Dynamics of supply chain networks with corporate social responsibility through integrated environmental decision-making

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Abstract

In this paper, we develop a dynamic framework for the modeling and analysis of supply chain networks with corporate social responsibility through integrated environmental decision-making. Through a multilevel supply chain network, we model the multicriteria decision-making behavior of the various decision-makers (manufacturers, retailers, and consumers), which includes the maximization of profit, the minimization of emission (waste), and the minimization of risk. We explore the dynamic evolution of the product flows, the associated product prices, as well as the levels of social responsibility activities on the network until an equilibrium pattern is achieved. We provide some qualitative properties of the dynamic trajectories, under suitable assumptions, and propose a discrete-time algorithm which is then applied to track the evolution of the levels of social responsibility activities, product flows and prices over time. We illustrate the model and computational procedure with several numerical examples.

Keywords: Supply chains network; Environment; Risk management; Multicriteria optimization; Variational inequalities

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1. Introduction

Corporate social responsibility (CSR) has been a theme of many researchers. Carroll (1999) traced the evolution of the CSR concept and found that the CSR construct originated in the 1950s (see Bowen, 1953). From 1953 to 1970, most of the literature focused on the responsibility of the businessman (see Bowen, 1953). From 1970 to 1980, the theme shifted to the characteristics of socially responsible behavior (see, e.g., Davis, 1973; Carroll, 1979). In the 1980s, the themes of stakeholder theory (cf. Freeman, 1984), business ethics (cf. Frederick, 1986), and corporate social performance (cf. Drucker, 1984) dominated the era. In the 1990s, a number of empirical studies attempted to correlate CRS with financial performance (Clarkson, 1991; Kotter and Heskett, 1992; Collins and Porras, 1995; Waddock and Graves, 1997; Berman et al., 1999; Roman et al., 1999).

Today, corporate social responsibility is not only a prominent research theme but it can also be found in corporate missions and value statements (Svendsen et al., 2001). Business in the Community defines CRS as: "a company's positive impact on society and the environment, through its operations, products or services and through its interaction with key stakeholders such as employees, customers, investors, communities and suppliers." Svendsen et al. (2001) argues that companies are increasingly learning their way into sustainability issues – whether it be through the rapid growth of ethical finance, the increasing interest of consumers in certified sustainable products and services, or the downward pressure on supply chain partners to demonstrate environmental and social responsibility (see also Elkington, 1998). Moreover, Svendsen et al. (2001) suggest that strong relationships with stakeholders are a prerequisite for innovation, good reputations, and necessary for the development of new markets and opportunities. In addition, they also argued that strong relationships can reduce shareholder risk and enhance brand value.

In this paper, we turn to the critical issue of supply chain network social responsibility. Our focus is on social responsibility activities and environmental decision-making. We model the multicriteria decision-making behavior of the various decision-makers, which includes: the maximization of profit, emission (waste) minimization, and the minimization of risk. We consider both business-to-business (B2B) and business-to-consumer (B2C) transactions and environmental decision making. The decision-makers, consist of: the manufacturers, the retailers, as well as consumers associated with the demand markets.

In recent years, there has been a considerable shift in thinking with regard to improving the social and environmental performance of companies (UNRISD, 2002). On one hand, there are those that argue that the government should regulate the social and environmental performance of companies. Porter and van der Linde (1995) noted that in many cases properly designed legal environmental standards could still trigger innovations that lower the total cost of a product or improve its value. On the other hand, there are those that believe that the private sector generally prefers the flexibility of self-designed voluntary standards (UNCTAD, 1999). Many researchers have tried to understand business motivation to adopt voluntary environmental programs (Delmas and Terlaak, 2001; Marcus et al., 2002). Mazurkiewicz (2004) suggest that an earlier emphasis on strict governmental regulations laid the foundation for corporate self-regulation and voluntary initiatives. Hart (1997) indicates that today many companies have accepted their responsibility to do no harm to the environment. Furthermore, the private sector is becoming a decisive factor in influencing environmental performance and long-term environmental sustainability (World Bank, 2002; Mazurkiewicz, 2004).

Environmental issues surrounding supply chains have only recently come to the fore, notably, in the context of conceptual and survey studies (cf. Hill, 1997 and the references therein) as well as applied studies (see Hitchens et al., 2000). In response to growing environmental concerns, researchers have begun to deal with environmental risks (Batterman, 1991; Buck et al., 1999; Quinn, 1999; Qio et al., 2001). More significantly, the increased focus on the environment is significantly influencing supply chains. Legal requirements and changing consumer preferences increasingly make suppliers, manufacturers, and distributors responsible for their products beyond their sales and delivery locations (cf. Bloemhof-Ruwaard et al., 1995).

Indeed, environmental pressure from consumers has, in part, affected the behavior of certain manufacturers so that they attempt to minimize their emissions; produce more environmentally friendly products; and/or establish sound recycling network systems (see, e.g., Bloemhof-Ruwaard et al., 1995; Hill, 1997). Fabian (2000) asserts that poor environmental performance, at any stage of the supply chain process, may damage a company's most important asset - its reputation. As a result, organizations are expanding their responsibility to include managing the corporate social responsibilities of their partners within the supply chain (Kolk and Tudder, 2002; Emmelhainz and Adams, 1999). Simpson and Power (2005) indicates that the supply relationship is capable of leading to programs of collaborative waste reduction, environmental innovation at the interface, cost-effective environmental solutions, the rapid development and uptake of innovation in environmental technologies, and allows firms to better understand the environmental impact of their supply chains (see also, e.g., Lamming and Hampson, 1996; Florida, 1996; Clift and Wright, 2000; Geffen and Rothenberg, 2000; Hall, 2000). Svendsen et al. (2001) indicate that a sound environment policy can foster a good relationship between a company and its stakeholders and can create a competitive advantage.

Nagurney and Toyasaki (2003) developed a network model for supply chain decision-making with environmental criteria that also included the possibility of electronic commerce, but did not include risk management. In this paper, in addition to the concept of environmental decision-making, we also consider corporate social responsibility activities and risk management. Increasing levels of social responsibility activities are assumed to reduce transaction costs, risk, and environmental emissions. This framework makes it possible to simulate different scenarios depending on how concerned (or not) the decision-makers are about environmental issues, and risk over all. Moreover, since the dynamic network model is also computable, it allows for the explicit determination of the equilibrium levels of social responsibility activities between the decision-makers, as well as, product transactions and prices.

This paper is organized as follows. In Section 2, we develop the model and describe the decision-makers' multicriteria decision-making behavior. We establish the governing equilibrium conditions along with the corresponding variational inequality formulation. In Section 3, we propose the disequilibrium dynamics of the product flows, the prices, and the levels of social responsibility activities, as they evolve over time and formulate the dynamics as a projected dynamical system (cf. Nagurney and Zhang, 1996). We establish that the set of stationary points of the projected dynamical system coincides with the set of solutions to the derived variational inequality problem (cf. Nagurney and Matsypura, 2004). In Section 4, we present a discrete-time algorithm to approximate (and track) trajectories of the product flows, prices, and the levels of social responsibility activities over time until the equilibrium values are reached. We then apply the discrete-time algorithm in Section 5 to several numerical examples to further illustrate the model and computational procedure. We conclude with Section 6, in which we summarize and suggest possibilities for future research.

2. The supply chain network equilibrium model with corporate social responsibility

In this Section, we develop the network model with manufacturers, retailers, and demand markets in which we explicitly integrate levels of social responsibility activities between buyers and sellers and risk management. We focus on the presentation of the model within an equilibrium context, whereas in Section 3, we provide the disequilibrium dynamics and the evolution of the supply chain flows, the prices, as well as the levels of social responsibility activities between tiers of decision-makers over time.

The model assumes that the manufacturing firms are involved in the production of a homogeneous product and considers m manufacturers, and n retailers, which can be either physical or virtual, as in the case of electronic commerce. There are o demand markets for the homogeneous product in the economy. We denote a typical manufacturer by i and a typical retailer by j. A typical demand market is denoted by k. We assume, for the sake of generality, that each manufacturer can transact electronically directly with the consumers at the demand market through the Internet and can also conduct transactions with the retailers either physically or electronically. We let l refer to a mode of transaction with $l = 1$ denoting a physical transaction and $l = 2$ denoting an electronic transaction via the Internet.

We note that the revolution in electronic commerce has affected not only consumers and their decision-making but has also influenced the producers/manufacturers, the distributors, as well as the retailers (be they physical or virtual), and, in effect, the entire product supply chain. Indeed, both business-to-consumer (B2C) commerce and business-to-business (B2B) commerce via the Internet are thriving. For example, the US Department of Commerce (DOC, 2006) recently stated that the total (B2C) ecommerce sales for 2005 were \$86.3 billion and that is an increase of 24 percent over 2004. Moreover, according to US research group Gartner (Gartner Group, 2001), the worldwide B2B internet commerce market was expected to reach \$8.5 trillion in 2005. Due to the importance of e-commerce we include it into our framework. Both B2B and B2C ecommerce are considered, in the form of electronic transactions via the Internet links between manufacturers and retailers, between manufacturers demand market and between retailers and demand market.

The depiction of the network is given in Figure 1. As this figure illustrates, the network consists of three tiers of decision-makers. The top tiers of m nodes consist of the manufacturers, with manufacturer i associated with each network node i. The middle tier of nodes consists of the n retailers who act as intermediaries between the manufacturers and the demand markets, with retailer j associated with each node j. The bottom tier of nodes consists of the demand markets, with a typical demand market k being associated with node k in the bottom tier of nodes. There are o bottom (or third) tiered nodes in the supply chain network. Internet links to denote the possibility of electronic transactions are denoted in the figure by dotted arcs.

We have identified the nodes in the network and now we turn to the identification of the links joining the nodes in a given tier with those in the next tier of supply chain network. We also associate the product shipments with the appropriate links. We assume that each manufacturer i can transact with a given retailer, as represented by the links joining each top tier node with each middle tier node j; $j = 1, \ldots, n$. The flow on such a link joining node i with node j is denoted by q_{ijl} and represents the nonnegative amount of the product produced by manufacturer i transacted with retailer j via mode l. We group all such transactions for all manufacturers into the column vector $Q^1 \in R_+^{2mn}$. Note that if a retailer is virtual, then the transaction takes

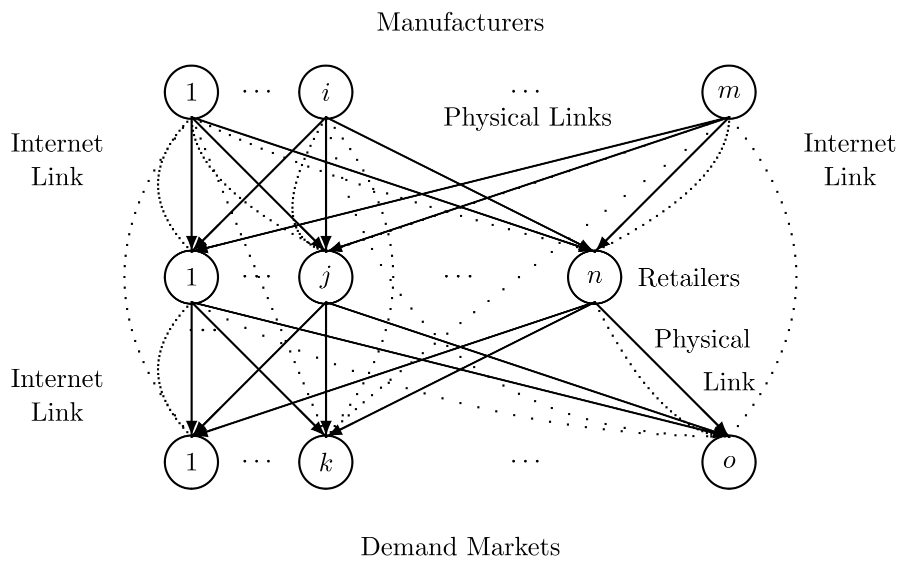

Figure 1: The structure of the supply chain network with electronic commerce

place electronically, although, of course, the product itself may be delivered physically. The manufacturer also may transact with the demand markets directly via Internet. The flow joining node i with node k is denoted by q_{ik}. We group all such transactions for all manufacturers into the column vector $Q^2 \in R_+^{mo}$

From each retailer node j; $j = 1, \ldots, n$, we construct a single link to each node k with the flow on such a link being denoted by q_{jkl} and corresponding to the amount of the product transacted between retailer j and demand market k via mode l. The product shipments for all the retailers are grouped into the column vector $Q^3 \in R_+^{2no}$.

The notation for the prices is now given. Note that there will be prices associated with each of the tiers of nodes in the supply chain network. Let ρ_{1ijl} denote the price associated with the product transacted between manufacturer i and retailer j via mode l and group these top tier prices into the column vector $\rho_1 \in R_+^{2mn}$. Let ρ_{1ik} denote the price associated with manufacturer i and demand market k and group all such prices into the column vector $\rho_{12} \in R_+^{mo}$. Let ρ_{2jkl}, in turn, denote the price associated with retailer j and demand market k via mode l and group all such prices into the column vector $\rho_2 \in R_+^{2no}$. Finally, let ρ_{3k} denote the price of the product at demand market k, and group all such prices into the column vector $\rho_3 \in R_+^{o}$.

We now turn to describing the behavior of the various decision-makers. We first focus on the manufacturers. We then turn to the retailers, and, subsequently, to the consumers at the demand markets.

