An Integrated Financial and Logistical Game Theory Model for Humanitarian Organizations with Purchasing Costs, Multiple Freight Service Providers, and Budget, Capacity, and Demand Constraints

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Abstract: In this paper, a game theory model for disaster relief is constructed that incorporates both financial and logistical aspects of humanitarian organizations involved in the purchasing and delivery of relief items, post-disaster, using freight services. The model allows for the purchasing of the relief items, both locally and nonlocally, includes a budget constraint for each humanitarian organization, along with imposed lower and upper bound demand constraints at each point of demand by a higher level organization. The governing concept is that of Generalized Nash Equilibrium, since not only does the utility function of a given humanitarian organization depend on its own strategies and the strategies of the other humanitarian organizations, but the constraints do as well. The concept of a variational equilibrium is utilized to derive the variational inequality formulation of the governing equilibrium conditions and the model is analyzed qualitatively. Lagrange analysis of the marginal utilities is conducted to gain insights on the impact of the constraints and an alternative variational inequality constructed, with nice features for computations. An algorithm is proposed and explicit formulae provided for the logistical flows and Lagrange multipliers at each iteration. Numerical examples, inspired by Hurricane Harvey hitting Houston, Texas, as occurred in August 2017, illustrate the framework. This work adds to the still nascent literature on game theory and disaster relief and also to the literature on variational inequality models with nonlinear constraints, which is limited.

Keywords:: game theory; logistics; Generalized Nash Equilibrium; disaster relief; budget constraints; variational inequalities

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1. Introduction

The Emergency Events Database (EM-DAT) defines a disaster as a natural situation or event that overwhelms local capacity and/or necessitates a request for external assistance. For a disaster to be entered into the EM-DAT database at least one of the following criteria must be met: (1) 10 or more people are reported killed; (2) 100 people are reported affected; (3) a state of emergency is declared; or (4) a call for international assistance is issued (Cavallo and Noy (2011)). Natural disasters can be sudden-onset such as, for example, earthquakes and hurricanes, or they can be slow-onset, such as droughts, which may result in famine and migrations (see Worland (2017)). Man-made crises can be the result of political situations such as the continuing Syria refugee crisis (cf. Tisdall (2017)) or because of human faults and intentions such as terrorist attacks as in 9/11 (see Argenti (2002)). The number of disasters is growing as well as the number of people affected by them (Nagurney and Qiang (2009)). Indeed, natural disasters, as defined in the EM-DAT database, are now fairly common events (Cavallo and Noy (2011)) with climate change exacerbating their frequency and severity (see Harvey (2018)).

Due to the prevalence and the impacts of disasters, researchers as well as practitioners have increased their efforts to find solutions to reduce the associated disaster losses and casualties (cf. Kotsireas, Nagurney, and Pardalos (2016) and the references therein) with work taking place on all aspects of the four-step disaster management concept consisting of: mitigation, preparedness, response, and recovery. However, even developed countries still are not able to prevent disasters from negatively affecting them.

When a disaster strikes a region, there may be very little time to save lives and to minimize losses through an appropriate response. After severe disasters, victims may face a shortage of critical essentials such as: water, food, medicine, blood, clothing, and appropriate shelter from the elements. Given the characteristics of disasters, providing such primary relief supplies is crucial; however, in many cases, it may be very difficult and challenging, since the infrastructure may be severely compromised, if not destroyed, and numerous services unavailable (see, e.g., Nagurney, Yu, and Qiang (2011)). Moreover, governments alone may not be able to assume the full responsibility of providing the essentials due to huge costs, limited human resources, and many other challenges going beyond governments' abilities and resources. Furthermore, in certain disasters, such as the 2010 Haiti earthquake, the local governmental facilities themselves may be severely compromised due to the disaster (Miller Liana (2010)). In such situations, the call for humanitarian organizations to assist, both national as well as international ones, is reasonable and also inevitable.

Humanitarian organizations, nevertheless, also face difficulties on the road to assisting in disaster response and recovery. As noted by Murray (2005), instead of the relatively stable demand for numerous products enjoyed by the private sector in the context of commercial supply chains, relief organizations face surges in demand whose timing, location, and scale are unpredictable. To overcome such obstacles, it is imperative that they focus on logistics. One of the notable aspects of the relief efforts following the Asian tsunami in 2004, for example, was that logistics was publicly acknowledged to play an extremely important role in relief (Thomas and Kopczak (2005)). Humanitarian logistics is defined as the process of planning, implementing, and controlling the efficient, cost-effective flow and storage of goods and materials and also the related information from the point of origin to the point of demand in order to reduce the suffering of the victims.

One of the most important restrictions in humanitarian operations is the cost, with logistics estimated to account for 80% of the total cost associated with disaster relief (cf. Van Wassenhove (2006)). HOs (Humanitarian Organizations), which are non-governmental organizations (NGOs), in turn, are nonprofit, and, as a result, they are dependent on donations, especially financial ones, for their ultimate effectiveness and sustainability (see also Toyasaki and Wakolbinger (2014)). Hence, competition is natural in an environment in which HOs are competing for donor funding with additional challenges including to enable mechanisms for collaboration within the competitive environment (Beamon and Kotleba (2006)). Indeed, a lack of coordination among agencies may lead to the duplication of efforts, and associated poor utilization of scarce resources, as well as confusion at the "last mile" (cf. Balcik, Beamon, and Smilowitz (2008)) along with issues of materiel convergence (cf. Nagurney, Alvarez Flores, and Soylu (2016) and the references therein). Initiatives have emerged, such as the United Nations Joint Logistics Center, which promotes cooperation among agencies by, for example, gathering and disseminating information and enabling the pooling of expensive assets such as aircraft (Murray (2005)). However, as eloquently noted in Kaatrud, Samii, and Van Wassenhove (2003), no single agency in a large-scale disaster has the resources to respond to immense areas in terms of determining the status of infrastructure, accessibility, availability, and even the relief item prices. This was especially vivid in the case of the response of FEMA to Hurricane Maria and Puerto Rico (Robles (2018)).

In this paper, hence, the focus is on game theory. The use of game theory enables the modeling and analysis of multiple players / decision-makers, each of whom is involved in his own optimization problem but with interactions with other decision-makers through objective functions, and, perhaps, constraints, and allows the modeler/analyst to capture the complexity and scale of humanitarian operations post-disaster in a more realistic and insightful manner. The new model constructed in this paper adds to the literature on game theory frameworks for the integration of financial and logistical issues faced by humanitarian organizations engaged in disaster relief. The model includes multiple humanitarian organizations (HOs) with associated benefit and logistical cost functions and relief item purchase costs, multiple freight service providers (FSPs) engaged, respectively, in the procurement and delivery of relief items with the HOs faced with budget constraints and the FSPs faced with capacity constraints. The HOs also are also subject to demand lower and upper bounds at demand points for the relief items with the latter ensuring that the minimal demands are satisfied and the former included with an eye towards reducing materiel convergence. The new model also adds to the literature on variational inequality problem with nonlinear constraints, which have been increasingly of interest, but with only a few papers to-date such as, for example, Toyasaki, Daniele, and Wakolbinger (2014) with an application to sustainable supply chains, and Nagurney, Daniele, and Shukla (2017) and Colojanni et al. (2018) with applications to cybersecurity investments.

2. Literature Review with a Focus on Game Theory and Humanitarian Operations

To address some of the above challenges, Nagurney, Alvarez Flores, and Soylu (2016) developed the first Generalized Nash Equilibrium (GNE) model for disaster relief, which integrated the financial side, in terms of the securing of donations, based on the visibility of HOs, and the logistical side, in terms of the cost-effectiveness of the delivery of relief items. The model contained, as a special case, its Nash Equilibrium (Nash (1950, 1951)) counterpart, which was achieved with the removal of the imposed lower and upper bound demands at points of demand, which result in common/shared constraints. In the case of game theory, recall that each HO as a player has a utility and tries to maximize it by selecting its optimal strategies. We should mention that each player's strategy has an effect on the others' utilities, and, if they do not share the constraints, then we are dealing with a Nash Equilibrium in the case of noncooperation. On the other hand, if some of the constraints are common/shared, then we have a Generalized Nash Equilibrium (Debreu (1952), Rosen (1965)). The Nagurney, Alvarez Flores, and Soylu (2016) model, because of its special structure, enabled an optimization reformulation of the GNE conditions. A case study focusing on Hurricane Katrina demonstrated that humanitarian organizations may end up delivering supplies where it is easier/cheaper to do so, unless the demand bounds are imposed by a higher level authority, with the consequence that demands may not be met and there may be an oversupply at demand points that are easier to reach. Subsequently, Nagurney et al. (2018) provided an extension of the earlier model, which does not allow for an optimization reformulation, but, instead, was reformulated as a variational inequality using the concept of a Variational Equilibrium. The model has more general logistical cost functions as well as financial funding functions and altruism functions associated with the humanitarian organizations than the earlier model. A case study applied to tornadoes hitting western Massachusetts, as occurred in June 2011, demonstrates the robustness of the previous case study results and further reinforces the need for coordination by a high level authority. The bounds on the demands in the affected areas, for example, could be applied by an upper level organization such as the United Nations Joint Logistics Center in order to manage the relief operation properly, even in the presence of competition among the humanitarian organizations.

It is important to mention that there have been numerous studies focusing on optimization frameworks in the context of disaster relief as well as humanitarian logistics. In these contexts it is imperative to construct the appropriate objective functions and constraints with the former being distinctly different from, for example, profit maximization used widely in commercial supply chains (cf. Nagurney (2006) and the references therein). For example, Tzeng, Cheng, and Huang (2007) proposed a dynamic selection of the amount of relief items to be transported from depots to demand points such that three objectives are achieved: minimum total cost, minimum travel time, and maximum demand satisfaction. Haghani and Oh (1996) considered commodity carry-over, routing, and mode transfer. Ozdamar, Ekinci, and Kkyazici (2004), Yi and Kumar (2007), and Yi and Ozdamar (2007) included split delivery and the sum of unmet demands in their models. Vitoriano et al. (2011) constructed a multicriteria model for humanitarian relief distribution with criteria of time of response, equity, and security. Huang, Smilowitz, and Balcik (2012), in their model, incorporated multiple relevant disaster relief objectives of efficiency, efficacy, and equity with the consideration of both vehicle routing and resource distribution.

Ertem and Buyurgan (2013) emphasized that procurement is one of the first and perhaps the most overlooked step in disaster relief operations and, in this paper, we include product procurement as well as freight service provision procurement decisions, but in a game theory framework. Falasca and Zobel (2011), in turn, constructed a two-stage stochastic model for procurement in humanitarian relief supply chains in which purchasing decisions are made after a disaster strikes (which is also the case in our model). Rancourt, Bellavance, and Goentzel (2014), on the other hand, focused on transportation procurement for food aid in Ethiopia through the construction of econometric models. Tofighi, Torabi, and Mansouri (2016) focused on a two-echelon humanitarian logistics network design problem, for handling pre-disaster and post-disaster challenges, with the city of Tehran in mind, and considering uncertainty and probabilistic characteristics of a potential earthquake. In their research, they deal with decisive criteria such as response time. The authors seek to minimize the total distribution time, the total cost of unused inventories, as well as other objectives. Ali Torabi et al. (2018), subsequently, introduced a novel two-stage scenario-based mixed fuzzy-stochastic programming model for integrated relief pre-positioning and procurement planning based on a quantity flexibility contract under a mixture of uncertain data, accompanied by an effective multi-step solution method.

We refer the reader to the special issue of the International Journal of Production Economics focused on *improving disaster supply chain management - key factors for humanitarian relief*, with an editorial by Boin, Kelle, and Clay Whybark (2010). Also we note the survey of optimization models in emergency logistics by Caunhye, Nie, and Pokharel (2012) and for additional references on models in humanitarian logistics, see Duran et al. (2013) and the survey by Ortuño et al. (2013).

