International Human Migration Networks Under Regulations

Anna Nagurney*
Department of Operations and Information Management
Isenberg School of Management
University of Massachusetts
Amherst, Massachusetts 01003

Patrizia Daniele
Department of Mathematics and Computer Science
University of Catania, Italy

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Abstract: International human migration has transformed economies and societies. The new millennium, with climate change and its impacts, and increasing conflicts and displacements, has experienced a great increase in international migrants, with associated challenges faced by governments. In this paper, we advance the modeling, analysis, and solution of international human migration problems by developing a network model with regulations. The formalism uses the theory of variational inequalities, coupled with Lagrange analysis, in order to gain insights as to the impacts of the regulations on utilities of multiple classes of migrants, and on the equilibrium flows. Our results add to the literature on operations research for societal impact, inspired by the real world.

Keywords: human migration, networks, variational inequalities, regulations

* corresponding author; email: nagurney@isenberg.umass.edu

1. Introduction

Humans have engaged in migration since time immemorial. Reasons for human migration are numerous, from individuals seeking better economic opportunities and enhanced prosperity for themselves and their families, to those fleeing conflict, violence, and persecution. With climate change and the increasing number and severity of natural disasters, including hurricanes, floods, tornados, earthquakes, etc., some migrants are seeking locations of greater expected safety and security. In 2017, the number of international migrants was an estimated 258 million persons (3.4% of the global population), with the total number of international migrants increasing by almost 50% since 2000 (United Nations (2017)). As also reported therein, the number of international migrants is growing faster than the global population. Moreover, in the same almost two decade period, the number of refugees and asylum seekers increased from 16 to 26 million, comprising about 10% of the international migrants.

Vivid depictions of people fleeing their origin locations permeate the news, whether attempting to escape the great strife and suffering in Syria (United Nations Refugee Agency (2019)); the violence in parts of Central America (Bartenstein and McDonald (2019)), the economic collapse of Venezuela (Kennedy (2019)), and even flooding in parts of Asia (Dash and Paul (2017)), as well as droughts in parts of Africa (cf. Linke et al. (2018)). Interestingly, since the onset of the new millennium, the countries experiencing the largest increase in their diaspora populations are the Syrian Arab Republic (872%), India (108%), and the Philippines (85%) (cf. United Nations (2017)). At times, refugees will travel in extremely dangerous conditions to escape the dire circumstances at their origin nodes. For example, the United Nations Refugee Agency (2015) reported a maritime refugee crisis with, in the first half of that year, 137,000 refugees crossing the Mediterranean Sea to Europe, via very risky transport modes, and with many more unsuccessfully attempting such a passage. 800 died in the largest refugee shipwreck on record that April. Clearly, the issue of human migration is one of multiple dimensions, including economic, political, sociological, and even environmental aspects.

Governments of various nations, hence, are increasingly being faced with multiple challenges associated with human migration flows. In response to challenges, they are adopting different regulations. According to Bertossi (2008), since the 1990s, unprecedented efforts have been made to control the movement of people across state borders. As argued eloquently by Helbling and Leblang (2019), migration regulations are core to national sovereignty since they directly impact the composition of the community that makes up the country. According to the informative report by the United Nations (2013), migration policies in both origin and destination countries play an important role in determining the migratory flows. Moreover, as noted therein, in managing international migration flows, governments usually focus on different types of migrants, of which the most salient are highly skilled workers, dependents of migrant workers, irregular migrants, and refugees and asylum seekers (cf. Karagiannis (2016)).

As an illustration of some of the challenges, in terms of refugees, Sweden, a country with only 10 million residents, responding to the refugee crisis of 2015, in the past five years, accepted more than 400,000 asylum-seekers and relatives of previous immigrants (Billner and Jefferson (2019)). This is the highest per capita number of refugees of all European countries. Sweden, along with Germany, was the preferred destination for a wave of Syrians, Afghans, and others who reached Europe in search of protection and better lives. However, as noted in Skodo (2018), eventually, and, in response to some backlash, the Swedish government introduced border controls. In mid-2016, with concerns over immigration growing among the Swedish public, Sweden instituted a highly restrictive asylum and reunification law – a major policy shift.

On the other hand, certain governments, including Canada and Australia, have been interested in acquiring highly skilled workers and have adopted highly-skilled migration policy regimes (Kerr et al. (2017)), with others, including the European Union, involved in studies to identify the best processes (see, e.g., Casarico and Uebelmesser (2018)). Some countries, including the United States, nevertheless, put a ceiling on the volume of immigrants from a given country (see American Immigration Council (2016)). Furthermore, there is now discussion that the US may be reducing the number of refugees allowed to enter the country (cf. Sullivan (2019)). As one might expect, due to different needs, pressures, etc., immigration laws and rules in the United States have changed through history (see Cohn (2015)).

Given that the United Nations (2013) identifies international migration as a global phenomenon that is growing in complexity, scope, and impact, it is quite relevant to revisit the modeling of international human migration networks in the context of different regulations. That is the goal of this paper.

2. Literature Review with a Focus on Networks and Migration

In this Section, we provide an overview of the relevant literature on human migration, with a focus on networks. For a critical review of a variety of theoretical frameworks for human migration models, which also reference foundational network equilibrium models, see Rahmati and Tularam (2017).

Nagurney (1989) introduced a multiclass migration equilibrium model, which did not include migration/movement costs, and was isomorphic to a traffic network equilibrium with special structure (see also Nagurney (1999)). In that model, there was a fixed population of each class of migrant and migrants distributed themselves among the locations according to maximal utilities associated with the locations. The utility functions were assumed to be concave and a function of the populations at the locations. The model was then extended to include flow-dependent migration costs and an expanded set of equilibrium conditions in Nagurney (1990). Subsequently, Nagurney, Pan, and Zhao (1992a), building on the latter work, proposed a multiclass human migration model, which was then further generalized to include class transformations in Nagurney, Pan, and Zhao (1992b). Note that a class of migrant was assumed to perceive utility associated with a location for migration in an individual way. The utility functions were functions of the populations of the different classes at the locations. Pan and Nagurney (1994), in turn, considered chain migration (unlike the earlier work) and introduced a multi-stage (but single class) Markov chain model. The authors established a connection between a sequence of variational inequalities and a non-homogeneous Markov chain. They also proved that, under certain assumptions, the stability of the one-step transition matrix guarantees the stability of the n-step transition matrix. In all of the above papers, the specific governing equilibrium conditions were formulated as finite-dimensional variational inequality problems. Pan and Nagurney (2006) utilized the methodology of evolution variational inequalities for the first time to model the dynamic adjustment of a socio-economic process in the context of human migration. The question of convergence of algorithms in this framework, which is infinite-dimensional, was also addressed (see also Daniele (2006)).

Interestingly, many of the network equilibrium models of human migration, as highlighted above, have also found application to the migration of animals in ecology with a focus on fish and maritime ecosystems (see Mullon and Nagurney (2012), Mullon (2014), Mariani et al. (2016)). Therein, utility functions as well as cost of migration, inspired by the above human migration models, are seminal constructs. Roy and Roy (2016) analyze the dynamics and the equilibria of a predator-prey fishery model that includes prey refuge and migration and harvesting. They also present stability results. Belen and Weber (2019) study the modelling and the limiting behaviors of stochastic information and stochastic prey-predators.

Kalashnikov et al. (2008) constructed a human migration model with a conjectural variations equilibrium (CVE). In contrast to previous research (see, e.g., Isac, Bulavsky, and Kalashnikov (2002)), they considered an extension in which the conjectural variations coefficients may not only be constants, but are (continuously differentiable) functions of the total population at the destination and of the group's fraction in it. They also conducted numerical experiments based on population data in locations in Mexico. Capello and Daniele (2019) developed a Nash equilibrium model of human migration with features of conjectural variations. The utility and migration cost functions for a given class depend on the populations and flows, respectively, of that class. The authors also provided a numerical example with sensitivity analysis focusing on the flow of migrants from Africa through the Mediterranean sea to Italy in 2018.

Causa, Jadamba, and Raciti (2017), in turn, extended the model of Nagurney (1990) to include uncertainty in the utility functions, the migration cost functions, and the populations. However, none of the earlier models noted above include regulations, as we do here.

Other OR-based research that is focused on refugees is that of Kjamili and Weber (2019), who, in their novel work on social entrepreneurship in developing and emerging countries, highlight Migport, which began as a mobile app serving as a meeting point between locals and refugees. Problems of refugees could then be identified by giving them a chance to talk about themselves to ensure social integration with given solutions based on a database of 100 thousand people. Pedamallu et al. (2012), in turn, constructed a system dynamics model based on primary education data obtained in a survey conducted in Turkey in order to address the challenges the schools located in migrant neighborhoods are faced with. Trapp et al. (2018) integrated machine learning and integer optimization into a software tool that assists a resettlement agency in the United States with matching refugees to their initial placements. The authors take what can be viewed as a system-optimization approach to refugee resettlement, whereas our framework is on the international human migration aspects of refugees from a user-optimization perspective, with the inclusion of regulations. Another stream of literature is that on human trafficking but that is out of the scope of our research (cf. Konrad et al. (2017) and Bhaumik, Roy, and Weber (2019)) since here we are interested in different classes of migrants selecting their destinations individually and independently, under regulations.