2.1. The Behavior of the manufacturers

Each manufacturer faces three criteria: the maximization of profit, the minimization of emission, and the minimization of risk. We first assume that each manufacturer seeks to maximize his profit. Here it is assumed that each manufacturer i is faced with a production cost function f_i, which can depend, in general, on the total quantity of the product produced by manufacturer i, that is,

$$f_i = f_i(q_i), \quad \forall i, \tag{1}$$

where q_i is quantity of the product produced by manufacturer i. It must satisfy the following conservation of flow equation:

$$q_i = \sum_{j=1}^{n} \sum_{l=1}^{2} q_{ijl} + \sum_{k=1}^{o} q_{ik}, \tag{2}$$

which states that the quantity produced by manufacturer i is equal to the sum of the quantities shipped from the manufacturer to all retailers (via the two modes) and to all demand markets.

Furthermore, let η_{ijl} denote the nonnegative level of the social responsibility activities between manufacturer i and retailer j via mode of transaction l and let η_{ik} denote the nonnegative level of social responsibility activities associated with the virtual mode of transaction between manufacturer i and "demand market" k. Each manufacturer i may actively try to achieve a certain level of social responsibility activities with a retailer and/or a demand market. We group the $\eta_{ijl}s$ for all manufacturer/retailer/mode combinations into the column vector $\eta^1 \in R_+^{2mn}$ and the $\eta_{ik}s$ for all the manufacturer/demand market/combinations into the column vector $\eta^2 \in R_+^{mo}$. Moreover, we assume that these levels of social responsibility activities take on a value that lies in the range $[0, 1]$. No social responsibility activity is indicated by a level of zero and the strongest possible level of social responsibility activities is indicated by a level of one. In the network depicted in Figure 1, the vector η^1 corresponds to the links between the manufacturers and the retailers, whereas the vector η^2 corresponds to the links between the manufacturers and the demand markets. The levels of social responsibility activities, along with the product flows, are endogenously determined in the model.

The manufacturer may spend money, for example, in the form of time/service, investment in new technology, training employees, and information sharing in order to promote a sound environmental policy. Here social responsibility activities are activities that promote quality assurance, environmental preservation, and compliance. According to Simpson and Power (2005), positive relationships have been established between environmental performance and improvements to the manufacturing quality management (Klassen, 2000; Kitazawa and Sarkis, 2000), lean manufacturing practice (Rothenberg et al., 2001; King and Lenox, 2001; Klassen, 2000) and worker involvement (Geffen and Rothenberg, 2000; Kitazawa and Sarkis, 2000; Rothenberg 2003). The production cost functions for social responsibility activities are denoted by b_{ijl} and b_{ik} and represent, respectively, how much money a manufacturer i has to spend in order to achieve a certain level of social responsibility activities with retailer j transacting through mode l or in order to achieve a certain level of social responsibility activities with demand market k. We assume that they are distinct for each such combination.

The social responsibility activities production cost functions are assumed, hence, to be a function of the level of social responsibility activities between the manufacturer i and retailer j transacting via mode l or with the consumers at demand market k, that is,

$$b_{ijl} = b_{ijl}(\eta_{ijl}), \quad \forall i, j, l, \tag{3}$$

$$b_{ik} = b_{ik}(\eta_{ik}), \quad \forall i, k. \tag{4}$$

We note that any level of social responsibility activities between any two parties in the supply chain requires a strong level of collaboration/cooperation between them. Here we define collaboration as any kind of joint, coordinated effort between decision-makers in supply chain in order to achieve a common goal. Many researchers have argued that collaboration can reduce waste in the supply chain, but can also increase market responsiveness, customer satisfaction, and competitiveness among all members of the partnership (Klassen and Vachon, 2003; Poter and Van der Linde 1995). Ashford (1993) and Kemp (1993) indicate that through knowledge sharing, collaborative activities reduce uncertainty, willingness to change, and other sources of resistance frequently associated with lack of investment in social responsibility activities such as investment in environmental technology. Bonifant et al. 1995, asserts that collaboration along the supply chain also helps management to identify and evaluate a greater variety of options that might address particular environmental challenges. Moreover, it can also alter the means by which any negative environmental impact is reduced. These include activities like reduced packaging, joint recycling of parts and components, and process changes that reduce the use of hazardous materials (Klassen and Vachon, 2003).

Each decision-maker is also faced with certain transaction costs which are the costs of making an economic exchange. These costs may include the costs of coordinating the exchange actions between decision-makers (Stigler 1961) and the costs of motivating decision-makers to align their interests, costs of cheating or costs of opportunistic behavior (Williamson 1975, 1985).

We denote the transaction cost associated with manufacturer i transacting with intermediary j via mode l by c_{ijl} and assume that:

$$c_{ijl} = c_{ijl}(q_{ijl}, \eta_{ijl}), \quad \forall i, j, l, \tag{5}$$

that is, the cost associated with manufacturer i transacting with retailer j via mode l depends on the volume of transactions between the particular pair via the particular mode, and on the levels of social responsibility activities between them. If the levels of social responsibility activities increases, the transaction cost may be expected to decrease. High levels of social responsibility activities between decision-makers imply high levels of collaboration. Increased levels of collaboration between decision makers lead to higher levels of trust which can affect transaction costs (Williamson 1975, 1985). Several authors have suggested that an established and reliable inter-firm collaboration (relationship) can lead to reduced transaction costs and improvements to supplier manufacturing performance (Dyer, 1997; Handfield and Bechtel, 2002; Dyer and Chu, 2003). Furthermore, Florida (1996) found evidence that customer supplier relationships facilitated the adoption and diffusion of environmental innovations in manufacturing practices.

We denote the transaction cost associated with manufacturer i transacting with demand market k via the Internet link by c_{ik} and assume that:

$$c_{ik} = c_{ik}(q_{ik}, \eta_{ik}), \quad \forall i, k. \tag{6}$$

We assume that the production cost (1) and transaction cost functions (3) through (6) are convex and continuously differentiable.

The manufacturer i faces total costs that equal the sum of the manufacturer's production cost plus total transaction costs and plus the costs that he incurs for establishing levels of social responsibility activities. His revenue, in turn, is equal to the sum of the price that the manufacturer can obtain times the total quantity obtained/purchased. Let now ρ^*_{1ijl} denote the actual price charged by manufacturer i for the product by retailer j transacting via mode l and let ρ^*_{1ik}, in turn, denote the actual price associated with manufacturer i transacting electronically with demand market k. We later discuss how such prices are recovered.

We assume that each manufacturer seeks to maximize his profit with the profit maximization problem for manufacturer i being given by:

$$\text{Maximize} \quad \sum_{j=1}^{n}\sum_{l=1}^{2}\rho^*_{1ijl}q_{ijl} + \sum_{k=1}^{o}\rho^*_{1ik}q_{ik} - f_i(q_i) - \sum_{j=1}^{n}\sum_{l=1}^{2}c_{ijl}(q_{ijl},\eta_{ijl})$$

$$-\sum_{k=1}^{o}c_{ik}(q_{ik},\eta_{ik}) - \sum_{j=1}^{n}\sum_{l=1}^{2}b_{ijl}(\eta_{ijl}) - \sum_{k=1}^{o}b_{ik}(\eta_{ik}), \tag{7}$$

subject to: $q_{ijl} \geq 0, \quad q_{ik} \geq 0, \quad 0 \leq \eta_{ijl} \leq 1, \quad 0 \leq \eta_{ik} \leq 1, \quad \forall j,k,l.$

Note that in (7), the first two terms represent the revenue whereas the subsequent five terms represent the various costs.

In addition to the criterion of profit maximization, we assume that each manufacturer also seeks to minimize the total emissions (waste) generated in the production of the product as well as its delivery to the next tier of decision-makers, whether retailers or consumers at the demand markets. Here, we also assume that the emission function depends on the volume of transactions between the particular pair via the particular mode, and on the levels of social responsibility activities between decision-makers. This is a reasonable assumption since one would generally expect the emissions (waste) to increase as the level of production and volume of transactions increases if a company is not environmentally friendly. However, as the companies become more environmentally socially responsible, greater innovation and environmental policy are introduced which in term decrease production inefficiencies and waste and may decrease environmental impact and future environmental risk (see, e.g., Lamming and Hampson, 1996; Florida, 1996; Clift and Wright, 2000; Geffen and Rothenberg, 2000; Hall, 2000). By assuming that the total emissions (waste) generated in the supply chain are functions of levels of social responsibility activities between supply chain partners we are trying to capture the correlation between CSR and environmental decision-making. Many studies have shown a positive relation between CRS and a corporation's environmental impact (Feldman et al., 1996; Kolk and Tudder, 2002; Emmelhainz and Adams, 1990).

We assume that the emissions function for manufacturer i is convex and continuously differentiable and given by the function e^i, where

$$e^i = e^i(Q^1, Q^2, \eta_1, \eta_2), \quad \forall i. \tag{8}$$

Hence, the second criterion of each manufacturer can be expressed mathematically as:

$$\text{Minimize } e^i(Q^1, Q^2, \eta_1, \eta_2) \tag{9}$$

subject to: $q_{ijl} \geq 0, \quad q_{ik} \geq 0, \quad 0 \leq \eta_{ijl} \leq 1, \quad 0 \leq \eta_{ik} \leq 1, \quad \forall j, k, l.$

Finally, we also assume that each manufacturer is concerned with risk minimization. Risk is defined as the possibility for companies to suffer harm or loss for their activities and also for the activities of their partners in the supply chain. In terms of CSR risk, companies may be found liable for pollution, compliance with regulation, dangerous operations, use of hazardous raw materials, production of hazardous waste, and for health and safety issues. As a result of these liabilities companies may lose their reputation (Fabian, 2000; Dowling, 2001; Fombrun, 2001), brand image, sales, access to markets and financial investments (Feldman et al., 1996). However, by investing in CSR activities and collaborating with their supply chain partners, companies may avoid the costs of future lawsuits, negative media coverage, poor workmanship, unreliable business relationships, financial mismanagement, and operations disruptions.

We note that the risk functions in our model are functions of both the product transactions and the levels of social responsibility activities. Jüttner et al. (2003) suggest that supply chain-relevant risk sources fall into three categories: environmental risk sources (e.g., fire, social-political actions (CSR), or acts of God), organizational risk sources (e.g., production uncertainties), and network-related risk sources. Johnson (2001) and Norrman and Jansson (2004) argue that network-related risk arises from the interaction between organizations within the supply chain, e.g., due to insufficient interaction and cooperation. Here, we model supply chain organizational risk, environmental risk, and network-related risk by defining the risk as a function of product flows as well as the levels of social responsibility activities. We use levels of social responsibility activities as a way of possibly mitigating these risks.

Hence, high levels of social responsibility activities are assumed to reduce risk and transactional uncertainty. The results in Feldman et al. (1996) suggest that adopting a more environmentally proactive posture has, in addition to any direct environmental and cost reduction benefits, a significant and favorable impact on the firm's perceived riskiness to investors and, accordingly, its cost of equity capital and value in the market place. Here, for the sake of generality, we assume, as given, a risk function r^i, for manufacturer i transacting with retailer j and demand market k, which is assumed to be continuous and convex. Hence, we assume that

$$r^i = r^i(Q^1, Q^2, \eta_1, \eta_2), \quad \forall i. \tag{10}$$

The third criterion faced by manufacturer i, thus, corresponds to risk minimization and can be expressed mathematically as:

$$\text{Minimize } r^i(Q^1, Q^2, \eta_1, \eta_2), \tag{11}$$

subject to: $q_{ijl} \geq 0, \quad q_{ik} \geq 0, \quad 0 \leq \eta_{ijl} \leq 1, \quad 0 \leq \eta_{ik} \leq 1, \quad \forall j, k, l.$

2.1.1. The multicriteria decision-making problem faced by a manufacturer

We can now construct the multicriteria decision-making problem facing a manufacturer which allows him to weight the criteria of profit maximization (cf. (7)), total emissions minimization (cf. (9)), and total risk minimization (see (11)) in an individual manner. Many researchers have studied the relationship between these three concepts, CSR, risk and profit (Dowling, 2001; Fombrun, 2001; Clarkson, 1991; Kotter and Heskett, 1992; Collins and Porras, 1995; Waddock and Graves, 1997; Berman et al., 1999; Roman et al., 1999). For example, according to Social Investment Forum (2005) report over the last ten years, socially responsible investment assets grew four percent faster than the entire universe of managed assets in the United States. CSR can potentially decrease production inefficiencies, reduce cost and risk and at the same time allow companies to increase sales, increase access to capital, new markets, and brand recognition. As a result of lower cost, lower risk and increase in sales, companies become more profitable. However, as illustrated in in Figure 2, we also expect that as the investment in CSR activities increases, the percentage increase of the return of investment will be smaller. In our model we try to determine the level of CSR activity that will maximize profit and minimize risk.

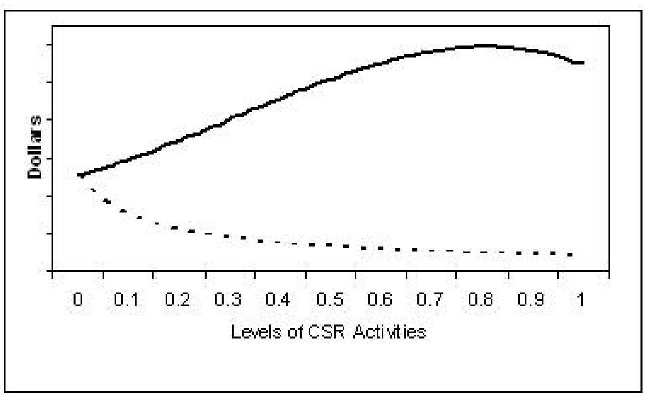

Figure 2: Relationship between levels of CSR activities, risk and profit (...Risk, —Profit)

Manufacturer is multicriteria decision-making objective function is denoted by U_i. Assume that manufacturer i assigns a nonnegative weight α_i to total emissions, and the nonnegative weight ω_i to risk generated.

