Supply Chain Game Theory Network Modeling Under Labor Constraints: Applications to the Covid-19 Pandemic

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Abstract: The Covid-19 pandemic has brought attention to supply chain networks due to disruptions for many reasons, including that of labor shortages as a consequences of illnesses, death, risk mitigation, as well as travel restrictions. Many sectors of the economy from food to healthcare have been competing for workers, as a consequence. In this paper, we construct a supply chain game theory network framework that captures labor constraints under three different scenarios. The appropriate equilibrium constructs are defined, along with their variational inequality formulations. Computed solutions to numerical examples inspired by shortages of migrant labor to harvest fresh produce; specifically, blueberries, in the United States, reveal the impacts of a spectrum of disruptions to labor on the product flows and the profits of the firms in the supply chain network economy. This research adds to the literature in both economics and operations research.

Keywords: Supply chain management, labor, game theory, pandemic, variational inequalities

1. Introduction

The Covid-19 pandemic, officially declared by the World Health Organization on March 11, 2020, has demonstrated dramatically the importance of the health of workers to the global economy (World Health Organization (2020)). Product supply chains as varied as those for Personal Protective Equipment (PPEs) (Burki (2020)) and other medical supplies (Ranney, Griffieth, and Jha (2020)), toilet paper and cleaning supplies (Gao (2020)), meat (Corkery and Yaffe-Bellany (2020)) and fresh produce (Laborde et al. (2020), Knight (2020)), and even blood (Nagurney (2020a), have been disrupted for reasons including that of the reduction of labor availability because of illnesses and death of workers, along with workers' fear of contracting the disease. Some workers have not been able to travel for seasonal employment because of travel restrictions to mitigate the spread of the coronavirus that causes the Covid-19 disease, resulting in both economic and personal losses (IHS Markit (2020)). According to Elflein (2020), as of August 14, 2020, Covid-19 had spread to six continents, with over 750,000 succumbing to the coronavirus that causes the Covid-19 disease. Just over 2 months after, as of October 27, 2020, there were 1,160,421 documented deaths globally due to Covid-19 (JHU CSSE (2020)). The negative impact of this global healthcare and economic crisis has affected different tiers of supply chains and associated network economic activities of production, transportation, storage, and distribution.

Labor has emerged as an essential factor in the functioning of supply chains in the pandemic. Without farmers, food cannot be produced. Without workers to harvest, food may end up rotting in the fields or be discarded (see Associated Press (2020)); without freight service provision food cannot be delivered to food processors; without food processors food cannot be processed, resulting in additional waste (see Polansek and Huffstutter (2020)). Ultimately, freight service provision is also essential for deliveries to retail outlets for purchase by consumers or even for direct transport to consumers using, for example, electronic commerce. Freight service workers have also been impacted by Covid-19 (Parker (2020)). Employees at distribution centers have been felled by Covid-19 (Daniels (2020)), as well, and Covid-19 has further exacerbated worker shortages at a time of greater demand because of electronic commerce (see Hardwick (2020)). Shortages of labor in manufacturing facilities were an issue even earlier on in February 2020 in China, since that country was impacted first by the coronavirus, which was to have originated in Wuhan, with implications for global supply chains (see Bloomberg (2020)). Labor shortages arose due to illnesses, need for social distancing in facilities, as well as adherence to quarantines. Each link of a product supply chain requires labor, and, therefore, reduction in labor capacity can propagate through paths of a supply chain network, affecting product flows as well as prices (see, e.g., OECD (2020)). Furthermore, unhealthy workers cannot be fully productive.

In this paper, we take up the challenge of constructing a supply chain network game theory framework for the modeling of competition among firms that produce a differentiated but substitutable product. The novelty of the framework lies in that it explicitly includes labor availability. Economists have emphasized the use of labor, as well as capital, as essential components of production functions (see Mishra (2007) and the references therein). The inclusion of labor into a competitive supply chain network for differentiated products, however, has not been addressed. This is an important area of research since only through a full supply chain network perspective can one identify the impacts of labor availability and possible capacity disruptions in the pandemic on profits, costs, and consumer prices.

Three different scenarios of labor availability are considered in our modeling framework, with accompanying constraints. In the first scenario, there is a bound on labor availability associated with each link of the supply chain network of each firm in the supply chain network economy. This is the most restrictive scenario in that labor capacities are imposed on individual links and, hence, there is not the freedom of movement from firm to firm and across tiers as in the other scenarios. In the second scenario, there is a bound on the labor availability associated with an activity tier of the supply chain networks, such as production, transportation, storage, and distribution. This scenario is relevant to the farming sector, since farmers compete for seasonal migrant workers for harvesting of the products, with the Covid-19 creating shortfalls of such labor in many parts of the globe (see, e.g., Corbishley (2020)). In the third scenario, there is a single bound on labor availability in the supply chain network economy and labor is free to move across a tier or between tiers. Barrero, Bloom, and Davis (2020) argue for the reallocation of labor because of the economic impacts of the Covid-19 crisis, which this scenario enables the evaluation and quantification of.

The governing equilibrium conditions for the first scenario correspond to a Nash (1950, 1951) Equilibrium, whereas those for the second and third scenarios correspond to a Generalized Nash Equilibrium (GNE) (cf. Debreu (1952) and Arrow and Debreu (1954)), since the feasible sets associated with the firms' strategies are common, that is, shared, because of the respective labor constraints. Although game theory supply chain network models are now fairly well-established (cf. Nagurney, Dong, and Zhang (2002), Nagurney (2006), Qiang et al. (2013), Toyasaki, Daniele, and Wakolbinger (2014), Nagurney and Li (2016), Saberi (2018), Saberi et al. (2018), Yu, Cruz, and Li (2019), and the references therein) and advances continue to be made (see, e.g., Gupta, Ivanov, and Choi (2020) and Gupta and Ivanov (2020a)), Generalized Nash Equilibrium models have only recently been applied to supply chains. For example, Nagurney, Yu, and Besik (2017) introduced a competitive supply chain network equilibrium model with outsourcing in which firms competed for limited capacity at shared distribution facilities. However, labor was not explicitly considered.

Also related to our theme here, in part, is the use of GNE supply chain models in disaster relief (cf. Nagurney, Alvarez-Flores, and Soylu (2016), Nagurney et al. (2018), Nagurney, Salarpour, and Daniele (2019), and Nagurney et al. (2020a), (2020b)) and in healthcare (Nagurney and Dutta (2019)). We emphasize that both Nash Equilibrium and Generalized Nash Equilibrium are applied in this paper, for the first time, for supply chain network competition with labor. Nagurney (2020b) introduced labor into a supply chain network optimization model for perishable food in the Covid-19 pandemic. Therein, arc multipliers captured perishability, akin to the work of Yu and Nagurney (2013), but only labor bounds on links were considered. Nagurney (2020c) considered a nonperishable product and also provided a system-optimization perspective for the supply chain network in the case of labor under different scenarios and in the case of elastic or fixed demands for the firm's product. In contrast to the framework in our paper, there was no competition among firms and no game theory concepts were utilized.

The Covid-19 pandemic was declared only several months ago and already, over this relatively short period of time, has completely disrupted both lives and the economy. In terms of additional related literature, which is nascent, but, nevertheless, growing, we note that Queiroz et al. (2020) detailed a research agenda through a structured literature review of Covid-19 related work and supply chain research on earlier epidemics. Ivanov (2020b), in turn, described simulation-based research concentrating on the potential impacts on global supply chains of the Covid-19 pandemic. Ivanov and Dolgui (2020) argued for the necessity of a new perspective due to the coronavirus Covid-19 outbreak through the use of intertwined supply networks (ISNs). An ISN is an entirety of interconnected supply chains which, in their integrity, yield the provision of society and markets with goods and services. Currie et al. (2020) in their paper identified multiple, complex challenges due to the COVID-19 pandemic and elaborated on how simulation modelling could help to support informed decision-making. Dolgui (2020), in turn, introduced a novel notion - that of a viable supply chain (VSC), in which viability is considered as an underlying supply chain property spanning three perspectives of agility, resilience, and sustainability. The contributions in the paper can assist firms in guiding their decisions on the recovery and re-building of their supply chains after crises of long duration such as the COVID-19 pandemic. Ivanov and Das (2020) captured the ripple effect of an epidemic outbreak in global supply chains in their model, with consideration of the velocity of pandemic propagation, the duration of production, distribution and market disruption, and a demand decline. In addition, the authors analyzed pandemic supply risk mitigation measures and possible recovery paths, accompanied by a relevant discussion of prospective global supply chain (re)-designs.