In terms of models with multiple organizations, Rodriguez-Espindola, Albores, and Brewster (2018) constructed a dynamic, multi-objective optimization model for humanitarian response operations incorporating multiple organizations. Muggy and Heier Stamm (2014), in turn, in their review of game theory and humanitarian operations emphasized that the applications of game theory to this important domain have been limited to-date. They highlight that relief agencies compete for media exposure as well as for financial funds (see also, Zhuang, Saxton, and Wu (2014) and Toyasaki and Wakolbinger (2014)). Seaberg, Devine, and Zhuang (2017) provided an excellent, panoramic review of 57 papers over a ten year horizon from 2006 to 2016 of papers on disaster management and game theory and found that the response phase of disaster relief has been researched most extensively. However, none of the papers reviewed therein consider a noncooperative game with multiple decision-makers, in the form of HOs, that includes procurement, freight service selection, and relief item distribution to multiple locations with the inclusion of benefit functions of HOs to measure the effectiveness of HO activities, along with logistical costs, as we do in our new model in this paper. Coles and Zhuang (2011) discussed the potential of cooperative game theory in disaster recovery operations, along with a critique and references, which is a promising direction for future research. Muggy and Heier Stamm (2014) also emphasized that cooperative models may assist in the identification of methods for partnering agencies to achieve greater impact than what is possible individually and independently. Nagurney and Qiang (2009) identified potential synergies associated with the possible teaming of humanitarian organizations from a supply chain network perspective.

Nagurney (2018) constructed a multitiered disaster relief network equilibrium model of humanitarian organizations and freight service providers assuming pre-positioned supplies and with fixed demands for deliveries. The author utilized variational inequality theory, which is also the formalism for the model in this paper. Gossler et al. (2018), in turn, focused on slow-onset disasters in which framework agreements are available for freight service provision and constructed a game theory model consisting of two submodels. They also made use of a variational equilibrium as was done in Nagurney et al. (2018). We remark that, to-date, the only papers on humanitarian operations and disaster relief that utilize Generalized Nash Equilibrium are those by Nagurney, Alvarez Flores, Soylu (2017), Nagurney et al. (2018), Gossler et al. (2018), and this paper.

2.1 Our Contributions

Specifically, in this paper, we construct a game theory model to capture the competition among humanitarian organizations with the goal of bringing greater realism in terms of the constraints that they face, such as budget ones, and in expanding the scope as we continue to integrate logistical and financial aspects. In particular, we extend the model of Nagurney et al. (2018), in substantive and significant ways, in order to incorporate and quantify the following issues:

1. **Purchasing of the products.** An important part of the humanitarian organizations' costs is the purchasing of the relief items that are then transported to the demand areas. There are different suppliers which HOs may be able to purchase products from and these may be local to the disaster or nonlocal with distinct associated prices. There may, hence, be trade-offs, as noted, for example, by Balcik and Beamon (2008): although local supplies may not be available in the quantity and quality needed, local procurement is, typically, characterized by shorter lead times and has lower logistics costs. The Nagurney et al. (2018) model assumed that the products were prepositioned. The model in this paper, in contrast, allows for the purchasing of the relief items post-disaster.

2. **Multiple freight service providers.** As we are focusing on logistics, it is important to consider different freight service providers (FSPs) that can ship relief items from the purchase locations of the HOs to the demand points. Each freight service provider may have specific advantages and disadvantages and may charge the HOs accordingly (see also Nagurney (2018)). Procuring globally may have the advantage of lower prices, higher quality and capacity, but brings the disadvantages of longer response time and higher transportation costs due to long distances (Duran et al. (2013)). In this paper, unlike in the model in Nagurney et al. (2018), there are multiple possible freight service providers the HOs can select services from.

3. **Capacity constraints.** The humanitarian organizations, in disaster situations, may be constrained by the freight service providers' shipment capacities due to their available facilities and the impacted regions' infrastructures. No such capacity constraints were considered in Nagurney et al. (2018).

4. **Budget constraints.** The budget constraint faced by an HO is the most critical constraint in any humanitarian relief operation. HOs do not have an unlimited budget; hence, they must determine the optimal way in which to allocate their resources without exceeding the limitation. This also demonstrates to donors and stakeholders that they are financially responsible. Since relief agencies have limited funding, procurement procedures in the humanitarian aid sector are principally accomplished through price-based competitive bidding (Balcik et al. (2010) and Gossler et al. (2018)). No budget constraints were included in the model in Nagurney et al. (2018).

One of the novel features of the model introduced in this paper is that humanitarian organizations have the flexibility of purchasing the relief items from different locations at associated prices and then having them shipped by freight service providers from the purchase locations to the points of demand, also at associated prices. This allows for making optimal resource allocations, given a budget constraint, based on local/nonlocal purchase prices and freight service provision costs. Furthermore, our framework captures the fact that multiple HOs may compete for freight service provision and the freight service providers have limited capacity for shipping. Although, recently, there have been freight service models introduced for humanitarian operations using game theory (cf. Nagurney (2016, 2018) and Gossler et al. (2018)), this paper differs from the previous work in that we include, among other features, purchasing and shipment from multiple locations, under budget constraints, and also objective functions that include benefit/altruism, since HOs are nonprofits. In addition, in the new model in this paper, in contrast to the models of Nagurney, Alvarez Flores, and Soylu (2016) and Nagurney et al. (2018), we do not assume that the relief items are prepositioned, but, rather, that they must be purchased. Moreover, we have explicit budget constraints whereas in the earlier papers the financial aspect was captured through functions that quantified donations based on the visibility of the HOs in terms of relief item deliveries and concomitant media attention. Finally, the budget constraints can handle nonlinear freight service costs.

As mentioned earlier, theresults in this paper also contribute to the literature on variational inequalities with nonlinear constraints, with a focus on game theory, which, in terms of modeling and analysis contributions is fairly recent. Moreover, for the model in this paper, we also provide a Lagrange analysis of the marginal utilities of the humanitarian organizations. To-date, the only other work that includes Lagrange analysis for a humanitarian logistics model in the context of game theory, is the paper by Nagurney et al. (2018) and therein all the constraints were linear and there were no purchasing costs nor budget constraints.

The remainder of the paper is organized as follows. In Section 3, we describe how the humanitarian organizations compete noncooperatively to maximize their respective utilities from the disaster relief operation, and present the components of the utility functions and the constraints. We construct the Generalized Nash Equilibrium model and present the Variational Equilibrium, followed by the derivation of the variational inequality formulation. For background on variational inequalities, see the books by Nagurney (1999, 2006). In Section 4, we utilize Lagrange theory to investigate the role of each constraint and to gain insights and we also provide alternative variational inequality formulations, one of which we then utilize for computational purposes. In Section 5, we present an algorithm, which yields closed form expressions for each of the relief item flows and Lagrange multipliers at a given iteration, along with numerical examples. The numerical examples are inspired by Hurricane Harvey hitting Houston, Texas, as occurred in August 2017. We summarize the results and present the conclusions in Section 6.

3. The Integrated Financial and Logistical Game Theory Model for Humanitarian Organizations with Purchasing Costs, Multiple Freight Service Providers, and Budget, Capacity, and Demand Constraints

In this section, we present a new game theory model of competition among humanitarian organizations in disaster relief. There are m humanitarian organizations, with a typical one denoted by i, involved in delivering relief supplies to n locations, with a typical location denoted by j. The humanitarian organizations may avail themselves of t freight service providers (FSPs) with a typical freight service provider denoted by l. Each humanitarian organization (HO) may purchase the relief item at o possible locations, with a typical such location denoted by k. The network structure of the problem is given in Figure 1.

We denote the volume of relief items purchased by HO i at location k and shipped to demand location j by FSP l by $q_{ijk,l}$. We group the relief item shipments of HO i into the vector $q_i \in R_+^{not}$. This is the strategy vector of HO i. We then further group the strategy vectors of all the HOs into the vector $q \in R_+^{mnot}$.

The price associated with purchasing the relief item at location k is denoted by ρ_k. Hence, the total financial outlay for purchasing the relief items at the various locations for HO i;

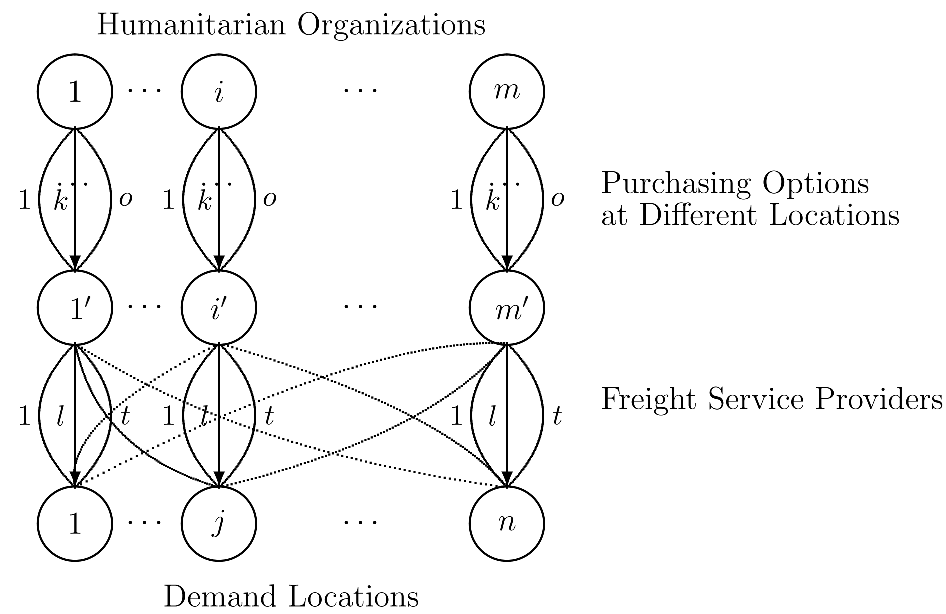

Figure 1: The Network Structure of the Game Theory Model with Multiple Purchasing Options and Multiple Freight Service Providers

$i = 1, \ldots, m$, is

$$\sum_{k=1}^{o} \rho_k \sum_{j=1}^{n} \sum_{l=1}^{t} q_{ijk,l}. \tag{1}$$

In addition to the purchasing outlay, each HO i; $i = 1, \ldots, m$, is responsible for having the relief items transported to the various points of demand following the disaster. Let $c_{ijk,l}$ denote the transportation cost that HO i pays to get its relief items delivered to the demand point j by freight service provider l from purchase location k. The total outlay associated with the logistical costs, hence, can be expressed for HO i; $i = 1, \ldots, m$, as:

$$\sum_{j=1}^{n} \sum_{k=1}^{o} \sum_{l=1}^{t} c_{ijk,l}(q). \tag{2}$$

Note that, according to expression (2), for the sake of generality, and also to further capture the competitive aspects for freight service provision, we have that, in general, the cost can depend on the vector of relief item shipments. Hence, the logistical costs of shipment depend not only on the procurement/shipment volumes of a specific HO, but also on those of the other HOs. These cost functions are assumed to be convex and continuously differentiable.

Since HOs are nonprofit organizations, they cover their costs by attracting donations. However, in doing so they compete with each other. The more effectively they provide relief at the demand points, the more attention they receive from potential and existing donors. Hence, we utilize benefit functions that measure the effectiveness of the HOs' activities. According to Nagurney, Alvarez Flores, and Soylu (2016), HOs may benefit not only from their own efforts but also from other HOs' visibility at the demand points. A benefit function associated with HO i; $i = 1, \ldots, m$, is denoted by $B_i(q)$, and with it we associate a nonnegative monetization weight ω_i as follows:

$$\omega_i B_i(q). \tag{3}$$

The benefit functions, which also may be interpreted as altruism functions, are assumed to be concave and continuously differentiable. Note that the benefit functions may, in general, depend on the vector of all the procurement/shipment strategies of all the HOs.

The utility function $U_i(q)$ for HO i; $i = 1, \ldots, m$, given the above, can be expressed as:

$$U_i(q) = \omega_i B_i(q) - \sum_{k=1}^{o} \rho_k \sum_{j=1}^{n} \sum_{l=1}^{t} q_{ijk,l} - \sum_{j=1}^{n} \sum_{k=1}^{o} \sum_{l=1}^{t} c_{ijk,l}(q). \tag{4}$$

We now describe the constraints faced by the HOs. Some of these are specific to each HO, whereas others are common, that is, shared. It is the latter type of constraint that makes the competitive game theory model one that is governed by a Generalized Nash Equilibrium and not simply by a Nash Equilibrium.

The first set of constraints is that the volume of the relief item of each HO i; $i = 1, \ldots, m$, to any demand point $j; j = 1, \ldots, n$, purchased at location k; $k = 1, \ldots, o$, and transported by FSP $l; l = 1, \ldots, t$, consists of the nonnegativity constraints:

$$q_{ijk,l} \geq 0, \quad \forall j, k, l. \tag{5}$$

The next constraint faced by each HO i; $i = 1, \ldots, m$, is its budget constraint:

$$\sum_{k=1}^{o} \rho_k \sum_{j=1}^{n} \sum_{l=1}^{t} q_{ijk,l} + \sum_{j=1}^{n} \sum_{k=1}^{o} \sum_{l=1}^{t} c_{ijk,l}(q) \leq b_i, \tag{6}$$

where b_i is HO i's budget allocated to both procurement of the disaster relief items and transportation to points of demand.