The weight associated with profit maximization serves as the numeraire and is set equal to 1. The nonnegative weights measure the importance of emission and risk, and, in addition, transform these values into monetary units. We can now construct a value function for each manufacturer (cf. Keeney and Raiffa, 1993) using a constant additive weight value function. Therefore, the multicriteria decision-making problem of manufacturer i can be expressed as:

$$\text{Maximize} \quad \sum_{j=1}^{n}\sum_{l=1}^{2}\rho_{1ijl}^{*}q_{ijl}+\sum_{k=1}^{o}\rho_{1ik}^{*}q_{ik}-f_i(q_i)-\sum_{j=1}^{n}\sum_{l=1}^{2}c_{ijl}(q_{ijl},\eta_{ijl})-\sum_{k=1}^{o}c_{ik}(q_{ik},\eta_{ik})$$

$$-\sum_{j=1}^{n}\sum_{l=1}^{2}b_{ijl}(\eta_{ijl})-\sum_{k=1}^{o}b_{ik}(\eta_{ik})-\alpha_i e^i(Q^1,Q^2,\eta_1,\eta_2)-\omega_i r^i(Q^1,Q^2,\eta_1,\eta_2) \tag{12}$$

subject to: $\quad q_{ijl}\geq 0, \quad q_{ik}\geq 0, \quad 0\leq\eta_{ijl}\leq 1, \quad 0\leq\eta_{ik}\leq 1, \quad \forall j,k,l.$

The first seven terms in (12) represent the profit which is to be maximized, the next term represent weighted total emission, which is to be minimized, and the last term represent the weighted total risk, which is to be minimized.

2.1.2. The optimality conditions of manufacturers

Here we assume that the manufacturers compete in a noncooperative fashion following Nash (1950, 1951). Hence, each manufacturer seeks to determine his optimal strategies, that is production outputs (and shipments), given those of the other manufacturers. The optimality conditions of all manufacturers i; $i=1,\dots,m$ simultaneously, under the above assumptions (cf. Bazaraa et al., 1993; Gabay and Moulin, 1980; Nagurney, 1999), can be compactly expressed as:
determine $(Q^{1*},Q^{2*},\eta^{1*},\eta^{2*})\in\mathcal{K}^1$, satisfying

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\left[\frac{\partial f_i(q_i^*)}{\partial q_{ijl}}+\frac{\partial c_{ijl}(q_{ijl}^*,\eta_{ijl}^*)}{\partial q_{ijl}}+\alpha_i\frac{\partial e^{i*}}{\partial q_{ijl}}+\omega_i\frac{\partial r^{i*}}{\partial q_{ijl}}-\rho_{1ijl}^*\right]\times\left[q_{ijl}-q_{ijl}^*\right]$$

$$+\sum_{i=1}^{m}\sum_{k=1}^{o}\left[\frac{\partial f_i(q_i^*)}{\partial q_{ik}}+\frac{\partial c_{ik}(q_{ik}^*,\eta_{ik}^*)}{\partial q_{ik}}+\alpha_i\frac{\partial e^{i*}}{\partial q_{ik}}+\omega_i\frac{\partial r^{i*}}{\partial q_{ik}}-\rho_{1ik}^*\right]\times\left[q_{ik}-q_{ik}^*\right]$$

$$+\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\left[\frac{\partial c_{ijl}(q_{ijl}^*,\eta_{ijl}^*)}{\partial \eta_{ijl}}+\frac{\partial b_{ijl}(\eta_{ijl}^*)}{\partial \eta_{ijl}}+\alpha_i\frac{\partial e^{i*}}{\partial \eta_{ijl}}+\omega_i\frac{\partial r^{i*}}{\partial \eta_{ijl}}\right]\times\left[\eta_{ijl}-\eta_{ijl}^*\right]$$

$$+\sum_{i=1}^{m}\sum_{k=1}^{o}\left[\frac{\partial c_{ik}(q_{ik}^*,\eta_{ik}^*)}{\partial \eta_{ik}}+\frac{\partial b_{ik}(\eta_{ik}^*)}{\partial \eta_{ik}}+\alpha_i\frac{\partial e^{i*}}{\partial \eta_{ik}}+\omega_i\frac{\partial r^{i*}}{\partial \eta_{ik}}\right]\times\left[\eta_{ik}-\eta_{ik}^*\right]\geq 0,$$

$$\forall(Q^1,Q^2,\eta^1,\eta^2)\in\mathcal{K}^1, \tag{13}$$

where $e^{i*} = e^i(Q^{1*}, Q^{2*}, \eta^{1*}, \eta^{2*})$, $r^{i*} = r^i(Q^{1*}, Q^{2*}, \eta^{1*}, \eta^{2*})$ and

$$\mathcal{K}^1 \equiv \left[(Q^1, Q^2, \eta^1, \eta^2)|q_{ijl} \geq 0,\ q_{ik} \geq 0,\ 0 \leq \eta_{ijl} \leq 1,\ 0 \leq \eta_{ik} \leq 1,\ \forall i, j, k, l\right]. \tag{14}$$

The inequality (13), which is a variational inequality (cf. Nagurney, 1999) has a meaningful economic interpretation. From the first term we can see that, if there is a positive transaction of the product transacted either in a classical manner or via the Internet from a manufacturer to a retailer, then the marginal cost of production plus the marginal cost of transacting plus the weighted marginal cost of risk and emission must be equal to the price that the retailer is willing to pay for the product. If that sum, in turn, exceeds the price then there will be no product transacted.

The second term in (13) states that there will be a positive flow of the product from a manufacturer to a demand market if the marginal cost of production of the manufacturer plus the marginal cost of transacting via the Internet for the manufacturer with consumers and the weighted marginal cost of risk and emission must be equal to the price the consumers are willing to pay for the product at the demand market.

The third and the fourth term in (13) show that if there is a positive level of social responsibility activities (and that level is less than one) then the marginal cost of establishing this level is equal to the marginal reduction in transaction cost plus the weighted marginal reduction in risk and emission.

2.2. The behavior of the retailers

The retailers, in turn, are involved in transactions both with the manufacturers since they wish to obtain the product for their retail outlets, as well as with the consumers, who are the ultimate purchasers of the product. Thus, as depicted in Figure 1, a retailer conducts transactions both with the manufacturers and with the consumers. The retailers are also assumed to be multicriteria decision-makers in that they seek to maximize profits with manufacturers and consumers, to minimize their individual risk associated with their transactions and to minimize the emissions generated from the perspective of the amounts of the product that they purchase from the manufacturers and the manner in which the transactions occur and the products are shipped.

As in the case of manufacturers, the retailers have to bear some costs to establish and maintain levels of social responsibility activities with manufacturers and with the consumers, who are the ultimate purchasers/buyers of the product. We denote the level of social responsibility activities between retailer j and demand market k transacting through mode l by η_{jkl}. We group the levels of social responsibility activities for all retailer/demand market pairs into the column vector $\eta^3 \in R^{2no}_+$. We assume that the levels of social responsibility activities are nonnegative and that they may assume a value from 0 through 1. These levels of social responsibility activities are associated with the links between the retailers and the demand market nodes in the network in Figure 1.

Let $\hat{b}_{ijl}$ denote the cost function associated with the level of social responsibility activities between retailer j and manufacturer i via mode l and let b_{jkl} denote the analogous cost function but associated with retailer j, demand market k, and mode l. Note that these functions are from the perspective of the retailer (whereas (3) and (4) are from the perspective of the manufacturers). These cost functions are a function of the levels of social responsibility activities (as in the case of the manufacturers) and are given by:

$$\hat{b}_{ijl} = \hat{b}_{ijl}(\eta_{ijl}), \quad \forall i, j, l, \tag{15}$$

$$b_{jkl} = b_{jkl}(\eta_{jkl}), \quad \forall j, k, l. \tag{16}$$

As discussed in Nagurney and Dong (2002), a retailer j is faced with what we term a *handling/conversion* cost, which may include, for example, the cost of handling and storing the product. We denote such a cost faced by retailer j by c_j and, in the simplest case, we would have that c_j is a function of $q_j = \sum_{i=1}^{m} \sum_{l=1}^{2} q_{ijl}$, that is, the handling/conversion cost of a retailer is a function of how much product he has obtained from various manufacturers. We may write:

$$c_j = c_j(q_j), \quad \forall j. \tag{17}$$

The retailers, which can be either physical or virtual, also have associated transaction costs in regards to transacting with the manufacturers, which we assume can be dependent on the type of the manufacturer. We denote the transaction cost associated with retailer j transacting with manufacturer i and via mode m by $\hat{c}_{ijl}$ and we assume that it is of the form

$$\hat{c}_{ijl} = \hat{c}_{ijl}(q_{ijl}, \eta_{ijl}), \quad \forall i, j, l, \tag{18}$$

that is, such a transaction cost is allowed to depend upon the amount of the product transacted by the manufacturer/retailer pair via the mode, and on the level of social responsibility activities established between the pair. In addition, we assume that a retailer j also incurs a transaction cost c_{jkl} associated with transacting with demand market k, where

$$c_{jkl} = c_{jkl}(q_{jkl}, \eta_{jkl}), \quad \forall j, k, l. \tag{19}$$

Hence, the transaction costs given in (19) can vary according to the retailer and are a function of the volume of the product transacted, and levels of social responsibility activities. We assume that the cost functions (15) – (19) are convex and continuously differentiable.

The actual price charged for the product by retailers j is denoted by ρ_{2jkl}, and is associated with transacting with consumers at demand market k via mode l. Similarly, as in the case of manufacturers, later, we discuss how such prices are arrived at. We assume that the retailers are also profit maximizers.

The utility maximization problem for retailer j can, hence, be expressed as:

$$\text{Maximize} \quad \sum_{k=1}^{o}\sum_{l=1}^{2}\rho^{*}_{2jkl}q_{jkl} - c_j(q_j) - \sum_{i=1}^{m}\sum_{l=1}^{2}\hat{c}_{ijl}(q_{ijl},\eta_{ijl}) - \sum_{k=1}^{o}\sum_{l=1}^{2}c_{jkl}(q_{jkl},\eta_{jkl})$$

$$-\sum_{i=1}^{m}\sum_{l=1}^{2}\hat{b}_{ijl}(\eta_{ijl}) - \sum_{k=1}^{o}\sum_{l=1}^{2}b_{jkl}(\eta_{jkl}) - \sum_{i=1}^{m}\sum_{l=1}^{2}\rho^{*}_{1ijl}q_{ijl} \tag{20}$$

subject to:

$$\sum_{k=1}^{o}\sum_{l=1}^{2}q_{jkl} \leq \sum_{i=1}^{m}\sum_{l=1}^{2}q_{ijl} \tag{21}$$

and the non-negativity constraints: $q_{ijl} \geq 0, \quad q_{jkl} \geq 0, \quad 0 \leq \eta_{ijl} \leq 1, \quad 0 \leq \eta_{jkl} \leq 1, \quad \forall i,k,l.$

Objective function (20) expresses that the difference between the revenue minus the handling cost and the transaction costs in dealing with manufacturers and the demand markets and the costs for establishing levels of social responsibility activities with manufacturers and demand markets and the payout to the manufacturers should be maximized. Constraint (21) states that consumers cannot purchase more from a retailer than is held in stock.

In addition, we assume that each retailer seeks to minimize the emissions and waste associated with his transactions with manufacturers and the demand markets. We assume that the emissions function is given by e^j, such that

$$e^j = e^j(Q^1, Q^3, \eta_1, \eta_3), \quad \forall j. \tag{22}$$

Hence, the second criterion of each retailer can be expressed mathematically as:

$$\text{Minimize } e^j(Q^1, Q^3, \eta_1, \eta_3) \tag{23}$$

subject to: (21) and the non-negativity constraints $q_{ijl} \geq 0, q_{jkl} \geq 0, \quad 0 \leq \eta_{ijl} \leq 1, \quad 0 \leq \eta_{jkl} \leq 1, \quad \forall i,k,l.$

Furthermore, we assume that each retailer is also concerned with risk minimization. For the sake of generality, we assume, as given, a risk function r^j, for retailer j in transacting with manufacturer i and with consumers at demand market k through mode l. The risk function is assumed to be continuous and convex and a function of both the product transactions and the levels of social responsibility activities.

The risk function is given by:

$$r^j = r^j(Q^1, Q^3, \eta_1, \eta_3), \quad \forall i, \tag{24}$$

Since a retailer j is assumed to minimize his total risk, he is also faced with the optimization problem given by:

$$\text{Minimize} \quad r^j(Q^1, Q^3, \eta_1, \eta_3) \tag{25}$$

subject to: (21) and $\quad q_{ijl} \geq 0, \quad q_{jkl} \geq 0, \quad 0 \leq \eta_{ijl} \leq 1, \quad 0 \leq \eta_{jkl} \leq 1, \quad \forall i, k, l.$

2.2.1. A retailer's multicriteria decision-making problem

Retailer j assigns a nonnegative weight α_j to total emissions, and the nonnegative weight ω_j to risk generated. The weight associated with profit maximization is set equal to 1 and serves as the numeraire (as in the case of the manufacturers). We are now ready to construct the multicriteria decision-making problem faced by a retailer, which combines with appropriate individual weights to the criteria of profit maximization given by (20); emission minimization given by (23); and risk minimization, given by (25). Let U_j denote the multicriteria objective function associated with intermediary j with his multicriteria decision-making problem expressed as:

$$\text{Maximize} \quad \sum_{k=1}^{o}\sum_{l=1}^{2} \rho^*_{2jkl} q_{jkl} - c_j(q_j) - \sum_{i=1}^{m}\sum_{l=1}^{2} \hat{c}_{ijl}(q_{ijl}, \eta_{ijl}) - \sum_{k=1}^{o}\sum_{l=1}^{2} c_{jkl}(q_{jkl}, \eta_{jkl})$$

$$-\sum_{i=1}^{m}\sum_{l=1}^{2} \hat{b}_{ijl}(\eta_{ijl}) - \sum_{k=1}^{o}\sum_{l=1}^{2} b_{jkl}(\eta_{jkl}) - \sum_{i=1}^{m}\sum_{l=1}^{2} \rho^*_{1ijl} q_{ijl} - \alpha_j e^j(Q^1, Q^3, \eta_1, \eta_3) - \omega_j r^j(Q^1, Q^3, \eta_1, \eta_3) \tag{26}$$

subject to: (21) and the non-negativity constraints: $q_{ijl} \geq 0, \quad q_{jkl} \geq 0, \quad 0 \leq \eta_{ijl} \leq 1, \quad 0 \leq \eta_{jkl} \leq 1, \quad \forall i, k, l.$

2.2.2. The optimality conditions of retailers

Now we turn to the optimality conditions of the retailers. Each retailer faces the multicriteria decision-making problem (26), subject to (21) and the nonnegativity assumption on the variables. As in the case of manufacturers, we assume that the retailers compete in a noncooperative manner, given the actions of the other retailers. Retailers seek to determine the optimal transactions associated with the demand markets and with the manufacturers. In equilibrium, all the transactions between the tiers of network decision-makers will have to coincide, as we will see later in this section.

