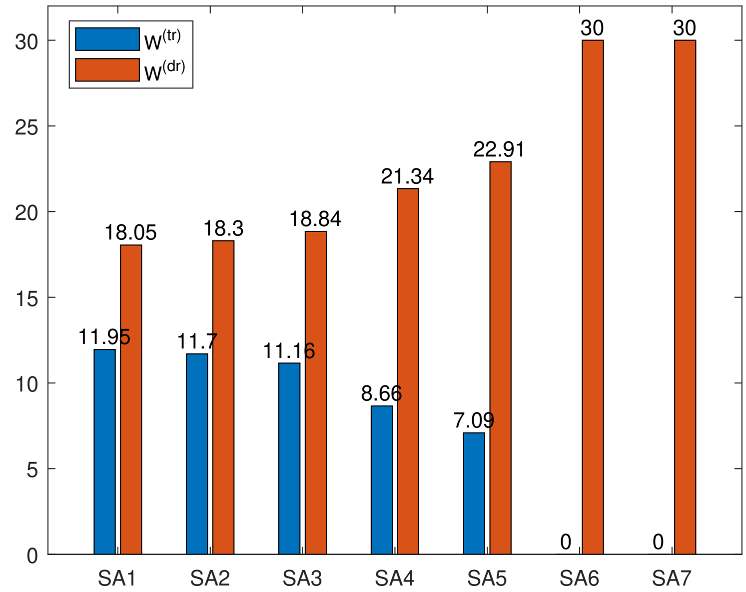

Figure 8: Total amount of product sent via trucks ($W^{(tr)}$) and via drones ($W^{(dr)}$) in the sensitivity analysis simulations

the unit incentive for using drones is less than or equal to 8 ($I_{22} \leq 8$). Therefore, we see that the demand at the first customer location is satisfied only by drones, if the incentive for using UAVs is greater than or equal to 4; otherwise, trucks are used. Moreover, the demand at the second customer location is satisfied only by drones, if the incentive for using UAVs is greater than or equal to 16. And, the requests from the third customer locations are always satisfied by UAVs.

We denote by

$$W^{(tr)} = \sum_{\substack{s=1,2 \\ k=1,2,3}} w_{sk}^{1*} \text{ and } W^{(dr)} = \sum_{\substack{s=1,2 \\ k=1,2,3}} w_{sk}^{2*},$$

the total amount of product sent via trucks and that sent via drones, respectively.

	SA1	SA2	SA3	SA4	SA5	SA6	SA7
I_{sk}	0	1	2	4	8	16	32
$W^{(tr)}$	11.95	11.70	11.16	8.66	7.09	0.00	0.00
$W^{(dr)}$	18.05	18.30	18.84	21.34	22.91	30.00	30.00
$I^{(SM)}$	0.00	18.30	37.68	85.34	183.25	480.00	960.00

Table 14: Sensitivity analysis: amount of product sent via trucks ($W^{(tr)}$) and via drones ($W^{(dr)}$) and incentive for sustainable mobility ($I^{(SM)}$)

In order to analyze in detail the amount of product sent via trucks, $W^{(tr)}$, and that sent via drones, $W^{(dr)}$, and the incentive for sustainable mobility, $I^{(SM)}$, for each simulation, we report them in Table 14.

Figure 8 shows the amounts of product sent via trucks (see the blue bars) and via drones (see the red bars), for each simulation. It is clear that, by increasing the incentive for using UAVs, the amount of product transported via trucks decreases, while that shipped via drones increases. In particular, observe that varying the unit incentive (I_{sk}, $\forall s, k$) from 8 to 16, we obtain that an amount of 7.09 (previously transported via trucks) is now delivered via drones, and that the total incentive for sustainable mobility, $I^{(SM)}$, increases by 296.75 (see Table 14, columns SA5 and SA6). Increasing the unit incentive from 16 to 32 (see Table 14, columns SA6 and SA7), we have a higher incentive for sustainable mobility (which differs by 480.00), but we obtain no difference in the distribution of products via trucks and drones. Indeed, no trucks are used in both the two simulations. Hence, we can conclude that the National Institution that disburses incentives for reducing pollution and congestion, in this case, has no advantage in increasing the incentives from 16 to 32, while it would get positive environmental impacts in varying the incentives from 8 to 16.

The results show how the choice of incentives can impact on environmental aspects (such as congestion and pollution), since companies could change their decision-making strategies, distributing more products via drones.