For a broad literature review on humanitarian supply chain management and refugees, but not on migration, see Seifert, Kunz, and Gold (2018). The authors note the relative scarcity of the literature on this topic.

Clearly, the outlook for further study and analysis of international human migration networks holds great promise because of its timeliness.

2.1 Our Contributions

The above highlighted human migration models are sufficiently general to be applicable to migration within a nation or across boundaries of nations. However, they do not include features relevant specifically to international human migration networks in an age of regulations. We address this research gap in this paper. In particular, in this paper, we use as the baseline the multiclass human migration network model of Nagurney, Pan, and Zhao (1992a), since it has been the inspiration for other models. We first propose alternative conservation of flow equations and a different underlying network structure. We then introduce additional constraints to capture distinct types of regulations associated with international human migration networks. We provide alternative variational inequality formulations. Alternative variational inequality formulations allow for different algorithms to be applied. We also conduct Lagrange analysis. The Lagrange analysis for the model, with accompanying interpretations, adds to the literature on operations research with societal impact in the context of a plethora of applications such as organ donation management (cf. Caruso and Daniele (2018)), supply chain network competition for blood services (Nagurney and Dutta (2019)), disaster relief (see Nagurney, Salarpour, and Daniele (2019)), and cybersecurity (cf. Colajanni et al. (2019)). The proposed algorithm is applied to a series of numerical examples, and insights provided.

The paper is organized as follows. In Section 3, the models without and with regulations are developed. In Section 4, we then provide Lagrange theory for the international human migration network model under regulations, along with analysis and interpretation. In Section 5, we propose an algorithm, with accompanying convergence results, which is then applied to a series of numerical examples. The algorithm solves an alternative variational inequality and yields closed form expressions, at each iteration, for the migration flows and the Lagrange multipliers. We summarize our results and present our conclusions in Section 6.

3. The International Human Migration Models

In this Section, we extend the model of Nagurney, Pan, and Zhao (1992a) to the international arena by including a spectrum of constraints, with a focus on regulations. We also propose equivalent conservation of flow equations, which yield an alternative network structure of the multiclass human migration problem, that enables alternative algorithmic schemes. We first present the model without regulations and then the one with.

The common notation for both international human migration network models is reported in Table 1 and their network structure depicted in Figure 1. Observe from the network in Figure 1 that a volume of population of a class may also decide not to migrate, but to remain in its origin country/location.

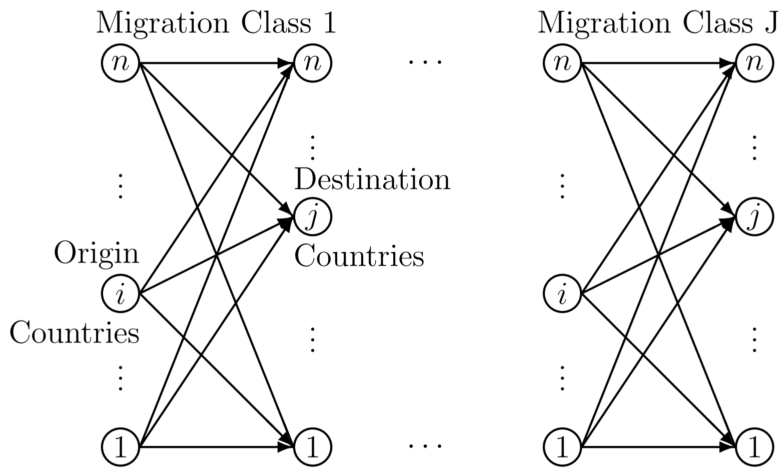

Figure 1: The Network Structure of International Human Migration

We assume a closed economy in which there are n countries under consideration, with a typical origin country denoted by i and a typical destination one denoted by j. There are J classes of international migrants, with a typical class denoted by k. It is important to recognize that a class of migrant may correspond, for example, to a highly skilled, a skilled, or an unskilled class of worker. It can also correspond to a refugee (or asylum seeker), an irregular migrant, etc. (cf. Karagiannis (2016)). Our model is sufficiently general to handle different types of migration/refugee settings since it includes multiple classes of migrant, each of which perceives the utility associated with a country in an individual way, along with the migration cost associated with changing countries/locations. Furthermore, as can be seen from Table 1, the utility associated with a country can depend, in general, on the vector of populations of different classes. Similarly, the migration costs can depend, in general, on the vector of international migratory flows. The former is very reasonable since attractiveness of a country for a specific class one would expect to depend not only on the population of that class, but also on other classes both in that country and at others, notably, on ones in proximity. Moreover, the cost associated with international migration can be expected to depend not only on the international flows of a particular class but also on those of other classes. The generality of our utility and migration cost functions, hence, captures competition for jobs, housing, social services, etc., for the former and competition for transportation and movement/processing resources for the latter (in addition to psychic costs associated with dislocation).

Table 1: Common Notation for the International Human Migration Models

Notation	Definition
f_{ij}^k	the flow of migrants of class k from country i to country j. The $\{f_{ij}^k\}$ elements for all i and j and fixed k are grouped into the vector $f^k \in R_+^{nn}$. We then further group the f^k vectors; $k = 1, \ldots, J$, into the vector $f \in R_+^{Jnn}$.
p_i^k	the nonnegative population of migrant class k in country i. We group the populations of class k; $k = 1, \ldots, J$, into the vector $p^k \in R_+^n$. We then further group all such vectors into the vector $p \in R_+^{Jn}$.
$\bar{p}_i^k$	the initial fixed population of class k in country i; $i = 1, \ldots, n$; $k = 1, \ldots, J$.
$u_i^k(p)$	the utility perceived by class k in country i; $i = 1, \ldots, n$; $k = 1, \ldots, J$.
$c_{ij}^k(f)$	the cost of international migration, which includes economic, psychological, and social costs encumbered by class k in migrating from country i to country j; $i = 1, \ldots, n$; $j = 1, \ldots, n$; $k = 1, \ldots, J$.

In our models we assume no repeat or chain migration. Also, all vectors are column vectors.

In Nagurney, Pan, and Zhao (1992a), the following conservation of flow equations were proposed:

$$p_i^k = \bar{p}_i^k + \sum_{l \neq i} f_{li}^k - \sum_{l \neq i} f_{il}^k, \quad \forall i, \forall k, \tag{1}$$

and

$$\sum_{l \neq i} f_{il}^k \leq \bar{p}_i^k, \quad \forall i, \forall k, \tag{2}$$

where $f_{il}^k \geq 0$, for all $k = 1, \ldots, J$; $\forall l$. Note that, according to (1), the population of a class in a country is equal to the initial population plus the inflow minus the outflow of that class. Equation (2), on the other hand, states that the flow out of country i by class k cannot exceed the initial population of class k at i, since no chain migration is allowed.

Here, we utilize, instead, the following conservation of flow equations, using also the network structure in Figure 1 as a guideline:

$$\bar{p}_i^k = \sum_l f_{il}^k, \quad \forall i, \forall k, \tag{3}$$

and

$$p_i^k = \sum_l f_{li}^k, \quad \forall i, \forall k, \tag{4}$$

with the nonnegativity assumption on the international migratory flows, following (2). According to (3), the fixed population of each class of migrant is equal to the sum of the flow remaining in the country of the class and the migration flows to other countries of that class. Equation (4), on the other hand, states that the final population of a class in a country is equal to the flow of that class that remains in the country plus the migratory flows of that class from other countries into the country.

Clearly, using (3) and (4), we obtain:

$$p_i^k - \bar{p}_i^k = \sum_l f_{li}^k - \sum_l f_{il}^k, \quad \forall i, \forall k, \tag{5}$$

which is equivalent to (1), since $f_{ii}^k - f_{ii}^k = 0$. Similarly, (3) is equivalent to (2).

We define the feasible set $K^1 \equiv \{(p, f) | f \geq 0 \,\text{and}\, (3) \,\text{and}\, (4) \,\text{hold}\}$.

2.1 Equilibrium Conditions for the International Human Migration Model without Regulations

We now state the equilibrium conditions for the model without regulations and then derive the variational inequality (VI) formulation.

As in our previous work, we assume that migrants are rational and that international migration will continue until no individual has any incentive to move since a unilateral decision will no longer yield a positive net gain (gain in utility minus migration cost).