If one assumes that the handling, transaction cost, production function for levels of social responsibility activities, emission and risk functions are continuously differentiable and convex, then the optimality conditions for all the retailers satisfy the variational inequality: determine $(Q^{1*}, Q^{3*}, \eta^{1*}, \eta^{3*}, \lambda_j^*) \in \mathcal{K}^2$, such that

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\left[\frac{\partial c_j(q_j^*)}{\partial q_{ijl}} + \frac{\partial \hat{c}_{ijl}(q_{ijl}^*, \eta_{ijl}^*)}{\partial q_{ijl}} + \alpha_j \frac{\partial e^{j*}}{\partial q_{ijl}} + \omega_j \frac{\partial r^{j*}}{\partial q_{ijl}} + \rho_{1ijl}^* - \lambda_j^*\right] \times \left[q_{ijl} - q_{ijl}^*\right]$$
$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{2}\left[\frac{\partial c_{jkl}(q_{jkl}^*, \eta_{jkl}^*)}{\partial q_{jkl}} + \alpha_j \frac{\partial e^{j*}}{\partial q_{jkl}} + \omega_j \frac{\partial r^{j*}}{\partial q_{jkl}} - \rho_{2jkl}^* + \lambda_j^*\right] \times \left[q_{jkl} - q_{jkl}^*\right]$$
$$+\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\left[\frac{\partial \hat{c}_{ijl}(q_{ijl}^*, \eta_{ijl}^*)}{\partial \eta_{ijl}} + \alpha_j \frac{\partial e^{j*}}{\partial \eta_{ijl}} + \omega_j \frac{\partial r^{j*}}{\partial \eta_{ijl}} + \frac{\partial \hat{b}_{ijl}(\eta_{ijl}^*)}{\partial \eta_{ijl}}\right] \times \left[\eta_{ijl} - \eta_{ijl}^*\right]$$
$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{2}\left[\frac{\partial c_{jkl}(q_{jkl}^*, \eta_{jkl}^*)}{\partial \eta_{jkl}} + \alpha_j \frac{\partial e^{j*}}{\partial \eta_{jkl}} + \omega_j \frac{\partial r^{j*}}{\partial \eta_{jkl}} + \frac{\partial b_{jkl}(\eta_{jkl}^*)}{\partial \eta_{jkl}}\right] \times \left[\eta_{jkl} - \eta_{jkl}^*\right]$$
$$+\sum_{j=1}^{n}\left[\sum_{i=1}^{m}\sum_{l=1}^{2} q_{ijl}^* - \sum_{k=1}^{o}\sum_{l=1}^{2} q_{jkl}^*\right] \times \left[\lambda_j - \lambda_j^*\right] \geq 0, \quad \forall (Q^1, Q^3, \eta^1, \eta^3, \lambda) \in \mathcal{K}^2, \tag{27}$$

where $e^{j*} = e^j(Q^{1*}, Q^{3*}, \eta^{1*}, \eta^{3*})$, $r^{j*} = r^j(Q^{1*}, Q^{3*}, \eta^{1*}, \eta^{3*})$ and

$$\mathcal{K}^2 \equiv \left[(Q^1, Q^3, \eta^1, \eta^3, \lambda) | q_{ijl} \geq 0,\, q_{jkl} \geq 0,\, 0 \leq \eta_{ijl} \leq 1,\, 0 \leq \eta_{jkl} \leq 1,\, \lambda_j \geq 0, \forall i, j, k, l\right]. \tag{28}$$

Here λ_j denotes the Lagrange multiplier associated with constraint (21) and λ is the column vector of all the intermediaries' Lagrange multipliers. These Lagrange multipliers can also be interpreted as shadow prices. Indeed, according to the last term in (27), λ_j^* serves as the price to "clear the market" at retailer j.

2.3. The consumers at the demand markets

We now describe the consumers located at the demand markets. The consumers take into account in making their consumption decisions not only the price charged for product by the manufacturers and retailers but also their transaction costs associated with obtaining the product as well as the levels of social responsibility activities of the manufacturers and retailers.

Let $\hat{c}_{jkl}$ denote the transaction cost associated with demand market k via mode l from retailer j and recall that q_{jkl} is the amount of product flowing between intermediary j and consumers at the demand market k via mode l. We assume that the transaction cost is continuous, and of the general form:

$$\hat{c}_{jkl} = \hat{c}_{jkl}(Q^2, Q^3, \eta_2, \eta_3), \quad \forall j, k, l. \tag{29}$$

Hence, the cost of transacting between a retailer and a demand market via a specific mode, from the perspective of the consumers, can depend upon the volume of product flows transacted either physically and/or electronically from retailers as well as from manufacturers and the associated levels of social responsibility activities. As in the case of the manufacturers and the retailers, higher levels of social responsibility activities potentially reduce transaction costs, which means that they can lead to quantifiable cost reductions. The generality of this cost function structure enables the modeling of competition on the demand side.

In addition, let $\hat{c}_{ik}$ denote the transaction cost associated with obtaining the product electronically from manufacturer i by the consumer at demand market k, where we assume that the transaction cost is continuous and of the general form:

$$\hat{c}_{ik} = \hat{c}_{ik}(Q^2, Q^3, \eta_2, \eta_3), \quad \forall i, k. \tag{30}$$

Hence, the transaction cost associated with transacting directly with manufacturers is of a form of the same level of generality as the transaction costs associated with transacting with the retailers.

Denote the demand for product at the demand market k by d_k and assume, as given, the continuous demand functions:

$$d_k = d_k(\rho_3), \quad \forall k. \tag{31}$$

Thus, according to (31), the demand of consumers for the product depends, in general, not only on the price of the product at that demand market but also on the prices of the product at the other demand markets. Consequently, consumers at a demand market, in a sense, also compete with consumers at other demand markets.

The consumers at the demand market k take the price charged by the retailer, which was denoted by ρ^*_{2jkl} for retailer j, via mode l, the price charged by manufacturer i, which was denoted by ρ^*_{1ik}, plus the transaction costs, in making their consumption decisions (Nagurney and Dong, 2002). The equilibrium conditions for the consumers at demand market k, thus, take the form: for all retailers: $j = 1, \ldots, n$ and all mode l; $l = 1, 2$:

$$\rho^*_{2jkl} + \hat{c}_{jkl}(Q^{2*}, Q^{3*}, \eta^*_2, \eta^*_3) \begin{cases} = \rho^*_{3k}, & \text{if} \quad q^*_{jkl} > 0 \\ \geq \rho^*_{3k}, & \text{if} \quad q^*_{jkl} = 0, \end{cases} \tag{32}$$

and for all source agents i; $i = 1, \ldots, m$:

$$\rho^*_{1ik} + \hat{c}_{ik}(Q^{2*}, Q^{3*}, \eta^*_2, \eta^*_3) \begin{cases} = \rho^*_{3k}, & \text{if} \quad q^*_{ik} > 0 \\ \geq \rho^*_{3k}, & \text{if} \quad q^*_{ik} = 0. \end{cases} \tag{33}$$

In addition, we must have that

$$d_k(\rho^*_3) \begin{cases} = \sum_{j=1}^{n} \sum_{l=1}^{2} q^*_{jkl} + \sum_{i=1}^{m} q^*_{ik}, & \text{if} \quad \rho^*_{3k} > 0 \\ \leq \sum_{j=1}^{n} \sum_{l=1}^{2} q^*_{jkl} + \sum_{i=1}^{m} q^*_{ik}, & \text{if} \quad \rho^*_{3k} = 0. \end{cases} \tag{34}$$

Conditions (32) state that consumers at demand market k will purchase the product from retailer j, if the price charged by the retailer for the product plus the transaction cost (from the perspective of the consumer) does not exceed the price that the consumers are willing to pay for the product, i.e., ρ^*_{3k}. Note that, according to (32), if the transaction costs are identically equal to zero, then the price faced by the consumers for a given product is the price charged by the retailer. Condition (33) state the analogue, but for the case of electronic transactions with the manufacturers.

Condition (34), on the other hand, states that, if the price the consumers are willing to pay for the product at a demand market is positive, then the quantity purchased/consumed by the consumers at the demand market is precisely equal to the demand.

In equilibrium, conditions (32), (33), and (34) will have to hold for all demand markets and these, in turn, can be expressed also as an inequality analogous to those in (13) and (27) and given by: determine $(Q^{2*}, Q^{3*}, \rho^*_3) \in R^{(m+2n+1)o}_+$, such that

$$\sum_{k=1}^{o} \sum_{j=1}^{n} \sum_{l=1}^{2} \left[\rho^*_{2jkl} + \hat{c}^*_{jkl} - \rho^*_{3k}\right] \times \left[q_{jkl} - q^*_{jkl}\right] + \sum_{i=1}^{m} \sum_{k=1}^{o} \left[\rho^*_{1ik} + \hat{c}^*_{ik} - \rho^*_{3k}\right] \times \left[q_{ik} - q^*_{ik}\right]$$

$$+ \sum_{k=1}^{o} \left[\sum_{j=1}^{n} \sum_{l=1}^{2} q^*_{jkl} + \sum_{i=1}^{m} q^*_{ik} - d_k(\rho^*_3) \right] \times \left[\rho_{3k} - \rho^*_{3k}\right] \geq 0, \qquad \forall (Q^2, Q^3, \rho_3) \in R^{(m+2n+1)o}_+. \tag{35}$$

where $\hat{c}^*_{jkl} = \hat{c}_{jkl}(Q^{2*}, Q^{3*}, \eta^*_2, \eta^*_3)$ and $\hat{c}^*_{ik} = \hat{c}_{ik}(Q^{2*}, Q^{3*}, \eta^*_2, \eta^*_3)$.

In the context of the consumption decisions, we have utilized demand functions, whereas profit functions, which correspond to objective functions, were used in the case of the manufacturers and the retailers. Since we can expect the number of consumers to be much greater than that of the manufacturers and retailers we believe that such a formulation is more natural.

2.4. The equilibrium conditions of the supply chain network

In equilibrium, the product flows that the manufacturers transact with the retailers must coincide with those that the retailers actually accept from them. In addition, the amounts of the products that are obtained by the consumers must be equal to the amounts that both the manufacturers and the retailers actually provide. Hence, although there may be competition between decision-makers at the same level of tier of nodes of the supply chain network there must be, in a sense, cooperation between decision-makers associated with pairs of nodes (through positive flows on the links joining them). Thus, in equilibrium, the prices and product flows must satisfy the sum of the optimality conditions (13) and (27) and the equilibrium conditions (35). We make these relationships rigorous through the subsequent definition and variational inequality derivation below.

Definition 1 (Supply chain network equilibrium). The equilibrium state of the supply chain network is one where the flows and levels of social responsibility activities between the tiers of the network coincide and the product transactions, levels of social responsibility activities and prices satisfy the sum of conditions (13), (27), and (35).

