Optimization of Investments in Labor Productivity in Supply Chain Networks

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Abstract: Labor is an essential resource in the functionality of supply chains. The COVID-19 pandemic has demonstrated the varied impacts of disruptions to supply chains because of labor issues. Shortages of labor continue even now as economies begin to open up with progress on vaccinations. Investing in labor productivity is a possible mechanism in moderating shortfalls in labor. This paper constructs a supply chain network optimization model, whose solution yields optimal product path flows to demand markets, the optimal investments in link labor productivity, as well as labor hours needed, and the optimal wages of the workers in production, transportation, storage, and distribution. The model includes a budget constraint on the investments, along with maximum bounds on investments on the supply chain network links. The theoretical framework, which includes Lagrange analysis, and the computational approach, are based on the theory of variational inequalities. Managerial insights are provided obtained via the Lagrange analysis and a series of numerical examples, which demonstrate that such investments can help both the firm and the consumers.

Keywords: labor, productivity, investments, supply chain networks, optimization, variational inequalities

1. Introduction

Labor is a critical resource in every supply chain network activity. Without labor, products cannot be produced, transported, stored, and distributed. The COVID-19 pandemic has vividly demonstrated the importance of labor resources, with disruptions to labor, due to illnesses, deaths, the need for social distancing and other mitigation procedures, and even certain labor-related trade measures, affecting the availability of products around the globe as well as product prices (see, e.g., Reiley (2020), Rosane (2020), Russell (2020), Nagurney, Salarpour, and Dong (2022)). Conerly (2021) emphasizes that the supply chain disruptions that have been pervasive in the pandemic are due, primarily, to labor shortages.

There are many relevant issues associated with labor in supply chains, which have been exacerbated in the pandemic. With vaccinations increasing and certain economies rebounding, labor issues continue in the pandemic. Many firms and organizations have had difficulty in attracting workers (cf. Rosenberg (2021), Morath (2021)). And this is not just a United States phenomenon. Weber (2021) emphasizes that the labor shortage that is impacting the U.S. is also coming to Europe, where it could prove even more difficult to repair. The productivity of labor has also decreased in different sectors, with additional studies being warranted (see Bloom et al. (2020)). Firms are trying to identify the wages that should be paid, and whether wages can serve as a mechanism to attract labor during shortages and shortfalls (cf. Sanandanji et al. (2021), Simon (2021)). In the COVID-19 pandemic, major electronic commerce retailers, such as Amazon, have also experienced labor shortages, due, in part, to immense demand for online deliveries and are seeking many new employees (Del Ray (2020)). They are also increasing the wages that they pay (Herrera (2021)).

Hintzmann, Llads-Masllorens, and Ramos (2021) state that many economies had barely recovered from the last crisis of 2007-2008, which was financial in nature, and then were walloped by the COVID-29 pandemic. The authors note that industries in the European Union are suffering, as many have had to shut down or reduce their production as well as to decrease their labor force, while under severe financial pressure. Furthermore, the companies have had to find strategies to survive. In their empirical study, that focused on 18 European countries between 1995 and 2017, the authors are concerned with labor productivity and industrial policy. They find that, among the variables considered significant, investments in advertising and marketing, organizational capital, R&D investment, and design are the ones that contribute individually the most to productivity growth in European manufacturing, whereas computerized information has a complementary effect with other such assets. Stundziene and Saboniene (2019) argue that increases in labor productivity is one of the key drivers of higher welfare in every economy. The goal of their research is to test if the investment in tangible assets enhances labor productivity in the European manufacturing sector. Their results show that, with consideration of all European countries, a 1% increase in gross investment in tangible goods per person employed has a 0.0373% long-run effect on apparent labor productivity.

Garton (2017) notes that there is a positive cycle between productivity and people with higher levels of productivity allowing society to reinvest in human capital, and with proper investments resulting in higher labor productivity. He notes that in the years between 2005 and 2015, labor productivity in the United States, as measured by GDP per labor hour was less than 1% for 7 of the 10 years, according to the OECD. Furthermore, he believes that productivity could be improved if we stopped the underinvestment in human capital. Chaney Cambon (2021) emphasizes that, after a decade of minimal increases in labor productivity, worker productivity might be about to accelerate, as a consequence of pandemic-induced technological adoption, which could raise economic growth and wages in coming years while keeping away inflation pressure. In her article, she highlights a study by McKinsey, wherein approximately 75% of the respondents at North American and European companies to the survey conducted in December 2020 expected to speed up investment in new technology in 2020-24, higher than the 55% who said that they increased such investments in 2014-19. There is a synergistic cycle between productivity and people: Higher levels of productivity allow society to reinvest in human capital and smart investments result in higher labor productivity.

Economists have, historically, included labor, along with capital, in the construction of production functions but their analyses have, typically, not considered supply chains holistically. It is important to capture the latter since local disruptions can permeate much farther afield. Furthermore, given the timeliness, it is critical to also identify the possible benefits of investments in labor productivity. Jorgenson (1991) eloquently argues that investments in productivity of labor can take many forms from investment in tangible assets, which he terms “hardware,” to investment in intangible ones, such as R&D, which he refers to as “software,” as well as investment in human capital through the acquisition of skills and education. And, in the pandemic, investments in health and safety as to work environments, can reduce stress and enhance workers’ productivity (see Igoe (2021)).

2. Literature Review and Contributions

In this paper, we propose a network optimization approach for identifying investments in productivity of labor in any/all supply chain network links. We consider a firm that seeks to determine its profit-maximizing product path flows from production sites to points of demand, along with the labor required, the wages that should be paid the workers, and the optimal investments in labor productivity. The model extends some recent optimization research on the integration of labor into supply chain networks but with a crucial distinction - that of the optimal allocation of investments, subject to a budget constraint and bounds on the link productivity investments. In particular, Nagurney (2021a) constructed a supply chain network optimization model with labor with a focus on a perishable product, specifically, food. Therein there was a bound on labor availability on each of the links. That work was, subsequently, adapted and extended in Nagurney (2021b) to consider several distinct sets of constraints on labor. In the latter paper, both fixed demands for the products, which are relevant in the case of PPEs, for example, as well as elastic demand functions were considered. That work was then extended using game theory in the case of elastic demands to multiple competing firms by Nagurney (2021c). In the supply chain network optimization model introduced in this paper, in contrast, there are no bounds on labor, but labor availability is wage-dependent. This is relevant since many companies are now looking at raising wages in order to attract workers. Furthermore, in contrast to the earlier work, the firm can invest in labor productivity on its supply chain network, subject to a budget constraint and also a maximum on the investment allowable on each link. The latter is important since there may be a maximum that a firm may wish to allocate for productivity enhancement on a link. Plus, there may be a maximum that may be achievable, regardless of the investment, because of human limitations. Several classical models in economics that focus on productivity and growth are highlighted in Stiroh (2001). However, none of these consider a supply chain network perspective. For a recent survey on COVID-19 and supply chains, see Queiroz et al. (2020).

In addition, in this paper, we provide a Lagrange analysis, which yields alternative variational inequality formulations, along with deeper managerial insights. This is the first time that such analysis is conducted in a supply chain network optimization model with labor. One of the variational inequality formulations is then used for computational purposes, since the proposed algorithm provides us with closed form iterations of the product path flows and the Lagrange multiplier associated with the investment budget constraint at each iteration. For Lagrange analysis on other network-based applications, see: Daniele (2001, 2004, 2006), Barbagallo, Daniele, and Maugeri (2012), Toyasaki et al. (2014), Caruso and Daniele (2018), Colajanni et al. (2018), Daniele and Sciacca (2021), and Nagurney and Daniele (2021).

This paper is organized as follows. In Section 3, the supply chain network optimization model is constructed. The model consists of a single firm interested in determining its profit-maximizing optimal product path flows to the demand markets, along with the optimal investments in link productivity. The investments are subject to a budget constraint and upper bounds on each link. The link productivity factors are generalized from those in Nagurney (2021a,b,c) to allow for investments, and relate the number of labor hours on a link to the product output on the link, whether the links are production links, or transportation, storage, or distribution ones. In the optimization model, labor is also wage-dependent, in that the higher the wage, the greater the labor availability. The solution of the model yields the optimal product path and link flows, as well as the optimal link productivity investments, along with the labor hours needed on each link and the wages to be paid the workers. We provide the variational inequality formulation of the optimal solution and then, in Section 4, we conduct Lagrange analysis. The analysis enables the construction of alternative variational inequality formulations, one of which is on the nonnegative orthant, and very amenable to solution via the algorithm we outline in Section 5. Section 4 also makes use of the Lagrange analysis to obtain managerial insights of an economics nature. Section 5 then provides solutions to a series of numerical examples, for which full results are reported. Section 6 summarizes the results in this paper and presents the conclusions.

3. The Labor Productivity Investment Supply Chain Network Model

We now introduce the optimization model for labor productivity investments in supply chain networks. Many firms in different industrial sectors, including agriculture and manufacturing, are dealing with shortfalls in labor and, hence, enhancing labor productivity may be an avenue for increasing product availability.

The optimization model considers a supply chain network topology, as depicted in Figure 1, which can be adapted, depending upon the specific application and circumstances under study. The top node corresponds to the firm, with the subsequent directed links corresponding to production in the first tier; to transportation in the second tier; storage at the distribution centers, and, with the bottom tier of links denoting the distribution to points of demand, which are represented by the bottom nodes: $1, \ldots, n_R$, corresponding to the demand markets.