Definition 1: International Human Migration Equilibrium without Regulations

A vector of populations and international migration flows $(p^*, f^*) \in K^1$ is in equilibrium if it satisfies the equilibrium conditions: For each class k; $k = 1, \ldots, J$ and each pair of countries i, j; $i = 1, \ldots, n$; $j = 1, \ldots, n$:

$$u_i^k(p^*) + c_{ij}^k(f^*) \begin{cases} = u_j^k(p^*) - \lambda_i^{k*}, & \textit{if} \quad f_{ij}^{k*} > 0, \\ \geq u_j^k(p^*) - \lambda_i^{k*}, & \textit{if} \quad f_{ij}^{k*} = 0, \end{cases} \tag{6}$$

and

$$\lambda_i^{k*} \begin{cases} \geq 0, & \textit{if} \quad \sum_{l \neq i} f_{il}^{k*} = \bar{p}_i^k, \\ = 0, & \textit{if} \quad \sum_{l \neq i} f_{il}^{k*} < \bar{p}_i^k. \end{cases} \tag{7}$$

The above equilibrium conditions are due to Nagurney, Pan, and Zhao (1992a). We assume that $c_{ii}^k(f) = 0$, for all i and k, since those who elect to stay in their origin countries and not migrate encumber a migration cost of zero.

We now recall the necessity of λ_i^{k*}. As emphasized in Nagurney, Pan, and Zhao (1992a), unlike spatial price equilibrium problems (or the transportation network equilibrium problem with elastic demand; see, e.g., Zhang and Nagurney (1997), Nagurney (1999), Daniele (2006), Nagurney, Li, and Nagurney (2014), Nagurney, Besik, and Dong (2019), and the references therein), the level of the population $\bar{p}_i^k$ may not be sufficiently large so that the gain in utility $u_j^k - u_i^k$ is exactly equal to the migration cost c_{ij}^k. Nevertheless, the utility gain minus the migration cost will be maximal and nonnegative. Furthermore, the net gain will be equalized for all countries and classes which have a positive flow out of a country.

The variational inequality formulation of the international human migration equilibrium conditions according to Definition 1 is given in Theorem 1 below.

Theorem 1: Variational Inequality Formulation of the International Human Migration Model without Regulations

A population and migration flow pattern $(p^*, f^*) \in K^1$ is an international human migration equilibrium without regulations according to Definition 1, if and only if it satisfies the variational inequality problem

$$-\langle u(p^*), p - p^* \rangle + \langle c(f^*), f - f^* \rangle \geq 0, \quad \forall (p, f) \in K^1. \tag{8}$$

Proof: We first show that if a pattern $(p^*, f^*) \in K^1$ satisfies equilibrium conditions (6) and (7), then it also satisfies the variational inequality in (8).

Suppose that $(p^*, f^*) \in K^1$ satisfies the equilibrium conditions. We know, from the conservation of flow equations, that $f_{ij}^{k*} \geq 0$, and $\sum_{l \neq i} f_{il}^{k*} \leq \bar{p}_i^k$, for all i, j, and k.

For fixed class k and country i, it follows from (6) that for $f \in K^1$:

$$\sum_l \left[u_i^k(p^*) + c_{il}^k(f^*) - u_l^k(p^*) \right] \times \left[f_{il}^k - f_{il}^{k*} \right]$$
$$\geq -\lambda_i^{k*} \sum_l (f_{il}^k - f_{il}^{k*}) = 0, \tag{9}$$

with the last equality holding because of the conservation of flow equation (3).

Therefore, for all classes k and all countries i, it follows that for $(p^*, f^*) \in K^1$:

$$\sum_k \sum_i \sum_l \left[u_i^k(p^*) + c_{il}^k(f^*) - u_l^k(p^*) \right] \times \left[f_{il}^k - f_{il}^{k*} \right] \geq 0, \quad \forall (p, f) \in K^1. \tag{10}$$

Note that inequality (10) can be re-expressed as:

$$\sum_k \sum_l u_l^k(p^*) \times ((\sum_j f_{lj}^k - \sum_j f_{jl}^k) - (\sum_j f_{lj}^{k*} - \sum_j f_{jl}^{k*})) + \sum_k \sum_i \sum_l c_{il}^k(f^*) \times (f_{il}^k - f_{il}^{k*}) \geq 0. \tag{11}$$

Using now constraints (3) and (4), substitution into (11), yields

$$-\sum_k \sum_l u_l^k(p^*) \times (p_l^k - p_l^{k*}) + \sum_k \sum_i \sum_l c_{il}^k(f^*) \times (f_{il}^k - f_{il}^{k*}) \geq 0, \tag{12}$$

or, equivalently, in vector notation:

$$-\langle u(p^*), p - p^* \rangle + \langle c(f^*), f - f^* \rangle \geq 0, \quad \forall (p, f) \in K^1. \tag{13}$$

Hence, necessity has been established.

We now show that if a pattern $(p^*, f^*) \in K^1$ satisfies variational inequality (8), then it also satisfies equilibrium conditions (6) and (7).

Suppose that (p^*, f^*) satisfies variational inequality (8). Since $(p^*, f^*) \in K^1$ satisfies VI (8), then

$$\langle -u(p^*), p \rangle + \langle c(f^*), f \rangle \geq \langle -u(p^*), p^* \rangle + \langle c(f^*), f^* \rangle, \quad \forall (p, f) \in K^1. \tag{14}$$

Therefore, (p^*, f^*) solves the minimization problem

$$\mathrm{Min}_{(p,f) \in K^1} \langle -u(p^*), p \rangle + \langle c(f^*), f \rangle. \tag{15}$$

In view of (4), we can express (15), exclusively, in terms of flows f:

$$\mathrm{Min}_{f \in \hat{K}^1} \langle -\hat{u}(f^*), f \rangle + \langle c(f^*), f \rangle, \tag{16}$$

where $\hat{K}^1 \equiv \{f | f \geq 0 \, \text{and satisfies} \, (3)\}$, A is the arc-node incidence matrix in (4), and $\hat{u}(f^*) \equiv u(p^*)$. Since the constraints are linear, one has the following Kuhn Tucker conditions, for all i, j, k; there exist:

$$\lambda_i^{k*} \geq 0, \tag{17}$$

such that

$$\lambda_i^{k*} \left(\sum_{l \neq i} f_{il}^{k*} - \bar{p}_i^k \right) = 0, \tag{18}$$

and

$$\hat{u}_i^k(f^*) + c_{ij}^k(f^*) - \hat{u}_j^k(f^*) + \lambda_i^{k*} \geq 0, \tag{19}$$

$$(\hat{u}_i^k(f^*) + c_{ij}^k(f^*) - \hat{u}_j^k(f^*) + \lambda_i^{k*}) f_{ij}^{k*} = 0. \tag{20}$$

Obviously, equilibrium conditions (6) and (7) follow from (17) - (20). The proof is complete. □

Existence of at least one solution to variational inequality (8) follows from the standard theory of variational inequalities (see Kinderlehrer and Stampacchia (1980) Theorem 3.1) under the assumption of continuity of the utility functions u and the international migration cost functions c, since the feasible convex set K^1 is compact. Uniqueness of the equilibrium population and international migration flow pattern (p^*, f^*), in turn, is guaranteed under the assumption that minus the utility functions and the international migration cost functions are strictly monotone, i.e.,

$$-\langle u(p^1) - u(p^2), p^1 - p^2 \rangle + \langle c(f^1) - c(f^2), f^1 - f^2 \rangle > 0, \qquad (21)$$
$$\forall (p^1, f^1), (p^2, f^2) \in K^1,\ (p^1, f^1) \neq (p^2, f^2).$$

We now provide an interpretation of the monotonicity condition (21), which we expect to hold under reasonable economic situations. Specifically, we assume that the migration network consisting of origin and destination country locations/nodes as well as migratory links/routes is subject to congestion. Consequently, the utility functions are decreasing with larger populations, and the international migration cost functions are increasing with larger migration flows. Furthermore, we assume that each utility function $u_i^k(p)$ depends mainly on the population p_i^k and that each migration cost function $c_{ij}^k(f)$ depends primarily on the flow f_{ij}^k. Mathematically, the strict monotonicity condition (21) will hold, for example, if the functions u and c are continuously differentiable and the Jacobian matrices $-\nabla u$ and ∇c are positive definite over K^1 (see Theorem 5.4.3 in Ortega and Rheinboldt (1970)).

2.2 Variational Inequality Formulation of the International Human Migration Model with Regulations

We now construct the constraint set that captures a plethora of international migration regulations. For definiteness, we consider regulations imposed by a single country $\bar{j}$. We define the set C^1 consisting of classes $\{k\}$ and countries $\{i\}$, with $i \neq \bar{j}$, subject to an upper bound on the international migration flows into country $\bar{j}$, denoted by $U_{\bar{j}}$.

The constraint can then be stated as follows:

$$\sum_{i \in C^1} \sum_{k \in C^1} f_{i\bar{j}}^k \leq U_{\bar{j}}. \qquad (22)$$

We now highlight the different types of regulations that (22) captures.