The equilibrium state is equivalent to the following:

Theorem 1 (Variational inequality formulation). *The equilibrium conditions governing the supply chain network model according to Definition 1 are equivalent to the solution of the variational inequality given by: determine* $(Q^{1*}, Q^{2*}, Q^{3*}, \eta^{1*}, \eta^{2*}, \eta^{3*}, \lambda^*, \rho_3^*) \in \mathcal{K}$*, satisfying:*

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\left[\frac{\partial f_i(q_i^*)}{\partial q_{ijl}} + \frac{\partial c_{ijl}(q_{ijl}^*, \eta_{ijl}^*)}{\partial q_{ijl}} + \frac{\partial c_j(q_j^*)}{\partial q_{ijl}} + \frac{\partial \hat{c}_{ijl}(q_{ijl}^*, \eta_{ijl}^*)}{\partial q_{ijl}} + \alpha_i\frac{\partial e^{i*}}{\partial q_{ijl}}\right.$$

$$\left. +\alpha_j\frac{\partial e^{j*}}{\partial q_{ijl}} + \omega_i\frac{\partial r^{i*}}{\partial q_{ijl}} + \omega_j\frac{\partial r^{j*}}{\partial q_{ijl}} - \lambda_j^*\right] \times \left[q_{ijl} - q_{ijl}^*\right]$$

$$+\sum_{i=1}^{m}\sum_{k=1}^{o}\left[\frac{\partial f_i(q_i^*)}{\partial q_{ik}} + \frac{\partial c_{ik}(q_{ik}^*, \eta_{ik}^*)}{\partial q_{ik}} + \alpha_i\frac{\partial e^{i*}}{\partial q_{ik}} + \omega_i\frac{\partial r^{i*}}{\partial q_{ik}} + \hat{c}_{ik}^* - \rho_{3k}^*\right] \times \left[q_{ik} - q_{ik}^*\right]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{2}\left[\frac{\partial c_{jkl}(q_{jkl}^*, \eta_{jkl}^*)}{\partial q_{jkl}} + \alpha_j\frac{\partial e^{j*}}{\partial q_{jkl}} + \omega_j\frac{\partial r^{j*}}{\partial q_{jkl}} + \hat{c}_{jkl}^* + \lambda_j^* - \rho_{3k}^*\right] \times \left[q_{jkl} - q_{jkl}^*\right]$$

$$+\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\left[\frac{\partial c_{ijl}(q^*_{ijl},\eta^*_{ijl})}{\partial \eta_{ijl}}+\frac{\partial b_{ijl}(\eta^*_{ijl})}{\partial \eta_{ijl}}+\frac{\partial \hat{b}_{ijl}(\eta^*_{ijl})}{\partial \eta_{ijl}}+\frac{\partial \hat{c}_{ijl}(q^*_{ijl},\eta^*_{ijl})}{\partial \eta_{ijl}}+\alpha_i\frac{\partial e^{i*}}{\partial \eta_{ijl}}\right.$$

$$\left.+\alpha_j\frac{\partial e^{j*}}{\partial \eta_{ijl}}+\omega_i\frac{\partial r^{i*}}{\partial \eta_{ijl}}+\omega_j\frac{\partial r^{j*}}{\partial \eta_{ijl}}\right]\times\left[\eta_{ijl}-\eta^*_{ijl}\right]$$

$$+\sum_{i=1}^{m}\sum_{k=1}^{o}\left[\frac{\partial c_{ik}(q^*_{ik},\eta^*_{ik})}{\partial \eta_{ik}}+\frac{\partial b_{ik}(\eta^*_{ik})}{\partial \eta_{ik}}+\alpha_i\frac{\partial e^{i*}}{\partial \eta_{ik}}+\omega_i\frac{\partial r^{i*}}{\partial \eta_{ik}}\right]\times\left[\eta_{ik}-\eta^*_{ik}\right]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{2}\left[\frac{\partial c_{jkl}(q^*_{jkl},\eta^*_{jkl})}{\partial \eta_{jkl}}+\frac{\partial b_{jkl}(\eta^*_{jkl})}{\partial \eta_{jkl}}+\alpha_j\frac{\partial e^{j*}}{\partial \eta_{jkl}}+\omega_j\frac{\partial r^{j*}}{\partial \eta_{jkl}}\right]\times\left[\eta_{jkl}-\eta^*_{jkl}\right]$$

$$+\sum_{j=1}^{n}\left[\sum_{i=1}^{m}\sum_{l=1}^{2}q^*_{ijl}-\sum_{k=1}^{o}\sum_{l=1}^{2}q^*_{jkl}\right]\times\left[\lambda_j-\lambda^*_j\right]$$

$$+\sum_{k=1}^{o}\left[\sum_{j=1}^{n}\sum_{l=1}^{2}q^*_{jkl}+\sum_{i=1}^{m}q^*_{ik}-d_k(\rho^*_3)\right]\times\left[\rho_{3k}-\rho^*_{3k}\right]\geq 0,\quad \forall (Q^1,Q^2,Q^3,\eta^1,\eta^2,\eta^3,\lambda,\rho_3)\in\mathcal{K}, \tag{36}$$

where

$$\mathcal{K}\equiv\left[(Q^1,Q^2,Q^3,\eta^1,\eta^2,\eta^3,\lambda,\rho_3)|q_{ijl}\geq 0,\ q_{ik}\geq 0,\ q_{jkl}\geq 0,\ 0\leq\eta_{ijl}\leq 1,\right.$$

$$\left.0\leq\eta_{ik}\leq 1,\ 0\leq\eta_{jkl}\leq 1,\ \rho_{3k}\geq 0,\ \lambda_j\geq 0,\forall i,j,k,l\right]. \tag{37}$$

Proof. Summation of inequalities (13), (27), and (35), yields, after algebraic simplification, the variational inequality (36). We now establish the converse, that is, that a solution to variational inequality (36) satisfies the sum of conditions (13), (27), and (35) and is, hence, an equilibrium according to Definition 1. To inequality (36) add the term $+\rho^*_{1ijl}$ - ρ^*_{1ijl} to the first set of brackets preceding the multiplication sign. Similarly, add the term $+\rho^*_{1ik}$ - ρ^*_{1ik} to the term in brackets preceding the second multiplication sign. Finally, add the term $+\rho^*_{2jkl}-\rho^*_{2jkl}$ to the term preceding the third multiplication sign in (36). The addition of such terms does not alter (36) since the value of these terms is zero. The resulting inequality can be rewritten to become equivalent to the price and material flow pattern satisfying the sum of the conditions (13), (27), and (35). The proof is complete. □

We now put variational inequality (36) into standard form which will be utilized in the subsequent sections. For additional background on variational inequalities and their applications, see the book by Nagurney (1999). In particular, we have that variational inequality (36) can be expressed as:

$$\langle F(X^*),X-X^*\rangle\geq 0,\quad \forall X\in\mathcal{K}, \tag{38}$$

where $X \equiv (Q^1, Q^2, Q^3, \eta^1, \eta^2, \eta^3, \lambda, \rho_3)$ and $F(X) \equiv (F_{ijl}, F_{ik}, F_{jkl}, \hat{F}_{ijl}, \hat{F}_{ik}, \hat{F}_{jkl}, F_j, F_k)$ with indices: $i = 1, \ldots, m$; $j = 1, \ldots, n$; $k = 1, \ldots, o$; $l = 1, 2$, and the specific components of F given by the functional terms preceding the multiplication signs in (36), respectively. The term $\langle \cdot, \cdot \rangle$ denotes the inner product in N-dimensional Euclidean space.

We now describe how to recover the prices associated with the first two tiers of nodes in the supply chain network. Clearly, the components of the vector ρ_3^* are obtained directly from the solution of variational inequality (36). In order to recover the second tier prices associated with the retailers one can (after solving variational inequality (36) for the particular numerical problem) *either* (cf. (32)) set $\rho^*_{2jkl} = \left[\rho^*_{3k} - \hat{c}^*_{jkl}\right]$, for any j, k, l such that $q^*_{jkl} > 0$, *or* (cf. (27)) for any $q^*_{jkl} > 0$, set $\rho^*_{2jkl} = \left[\frac{\partial c_{jkl}(q^*_{jkl}, \eta^*_{jkl})}{\partial q_{jkl}} + \alpha_j \frac{\partial e^{j*}}{\partial q_{jkl}} + \omega_j \frac{\partial r^{j*}}{\partial q_{jkl}} + \lambda^*_j\right]$.

Similarly, from (13) we can infer that the top tier prices comprising the vector ρ_1^* can be recovered (once the variational inequality (36) is solved with particular data) thus: for any i, j, l, such that $q^*_{ijl} > 0$, set $\rho^*_{1ijl} = \left[\frac{\partial f_i(q^*_i)}{\partial q_{ijl}} + \frac{\partial c_{ijl}(q^*_{ijl}, \eta^*_{ijl})}{\partial q_{ijl}} + \alpha_i \frac{\partial e^{i*}}{\partial q_{ijl}} + \omega_i \frac{\partial r^{i*}}{\partial q_{ijl}}\right]$, or, equivalently to $\left[\lambda^*_j - \frac{\partial c_j(q^*_j)}{\partial q_{ijl}} - \frac{\partial \hat{c}_{ijl}(q^*_{ijl}, \eta^*_{ijl})}{\partial q_{ijl}} - \alpha_j \frac{\partial e^{j*}}{\partial q_{ijl}} - \omega_j \frac{\partial r^{j*}}{\partial q_{ijl}}\right]$ (cf. (27)).

In addition, in order to recover the first tier prices associated with the demand market one can (after solving variational inequality (36) for the particular numerical problem) *either* (cf. (13)) set $\rho^*_{1ik} = \left[\frac{\partial f_i(q^*_i)}{\partial q_{ik}} + \frac{\partial c_{ik}(q^*_{ik}, \eta^*_{ik})}{\partial q_{ik}} + \alpha_i \frac{\partial e^{i*}}{\partial q_{ik}} + \omega_i \frac{\partial r^{i*}}{\partial q_{ik}}\right]$, for any i, k such that $q^*_{ik} > 0$, *or* (cf. (33)) for any $q^*_{ik} > 0$, set $\rho^*_{1ik} = \left[\rho^*_{3k} - \hat{c}^*_{ik}\right]$.

Under the above pricing mechanism, the optimality conditions (13) and (27) as well as the equilibrium conditions (35) also hold separately (as well as for each individual decision-maker).

3. The Dynamic Adjustment Process

In this section, we describe the dynamics associated with the network model developed in Section 2 and formulate the corresponding dynamic model as a projected dynamical system (cf. Nagurney and Zhang, 1996; Nagurney and Matsypura, 2004). Importantly, the set of stationary points of the projected dynamical system which formulates the dynamic adjustment process will coincide with the set of solutions to the variational inequality problem (36). In particular, we describe the disequilibrium dynamics of the product flows, the levels of social responsibility activities, as well as the prices.

3.1. The Dynamics of the product shipments

3.1.1. The Dynamics of the product shipments between manufacturers and retailers

The dynamics of the product transactions between manufacturers and the retailers in the different modes are now described. Note that in order for a transaction between nodes in these two tiers to take place there must be agreement between the pair of decision-makers. Towards that end, we let $\dot{q}_{ijl}$ denote the rate of change of the product transaction between manufacturer i and retailer j transacted via mode l and mathematically can express it in the following way: $\forall i, j, l$,

$$\dot{q}_{ijl} = \begin{cases} \phi_{ijl}(\lambda_j - \frac{\partial f_i}{\partial q_{ijl}} - \frac{\partial c_{ijl}}{\partial q_{ijl}} - \frac{\partial c_j}{\partial q_{ijl}} - \frac{\partial \hat{c}_{ijl}}{\partial q_{ijl}} - \alpha_i \frac{\partial e^i}{\partial q_{ijl}} - \alpha_j \frac{\partial e^j}{\partial q_{ijl}} - \omega_i \frac{\partial r^i}{\partial q_{ijl}} - \omega_j \frac{\partial r^j}{\partial q_{ijl}}), \quad \text{if} \quad q_{ijl} > 0, \\ \max\{0, \phi_{ijl}(\lambda_j - \frac{\partial f_i}{\partial q_{ijl}} - \frac{\partial c_{ijl}}{\partial q_{ijl}} - \frac{\partial c_j}{\partial q_{ijl}} - \frac{\partial \hat{c}_{ijl}}{\partial q_{ijl}} - \alpha_i \frac{\partial e^i}{\partial q_{ijl}} - \alpha_j \frac{\partial e^j}{\partial q_{ijl}} - \omega_i \frac{\partial r^i}{\partial q_{ijl}} - \omega_j \frac{\partial r^j}{\partial q_{ijl}})\}, \\ \quad \text{if} \quad q_{ijl} = 0. \end{cases} \tag{39}$$

Here ϕ_{ijl} denotes the (positive) speed of adjustment associated with the particular transaction.

The expression above states that whenever there is a positive difference between the shadow price of the retailer and the aggregated marginal costs, weighted marginal risk and emission associated with a transaction, the amount of "flow" on that particular link will increase with the speed ϕ_{ijl}. Otherwise, the amount of flow will either decrease with the same speed or remain the same. Moreover, we guarantee that the volume of product transacted never becomes negative.

Note that, as in Nagurney and Matsypura (2004), the adjustment assumptions presented here were suggested by a number of prominent economists (see, e.g., Fisher, 1961) and are based on the following reasoning. The rate of change in the amount of good produced/transacted by a particular decision-maker is proportional to his marginal profit and has the same sign. In other words, a typical decision-maker will try to increase his output for as long as the marginal profit is nonnegative, and will try to decrease his output for as long as the marginal profit is negative until the output is zero (that is, hits the boundary for the variable).

3.1.2. The dynamics of the product shipments between manufacturers and demand markets

The rate of change of the product transactions between a manufacturer and demand market pair is assumed to be proportional to the price the consumers are willing to pay minus the various costs, including marginal ones, that the manufacturer incurs when transacting with the demand market and the weighted marginal risk and emission. We denote this rate of change by $\dot{q}_{ik}$, and mathematically, express it in the following way: $\forall i,k$

$$\dot{q}_{ik} = \begin{cases} \phi_{ik}(\rho_{3k} - \frac{\partial f_i}{\partial q_{ik}} - \frac{\partial c_{ik}}{\partial q_{ik}} - \alpha_i \frac{\partial e^i}{\partial q_{ik}} - \omega_i \frac{\partial r^i}{\partial q_{ik}} - \hat{c}_{ik}), & \text{if} \quad q_{ik} > 0, \\ \max\{0, \phi_{ik}(\rho_{3k} - \frac{\partial f_i}{\partial q_{ik}} - \frac{\partial c_{ik}}{\partial q_{ik}} - \alpha_i \frac{\partial e^i}{\partial q_{ik}} - \omega_i \frac{\partial r^i}{\partial q_{ik}} - \hat{c}_{ik})\}, & \text{if} \quad q_{ik} = 0. \end{cases} \tag{40}$$

Here, similar to (39), the term ϕ_{ik} for all i,k, which is assumed to be positive, is a speed of adjustment. Note that (40) guarantees that the volume of product transacted will not take on a negative value.

Hence, (40) expresses that the volume of transactions between a manufacturer and demand market pair will increase if the price associated with the product at the demand market exceeds the various marginal costs and risks.