6 Conclusions and Suggestions for Future Research

UAVs have become increasingly popular in recent years, with their usage ranging from military operations to commercial applications such as aerial photography, mapping, and package delivery. With advancements in technology, drones have become more sophisticated and capable of performing a wide range of tasks. However, one of the main challenges facing drone technology is their optimal management in supply chain networks for product delivery. Flow optimization is a critical aspect of supply chain management, as it plays a crucial role in improving the efficiency and effectiveness of the whole delivery process. It involves finding the best possible solution to a given problem, such as the optimal quantities of products a company has to buy from third party sellers, produce iself, sell and deliver via trucks and/or drones. UAVs usage can help to reduce the time, energy and resources required for delivery operations, making them more cost-effective and environmentally sustainable. Therefore, the implementation of drones in the last mile of a supply chain network can result in significant optimization benefits. By utilizing drones for last mile deliveries, companies can reduce delivery times, increase delivery efficiency and reduce costs associated with traditional delivery methods.

This research paper has explored the use of drones for the last mile in the framework of optimizing a supply chain network. The proposed nonlinear optimization model is able to take into account some important key features such as the number of drones which are available, their maximum capacity (that is, the maximum weight they are able to carry, each drone individually and in total) and the maximum distance that each drone can reach (which is mostly conditioned by its limited battery duration). Furthermore, the study has shown that the use of drones can significantly reduce the delivery cost of the supply chain. Indeed, the analysis of numerical simulations has demonstrated that implementing a drone delivery system can reduce the cost by up to 60%. A sensitivity analysis on incentive values is also provided to highlight their impact on optimal solutions, and, hence, on environmental aspects.

However, despite the potential benefits of drone delivery, there are also several challenges that must be addressed. These challenges include regulatory issues. There are some restrictions and constraints to which the use of UAVs is subject, especially in urban areas; there are also technological limitations, delivery area issues and public acceptance. Therefore, it is crucial that companies work closely with researchers, regulators and the public to develop, investigate and implement drone delivery solutions that are safe, reliable and accepted by all stakeholders. In our further research, we aim to introduce infrastructures that support drone delivery, such as green charging stations and landing pads, as well as the use of renewable energy and collection points in the supply chain. Collection points could be modeled as two different location types: the locker locations and the hub pick-up point locations. The first are locations where parcel lockers (an automated postal box that allows users to self-collect parcels) are positioned. External parcel lockers could be used at any time (in three days, when the customer receives his code to open the locker), but they have limited dimensions. Hence, the hub pick-up points, where there are no dimension limits, could be used, but only during some allowed delivery time windows. The optimal locations, sizing and numbers, as well as the reverse chain to return items (distinguishing the collection points where packaging is also carried out or not) could be analyzed.

Overall, our research suggests that the optimization of supply chains with drones in the last mile can result in significant benefits for companies. As such, we recommend that companies continue to explore the potential of drone delivery as a means of improving their supply chain operations, reducing their costs, as well as pollution and congestion (from which consumers and society, as a whole, will obtain benefits). In conclusion, the findings of this research suggest that the use of drones in the last mile of a supply chain network can be an effective and efficient solution for businesses to improve their delivery operations and reduce the environmental emissions.

Another challenge in last mile delivery, which could be addressed in a future work, is ensuring timely delivery. Customers expect their orders to be delivered within a specific timeframe, and delays or missed deliveries can lead to customer dissatisfaction and lost business. Delivery companies are under pressure to meet these expectations while also managing the complexity of last mile logistics, including traffic congestion, unexpected road closures, and other unforeseen events. To address these challenges, delivery companies are using a range of technologies and strategies to improve the timeliness of deliveries, but they are also investing in new delivery models that can help improve the timeliness of deliveries. For example, some companies are experimenting with crowdsourcing delivery, where independent contractors or gig workers deliver packages using their own vehicles. Such a model can be particularly effective in urban areas where traffic congestion may slow down traditional delivery vehicles.

Declarations of interest

None

CRediT authorship contribution statement

Gabriella Colajanni: Methodology, Writing, Formal Analysis, Software. **Patrizia Daniele:** Conceptualization, Methodology, Writing. **Anna Nagurney:** Methodology, Writing, Supervision.

Acknowledgements

The research was partially supported by the research project “Programma ricerca di ateneo UNICT 2020–22 linea 2-OMNIA” of Catania, by the Italian Ministry of University and Research (MUR) and the European Union for the PON project on Research and Innovation 2014– 2020, D.M. 1062/2021. The first author is grateful for the collaboration with the Virtual Center for Supernetworks. These supports are gratefully acknowledged.

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