We define the feasible set K_i corresponding to HO i as:

$$K_i \equiv \{q_i | (5) \text{ holds}\} \tag{7}$$

and we let $K \equiv \prod_{i=1}^{m} K_i$.

The HOs compete among themselves for freight capacity and, hence, are faced with the following common set of constraints. Indeed, while the HOs may be willing to send as much of the relief item as they can, FSPs have limited capacity due to their facilities, vehicle portfolio and availability, and also the disaster regions' infrastructures, which may have been severely compromised, if not destroyed. Therefore, the HOs cannot send more than a certain volume of relief items through a specific freight service provider from a particular purchase location. With $u_{k,l}$ denoting the shipment capacity from purchase location k of FSP l, the common capacity constraints are:

$$\sum_{i=1}^{m}\sum_{j=1}^{n} q_{ijk,l} \leq u_{k,l}, \quad k = 1, \ldots, o; l = 1, \ldots, t. \tag{8}$$

Furthermore, to ensure that the minimum demands are satisfied at the demand points, while not exceeding the maximum demands, in order to reduce possible congestion as well as materiel convergence, as in Nagurney, Alvarez Flores, and Soylu (2016), we have the following constraints, which are imposed by a higher authority. By applying lower bound and upper bounds on the demands we guarantee that at each demand point j; $j = 1, \ldots, n$, the volume of relief items will not be less than $\underline{d}_j$ and, at the same time, it will not exceed $\bar{d}_j$, that is,

$$\sum_{i=1}^{m}\sum_{k=1}^{o}\sum_{l=1}^{t} q_{ijk,l} \geq \underline{d}_j, \quad j = 1, \ldots, n, \tag{9}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{o}\sum_{l=1}^{t} q_{ijk,l} \leq \bar{d}_j \quad j = 1, \ldots, n. \tag{10}$$

In Nagurney, Alvarez Flores, and Soylu (2016) and in Nagurney et al. (2018), the importance of such constraints was demonstrated in case studies, in which their removal resulted in an oversupply in certain regions and an undersupply in other (typically, in those regions where it was more difficult and costly to deliver relief items). Hence, we retain these important constraints in our model.

We define the feasible set $\mathcal{S}$ of shared constraints as:

$$\mathcal{S} \equiv \{q | (6) \text{ holding for all } i, \text{ and } (8), (9), (10) \text{ hold}\}. \tag{11}$$

Here we assume that the sum of the budgets of all the HOs, i.e., $\sum_{i=1}^{m} b_i$, is sufficient to meet the sum of all the minimum demands, that is, $\sum_{j=1}^{n} \underline{d}_j$ so that the set $\mathcal{K} \equiv K \cap \mathcal{S}$ will be nonempty.

Now we are ready to state the formal definition.

Definition 1: Generalized Nash Equilibrium for the Humanitarian Organizations

A relief item flow vector $q^* \in K, q^* \in \mathcal{S}$ is a Generalized Nash Equilibrium if for each HO $i; i = 1, \dots, m$:

$$U_i(q_i^*, \hat{q_i^*}) \geq U_i(q_i, \hat{q_i^*}), \quad \forall q_i \in K_i, \forall q \in \mathcal{S}, \tag{12}$$

where $\hat{q_i^*} \equiv (q_1^*, \dots, q_{i-1}^*, q_{i+1}^*, \dots, q_m^*)$.

The above definition means that not one of the HOs is willing to deviate from his current relief item flow pattern, given the relief flow item patterns of the other HOs. Observe that each HO's utility depends not only on his own strategies but also on those of the others' strategies, and so do their feasible sets, since their feasible sets are intertwined. The latter condition makes the problem a Generalized Nash Equilibrium model (Debreu (1952)). We know that the feasible sets K_i are convex for each i, as is the set $\mathcal{S}$, under the imposed assumptions.

It is well-known that Generalized Nash Equilibria can be formulated as quasivariational inequality problems (cf. Fischer, Herrich, and Schonefeld (2014)); however, the state of the art of algorithms for the solution of such problems is not as advanced as for variational inequality problems. As noted in Nagurney, Yu, and Besik (2017), one may take advantage of a refinement of the GNE known as a Variational Equilibrium, which is a specific type of GNE (cf. Kulkarni and Shabhang (2012)), and enables a variational inequality formulation.

Definition 2: Variational Equilibrium

A relief item flow vector q^* is a Variational Equilibrium of the above Generalized Nash Equilibrium problem if $q^* \in K, q^* \in \mathcal{S}$ is a solution to the following variational inequality:

$$-\sum_{i=1}^{m} \langle \nabla_{q_i} U_i(q^*), q_i - q_i^* \rangle \geq 0, \quad \forall q \in K, \forall q \in \mathcal{S}. \tag{13}$$

In particular, it is worth mentioning that the variational equilibrium corresponds to Lagrange multipliers associated with the common constraints being the same for all the HOs. This has a nice fairness and equity interpretation and is very reasonable for humanitarian organizations involved in disaster relief.

Expanding variational inequality (13), we obtain:

$$\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{o} \sum_{l=1}^{t} \left[\sum_{r=1}^{n} \sum_{p=1}^{o} \sum_{s=1}^{t} \frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}} + \rho_k - \omega_i \frac{\partial B_i(q^*)}{\partial q_{ijk,l}} \right] \times \left[q_{ijk,l} - q_{ijk,l}^* \right] \geq 0, \tag{14}$$

$$\forall q \in K, \forall q \in \mathcal{S}.$$

We now put variational inequality (14) into standard form (cf. Nagurney (1999)): determine $X^* \in \mathcal{K}$, such that:

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{15}$$

where $\langle \cdot, \cdot \rangle$ is the inner product in N-dimensional Euclidean space, where $N = mnot$ for our model. We define $X \equiv q$ and $F(X)$ as having components:

$$F_{ijk,l}(X) \equiv \sum_{r=1}^{n} \sum_{p=1}^{o} \sum_{s=1}^{t} \frac{\partial c_{irp,s}(q)}{\partial q_{ijk,l}} + \rho_k - \omega_i \frac{\partial B_i(q)}{\partial q_{ijk,l}}, \quad \forall i, j, k, l, \tag{16}$$

with $\mathcal{K} \equiv K \cap \mathcal{S}$, and the standard form follows.

Remark: Existence and Uniqueness

Since the function $F(X)$ that enters our variational inequality problem (15) with components as in (16) is, under the imposed conditions, continuous and, clearly, the feasible set $\mathcal{K}$ is not only convex, but compact because of the demand and budget constraints, we know that a solution X^* exists from the standard theory of variational inequalities (Kinderlehrer and Stampacchia (1980)).

4. Lagrange Theory and Analysis of the Marginal Utilities:

In this section, we investigate the Lagrange theory associated with the variational inequality (14). Then, through the use of the Lagrange multipliers, we analyze the marginal utilities and the role of each constraint in the model. We also derive alternative variational inequalities to the one in (14), one of which we then utilize for computational purposes.

By setting:

$$C(q) = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{o} \sum_{l=1}^{t} \left[\sum_{r=1}^{n} \sum_{p=1}^{o} \sum_{s=1}^{t} \frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}} + \rho_k - \omega_i \frac{\partial B_i(q^*)}{\partial q_{ijk,l}} \right] \times \left[q_{ijk,l} - q^*_{ijk,l} \right], \tag{17}$$

variational inequality (14) can be rewritten as the following minimization problem:

$$\min_{\mathcal{K}} C(q) = C(q^*) = 0. \tag{18}$$

Based on the previous assumptions, all the involved functions in (18) are convex and continuously differentiable.

In order to construct the Lagrange function, we reformulate the constraints as below, with the associated Lagrange multiplier next to the corresponding constraint:

$$
\begin{aligned}
e_{k,l} &= \sum_{i=1}^{m}\sum_{j=1}^{n} q_{ijk,l} - u_{k,l} \le 0, \quad \epsilon_{k,l}, \forall k, \forall l, \\
f_i &= \sum_{k=1}^{o} \rho_k \sum_{j=1}^{n}\sum_{l=1}^{t} q_{ijk,l} + \sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t} c_{ijk,l}(q) - b_i \le 0, \quad \gamma_i, \forall i, \\
g_{ijk,l} &= -q_{ijk,l} \le 0, \quad \lambda_{ijk,l}, \forall i, \forall j, \forall k, \forall l, \\
a_j &= \underline{d}_j - \sum_{i=1}^{m}\sum_{k=1}^{o}\sum_{l=1}^{t} q_{ijk,l} \le 0, \quad \alpha_j, \forall j, \\
b_j &= \sum_{i=1}^{m}\sum_{k=1}^{o}\sum_{l=1}^{t} q_{ijk,l} - \bar{d}_j \le 0, \quad \beta_j, \forall j,
\end{aligned}
\tag{19}
$$

and

$$
\Gamma(q) = (e_{k,l}, f_i, g_{ijk,l}, a_j, b_j)_{i=1,\dots,m; j=1,\dots,n; k=1,\dots,0; l=1,\dots,t}. \tag{20}
$$

We now construct the Lagrange function:

$$
\begin{aligned}
\mathcal{L}(q, \epsilon, \gamma, \lambda, \alpha, \beta) = &\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t} \left[\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t} \frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}} + \rho_k - \omega_i \frac{\partial B_i(q^*)}{\partial q_{ijk,l}}\right] \times \left[q_{ijk,l} - q^*_{ijk,l}\right] \\
&+ \sum_{k=1}^{o}\sum_{l=1}^{t} e_{k,l}\epsilon_{k,l} + \sum_{i=1}^{m} f_i\gamma_i + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t} g_{ijk,l}\lambda_{ijk,l} + \sum_{j=1}^{n} a_j\alpha_j + \sum_{j=1}^{n} b_j\beta_j, \\
&\forall q \in R_+^{mnot}, \forall \alpha \in R_+^n, \forall \beta \in R_+^n, \forall \epsilon \in R_+^{ot}, \forall \gamma \in R_+^m, \forall \lambda \in R_+^{mnot},
\end{aligned}
\tag{21}
$$

where α is the vector of all α_js, β is the vector of all β_js, ϵ is the vector of all $\epsilon_{k,l}$s, γ is the vector of all γ_is, and λ is the vector of all $\lambda_{ijk,l}$s.

Since the feasible set $\mathcal{K}$ is convex and the Slater condition is satisfied, if q^* is a minimal solution to problem (18) there exist $\epsilon^* \in R_+^{ot}$, $\gamma^* \in R_+^m$, $\lambda^* \in R_+^{mnot}$, $\alpha^* \in R_+^n$, $\beta^* \in R_+^n$, such that the vector (q^*, ϵ^*, γ^*, λ^*, α^*, β^*) is a saddle point of the Lagrange function (21):

$$
\mathcal{L}(q^*, \epsilon, \gamma, \lambda, \alpha, \beta) \le \mathcal{L}(q^*, \epsilon^*, \gamma^*, \lambda^*, \alpha^*, \beta^*) \le \mathcal{L}(q, \epsilon^*, \gamma^*, \lambda^*, \alpha^*, \beta^*) \tag{22}
$$

and

$$
\begin{aligned}
e^*_{k,l}\epsilon^*_{k,l} &= 0, \qquad \forall k, \forall l, \\
f^*_i\gamma^*_i &= 0, \qquad \forall i, \\
g^*_{ijk,l}\lambda^*_{ijk,l} &= 0, \qquad \forall i, \forall j, \forall k, \forall l,
\end{aligned}
$$

$$a_j^* \alpha_j^* = 0, \qquad \forall j,$$

$$b_j^* \beta_j^* = 0, \qquad \forall j. \tag{23}$$

From the right-hand side of (22), it follows that $q^* \in R_+^{mnot}$ is a minimal point of the function $\mathcal{L}(q, \epsilon^*, \gamma^*, \lambda^*, \alpha^*, \beta^*)$ in the whole space R^{mnot}, and, hence, for all $i = 1, \ldots, m$, for all $j = 1, \ldots, n$, for all $k = 1, \ldots, o$, and for all $l = 1, \ldots, t$, we have that:

$$\frac{\partial \mathcal{L}(q^*, \epsilon^*, \gamma^*, \lambda^*, \alpha^*, \beta^*)}{\partial q_{ijk,l}} = \left[\sum_{r=1}^{n} \sum_{p=1}^{o} \sum_{s=1}^{t} \frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}} + \rho_k - \omega_i \frac{\partial B_i(q^*)}{\partial q_{ijk,l}} \right]$$

$$+\epsilon_{k,l}^* + \gamma_i^* \left(\rho_k + \sum_{r=1}^{n} \sum_{p=1}^{o} \sum_{s=1}^{t} \frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}} \right) - \lambda_{ijk,l}^* - \alpha_j^* + \beta_j^* = 0, \tag{24}$$

together with conditions (23).