3.1.3. The dynamics of the product shipments between the retailers and the demand markets

The rate of change of the product flow q_{jkl}, denoted by $\dot{q}_{jkl}$, is assumed to be proportional to the difference between the price the consumers are willing to pay for the product at the demand market minus the price charged and the various transaction costs and the weighted marginal risk and emission associated with the transaction. Here the rate of proportionality (speed of adjustment) is $\phi_{jkl} > 0$ and we also guarantee that the product flows do not become negative. Hence, we may write: for every j,k,l:

$$\dot{q}_{jkl} = \begin{cases} \phi_{jkl}(\rho_{3k} - \frac{\partial c_{jkl}}{\partial q_{jkl}} - \alpha_j \frac{\partial e^j}{\partial q_{jkl}} - \omega_j \frac{\partial r^j}{\partial q_{jkl}} - \hat{c}_{jkl} - \lambda_j), & \text{if} \quad q_{jkl} > 0, \\ \max\{0, \phi_{jkl}(\rho_{3k} - \frac{\partial c_{jkl}}{\partial q_{jkl}} - \alpha_j \frac{\partial e^j}{\partial q_{jkl}} - \omega_j \frac{\partial r^j}{\partial q_{jkl}} - \hat{c}_{jkl} - \lambda_j)\}, & \text{if} \quad q_{jkl} = 0. \end{cases} \tag{41}$$

According to (41), if the price that the consumers are willing to pay for the product exceeds the price that the retailer charges and the various transaction costs and weighted marginal risk, then the volume of flow of the product to that demand market will increase; otherwise, it will decrease (or remain unchanged).

3.2. The dynamics of the levels of social responsibility activities

3.2.1. The dynamics of the levels of social responsibility activities between the manufacturers and the retailers

Now the dynamics of the levels of social responsibility activities between the manufacturers and the retailers are described. The rate of change of the level of social responsibility activities η_{ijl}, denoted by $\dot{\eta}_{ijl}$, is assumed to be proportional to the negative of the sum of the marginal costs and the weighted marginal risks and emission. Again, one must also guarantee that levels of social responsibility activities do not become negative. Moreover, they may not exceed the level equal to one. Hence, we can immediately write:

$$\dot{\eta}_{ijl} = \begin{cases} \hat{\phi}_{ijl}(-\frac{\partial c_{ijl}}{\partial \eta_{ijl}} - \frac{\partial b_{ijl}}{\partial \eta_{ijl}} - \frac{\partial \hat{b}_{ijl}}{\partial \eta_{ijl}} - \frac{\partial \hat{c}_{ijl}}{\partial \eta_{ijl}} - \alpha_i \frac{\partial e^i}{\partial \eta_{ijl}} - \alpha_j \frac{\partial e^j}{\partial \eta_{ijl}} - \omega_i \frac{\partial r^i}{\partial \eta_{ijl}} \\ -\omega_j \frac{\partial r^j}{\partial \eta_{ijl}}), \qquad \text{if} \quad 0 < \eta_{ijl} < 1, \\ \\ \min\{1, \max\{0, \hat{\phi}_{ijl}(-\frac{\partial c_{ijl}}{\partial \eta_{ijl}} - \frac{\partial b_{ijl}}{\partial \eta_{ijl}} - \frac{\partial \hat{b}_{ijl}}{\partial \eta_{ijl}} - \frac{\partial \hat{c}_{ijl}}{\partial \eta_{ijl}} - \alpha_i \frac{\partial e^i}{\partial \eta_{ijl}} \\ -\alpha_j \frac{\partial e^j}{\partial \eta_{ijl}} - \omega_i \frac{\partial r^i}{\partial \eta_{ijl}} - \omega_j \frac{\partial r^j}{\partial \eta_{ijl}})\}\}, \qquad \text{otherwise,} \end{cases} \tag{42}$$

where $\hat{\phi}_{ijl} > 0$ is the speed of adjustment.

This shows that if the marginal reduction of transaction costs, weighted risk and emission is greater than the marginal cost of social responsibility activities between the manufacturer and the retailer, then the level of social responsibility activities between that manufacturer and the retailer pair will increase with speed $\hat{\phi}_{ijl}$. If it is lower, the level of social responsibility activities will decrease with the same speed $\hat{\phi}_{ijl}$. Note that the increase on the levels of social responsibility activities are supposed to reduce transaction costs, risk and emissions generated in the production and delivery of the product.

3.2.2. The dynamics of the the levels of social responsibility activities between the manufacturers and the demand markets

Here we describe the dynamics of the levels of social responsibility activities between the manufacturers and the demand markets. The rate of change of level of social responsibility activities η_{ik} in turn, is assumed to be proportional to the negative of the sum of the marginal cost and weighted marginal risks and emission. One also must guarantee that these levels of social responsibility activities do not become negative (nor higher than one). Hence, one may write:

$$\dot{\eta}_{ik} = \begin{cases} \hat{\phi}_{ik}(-\frac{\partial c_{ik}}{\partial \eta_{ik}} - \frac{\partial b_{ik}}{\partial \eta_{ik}} - \alpha_i \frac{\partial e^i}{\partial \eta_{ik}} - \omega_i \frac{\partial r^i}{\partial \eta_{ik}}), & \text{if} \quad 0 < \eta_{ik} < 1, \\ \\ \min\{1, \max\{0, \hat{\phi}_{ik}(-\frac{\partial c_{ik}}{\partial \eta_{ik}} - \frac{\partial b_{ik}}{\partial \eta_{ik}} - \alpha_i \frac{\partial e^i}{\partial \eta_{ik}} - \omega_i \frac{\partial r^i}{\partial \eta_{ik}})\}\}, & \text{otherwise,} \end{cases} \tag{43}$$

where $\dot{\eta}_{ik}$ denotes the rate of change of the level of social responsibility activities η_{ik} and $\hat{\phi}_{ik} > 0$ is the speed of adjustment.

This shows that if the marginal reduction of transaction cost, weighted risk and emission for the manufacturer is greater than the marginal cost of social responsibility activities, then the level of social responsibility activities between that manufacturer and demand market pair will increase with speed $\hat{\phi}_{ik}$. If it is lower, the level of social responsibility activities will decrease with the same speed $\hat{\phi}_{ik}$. Of course, the bounds on the level of social responsibility activities must also hold.

3.2.3. The dynamics of the levels of social responsibility activities between the retailers and the demand markets

The dynamics of the levels of social responsibility activities between the retailers and demand markets are now described. The rate of change of the level of social responsibility activities $\dot{\eta}_{jkl}$ transacted via mode l is assumed to be proportional to the negative of the sum of the marginal cost and weighted marginal risk and emission, where, of course, one also must guarantee that the levels of social responsibility activities do not become negative nor exceed one. Hence, one may write:

$$\dot{\eta}_{jkl} = \begin{cases} \hat{\phi}_{jkl}(-\frac{\partial c_{jkl}}{\partial \eta_{jkl}} - \frac{\partial b_{jkl}}{\partial \eta_{jkl}} - \alpha_j \frac{\partial e^j}{\partial \eta_{jkl}} - \omega_j \frac{\partial r^j}{\partial \eta_{jkl}}), \text{ if } \quad 0 < \eta_{jkl} < 1, \\ \\ \min\{1, \max\{0, \hat{\phi}_{jkl}(-\frac{\partial c_{jkl}}{\partial \eta_{jkl}} - \frac{\partial b_{jkl}}{\partial \eta_{jkl}} - \alpha_j \frac{\partial e^j}{\partial \eta_{jkl}} - \omega_j \frac{\partial r^j}{\partial \eta_{jkl}})\}\}, \qquad \text{otherwise,} \end{cases} \tag{44}$$

where $\hat{\phi}_{jkl} > 0$ is the speed of adjustment and $\dot{\eta}_{jkl}$ denotes the rate of change of the level of social responsibility activities η_{jkl}. Expression (44) reveals that if the marginal reduction of transaction cost, weighted risk and emission for the retailer with the demand market is higher than the total marginal cost of social responsibility activities, then the level of social responsibility activities between that retailer and demand market pair will increase with the speed $\hat{\phi}_{jkl}$. If it is lower, the level of social responsibility activities will decrease the same speed.

3.3. Price dynamics

3.3.1. Demand market price dynamics

As in Nagurney and Matsypura (2004), we assume that the rate of change of the price ρ_{3k}, denoted by $\dot{\rho}_{3k}$, is proportional to the difference between the demand for the product at the demand market and the amount of the product actually available at that particular market (see also Samuelson, 1941; Metzler, 1945; Arrow and Hurwicz, 1958). Moreover, the rate of change of the price ρ_{3k} has the same sign as the difference between the demand and the total amount transacted with the demand market. Let ϕ_k denote the speed of adjustment associated with the price at demand market k. This term or factor is assumed to be positive.

Thus, the dynamics of the price ρ_{3k} for each k can be expressed as:

$$\dot{\rho}_{3k} = \begin{cases} \phi_k(d_k(\rho_3) - \sum_{j=1}^n \sum_{l=1}^2 q_{jkl} - \sum_{i=1}^m q_{ik}), & \text{if} \quad \rho_{3k} > 0 \\ \max\{0, \phi_k(d_k(\rho_3) - \sum_{j=1}^n \sum_{l=1}^2 q_{jkl} - \sum_{i=1}^m q_{ik})\}, & \text{if} \quad \rho_{3k} = 0. \end{cases} \tag{45}$$

Note that ϕ_k may also be interpreted as the sensitivity of the specific demand market to the changes in the supply and/or demand of the product.

Hence, if the demand for the product at the demand market (at an instant in time) exceeds the amount available, the price of the product at that demand market will increase with the speed ϕ_k; if the amount available exceeds the demand at the price, then the price at the demand market will decrease with the same speed ϕ_k. Furthermore, we guarantee that the prices do not become negative.

3.3.2. The dynamics of the prices at the retailers

The prices at the retailers, whether they are physical or virtual, must reflect supply and demand conditions as well. In particular, we let $\dot{\lambda}_j$ denote the rate of change in the market clearing price associated with retailer j and we propose the following dynamic adjustment for every retailer j:

$$\dot{\lambda}_j = \begin{cases} \phi_j(\sum_{k=1}^{o}\sum_{l=1}^{2} q_{jkl} - \sum_{i=1}^{m}\sum_{l=1}^{2} q_{ijl}), & \text{if} \quad \lambda_j > 0 \\ \max\{0, \phi_j(\sum_{k=1}^{o}\sum_{l=1}^{2} q_{jkl} - \sum_{i=1}^{m}\sum_{l=1}^{2} q_{ijl})\}, & \text{if} \quad \lambda_j = 0. \end{cases} \tag{46}$$

Here, the term ϕ_j is the speed of adjustment of the price for the product at retailer j. This term is also assumed to be positive for all j's.

Hence, if the product flows from the manufacturers into a retailer exceed the amount demanded at the demand markets from the retailer, then the market-clearing price at that retailer will decrease with the speed ϕ_j; if, on the other hand, the volume of product flows into a retailer is less than that demanded by the consumers at the demand markets (and handled by the retailer), then the market-clearing price at that retailer will increase with the same speed ϕ_j. As in the case of the demand market prices, we guarantee that the prices charged by the retailers remain nonnegative through the above projection operation.

3.4. The projected dynamical system

We now turn to stating the complete dynamic model. In the dynamic model the flows evolve according to the mechanisms described above; specifically, the product shipments between manufacturers and retailers evolve according to (39) and the product shipments between manufacturers and demand markets evolve according to (40) for all manufacturers i. The product shipments between retailers and demand markets evolve according to (41) for all retailers j, demand markets k, and modes l. The levels of social responsibility activities between manufacturers and retailers for all modes l evolve according to (42), the levels of social responsibility activities between manufacturers i and demand markets k evolve according to (43), and the levels of social responsibility activities between retailers j and demand markets k for all modes l evolve according to (44). Furthermore, the prices associated with the retailers evolve according to (46) for all retailers j, and the demand market prices evolve according to (45) for all k.

Let now X denote the aggregate column vector $(Q^1, Q^2, Q^3, \eta^1, \eta^2, \eta^3, \lambda, \rho_3)$ in the feasible set $\mathcal{K}$. Define the column vector

$$\hat{F}(X) \equiv (\phi_{ijl}F_{ijl}, \phi_{ik}F_{ik}, \phi_{jkl}F_{jkl}, \hat{\phi}_{ijl}\hat{F}_{ijl}, \hat{\phi}_{il}\hat{F}_{ik}, \hat{\phi}_{jkl}\hat{F}_{jkl}, \phi_j F_j, \phi_k F_k)$$

with indices: $i = 1, \ldots, m$; $j = 1, \ldots, n$; $k = 1, \ldots, o$; $l = 1, 2$, and the specific components of $F(X) \equiv (F_{ijl}, F_{ik}, F_{jkl}, \hat{F}_{ijl}, \hat{F}_{ik}, \hat{F}_{jkl}, F_j, F_k)$ given by the functional terms preceding the multiplication signs in (36), respectively. Then the dynamic model described by (39)–(46) can be rewritten as a *projected dynamical system* (Nagurney and Zhang, 1996) defined by the following initial value problem:

$$\dot{X} = \Pi_{\mathcal{K}}(X, -\hat{F}(X)), \quad X(0) = X_0, \tag{47}$$

where $\Pi_{\mathcal{K}}$ is the projection operator of $-\hat{F}(X)$ onto $\mathcal{K}$ at X and
$X_0 = (Q^{10}, Q^{20}, Q^{30}, \eta^{10}, \eta^{20}, \eta^{30}, \lambda^0, \rho_3^0)$ is the initial point corresponding to the initial product flow and price pattern.

The trajectory of (47) describes the dynamic evolution of the levels of social responsibility activities and the product transactions on the supply chain network, the demand market prices and the Lagrange multipliers or shadow prices associated with the retailers. The projection operation guarantees the constraints underlying the network system are not violated. Recall that the constraint set $\mathcal{K}$ consists of the nonnegativity constraints associated with all the product flows, the prices, as well as the levels of social responsibility activities. Moreover, the levels of social responsibility activities are assumed to not exceed the value of one.

Following Dupuis and Nagurney (1993) and Nagurney and Zhang (1996), the following result is immediate.