The supply chain network in Figure 1 is abstracted as the graph $G = [N, L]$, where N denotes the set of nodes and L denotes the set of links. A path p in the supply chain network joins the top-tiered node 1, to a bottom-tiered demand market node. The paths are acyclic and each path consists of a sequence of links representing the supply chain network activities of production, transportation, storage, and distribution to a demand market. A demand market may be a business, an organization, a retailer, or even consumers at their home. We let P_k denote the set of paths, representing alternative supply chain network processes, joining the pair of nodes $(1, k)$, with k denoting a typical demand market node. P then denotes the set of all paths joining node 1 to the demand market nodes. There are n_P paths in the supply chain network and n_L links. We denote a typical link in the supply chain network by a.

The firm is interested in maximizing its profits by identifying its optimal product path flows plus the investments in labor productivity on the links.

The additional notation for the model is given in Table 1. All vectors are assumed to be column vectors.

We now provide the constraints and then construct the objective function representing the profit that the firm wishes to maximize.

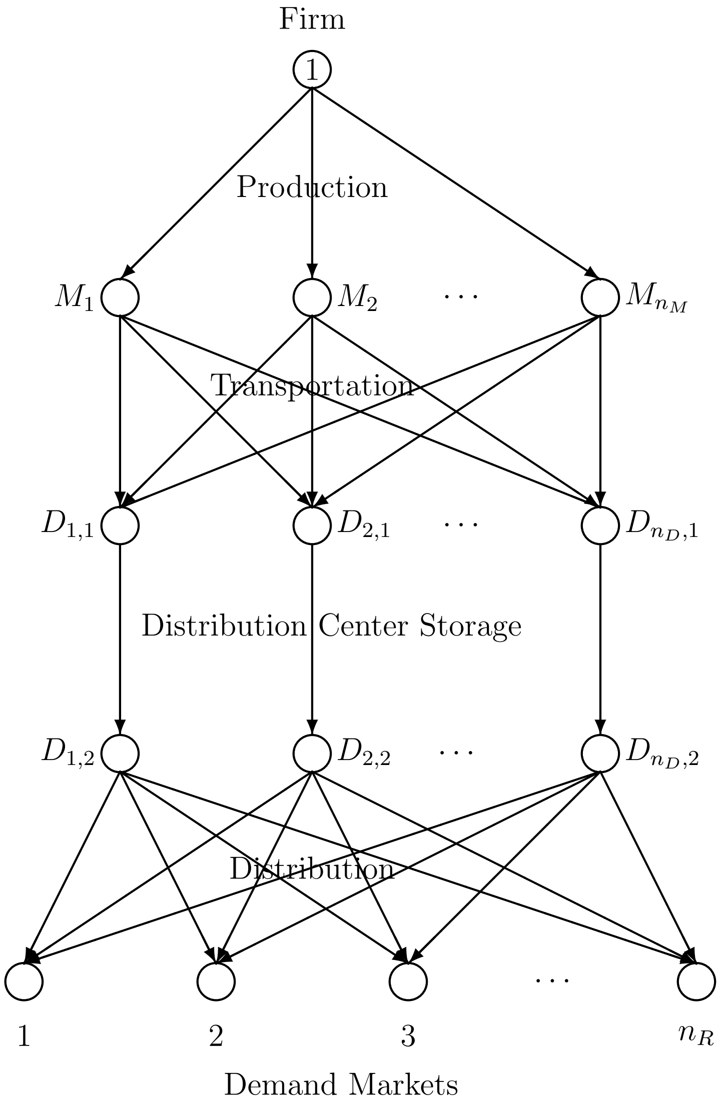

Figure 1: The Supply Chain Network Topology for Optimization of Product Path Flows and Investments in Labor Productivity

Table 1: Notation for the Supply Chain Network Models with Labor

Notation	Variable Definition
x_p	the product flow on path p. We group all the path flows into the vector $x \in R_+^{n_P}$.
f_a	the product flow on link a. We group all the link flows into the vector $f \in R_+^{n_L}$.
l_a	the labor available for link a activity, $\forall a \in L$.
d_k	the demand for the product at demand market k; $k = 1, \dots, n_R$. We group the demands into the vector $d \in R_+^{n_R}$.
v_a	the investment in labor productivity on link a, $\forall a \in L$. We group all the investments in links into the vector $v \in R_+^{n_L}$.
w_a	the (hourly) wage paid for a unit of labor on link a, $\forall a \in L$.
Notation	**Parameter Definition**
γ_a	positive factor relating wage to labor on link a, $\forall a \in L$.
v_a^{max}	the maximum investment possible in labor productivity on link a, $\forall a \in L$.
B	the budget of the firm for labor productivity investments.
Notation	**Function Definition**
$\hat{c}_a(f, v_a)$	the total operational cost associated with link a, excluding the labor cost, $\forall a \in L$.
$\rho_k(d)$	the demand price for the product at demand market k; $k = 1, \dots, n_R$.
$\alpha_a + \beta_a v_a$	the link productivity function relating input of labor to product flow on link a, $\forall a \in L$. We let $\pi_a = \alpha_a + \beta_a v_a, \forall a \in L$.

The path flows must be nonnegative, that is,

$$x_p \geq 0, \quad \forall p \in P. \tag{1}$$

The demand at each demand market must be satisfied by the sum of the product flows of the firm on paths to each demand market, that is,

$$\sum_{p \in P_k} x_p = d_k, \quad k = 1, \dots, n_R. \tag{2}$$

The product flow on each link is equal to the sum of flows on paths that contain that link:

$$f_a = \sum_{p \in P} x_p \delta_{ap}, \quad \forall a \in L, \tag{3}$$

where $\delta_{ap} = 1$, if link a is contained in path p, and is 0, otherwise.

Furthermore, since the product output on a link is equal to the labor input on the link times the productivity on the link, which is no longer fixed as in Nagurney (2021a,b,c), but, rather, is a function of the investment in productivity on the link, we have that:

$$f_a = (\alpha_a + \beta_a v_a) l_a, \quad \forall a \in L, \tag{4}$$

where

$$0 \leq v_a \leq v_a^{max}, \quad \forall a \in L. \tag{5}$$

According to (5), the investment on each link in terms of labor productivity must be nonnegative and cannot exceed the imposed maximum investment desired on a link by the firm. Having the constraints in (5) is important since there may be a maximum achievable productivity for a given link. (4), in turn, is an extension of a linear production function (cf. Mishra (2007)) to include the labor productivity enhancement due to investment. Such an investment can be for education, improvement in hardware or technology, or even software as well as investment in health and safety in the pandemic.

Also, we must have that the firm does not exceed the budget that it has allocated for labor productivity investments. Hence, the following constraint also applies:

$$\sum_{a \in L} v_a \leq B. \tag{6}$$

We emphasize the flexibility of the model in that a firm can invest in any or all of its supply chain network links. The solution of the full supply chain network optimization model will yield which links the firm should invest in and also at what level.

In addition, we assume that the availability of labor is wage-dependent, so that

$$l_a = \gamma_a w_a, \quad \forall a \in L. \tag{7}$$

Note that according to (7), the higher the wage, the greater the labor availability. This is also reasonable, and higher wages are now being used by many companies to attract workers.

The firm seeks to maximize its profit, with the profit denoted by U, being the difference between its revenue and the total cost, with the total cost consisting of the sum of the total operational costs on all the links and the investments in labor productivity on the links plus the total wages paid. Hence, the profit is expressed as:

$$U=\sum_{k=1}^{n_R}\rho_k(d)d_k-\sum_{a\in L}\hat{c}_a(f,v_a)-\sum_{a\in L}v_a-\sum_{a\in L}w_al_a. \tag{8}$$

and the maximization problem is subject to the above constraints.

We now demonstrate that (8) can be expressed solely in terms of product path flows and link labor productivity investments. Indeed, in view of (3), we can define link total operational cost $\tilde{c}_a(x,v_a)\equiv\hat{c}_a(f,v_a)$, for all links $a\in L$. Also, in view of (2), we can define demand price function $\tilde{\rho}_k(x)\equiv\rho_k(d)$, for all k. Also, using (4) and (7) and then (3), we deduce that

$$w_al_a=\left(\frac{\sum_{p\in P}x_p\delta_{ap}}{(\alpha_a+\beta_av_a)}\right)^2\frac{1}{\gamma_a},\quad \forall a\in L. \tag{9}$$

We now rewrite (8) as:

$$U(x,v)=\sum_{k=1}^{n_R}\tilde{\rho}_k(x)\sum_{p\in P_k}x_p-\sum_{a\in L}\tilde{c}_a(x,v_a)-\sum_{a\in L}v_a-\sum_{a\in L}\left(\frac{\sum_{p\in P}x_p\delta_{ap}}{(\alpha_a+\beta_av_a)}\right)^2\frac{1}{\gamma_a}. \tag{10}$$