For example, the set C^1 can be defined to restrict the migratory flow from a specific country $\bar{i}$ and specific class of migrant $\bar{k}$, in which case, (22) collapses to:

$$f_{\bar{i}\bar{j}}^{\bar{k}} \leq U_{\bar{j}}. \qquad (23)$$

On the other hand, an upper bound on all incoming migrants from a specific country $\bar{i}$, irrespective of class, reduces constraint (22) to:

$$\sum_{k} f^{k}_{\bar{i}\bar{j}} \leq U_{\bar{j}}. \tag{24}$$

Also, a regulation restricting the number of all incoming migrants of class $\bar{k}$ from a group of countries, reduces constraint (22) to:

$$\sum_{i \in C^1} f^{\bar{k}}_{i\bar{j}} \leq U_{\bar{j}}. \tag{25}$$

For the international human migration model with regulations, the equilibrium conditions (6) and (7) are still relevant but with a new feasible set K^2 defined as below to include the constraint (22):

$$K^2 \equiv \{(p, f) | f \geq 0 \,\text{and}\, (3),\ (4),\ \text{ and } (22) \text{ hold}\}. \tag{26}$$

Hence, it follows immediately that the variational inequality for the international human migration model with regulations is as given in Theorem 2 below.

Theorem 2: Variational Inequality Formulation of the International Human Migration Model with Regulations

A population and migration flow pattern $(p^*, f^*) \in K^2$ is an international human migration equilibrium with regulations, if and only if it satisfies the variational inequality problem

$$-\langle u(p^*), p - p^* \rangle + \langle c(f^*), f - f^* \rangle \geq 0, \quad \forall (p, f) \in K^2. \tag{27}$$

Proof: See Smith (1979).

Observe that VI (27) differs from VI (8) in that the feasible set K^2 is distinct from K^1.

We now provide a VI equivalent to the one in (27) but in flows only. We define a new feasible set $K^3 \equiv \{f | f \in R^{Jnn}_{+} \,\text{and}\, (3) \,\text{and}\, (22) \,\text{hold}\}$. Recall that $\hat{u}^k_i(f) \equiv u^k_i(p)$, $\forall i$, $\forall k$.

Observe now that VI (27) can be expressed in flows, since the first term in (27) is as below:

$$-\sum_{i}\sum_{k} u^k_i(p^*) \times (p^k_i - p^{k*}_i) = -\sum_{i}\sum_{k} \hat{u}^k_i(f^*) \times (\sum_{l} f^k_{li} - \sum_{l} f^{k*}_{li}) \tag{28}$$

$$= -\sum_j \sum_k \hat{u}_j^k(f^*) \times (\sum_i f_{ij}^k - \sum_i f_{ij}^{k*}). \tag{29}$$

It, thus, follows that VI (27) is equivalent to the VI: determine $f^* \in K^3$ such that

$$\sum_i \sum_j \sum_k (-\hat{u}_j^k(f^*) + c_{ij}^k(f^*)) \times (f_{ij}^k - f_{ij}^{k*}) \geq 0, \quad \forall f \in K^3. \tag{30}$$

Existence of a solution to variational inequality (30) is also guaranteed.

4. Lagrange Theory and Analysis

The aim of this section is to find an equivalent formulation of variational inequality (30), by means of the Lagrange multipliers associated with the constraints defining the feasible set K^3. We also provide an interpretation of the Lagrange analysis obtained and present several illustrative examples.

4.1 Lagrange Theory

First of all, we observe that K^3 can be rewritten as follows:

$$K^3 = \left\{ f : -f \leq 0; \sum_j f_{ij}^k - \bar{p}_i^k = 0, \ \forall i, \ \forall k; \ \sum_{i \in C^1} \sum_{k \in C^1} f_{i\bar{j}}^k - U_{\bar{j}} \leq 0 \right\}. \tag{31}$$

Also, variational inequality (30) can be rewritten as a minimization problem, since if we set:

$$V(f) = \sum_i \sum_j \sum_k (-\hat{u}_j^k(f^*) + c_{ij}^k(f^*)) \times (f_{ij}^k - f_{ij}^{k*}),$$

then we have:

$$V(f) \geq 0 \, \text{for} \, f \in K^3 \text{ and } \min_{f \in K^3} V(f) = V(f^*) = 0.$$

Making use of the classical linear optimization duality, if f^* is a solution to variational inequality (30), then the following conditions (32) – (34) hold and, vice versa; moreover, also strong duality (35) holds.

Theorem 3

If $f^* \in K^3$ is a solution to variational inequality (30), then the Lagrange multipliers $\bar{\gamma} \in R_+^{Jnn}$, $\bar{\delta} \in R^{Jn}$, and $\bar{\mu}_{\bar{j}} \in R_+$ do exist, and for all i, j, k, and $\bar{j}$, the following conditions hold true:

$$\bar{\gamma}_{ij}^k(-f_{ij}^{k*}) = 0, \quad \bar{\delta}_{ik} \left(\sum_j f_{ij}^{k*} - \bar{p}_i^k \right) = 0, \quad \bar{\mu}_{\bar{j}} \left(\sum_{i \in C^1} \sum_{k \in C^1} f_{i\bar{j}}^{k*} - U_{\bar{j}} \right) = 0, \tag{32}$$

$$-\hat{u}_j^k(f^*) + c_{ij}^k(f^*) - \overline{\gamma}_{ij}^k + \overline{\delta}_{ik} = 0, \qquad if\ j \neq \bar{j}, \tag{33}$$

$$-\hat{u}_{\bar{j}}^k(f^*) + c_{i\bar{j}}^k(f^*) - \overline{\gamma}_{i\bar{j}}^k + \overline{\delta}_{ik} + \bar{\mu}_{\bar{j}} = 0, \qquad if\ j = \bar{j}. \tag{34}$$

Moreover, the strong duality also holds true; namely:

$$V(f^*) = \min_{f \in K^3} V(f) = \max_{\gamma \in R_+^{Jnn}, \delta \in R^{Jn}, \mu_{\bar{j}} \in R_+} \min_{f \in R^{Jnn}} \mathcal{L}(f, \gamma, \delta, \mu_{\bar{j}}). \tag{35}$$

Conditions (32)–(34) represent an equivalent formulation of variational inequality (30). Indeed, let $f \in K^3$ and let us multiply (34) by $(f_{ij}^k - f_{ij}^{k*})$. Then we obtain:

$$\left(-\hat{u}_j^k(f^*) + c_{ij}^k(f^*)\right)(f_{ij}^k - f_{ij}^{k*}) = \left(\overline{\gamma}_{ij}^k - \overline{\delta}_{ik}\right)(f_{ij}^k - f_{ij}^{k*}). \tag{36}$$

Analogously, by multiplying (34) by $(f_{i\bar{j}}^k - f_{i\bar{j}}^{k*})$, we have:

$$\left(-\hat{u}_{\bar{j}}^k(f^*) + c_{i\bar{j}}^k(f^*)\right)(f_{i\bar{j}}^k - f_{i\bar{j}}^{k*}) = \left(\overline{\gamma}_{i\bar{j}}^k - \overline{\delta}_{ik} - \bar{\mu}_{\bar{j}}^*\right)(f_{i\bar{j}}^k - f_{i\bar{j}}^{k*}). \tag{37}$$

Then, summing (36) and (37) with respect to i, j, and k, we get:

$$\sum_{i \notin C^1} \sum_{k \notin C^1} \sum_{j \neq \bar{j}} \left(-\hat{u}_j^k(f^*) + c_{ij}^k(f^*)\right)(f_{ij}^k - f_{ij}^{k*}) + \sum_{i \in C^1} \sum_{k \in C^1} \left(-\hat{u}_{\bar{j}}^k(f^*) + c_{i\bar{j}}^k(f^*)\right)(f_{i\bar{j}}^k - f_{i\bar{j}}^{k*})$$

$$= \sum_{i \notin C^1} \sum_{k \notin C^1} \sum_{j \neq \bar{j}} \overline{\gamma}_{ij}^k (f_{ij}^k - f_{ij}^{k*}) + \sum_{i \in C^1} \sum_{k \in C^1} \overline{\gamma}_{i\bar{j}}^k (f_{i\bar{j}}^k - f_{i\bar{j}}^{k*})$$

$$- \sum_{i \notin C^1} \sum_{k \notin C^1} \sum_{j \neq \bar{j}} \overline{\delta}_{ik}(f_{ij}^k - f_{ij}^{k*}) - \sum_{i \in C^1} \sum_{k \in C^1} \overline{\delta}_{ik}(f_{i\bar{j}}^k - f_{i\bar{j}}^{k*}) - \sum_{i \in C^1} \sum_{k \in C^1} \bar{\mu}_{\bar{j}}(f_{i\bar{j}}^k - f_{i\bar{j}}^{k*}). \tag{38}$$

Since $f_{ij}^k \geq 0$, $\forall i, j, k$, it follows, by also making use of the first equality in (32), that the sum of the first two terms after the equal sign in (38) is nonnegative. Furthermore, since $f \in K^3$ and $f^* \in K^3$, $-\sum_{i \notin C^1} \sum_{k \notin C^1} \sum_{j \neq \bar{j}} \overline{\delta}_{ik}(f_{ij}^k - f_{ij}^{k*}) - \sum_{i \in C^1} \sum_{k \in C^1} \overline{\delta}_{ik}(f_{i\bar{j}}^k - f_{i\bar{j}}^{k*})$ in (38) is zero. Finally, the last term after the equal sign in (38) is nonnegative, that is, $-\sum_{i \in C^1} \sum_{k \in C^1} \bar{\mu}_{\bar{j}}(f_{i\bar{j}}^k - f_{i\bar{j}}^{k*}) \geq 0$, in view of the third term in (32) and since $f \in K^3$. Hence, the assertion follows.