Theorem 2 (Set of stationary points coincides with the set of solutions of a variational inequality problem). *Since the feasible set $\mathcal{K}$ is a convex polyhedron, the set of stationary points of the projected dynamical system given by (47), that is, X^* such that $0 = \Pi_{\mathcal{K}}(X^*, -\hat{F}(X^*))$, coincides with the set of solutions to the variational inequality problem given by: Determine $X^* \in \mathcal{K}$, such that*

$$\langle \hat{F}(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{48}$$

where $\hat{F}(X)$ and X are as defined above.

Corollary 1. *If the speeds of adjustment: $(\phi_{ijl}, \phi_{ik}, \phi_{jkl}, \hat{\phi}_{ijl}, \hat{\phi}_{il}, \hat{\phi}_{jkl}, \phi_j, \phi_k)$ are identically equal to 1, for all i, j, k, l, the variational inequality (48) takes the form of the variational inequality (38).*

Proof. The proof is trivial. $\square$

Even though the statement of the corollary may seem obvious, it provides interesting insights into the relationship between the equilibrium state of the economic system and the underlying dynamics and also raises further questions. For example, what can be said regarding the set of stationary points of the projected dynamical system (47), which coincides with the set of solutions to variational inequality (48), and the set of stationary points associated with a projected dynamical system in which all of the speeds of adjustment are equal to one (and the equivalent equilibria) (38)?

We provide the answer in the form of the following theorem, but, first, we need to introduce some notation. We express the components of $F(X)$ above, for simplicity, as $F_1, \ldots, F_N$ and recall that the feasible set is the nonnegative orthant. We also denote these equivalent variational inequalities by VI$(F, \mathcal{K})$.

Theorem 3. *Assume that $\mathcal{K}$ is the convex polyhedron given by R^N_+ and that $\phi \equiv (\phi_1, \ldots, \phi_N)$ is a vector of positive terms. Then, the set of stationary points of the ordinary differential equation given in (47), which coincides with the set of solutions X^* to variational inequality (48), are equivalent to the set of solutions to the variational inequality VI $(F, \mathcal{K})$ $(cf.(38))$ where:*

$$F \equiv (F_1, \ldots, F_N)$$

and

$$\hat{F} \equiv (\phi_1 F_1, \ldots, \phi_n F_N).$$

Proof. From Theorem 2 we know that the set of stationary points of the ODE (47) coincide with the set of solutions to VI$(\hat{F}, \mathcal{K})$. For the prove that the solutions to VI$(\hat{F}, \mathcal{K})$ coincide with the solutions to VI$(F, \mathcal{K})$ see Nagurney and Matsypura (2004). $\square$

This theoretical result can be summarized as follows. The set of equilibria of the supply chain network are *independent* of the speeds of adjustment. Hence, if one is interested merely in the computation of the equilibrium product flow and price patterns one may compute the solution to either variational inequality problem (36) or to variational inequality problem (38). However, if one is interested in tracking the trajectories to the equilibrium state then one should include the speeds of adjustment. We also note that the factors which we term "speeds of adjustment" can also capture conversion factors, if need be.

To conclude this Section, we present several properties of the PDS defined by the initial value problem (47) that follow from the theory of projected dynamical systems.

Theorem 4 (Existence and uniqueness of a solution to the initial value problem). *Assume that $F(X)$ is Lipschitz continuous, that is, that*

$$\|F(X') - F(X'')\| \leq \mathcal{L}\|X' - X''\|, \quad \forall X', X'' \in \mathcal{K}, \textit{ where } \mathcal{L} > 0, \tag{49}$$

Then, for any $X_0 \in \mathcal{K}$, there exists a unique solution $X_0(t)$ to the initial value problem (47).

Proof. Lipschitz continuity of the function F is sufficient for the result following Theorem 2.5 in Nagurney and Zhang (1996). □

Theorem 4 gives conditions under which the trajectories associated with the initial value problem are well-defined. Note that Lipschitz continuity is not an unreasonable condition for F to satisfy in our model (see Nagurney and Dong, 2002).

Under suitable conditions on the underlying functions,in particular, under monotonicity of the function F in (38), one can obtain stability results for the network.

4. The algorithm

In this section, we consider the computation of a stationary of (36). The algorithm that is proposed is the Euler-type method, which is induced by the general iterative scheme of Dupuis and Nagurney (1993). It has been applied to-date to solve a plethora of dynamic network models (see, e.g., Nagurney and Zhang, 1996; Nagurney and Dong 2002).

4.1. The Euler method

Step 0: Initialization

Set $X^0 = (Q^{10}, Q^{20}, Q^{30}, \eta^{10}, \eta^{20}, \eta^{30}, \lambda^0, \rho_3^0) \in \mathcal{K}$. Let $\mathcal{T}$ denote an iteration counter and set $\mathcal{T} = 1$. Set the sequence $\{a_\mathcal{T}\}$ so that $\sum_{\mathcal{T}=1}^{\infty} a_\mathcal{T} = \infty$, $a_\mathcal{T} > 0$, $a_\mathcal{T} \to 0$, as $\mathcal{T} \to \infty$ (which is a requirement for convergence).

Step 1: Computation

Compute $X^{\mathcal{T}} = (Q^{1\mathcal{T}}, Q^{2\mathcal{T}}, Q^{3\mathcal{T}}, \eta^{1\mathcal{T}}, \eta^{2\mathcal{T}}, \eta^{3\mathcal{T}}, \lambda^{\mathcal{T}}, \rho_3^{\mathcal{T}}) \in \mathcal{K}$ by solving the variational inequality subproblem:

$$\langle X^{\mathcal{T}} + a_{\mathcal{T}} F(X^{\mathcal{T}-1}) - X^{\mathcal{T}-1}, X - X^{\mathcal{T}} \rangle \geq 0, \qquad \forall X \in \mathcal{K}. \tag{50}$$

Step 2: Convergence Verification

If $|X^{\mathcal{T}} - X^{\mathcal{T}-1}| \leq \epsilon$, with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $\mathcal{T} := \mathcal{T} + 1$, and go to Step 1.

Convergence results for the Euler method can be found in Dupuis and Nagurney (1993). See the book by Nagurney and Dong (2002) for applications of this algorithm to other network problems in the context of dynamic supply chains and financial networks with intermediation.

Variational inequality subproblem (36) can be solved explicitly and in closed form. For completeness, and also to illustrate the simplicity of the proposed computational procedure in the context of the supply chain network model, we provide the explicit formulae for the computation of the $Q^{1\mathcal{T}}$, the $Q^{2\mathcal{T}}$, the $Q^{3\mathcal{T}}$, the $\eta^{1\mathcal{T}}$, the $\eta^{2\mathcal{T}}$ the $\eta^{3\mathcal{T}}$, the $\lambda^{\mathcal{T}}$, and the $\rho_3^{\mathcal{T}}$ (cf. (36)) below.

4.1.1. Computation of the products flows

At iteration $\mathcal{T}$, the $q_{ijl}^{\mathcal{T}}$s according to:

$$q_{ijl}^{\mathcal{T}} = \max\{0, q_{ijl}^{\mathcal{T}-1} - a_{\mathcal{T}} \phi_{ijl} (\frac{\partial f_i(q_i^{\mathcal{T}-1})}{\partial q_{ijl}} + \frac{\partial c_{ijl}(q_{ijl}^{\mathcal{T}-1}, \eta_{ijl}^{\mathcal{T}-1})}{\partial q_{ijl}} + \frac{\partial c_j(q_j^{\mathcal{T}-1})}{\partial q_{ijl}} + \frac{\partial \hat{c}_{ijl}(q_{ijl}^{\mathcal{T}-1}, \eta_{ijl}^{\mathcal{T}-1})}{\partial q_{ijl}}$$
$$+\alpha_i \frac{\partial e^i(Q^{1\mathcal{T}-1}, Q^{2\mathcal{T}-1}, \eta_1^{\mathcal{T}-1}, \eta_2^{\mathcal{T}-1})}{\partial q_{ijl}} + \alpha_j \frac{\partial e^j(Q^{1\mathcal{T}-1}, Q^{3\mathcal{T}-1}, \eta_1^{\mathcal{T}-1}, \eta_3^{\mathcal{T}-1})}{\partial q_{ijl}}$$
$$+\omega_i \frac{\partial r^i(Q^{1\mathcal{T}-1}, Q^{2\mathcal{T}-1}, \eta_1^{\mathcal{T}-1}, \eta_2^{\mathcal{T}-1})}{\partial q_{ijl}} + \omega_j \frac{\partial r^j(Q^{1\mathcal{T}-1}, Q^{3\mathcal{T}-1}, \eta_1^{\mathcal{T}-1}, \eta_3^{\mathcal{T}-1})}{\partial q_{ijl}} - \lambda_j^{\mathcal{T}-1})\}, \quad \forall i, j, l \tag{51}$$

at iteration $\mathcal{T}$, the $q_{ik}^{\mathcal{T}}$s according to:

$$q_{ik}^{\mathcal{T}} = \max\{0, q_{ik}^{\mathcal{T}-1} - a_{\mathcal{T}} \phi_{ik} (\frac{\partial f_i(q_i^{\mathcal{T}-1})}{\partial q_{ik}} + \frac{\partial c_{ik}(q_{ik}^{\mathcal{T}-1}, \eta_{ik}^{\mathcal{T}-1})}{\partial q_{ik}} + \alpha_i \frac{\partial e^i(Q^{1\mathcal{T}-1}, Q^{2\mathcal{T}-1}, \eta_1^{\mathcal{T}-1}, \eta_2^{\mathcal{T}-1})}{\partial q_{ik}}$$
$$+\omega_i \frac{\partial r^i(Q^{1\mathcal{T}-1}, Q^{2\mathcal{T}-1}, \eta_1^{\mathcal{T}-1}, \eta_2^{\mathcal{T}-1})}{\partial q_{ik}} + \hat{c}_{ik}(Q^{2\mathcal{T}-1}, Q^{3\mathcal{T}-1}, \eta_2^{\mathcal{T}-1}, \eta_3^{\mathcal{T}-1}) - \rho_{3k}^{\mathcal{T}-1})\}, \quad \forall i, k, \tag{52}$$

and at iteration $\mathcal{T}$, the $q_{jkl}^{\mathcal{T}}$s, according to:

$$q_{jkl}^{\mathcal{T}} = \max\{0, q_{jkl}^{\mathcal{T}-1} - a_{\mathcal{T}} \phi_{jkl} (\alpha_j \frac{\partial e^j(Q^{1\mathcal{T}-1}, Q^{3\mathcal{T}-1}, \eta_1^{\mathcal{T}-1}, \eta_3^{\mathcal{T}-1})}{\partial q_{jkl}} + \omega_j \frac{\partial r^j(Q^{1\mathcal{T}-1}, Q^{3\mathcal{T}-1}, \eta_1^{\mathcal{T}-1}, \eta_3^{\mathcal{T}-1})}{\partial q_{jkl}}$$

$$+\frac{\partial c_{jkl}(q_{jkl}^{\mathcal{T}-1},\eta_{jkl}^{\mathcal{T}-1})}{\partial q_{jkl}}+\hat{c}_{jkl}(Q^{2\mathcal{T}-1},Q^{3\mathcal{T}-1},\eta_2^{\mathcal{T}-1},\eta_3^{\mathcal{T}-1})+\lambda_j^{\mathcal{T}-1}-\rho_{3k}^{\mathcal{T}-1})\},\quad \forall j,k,l. \tag{53}$$

4.1.2. Computation of the levels of social responsibility activities

At iteration $\mathcal{T}$ compute the $\eta_{ijl}^{\mathcal{T}}s$ according to:

$$\eta_{ijl}^{\mathcal{T}}=\min\{1,\max\{0,\eta_{ijl}^{\mathcal{T}-1}-a_{\mathcal{T}}\hat{\phi}_{ijl}(\frac{\partial c_{ijl}(q_{ijl}^{\mathcal{T}-1},\eta_{ijl}^{\mathcal{T}-1})}{\partial \eta_{ijl}}+\frac{\partial b_{ijl}(\eta_{ijl}^{\mathcal{T}-1})}{\partial \eta_{ijl}}+\frac{\partial \hat{b}_{ijl}(\eta_{ijl}^{\mathcal{T}-1})}{\partial \eta_{ijl}}+\frac{\partial \hat{c}_{ijl}(q_{ijl}^{\mathcal{T}-1},\eta_{ijl}^{\mathcal{T}-1})}{\partial \eta_{ijl}}$$

$$+\alpha_i\frac{\partial e^i(Q^{1\mathcal{T}-1},Q^{2\mathcal{T}-1},\eta_1^{\mathcal{T}-1},\eta_2^{\mathcal{T}-1})}{\partial \eta_{ijl}}+\alpha_j\frac{\partial e^j(Q^{1\mathcal{T}-1},Q^{3\mathcal{T}-1},\eta_1^{\mathcal{T}-1},\eta_3^{\mathcal{T}-1})}{\partial \eta_{ijl}}$$

$$+\omega_i\frac{\partial r^i(Q^{1\mathcal{T}-1},Q^{2\mathcal{T}-1},\eta_1^{\mathcal{T}-1},\eta_2^{\mathcal{T}-1})}{\partial \eta_{ijl}}+\omega_j\frac{\partial r^j(Q^{1\mathcal{T}-1},Q^{3\mathcal{T}-1},\eta_1^{\mathcal{T}-1},\eta_3^{\mathcal{T}-1})}{\partial \eta_{ijl}})\}\},\quad \forall i,j,l. \tag{54}$$