Theorem: Alternative Variational Inequality Formulations

Conditions (23) and (24) represent an equivalent formulation of variational inequality (14) given by: determine $(q^*, \epsilon^*, \gamma^*, \lambda^*, \alpha^*, \beta^*) \in R_+^{2mnot+ot+m+2n}$ *such that*

$$\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{o} \sum_{l=1}^{t} \left[(\sum_{r=1}^{n} \sum_{p=1}^{o} \sum_{s=1}^{t} \frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}} + \rho_k)(1 + \gamma_i^*) - \omega_i \frac{\partial B_i(q^*)}{\partial q_{ijk,l}} + \epsilon_{k,l}^* - \lambda_{ijk,l}^* - \alpha_j^* + \beta_j^* \right]$$

$$\times \left[q_{ijk,l} - q_{ijk,l}^* \right] + \sum_{k=1}^{o} \sum_{l=1}^{t} \left[u_{k,l} - \sum_{i=1}^{m} \sum_{j=1}^{n} q_{ijk,l}^* \right] \times \left[\epsilon_{k,l}^* - \epsilon_{k,l}^* \right]$$

$$+ \sum_{i=1}^{m} \left[b_i - \sum_{k=1}^{o} \rho_k \sum_{j=1}^{n} \sum_{l=1}^{t} q_{ijk,l}^* - \sum_{j=1}^{n} \sum_{k=1}^{o} \sum_{l=1}^{t} c_{ijk,l}(q^*) \right] \times \left[\gamma_i - \gamma_i^* \right]$$

$$+ \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{o} \sum_{l=1}^{t} q_{ijk,l}^* \times (\lambda_{ijk,l} - \lambda_{ijk,l}^*)$$

$$+ \sum_{j=1}^{n} \left[\sum_{i=1}^{m} \sum_{k=1}^{o} \sum_{l=1}^{t} q_{ijk,l}^* - \underline{d}_j \right] \times \left[\alpha_j - \alpha_j^* \right] + \sum_{j=1}^{n} \left[\sum_{i=1}^{m} \sum_{k=1}^{o} \sum_{l=1}^{t} \bar{d}_j - q_{ijk,l}^* \right] \times \left[\beta_j - \beta_j^* \right] \geq 0,$$

$$\forall (q, \epsilon, \gamma, \lambda, \alpha, \beta) \in R_+^{2mnot+ot+m+2n}, \tag{25a}$$

or, more simply: determine $(q^*, \epsilon^*, \gamma^*, \alpha^*, \beta^*) \in R_+^{mnot+m+2n}$ *such that*

$$\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{o} \sum_{l=1}^{t} \left[(\sum_{r=1}^{n} \sum_{p=1}^{o} \sum_{s=1}^{t} \frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}} + \rho_k)(1 + \gamma_i^*) - \omega_i \frac{\partial B_i(q^*)}{\partial q_{ijk,l}} + \epsilon_{k,l}^* - \alpha_j^* + \beta_j^* \right] \times \left[q_{ijk,l} - q_{ijk,l}^* \right]$$

$$+\sum_{k=1}^{o}\sum_{l=1}^{t}\left[u_{k,l}-\sum_{i=1}^{m}\sum_{j=1}^{n}q^*_{ijk,l}\right]\times\left[\epsilon^*_{k,l}-\epsilon^*_{k,l}\right]$$

$$+\sum_{i=1}^{m}\left[b_i-\sum_{k=1}^{o}\rho_k\sum_{j=1}^{n}\sum_{l=1}^{t}q^*_{ijk,l}-\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}c_{ijk,l}(q^*)\right]\times\left[\gamma_i-\gamma^*_i\right]$$

$$+\sum_{j=1}^{n}\left[\sum_{i=1}^{m}\sum_{k=1}^{o}\sum_{l=1}^{t}q^*_{ijk,l}-\underline{d}_j\right]\times\left[\alpha_j-\alpha^*_j\right]+\sum_{j=1}^{n}\left[\sum_{i=1}^{m}\sum_{k=1}^{o}\sum_{l=1}^{t}\bar{d}_j-q^*_{ijk,l}\right]\times\left[\beta_j-\beta^*_j\right]\geq 0,$$

$$\forall(q,\epsilon,\gamma,\alpha,\beta)\in R^{mnot+ot+m+2n}_{+}. \qquad (25b)$$

Proof: See the Appendix.

We now provide interpretations of the Lagrange multipliers. We focus on the case where $q^*_{ijk,l}>0$; namely, the volume of relief items purchased by HO i at location k and shipped to demand location j by FSP l is positive; otherwise, if $q^*_{ijk,l}=0$, the problem is not interesting. Then, from the third line in (23), we have that $\lambda^*_{ijk,l}=0$.

Let us consider the situation when the constraints are not active, that is, $\sum_{i=1}^{m}\sum_{j=1}^{n}q^*_{ijk,l}<u_{k,l}$, $\forall k$, $\forall l$, $\sum_{k=1}^{o}\rho_k\sum_{j=1}^{n}\sum_{l=1}^{t}q^*_{ijk,l}+\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}c_{ijk,l}(q^*)<b_i$, $\forall i$, and $\underline{d}_j<\sum_{i=1}^{m}\sum_{k=1}^{o}\sum_{l=1}^{t}q^*_{ijk,l}<\bar{d}_j$, $\forall j$.

In this case, all the associated Lagrange multipliers are equal to zero, specifically:

$$\epsilon^*_{k,l}=0,\ \forall k,\ \forall l;\ \ \gamma^*_i=0,\ \forall i;\ \ \alpha^*_j=0,\ \forall j;\ \ \beta^*_j=0,\ \forall j;$$

hence, (24) yields:

$$\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t}\frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}}+\rho_k=\omega_i\frac{\partial B_i(q^*)}{\partial q_{ijk,l}}. \qquad (26)$$

This means that the weighted marginal altruism is equal to the sum of the marginal logistical cost and the relief item purchase price (for the respective i,j,k,l).

If, now, constraint (8) is active for k,l; namely, $\sum_{i=1}^{m}\sum_{j=1}^{n}q^*_{ijk,l}=u_{k,l}$, then, from the first line of (23), the associated Lagrange multiplier $\epsilon^*_{k,l}$ is greater than zero and (24) becomes:

$$\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t}\frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}}+\rho_k+\epsilon_{k,l}=\omega_i\frac{\partial B_i(q^*)}{\partial q_{ijk,l}}, \qquad (27)$$

which means that the weighted marginal altruism exceeds the sum of the marginal logistical cost and the relief item purchase price (for the respective i, j, k, l) and this is a desirable situation since we are dealing with a humanitarian organization.

If constraint (6) is active for i; namely, $\sum_{k=1}^{o} \rho_k \sum_{j=1}^{n} \sum_{l=1}^{t} q^*_{ijk,l} + \sum_{j=1}^{n} \sum_{k=1}^{o} \sum_{l=1}^{t} c_{ijk,l}(q^*) = b_i$, then from the second line of (23), the associated Lagrange multiplier γ_i^* is greater than zero and (24) becomes:

$$(1+\gamma_i^*)\left[\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t}\frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}} + \rho_k\right] = \omega_i \frac{\partial B_i(q^*)}{\partial q_{ijk,l}}, \tag{28}$$

which implies also a desirable situation.

If constraint (9) is active for j; namely, $\sum_{i=1}^{m} \sum_{k=1}^{n} \sum_{l=1}^{t} q^*_{ijk,l} = \underline{d}_j$, then, from the fourth line of (23), the associated Lagrange multiplier α_j^* is greater than zero and (24) becomes:

$$\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t}\frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}} + \rho_k - \alpha_j = \omega_i \frac{\partial B_i(q^*)}{\partial q_{ijk,l}}, \tag{29}$$

which is an undesirable situation. Indeed, one can see that, in this case, the volume of disaster relief items to the demand point is only at its lower bound and, hence, the marginal logistical cost and the relief item purchase cost exceed the weighted marginal altruism.

If, on the other hand, constraint (9) is active for j; namely, $\sum_{i=1}^{m} \sum_{k=1}^{n} \sum_{l=1}^{t} q^*_{ijk,l} = \overline{d}_j$, then, from the fifth line of (23), the associated Lagrange multiplier β_j^* is greater than zero and (24) becomes:

$$\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t}\frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}} + \rho_k + \beta_j = \omega_i \frac{\partial B_i(q^*)}{\partial q_{ijk,l}}, \tag{30}$$

which means that the weighted marginal altruism exceeds the sum of the marginal logistical cost and the relief item purchase price (for the respective i, j, k, l) and this is again a desirable situation. Observe that, in this case, the victims of the disaster at that demand point receive a volume of relief items at the upper bound.

From the above analysis of the Lagrange multipliers and marginal utilities at the equilibrium solution, we can conclude that the most desirable case, in terms of weighted altruism, is the one when $\sum_{i=1}^{m}\sum_{j=1}^{n} q^*_{ijk,l} = u_{k,l}$, $\sum_{i=1}^{m} \sum_{k=1}^{n} \sum_{l=1}^{t} q^*_{ijk,l} = \overline{d}_j$, and $(1+\gamma_i^*)\left[\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t}\frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}} + \rho_k\right] = \omega_i \frac{\partial B_i(q^*)}{\partial q_{ijk,l}}$. Indeed, in this case, the full resources of the freight service provider for shipping to the demand point are utilized and the victims of the disasters at that demand point acquire the volume of relief items at the maximum amount demanded, that is, at the imposed upper bound.

5. Algorithm and Numerical Examples

In this section, before discussing the numerical examples, we describe the algorithm that we use for the computations. The algorithm is the Euler method, which is induced by the general iterative scheme of Dupuis and Nagurney (1993). As Dupuis and Nagurney (1993) establish, for convergence of the general iterative scheme, the sequence $\{a_\tau\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \to 0$, as $\tau \to \infty$. Convergence conditions for different types of network-based problems are presented in Nagurney and Zhang (1996) and Nagurney (2006).

Specifically, we utilize variational inequality (25b) for the computations.

The Euler method yields a specific closed form expression for each relief item flow and Lagrange multiplier at iteration $\tau + 1$ as follows.

5.1 Explicit Formulae for the Euler Method Applied to the Game Theory Model

Specifically, at an iteration $\tau + 1$, we have the following closed form expression for the relief item flow that each HO $i = 1, \dots, m$, purchases at location $k = 1, \dots, o$, and has then transported to the demand point $j = 1, \dots, n$, by FSP $l = 1, \dots, t$:

$$q^{\tau+1}_{ijk,l} = \max\{0, q^{\tau}_{ijk,l} + a_\tau(\omega_i \frac{\partial B_i(q^\tau)}{\partial q_{ijk,l}} - (\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t} \frac{\partial c_{irp,s}(q^\tau)}{\partial q_{ijk,l}} + \rho_k)(1+\gamma^\tau_i) + \alpha^\tau_j - \beta^\tau_j - \epsilon^\tau_{k,l})\}. \tag{31}$$

The explicit formula for the Lagrange multipliers associated with the budget constraint (6), respectively, for $i = 1, \dots, m$, at iteration $\tau + 1$, is:

$$\gamma^{\tau+1}_i = \max\{0, \gamma^\tau_i + a_\tau(-b_i + \sum_{k=1}^{o}\rho_k \sum_{j=1}^{n}\sum_{l=1}^{t} q^\tau_{ijk,l} + \sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t} c_{ijk,l}(q^\tau))\}. \tag{32}$$

The closed form expression for the Lagrange multiplier for each capacity constraint (8) for $k = 1, \dots, o$; $l = 1, \dots, t$, in turn, at iteration $\tau + 1$, is:

$$\epsilon^{\tau+1}_{k,l} = \max\{0, \epsilon^\tau_{k,l} + a_\tau(-u_{k,l} + \sum_{i=1}^{m}\sum_{j=1}^{n} q^\tau_{ijk,l})\}. \tag{33}$$

The Lagrange multiplier for the demand lower bound constraint (9) at demand points $j =$

$1, \ldots, n$, at iteration $\tau + 1$, is computed according to:

$$\alpha_j^{\tau+1} = \max\{0, \alpha_j^{\tau} + a_{\tau}(-\sum_{i=1}^{m}\sum_{k=1}^{o}\sum_{l=1}^{t} q_{ijk,l}^{\tau} + \underline{d}_j)\}. \quad (34)$$

The Lagrange multiplier for the demand upper bound constraint (10) at demand points $j = 1, \ldots, n$, at iteration $\tau + 1$, on the other hand, is computed as follows:

$$\beta_j^{\tau+1} = \max\{0, \beta_j^{\tau} + a_{\tau}(-\bar{d}_j + \sum_{i=1}^{m}\sum_{k=1}^{o}\sum_{l=1}^{t} q_{ijk,l}^{\tau})\}. \quad (35)$$

5.2 Numerical Examples

The examples in this section are inspired by Hurricane Harvey. Hurricane Harvey was a Category 4 storm that hit Texas on August 25, 2017 with epic flooding. Hurricane Harvey caused \$125 billion in damage, according to the National Hurricane Center, and affected almost 13 million people. That is more than any other natural disaster in U.S history except for Hurricane Katrina (Amadeo (2018)).