We now proceed to provide a deeper interpretation of the above Lagrange analysis, which also yields equilibrium conditions for the model with regulations.

Let us consider the case where $f_{ij}^{k*} > 0$ for some $j \neq \bar{j}$; from (32) we know that then $\bar{\gamma}_{ij}^k = 0$. It then follows from (34) that:

$$\bar{\delta}_{ik} + c_{ij}^k(f^*) = \hat{u}_j^k(f^*). \tag{39}$$

If, on the other hand, $f_{ij}^{k*} = 0$, for some $j \neq \bar{j}$, then $\bar{\gamma}_{ij}^k \geq 0$, and, from (34), we can infer that:

$$\bar{\delta}_{ik} + c_{ij}^k(f^*) = \hat{u}_j^k(f^*) + \bar{\gamma}_{ij}^k;$$

equivalently:

$$\bar{\delta}_{ik} + c_{ij}^k(f^*) \geq \hat{u}_j^k(f^*). \tag{40}$$

Also, note that, if $f_{ii}^* > 0$, then from (34) it follows, since $c_{ii} = 0$, that

$$0 + \bar{\delta}_{ik} = \hat{u}_i^k(f^*)$$

and, hence,

$$\bar{\delta}_{ik} = \hat{u}_i^k(f^*).$$

It is easy to see that the equilibrium conditions (6) and (7) hold true. Indeed, expressions (39) and (40) can, hence, be interpreted as follows: migrants of a class will continue to migrate to a destination node until the utility at it is equal to the utility at the origin node plus the migration cost, assuming that there is sufficient population to equalize these. There will be no flow, in equilibrium, to a "destination" country node if the utility there minus the utility at the origin country node is insufficient to cover the cost of migration.

In contrast, if we consider a destination node $\bar{j}$, under the regulation, and, if $f_{i\bar{j}}^{k*} > 0$, then (34) applies and we obtain, from (34), that:

$$\bar{\delta}_{ik} + c_{i\bar{j}}^k(f^*) = \hat{u}_{\bar{j}}^k(f^*) - \bar{\mu}_{\bar{j}}. \tag{41}$$

Using now (32), we can infer from (41) that, if the migration upper bound $U_{\bar{j}}$ is not met, then the condition (40) coincides with (39). On the other hand, if the upper bound holds tightly, then the migrants incur a higher utility at destination node $\bar{j}$ than just the sum of the origin node utility and the migration cost. In other words, more were seeking to migrate to that destination, but were prevented from doing so because of the regulation and, interestingly, the migrants that did manage to reach $\bar{j}$, are, in fact, "better off". However, those left at origin node i encounter a lower utility.

We now define $K^4 \equiv \{(f, \delta, \mu_{\bar{j}}) | f \in R_+^{Jnn}, \delta \in R^{Jn}, \mu_{\bar{j}} \in R_+\}$. Using (32) – (34) we can construct an alternative VI to (27): determine $(f^*, \delta^*, \mu_{\bar{j}}^*) \in K^4$ such that

$$\sum_{i;i \notin C^1} \sum_{j \neq \bar{j}} \sum_{k;k \notin C^1} (-\hat{u}_j^k(f^*) + c_{ij}^k(f^*) + \delta_{ik}^*) \times (f_{ij}^k - f_{ij}^{k*})$$

$$+ \sum_{i \in C^1} \sum_{k \in C^1} (-\hat{u}_{\bar{j}}^k(f^*) + c_{i\bar{j}}^k(f^*) + \delta_{ik}^* + \mu_{\bar{j}}^*) \times (f_{i\bar{j}}^k - f_{i\bar{j}}^{k*})$$

$$+\sum_i \sum_k (\bar{p}_i^k - \sum_j f_{ij}^{k*}) \times (\delta_{ik} - \delta_{ik}^*)$$

$$+(U_{\bar{j}} - \sum_{i \in C^1} \sum_{k \in C^1} f_{i\bar{j}}^{k*}) \times (\mu_{\bar{j}} - \mu_{\bar{j}}^*) \geq 0, \quad \forall (f, \delta, \mu_{\bar{j}}) \in K^4. \tag{42}$$

We now put variational inequality (42) into standard form (cf. Nagurney (1999)): determine $X^* \in \mathcal{K}$ such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{43}$$

where F is a given continuous function from $\mathcal{K}$ to R^N, $\mathcal{K}$ is a given closed convex set, and $\langle \cdot, \cdot \rangle$ denotes the inner product in N-dimensional Euclidean space.

Indeed, we set $\mathcal{K} \equiv K^4$, $X \equiv (f, \delta, \mu_{\bar{j}})$, and $N = Jnn + Jn + 1$. Also, we define the vector $F \equiv (F_1, F_2, F_3, F_4)$, where the components of F_1 consist of the elements: $-\hat{u}_j^k(f) + c_{ij}^k(f) + \delta_{ik}$, for $i; i \notin C^1$ and $j \neq \bar{j}$, and $k; k \notin C^1$; the components of F_2 consist of the elements: $-\hat{u}_{\bar{j}}^k(f) + c_{i\bar{j}}^k(f) + \delta_{ik} + \mu_{\bar{j}}$, for $i \in C^1$ and $k \in C^1$; F_3 consists of the elements: $\bar{p}_i^k - \sum_j f_{ij}^k$, $\forall i, k$, and, finally, F_4 consists of the single element: $U_{\bar{j}} - \sum_{i \in C^1} \sum_{k \in C^1} f_{i\bar{j}}^k$.

4.2 Illustrative Examples

We now proceed to illustrate the above results in several simple examples. The migration network consists of the topology depicted in Figure 2.

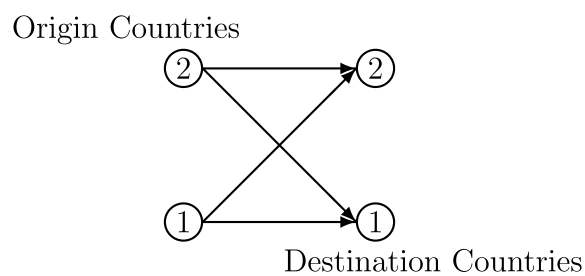

Figure 2: International Migration Network for Illustrative Examples

There are two origin country nodes and the same two country destination nodes. We consider a single class of migrant. We, hence, suppress the superscript 1 in the notation. The data are as follows: $\bar{p}_1 = 50$ and $\bar{p}_2 = 0$ with the utility functions given by: $u_1(p) = -p_1 + 100$ and $u_2(p) = -p_2 + 120$. The migration cost functions are: $c_{11}(f) = c_{22}(f) = 0$, $c_{12}(f) = .1f_{12} + 7$, $c_{21}(f) = f_{21} + 10$.

We first consider the case without regulations. It is easy to compute the equilibrium solution, using simple algebra. Indeed, we find that:

$$f_{12}^* = 30, \quad f_{11}^* = 20, \quad f_{21}^* = 0, \quad f_{22}^* = 0;$$

hence,

$$p_1^* = 20, \quad p_2^* = 30,$$

with associated utilities being:

$$\hat{u}_1(f^*) = u_1(p^*) = 80, \quad \hat{u}_2(f^*) = u_2(p^*) = 90,$$

and the migration costs: $c_{11}(f^*) = c_{22}(f^*) = 0$, $c_{12}(f^*) = 10$, and $c_{21}(f^*) = 10$.

Moreover, $\bar{\delta}_{11} = 80$, $\bar{\delta}_{21} = 90$, and $\bar{\gamma}_{11} = \bar{\gamma}_{21} = \bar{\gamma}_{12} = \bar{\gamma}_{22} = 0$.

We now verify that the equilibrium conditions hold. Indeed, for the node pair $(1,2)$: $f_{12}^* > 0$, and

$$\bar{\delta}_{11} + c_{12}(f^*) = \hat{u}_2(f^*) \quad \text{or} \quad u_1(p^*) + c_{12}(f^*) = u_2(p^*),$$

since $80 + 10 = 90$. Moreover, for node pair $(1,1)$ and node pair $(2,2)$:

$$u_1(p^*) + c_{11}(f^*) = u_1(p^*), \quad u_2(p^*) + c_{22}(f^*) = u_2(p^*),$$

since the $c_{ii}(f)$s are equal to 0 for $i = 1, 2$. Finally, since $f_{21}^* = 0$:

$$\bar{\delta}_{21} + c_{21}(f^*) \geq \hat{u}_1 \quad \text{or} \quad u_2(p^*) + c_{21}(f^*) \geq u_1(p^*),$$

with notice that, indeed, $90 + 10 \geq 80$.

It is also easy to verify that VI (8) is satisfied.