The firm's goal is to maximize (10) subject to the nonnegativity constraints (1), the budget constraint (6), and the bounds on the investments on the links (5) (since we have embedded constraints (2), (3), (4), and (7) into the objective function). We define the feasible set $K^1\equiv\{(x,v)|\,(1),(5),\text{ and }(6)\text{ hold}\}$. Observe that the feasible set K^1 is convex. Also, we assume that the profit function $U(x,v)$ is continuously differentiable and concave. Then, it follows that an optimal solution to the above network optimization problem coincides with the solution of the following variational inequality problem (cf. Kinderlehrer and Stampacchia (1980) and Nagurney (1999)): determine $(x^*,v^*)\in K^1$ such that

$$-\sum_{p\in P}\frac{\partial U(x^*,v^*)}{\partial x_p}\times(x_p-x_p^*)-\sum_{a\in L}\frac{\partial U(x^*,v^*)}{\partial v_a}\times(v_a-v_a^*)\geq 0,\quad\forall(x,v)\in K^1, \tag{11a}$$

or, equivalently, by expanding out (11a), determine $(x^*,v^*)\in K^1$, such that

$$\sum_{k=1}^{n_R}\sum_{p\in P_k}\left[\frac{\partial\tilde{C}_p(x^*,v^*)}{\partial x_p}+\sum_{a\in L}\frac{2}{\gamma_a}\frac{\sum_{q\in P}x_q^*\delta_{aq}}{(\alpha_a+\beta_av_a^*)(\alpha_a+\beta_av_a^*)}\delta_{ap}-\tilde{\rho}_k(x^*)-\sum_{l=1}^{n_R}\frac{\partial\rho_l(x^*)}{\partial x_p}\sum_{q\in P_l}x_q^*\right]\times\left[x_p-x_p^*\right]$$

$$+\sum_{a\in L}\left[\frac{\partial \tilde{c}_a(x^*,v_a^*)}{\partial v_a}+1-2(\alpha_a+\beta_a v_a^*)^{-3}\frac{\beta_a}{\gamma_a}(\sum_{p\in P}x_p^*\delta_{ap})^2\right]\times[v_a-v_a^*]\geq 0,\quad \forall (x,v)\in K^1, \tag{11b}$$

where

$$\frac{\partial \tilde{C}_p(x,v)}{\partial x_p}\equiv\sum_{a\in L}\sum_{b\in L}\frac{\partial \hat{c}_b(f,v_b)}{\partial f_a}\delta_{ap},\quad \forall p\in P,\quad \text{and}\quad \frac{\partial \tilde{\rho}_l(x)}{\partial x_p}\equiv\frac{\partial \rho_l(d)}{\partial d_k},\forall p\in P_k,\forall k. \tag{12}$$

A solution $(x^*,v^*)\in K^1$ is guaranteed to the above variational inequalities since the feasible set is compact and the underlying functions, under our imposed assumptions, are continuous.

4. Lagrange Analysis and Alternative Variational Inequality Formulations

We now turn to Lagrange analysis, which enables us to construct alternative variational inequality formulations, one of which we will use for computational purposes in the next Section. Lagrange analysis also provides us with deeper insights.

By setting

$$V(x,v)$$

$$=\sum_{k=1}^{n_R}\sum_{p\in P_k}\left[\frac{\partial \tilde{C}_p(x^*,v^*)}{\partial x_p}+\sum_{a\in L}\frac{2}{\gamma_a}\frac{\sum_{q\in P}x_q^*\delta_{aq}}{(\alpha_a+\beta_a v_a^*)(\alpha_a+\beta_a v_a^*)}\delta_{ap}-\tilde{\rho}_k(x^*)-\sum_{l=1}^{n_R}\frac{\partial \rho_l(x^*)}{\partial x_p}\sum_{q\in P_l}x_q^*\right]$$

$$\times\left[x_p-x_p^*\right]+\sum_{a\in L}\left[\frac{\partial \tilde{c}_a(x^*,v_a^*)}{\partial v_a}+1-2(\alpha_a+\beta_a v_a^*)^{-3}\frac{\beta_a}{\gamma_a}(\sum_{p\in P}x_p^*\delta_{ap})^2\right]\times[v_a-v_a^*], \tag{13}$$

variational inequality (11b) can be rewritten as the following minimization problem:

$$\min_{K^1}V(x,v)=V(x^*,v^*)=0. \tag{14}$$

Given the previous assumptions, all the functions in (14) are convex and continuously differentiable.

The constraints are reformulated as below in order to construct the Lagrange function, with the associated Lagrange multipliers stated immediately afterwards.

$$g=\sum_{a\in L}v_a-B\leq 0,\quad \eta,$$

$$h_a^1 = -v_a \le 0, \quad \lambda_a^1, \forall a,$$
$$h_a^2 = v_a - v_a^{max} \le 0, \quad \lambda_a^2, \forall a,$$
$$e_p = -x_p \le 0, \quad \mu_p, \forall p. \tag{15}$$

We now construct the Lagrange function $\mathcal{L}(x, v, \eta, \lambda^1, \lambda^2, \mu)$, where λ^1 and λ^2 are, respectively, the vectors of all the λ_a^1s and λ_a^2s, and μ is the vector of all the μ_ps, as follows:

$$\begin{aligned}
&\mathcal{L}(x, v, \eta, \lambda^1, \lambda^2, \mu) \\
&= \sum_{k=1}^{n_R} \sum_{p \in P_k} \left[\frac{\partial \tilde{C}_p(x^*, v^*)}{\partial x_p} + \sum_{a \in L} \frac{2}{\gamma_a} \frac{\sum_{q \in P} x_q^* \delta_{aq}}{(\alpha_a + \beta_a v_a^*)(\alpha_a + \beta_a v_a^*)} \delta_{ap} - \tilde{\rho}_k(x^*) - \sum_{l=1}^{n_R} \frac{\partial \rho_l(x^*)}{\partial x_p} \sum_{q \in P_l} x_q^* \right] \\
&\times \left[x_p - x_p^* \right] + \sum_{a \in L} \left[\frac{\partial \tilde{c}_a(x^*, v_a^*)}{\partial v_a} + 1 - 2(\alpha_a + \beta_a v_a^*)^{-3} \frac{\beta_a}{\gamma_a} (\sum_{p \in P} x_p^* \delta_{ap})^2 \right] \times [v_a - v_a^*] \\
&+ g\eta + \sum_{a \in L} h_a^1 \lambda_a^1 + \sum_{a \in L} h_a^2 \lambda_a^2 + \sum_{p \in P} e_p \mu_p,
\end{aligned} \tag{17}$$
$$\forall x \in R_+^{n_P}, \forall v \in R_+^{n_L}, \forall \eta \ge 0, \forall \lambda^1 \in R_+^{n_L}, \forall \lambda^2 \in R_+^{n_L}, \forall \mu \in R_+^{n_P}.$$

Since the feasible set K^1 is convex and the Slater condition is satisfied, if (x^*, v^*) is a minimal point of (14), there exist $\eta^* \ge 0$, $\lambda^{1*} \in R_+^{n_L}$, $\lambda^{2*} \in R_+^{n_L}$, $\mu^* \in R_+^{n_P}$, such that the vector $(x^*, v^*, \eta^*, \lambda^{1*}, \lambda^{2*}, \mu^*)$ is a saddle point of the Lagrange function (17):

$$\mathcal{L}(x^*, v^*, \eta, \lambda^1, \lambda^2, \mu) \le \mathcal{L}(x^*, v^*, \eta^*, \lambda^{1*}, \lambda^{2*}, \mu^*) \le \mathcal{L}(x, v, \eta^*, \lambda^{1*}, \lambda^{2*}, \mu^*) \tag{18}$$

and

$$g^* \eta^* = 0,$$
$$h_a^{1*} \lambda_a^{1*} = 0, \quad \forall a \in L,$$
$$h_a^{2*} \lambda_a^{2*} = 0, \quad \forall a \in L,$$
$$e_p^* \mu_p^* = 0, \quad \forall p \in P. \tag{19}$$

From the right-hand side of (18) it follows that $x^* \in R_+^{n_P}$ and $v^* \in R_+^{n_L}$ is a minimal point of $\mathcal{L}(x, v, \eta^*, \lambda^{1*}, \lambda^{2*}, \mu^*)$ in the whole space and therefore, we have that, for all paths $p \in P$:

$$\frac{\partial \mathcal{L}(x^*, v^*, \eta^*, \lambda^{1*}, \lambda^{2*}, \mu^*)}{\partial x_p}$$

$$= \left[\frac{\partial \tilde{C}_p(x^*, v^*)}{\partial x_p} + \sum_{a \in L} \frac{2}{\gamma_a} \frac{\sum_{q \in P} x_q^* \delta_{aq}}{(\alpha_a + \beta_a v_a^*)(\alpha_a + \beta_a v_a^*)} \delta_{ap} - \tilde{\rho}_k(x^*) - \sum_{l=1}^{n_R} \frac{\partial \rho_l(x^*)}{\partial x_p} \sum_{q \in P_l} x_q^* \right] - \mu_p^* = 0. \tag{20}$$

and for all links $a \in L$:

$$\frac{\partial \mathcal{L}(x^*, v^*, \eta^*, \lambda^{1*}, \lambda^{2*}, \mu^*)}{\partial v_a}$$

$$= \left[\frac{\partial \tilde{c}_a(x^*, v_a^*)}{\partial v_a} + 1 - 2(\alpha_a + \beta_a v_a^*)^{-3} \frac{\beta_a}{\gamma_a} (\sum_{p \in P} x_p^* \delta_{ap})^2 \right] + \eta^* - \lambda_a^{1*} + \lambda_a^{2*} = 0, \tag{21}$$

together with conditions (19).