The Episcopal Health Foundation analyzed the FEMA assistance applications. There were over 880,000 applications across 41 Texas counties (Keyser (2017)). In the analysis, it was clear that some regions of Texas had needed more help, such as Port Arthur, where over 13,654 applications were submitted, whereas other locations had fewer recorded cases, such as Bay City, with 6500, and Silsbee, with 3,232 registered applications.

One of the active organizations in disaster relief in Houston was the American Red Cross (ARC) (cf. FEMA (2017)). The Salvation Army was another humanitarian organization that had a role in disaster relief post Hurricane Harvey (The Salvation Army (2017)). Hence, we utilize these two humanitarian organizations in our numerical examples.

Example 1

In Example 1, we consider the basic disaster relief supply chain network depicted in Figure 2. It consists of two humanitarian organizations, The Salvation Army and the American Red Cross, respectively. The Salvation Army is a smaller relief organization as compared to the American Red Cross, which has a larger budget. There are three demand points: Port Arthur, Bay City, and Silsbee, respectively. The major devastation occurred in the Port Arthur region and, therefore, the need for relief items at this demand point was much greater than at the two other areas.

The HOs are involved in the purchasing and delivery of relief item kits. Here we use "items" and "kits" interchangeably. The HOs have two location options at which to purchase the relief items: Purchasing Location 1 (PL 1) and Purchasing Location 2 (PL 2). PL 1, unlike PL 2, offers the relief items at reasonable prices because the market is far from the affected area and the disaster has not had an effect on its prices. However, PL 2, which is a local market, offers a similar product at a higher price due to the increased demand after the disaster. The relief item kits are sold at the two purchasing locations at the following prices:

$$\rho_1 = 50, \quad \rho_2 = 70.$$

The above prices are reasonable, based on data from FEMA (2012) and the American Red Cross (2018).

Hence, we assume that there are two Freight Service Providers, FSP 1 and FSP 2, involved in the shipment of the relief items from the purchase locations to the demand points. In contrast to FSP 2, FSP 1 has less equipment and capability, so it provides service at a lower capacity as compared to FSP 1.

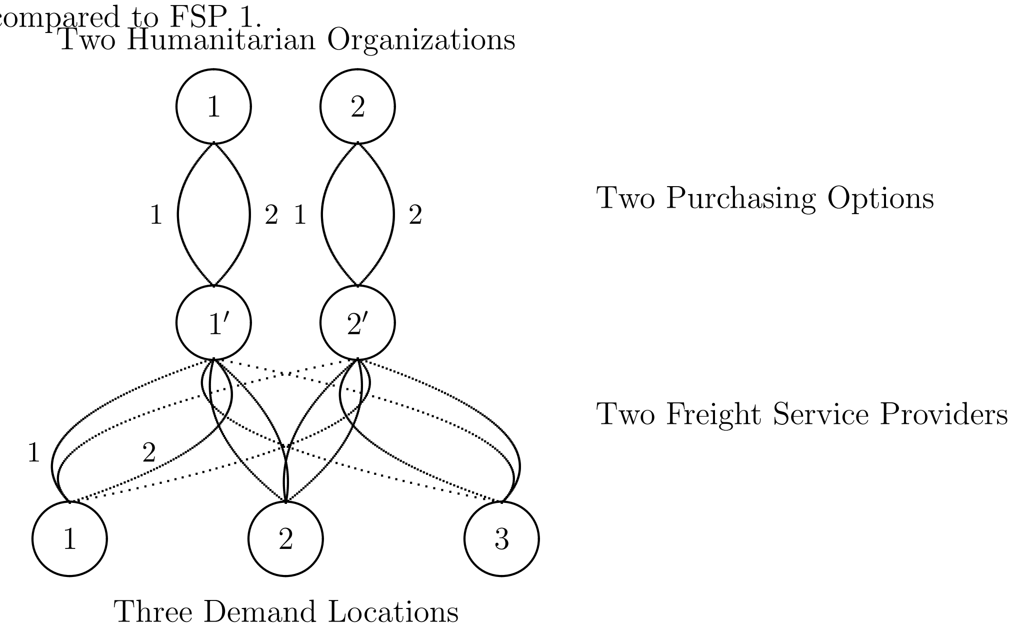

Figure 2: Example 1: Two Humanitarian Organizations, Three Demand Locations, Two Purchasing Options, Two Freight Service Providers

The data for Example 1 are as follows.

The humanitarian organizations' budgets, in dollars, are:

$$b_1 = 3 \times 10^6, \quad b_2 = 6 \times 10^6.$$

According to FEMA (2017), the American Red Cross provided \$45 million in disaster relief immediately after Hurricane Harvey hit Houston. Given that the three demand points in the numerical example are representative, although a fraction of the sites impacted, and The Salvation Army's relative size to the ARC, the budgets that we utilize for the two humanitarian organizations are reasonable.

The HOs' altruism functions are:

$$B_1(q)=\sum_{k=1}^{2}\sum_{l=1}^{2}(300q_{11k,l}+200q_{12k,l}+100q_{13k,l}),\quad B_2(q)=\sum_{k=1}^{2}\sum_{l=1}^{2}(400q_{21k,l}+300q_{22k,l}+200q_{23k,l}),$$

and the monetization weights associated with these benefit functions are:

$$\omega_1=1,\quad \omega_2=1.$$

Similar altruism functions were used in Nagurney et al. (2018) and are quite reasonable. Indeed, Demand Point 1 has had more devastation and, as a result, attracts the most media attention, so the coefficient of $q_{i1k,l}$ is the highest one. Demand Points 2 and 3 have the next highest such coefficients, accordingly. And, generally, HO 2 has higher coefficients than HO 1 because it has a more recognized brand.

The lower and upper bounds for the relief items at the demand points are:

$$\underline{d}_1=10000,\quad \bar{d}_1=20000,$$

$$\underline{d}_2=1000,\quad \bar{d}_2=10000,$$

$$\underline{d}_3=1000,\quad \bar{d}_3=10000.$$

These lower and upper bound demands are in concert with the number of FEMA assistance applications for these locations, as noted at the beginning of Section 5.2.

The FSPs' capacities are as follows:

$$u_{1,1}=3000,\quad u_{1,2}=6000,$$

$$u_{2,1}=5000,\quad u_{2,2}=8000.$$

Each FSP, according to its facilities and location of origin of purchase and destination of delivery, encumbers different logistical costs, with these cost functions being:

$$c_{i11,1}(q)=0.2q_{i11,1}^2+2q_{i11,1}+q_{j11,1},\quad c_{i21,1}(q)=0.2q_{i21,1}^2+5q_{i21,1}+2.5q_{j21,1},$$

$$c_{i31,1}(q) = 0.2q_{i31,1}^2 + 7q_{i31,1} + 3.5q_{j31,1},$$

$$c_{i12,1}(q) = 0.15q_{i12,1}^2 + 2q_{i12,1} + q_{j12,1}, \quad c_{i22,1}(q) = 0.15q_{i22,1}^2 + 5q_{i22,1} + 2.5q_{j22,1},$$

$$c_{i32,1}(q) = 0.15q_{i32,1}^2 + 7q_{i32,1} + 3.5q_{j32,1},$$

$$c_{i11,2}(q) = 0.15q_{i11,2}^2 + 2q_{i11,2} + q_{j11,2}, \quad c_{i21,2}(q) = 0.15q_{i21,2}^2 + 5q_{i21,2} + 2.5q_{j21,2},$$

$$c_{i31,2}(q) = 0.15q_{i31,2}^2 + 7q_{i31,2} + 3.5q_{j31,2},$$

$$c_{i12,2}(q) = 0.1q_{i12,2}^2 + 2q_{i12,2} + q_{j12,2}, \quad c_{i22,2}(q) = 0.1q_{i22,2}^2 + 5q_{i22,2} + 2.5q_{j22,2},$$

$$c_{i32,2}(q) = 0.1q_{i32,2}^2 + 7q_{i32,2} + 3.5q_{j32,2},$$

$$\forall i = 1, 2; j \neq i.$$

Observe that FSP 2 has lower costs than FSP 1 and higher shipment capacities.

The Euler method was implemented in FORTRAN and a Linux system at the University of Massachusetts Amherst used for the computations. We initialized the algorithm so that the flows on the links leading to each demand point are equal, and their summation is equal to the lower bound on the demand at that point. All the Lagrange multipliers are set to 0.00. The algorithm is considered to have converged when the absolute value of all the computed variables at two successive iterates are less than or equal to 10^{-5}. The sequence $\{a_\tau\} = .1(1, 1/2, 1/2, 1/3, 1/3, 1/3, \ldots)$.

The results for Example 1 are given below.

The computed equilibrium relief item flows for $i = 1, 2; j = 1, 3$, are:

$$q_{ij1,1}^* = \begin{bmatrix} 874.22 & 362.42 & 107.52 \\ 134.20 & 0.00 & 357.42 \end{bmatrix}, \quad q_{ij2,1}^* = \begin{bmatrix} 1098.80 & 416.54 & 76.70 \\ 1432.14 & 749.74 & 409.87 \end{bmatrix},$$

$$q_{ij1,2}^* = \begin{bmatrix} 1165.47 & 483.20 & 143.28 \\ 1498.80 & 816.10 & 476.53 \end{bmatrix}, \quad q_{ij2,2}^* = \begin{bmatrix} 1648.22 & 624.48 & 115.05 \\ 2148.17 & 1123.9 & 614.48 \end{bmatrix}.$$

The amount of the relief item kits delivered to each demand point is:

$$\sum_{i=1}^{2}\sum_{k=1}^{2}\sum_{l=1}^{2} q_{i1k,l}^* = 10000.00, \quad \sum_{i=1}^{2}\sum_{k=1}^{2}\sum_{l=1}^{2} q_{i2k,l}^* = 4576.76, \quad \sum_{i=1}^{2}\sum_{k=1}^{2}\sum_{l=1}^{2} q_{i3k,l}^* = 2300.86.$$

Since the demand is at the lower bound at the first demand point, α_1^* is positive, while the other Lagrange multipliers associated with the lower bound demand constraints are equal to zero:

$$\alpha_1^* = 101.72, \quad \alpha_2^* = \alpha_3^* = 0.$$

Since all the demand points receive volumes of relief items less than their corresponding upper bound, all the Lagrange multipliers associated with the upper demand bound constraints are equal to zero, that is,

$$\beta_1^* = \beta_2^* = \beta_3^* = 0.$$

The volumes of relief items carried by each FSP $l; l = 1, 2$, from each purchasing location $k; k = 1, 2$, are:

$$\sum_{i=1}^{2} \sum_{j=1}^{3} q_{ijk,l}^* = \begin{bmatrix} 1835.79 & 4583.70 \\ 4183.79 & 6274.38 \end{bmatrix}.$$

None of the FSPs have consumed their full capacities and, therefore:

$$\epsilon_{1,1}^* = \epsilon_{1,2}^* = \epsilon_{2,1}^* = \epsilon_{2,2}^* = 0.$$

The logistical costs for the humanitarian organization are, respectively,

$$\sum_{j=1}^{3} \sum_{k=1}^{2} \sum_{l=1}^{2} c_{1jk,l}(q^*) = 976,436.92$$

and

$$\sum_{j=1}^{3} \sum_{k=1}^{2} \sum_{l=1}^{2} c_{2jk,l}(q^*) = 1,598,108.48.$$

The total expenditure of each humanitarian organization in this disaster relief operation, which includes the cost of purchasing and shipping the relief items, is, respectively:

$$\sum_{k=1}^{2} \rho_k \sum_{j=1}^{3} \sum_{l=1}^{2} q_{1jk,l}^* + \sum_{j=1}^{3} \sum_{k=1}^{2} \sum_{l=1}^{2} c_{1jk,l}(q^*) = 1,419,224.00,$$

$$\sum_{k=1}^{2} \rho_k \sum_{j=1}^{3} \sum_{l=1}^{2} q_{2jk,l}^* + \sum_{j=1}^{3} \sum_{k=1}^{2} \sum_{l=1}^{2} c_{2jk,l}(q^*) = 2,208,465.25.$$

Observe that HO 1's logistical costs are 69% of its total costs, whereas the logistical costs of HO 2 are 72% of its total costs. This is reasonable since, as noted in the Introduction, logistics is estimated to account for 80% of the total cost associated with disaster relief.