Furthermore, at iteration $\mathcal{T}$ compute the $\eta_{ik}^{\mathcal{T}}$ according to:

$$\eta_{ik}^{\mathcal{T}}=\min\{1,\max\{0,\eta_{ik}^{\mathcal{T}-1}-a_{\mathcal{T}}\hat{\phi}_{ik}(\frac{\partial c_{ik}(q_{ik}^{\mathcal{T}-1},\eta_{ik}^{\mathcal{T}-1})}{\partial \eta_{ik}}+\alpha_i\frac{\partial e_{ik}(q_{ik}^{\mathcal{T}-1},\eta_{ik}^{\mathcal{T}-1})}{\partial \eta_{ik}}$$

$$+\frac{\partial b_{ik}(\eta_{ik}^{\mathcal{T}-1})}{\partial \eta_{ik}}+\omega_i\frac{\partial r_{ik}(q_{ik}^{\mathcal{T}-1},\eta_{ijl}^{\mathcal{T}-1})}{\partial \eta_{ik}})\}\},\quad \forall i,k. \tag{55}$$

At iteration $\mathcal{T}$ compute the $\eta_{jkl}^{\mathcal{T}}$ according to:

$$\eta_{jkl}^{\mathcal{T}}=\min\{1,\max\{0,\eta_{jkl}^{\mathcal{T}-1}-a_{\mathcal{T}}\hat{\phi}_{jkl}(\frac{\partial c_{jkl}(q_{jkl}^{\mathcal{T}-1},\eta_{jkl}^{\mathcal{T}-1})}{\partial \eta_{jkl}}+\alpha_j\frac{\partial e^j(Q^{1\mathcal{T}-1},Q^{3\mathcal{T}-1},\eta_1^{\mathcal{T}-1},\eta_3^{\mathcal{T}-1})}{\partial \eta_{jkl}}$$

$$+\omega_j\frac{\partial r^j(Q^{1\mathcal{T}-1},Q^{3\mathcal{T}-1},\eta_1^{\mathcal{T}-1},\eta_3^{\mathcal{T}-1})}{\partial \eta_{jkl}}+\frac{\partial b_{jkl}(\eta_{jkl}^{\mathcal{T}-1})}{\partial \eta_{jkl}})\}\},\quad \forall j,k,l. \tag{56}$$

4.1.3. Computation of the prices

At iteration $\mathcal{T}$, compute the $\lambda_j^{\mathcal{T}}$s (shadow prices) according to:

$$\lambda_j^{\mathcal{T}}=\max\{0,\lambda_j^{\mathcal{T}-1}-a_{\mathcal{T}}\phi_j(\left[\sum_{i=1}^{m}\sum_{l=1}^{2}q_{ijl}^{\mathcal{T}-1}-\sum_{k=1}^{o}\sum_{l=1}^{2}q_{jkl}^{\mathcal{T}-1}\right])\},\quad \forall j, \tag{57}$$

Finally, at iteration $\mathcal{T}$ compute the demand market prices, the $\rho_{3k}^{\mathcal{T}}$s, according to:

$$\rho_{3k}^{\mathcal{T}}=\max\{0,\rho_{3k}^{\mathcal{T}-1}-a_{\mathcal{T}}\phi_k(\sum_{j=1}^{n}\sum_{l=1}^{2}q_{jkl}^{\mathcal{T}-1}+\sum_{i=1}^{m}q_{ik}^{\mathcal{T}-1}-d_k(\rho_3^{\mathcal{T}-1})))\},\quad \forall k. \tag{58}$$

As one can see in the discrete-time adjustment process(es) described above the algorithm is initialized with a vector of product flows, levels of social responsibility activities, and prices. For example, the levels of social responsibility activities may be set to zero (and the same holds for the prices, initially). The product flows, shadow prices, and the demand market prices are computed. In particular, the product transactions between manufacturers and retailers are computed according to (51), the product transactions between manufacturers and demand markets are computed according to (52) and the product transactions between retailers and demand markets are computed according to (53). The levels of social responsibility activities are computed according to (54), (55), and (56), respectively. Finally, the shadow prices are computed according to (57) and the demand market prices are computed according to (58).

The dynamic network system will then evolve according to the discrete-time adjustment process (51) through (58) until a stationary/equilibrium point of the projected dynamical system (47) (equivalently, and a solution to variational inequality (36)) is achieved. Once the convergence tolerance has been reached then the equilibrium conditions according to Definition 1 are satisfied as one can see from (51) through (58).

5. Numerical Examples

In this Section, we apply the Euler method described in Section 4 to several numerical examples. The Euler method was implemented in MATLAB 7 and the computer used was an ordinary IBM ThinkPad R51. The convergence criterion utilized was that the absolute value of the product transactions, levels of social responsibility activities, and prices between two successive iterations differed by no more than 10^{-4}. The parameter $a_{\mathcal{T}}$ in the Euler method (cf. (50)) was set to $.1\{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, ...\}$ for all the examples. The algorithm was initialized as follows: all the initial product transactions and prices were set equal to 1. The levels of social responsibility activities were set equal to 0.

The purpose of these examples is to illustrate the effects of social responsibility activities on the supply chain network. Detailed descriptions of the specific data for the examples are given below.

Example 1

The first numerical example consisted of two manufacturers, two retailers, two demand markets, with only physical transactions between manufacturers and retailers and the retailers and demand markets. Electronic transactions were not allowed. Hence, $m = 2$, $n = 2$, $o = 2$ with $l = 1$. This yielded a numerical example in which (cf. Figure 3) there were two top tier nodes in the supply chain network and two middle tier nodes and two bottom tier nodes. There was a single link from each top-tiered node to each middle-tiered node. There was a single link joining each of the two middle-tiered nodes with each bottom-tiered node. The data for the first example were constructed for easy interpretation purposes and to serve as a base line.

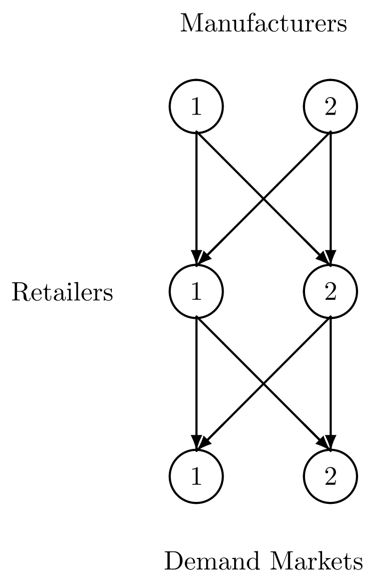

Figure 3: The network structure of the supply chain

The production cost functions faced by the manufacturers were

$$f^i(q_i) = 2.5(\sum_{j=1}^{2} q_{ijl})^2, \quad \text{for} \quad i = 1, 2.$$

The transaction cost functions faced by the manufacturers associated with transacting with the retailers were given by:

$$c_{ijl}(q_{ijl}, \eta_{ijl}) = (0.5 - 0.4\eta_{ijl})(q_{ijl})^2 + (3.5 - \eta_{ijl})q_{ijl}, \quad \text{for} \quad i = 1, 2; j = 1, 2; l = 1.$$

The handling costs of the retailers were given by:

$$c_j(q_j) = .5(\sum_{i=1}^{2} q_{ijl})^2, \quad \text{for} \quad j = 1, 2.$$

The transaction costs of the retailers associated with transacting with the manufacturers were given by:

$$\hat{c}_{ijl}(q_{ijl}, \eta_{ijl}) = 1.5{q_{ijl}}^2 + 3q_{ijl}, \quad \text{for} \quad i = 1, 2; l = 1; j = 1, 2.$$

The demand functions at the demand markets were:

$$d_1(\rho_3) = -2\rho_{31} - 1.5\rho_{32} + 1000, \quad d_2(\rho_3) = -2\rho_{32} - 1.5\rho_{31} + 1000,$$

and the transaction costs between the retailers and the consumers at the demand markets were given by:

$$\hat{c}_{jkl}(Q^2, Q^3, \eta^2, \eta^3) = (1 - \eta_{jkl})q_{jkl} + 5, \quad \text{for} \quad j = 1, 2; k = 1, 2; l = 1.$$

The emission functions were as follows:

$$e^i(Q^1, Q^2, \eta^1, \eta^2) = \sum_{j=1}^{2}(1 - \eta_{ijl})q_{ijl}, \quad \text{for} \quad i = 1, 2.$$

$$e^j(Q^1, Q^3, \eta^1, \eta^3) = \sum_{i=1}^{2}(1 - \eta_{ijl})q_{ijl} + \sum_{k=1}^{2}(1 - \eta_{jkl})q_{jkl}, \quad \text{for} \quad j = 1, 2.$$

The social responsibility activities cost functions were:

$$b_{ijl}(\eta_{ijl}) = 2\eta_{ijl}, \quad \text{for} \quad i = 1, 2; j = 1, 2; l = 1,$$

$$b_{jkl}(\eta_{jkl}) = 2\eta_{jkl}, \quad \text{for} \quad j = 1, 2; k = 1, 2; \hat{l} = 1.$$

All other functions were set equal to zero.

For the first example we assumed that all the weights associated with the different criteria were set equal to one by all the decision-makers. Hence, in this example, the manufacturers and the retailers assigned the same weight to profit maximization, risk minimization, and emissions value minimization. In addition, all levels of social responsibility variables η were kept at 0, which can be interpreted as that none of the decision makers has a possibility to invest into activities that promote quality assurance, environmental preservation, and/or compliance.

The computational results are presented in Table 1. The total emissions generated by each manufacturer were 30.8696 and for each retailer 61.7401.

Example 2

In the second example, the data was as in Example 1 except for the following changes: Manufacturer 1 was able to invest in the social responsibility activities on the link between him and Retailer 1. In other

Table 1: Solutions to Examples 1–3

Variable	Example 1	Example 2	Example 3
Product Transactions between Manufacturers and Retailers			
q_{111}	15.43	17.55	19.93
q_{121}	15.43	14.30	13.31
q_{211}	15.43	14.91	16.88
q_{221}	15.43	15.83	15.00
η_{111}	0	1	1
η_{121}	0	0	0
η_{211}	0	0	0
η_{221}	0	0	0
Product Transactions between Retailers and Demand Markets			
q_{111}	15.43	16.29	30.58
q_{121}	15.43	16.29	6.23
q_{211}	15.43	15.06	6.04
q_{221}	15.43	15.06	22.27
η_{111}	0	0	1
η_{121}	0	0	0
η_{211}	0	0	0
η_{221}	0	0	0
Shadow Prices at Retailers			
λ_1	255.45	254.46	272.29
λ_2	255.45	255.69	256.25
Prices at Demand Markets			
ρ_{31}	276.89	276.75	268.29
ρ_{32}	276.89	276.75	284.52

words variable η_{111} that corresponds to the link joining Manufacturer 1 and Retailer 1 can now take any value from 0 to 1.

As a result Manufacturer 1 decided to increase the level of social responsibility to its maximum (see Table 1). This resulted in the increase of the product flow between Manufacturer 1 and Retailer 1 and a decrease of product flow between Manufacturer 1 and Retailer 2. The effect on the product transactions from Manufacturer 2 was the opposite. The total emissions generated by Manufacturer 1 were now equal to 14.3019 and for Retailer 1 it was 47.4930. This represents a 52.3% reduction in total emissions generated for Manufacturer 1 and 23.07% reduction in total emissions for Retailer 1. Hence, as expected, environmentally conscious decision-makers can reduce the total emissions generated while increasing the levels of productions and transactions.

Example 3

In the third example, the data was as in Example 2 except for the following changes: Retailer 1 was now able to invest in the social responsibility activities on the link joining Retailer 1 with Demand Market 1. In other words, variable η_{111} that corresponds to the link joining Retailer 1 and Demand Market 1 can now take any value from 0 to 1.

Observe from Table 1 that the levels of social responsibility activities on the these links were at there maximum value of 1. As a result, the volumes of flow on the links from the Manufacturer 1 to Retailer 1 and from Retailer 1 to Demand Market 1 are further increased. Moreover, the total emissions generated were now further reduced and were equal to 13.31 for Manufacturer 1 and 23.11 for Retailer 1. Hence, environmentally conscious consumers could significantly reduce the environmental emissions through the economics and the underlying decision-making behavior in the supply chain network.

These examples (although stylized) have been presented to show both the model and the computational procedure. Obviously, different input data and dimensions of the problems solved will affect the equilibrium product transaction, levels of social responsibility activities, and price patterns. One now has a powerful environmental decision-making tool with which to explore the effects of perturbations to the data as well as the effects of changes in the number of manufacturers, retailers, demand markets, and levels of social responsibility activities.

6. Summary and Conclusions

In this paper, we developed a dynamic of supply chain networks with corporate social responsibility through integrated environmental decision-making. We model the multicriteria decision-making behavior of the various decision-makers, which includes the maximization of profit, the emission (waste) minimization, and the minimization of risk, in the presence of both business-to-business (B2B) and business-to-consumer (B2C) transactions. The network had three tiers of decision-makers, consisting of: manufacturers, retailers, as well as consumers associated with the demand markets. We allowed for physical as well as electronic transactions between the decision-makers in the network. The levels of social responsibility activities were allowed to affect not only risk and emission but also the transaction costs (by reducing them, in general) and did have associated costs.

We first modeled the network in equilibrium, in which the product transactions between the tiers as well as the levels of social responsibility activities coincide and established the variational inequality formulation of the governing equilibrium conditions. We then proposed the underlying (disequilibrium) dynamics associated with the continuous-time adjustment process(es) and constructed the projected dynamical system formulation. We proved that the set of stationary points of the projected dynamical system coincides with the set of solutions of the variational inequality problem. We also provided conditions under which the dynamic trajectories of the product flows, levels of social responsibility activities, and prices are well-defined. We proposed a discrete-time algorithm to approximate the continuous-time adjustment process.

The theoretical results established in this paper were then exploited in the computation of supply chain numerical examples with distinct weights associated with the environmental criteria and risk. The numerical examples illustrate the flexibility of the modeling schema. Moreover, the results in this paper add to the timely topic of rigorous model and quantitative tool development for combined supply chain network analysis and corporate social responsibility.

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