The paper is organized as follows. In Section 2, we construct the supply chain network game theory framework for differentiated products under three distinct labor scenarios. We identify the supply chain network structure, describe the behavior of the profit-maximizing firms, and the underlying constraints. For each scenario, we present two alternative variational inequality formulations of the governing equilibrium conditions. For Scenarios 2 and 3, which are Generalized Nash Equilibrium problems, we use the concept of a Variational Equilibrium to enable the variational inequality formulations. We also discuss existence results. The alternative variational inequality formulations allow for the implementation of an effective algorithm, which we present in Section 3, along with computes solutions to a series of numerical examples. The numerical examples are motivated by shortages of migrant labor to harvest fresh produce, in particular, blueberries in the United States. The results of the paper are summarized in Section 4, along with our conclusions, and suggestions for future research.

2. Supply Chain Network Game Theory Modeling Under Labor Constraints

There are I firms in the supply chain network economy that produce a substitutable product and compete noncooperatively in the production, transportation, storage, and distribution of their products to demand markets. The firms also compete with one another for labor, since labor is essential to the above network economic activities. Each firm is represented as a network of its economic activities as drawn in Figure 1. Observe that, according to Figure 1, the networks of the individual firms do not have any links in common. Table 1 contains the basic notation for the model. All vectors are column vectors.

Each firm i; $i = 1, \ldots, I$, owns n^i_M production facilities; can make use of n^i_D distribution centers, and can provide its product to the n_R demand markets. Let L^i denote the links comprising the supply chain network of firm i; $i = 1, \ldots, I$, that it owns/controls, with a total of n_{L^i} elements. The links of L^i include firm i's links to its production nodes; the links from production nodes to the distribution centers, the storage links, and the links from the distribution centers to the demand markets. L then denotes the full set of links in the supply chain network economy with $L = \cup^I_{i=1} L^i$ with a total of n_L elements. Let now $G = [N, L]$ denote the graph consisting of the set of nodes N and the set of links L in Figure 1. Each firm seeks to determine its optimal product quantities that maximize its profits by using Figure 1 as a schematic, coupled with the labor volumes, which we discuss the constraints on, further below.

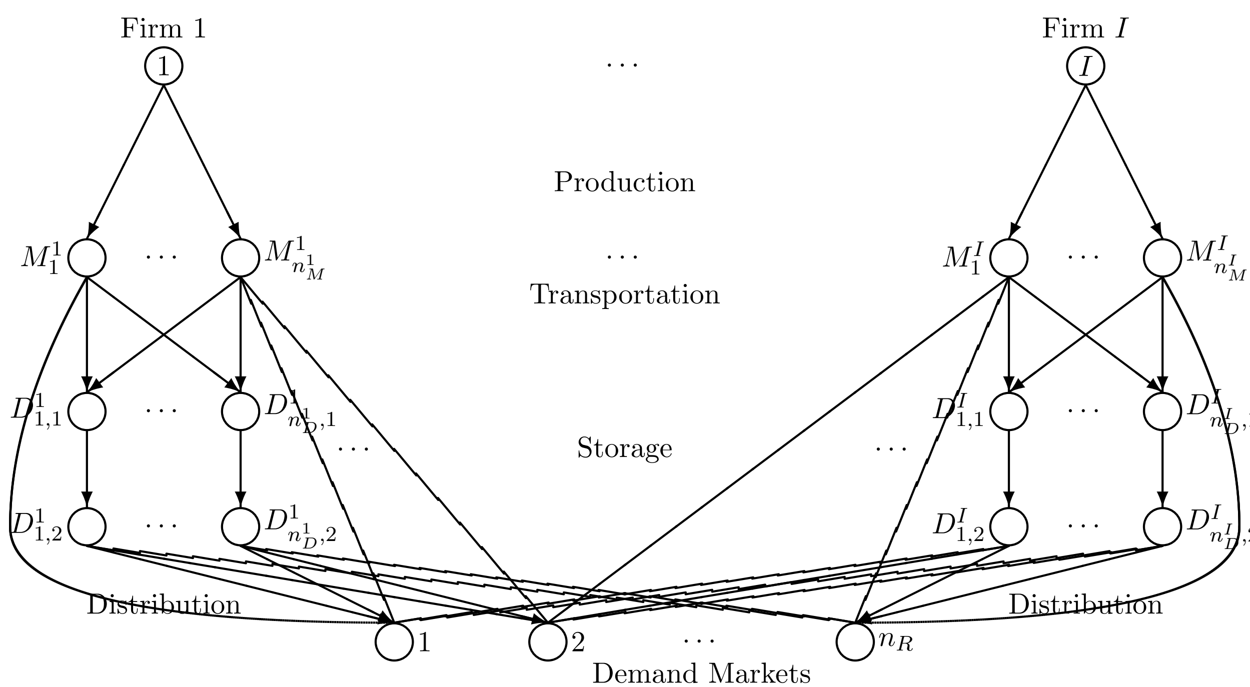

Figure 1: The Supply Chain Network Topology of the Model with Labor

Table 1: Notation for the Supply Chain Game Theory Modeling Framework with Labor

Notation	**Definition**
P_k^i	the set of paths in firm i's supply chain network terminating in demand market k; $i = 1, \ldots, I$; $k = 1, \ldots, n_R$.
P^i	the set of all n_{P^i} paths of firm i; $i = 1, \ldots, I$.
P	the set of all n_P paths in the supply chain network economy.
$x_p; p \in P_k^i$	the nonnegative flow on path p originating at firm node i and terminating at demand market k; $i = 1, \ldots, I$; $k = 1, \ldots, n_R$. We group firm i's product path flows into the vector $x^i \in R_+^{n_{P^i}}$. We emphasize that x^i is the vector of strategic variables of firm i. We then group all the firms' product path flows into the vector $x \in R_+^{n_P}$.
f_a	the nonnegative flow of the product on link a, $\forall a \in L$. We group all the link flows into the vector $f \in R_+^{n_L}$.
l_a	the labor on link a (usually denoted in person hours).
α_a	positive factor relating input of labor to output of product flow on link a, $\forall a \in L$.
$\bar{l}_a$	the upper bound on the availability of labor on link a under Scenario 1, $\forall a \in L$
$\bar{l}^t$	the upper bound on labor availability for tier t activities under Scenario 2, with tier $t = 1$ being production; tier $t = 2$ refers to transportation, and, so on, until $t = T$, which corresponds to distribution. Here, $T + 1$ corresponds to the electronic commerce tier.
$\bar{l}$	the upper bound on labor availability under Scenario 3.
d_{ik}	the demand for the product of firm i at demand market k; $i = 1, \ldots, I$; $k = 1, \ldots, n_R$. We group the $\{d_{ik}\}$ elements for firm i into the vector $d^i \in R_+^{n_R}$ and all the demands into the vector $d \in R_+^{I \times n_R}$.
$\hat{c}_a(f)$	the total operational cost associated with link a, $\forall a \in L$.
π_a	cost of a unit of labor on link a
$\rho_{ik}(d)$	the demand price function for the product of firm i at demand market k; $i = 1, \ldots, I$; $k = 1, \ldots, n_R$.

Specifically, production links from the top-tiered nodes i; $i = 1, \ldots, I$, representing firm i, in Figure 1 are connected to the production nodes of firm i, which are denoted, respectively, by: $M_1^i, \ldots, M_{n_M^i}^i$. The links from the production nodes, in turn, are joined with the distribution center nodes of each firm i; $i = 1, \ldots, I$, and correspond to transportation links. These nodes are denoted by $D_{1,1}^i, \ldots, D_{n_D^i,1}^i$. The links joining nodes $D_{1,1}^i, \ldots, D_{n_D^i,1}^i$ with nodes $D_{1,2}^i, \ldots, D_{n_D^i,2}^i$ correspond to the storage links. There are also distribution links connecting the nodes $D_{1,2}^i, \ldots, D_{n_D^i,2}^i$ for $i = 1, \ldots, I$, with the bottom-tiered demand market nodes: $1, \ldots, n_R$. Finally, there are links joining the production nodes with the demand market nodes and these links correspond to direct shipments to the demand markets. For example, in the case of demand markets corresponding to homes, such links can capture electronic commerce. They also can denote direct deliveries to consumers at demand markets where the producers are farms and such distribution channels have been initiated because of the pandemic (see, for example, Shea (2020)). Of course, the supply chain network topology in Figure 1 can be adapted for the specific application under consideration, with the appropriate addition/deletion of nodes, links, and/or supply chain network tiers.