4.1 Alternative Variational Inequality Formulations

The variational inequalities that we now present are defined on the nonnegative orthant, which enables the resolution of our proposed computational procedure into steps yielding closed form expressions in the variables.

Theorem: Alternative Variational Inequality Formulations

Conditions (19), (20), and (21) represent an alternative form of variational inequality (11b) given by: determine $x^* \in R_+^{n_P}, v^* \in R_+^{n_L}, \eta^* \geq 0$, $\lambda^{1*} \in R_+^{n_L}$, $\lambda^{2*} \in R_+^{n_L}$, $\mu^* \in R_+^{n_P}$, such that

$$\sum_{p \in P} \left[\frac{\partial \tilde{C}_p(x^*, v^*)}{\partial x_p} + \sum_{a \in L} \frac{2}{\gamma_a} \frac{\sum_{q \in P} x_q^* \delta_{aq}}{(\alpha_a + \beta_a v_a^*)(\alpha_a + \beta_a v_a^*)} \delta_{ap} - \tilde{\rho}_k(x^*) - \sum_{l=1}^{n_R} \frac{\partial \rho_l(x^*)}{\partial x_p} \sum_{q \in P_l} x_q^* - \mu_p^* \right] \times \left[x_p - x_p^* \right]$$

$$+ \sum_{a \in L} \left[\frac{\partial \tilde{c}_a(x^*, v_a^*)}{\partial v_a} + 1 - 2(\alpha_a + \beta_a v_a^*)^{-3} \frac{\beta_a}{\gamma_a} (\sum_{p \in P} x_p^* \delta_{ap})^2 + \eta^* - \lambda_a^{1*} + \lambda_a^{2*} \right] \times [v_a - v_a^*]$$

$$+ \left[B - \sum_{a \in L} v_a^* \right] \times [\eta - \eta^*] + \sum_{a \in L} v_a^* \times \left[\lambda_a^1 - \lambda_a^{1*} \right] + \sum_{a \in L} [v_a^{max} - v_a^*] \times \left[\lambda_a^2 - \lambda_a^{2*} \right] + \sum_{p \in P} x_p^* \times \left[\mu_p - \mu_p^* \right] \geq 0,$$

$$\forall x \in R_+^{n_P}, v \in R_+^{n_L}, \forall \eta \geq 0, \lambda^1 \in R_+^{n_L}, \lambda^2 \in R_+^{n_L}, \mu \in R_+^{n_P}, \tag{22}$$

or, more simply, determine $x^* \in R_+^{n_P}, v_a^*$, where $0 \leq v_a^* \leq v_a^{max}, \forall a$, and $\eta^* \geq 0$, such that

$$\sum_{p \in P} \left[\frac{\partial \tilde{C}_p(x^*, v^*)}{\partial x_p} + \sum_{a \in L} \frac{2}{\gamma_a} \frac{\sum_{q \in P} x_q^* \delta_{aq}}{(\alpha_a + \beta_a v_a^*)(\alpha_a + \beta_a v_a^*)} \delta_{ap} - \tilde{\rho}_k(x^*) - \sum_{l=1}^{n_R} \frac{\partial \rho_l(x^*)}{\partial x_p} \sum_{q \in P_l} x_q^* \right] \times \left[x_p - x_p^* \right]$$

$$+\sum_{a\in L}\left[\frac{\partial \tilde{c}_a(x^*,v_a^*)}{\partial v_a}+1-2(\alpha_a+\beta_a v_a^*)^{-3}\frac{\beta_a}{\gamma_a}(\sum_{p\in P}x_p^*\delta_{ap})^2+\eta^*\right]\times[v_a-v_a^*]$$

$$+\left[B-\sum_{a\in L}v_a^*\right]\times[\eta-\eta^*]\geq 0,$$

$$\forall x\in R_+^{n_P},\forall v_a,\ \textit{where } 0\leq v_a\leq v_a^{max},\forall a,\forall\eta\geq 0. \tag{23}$$

Proof: It follows directly from (19), (20), (21) that for $x^*\in R_+^{n_P}, v^*\in R_+^{n_L}, \eta^*\geq 0$, $\lambda^{1*}\in R_+^{n_L}$, $\lambda^{2*}\in R_+^{n_L}$, $\mu^*\in R_+^{n_P}$ satisfying those expressions also satisfies variational inequality (22). We now prove that such vectors also satisfy variational inequality (11b).

Multiplying (20) by $(x_p-x_p^*)$, we obtain:

$$\left[\frac{\partial \tilde{C}_p(x^*,v^*)}{\partial x_p}+\sum_{a\in L}\frac{2}{\gamma_a}\frac{\sum_{q\in P}x_q^*\delta_{aq}}{(\alpha_a+\beta_a v_a^*)(\alpha_a+\beta_a v_a^*)}\delta_{ap}-\tilde{\rho}_k(x^*)-\sum_{l=1}^{n_R}\frac{\partial\rho_l(x^*)}{\partial x_p}\sum_{q\in P_l}x_q^*\right]\times(x_p-x_p^*)$$

$$=\mu_p^*\times(x_p-x_p^*), \tag{24}$$

and, since $\mu_p^*x_p\geq 0$, for all p, and $\mu_p^*x_p^*=0$, for all p, summation of the left-hand side of (24) over all paths p, yields:

$$\sum_{p\in P}\left[\frac{\partial \tilde{C}_p(x^*,v^*)}{\partial x_p}+\sum_{a\in L}\frac{2}{\gamma_a}\frac{\sum_{q\in P}x_q^*\delta_{aq}}{(\alpha_a+\beta_a v_a^*)(\alpha_a+\beta_a v_a^*)}\delta_{ap}-\tilde{\rho}_k(x^*)-\sum_{l=1}^{n_R}\frac{\partial\rho_l(x^*)}{\partial x_p}\sum_{q\in P_l}x_q^*\right]$$

$$\times(x_p-x_p^*)\geq 0,\quad \forall x\in R_+^{n_P}. \tag{25}$$

Multiplying (21) by $(v_a-v_a^*)$, in turn, yields:

$$\left[\frac{\partial \tilde{c}_a(x^*,v_a^*)}{\partial v_a}+1-2(\alpha_a+\beta_a v_a^*)^{-3}\frac{\beta_a}{\gamma_a}(\sum_{p\in P}x_p^*\delta_{ap})^2\right]\times(v_a-v_a^*)$$

$$=(-\eta^*+\lambda_a^{1*}-\lambda_a^{2*})\times(v_a-v_a^*). \tag{26}$$

Summation, in turn, over all links $a\in L$ of (26), and the use of (19), gives us:

$$\sum_{a\in L}\left[\frac{\partial \tilde{c}_a(x^*,v_a^*)}{\partial v_a}+1-2(\alpha_a+\beta_a v_a^*)^{-3}\frac{\beta_a}{\gamma_a}(\sum_{p\in P}x_p^*\delta_{ap})^2\right]\times(v_a-v_a^*)$$

$$= -\eta^* \sum_{a \in L} v_a + \eta^* B - \sum_{a \in L} \lambda_a^{2*} v_a + \sum_{a \in L} \lambda_a^{2*} v_a^{max} + \sum_{a \in L} \lambda_a^{1*} v_a - \sum_{a \in L} \lambda_a^{1*} v_a^*. \tag{27}$$

The first two terms on the right-hand side of (27) result in a nonnegative value, as do the second two terms. The next to the final term in (27) is also nonnegative, whereas the last term is equal to zero. Hence, the first conclusion follows.

Furthermore, variational inequality (23) follows from variational inequality (22) since the feasible set underlying the former captures the nonnegativity assumption on the product path flows and on the productivity link investments with the latter not exceeding the respective imposed upper bounds. The proof is complete. □

4.2 Additional Lagrange Analysis with Interpretations

We now utilize the above Lagrange analysis results to obtain deeper insights. Making use of (20), we consider the case where the optimal product flow on a path p, $p \in P_k$, is positive; that is, $x_p^* > 0$, which means that $\mu_p^* = 0$. From (20), we then get:

$$\left[\frac{\partial \tilde{C}_p(x^*, v^*)}{\partial x_p} + \sum_{a \in L} \frac{2}{\gamma_a} \frac{\sum_{q \in P} x_q^* \delta_{aq}}{(\alpha_a + \beta_a v_a^*)(\alpha_a + \beta_a v_a^*)} \delta_{ap} = \tilde{\rho}_k(x^*) + \sum_{l=1}^{n_R} \frac{\partial \rho_l(x^*)}{\partial x_p} \sum_{q \in P_l} x_q^* \right]. \tag{28}$$

Equation (28) has the interpretation that the marginal total costs, which include what we refer to as the marginal total operational cost on a path and the marginal cost associated with labor on the path (see also (9) and (10)), are precisely equal to the marginal revenue. This is a good result in terms of economics.

If the optimal product flow on the path is still positive and if there is a link a on which the optimal investment on the link is neither at its upper bound nor at its lower bound, and the budget is not exhausted, then we know, from (21), that:

$$\left[\frac{\partial \tilde{c}_a(x^*, v^*)}{\partial v_a} + 1 = 2(\alpha_a + \beta_a v_a^*)^{-3} \frac{\beta_a}{\gamma_a} (\sum_{p \in P} x_p^* \delta_{ap})^2 \right]. \tag{29}$$

Expression (29) has the interpretation that the marginal cost associated with investing in the productivity of the link is equal to the marginal return of the investment. On the other hand, if the budget is exhausted, then we can conclude that the marginal cost associated with investing in productivity on the link is greater than the marginal return of the investment and that is not a good situation. The Lagrange multiplier η^* is then greater than zero, and its interpretation as a shadow price reflects how much can be gained in terms of the profit by increasing the budget by a unit. Of course, if the investment on the link is at its upper bound, and the budget is exhausted, and the path has positive flow at optimality, then the marginal investment cost on the link exceeds the marginal return by even a greater amount.