We now suppose that a regulation is imposed on destination node 2, such that

$$f_{1\bar{2}} \leq U_{\bar{2}} = 20.$$

Following our notation, and since the underlying functions are not changed, we have that

$$c_{1\bar{2}}(f) = c_{12}(f)$$

and that

$$\hat{u}_{\bar{2}}(f) = u_2(p).$$

The new equilibrium solution is:

$$f_{11}^* = 30, \quad f_{1\bar{2}}^* = 20, \quad f_{21}^* = 0, \quad f_{22}^* = 0,$$

with $p_1^* = 30$ and $p_{\bar{2}}^* = 20$, and associated utilities:

$$\hat{u}_1(f^*) = u_1(p^*) = 70, \quad \hat{u}_{\bar{2}}(f^*) = u_{\bar{2}}(p^*) = 100,$$

and incurred migration costs: $c_{11}(f^*) = c_2(f^*) = 0$; $c_{1\bar{2}}(f^*) = 9$, $c_{21}(f^*) = 10$.

Also, cf. (32): $\bar{\mu}_{\bar{2}} = 21$.

Moreover, $\bar{\delta}_{11} = 70$, $\bar{\delta}_{21} = 100$, and $\bar{\gamma}_{11} = \bar{\gamma}_{21} = \bar{\gamma}_{12} = \bar{\gamma}_{22} = 0$.

One can see that, indeed, (34) holds since

$$\bar{\delta}_{11} + c_{1\bar{2}}(f^*) - \bar{\gamma}_{1\bar{2}} = \hat{u}_{\bar{2}}(f^*) - \bar{\mu}_{\bar{2}};$$

equivalently,

$$70 + 9 - 0 = 100 - 21.$$

Under this regulation, those who manage to migrate enjoy a higher utility than before, but those who are left behind in country 1 experience a lower utility than in the case without regulations (a utility of 70, as compared to 80).

5. Algorithm and Numerical Examples

In this section, we describe the algorithm, along with convergence results, and present several numerical examples solved by the algorithm.

5.1 The Modified Projection Method

For the solution of the numerical examples, we implemented the modified projection method (see Korpelevich (1977) and Nagurney (1999)). This algorithm is guaranteed to converge if the function F that enters the standard form of the variational inequality (cf. (42)) satisfies monotonicity and Lipschitz continuity (see Nagurney (1999)) and that a solution exists.

Recall that the function $F(X)$ is said to be monotone, if

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle \geq 0, \quad \forall X^1, X^2 \in \mathcal{K}, \tag{44}$$

and that the function $F(X)$ is Lipschitz continuous, if there exists a constant $L > 0$, known as the Lipschitz constant, such that

$$\|F(X^1) - F(X^2)\| \leq L\|X^1 - X^2\|, \quad \forall X^1, X^2 \in \mathcal{K}. \tag{45}$$

The steps of the modified projection method are given below, with t denoting an iteration counter:

The Modified Projection Method

Step 0: Initialization

Initialize with $X^0 \in \mathcal{K}$. Set $t := 1$ and let β be a scalar such that $0 < \beta \leq \frac{1}{L}$, where L is the Lipschitz constant.

Step 1: Computation

Compute $\bar{X}^t$ by solving the variational inequality subproblem:

$$\langle \bar{X}^t + \beta F(X^{t-1}) - X^{t-1}, X - \bar{X}^t \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{46}$$

Step 2: Adaptation

Compute X^t by solving the variational inequality subproblem:

$$\langle X^t + \beta F(\bar{X}^t) - X^{t-1}, X - X^t \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{47}$$

Step 3: Convergence Verification

If $|X^t - X^{t-1}| \leq \epsilon$, with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $t := t + 1$ and go to Step 1.

As mentioned earlier, because of the simplicity of the feasible set $\mathcal{K} = K^4$, both Steps 1 and 2 result in subproblems that can be solved explicitly in closed form in terms of the migration flows, the Lagrange multipliers associated with the population equality constraint, and the Lagrange multiplier associated with the regulation constraint. Indeed, at each iteration of the algorithm, we have the following explicit formulae for Step 1. The corresponding explicit formulae for Step 2 are similar in form.

Explicit Formulae for the Migration Flows in Step 1 of the Modified Projection Method

$$\bar{f}_{ij}^{kt} = \max\{0, f_{ij}^{k(t-1)} + \beta(\hat{u}_j^k(f^{t-1}) - c_{ij}^k(f^{t-1}) - \delta_{ik}^{t-1})\}, \quad i; i \notin C^1; j \neq \bar{j}; k; k \notin C^1, \tag{48}$$

$$\bar{f}_{i\bar{j}}^{kt} = \max\{0, f_{i\bar{j}}^{k(t-1)} + \beta(\hat{u}_{\bar{j}}^k(f^{t-1}) - c_{i\bar{j}}^k(f^{t-1}) - \delta_{ik}^{t-1} - \mu_{\bar{j}}^{t-1})\}, \quad i; i \in C^1; k; k \in C^1. \tag{49}$$

Explicit Formulae for the Lagrange Multipliers in Step 1 of the Modified Projection Method

$$\bar{\delta}_{ik}^t = \delta_{ik}^{t-1} + \beta(\sum_j f_{ij}^{k(t-1)} - \bar{p}_i^k), \quad \forall i, \forall k, \tag{50}$$

$$\bar{\mu}^t_{\bar{j}} = \max\{0, \mu^{t-1}_{\bar{j}} + \beta(\sum_{i\in C^1}\sum_{k\in C^1} f^{k(t-1)}_{i\bar{j}} - U_{\bar{j}})\}. \tag{51}$$

We now verify under what conditions on the specific $F(X)$ of our model in (43) the conditions for convergence will be satisfied.

We first construct $\langle F(X^1) - F(X^2), X^1 - X^2\rangle$ as in (44) for our model, which is:

$$\sum_{i;i\in C^1}\sum_{j\neq\bar{j}}\sum_{k;k\neq C^1}((-\hat{u}^k_j(f^1) + c^k_{ij}(f^1) + \delta^1_{ik}) - (-\hat{u}^k_j(f^2) + c^k_{ij}(f^2) + \delta^2_{ik})) \times (f^{k1}_{ij} - f^{k2}_{ij})$$

$$+\sum_{i\in C^1}\sum_{k\in C^1}((-\hat{u}^k_{\bar{j}}(f^1) + c^k_{i\bar{j}}(f^1) + \delta^1_{ik} + \mu^1_{\bar{j}}) - (-\hat{u}^k_{\bar{j}}(f^2) + c^k_{i\bar{j}}(f^2) + \delta^2_{ik} + \mu^2_{\bar{j}})) \times (f^{k1}_{i\bar{j}} - f^{k2}_{i\bar{j}})$$

$$+\sum_i\sum_k((\bar{p}^k_i - \sum_j f^{k1}_{ij}) - (\bar{p}^k_i - \sum_j f^{k2}_{ij})) \times (\delta^1_{ik} - \delta^2_{ik})$$

$$((U_{\bar{j}} - \sum_{i\in C^1}\sum_{k\in C^1} f^{k1}_{i\bar{j}}) - (U_{\bar{j}} - \sum_{i\in C^1}\sum_{k\in C^1} f^{k2}_{i\bar{j}}) \times (\mu^1_{\bar{j}} - \mu^2_{\bar{j}}), \tag{52}$$

which, after algebraic simplification, yields:

$$\sum_{i;i\in C^1}\sum_{j\neq\bar{j}}\sum_{k;k\neq C^1}((-\hat{u}^k_j(f^1) + c^k_{ij}(f^1)) - (-\hat{u}^k_j(f^2) + c^k_{ij}(f^2)) \times (f^{k1}_{ij} - f^{k2}_{ij})$$

$$+\sum_{i\in C^1}\sum_{k\in C^1}((-\hat{u}^k_{\bar{j}}(f^1) + c^k_{i\bar{j}}(f^1)) - (-\hat{u}^k_{\bar{j}}(f^2) + c^k_{i\bar{j}}(f^2))) \times (f^{k1}_{i\bar{j}} - f^{k2}_{i\bar{j}}). \tag{53}$$

But (53) can be further simplified, with the use of (4) and (28), to:

$$\sum_k\sum_j(-u^k_j(p^1) + u^k_j(p^2)) \times (p^{k1}_j - p^{k2}_j) + \sum_i\sum_j\sum_k(c^k_{ij}(f^1) - c^k_{ij}(f^2)) \times (f^{k1}_{ij} - f^{k2}_{ij}). \tag{54}$$

Clearly, if the functions $-u(p)$ and $c(f)$ are monotone in p and f, respectively, then (54) is greater than or equal to zero and $F(X)$ is, under these reasonable assumptions, monotone. Note that the numerical examples in this section satisfy this condition.

As for Lipschitz continuity of $F(X)$ for our model, the other condition for convergence of the algorithm, we assume that the utility functions and the migration cost functions have bounded first order derivatives. The result is then direct by applying a mid-value theorem from calculus to the vector function that enters the variational inequality problem (43); equivalently, variational inequality (42). Lipschitz continuity of $F(X)$ also holds in the case of our numerical examples.