The demand for each firm's product at each demand market must be satisfied by the product flows from the firm to that demand market. Hence, the following conservation of flow equations must hold for each firm i: $i = 1, \ldots, I$:

$$\sum_{p \in P_k^i} x_p = d_{ik}, \quad k = 1, \ldots, n_R. \tag{1}$$

Moreover, the path flows must be nonnegative; that is, for each firm i; $i = 1, \ldots, I$:

$$x_p \geq 0, \quad \forall p \in P^i. \tag{2}$$

The link flows of each firm i; $i = 1, \ldots, I$, are related to the path flows by the expression:

$$f_a = \sum_{p \in P} x_p \delta_{ap}, \quad \forall a \in L^i, \tag{3}$$

where $\delta_{ap} = 1$, if link a is contained in path p, and 0, otherwise. According to (3), the flow of a firm's product on a link is equal to the sum of that product's flows on paths that contain that link.

We now discuss how labor is related to product flow. In particular, we assume that the product output on each link is a linear function of the labor input. This corresponds to what is known as a linear production function in economics. Hence, we have that

$$f_a = \alpha_a l_a, \quad \forall a \in L^i, \quad i = 1, \ldots, I. \tag{4}$$

The greater the value of α_a, the more productive the labor on the link.

The utility function of firm i, U^i; $i = 1, \ldots, I$, is the profit, given by the difference between its revenue and its total costs:

$$U^i = \sum_{k=1}^{n_R} \rho_{ik}(d) d_{ik} - \sum_{a \in L^i} \hat{c}_a(f) - \sum_{a \in L^i} \pi_a l_a. \tag{5a}$$

The first expression after the equal sign in (5a) is the revenue of firm i. The second expression in (5) is the total operational costs for the supply chain network L^i of firm i and the third expression captures the total labor costs of firm i. The functions U_i; $i = 1, \ldots, I$, are assumed to be concave, with the demand price functions being monotone decreasing and continuously differentiable and the total link cost functions being convex and also continuously differentiable.

The optimization problem of each firm i; $i = 1, \ldots, I$, is, hence, for firm i as follows:

$$\text{Maximize} \quad \sum_{k=1}^{n_R} \rho_{ik}(d) d_{ik} - \sum_{a \in L^i} \hat{c}_a(f) - \sum_{a \in L^i} \pi_a l_a, \tag{5b}$$

subject to: (1), (2), (3), and (4).

We now consider three distinct scenarios in terms of the labor availability in the supply chain network economy and the associated bound(s) on specific links.

Labor Scenario 1 – A Bound on Labor on Each Supply Chain Network Link

In Scenario 1, the additional constraints on the fundamental model described above are:

$$l_a \leq \bar{l}_a, \quad \forall a \in L. \tag{6}$$

According to (6), there is an upper bound on labor associated with each link in the supply chain network of Figure 1. In this scenario, unlike the subsequent two scenarios, the feasible sets of the individual firms will depend only on their specific strategies and not on the strategies of the other firms, as we will show below.

Labor Scenario 2 – A Bound on Labor on Each Tier of Links in the Supply Chain Network

In Scenario 2, we consider that there is a bound on labor associated with each activity tier in the supply chain network economy, that is, in addition to the original constraints (1) through (4), the firms are now faced with the following constraints:

$$\sum_{a \in L^1} l_a \leq \bar{l}^1, \tag{7,1}$$

$$\sum_{a \in L^2} l_a \leq \bar{l}^2, \tag{7,2}$$

and so on, until

$$\sum_{a \in L^{T+1}} l_a \leq l^{T+1}. \tag{7,T+1}$$

Observe that, in Scenario 2, unlike in Scenario 1, the firms now have shared, that is, common labor constraints. Hence, their underlying feasible sets will no longer be disjoint. This will result in a Generalized Nash Equilibrium, rather than a Nash Equilibrium, and we will elaborate further below when we present the variational inequality formulations of the model under the different scenarios. Hence, in this scenario, labor is movable across a tier within a firm or across firms.

Scenario 2 allows for the movement of labor across the firms' different production facilities; across the distinct storage facilities, etc. Since one can expect that the skill set of a worker will be transferable across production facilities of a differentiated, but substitutable product; similarly, across distribution facilities or freight provision services, this scenario is quite reasonable in the pandemic.

Labor Scenario 3 – A Single Labor Bound on Labor for All the Links in the Supply Chain Network

Scenario 3 may be interpreted as being the least restrictive of the scenarios considered here in that labor can be transferable across different activities of production, transportation, storage, and distribution. In the pandemic, we are seeing that some employees are assuming different tasks in supply chain networks than they had been doing previously. This may enhance agility and flexibility, provided that labor has the requisite skills or the skills can be acquired fairly readily. For example, farmers now in the US have been innovating in terms of direct sales to consumers and taking on different tasks, such as deliveries of their farmed products to homes (see Woolever (2020)). This can benefit both producers and consumers.

In Scenario 3, in addition to constraints (1) through (4), the firms are now faced with the following single constraint:

$$\sum_{a \in L} l_a \leq \bar{l}. \tag{8}$$

We now reformulate the objective function of each firm i; $i = 1, \ldots, I$, given by (5b) in path flow variables exclusively. We are able to do this because of expressions (1), (2), and (4), which, recall, relates labor to product flow. Specifically, we can redefine the total operational cost link functions as: $\tilde{c}_a(x) \equiv \hat{c}_a(f)$, $\forall a \in L$, and the demand price functions as: $\tilde{\rho}_{ik}(x) \equiv \rho_{ik}(d)$, $\forall i$, $\forall k$. In addition, using a result of Nagurney et al. (2020b), we know that, in view of (3) and (4): $l_a = \frac{\sum_{p \in P} x_p \delta_{ap}}{\alpha_a}$, for all $a \in L$.

Recall also that, according to Table 1, x^i denotes the vector of strategies, which are the path flows, for each firm i; $i = 1, \ldots, I$. We can redefine the utility/profit functions $\tilde{U}^i(x) \equiv U^i$; $i = 1 \ldots, I$ and group the profits of all the firms into an I-dimensional vector $\tilde{U}$, such that

$$\tilde{U} = \tilde{U}(x). \tag{9}$$

Objective function (5b), in lieu of the above, can now be expressed as:

$$\text{Maximize} \quad \tilde{U}^i(x) = \sum_{k=1}^{n_R} \tilde{\rho}_{ik}(x) \sum_{p \in P_k^i} x_p - \sum_{a \in L^i} \tilde{c}_a(x) - \sum_{a \in L^i} \frac{\pi_a}{\alpha_a} \sum_{p \in P} x_p \delta_{ap}. \tag{10}$$

Furthermore, it readily follows that constraint (6) for Scenario 1 can be reexpressed exclusively in path flows; the same holds for constraints (7, 1) through (7,T+1) for Scenario 2, and for constraint (8) for Scenario 3.

2.1 Governing Equilibrium Conditions and Variational Inequality Formulations

We now state the governing equilibrium conditions for the different scenarios and provide alternative variational inequality formulations for each scenario.

2.1.1 Scenario 1 Nash Equilibrium Conditions and Variational Inequality Formulations

We define the feasible set K_i for firm i thus: $K_i \equiv \{x^i | x^i \in R_+^{n_{P^i}}, \frac{\sum_{p \in P^i} x_p \delta_{ap}}{\alpha_a} \leq \bar{l}_a, \forall a \in L^i\}$, for $i = 1, \ldots, I$. Also, we define $K \equiv \prod_{i=1}^{I} K_i$.

In Scenario 1, each firm competes noncooperatively until the following equilibrium is achieved.