Going back to (20), for completeness, we see that if, on the other hand, the optimal product flow on a path p is zero; that is, $x_p^* = 0$, then the marginal total costs on the path exceed the marginal revenue on the path, so it makes no sense, from a profit-standpoint, to use that path for product flow. We recall that a path consists of: production, transportation, storage, and distribution in our basic framework, The framework, as noted earlier, can be adapted from a supply chain topological standpoint, as need be, and can even incorporate the option of electronic commerce (or direct sales) as investigated in the context of food supply chains by Nagurney (2021a). We demonstrate this feature through numerical examples in the next section.

Also, from (21), one can see that, if the investment on a link a is zero; that is, $v_a^* = 0$, and the budget is not exhausted then the marginal costs associated with investing in the productivity on the link exceeds the marginal return of investing in the link productivity.

5. Computational Procedure and Numerical Examples

All of the above variational inequalities can be put into standard variational inequality form (cf. Nagurney (1999)), where the finite-dimensional variational inequality problem VI$(F, \mathcal{K})$, is to determine a vector $X^* \in \mathcal{K} \subset R^{\mathcal{N}}$, such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{30}$$

where F is a given continuous function from $\mathcal{K}$ to $R^{\mathcal{N}}$, $\mathcal{K}$ is a given closed, convex set, and $\langle \cdot, \cdot \rangle$ denotes the inner product in $\mathcal{N}$-dimensional Euclidean space.

However, we will be solving variational inequality (23), since the algorithm that we propose will resolve the problem into a series of subproblems in the variables, for which we give explicit formulae. Hence, for completeness, we now put variational inequality (23) into standard form (30). We define the vector $X \equiv (x, v, \eta)$ and the vector $F(X) \equiv (F^1(X), F^2(X), F^3(X))$ where the p-th component of $F^1(X)$, $F^1_p(X)$, is:

$$F^1_p(X) = \left[\frac{\partial \tilde{C}_p(x,v)}{\partial x_p} + \sum_{a\in L} \frac{2}{\gamma_a} \frac{\sum_{q\in P} x_q \delta_{aq}}{(\alpha_a + \beta_a v_a)(\alpha_a + \beta_a v_a)} \delta_{ap} - \tilde{\rho}_k(x) - \sum_{l=1}^{n_R} \frac{\partial \rho_l(x)}{\partial x_p} \sum_{q\in P_l} x_q \right];$$

the a-th component of $F^2(X) = \left[\frac{\partial \tilde{c}_a(x,v)}{\partial v_a} + 1 - 2(\alpha_a + \beta_a v_a)^{-3} \frac{\beta_a}{\gamma_a} (\sum_{p\in P} x_p \delta_{ap})^2 + \eta\right]$, and the single component of $F^3(X) = [B - \sum_{a\in L} v_a]$. $\mathcal{N} = n_P + n_L + 1$ and $\mathcal{K} = \{(x, v, \eta) | x \in R^{n_P}_+, 0 \le v_a \le v_a^{max}, \forall a, \eta \ge 0\}$.

5.1 Computational Procedure

The steps of the modified projection method, due to Korpelevich (1977), are stated below, with τ denoting an iteration counter. The algorithm is guaranteed to converge to a solution of variational inequality (30) if $F(X)$ is monotone and Lipschitz continuous, and a solution exists.

Recall that the function $F(X)$ is said to be monotone, if

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle \ge 0, \quad \forall X^1, X^2 \in \mathcal{K}, \tag{31}$$

and the function $F(X)$ is Lipschitz continuous, if there exists a constant $\bar{L} > 0$, known as the Lipschitz constant, such that

$$\|F(X^1) - F(X^2)\| \le \bar{L} \|X^1 - X^2\|, \quad \forall X^1, X^2 \in \mathcal{K}. \tag{32}$$

The Modified Projection Method

Step 0: Initialization

Initialize with $X^0 \in \mathcal{K}$. Set the iteration counter $\tau := 1$ and let ζ be a scalar such that $0 < \zeta \le \frac{1}{\bar{L}}$, where $\bar{L}$ is the Lipschitz constant.

Step 1: Computation

Compute $\bar{X}^\tau$ by solving the variational inequality subproblem:

$$\langle \bar{X}^\tau + \zeta F(X^{\tau-1}) - X^{\tau-1}, X - \bar{X}^\tau \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{33}$$

Step 2: Adaptation

Compute X^τ by solving the variational inequality subproblem:

$$\langle X^\tau + \zeta F(\bar{X}^\tau) - X^{\tau-1}, X - X^\tau \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{34}$$

Step 3: Convergence Verification

If $|X^\tau - X^{\tau-1}| \leq \epsilon$, with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $\tau := \tau + 1$ and go to Step 1.

The explicit formulae for all the variables for our model for Step 1 above are now provided. The analogues of Step 2 easily follow.

Explicit Formulae at Iteration τ for the Product Path Flows in Step 1

Specifically, we have the following closed form expressions for the path flows in Step 1 in the solution of variational inequality (23):

$$\bar{x}_p^\tau = \max\{0, x_p^{\tau-1} + \zeta(\tilde{\rho}_k(x^{\tau-1}) + \sum_{l=1}^{n_R} \frac{\partial \tilde{\rho}_l(x^{\tau-1})}{\partial x_p} \sum_{q \in P_l} x_q^{\tau-1} - \frac{\partial \tilde{C}_p(x^{\tau-1}, v^{\tau-1})}{\partial x_p}$$

$$- \sum_{a \in L} \frac{2}{\gamma_a} \frac{\sum_{q \in P} x_q^{\tau-1} \delta_{aq}}{(\alpha_a + \beta_a v_a)(\alpha_a + \beta_a v_a)} \delta_{ap})\}, \quad \forall p \in P_k; k = 1, \ldots, n_R. \tag{35}$$

Explicit Formulae at Iteration τ for the Link Productivity Investments Flows in Step 1

Also, we have the following closed form expressions for the link productivity investments in Step 1 in the solution of variational inequality (23):

$$\bar{v}_a^\tau = \max\{0, \min\{v_a^{\tau-1} + \zeta(2(\alpha_a + \beta_a v_a^{\tau-1})^{-3} \frac{\beta_a}{\gamma_a} (\sum_{p \in P} x_p^{\tau-1} \delta_{ap})^2 - \frac{\partial \tilde{c}_a(x^{\tau-1}, v_a^{\tau-1})}{\partial v_a} - 1 - \eta^{\tau-1}, v_a^{max}\},$$

$$\forall a \in L. \tag{36}$$

Explicit Formula at Iteration τ for the Lagrange Multiplier in Step 1

Finally, we have the following closed form expression for the Lagrange multiplier in Step 1 at an iteration τ:

$$\bar{\eta}^{\tau} = \max\{0, \{\eta^{\tau-1} + \zeta(\sum_{a \in L} v_a^{\tau-1}\delta_{ap} - B)\}. \tag{37}$$

It is straightforward to adapt the above closed form expressions for the special case of the model without link investment upper bounds and/or without a budget constraint.

5.2 Numerical Examples

The modified projection method was implemented in FORTRAN and a Linux system at the University of Massachusetts Amherst used for computing solutions to the subsequent numerical examples. The computational procedure was initialized as follows. All the link investments and the Lagrange multiplier associated with the budget constraint were set to 0.00. The initial demand at each market was set to 40 with the demand equally distributed among the paths terminating in each demand market. The convergence tolerance was 10^{-7}; that is, the algorithm was considered to have converged when the absolute value of the difference between each of the variables at two successive iterations differed by no more than this value. The parameter ζ was set to .01 for each of the numerical examples. The numerical examples are used as a "proof of concept" and are not focused on a specific application but, nevertheless, yield broader insights. Specific applications, with particular features, and parameterized accordingly, can be readily solved using the algorithm.

5.2.1 Examples 1, 2, and 3

The supply chain network topology for Examples 1, 2, and 3 is given in Figure 2. The firm has two production sites, one distribution center for storage, and sells its product at two demand markets. Example 1 assumes that there is no investment possible (and, hence, there is no investment budget and no bounds on the link productivity investments. Example 2 then has the identical data to the data in Example 2 but with the investments added and the beta values as given under Example 2 below. Example 3, in turn, has the identical data to the data in Example 2 but with the addition of bounds on the link productivity investments as well as a budget.