Since both organizations spend less than their budgets, the Lagrange multipliers associated with their budget constraints are equal to zero:

$$\gamma_1^* = \gamma_2^* = 0.$$

The benefit/altruism that each humanitarian organization gains from helping the disaster victims is:

$$B_1(q^*) = 1,857,600.50, \quad B_2(q^*) = 3,264,018.25.$$

Putting all the terms in the respective objective functions together, the utility of each HO, after the disaster relief operation, is:

$$U_1(q^*) = 438,376.50, \quad U_2(q^*) = 1,055,553.00.$$

Observe that the American Red Cross, which is the larger organization, tends to be more active than The Salvation Army. The American Red Cross delivers 9,761.73 relief item kits, which is more than the 7,115.92 that The Salvation Army delivers. The high volume of relief item kit shipments encumbers a high cost to the ARC, but, at the same time, it brings more benefits to it and, ultimately, the American Red Cross has a higher utility than The Salvation Army.

FSP 2 achieves a large share of the transportation market by benefiting from its lower costs and larger shipment capacities. FSP 2 carries a volume of 10,858.08 disaster relief item kits, while FSP 1 carries just 6,019.58.

In the relief item kit sales market, 10.458.16 relief item kits are purchased from PL 2 and, despite having a higher price, PL2 is preferred by the HOs due to the lower shipping costs. PL 1 also, because of its lower item price, still enjoys a good share of market with 6,419.48 relief item kits delivered to the demand points from PL 1.

Example 2

In this example, the HOs have a new location option for purchasing the relief items. The new purchasing location, denoted by PL 3, is a local one. It charges a lower price than the existing local purchasing location, PL 2, in order to compete and gain a good market share but its price is still higher than PL 1's price. The supply chain network topology for Example 2 is depicted in Figure 3.

The data on the new purchasing location, added to Example 1, are presented below.

The relief items are sold at PL 3 at the price:

$$\rho_3 = 60.$$

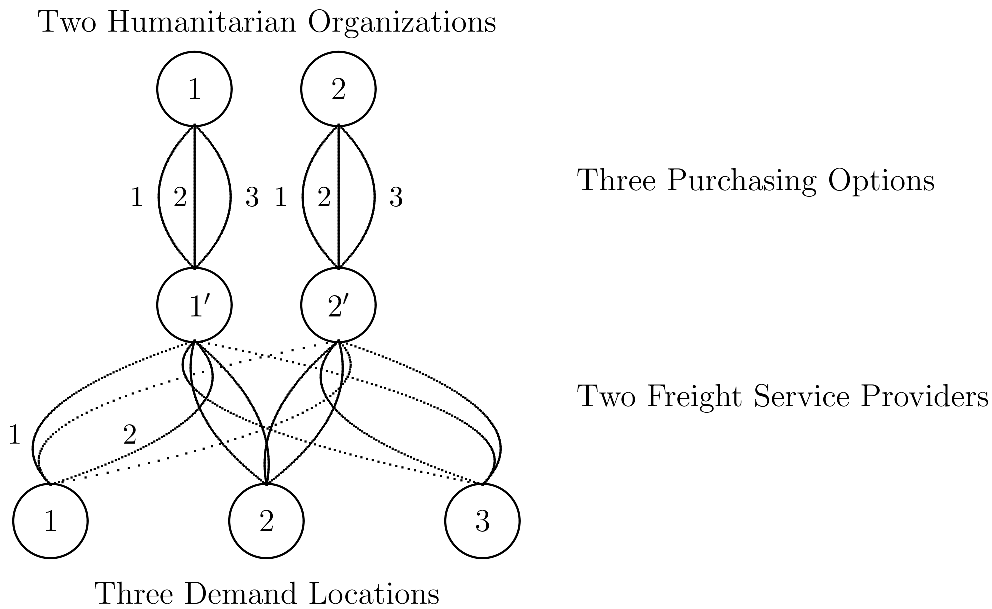

Figure 3: Example 2: Two Humanitarian Organizations, Three Demand Locations, Three Purchasing Options, Two Freight Service Providers

The FSPs' capacities for shipping products from the purchasing locations to the affected area have been updated as below:

$$u_{1,1} = 3000, \quad u_{1,2} = 6000,$$

$$u_{2,1} = 4000, \quad u_{2,2} = 7000.$$

$$u_{3,1} = 4000, \quad u_{3,2} = 7000.$$

The HOs' altruism functions are:

$$B_1(q) = \sum_{k=1}^{3}\sum_{l=1}^{2}(300q_{11k,l}+200q_{12k,l}+100q_{13k,l}), \quad B_2(q) = \sum_{k=1}^{3}\sum_{l=1}^{2}(400q_{21k,l}+300q_{22k,l}+200q_{23k,l}).$$

The logistical cost functions from the new purchasing location PL 3 to the affected area are as follows:

$$c_{i13,1}(q) = 0.15q_{i13,1}^2 + 2q_{i13,1} + q_{j13,1}, \quad c_{i23,1}(q) = 0.15q_{i23,1}^2 + 5q_{i23,1} + 2.5q_{j23,1},$$

$$c_{i33,1}(q) = 0.15q_{i33,1}^2 + 7q_{i33,1} + 3.5q_{j33,1},$$

$$c_{i13,2}(q) = 0.1q_{i13,2}^2 + 2q_{i13,2} + q_{j13,2}, \quad c_{i23,2}(q) = 0.1q_{i23,2}^2 + 5q_{i23,2} + 2.5q_{j23,2},$$

$$c_{i33,2}(q) = 0.1q_{i33,2}^2 + 7q_{i33,2} + 3.5q_{j33,2}.$$

$$\forall i = 1, 2; j \neq i.$$

The computed equilibrium relief item flows for $i = 1, 2; j = 1, 3$ are now:

$$q^*_{ij1,1} = \begin{bmatrix} 620.14 & 362.42 & 107.48 \\ 0.00 & 0.00 & 357.42 \end{bmatrix}, \quad q^*_{ij2,1} = \begin{bmatrix} 760.20 & 416.54 & 76.69 \\ 1092.91 & 749.74 & 409.87 \end{bmatrix},$$

$$q^*_{ij1,2} = \begin{bmatrix} 826.80 & 483.20 & 143.28 \\ 1159.56 & 816.40 & 476.53 \end{bmatrix}, \quad q_{ij2,2} = \begin{bmatrix} 1139.12 & 624.48 & 114.94 \\ 1639.00 & 1123.96 & 614.48 \end{bmatrix},$$

$$q^*_{ij3,1} = \begin{bmatrix} 793.51 & 449.87 & 109.97 \\ 1126.623 & 783.07 & 443.20 \end{bmatrix},$$

$$q^*_{ij3,2} = \begin{bmatrix} 1189.10 & 674.47 & 164.88 \\ 1688.99 & 1173.96 & 664.47 \end{bmatrix}.$$

The volume of relief item kits received at each demand point is:

$$\sum_{i=1}^{2}\sum_{k=1}^{3}\sum_{l=1}^{2} q^*_{i1k,l} = 12,035.55, \quad \sum_{i=1}^{2}\sum_{k=1}^{3}\sum_{l=1}^{2} q^*_{i2k,l} = 7658.10, \quad \sum_{i=1}^{2}\sum_{k=1}^{3}\sum_{l=1}^{2} q^*_{i3k,l} = 3,683.22.$$

Observe that all the demand points receive a volume of relief item kits greater than their respective demand lower bound and, therefore, all the Lagrange multipliers for the lower bound demand constraints are equal to zero:

$$\alpha^*_1 = \alpha^*_2 = \alpha^*_3 = 0.$$

Also, the demand points receive an amount of relief items less than their demand upper bound. Hence, all the Lagrange multipliers associated with the upper bound demand constraints are also equal to zero:

$$\beta^*_1 = \beta^*_2 = \beta^*_3 = 0.$$

The relief item kit volumes carried from each purchasing location $k; k = 1, 2, 3$, by each FSP $l; l = 1, 2$ are:

$$\sum_{i=1}^{2}\sum_{j=1}^{3} q^*_{ijk,l} = \begin{bmatrix} 1447.47 & 3905.76 \\ 3505.94 & 5255.98 \\ 3705.85 & 5555.87 \end{bmatrix}.$$

Not one of the FSPs has reached its capacity and, hence, we have that:

$$\epsilon^*_{1,1} = \epsilon^*_{1,2} = \epsilon^*_{2,1} = \epsilon^*_{2,2} = \epsilon^*_{3,1} = \epsilon^*_{3,2} = 0.$$

The total cost of each organization in this operation, which includes the cost of purchasing and shipping the relief items, is:

$$\sum_{k=1}^{3} \rho_k \sum_{j=1}^{3}\sum_{l=1}^{2} q^*_{1jk,l} + \sum_{j=1}^{3}\sum_{k=1}^{3}\sum_{l=1}^{2} c_{1jk,l}(q^*) = 1,454,783.75,$$

$$\sum_{k=1}^{3} \rho_k \sum_{j=1}^{3} \sum_{l=1}^{2} q^*_{2jk,l} + \sum_{j=1}^{3} \sum_{k=1}^{3} \sum_{l=1}^{2} c_{2jk,l}(q^*) = 2,821,781.50.$$

Since both organizations spend less than their budgets, the Lagrange multipliers associated with the budget constraints are equal to zero:

$$\gamma_1^* = \gamma_2^* = 0.$$

The benefit/altruism of each organization is:

$$B_1(q^*) = 2,272,585.50, \quad B_2(q^*) = 4,670,005.50.$$

The utility of each HO, after the disaster relief operation, is:

$$U_1(q^*) = 817,796.75, \quad U_2(q^*) = 1,848,224.00.$$

In Example 2, with the addition of a new purchasing location, both organizations take advantage of this opportunity and purchase more relief item kits for delivery to the affected areas. The American Red Cross, the larger organization, manages to increase the amount of items shipped to 14,319.79 - an increase of almost 5,000, as compared to that in Example 1. The Salvation Army provides 9,057.09 relief item kits, while its contribution was only slightly above 7,000 kits in Example 1. Both humanitarian organizations pay more, after the addition of a new PL, but this higher cost has led to a significant increase in their utilities.

In the relief item kit sales market, as expected, the new PL is able to take a great market share due to its lower price than the other local PL and with lower associated logistical costs than the nonlocal PL. Both of the previous purchasing locations drop sales with the arrival of the new PL. PL 1 and PL 2 sell 5,353.23 and 8,761.92 relief item kits, respectively, while PL 3 is very successful at selling 9,261.72 items.

The increase in the purchasing power of the HOs has also boosted the logistical / transportation market. Both FSPs ship higher volumes of relief items as compared to Example 1. FSP 1 and FSP 2 ship 14,717.61 and 8,659.26 relief items, respectively, with the major increase being in the shipments of the relief items from the newly added PL to the affected region. All of the purchasing locations experience a drop in sales for the disaster with the total volume purchased at PL 1 being: 4,157.66; that at PL 2: 6,996.47, and at PL 3: 7,413.07.

Example 3

Example 3 has the same data as that in Example 2 except that we now consider additional disruptions in transportation so that all the logistical costs are as in Example 2 except that the nonlinear component is multiplied by a factor of 10.