Definition 1: Supply Chain Network Nash Equilibrium for Scenario 1

A path flow pattern $x^* \in K$ is a supply chain network Nash Equilibrium if for each firm i; $i = 1, \ldots, I$:

$$\tilde{U}^i(x^{i*}, \hat{x}^{i*}) \geq \tilde{U}^i(x^i, \hat{x}^{i*}), \quad \forall x^i \in K_i, \tag{11}$$

where $\hat{x}^{i*} \equiv (x^{1*}, \ldots, x^{i-1*}, x^{i+1*}, \ldots, x^{I*})$.

According to (11), a supply chain Nash Equilibrium is established if no firm can improve upon its profits unilaterally. We know that K is a convex set.

Applying the classical theory of Nash equilibria and variational inequalities, under our imposed assumptions on the underlying functions, it follows that (cf. Gabay and Moulin (1980) and Nagurney (1999)) the solution to the above Nash Equilibrium problem (see Nash (1950, 1951)) coincides with the solution of the variational inequality problem: determine $x^* \in K$, such that

$$-\sum_{i=1}^{I} \langle \nabla_{x^i} \tilde{U}^i(x^*), x^i - x^{i*} \rangle \geq 0, \quad \forall x \in K, \tag{12}$$

where $\langle \cdot, \cdot \rangle$ represents the inner product in the corresponding Euclidean space, which here is of dimension n_P, and $\nabla_{x^i} \tilde{U}^i(x)$ is the gradient of $\tilde{U}^i(x)$ with respect to x^i.

Existence of a solution to variational inequality (12) is guaranteed since the feasible set K is compact and the utility functions are continuously differentiable, under our imposed assumptions (cf. Kinderlehrer and Stampacchia (1980)).

We now provide an alternative variational inequality to (12) over a simpler feasible set. We introduce Lagrange multipliers λ_a associated with the constraint (6) for each link $a \in L$ and we group the Lagrange multipliers for each firm i's network L^i into the vector λ^i. We then group all such vectors for the firms into the vector $\lambda \in R_+^{n_L}$. Also, we define the feasible sets: $K_i^1 \equiv \{(x^i, \lambda^i) | (x^i, \lambda^i) \in R_+^{n_{P^i}+n_{L^i}}\}$; $i = 1, \ldots, I$, and $K^1 \equiv \prod_{i=1}^I K_i^1$.

Then, using similar arguments as in Theorem 1 in Nagurney, Yu, and Besik (2017), the following result is immediate.

Theorem 1: Alternative Variational Inequality Formulation of Nash Equilibrium for Scenario 1

The supply chain network Nash Equilibrium satisfying Definition 1 is equivalent to the solution of the variational inequality: determine the vector of equilibrium path flows and the vector of optimal Lagrange multipliers, $(x^*, \lambda^*) \in K^1$, such that:

$$\sum_{i=1}^{I}\sum_{k=1}^{n_R}\sum_{p\in P_k^i}\left[\frac{\partial \tilde{C}_p(x^*)}{\partial x_p} + \sum_{a\in L^i}\frac{\lambda_a^*}{\alpha_a}\delta_{ap} + \sum_{a\in L^i}\frac{\pi_a}{\alpha_a}\delta_{ap} - \tilde{\rho}_{ik}(x^*) - \sum_{l=1}^{n_R}\frac{\partial \tilde{\rho}_{il}(x^*)}{\partial x_p}\sum_{q\in P_l^i} x_q^*\right] \times [x_p - x_p^*]$$

$$+ \sum_{a\in L}\left[\bar{l}_a - \frac{\sum_{p\in P} x_p^* \delta_{ap}}{\alpha_a}\right] \times [\lambda_a - \lambda_a^*] \geq 0, \quad \forall (x, \lambda) \in K^1, \tag{13}$$

where

$$\frac{\partial \tilde{C}_p(x)}{x_p} \equiv \sum_{a\in L}\sum_{b\in L}\frac{\partial \hat{c}_b(f)}{\partial f_a}, \quad \forall p \in P. \tag{14}$$

The above feasible set K^1 is the nonnegative orthant. This feature enables the implementation of an algorithm, which we describe in the next section, which is an iterative procedure that yields closed form expressions at an iteration for the path flows and the link Lagrange multipliers.

2.1.2 Scenario 2 Generalized Nash Equilibrium Conditions and Variational Inequality Formulations

In Scenarios 2 and 3, on the other hand, not only do the utility functions of the firms depend on their own strategies and those of the other firms, but the feasible sets do, as well. This happens because of the corresponding labor constraints, which capture that labor is "shared." Therefore, in Scenarios 2 and 3, the governing concept is no longer tht of Nash Equilibrium, but, rather, it is that of Generalized Nash Equilibrium (see Debreu (1952) and Arrow and Debreu (1954)). This presents additional challenges, since, as noted in Nagurney, Yu, and Besik (2017), Generalized Nash Equilibrium problems can't be directly formulated as variational inequality problems, but, instead, quasi-variational inequalities are often used as the formulation (see, e.g., Facchinei and Kanzow (2010)). It is well-known (cf. Luna (2013) and the references therein) that quasi-variational inequality problems are, nevertheless, much harder to solve than finite-dimensional variational inequality problems.

We can utilize a refinement of the Generalized Nash Equilibrium (GNE) in order to secure a variational inequality formulation. The refinement is a *Variational Equilibrium* and it is a specific type of GNE (see Kulkarni and Shabhang (2012)). In a Generalized Nash Equilibrium defined by a Variational Equilibrium, the Lagrange multipliers associated with the common/shared constraints are all the same. As noted in Nagurney, Yu, and Besik (2017), this provides a fairness interpretation and is reasonable from an economic standpoint. We denote the set of shared constraints under Scenario 2 by $\mathcal{S}^1$, where $\mathcal{S}^1 \equiv \{x | \text{all constraints}\, (7,1) - (7,T+1)\, \text{hold}\}$.

We have the following definitions:

Definition 2: Generalized Nash Equilibrium Under Scenario 2

A product flow vector $x^* \in K \cap \mathcal{S}^1$ is a Generalized Nash Equilibrium Under Scenario 2, if for each firm i; $i = 1, \ldots, I$:

$$\tilde{U}^i(x^{i*}, \hat{x}^{i*}) \geq \tilde{U}^i(x^i, \hat{x}^{i*}), \quad \forall x^i \in K_i \cap \mathcal{S}^1, \tag{15}$$

where, as defined previously, $\hat{x}^{i*} \equiv (x^{1*}, \ldots, x^{i-1*}, x^{i+1*}, \ldots, x^{I*})$.

Definition 3: Variational Equilibrium Under Scenario 2

A strategy vector x^* is said to be a Variational Equilibrium of the above Generalized Nash Equilibrium according to Definition 2 if $x^* \in K \cap \mathcal{S}^1$ is a solution of the variational inequality:

$$-\sum_{i=1}^{I} \langle \nabla_{x_i} \tilde{U}^i(x^*), x^i - x^{i*} \rangle \geq 0, \quad \forall x \in K \cap \mathcal{S}^1. \tag{16}$$

Clearly, a solution x^* to variational inequality (16) exists.

For Scenario 2, we now associate the nonnegative Lagrange multiplier μ^t with labor constraint $(7,t)$, for $t = 1, \ldots, T+1$. We define the vector of Lagrange multipliers $\mu \in R_+^{T+1}$ and the feasible set $K^2 \equiv \{(x,\mu) | (x,\mu) \in R_+^{n_P+T+1}\}$. An alternative variational inequality formulation to that of (16), whose solution corresponds to the Variational Equilibrium under Scenario 2 is: determine $(x^*, \mu^*) \in K^2$, such that:

$$\sum_{i=1}^{I} \sum_{k=1}^{n_R} \sum_{p \in P_k^i} \left[\frac{\partial \tilde{C}_p(x^*)}{\partial x_p} + \sum_{a \in L^i} \frac{\pi_a}{\alpha_a} \delta_{ap} - \tilde{\rho}_{ik}(x^*) - \sum_{l=1}^{n_R} \frac{\partial \tilde{\rho}_{il}(x^*)}{\partial x_p} \sum_{q \in P_l^i} x_q^* + \sum_{t=1}^{T+1} \mu^{t*} \sum_{a \in L^t} \frac{1}{\alpha_a} \delta_{ap} \right]$$

$$\times[x_p - x_p^*] + \sum_{t=1}^{T+1}\left[\bar{l}^t - \sum_{a\in L^t}\frac{\sum_{p\in P} x_p^*\delta_{ap}}{\alpha_a}\right] \times \left[\mu^t - \mu^{t*}\right] \geq 0, \quad \forall (x,\mu) \in K^2. \tag{17}$$

The variational inequality (17) will also allow for the application of an effective and efficient computational procedure.