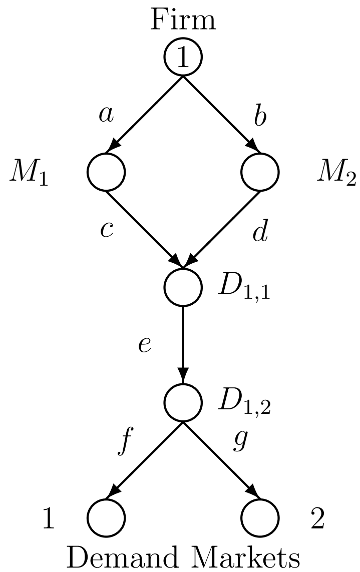

Figure 2: Supply Chain Network Topology for Examples 1, 2, and 3

Example 1 – No Investment Parameters Beta, No Budget Constraint, and No Bounds on Link Productivity Investments

The total operational link cost functions (since we do not consider investments until the next examples) are:

$$\hat{c}_a(f) = 2f_a^2, \quad \hat{c}_b(f) = 2f_b^2, \quad \hat{c}_c(f) = .5f_c^2, \quad \hat{c}_d(f) = .5f_d^2,$$

$$\hat{c}_e(f) = f_e^2 + 2f_e, \quad \hat{c}_f(f) = .5f_f^2, \quad \hat{c}_g(f) = .5f_g^2.$$

The demand price functions are:

$$\rho_1(d) = -5d_1 + 800, \quad \rho_2(d) = -5d_2 + 850.$$

The alpha link parameters are:

$$\alpha_a = 55, \quad \alpha_b = 50, \quad \alpha_c = 35, \quad \alpha_d = 35, \quad \alpha_e = 60, \quad \alpha_f = 38, \quad \alpha_g = 36,$$

the beta link parameters are:

$$\beta_a = 0, \quad \beta_b = 0, \quad \beta_c = 0, \quad \beta_d = 0, \quad \beta_e = 0, \quad \beta_f = 0, \quad \beta_g = 0,$$

and the gamma link parameters are:

$$\gamma_a = .1, \quad \gamma_b = .1, \quad \gamma_c = .09, \quad \gamma_d = .07, \quad \gamma_e = .08 \quad \gamma_f = .06, \quad \gamma_g = .08.$$

The paths are defined as: path $p_1 = (a, c, e, f)$, path $p_2 = (b, d, e, f)$, path $p_3 = (a, c, e, g)$, and path $p_4 = (b, d, e, g)$.

The modified projection method yields the following equilibrium product path flow pattern:

$$x^*_{p_1} = 19.39, \quad x^*_{p_2} = 19.36, \quad x^*_{p_3} = 21.66, \quad x^*_{p_4} = 21.63.$$

The equilibrium link flows and labor values are reported in Table 2, whereas the equilibrium productivity investments and hourly wages are reported in Table 3.

The demand price at the first demand market is 606.27 and at the second demand market the price is: 633.52, with the corresponding equilibrium demands of: 38.75 and 43.30.

The firm earns a profit of: 33,816.98.

Example 2 – Positive Investment Parameters Beta, No Budget Constraint, and No Bounds on Link Productivity Investments

Example 2 has the same data as Example 1, but now we include investments (but no bounds).

The total operational costs are as in Example 1 in terms of the link flow dependence, but they are now extended to have an investment component as follows:

$$\hat{c}_a(f, v_a) = 2f_a^2 + .05v_a^2, \quad \hat{c}_b(f, v_b) = 2f_b^2 + .1v_b^2, \quad \hat{c}_c(f, v_c) = .5f_c^2 + .05v_c^2, \quad \hat{c}_d(f, v_d) = .5f_d^2 + .05v_d^2,$$

$$\hat{c}_e(f, v_e) = f_e^2 + 2f_e + .1v_e^2, \quad \hat{c}_f(f, v_f) = .5f_f^2 + .1v_f^2, \quad \hat{c}_g(f, v_g) = .5f_g^2 + .1v_g^2.$$

The βs are no longer equal to zero, as they were in Example 1, but are, now, as given below:

$$\beta_a = 10, \quad \beta_b = 10, \quad \beta_c = 20, \quad \beta_d = 20, \quad \beta_e = 10, \quad \beta_f = 10, \quad \beta_g = 10.$$

The link productivity functions (cf. Table 1) are, thus, of the form $\pi_a = \alpha_a + \beta_a v_a$, for all links $a \in L$ with the *alpha* terms as in Example 1.

The modified projection method yields the following equilibrium product path flow pattern:

$$x^*_{p_1} = 19.38, \quad x^*_{p_2} = 19.36, \quad x^*_{p_3} = 21.66, \quad x^*_{p_4} = 21.63.$$

The equilibrium link flows and labor values for Example 2 are reported in Table 2, whereas the equilibrium productivity investments and hourly wages are reported in Table 3.

The demand price at the first demand market is 606.28 and at the second demand market the price is: 633.54, with the corresponding equilibrium demands of: 38.74 and 43.29.

The firm earns a profit of: 33,868.11. The total investment outlay of the firm is: 17.72. Note that there is a good return on investment, since the profit in Example 1 is 33,816.98, whereas now the profit is: 33,868.11.

We also, for completeness, report the values of the link productivity functions at the equilibrium:

$$\pi_a = 67.00, \quad \pi_b = 64.03, \quad \pi_c = 84.36, \quad \pi_d = 90.88, \quad \pi_e = 98.37, \quad \pi_f = 67.90, \quad \pi_g = 66.32.$$

Note that since the βs are all equal to zero in Example 1, the π_as there are not investment-dependent and collapse to the corresponding α on the link.

The labor hours needed on each link decrease, as compared to the respective result in Example 1, and the wage on each link also decreases.

Example 2 demonstrates the benefits for the firm of investing in link productivity. Note that the total operational link cost does depend on the investment on the link since there may be, for example, some maintenance and other related costs associated with the investment encumbered.

Example 3 – Positive Investment Parameters Beta, Budget Constraint, and Bounds on Link Productivity Investments

The data for Example 3 are as that in Example 2 with the budget $B = 15$ and $v_a^{max} = \ldots = v_g^{max} = 3$. The path flows remain essentially as in Example 2 since the demand price functions are not functions of the investments. The equilibrium link flows and labor values are reported in Table 2 with the equilibrium link productivity investments and the hourly wages reported in Table 3. Wages are now higher, in order to attract labor since more is needed due to a decrease in productivity as compared to that in Example 2. The investment on link e is at the upper bound. The total investment outlay is now 15, so the budget is exhausted with the Lagrange multiplier being positive and with a value of $\eta^* = .29$.

The link productivity functions evaluated at the computed equilibrium link investments are now:

$$\pi_a = 62.70, \quad \pi_b = 60.69, \quad \pi_c = 79.14, \quad \pi_d = 85.41, \quad \pi_e = 90.00, \quad \pi_f = 64.93, \quad \pi_g = 63.41.$$

The profit of the firm is: 33,867.59. Under the imposed budget constraint and the maximum bounds on investment links, the profit of the firm decreases from 33,868.11 in Example 2 to 33,867.59 in Example 3.

Notation	Equilibrium Value		
	Example 1	**Example 2**	**Example 3**
f_a^*	41.05	41.04	41.05
f_b^*	40.99	41.00	41.01
f_c^*	41.05	41.04	41.05
f_d^*	40.99	41.00	41.01
f_e^*	82.04	82.04	82.06
f_f^*	38.75	38.74	38.75
f_g^*	43.30	43.29	43.30
l_a^*	0.75	0.61	0.65
l_b^*	0.82	0.64	0.68
l_c^*	1.17	0.49	0.52
l_d^*	1.17	0.45	0.48
l_e^*	1.37	0.83	0.91
l_f^*	1.02	0.57	0.60
l_g^*	1.20	0.65	0.68

Table 2: Equilibrium Link Flows and Labor Values for Examples 1, 2, 3

Notation	Equilibrium Value		
	Example 1	**Example 2**	**Example 3**
v_a^*	–	1.20	0.77
v_b^*	–	1.40	1.07
v_c^*	–	2.47	2.21
v_d^*	–	2.79	2.52
v_e^*	–	3.84	3.00
v_f^*	–	2.99	2.69
v_g^*	–	3.03	2.74
w_a^*	7.46	6.13	6.55
w_b^*	8.20	6.40	6.76
w_c^*	13.03	5.41	5.76
w_d^*	16.73	6.44	6.86
w_e^*	17.09	10.42	11.40
w_f^*	16.99	9.51	9.95
w_g^*	15.03	8.16	8.54

Table 3: Equilibrium Link Productivity Investments and Hourly Wages for Examples 1, 2, and 3

5.2.2: Examples 4, 5, and 6

In the second series of examples we consider a supply chain network topology as depicted in Figure 3. There is now an additional production site available to the firm, but it still has a single distribution center and serves two demand markets. These examples follow a similar pattern to that of Examples 1, 2, and 3. Example 1 considers no investments so the βs are all zero. Example 7 then allows for investment but has no budget and no upper bounds on the link productivity investments and, finally, Example 9 introduces both a budget and investment bounds on the links.