The new computed equilibrium relief item flows $i = 1, 2; j = 1, 3$ are:

$$q^*_{ij1,1} = \begin{bmatrix} 575.14 & 249.96 & 0.00 \\ 0.00 & 0.00 & 266.55 \end{bmatrix}, \quad q^*_{ij2,1} = \begin{bmatrix} 700.21 & 266.54 & 0.00 \\ 1032.92 & 599.75 & 199.94 \end{bmatrix},$$

$$q^*_{ij1,2} = \begin{bmatrix} 766.86 & 333.21 & 0.00 \\ 1099.57 & 666.41 & 266.55 \end{bmatrix}, \quad q_{ij2,2} = \begin{bmatrix} 1049.17 & 399.73 & 0.00 \\ 1549.01 & 899.45 & 299.75 \end{bmatrix},$$

$$q^*_{ij3,1} = \begin{bmatrix} 0.00 & 299.88 & 0.00 \\ 1066.25 & 633.08 & 233.27 \end{bmatrix},$$

$$q^*_{ij3,2} = \begin{bmatrix} 0.00 & 499.73 & 0.00 \\ 1599.00 & 949.45 & 349.74 \end{bmatrix}.$$

Due to the increase in the logistical/transportation costs, an increasing number of the computed equilibrium flow variables are now equal to 0. In Example 2, there were two flows at level 0.00 and now there are ten such flows, including the same two as in Example 2. Some purchasing/delivery combinations are no longer attractive under the higher logistical costs.

The volume of relief item kits received at each demand point is now:

$$\sum_{i=1}^{2}\sum_{k=1}^{3}\sum_{l=1}^{2} q^*_{i1k,l} = 11,270.81, \quad \sum_{i=1}^{2}\sum_{k=1}^{3}\sum_{l=1}^{2} q^*_{i2k,l} = 5,747.18, \quad \sum_{i=1}^{2}\sum_{k=1}^{3}\sum_{l=1}^{2} q^*_{i3k,l} = 1,549.20.$$

All the demand points receive a volume of relief item greater than their demand lower bound and, therefore, all the Lagrange multipliers for the lower bound demand constraints are equal to zero:

$$\alpha^*_1 = \alpha^*_2 = \alpha^*_3 = 0.$$

Also, the demand points receive an amount of relief items less than their demand upper bound. As a result, all the Lagrange multipliers associated with the upper bound demand constraints are also equal to zero:

$$\beta^*_1 = \beta^*_2 = \beta^*_3 = 0.$$

The relief item kit volumes delivered from each purchasing location $k; k = 1, 2, 3$ by each FSP $l; l = 1, 2$ are now:

$$\sum_{i=1}^{2}\sum_{j=1}^{3} q^*_{ijk,l} = \begin{bmatrix} 1025.06 & 3132.59 \\ 2799.36 & 4197.11 \\ 2966.01 & 4447.06 \end{bmatrix}.$$

FSP 2 remains the dominant freight service provider in this disaster operation.

The FSPs have not exhausted their capacities and, therefore:

$$\epsilon^*_{1,1} = \epsilon^*_{1,2} = \epsilon^*_{2,1} = \epsilon^*_{2,2} = \epsilon^*_{3,1} = \epsilon^*_{3,2} = 0.$$

The total cost of each humanitarian organization is now:

$$\sum_{k=1}^{3} \rho_k \sum_{j=1}^{3}\sum_{l=1}^{2} q^*_{1jk,l} + \sum_{j=1}^{3}\sum_{k=1}^{3}\sum_{l=1}^{2} c_{1jk,l}(q^*) = 1,459,130.25,$$

$$\sum_{k=1}^{3} \rho_k \sum_{j=1}^{3}\sum_{l=1}^{2} q^*_{2jk,l} + \sum_{j=1}^{3}\sum_{k=1}^{3}\sum_{l=1}^{2} c_{2jk,l}(q^*) = 2,657,446.75.$$

Again, both organizations spend less than their full budgets, and, therefore, the Lagrange multipliers associated with the budget constraints are equal to zero:

$$\gamma^*_1 = \gamma^*_2 = 0.$$

HO 1 purchases, and has delivered, 6,923.11 relief item kits, whereas HO 2 purchases 11,644.08 relief item kits and has that volume shipped to the points of demand.

The benefit/altruism of each organization is:

$$B_1(q^*) = 1,877,028.75, \quad B_2(q^*) = 3,972,979.50.$$

The utility of each HO, after the disaster relief operation, is:

$$U_1(q^*) = 417,898.50, \quad U_2(q^*) = 1,315,532.75.$$

Both HOs have a reduced incurred altruism, as compared to those in Example 2, and also lower utilities.

Example 4

Example 4 is constructed from Example 3 and considers even a greater disruption in the transportation network, with even larger associated costs. The data remain as in Example 3, but now each nonlinear term in each logistical cost function is multiplied by a factor of 3.

The new computed equilibrium relief item flows for $i = 1, 2; j = 1, 3$ are:

$$q^*_{ij1,1} = \begin{bmatrix} 541.48 & 83.33 & 23.15 \\ 294.84 & 0.00 & 106.48 \end{bmatrix}, \quad q^*_{ij2,1} = \begin{bmatrix} 699.75 & 88.89 & 8.64 \\ 810.86 & 199.99 & 119.75 \end{bmatrix},$$

$$q^*_{ij1,2} = \begin{bmatrix} 721.97 & 111.11 & 30.86 \\ 833.08 & 222.21 & 141.98 \end{bmatrix}, \quad q_{ij2,2} = \begin{bmatrix} 1049.63 & 133.32 & 12.96 \\ 1216.30 & 299.96 & 179.63 \end{bmatrix},$$

$$q^*_{ij3,1} = \begin{bmatrix} 710.86 & 100.00 & 19.75 \\ 821.97 & 211.10 & 130.87 \end{bmatrix},$$

$$q^*_{ij3,2} = \begin{bmatrix} 1066.30 & 149.98 & 29.63 \\ 1232.97 & 316.63 & 196.30 \end{bmatrix}.$$

The volume of relief item kits received at each demand point is now:

$$\sum_{i=1}^{2}\sum_{k=1}^{3}\sum_{l=1}^{2} q^*_{i1k,l} = 10,000.00, \quad \sum_{i=1}^{2}\sum_{k=1}^{3}\sum_{l=1}^{2} q^*_{i2k,l} = 1,916.52, \quad \sum_{i=1}^{2}\sum_{k=1}^{3}\sum_{l=1}^{2} q^*_{i3k,l} = 1,000.00.$$

Observe that now, due to the increase in logistical costs, the demands at the first and third demand points are at their respective lower bounds and, hence, the corresponding Lagrange multipliers are now positive; that is:

$$\alpha^*_1 = 419.84, \quad \alpha^*_2 = 0.00, \quad \alpha^*_3 = 47.78.$$

The Lagrange multipliers associated with the upper bound demand constraints are, as in Example 3, again, all equal to zero:

$$\beta^*_1 = \beta^*_2 = \beta^*_3 = 0.$$

The relief item kit volumes delivered from each purchasing location $k; k = 1, 2, 3$, by each FSP $l; l = 1, 2$ are:

$$\sum_{i=1}^{2}\sum_{j=1}^{3} q^*_{ijk,l} = \begin{bmatrix} 1049.28 & 2061.21 \\ 1927.88 & 2891.81 \\ 1994.55 & 2994.55 \end{bmatrix}.$$

The FSPs have not reached their capacities and, hence, we have that:

$$\epsilon^*_{1,1} = \epsilon^*_{1,2} = \epsilon^*_{2,1} = \epsilon^*_{2,2} = \epsilon^*_{3,1} = \epsilon^*_{3,2} = 0.$$

The total cost of each humanitarian organization is now:

$$\sum_{k=1}^{3}\rho_k\sum_{j=1}^{3}\sum_{l=1}^{2}q^*_{1jk,l}+\sum_{j=1}^{3}\sum_{k=1}^{3}\sum_{l=1}^{2}c_{1jk,l}(q^*)=2,152,526.75,$$

$$\sum_{k=1}^{3}\rho_k\sum_{j=1}^{3}\sum_{l=1}^{2}q^*_{2jk,l}+\sum_{j=1}^{3}\sum_{k=1}^{3}\sum_{l=1}^{2}c_{2jk,l}(q^*)=2,782,132.00.$$

The total costs increase for both HOs, as compared to Example 3, with HO 1 encumbering the greater increase in its costs. HO 1 delivers a total of 5,581.62 relief item kits, whereas HO 2 delivers 7,334.92 relief item kits. These values are significantly lower than those obtained in Example 3. Moreover, HO 2 experiences the greater decrease in volume purchased and shipped than HO 1.

The volume of purchases at PL 1 is now: 3,110.49; that at PL 2 is: 4,819.69, whereas the amount purchased at Pl 3 is now: 4,986.36.

Again, both humanitarian organizations spend less than their full budgets, and, consequently, the Lagrange multipliers associated with the budget constraints are equal to zero:

$$\gamma_1^*=\gamma_2^*=0.$$

The benefit/altruism of each organization is now:

$$B_1(q^*)=1,582,822.00,\quad B_2(q^*)=2,633,978.00.$$

The utility of each HO, after the disaster relief operation, is now:

$$U_1(q^*)=-569,704.75,\quad U_2(q^*)=-148,154.00.$$

Observe that, due to the very high logistical costs associated with this disaster relief operation, the humanitarian organizations now encounter negative utilities. Nevertheless, as humanitarian organizations, and with the imposed minimum demands, they are required to meet the demands of the disaster victims.

Also, observe that, interestingly, in Example 3, eight of the flows were equal to 0.00, whereas now only one flow is equal to 0.00. With the increased nonlinear term factor, the HOs spread their logistical flows in order to reduce the costs, which is reasonable.

Example 5

In the fifth, and final numerical example, we explore the impacts of the removal of the common/shared demand constraints, that is, the upper and lower bounds on the demands at the demand points. Example 5, hence, has the same data as Example 4, except that the constraints (9) and (10), which we expect to be imposed by a higher level authority, are removed. It is easy to adapt the Euler method to handle the removal of such constraints and that is done.

The new computed equilibrium relief item kit flows are for $i = 1, 2; j = 1, 3$ now:

$$q^*_{ij1,1} = \begin{bmatrix} 191.67 & 83.33 & 0.00 \\ 0.00 & 0.00 & 66.67 \end{bmatrix}, \quad q^*_{ij2,1} = \begin{bmatrix} 233.34 & 88.89 & 0.00 \\ 344.46 & 199.99 & 66.67 \end{bmatrix},$$

$$q^*_{ij1,2} = \begin{bmatrix} 255.57 & 111.11 & 0.00 \\ 366.69 & 0.00 & 88.89 \end{bmatrix}, \quad q_{ij2,2} = \begin{bmatrix} 350.04 & 133.32 & 0.00 \\ 516.74 & 299.96 & 99.99 \end{bmatrix},$$

$$q^*_{ij3,1} = \begin{bmatrix} 244.45 & 100.00 & 0.00 \\ 355.57 & 211.10 & 77.78 \end{bmatrix},$$

$$q^*_{ij3,2} = \begin{bmatrix} 366.71 & 149.98 & 0.00 \\ 533.40 & 316.63 & 166.66 \end{bmatrix}.$$

The volume of relief item kits received at each demand point is now:

$$\sum_{i=1}^{2}\sum_{k=1}^{3}\sum_{l=1}^{2} q^*_{i1k,l} = 3,758.65, \quad \sum_{i=1}^{2}\sum_{k=1}^{3}\sum_{l=1}^{2} q^*_{i2k,l} = 1,916.52, \quad \sum_{i=1}^{2}\sum_{k=1}^{3}\sum_{l=1}^{2} q^*_{i3k,l} = 516.66.$$

Note that, without the imposition of the lower bounds on the demands, in order to guarantee sufficient relief for the disaster victims, both at Demand Point 1 and at Demand Point 3 the volumes of relief item kits received suffer a significant shortfall, since there was a lower bound of 10,000 at Demand Point 1 and a lower bound of 1,000 at Demand Point 3. Humanitarian organizations, hence, may not deliver the necessary supplies if such important constraints are not added.

The relief item kit volumes delivered from each purchasing location $k; k = 1, 2, 3$, by each FSP $l; l = 1, 2$ are:

$$\sum_{i=1}^{2}\sum_{j=1}^{3} q^*_{ijk,l} = \begin{bmatrix} 341.68 & 1044.46 \\ 933.35 & 1400.05 \\ 988.91 & 1483.38 \end{bmatrix}.$$

Since the FSPs, with lower volumes of shipments, do not reach their capacities, we have that:

$$\epsilon^*_{1,1} = \epsilon^*_{1,2} = \epsilon^*_{2,1} = \epsilon^*_{2,2} = \epsilon^*_{3,1} = \epsilon^*_{3,2} = 0.$$

The total cost of each humanitarian organization is now:

$$\sum_{k=1}^{3} \rho_k \sum_{j=1}^{3} \sum_{l=1}^{2} q^*_{1jk,l} + \sum_{j=1}^{3} \sum_{k=1}^{3} \sum_{l=1}^{2} c_{1jk,l}(q^*) = 486,594.13,$$

$$\sum_{k=1}^{3} \rho_k \sum_{j=1}^{3} \sum_{l=1}^{2} q^*_{2jk,l} + \sum_{j=1}^{3} \sum_{k=1}^{3} \sum_{l=1}^{2} c_{2jk,l}(q^*) = 886,547.00.$$

HO 1 delivers a total of 2,308.40 relief item kits, whereas HO 2 delivers 3,883.42 relief item kits. These values are approximately half of the respective amounts delivered by the HOs in Example 4, demonstrating, as well the importance of the demand lower bound constraints.