2.1.3 Scenario 3 Generalized Nash Equilibrium Conditions and Variational Inequality Formulations

Recall that in Scenario 3 there is a single labor constraint, and labor is allowed to move freely among the supply chain network economic activities across the firms. Proceeding in a similar manner as for Scenario 2, we define $\mathcal{S}^2$, where $\mathcal{S}^2 \equiv \{x|\,(8)\,\text{holds}\}$. We then can state the following definitions:

Definition 4: Generalized Nash Equilibrium Under Scenario 3

A product flow vector $x^* \in K \cap \mathcal{S}^2$ is a Generalized Nash Equilibrium Under Scenario 3, if for each firm i; $i = 1, \ldots, I$:

$$\tilde{U}^i(x^{i*}, \hat{x}^{i*}) \geq \tilde{U}^i(x^i, \hat{x}^{i*}), \quad \forall x^i \in K_i \cap \mathcal{S}^2. \tag{18}$$

Definition 5: Variational Equilibrium Under Scenario 3

A strategy vector x^* is said to be a Variational Equilibrium of the above Generalized Nash Equilibrium according to Definition 3 if $x^* \in K \cap \mathcal{S}^1$ is a solution of the variational inequality:

$$-\sum_{i=1}^{I}\langle \nabla_{x_i}\tilde{U}^i(x^*), x^i - x^{i*}\rangle \geq 0, \quad \forall x \in K \cap \mathcal{S}^2. \tag{19}$$

Of course a solution to variational inequality (19) is guaranteed to exist since the underlying feasible set $K \cap \mathcal{S}^2$ is compact.

As for Scenarios 1 and 2, we now provide an alternative variational inequality formulation. We denote the Lagrange multiplier associated with the labor constraint (8) by γ. Also, we define the feasible set $K^3 \equiv \{(x,\gamma)|(x,\gamma) \in R_+^{n_P+1}\}$. We then have the following result. An alternative variational inequality to that of (16) is: determine $(x^*, \gamma^*) \in K^3$ such that

$$\sum_{i=1}^{I}\sum_{k=1}^{n_R}\sum_{p\in P_k^i}\left[\frac{\partial \tilde{C}_p(x^*)}{\partial x_p} + \sum_{a\in L^i}\frac{\pi_a}{\alpha_a}\delta_{ap} - \tilde{\rho}_{ik}(x^*) - \sum_{l=1}^{n_R}\frac{\partial \tilde{\rho}_{il}(x^*)}{\partial x_p}\sum_{q\in P_l^i} x_q^* + \gamma^*\sum_{a\in L}\frac{1}{\alpha_a}\delta_{ap}\right] \times [x_p - x_p^*]$$

$$+\left[\bar{l}-\sum_{a\in L}\frac{\sum_{p\in P}x^*_p\delta_{ap}}{\alpha_a}\right]\times[\gamma-\gamma^*]\geq 0,\quad \forall (x,\gamma)\in K^3. \tag{20}$$

Observe that the feasible set K^3 is also the nonnegative orthant and this will allow effective solution of variational inequality (20).

Also, it is important to emphasize that the Lagrange multipliers associated with the respective constraint sets in Scenarios 1, 2, and 3, provide valuable information, upon problem solution. In particular, they quantify the value of an additional unit of the labor resource associated with the respective constraint. Having a computational procedure that can compute both the equilibrium product path flows (from which the labor values are then obtained from: $l_a=\frac{\sum_{p\in P}x_p\delta_{ap}}{\alpha_a}$, for all links $a\in L$) and the equilibrium Lagrange multipliers enriches the toolbox for managerial insights and decision-making. We present the algorithm in Section 3.

All of the above six variational inequalities can be put into standard variational inequality form, where (cf. Nagurney (1999)) the finite-dimensional variational inequality problem VI$(F,\mathcal{K})$, is to determine a vector $X^*\in\mathcal{K}\subset R^{\mathcal{N}}$, such that

$$\langle F(X^*),X-X^*\rangle\geq 0,\quad \forall X\in\mathcal{K}, \tag{21}$$

where F is a given continuous function from $\mathcal{K}$ to $R^{\mathcal{N}}$, $\mathcal{K}$ is a given closed, convex set, and $\langle\cdot,\cdot\rangle$ denotes the inner product in $\mathcal{N}$-dimensional Euclidean space.

3. The Algorithm and Examples

We, specifically, recommend and, then, apply the modified projection method of Korpelevich (1977) to compute solutions to the supply chain network game theory model under labor different scenarios. This algorithm was recently also applied to solve problem of human migration with policy interventions in Nagurney and Daniele (2020).

For the alternative variational inequality formulations of Scenarios 1, 2, and 3, given, respectively, by (13), (17), and (20), each iteration of the modified projection method yields closed form expressions for the path flows and for the associated Lagrange multipliers. The modified projection method is guaranteed to converge if the function $F(X)$ that enters the variational inequality (cf. (21)) is Lipschitz continuous and monotone. These are reasonable conditions for the supply chain network game theory model with labor under the scenarios considered here.

The definitions of monotonicity and Lipschitz continuity of $F(X)$ are as follow. The function $F(X)$ is said to be monotone, if

$$\langle F(X^1)-F(X^2),X^1-X^2\rangle\geq 0,\quad \forall X^1,X^2\in\mathcal{K}, \tag{22}$$

and the function $F(X)$ is Lipschitz continuous, if there exists a constant $L > 0$, known as the Lipschitz constant, such that

$$\|F(X^1) - F(X^2)\| \leq L\|X^1 - X^2\|, \quad \forall X^1, X^2 \in \mathcal{K}. \tag{23}$$

The steps of the modified projection method are given below, with τ denoting an iteration counter:

The Modified Projection Method

Step 0: Initialization

Initialize with $X^0 \in \mathcal{K}$. Set the iteration counter $\tau := 1$ and let β be a scalar such that $0 < \beta \leq \frac{1}{L}$, where L is the Lipschitz constant.

Step 1: Computation

Compute $\bar{X}^\tau$ by solving the variational inequality subproblem:

$$\langle \bar{X}^\tau + \beta F(X^{\tau-1}) - X^{\tau-1}, X - \bar{X}^\tau \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{24}$$

Step 2: Adaptation

Compute X^τ by solving the variational inequality subproblem:

$$\langle X^\tau + \beta F(\bar{X}^\tau) - X^{\tau-1}, X - X^\tau \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{25}$$

Step 3: Convergence Verification

If $|X^\tau - X^{\tau-1}| \leq \epsilon$, with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $\tau := \tau + 1$ and go to Step 1.

We now present the numerical examples. The modified projection method was implemented in FORTRAN and a Linux system at the University of Massachusetts Amherst used for the computation of solutions to the subsequent numerical examples. The algorithm was initialized as follows. We initialized the elastic demand for each demand market at 40 and equally distributed the demand among the paths connecting each demand market from each origin node (firm). The Lagrange multipliers were initialized to 0. The modified projection method was deemed to have converged if the absolute difference of the path flows differed by no more than 10^{-7} and the same for the Lagrange multipliers.

Our numerical examples are inspired by the summer harvest season for fresh produce and, specifically, the need to harvest berries, including blueberries. Much of the seasonal fresh produce harvesting in the United States is done by migrant workers. This is also the case in other parts of the world, including Europe. The harvesting of blueberries in the northeastern United States, with major growing areas being New Jersey and Maine, has been especially challenged this summer because of the Covid-19 pandemic (see Tully (2020) and Russell (2020)). Workers have become ill. Some have died, whereas others have not been able to secure permission to travel in time to harvest this perishable product. Interestingly, although operations researchers have been tackling human migration networks for several decades now, it is only recently that regulations have been incorporated in such network-based models (see, e.g., Nagurney and Daniele (2020) and Nagurney, Daniele, and Cappello (2021)). Here we focus on seasonal migrants, rather than those who wish to relocate permanently to new locations.