5.2 Numerical Examples

The modified projection method was implemented in FORTRAN and the computer system used was a Linux system at the University of Massachusetts Amherst. The algorithm was initialized as follows: the initial migration flow of a class from a given origin country to a destination country was set equal to the initial population of the class in the origin country divided by the number of countries, resulting in equal flows. The Lagrange multipliers were all initialized to be equal to 1. The convergence tolerance ϵ was set to 10^{-3}; in other words, the algorithm was deemed to have converge if the absolute value of successive flow and Lagrange multiplier iterates differed by no more than this value.

5.2.1 Single Class Example without and with a Regulation

The first example consists of three countries and no regulations. Since there is only a single class of migrant we suppress the superscript 1 in the notation. The initial populations are given, respectively, by $\bar{p}_1 = 10,000$, $\bar{p}_2 = 5,000$, and $\bar{p}_3 = 1,000$. The utility functions associated with the countries are:

$$u_1(p) = -p_1 - .5p_2 + 30,000, \quad u_2(p) = -2p_2 - p_1 + 20,000, \quad u_3(p) = -3p_3 + .5p_2 + 10,000.$$

The migration cost functions are:

$$c_{ii}(f) = 0, \quad i = 1, 2, 3;$$

$$c_{12}(f) = 2f_{12} + 20, \quad c_{13}(f) = f_{13} + 30,$$

$$c_{21}(f) = 5f_{21} + 40, \quad c_{23}(f) = 4f_{23} + 20,$$

$$c_{31}(f) = 6f_{31} + 80, \quad c_{32}(f) = 4f_{32} + 60.$$

We set $\beta = .1$ in the modified projection method.

The algorithm converged, yielding the following equilibrium migration flow pattern:

$$f_{11}^* = 10,000.00, \quad f_{12}^* = 0.00, \quad f_{13}^* = 0.00,$$

$$f_{21}^* = 2,447.90, \quad f_{22}^* = 1,519.66, \quad f_{23}^* = 1,032.44,$$

$$f_{31}^* = 1,000.00, \quad f_{32}^* = 0.00, \quad f_{33}^* = 0.00.$$

The incurred migration costs at equilibrium are:

$$c_{11}(f^*) = 0.00, \quad c_{12}(f^*) = 20.00, \quad c_{13}(f^*) = 30.00,$$

$$c_{21}(f^*) = 12,279.50, \quad c_{22}(f^*) = 0.00, \quad c_{23}(f^*) = 4,149.76,$$

$$c_{31}(f^*) = 6,080.09, \quad c_{32}(f^*) = 60.00, \quad c_{33}(f^*) = 0.00.$$

The above equilibrium flow pattern corresponds to the following equilibrium population distribution:

$$p_1^* = 13,447.92, \quad p_2^* = 1,519.66, \quad p_3^* = 1,032.44,$$

and associated utilities at the equilibrium given by:

$$u_1(p^*) = 15,792.25, \quad u_2(p^*) = 3,512.75, \quad u_3(p^*) = 7,662.51.$$

The computed equilibrium Lagrange multipliers are:

$$\delta_1^* = 15,792.25, \quad \delta_2^* = 3,512.75, \quad \delta_3^* = 9,712.17.$$

Country 1 is clearly a very attractive country. No-one migrates out of the country. Moreover, about half of those in Country 2 migrate to Country 1, and the entire original population of Country 3 migrates to Country 1. Less than a third of the original population of Country 2 remains, with others migrating also to Country 3, in addition to Country 1. Clearly, this example also verifies the theory in Section 4 associated with the international human migration model without regulations.

Furthermore, it is easy to verify that the equilibrium conditions (cf. (6) and (7)) hold for all migration outflows associated with Country 1 and with Country 2, and, clearly, $\lambda_1^* = 0$ and $\lambda_2^* = 0$. Furthermore, it follows that $\lambda_3^* = 2,049.65$, with notice that $u_3(p^*) + c_{31}(f^*) = u_1(p^*) - \lambda_3^*$, since: $7,622.51 + 6,080.09 = 15,792.25 - 2,049.65$.

We now proceed to impose a regulation on the above example. Specifically, Country 1 is concerned about overpopulation and imposes the following regulation (see also (22)):

$$f_{2\bar{1}} + f_{3\bar{1}} \leq 2,000.$$

The modified projection method converged, yielding the following equilibrium migration flow pattern:

$$f_{11}^* = 10,000.00, \quad f_{12}^* = 0.00, \quad f_{13}^* = 0.00,$$

$$f_{2\bar{1}}^* = 1,458.42, \quad f_{22}^* = 2,545.93, \quad f_{23}^* = 995.65,$$

$$f_{3\bar{1}}^* = 541.58, \quad f_{32}^* = 0.00, \quad f_{33}^* = 458.42.$$

The incurred migration costs at equilibrium are:

$$c_{11}(f^*) = 0.00, \quad c_{12}(f^*) = 20.00, \quad c_{13}(f^*) = 30.00,$$

$$c_{2\bar{1}}(f^*) = 7,332.10, \quad c_{22}(f^*) = 0.00, \quad c_{23}(f^*) = 4,002.61,$$

$$c_{3\bar{1}}(f^*) = 3,329.50, \quad c_{32}(f^*) = 60.00, \quad c_{33}(f^*) = 0.00.$$

The above equilibrium flow pattern corresponds to the following equilibrium population distribution:

$$p_1^* = 12,000.00, \quad p_2^* = 2,545.93, \quad p_3^* = 1,454.07,$$

and associated utilities at the equilibrium given by:

$$u_{\bar{1}}(p^*) = 16,727.04, \quad u_2(p^*) = 2,908.15, \quad u_3(p^*) = 6,910.75.$$

The computed equilibrium Lagrange multipliers are:

$$\delta_1^* = 10,240.25, \quad \delta_2^* = 2,908.15, \quad \delta_3^* = 6,910.75,$$

and

$$\mu_{\bar{1}}^* = 6,486.79.$$

Observe that the migration flows into Country 1 are precisely equal to the regulatory upper bound of 2,000 and, hence, the associated Lagrange multiplier $\mu_{\bar{1}}^* > 0$.

Note, also, that, under the regulation, denizens of Country 1 now enjoy a high utility of $16,727.04$, as opposed to a utility of $15,792.25$, without the regulation.

However, migrants who move to either Country 2 or to Country 3 now experience a lower utility; for Country 2: $2,908.15$ versus $3,512.75$ and, for Country 3: $6,910.76$ as opposed to $7,662.51$. Both Countries 2 and 3, under the regulation, have a higher final population than that under no regulation.

The equilibrium conditions hold with excellent accuracy for this example with the regulation, as well.

5.2.2 Multiclass Example without and with a Regulation

We now present examples consisting of two classes of migrants. The examples build on the examples in Section 5.2.1. There are, again, three countries. The initial populations of Class 1 in the countries are as in the preceding example as are the migration costs associated with the three countries. The utility functions, on the other hand, now capture interactions among classes. The superscripts 1 and 2 in the notation below refer to the respective class. The utility functions are as follows for both classes of migrants:

$$u_1^1(p) = -p_1^1 - .5p_2^1 - .5p_1^2 + 30{,}000,\ u_2^1(p) = -2p_2^1 - p_1^1 - p_2^2 + 20{,}000,\ u_3^1(p) = -3p_3^1 + .5p_2^1 - p_3^2 + 10{,}000,$$

$$u_1^2(p) = -2p_1^2 - p_1^1 + 25{,}000, \quad u_2^2(p) = -3p_2^2 - p_2^1 + 15{,}000, \quad u_3^2(p) = -p_3 - .5p_1^1 + 20{,}000.$$

The initial populations of Class 2 in the countries are: $\bar{p}_1^2 = 5{,}000$, $\bar{p}_2^2 = 3{,}000$, and $\bar{p}_3^2 = 500$. Again, we have that $c_{ii}^k(f) = 0$, $\forall i$, and for $k = 1, 2$. The additional migration cost functions for Class 2 are:

$$c_{12}^2(f) = 2f_{12}^2 + 10, \quad c_{13}^2(f) = f_{13}^2 + 20,$$

$$c_{21}^2(f) = 3f_{21}^2 + 10, \quad c_{23}^2(f) = 2f_{23}^2 + 30,$$

$$c_{31}^2(f) = f_{31}^2 + 25, \quad c_{32}^2(f) = 2f_{32}^2 + 15.$$

The remainder of the migration cost functions are as in the preceding example(s).

The modified projection method converged to the following multiclass equilibrium solution.