The volume of purchases at PL 1 is now: 3,110.49; that at PL 2 is: 4,819.69, whereas the amount purchased at Pl 3 is now: 4,986.36.

Again, both humanitarian organizations spend less than their full budgets, and, consequently, the Lagrange multipliers associated with the budget constraints are equal to zero:

$$\gamma_1^* = \gamma_2^* = 0.$$

The benefit/altruism of each organization is now:

$$B_1(q^*) = 625,858, \quad B_2(q^*) = 1,325,047.00.$$

The utility of each HO, after the disaster relief operation, is now:

$$U_1(q^*) = 139,264.75, \quad U_2(q^*) = 438,500.00.$$

The altruism is decreased for each HO, as compared to the altruism enjoyed in Example 4. However, the total costs are as well, yielding a positive utility for each humanitarian organization.

We note that the above examples, although stylized, illustrate the types of problems that can be addressed and analyzed using our modeling and computational game theory framework. We expect that larger-scale examples would also be effectively solvable using the Euler method given that it has performed well on large-scale network equilibrium problems, with problems with hundreds of variables solved in less than 1 CPU second (cf. Nagurney and Zhang (1996)).

6. Summary and Conclusions

The number of disasters is increasing as well as the number of people affected by them, posing great challenges to governments, the population, as well as humanitarian organizations that provide disaster relief. In this paper, we develop an integrated financial and logistical game theory model for humanitarian organizations that has several notable features. In particular, the new model extends existing models in the literature in several ways:

1. The model includes both relief item purchasing costs and freight service shipping costs, with the former being possible both locally and nonlocally, if feasible, and with the latter including competition, under capacity constraints, among the humanitarian organizations.

2. The governing equilibrium conditions, given common/shared constraints associated with the demands for relief items at the demand points, plus the freight capacity constraints, yield a Generalized Nash Equilibrium, which can be challenging to solve. Nevertheless, through the concept of a variational equilibrium, we construct a variational inequality formulation. This paper is one of only a handful, outside of the work of Nagurney, Alvarez Flores, and Soylu (2016), Nagurney et al. (2018) and Gossler et al. (2018) in which the GNE concept is applied to disaster relief.

3. The model is qualitatively analyzed and a Lagrange analysis provided - the latter is especially valuable since it yields insights on the impacts of the constraints - and is one of the very few such analyses conducted for variational inequality problems with nonlinear constraints. Moreover, we provide an alternative variational inequality formulation. The interpretation of the Lagrange analysis reveals which situations are beneficial to both humanitarian organizations and victims of disasters.

4. The proposed algorithm, when applied to the alternative variational inequality formulation, which is over the nonnegative orthant, yields closed form expressions, at each iteration, which enables ease of computer implementation.

The numerical examples, inspired by Hurricane Harvey, one of the most expensive natural disasters to ever hit the United States, illustrate the flexibility of the modeling and computational framework. The examples explore the impacts of the addition of a freight service provider; the further deterioration of the logistical infrastructure with the associated increasing costs, as well as the removal of demand constraints on the incurred costs, benefits, and utilities. The final example further reinforces the need for the imposition of lower and upper bounds on the volume of disaster relief supplies at demand points, since otherwise, there may be insufficient relief supplies delivered to deserving victims.

The framework adds to the literature on game theory and disaster relief as well as to the literature on variational inequalities with nonlinear constraints. Future research may include the incorporation of stochastic elements in terms of costs, prices, and demands within a game theory framework as well as capturing multiple stages associated with relief item procurement both prior to and post disasters, under competition. Such future research, we expect will build on the results in this paper and also on the published works of Nagurney and Nagurney (2016), Liu and Nagurney (2013), and Gossler et al. (2018), among others.

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Appendix

Proof of the Theorem.

If we multiply (24) by $(q_{ijk,l} - q^*_{ijk,l})$ and sum up with respect to i, j, k, and l we get:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}\left[\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t}\frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}} + \rho_k - \omega_i\frac{\partial B_i(q^*)}{\partial q_{ijk,l}}\right](q_{ijk,l} - q^*_{ijk,l})$$

$$= -\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}\epsilon^*_{k,l}q_{ijk,l} + \underbrace{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}\epsilon^*_{k,l}q^*_{ijk,l}}_{=\epsilon^*_{k,l}u_{k,l}}$$

$$-\sum_{i=1}^{m}\gamma^*_i\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}\left(\rho_k + \sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t}\frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}}\right)q_{ijk,l}$$

$$+\sum_{i=1}^{m}\gamma^*_i\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}\left(\rho_k + \sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t}\frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}}\right)q^*_{ijk,l}$$

$$+\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}\lambda^*_{jk,l}q_{ijk,l} - \underbrace{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}\lambda^*_{jk,l}q^*_{ijk,l}}_{=0}$$

$$+\sum_{j=1}^{n}\alpha^*_j\sum_{i=1}^{m}\sum_{k=1}^{o}\sum_{l=1}^{t}q_{ijk,l} - \underbrace{\sum_{j=1}^{n}\alpha^*_j\sum_{i=1}^{m}\sum_{k=1}^{o}\sum_{l=1}^{t}q^*_{ijk,l}}_{=\alpha^*_j\underline{d}_j}$$

$$-\sum_{j=1}^{n}\beta_j^*\sum_{i=1}^{m}\sum_{k=1}^{o}\sum_{l=1}^{t}q_{ijk,l}+\underbrace{\sum_{j=1}^{n}\beta_j^*\sum_{i=1}^{m}\sum_{k=1}^{o}\sum_{l=1}^{t}q^*_{ijk,l})}_{=\beta_j^*\overline{d}_j}$$

$$=-\sum_{k=1}^{o}\sum_{l=1}^{t}\epsilon^*_{k,l}\underbrace{\sum_{i=1}^{m}\sum_{j=1}^{n}(q_{ijk,l}-u_{k,l})}_{\leq 0}+\underbrace{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}\lambda^*_{ijk,l}q_{ijk,l}}_{\geq 0}$$

$$+\sum_{j=1}^{n}\alpha_j^*\underbrace{\left(\sum_{i=1}^{m}\sum_{k=1}^{o}\sum_{l=1}^{t}q_{ijk,l}-\underline{d}_j\right)}_{\geq 0}-\sum_{j=1}^{n}\beta_j^*\underbrace{\left(\sum_{i=1}^{m}\sum_{k=1}^{o}\sum_{l=1}^{t}q_{ijk,l}-\overline{d}_j\right)}_{\leq 0}$$

$$-\sum_{i=1}^{m}\gamma_i^*\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}\left(\rho_k+\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t}\frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}}\right)q_{ijk,l}$$

$$+\sum_{i=1}^{m}\gamma_i^*\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}\left(\rho_k+\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t}\frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}}\right)q^*_{ijk,l}. \qquad (A1)$$

Let us consider the last term in (A1):

$$-\sum_{i=1}^{m}\gamma_i^*\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}\left(\rho_k+\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t}\frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}}\right)q_{ijk,l}$$

$$+\sum_{i=1}^{m}\gamma_i^*\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}\left(\rho_k+\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t}\frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}}\right)q^*_{ijk,l}. \qquad (A2)$$

From the second line of (19), we obtain:

$$\sum_{k=1}^{o}\rho_k\sum_{j=1}^{n}\sum_{l=1}^{t}q_{ijk,l}+\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}c_{ijk,l}(q)-b_i\leq 0,\quad \forall i;$$

hence,

$$\sum_{i=1}^{m}\gamma_i^*\left(\sum_{k=1}^{o}\rho_k\sum_{j=1}^{n}\sum_{l=1}^{t}q_{ijk,l}+\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}c_{ijk,l}(q)-b_i\right)\leq 0. \qquad (A3)$$

From the second line of (23), we get:

$$\gamma_i^*\left(\sum_{k=1}^{o}\rho_k\sum_{j=1}^{n}\sum_{l=1}^{t}q^*_{ijk,l}+\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}c_{ijk,l}(q^*)-b_i\right)=0,\quad \forall i,$$

and, therefore, we have that:

$$\sum_{i=1}^{m}\gamma_i^*\left(\sum_{k=1}^{o}\rho_k\sum_{j=1}^{n}\sum_{l=1}^{t}q^*_{ijk,l}+\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}c_{ijk,l}(q^*)-b_i\right)=0. \qquad (A4)$$

As a consequence,

$$-\left[\sum_{i=1}^{m}\gamma_i^*\left(\sum_{k=1}^{o}\rho_k\sum_{j=1}^{n}\sum_{l=1}^{t}\left(q_{ijk,l}-q_{ijk,l}^*\right)+\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}\left(c_{ijk,l}(q)-c_{ijk,l}(q^*)\right)\right)\right]\geq 0. \quad (A5)$$

Since we have assumed that the logistical cost functions $c_{ijk,l}(q)$ are convex, $\forall i,j,k,l$, then, applying the convexity properties, for all i, j, k, and l, we have:

$$c_{ijk,l}(q)-c_{ijk,l}(q^*)\geq\left(\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t}\frac{\partial c_{ijk,l}(q)}{\partial q_{irp,s}}-\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t}\frac{\partial c_{ijk,l}(q^*)}{\partial q_{irp,s}}\right)(q_{irp,s}-q_{irp,s}^*). \quad (A6)$$

Summing (A6) with respect to i, j, k, and l, and multiplying by γ_i^*, we get:

$$\sum_{i=1}^{m}\gamma_i^*\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}c_{ijk,l}(q)-\sum_{i=1}^{m}\gamma_i^*\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}c_{ijk,l}(q^*)$$

$$\geq\sum_{i=1}^{m}\gamma_i^*\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}\left(\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t}\frac{\partial c_{ijk,l}(q)}{\partial q_{irp,s}}-\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t}\frac{\partial c_{ijk,l}(q^*)}{\partial q_{irp,s}}\right)\left(q_{irp,s}-q_{irp,s}^*\right). \quad (A7)$$

Therefore,

$$-\sum_{i=1}^{m}\gamma_i^*\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}\left(c_{ijk,l}(q)-c_{ijk,l}(q^*)\right)$$

$$\leq-\sum_{i=1}^{m}\gamma_i^*\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}\left(\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t}\frac{\partial c_{ijk,l}(q)}{\partial q_{irp,s}}-\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t}\frac{\partial c_{ijk,l}(q^*)}{\partial q_{irp,s}}\right)\left(q_{irp,s}-q_{irp,s}^*\right). \quad (A8)$$

Now, we add $-\sum_{i=1}^{m}\gamma_i^*\sum_{k=1}^{o}\rho_k\sum_{j=1}^{n}\sum_{l=1}^{t}\left(q_{ijk,l}-q_{ijk,l}^*\right)$ to each side of (A8) and obtain:

$$-\sum_{i=1}^{m}\gamma_i^*\sum_{k=1}^{o}\rho_k\sum_{j=1}^{n}\sum_{l=1}^{t}\left(q_{ijk,l}-q_{ijk,l}^*\right)$$

$$-\sum_{i=1}^{m}\gamma_i^*\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}\left(\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t}\frac{\partial c_{ijk,l}(q)}{\partial q_{irp,s}}-\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t}\frac{\partial c_{ijk,l}(q^*)}{\partial q_{irp,s}}\right)\left(q_{irp,s}-q_{irp,s}^*\right)$$

$$\geq-\sum_{i=1}^{m}\gamma_i^*\sum_{k=1}^{o}\rho_k\sum_{j=1}^{n}\sum_{l=1}^{t}\left(q_{ijk,l}-q_{ijk,l}^*\right)-\sum_{i=1}^{m}\gamma_i^*\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}\left(c_{ijk,l}(q)-c_{ijk,l}(q^*)\right)\geq 0. \quad (A9)$$

So, also the last term in (A1) is greater than or equal to zero. The conclusion follows that variational inequality (14) is equivalent to the variational inequality (25a). Variational inequality (25b) then follows from (25a) since the nonnegativity of q is guaranteed by the feasible set in (25b).