Our numerical examples are stylized but we did utilize Internet available resources to obtain blueberry price and picking data in the United States (see Galinato, Gallardo, and Hong (2016) and howmuchitis.org (2018)). The flow variables are in pounds of blueberries, the prices are in dollars per pound, and labor is in person hours. In Section 3.1, the numerical examples are of Scenario 1 and we solve the governing variational inequality (13). In Section 3.2, the examples are of Scenario 3, and we solve the governing variational inequality (20).

3.1 Scenario 1 Examples

Examples 1, 2, and 3 have the supply chain network topology given in Figure 2. In these examples there is a single firm, with Example 1 serving as the baseline, which is a blueberry farm. It has two planting sites (production locations), access to a single distribution center, and the blueberries are distributed to two demand markets.

Example 1 - Baseline Example

The total operational cost functions on the links are:

$$\hat{c}_a(f) = .0006f_a^2, \quad \hat{c}_b(f) = .0007f_b^2, \quad \hat{c}_c(f) = .001f_c^2, \quad \hat{c}_d(f) = .001f_d^2, \quad \hat{c}_e(f) = .002f_e^2,$$
$$\hat{c}_f(f) = .005f_f^2, \quad \hat{c}_g(f) = .005f_g^2.$$

The costs for labor are:

$$\pi_a = 10, \quad \pi_b = 10, \quad \pi_c = 15, \quad \pi_d = 15, \quad \pi_e = 20, \quad \pi_f = 17, \quad \pi_g = 18.$$

The link labor productivity factors are:

$$\alpha_a = 24, \quad \alpha_b = 25, \quad \alpha_c = 100, \quad \alpha_d = 100, \quad \alpha_e = 50, \quad \alpha_f = 100, \quad \alpha_g = 100.$$

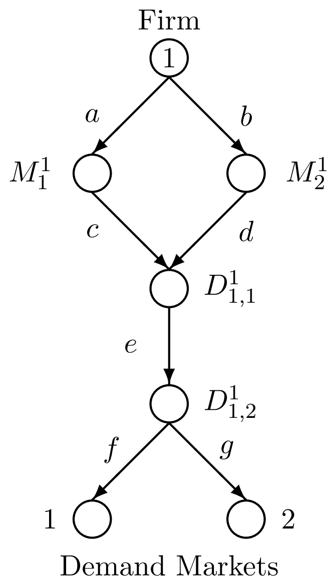

Figure 2: Supply Chain Network Topology for Examples 1, 2, and 3

The bounds on labor are:

$$\bar{l}_a = 200, \quad \bar{l}_b = 200, \quad \bar{l}_c = 300, \quad \bar{l}_d = 300, \quad \bar{l}_e = 100, \quad \bar{l}_f = 120, \quad \bar{l}_g = 120.$$

The labor bounds are set high since this is the baseline example.

The demand price functions are: $\rho_{11}(d) = -.0001d_{11} + 6$ and $\rho_{12}(d) = -.0002d_{12} + 8$.

The paths are as follows: $p_1 = (a, c, e, f)$, $p_2 = (b, d, e, f)$, $p_3 = (a, c, e, g)$, and path $p_r = (b, d, e, g)$.

The modified projection method yields the following solution: The equilibrium path flows are:

$$x^*_{p_1} = 91.46, \quad x^*_{p_2} = 85.77, \quad x^*_{p_3} = 185.40, \quad x^*_{p_4} = 179.77.$$

The equilibrium link labor values are:

$$l^*_a = 11.54, \quad l^*_b = 10.62, \quad l^*_c = 2.77, \quad l^*_d = 2.66, \quad l^*_e = 10.85, \quad l^*_f = 1.77, \quad l^*_g = 3.65.$$

As expected, all the Lagrange multipliers are equal to 0:

$$\lambda^*_a = \lambda^*_b = \lambda^*_c = \lambda^*_d = \lambda^*_e = \lambda^*_f = \lambda^*_g = 0.00.$$

The demand price at demand market 1 is: 5.98 and at demand market 2: 7.93 and the computed respective demands are: 177.23 and: 365.16.

The profit of the firm is: $1,684.47$.

Example 2 – Disruption in Labor Availability on Link a

Example 2 has the same data as Example 1, except that now we consider the following situation. Workers have been getting sick and are not as available to pick the blueberries on the first harvesting site. Specifically, now the bound on the labor on link a has been drastically reduced to: $\bar{l}_a = 10$.

The modified projection method yields the following equilibrium solution: The equilibrium path flow pattern is:

$$x^*_{p_1} = 72.99, \quad x^*_{p_2} = 99.19, \quad x^*_{p_3} = 167.01, \quad x^*_{p_4} = 193.21.$$

The equilibrium link labor values are:

$$l^*_a = 10.00, \quad l^*_b = 11.70, \quad l^*_c = 2.40, \quad l^*_d = 2.92, \quad l^*_e = 10.65, \quad l^*_f = 1.72, \quad l^*_g = 3.60.$$

The Lagrange multipliers are all equal to 0, except for:

$$\lambda^*_a = 5.0253.$$

The demand price at demand market 1 is: 5.98 and at demand market 2: 7.93 with the computed respective demands being, respectively: 172.18 and: 360.22.

The profit of the firm is: 1,680.61. Since the labor amount is at the bound on link a, the associated Lagrange multiplier is positive. Due to the labor disruption because of the pandemic, the profit of the firm decreases. The demands also decrease.

Example 3 – No Labor Availability on Link a

Example 3 is a Variant of Example 2 and has the same data except that now, due to illnesses, there is no labor available on link a; hence, $\bar{l}_a = 0.00$.

The modified projection method converges to the following equilibrium solution. The equilibrium path flow pattern is:

$$x^*_{p_1} = 0.00, \quad x^*_{p_2} = 139.38, \quad x^*_{p_3} = 0.00, \quad x^*_{p_4} = 327.99.$$

The equilibrium link labor values are:

$$l^*_a = 0.00, \quad l^*_b = 18.69, \quad l^*_c = 0.00, \quad l^*_d = 4.67, \quad l^*_e = 9.35, \quad l^*_f = 1.39, \quad l^*_g = 3.28.$$

The Lagrange multipliers are all equal to 0, except for:

$$\lambda^*_a = 162.6470.$$

We can see from this example the value of increasing labor availability on link a, as revealed through the significantly higher value of the Lagrange multiplier λ_a^*. The profit of the firm now drops to 1,466.78.

The demand price at demand market 1 is: 5.99 and at demand market 2 it is: 7.93. The computed respective demands are: 139.38 and: 327.99. The demand for blueberries drops at each demand market.

Clearly, the farmer should do everything possible to secure the health of the workers at his production/harvesting facilities, so that the blueberries can be harvested in a timely manner and so that profits do not suffer.

Example 4 – Addition of a Competitor

In Examples 4 through 6, we evaluate the impact of a competitor, that is, there is now another farm growing blueberries in the general area of the first farm. The underlying supply chain network topology is now as in Figure 3.