The equilibrium migration flows are:

for Class 1:

$$f_{11}^{1*} = 10{,}000.00, \quad f_{12}^{1*} = 0.00, \quad f_{13}^{1*} = 0.00,$$

$$f_{21}^{1*} = 2{,}649.57, \quad f_{22}^{1*} = 1{,}547.75, \quad f_{23}^{1*} = 802.68,$$

$$f_{31}^{1*} = 1{,}000.00, \quad f_{32}^{1*} = 0.00, \quad f_{33}^{1*} = 0.00,$$

for class 2:

$$f_{11}^{2*} = 2{,}343.67, \quad f_{12}^{2*} = 182.49, \quad f_{13}^{2*} = 2{,}473.85,$$

$$f_{21}^{2*} = 0.00, \quad f_{22}^{2*} = 1{,}955.57, \quad f_{23}^{2*} = 1{,}044.43,$$

$$f_{31}^{2*} = 0.00, \quad f_{32}^{2*} = 0.00, \quad f_{33}^{2*} = 500.00.$$

The incurred migration costs at equilibrium are:

for Class 1:

$$c_{11}^1(f^*) = 0.00, \quad c_{12}^1(f^*) = 20.00, \quad c_{13}^1(f^*) = 30.00,$$

$$c_{21}^1(f^*) = 13{,}287.86, \quad c_{22}^1(f^*) = 0.00, \quad c_{23}^1(f^*) = 3{,}230.72,$$

$$c^1_{31}(f^*) = 6,080.09, \quad c^1_{32}(f^*) = 60.00, \quad c^1_{33}(f^*) = 0.00,$$

for Class 2:

$$c^2_{11}(f^*) = 0.00, \quad c^2_{12}(f^*) = 374.98, \quad c^2_{13}(f^*) = 2,493.85,$$

$$c^2_{21}(f^*) = 10.00, \quad c^2_{22}(f^*) = 0.00, \quad c^2_{23}(f^*) = 2,118.86,$$

$$c^2_{31}(f^*) = 25.00, \quad c^2_{32}(f^*) = 15.00, \quad c^2_{33}(f^*) = 0.00.$$

The above equilibrium flow pattern corresponds to the following multiclass equilibrium population distribution:

$$p^{1*}_1 = 13,649.59, \quad p^{1*}_2 = 1,547.75, \quad p^{1*}_3 = 802.68,$$

$$p^{2*}_1 = 2,343.67, \quad p^{2*}_2 = 2,138.06, \quad p^{2*}_3 = 4,018.28,$$

and associated multiclass utilities at the equilibrium given by:

$$u^1_1(p^*) = 14,404.70, \quad u^1_2(p^*) = 1,116.84, \quad u^1_3(p^*) = 4,347.56,$$

$$u^2_1(p^*) = 6,663.08, \quad u^2_2(p^*) = 7,038.06, \quad u^2_3(p^*) = 9,156.92.$$

The computed equilibrium Lagrange multipliers are:

$$\delta^{1*}_1 = 14,404.70, \quad \delta^{1*}_2 = 1,116.84, \quad \delta^{1*}_3 = 8,324.62,$$

$$\delta^{2*}_1 = 6,663.08, \quad \delta^{2*}_2 = 7,038.06, \quad \delta^{2*}_3 = 9,156.92.$$

Country 1 remains the most popular for Class 1; whereas Country 3 has the greatest number of Class 2 residents. Class 2 prefers Country 3 whereas Class 1 prefers Country 1.

We now present the multiclass human migration network example under a regulation. The data are as in the multiclass example above except now we consider the following scenario. The government of Country 3 is concerned about overpopulation and imposes the following regulation, which bounds the number of migrants of either class to the country:

$$f^1_{1\bar{3}} + f^1_{2\bar{3}} + f^2_{1\bar{3}} + f^2_{2\bar{3}} \leq 2,000.$$

The modified projection method yielded the following multiclass equilibrium solution.

for Class 1:

$$f^{1*}_{11} = 10,000.00, \quad f^{1*}_{12} = 0.00, \quad f^{1*}_{1\bar{3}} = 0.00,$$

$$f^{1*}_{21} = 2,746.92, \quad f^{1*}_{22} = 1,788.86, \quad f^{1*}_{2\bar{3}} = 464.22,$$

$$f^{1*}_{31} = 1,000.00, \quad f^{1*}_{32} = 0.00, \quad f^{1*}_{33} = 0.00,$$

for Class 2:

$$f^{2*}_{11} = 3,581.93, \quad f^{2*}_{12} = 232.53, \quad f^{2*}_{1\bar{3}} = 1,185.54,$$

$$f^{2*}_{21} = 0.00, \quad f^{2*}_{22} = 2,649.76, \quad f^{2*}_{2\bar{3}} = 350.24,$$

$$f^{2*}_{31} = 0.00, \quad f^{2*}_{32} = 0.00, \quad f^{2*}_{33} = 500.00.$$

The incurred migration costs at equilibrium are:

for Class 1:

$$c^1_{11}(f^*) = 0.00, \quad c^1_{12}(f^*) = 20.00, \quad c^1_{1\bar{3}}(f^*) = 30.00,$$

$$c^1_{21}(f^*) = 13,774.62, \quad c^1_{22}(f^*) = 0.00, \quad c^1_{2\bar{3}}(f^*) = 1,876.90,$$

$$c^1_{31}(f^*) = 6.080.02, \quad c^1_{32}(f^*) = 60.00, \quad c^1_{33}(f^*) = 0.00,$$

for Class 2:

$$c^2_{11}(f^*) = 0.00, \quad c^2_{12}(f^*) = 475.06, \quad c^2_{1\bar{3}}(f^*) = 1,205.54,$$

$$c^2_{21}(f^*) = 10.00, \quad c^2_{22}(f^*) = 0.00, \quad c^2_{2\bar{3}}(f^*) = 730.47,$$

$$c^2_{31}(f^*) = 25.00, \quad c^2_{32}(f^*) = 15.00, \quad c^2_{33}(f^*) = 0.00.$$

The above equilibrium migration flow pattern corresponds to the following multiclass equilibrium population distribution:

$$p^{1*}_1 = 13,746.93, \quad p^{1*}_2 = 1,788.86, \quad p^{1*}_{\bar{3}} = 464.22,$$

$$p^{2*}_1 = 3,581.93, \quad p^{2*}_2 = 2,882.29, \quad p^{2*}_{\bar{3}} = 2,035.78,$$

and associated multiclass utilities at the equilibrium given by:

$$u^1_1(p^*) = 13,567.68, \quad u^1_2(p^*) = -206.93, \quad u^1_{\bar{3}}(p^*) = 7,465.98,$$

$$u^2_1(p^*) = 4,089.21, \quad u^2_2(p^*) = 4,564.27, \quad u^2_{\bar{3}}(p^*) = 11,090.76.$$

The computed equilibrium Lagrange multipliers are:

$$\delta^{1*}_1 = 13,567.69, \quad \delta^{1*}_2 = -206.94, \quad \delta^{1*}_3 = 7,487.66,$$

$$\delta^{2*}_1 = 4,089.20, \quad \delta^{2*}_2 = 4,564.27, \quad \delta^{2*}_3 = 11,090.75,$$

and for the regulation constraint:

$$\mu_3^* = 5,796.02.$$

Note that the regulation constraint holds tightly and, hence, the associated Lagrange multiplier is positive.

Both classes in Country 3, under the regulation, experience a higher utility than in the case of no regulation. However, with freedom of migration restricted, both classes in Countries 1 and 2 now experience lower utilities. Members of Class 2, again, enjoy the highest utility in Country 3, whereas members of Class 1, again, enjoy the highest utility in Country 1.

6. Summary and Conclusions

International human migration is a subject of global concern with the number of international migrants growing faster than the world's population. Challenges such as climate change and associated disruptions, along with wars, conflicts, and strife, are acting as push forces for humans to seek locations of greater safety and security. Others, on the other hand, are being pulled by the prospect of better economic conditions and enhanced prosperity in different countries.

Governments, in turn, are being forced to deal with increases in migratory flows across national boundaries. This has given rise to various regulations. In this paper, we provide both the theoretical and computational network framework to model and solve international human migration problems under regulations. We introduce a novel constraint that captures distinct possible regulations associated with bounding migratory flows of different countries and classes of migrants. The work builds on the earlier literature on operations research perspectives for human migration but with notable extensions:

1. We provide a new underlying network structure for problems of human migration and associated feasibility conditions.

2. A constraint is introduced to capture a plethora of regulations of migratory flows.

3. Lagrange analysis is conducted, with accompanying insights, on the associated utilities of migrants at origin and at destination nodes, along with incurred migration costs.

4. Alternative variational inequality formulations of the governing equilibrium conditions are constructed along with accompanying qualitative analysis, in terms of conditions for existence and uniqueness of a solution.

5. An algorithm, with nice features for implementation, is proposed, along with conditions

for convergence, and then applied to solve a series of single class and multiclass international human migration problems, in order to illustrate the framework.

The numerical examples support the theoretical result that, under the regulation, denizens of the country imposing the migratory flow bound, incur higher utility, whereas those in other countries can experience reduced utility, due to restrictions on their movement across national boundaries for relocation.

The results in this paper also add to the literature on operations research for problems of societal impact, with an emphasis on the impact of regulations.

Future research can advance in multiple directions, including: the investigation of human migration chaining between countries, under regulations; comparing system-optimal solutions to problems of human migration to the more user-optimal perspective delineated here, and even applying the model to problems of migration encountered in nature, such as that of fisheries, in order to reduce the depletion of stocks, a problem, in many parts of the globe. Of course, the general, theoretical framework in this paper can also be adapted to empirically study migratory flows in various parts of the world, with and without regulations imposed.

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