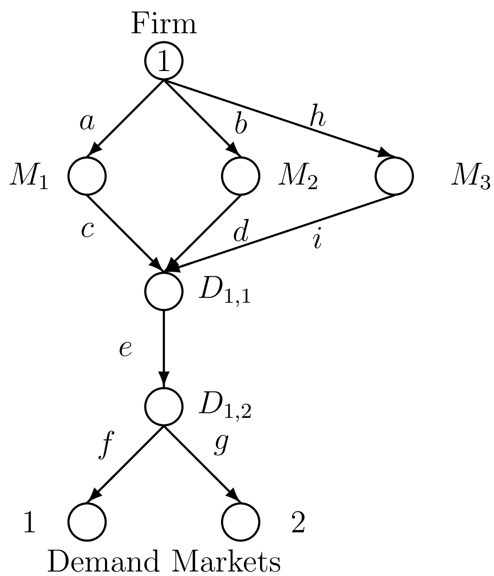

Figure 3: Supply Chain Network Topology for Examples 4, 5, and 6

Example 4: No Investment Parameters Beta, No Budget Constraint, and No Bounds on Link Productivity Investments

Example 4 has the identical data to that in Example 1 but with the additional data for links h and i as follows:

$$\hat{c}_h(f, v_h) = f_h^2 + .05v_h^2, \quad \hat{c}_i(f, v_i) = .5f_i^2 + .05v_i^2,$$

$$\alpha_h = 45, \quad \alpha_i = 30, \quad \gamma_h = .1, \quad \gamma_i = .08,$$

and

$$\beta_h = 0, \quad \beta_i = 0,$$

since in this example, as we did in Example 1, we consider the case that there are no investments in link productivity.

We also have two new paths: path $p_5 = (h, i, e, f)$, and path $p_6 = (h, i, e, g)$.

The modified projection method converges to the following equilibrium product path flow pattern:

$$x^*_{p_1} = 11.91, \quad x^*_{p_2} = 11.89, \quad x^*_{p_3} = 20.20, \quad x^*_{p_4} = 13.43,$$
$$x^*_{p_5} = 13.41, \quad x^*_{p_6} = 21.77.$$

The original paths have much lower volumes of product flow than they had in Example 1, with the two new paths having the largest volume of product flow. The profit is: 38,138.79. The profit increases substantially with the introduction of a new production site, by more than 4,000.

The demand price at the first demand market is 580.02 and at the second demand market the price is: 607.26, with the corresponding equilibrium demands of: 44.00 and 48.55. Consumers also benefit since the prices at the demand markets decrease. The computed equilibrium link flows and labor values are given in Table 4 and the equilibrium link productivity investments and the hourly wages are reported in Table 5.

Example 5 – Positive Investment Parameters Beta, No Budget Constraint, and No Bounds on Link Productivity Investments

Example 5 has the same data as Example 4 with the beta parameters now being positive and as in Example 2, with the addition of the following ones on the added two links:

$$\beta_h = 15, \quad \beta_i = 15.$$

As in Example 2, there are no bounds on the link productivity investments and no budget.

The modified projection method now converges to the following equilibrium product path flow pattern:

$$x^*_{p_1} = 11.88, \quad x^*_{p_2} = 11.87, \quad x^*_{p_3} = 20.24, \quad x^*_{p_4} = 13.40,$$

$$x^*_{p_5} = 13.39, \quad x^*_{p_6} = 21.76.$$

The additional equilibrium results are reported in Tables 4 and 5.

The firm invests a total amount of: 19.74.

The demand price at the first demand market is: 580.01 and at the second: 607.26 with equilibrium demands of: 44.00 and 48.55, respectively.

The firm earns a profit of 38,202.74. The firm gains in profit by investing in productivity in the supply chain links. In Example 4, the profit is 38,138.79, whereas now, with an investment of only 19.74, the profit has risen to: 38,202.74.

The values of the link productivity functions at the equilibrium, where recall that $\pi_a = \alpha_a + \beta_a v_a$, $\forall a \in L$, are:

$$\pi_a = 55.00, \quad \pi_b = 50.26, \quad \pi_c = 62.93, \quad \pi_d = 67.91, \quad \pi_e = 104.32, \quad \pi_f = 72.54, \quad \pi_g = 70.41,$$

$$\pi_h = 75.98, \quad \pi_i = 79.26.$$

Example 6 – Positive Investment Parameters Beta, Budget Constraint, and Bounds on Link Productivity Investments

Example 6 is constructed from Example 5 and has the same data but with the addition of the same budget and link investment bounds as in Example 3. Hence, the budget is 15 and all the v^{max}s are equal to 3, including on the two added links.

The modified projection method now converges to the following equilibrium product path flow pattern:

$$x^*_{p_1} = 11.88, \quad x^*_{p_2} = 11.88, \quad x^*_{p_3} = 20.25, \quad x^*_{p_4} = 13.40,$$

$$x^*_{p_5} = 13.39, \quad x^*_{p_6} = 21.77.$$

The additional equilibrium results are reported in Tables 4 and 5.

The firm invests a total amount of: 15.00 and the Lagrange multiplier $\eta^* = .64$.

Notation	Equilibrium Value		
	Example 4	**Example 5**	**Example 6**
f_a^*	25.33	25.28	25.29
f_b^*	25.30	25.26	25.27
f_c^*	25.33	25.28	25.29
f_d^*	25.30	25.26	25.27
f_e^*	92.54	92.55	92.58
f_f^*	44.00	44.00	44.02
f_g^*	48.55	48.55	48.56
f_h^*	41.91	42.00	42.02
f_i^*	41.91	42.00	42.02
l_a^*	0.46	0.46	0.46
l_b^*	0.51	0.50	0.51
l_c^*	0.72	0.40	0.46
l_d^*	0.72	0.37	0.43
l_e^*	1.54	0.89	1.03
l_f^*	1.16	0.61	0.66
l_g^*	1.35	0.69	0.75
l_h^*	0.93	0.55	0.63
l_i^*	1.40	0.53	0.60

Table 4: Equilibrium Link Flows and Labor Values for Examples 4, 5, and 6

The demand price at the first demand market is: 579.92 and at the second: 607.18 with equilibrium demands of: 44.02 and 48.56, respectively.

The firm now earns a profit of: 38,200.93, a decrease, but not a significant one, from the profit in Example 5.

The values of the link productivity functions at the equilibrium, where recall that $\pi_a = \alpha_a + \beta_a v_a$, $\forall a \in L$, are:

$$\pi_a = 55.00, \quad \pi_b = 50.00, \quad \pi_c = 54.67, \quad \pi_d = 59.17, \quad \pi_e = 90.00, \quad \pi_f = 66.38, \quad \pi_g = 64.39,$$

$$\pi_h = 66.70, \quad \pi_i = 70.26.$$

The investment on link a is at the imposed upper bound.

Notation	Equilibrium Value		
	Example 4	**Example 5**	**Example 6**
v_a^*	–	0.00	0.00
v_b^*	–	0.03	0.00
v_c^*	–	1.40	0.98
v_d^*	–	1.65	1.21
v_e^*	–	4.43	3.00
v_f^*	–	3.45	2.84
v_g^*	–	3.44	2.84
v_h^*	–	2.07	1.45
v_i^*	–	3.28	2.68
w_a^*	4.61	4.60	4.60
w_b^*	5.06	5.03	5.05
w_c^*	8.04	4.46	5.14
w_d^*	10.33	5.31	6.10
w_e^*	19.28	11.09	12.86
w_f^*	19.30	10.11	11.05
w_g^*	16.86	8.62	9.43
w_h^*	9.31	5.53	6.30
w_i^*	17.46	6.62	7.48

Table 5: Equilibrium Link Productivity Investments and Hourly Wages for Examples 4, 5, and 6

5.2.3: Examples 7, 8, and 9: Introduction of E-Commerce

In our third, and final series of numerical examples, we consider the impact of electronic commerce. Specifically, to the supply chain network topology in Figure 3 we now add direct links j and k to demand markets 1 and 2, respectively, from nodes M_1 and M_3 and consider numerical examples with the supply chain network topology in Figure 4.

Example 7 - Positive Investment Parameters Beta, No Budget Constraint, and No Bounds on Link Productivity Investments

From the previous numerical examples, it was clear that allowing for investments in link productivity could raise profits of the firm. Example 7 serves as the baseline from which we then construct in this series Examples 8 and 9. Hence, Example 7 (unlike Examples 1 and 4) has positive beta parameters on all of its links.

Example 7 has the identical data to that in Example 5 but with the additional data for links j and k as follows:

$$\hat{c}_j(f, v_j) = 1.5f_j^2 + .05v_j^2, \quad \hat{c}_k(f, v_k) = 2f_k^2 + .1v_k^2,$$

$$\alpha_j = 55, \quad \alpha_k = 60, \quad \gamma_j = .1, \quad \gamma_k = .1,$$

and

$$\beta_j = 20, \quad \beta_k = 20.$$

We also have two new paths: path $p_7 = (a, j)$, and path $p_6 = (h, k)$.

There is no budget and no v_a^{max} on all links $a \in L$.

The modified projection method converges to the following equilibrium product path flow pattern:

$$x_{p_1}^* = 0.00, \quad x_{p_2}^* = 11.80, \quad x_{p_3}^* = 8.77, \quad x_{p_4}^* = 0.00,$$

$$x_{p_5}^* = 14.09, \quad x_{p_6}^* = 11.06, \quad x_{p_7}^* = 34.93, \quad x_{p_8}^* = 34.88.$$

The additional equilibrium results are reported in Tables 6 and 7.