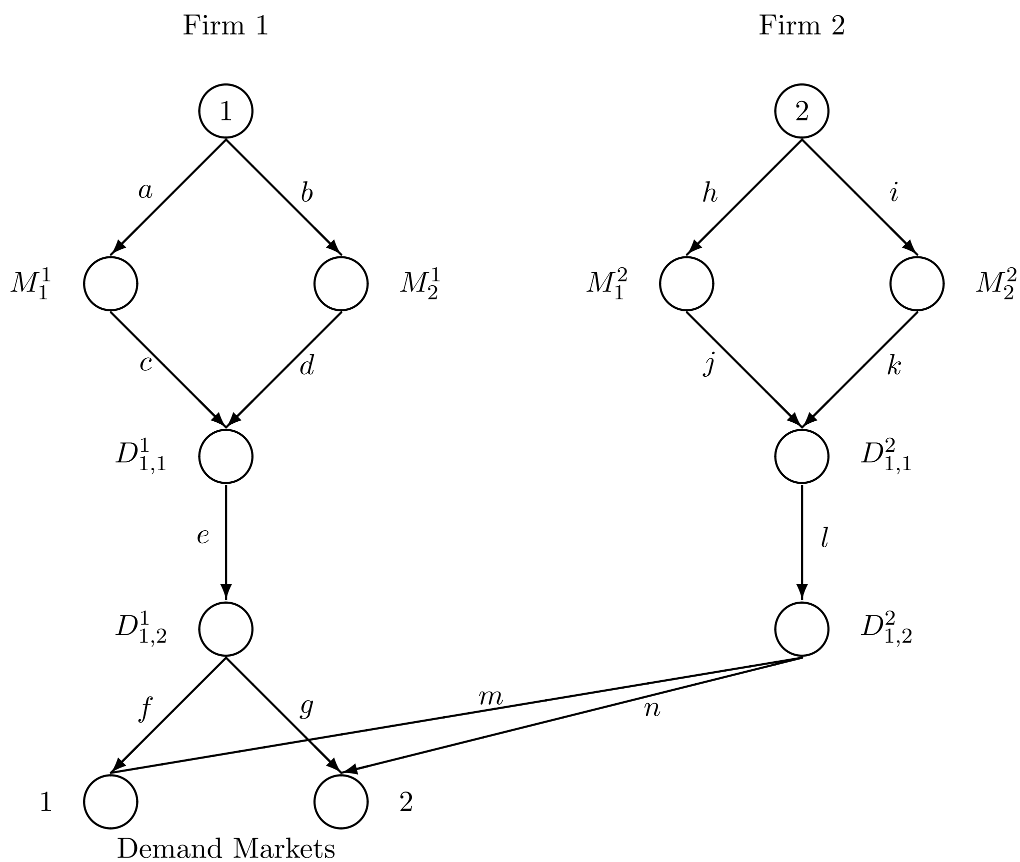

Figure 3: The Supply Chain Network Topology for the Numerical Examples 4 Through 6

The new firm also has two production sites, a single distribution center, and serves the same two locations as demand markets as Firm 1 does.

The data for the first firm are identical to the data in Example 2 except with a modification of the firm's demand price functions in order to capture competition with the new firm.

In particular, the demand price functions of Firm 1 are now:

$$\rho_{11}(d) = -.0001d_{11} - .00005d_{21} + 6, \quad \rho_{12}(d) = -.0002d_{12} - .0001d_{22} + 8.$$

The demand price functions of Firm 2 are:

$$\rho_{21}(d) = -.0003d_{21} + 7, \quad \rho_{22}(d) = -.0002d_{22} + 7.$$

Also, the total operational costs associated with Firm 2's supply chain network L^2 are:

$$\hat{c}_h(f) = .00075f_h^2, \quad \hat{c}_i(f) = .0008f_i^2, \quad \hat{c}_j(f) = .0005f_j^2, \quad \hat{c}_k(f) = .0005f_k^2, \quad \hat{c}_l(f) = .0015f_l^2,$$

$$\hat{c}_m(f) = .01f_m^2, \quad \hat{c}_n(f) = .01f_n^2.$$

The costs for labor for Firm 2 are:

$$\pi_h = 11, \quad \pi_i = 22, \quad \pi_j = 15, \quad \pi_k = 15, \quad \pi_l = 18, \quad \pi_m = 18, \quad \pi_n = 18.$$

The link labor productivity factors for Firm 2 on its supply chain network are:

$$\alpha_h = 23, \quad \alpha_i = 24, \quad \alpha_j = 100, \quad \alpha_k = 100, \quad \alpha_l = 70, \quad \alpha_m = 100, \quad \alpha_n = 100.$$

The bounds on labor, in turn, are:

$$\bar{l}_h = 800, \quad \bar{l}_i = 90, \quad \bar{l}_j = 200, \quad \bar{l}_k = 200, \quad \bar{l}_l = 300, \quad \bar{l}_m = 100, \quad \bar{l}_n = 100.$$

The four new paths associated with Firm 2 are:

$$p_5 = (h, j, l, m), \quad p_6 = (i, k, l, m), \quad p_7 = (h, j, l, n), \quad p_8 = (i, k, l, n).$$

The modified projection method yields the following equilibrium solution: The equilibrium path flow pattern is:

$$x^*_{p_1} = 73.23, \quad x^*_{p_2} = 98.85, \quad x^*_{p_3} = 166.77, \quad x^*_{p_4} = 192.38,$$

$$x^*_{p_5} = 142.85, \quad x^*_{p_6} = 53.08, \quad x^*_{p_7} = 143.81, \quad x^*_{p_8} = 54.04.$$

The equilibrium link labor values are:

$$l_a^* = 10.00, \quad l_b^* = 11.65, \quad l_c^* = 2.40, \quad l_d^* = 2.91, \quad l_e^* = 10.62, \quad l_f^* = 1.72, \quad l_g^* = 3.59,$$

$$l_h^* = 12.46, \quad l_i^* = 4.46, \quad l_j^* = 2.87, \quad l_k^* = 1.07, \quad l_l^* = 5.63, \quad l_m^* = 1.96, \quad l_n^* = 1.98.$$

The Lagrange multipliers are all equal to 0.00 except for $l_a^* = 4.9295$.

The product prices at equilibrium are:

$$\rho_{11} = 5.97, \quad \rho_{12} = 7.91, \quad \rho_{21} = 6.94, \quad \rho_{22} = 6.96,$$

with the equilibrium demands:

$$d_{11}^* = 172.07, \quad d_{12}^* = 359.15, \quad \rho_{21} = 195.94, \quad \rho_{22} = 197.86.$$

The profit for Firm 1 is: 1,671.80 and the profit for Firm 2 is: 1,145.06. With the addition of a competitor, the prices of Firm 1 for its blueberries drop at the demand markets and its profit is also negatively impacted.

Example 5 – Modification of Demand Price Functions

Example 5 has the same data as Example 4 except that we modified the demand price functions $\rho_{21}(d)$ and $\rho_{22}(d)$ to include a cross term, so that:

$$\rho_{21}(d) = -.0003d_{21} - .0001d_{11} + 6, \quad \rho_{22}(d) = -.0002d_{22} - .0001d_{12} + 7.$$

The new computed equilibrium path flow and Lagrange multiplier patterns are given below, as well as the equilibrium labor values on the links. The equilibrium path flow is now:

$$x_{p_1}^* = 73.22, \quad x_{p_2}^* = 98.85, \quad x_{p_3}^* = 166.78, \quad x_{p_4}^* = 192.38,$$

$$x_{p_5}^* = 142.62, \quad x_{p_6}^* = 52.86, \quad x_{p_7}^* = 143.12, \quad x_{p_8}^* = 53.36.$$

The equilibrium link labor values are:

$$l_a^* = 10.00, \quad l_b^* = 11.65, \quad l_c^* = 2.40, \quad l_d^* = 2.91, \quad l_e^* = 10.62, \quad l_f^* = 1.72, \quad l_g^* = 3.59,$$

$$l_h^* = 12.42, \quad l_i^* = 4.43, \quad l_j^* = 2.86, \quad l_k^* = 1.06, \quad l_l^* = 5.60, \quad l_m^* = 1.95, \quad l_n^* = 1.96.$$

The Lagrange multipliers are all equal to 0.00 except to $l_a^* = 4.93$.

The product prices at equilibrium are:

$$\rho_{11} = 5.97, \quad \rho_{12} = 7.91, \quad \rho_{21} = 6.92, \quad \rho_{22} = 6.92,$$

with the equilibrium demands:

$$d_{11}^* = 172.07, \quad d_{12}^* = 359.16, \quad d_{21}^* = 195.48, \quad d_{22}^* = 196.48.$$

The profit for Firm 1 is: 1,671.86 and the profit for Firm 2 is: 1,134.61. The profit for firm 1 rises ever so slightly, whereas that for Firm 2 decreases.

Example 6 – Disruptions in Storage Facilities

Example 6 has the same data as Example 5 except that we now consider a major disruption in terms of the spread of Covid-19 at the distribution centers of both firms with the bounds on labor corresponding to the associated respective links now being:

$$\bar{l}_e = 5, \quad \bar{l}_l = 5.$$

The modified projection method computes the following equilibrium path flow pattern:

$$x_{p_1}^* = 15.65, \quad x_{p_2}^* = 14.38, \quad x_{p_3}^* = 110.60, \quad x_{p_4}^* = 109.35,$$

$$x_{p_5}^* = 131.97, \quad x_{p_6}^* = 42.63, \quad x_{p_7}^* = 132.36, \quad x_{p_8}^* = 43.02.$$

The equilibrium link labor values are:

$$l_a^* = 5.26, \quad l_b^* = 4.95, \quad l_c^* = 1.26, \quad l_d^* = 1.24, \quad l_e^* = 5.00, \quad l_f^* = 0.30, \quad l_g^* = 2.20,$$

$$l_h^* = 11.49, \quad l_i^* = 3.57, \quad l_j^* = 2.64, \quad l_k^* = 0.86, \quad l_l^* = 5.00, \quad l_m^* = 1.75, \quad l_n^* = 1.75.$$

All computed equilibrium Lagrange multipliers are now equal to 0 except for those associated with the distribution center link labor bounds which are:

$$\lambda_e^* = 157.2138, \quad \lambda_l^* = 43.6537.$$

The product prices at equilibrium are now:

$$\rho_{11} = 5.99, \quad \rho_{12} = 7.94, \quad \rho_{21} = 6.94, \quad \rho_{22} = 6.94,$$

with the equilibrium demands:

$$d_{11}^* = 30.03, \quad d_{12}^* = 219.96, \quad d_{21} = 174.61, \quad d_{22} = 175.39.$$

The profit for Firm 1 is now drastically reduced to: 1,218.74 and the profit for Firm 2 also declines but by a much smaller amount to 1,126.73.

3.2 Scenario 3 Examples

In this subsection, we compute solutions via the modified projection method to numerical examples of Scenario 3.

Specifically, the supply chain network topology remains as in Figure 3. The data for Examples 7 through 11 in this set are identical to that in Example 6, except there are no longer any link bounds but, rather, only a single labor bound $\bar{l}$.

Example 7 has $\bar{l} = 80$; Example 8 has $\bar{l} = 70$; Example 9 has $\bar{l} = 60$; Example 10 has $\bar{l} = 30$, and Example 11 has $\bar{l} = 10$.

The computed equilibrium path flows for these examples are reported in Table 2 with the equilibrium labor values given in Table 3.

Equilibrium Product Path Flows	Ex. 7	Ex. 8	Ex. 9	Ex.10	Ex.11
$x^*_{p_1}$	91.35	83.08	42.05	1.03	0.00
$x^*_{p_2}$	85.67	78.87	45.09	11.31	0.00
$x^*_{p_3}$	184.84	176.75	136.58	96.40	24.01
$x^*_{p_4}$	179.18	172.54	139.60	106.66	60.82
$x^*_{p_5}$	142.63	136.59	106.62	76.65	20.51
$x^*_{p_6}$	52.84	48.42	26.50	4.58	0.00
$x^*_{p_7}$	143.14	137.04	106.81	76.58	20.30
$x^*_{p_8}$	53.34	48.84	26.69	4.51	0.00

Table 2: Equilibrium Product Path Flows for Examples 7 Through 11 Representing Scenario 3

The computed associated equilibrium Lagrange multiplier is as follows. For Example 7, the labor bound is not tight and, therefore, $\gamma^* = 0.00$. For Example 8, $\gamma^* = 3.99$; for Example 9: $\gamma^* = 23.79$; for Example 10: $\gamma^* = 43.59$ and for Example 11: $\gamma^* = 68.00$.

As for the profits, it is interesting to see how the disruptions in labor availability impact the profits negatively. In Example 7, Firm 1 earns a profit of 1,675.58, whereas Firm 2 earns a profit of 1,134.42. In Example 8, with labor being at the capacity, Firm 1 earns a profit of 1,671.23, whereas Firm 2 earns a profit of 1,131.76, which is only a small decrease. However, there is a big decrease in profit for both firms in Example 9. In Example 9, Firm 1 earns a profit of 1,506.94, whereas Firm 2 earns a profit of 1,022.56. In Example 10, with the labor capacity further reduced, Firm 1 earns only a profit of 1,105.06, whereas Firm 2 earns a profit of only 753.56. And, in our final example, Example 11, with the labor bound $\bar{l} = 10$, Firm 1's profit drops further to 523.19, whereas Firm

Equilibrium Link Labor Values	Ex. 7	Ex. 8	Ex. 9	Ex. 10	Ex. 11
l_a^*	11.51	10.83	7.44	4.06	1.00
l_b^*	10.59	10.06	7.39	4.72	2.43
l_c^*	2.76	2.60	1.79	0.97	0.24
l_d^*	2.65	2.51	1.85	1.18	0.61
l_e^*	10.82	10.22	7.27	4.31	1.70
l_f^*	1.77	1.62	0.8	0.12	0.00
l_g^*	3.64	3.49	2.76	2.03	0.85
l_h^*	12.42	11.90	9.28	6.66	1.77
l_i^*	4.42	4.05	2.22	0.38	0.00
l_j^*	2.86	2.74	2.13	1.53	0.41
l_k^*	1.06	0.97	0.53	0.09	0.00
l_l^*	5.60	5.30	3.81	2.32	0.58
l_m^*	1.95	1.85	1.33	0.81	0.21
l_n^*	1.96	1.86	1.34	0.81	0.20

Table 3: Equilibrium Link Labor Values for Examples 7 Through 11 Representing Scenario 3

2's profit drops to 228.91.

Again, we emphasize that, although the above examples are stylized, and, clearly, examples of Scenario 2 can also be effectively computed using the proposed algorithm, the supply chain network game theory framework provides decision-makers with the flexibility of evaluating impacts of labor reductions due to Covid-19 and, of course, also investments in terms maintaining labor bound values at high levels. The latter can be achieved, in part, through social distancing, proper sanitation, and providing uncrowded living facilities for migrant workers.

Furthermore, our numerical examples were inspired by crises faced by farmers due to labor shortages as a consequence of the Covid-19 pandemic. The supply chain network game theory framework can be adapted to different industrial sectors where labor plays an important role, such as, for example, in medical supply production from PPEs to ventilators and, in the future, if all goes well, also for medical treatments for Covid-19 patients and vaccines for everyone.

The computed solutions demonstrate that the proposed algorithm is effective for the game theory supply chain network model with labor constraints and that the model is illuminating. Furthermore, having such an algorithm allows for the calculation of the quantitative impacts on equilibrium product flows, product prices, and firm profits of labor availability and possible disruptions under different scenarios. Finally, the computed equilibrium Lagrange multipliers associated with the various labor constraints provide the value in relaxing a specific labor constraint.

4. Summary and Conclusions

In this paper, we introduced a supply chain network game theory framework consisting of multiple profit-maximizing firms competing noncooperatively in the production, storage, and ultimate distribution of their differentiated products in the presence of labor constraints. Specifically, we considered three different scenarios of labor constraints consisting of bounds on labor availability on supply chain links in the first scenario; bounds on labor availability across a tier of the supply chain networks of the first in the second scenario, and, finally, in the third scenario, a single bound on the labor in the supply chain network economy. The framework is a contribution to the operations research and to the economics literatures and is inspired by the challenges associated with labor and accompanying shortages in the Covid-19 pandemic.

The governing concept in the case of labor Scenario 1 is that of Nash Equilibrium, whereas, because of the shared constraints in terms of labor, the governing concept in the case of labor Scenarios 2 and 3 is that of a Generalized Nash Equilibrium. For each labor scenario, we provide two variational inequality formulations, with the variational inequality that includes the Lagrange multipliers associated with the labor constraints being especially amenable to solution via our proposed algorithm, since both the equilibrium product flows and the equilibrium Lagrange multipliers can be computed iteratively using explicit formulae.

Qualitative results are presented in terms of existence of the equilibrium patterns, along with solutions to a series of numerical examples. The numerical examples are grounded in disruptions taking place now in terms of shortages of migrant workers to harvest fresh produce, in particular, berries, and, specifically, blueberries in parts of the United States. Such labor shortfalls are not only a critical issue now in the United States but also in other parts of the globe, as a consequence of the Covid-19 pandemic.

We emphasize that the framework constructed in his paper is a partial equilibrium one since the focus was on oligopolistic competition in supply chain networks of relevance to the Covid-19 pandemic. In future research, we expect that the supply chain network game theory model(s) herein can be adapted to specific industrial sectors and the data fitted empirically. We also expect that different labor constraints may be identified and perhaps even different functions than the linear ones that we utilized here to relate labor to product flow. Also, interesting avenues for future research are the use of cooperative game theory as well as the incorporation of costs associated with training/education to enable the reallocation of labor with appropriate skills where they may be shortfalls. This is happening, for example, in practice (cf. Reuters (2020)), where we are seeing former airline workers being retrained as healthcare workers. Finally, it would be interesting to construct a general equilibrium model.

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