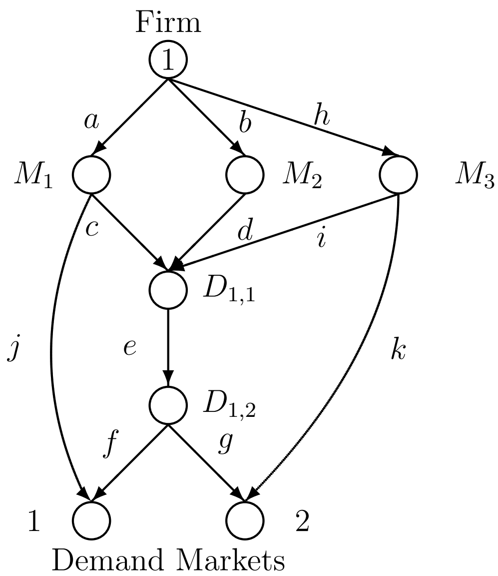

Figure 4: Supply Chain Network Topology for Examples 7, 8, and 9

One can see that the paths with the electronic commerce links have the highest product path flows and that both paths p_1 and p_4 are not even used (in contrast to the results in Example 5) and, hence, have zero flow.

The demand price at the first demand market is: 522.51 and at the second: 549.82 with equilibrium demands of: 55.50 and 60.04, respectively.

The firm earns a profit of: 47,685.11. The profit is higher than in Example 5 by more than 8,000. Electronic commerce benefits the firm in terms of profit and consumers, in terms of demand market prices, which are now lower.

The values of the link productivity functions at the equilibrium, where recall that $\pi_a = \alpha_a + \beta_a v_a$, $\forall a \in L$, are:

$$\pi_a = 61.25, \quad \pi_b = 50.89, \quad \pi_c = 35.00, \quad \pi_d = 68.93, \quad \pi_e = 74.14, \quad \pi_f = 48.77, \quad \pi_g = 49.85,$$

$$\pi_h = 86.81, \quad \pi_i = 50.61, \quad \pi_j = 76.13, \quad \pi_k = 75.06.$$

Example 8 – Positive Investment Parameters Beta, Budget Constraint, Bounds on Link Productivity Investments, and Demand Price Function Changes

In Example 8, we retain the budget of 15, as in earlier examples with a budget, and also we have that $v_a^{max} = 3$ for all links that are not e-commerce links (as in previous examples with bounds) but we now add the following bounds on the electronic commerce links:

$$v_j^{max} = 4, \quad v_k^{max} = 4.$$

Also, we now consider the case that consumers at the demand markets are grateful to for the e-commerce option and are willing to pay higher prices. The demand price function intercept terms are changed from 800 to 850 for the first demand market and from 850 to 900 for the second demand market.

The modified projection method now converges to the following equilibrium product path flow pattern:

$$x_{p_1}^* = 0.00, \quad x_{p_2}^* = 12.57, \quad x_{p_3}^* = 9.37, \quad x_{p_4}^* = 0.00,$$

$$x_{p_5}^* = 14.89, \quad x_{p_6}^* = 11.69, \quad x_{p_7}^* = 37.06, \quad x_{p_8}^* = 36.96.$$

Additional equilibrium results are reported in Tables 6 and 7.

One can see that the paths with the electronic commerce links, as in Example 7, have the highest product path flows and that both paths p_1 and p_4 have zero flow, as they did in Example 7.

The demand price at the first demand market is: 554.97 and at the second: 582.31 with equilibrium demands of: 59.01 and 63.54, respectively.

The firm earns a profit of: 53,640.90, which is higher than the profit in Example 7. This shows the potential benefit of having consumers being willing to pay higher prices and firms can achieve this through marketing, for example.

The values of the link productivity functions at the equilibrium are:

$\pi_a = 63.30, \quad \pi_b = 52.40, \quad \pi_c = 35.00, \quad \pi_d = 71.43, \quad \pi_e = 76.29, \quad \pi_f = 50.46, \quad \pi_g = 51.33,$

$$\pi_h = 90.00, \quad \pi_i = 52.50, \quad \pi_j = 78.88, \quad \pi_k = 77.48.$$

The total investments are: 13.87. The investment on link h is at the upper bound of 3.

Example 9 – Positive Investment Parameters Beta, Budget Constraint, Bounds on Link Productivity Investments, and Increase in Production Costs

Example 9 has the same data as Example 8 but we now consider a production disruption with the total costs on links a and b increasing to the following, respectively:

$$\hat{c}_a(f, v_a) = 3f_a^2 + .05v_a^2, \quad \hat{c}_b(f, v_b) = 3f_b^2 + .1v_b^2.$$

The modified projection method now converges to the following equilibrium product path flow pattern:

$$x_{p_1}^* = 0.00, \quad x_{p_2}^* = 11.51, \quad x_{p_3}^* = 13.70, \quad x_{p_4}^* = 0.00,$$

$$x_{p_5}^* = 10.83, \quad x_{p_6}^* = 13.02, \quad x_{p_7}^* = 31.44, \quad x_{p_8}^* = 37.96.$$

The additional equilibrium results are reported in Tables 6 and 7.

The demand price at the first demand market is: 566.71 and at the second: 590.97 with equilibrium demands of: 56.66 and 61.81, respectively.

The firm earns a profit of: 51,863.57. With higher costs at two of the three production sites, the profit now decreases.

The values of the link productivity functions at the equilibrium, where recall that $\pi_a = \alpha_a + \beta_a v_a$, $\forall a \in L$, are:

$$\pi_a = 57.74, \quad \pi_b = 50.00, \quad \pi_c = 35.00, \quad \pi_d = 63.02, \quad \pi_e = 76.70, \quad \pi_f = 50.28, \quad \pi_g = 48.46,$$

$$\pi_h = 90.00, \quad \pi_i = 60.58, \quad \pi_j = 71.49, \quad \pi_k = 78.61.$$

The total investments are: 13.01. The investment on link h remains at the upper bound of 3.

Notation	Equilibrium Value		
	Example 7	**Example 8**	**Example 9**
f_a^*	34.93	37.06	3‘.44
f_b^*	25.89	27.46	22.35
f_c^*	0.00	0.00	0.00
f_d^*	25.89	27.46	22.35
f_e^*	45.72	48.52	49.07
f_f^*	20.56	21.94	25.22
f_g^*	25.16	26.58	23.85
f_h^*	54.71	58.02	64.67
f_i^*	19.83	21.06	26.72
f_j^*	34.93	37.06	31.44
f_k^*	34.88	36.95	37.96
l_a^*	0.57	0.59	0.54
l_b^*	0.51	0.52	0.45
l_c^*	0.00	0.00	0.00
l_d^*	0.38	0.38	0.35
l_e^*	0.62	0.64	0.64
l_f^*	0.42	0.42	0.46
l_g^*	0.50	0.52	0.49
l_h^*	0.62	0.64	0.72
l_i^*	0.39	0.40	0.44
l_j^*	0.46	0.47	0.44
l_k^*	0.46	0.48	0.48

Table 6: Equilibrium Link Flows and Labor Values for Examples 7, 8, and 9

Notation	**Equilibrium Value**		
	Example 7	**Example 8**	**Example 9**
v_a^*	0.62	0.83	0.27
v_b^*	0.09	0.24	0.00
v_c^*	0.00	0.00	0.00
v_d^*	1.70	1.82	1.40
v_e^*	1.41	1.63	1.67
v_f^*	1.08	1.25	1.63
v_g^*	1.39	1.53	1.25
v_h^*	2.91	3.00	3.00
v_i^*	1.37	1.50	2.04
v_j^*	1.06	1.19	0.82
v_k^*	0.75	0.87	0.93
w_a^*	5.70	5.85	5.45
w_b^*	5.09	5.24	4.47
w_c^*	0.00	0.00	0.00
w_d^*	5.37	5.49	5.07
w_e^*	7.71	7.95	8.00
w_f^*	7.03	7.25	7.74
w_g^*	6.31	6.47	6.15
w_h^*	6.17	6.45	7.19
w_i^*	4.0	5.02	5.51
w_j^*	4.59	4.70	4.40
w_k^*	4.65	4.77	4.83

Table 7: Equilibrium Link Productivity Investments and Hourly Wages for Examples 7, 8, and 9

6. Summary and Conclusions

Disruptions to labor in the COVID-19 pandemic have been deep and widespread, affecting agriculture and manufacturing as well as freight service provision, among many other economic sectors. And now, with progress on vaccinations, many firms are dealing with difficulty in attracting labor, as economies start to rebound. The pandemic has also affected the productivity of labor with investments in productivity being a possible avenue to enable enhanced output.

In this paper, we construct an optimization model for supply chain networks that includes labor as a resource, allows for wage-dependent labor, as well as investments in supply chain link productivity. The investments are subject to a budget constraint and also to a bound on the investment on each link. The solution of the model yields the optimal product path flows from the firm to the demand markets, the optimal link productivity investments, as well as the labor hours needed on the supply chain network links, and the wages that should be paid to the workers on the links, which consist of production, transportation, storage, and distribution links.

We provide alternative variational inequality formulations of the optimal solution, and conduct Lagrange analysis. The proposed algorithmic scheme has nice features for implementation since it resolves the variational inequality formulation that we utilize into subproblems which yield closed form expressions in the product path flows and the Lagrange multiplier associated with the budget constraint. We apply the algorithm to compute solutions to three sets of numerical examples, including a set with electronic commerce, and report the full solutions, including the labor hours and the wages that should be paid. We find that investments of link productivity can enhance profits for the firm, and reduce the product price at the demand markets for the consumers. Also, adding a production site can enhance profits as well as taking advantage of electronic commerce. Firms, however, should be careful in moderating their operational costs since increases can have a big impact on the bottom line.

This work adds to the still nascent literature on the synthesis of operations research and economics for addressing fundamental questions and issues surrounding labor and supply chains.

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