Integrated Crop and Cargo War Risk Insurance: Application to Ukraine

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Abstract:

Conflicts and wars can significantly disrupt global agricultural supply chains, with severe impacts on food security worldwide. War risk insurance, on the other hand, can mitigate farmers' losses. In this paper, we use the theory of variational inequalities to construct a new multicommodity international trade network equilibrium model with exchange rates on general transportation networks joining supply markets with demand markets under different wartime scenarios. The model incorporates capacities on production and transportation, as well as potential commodity losses in transportation. We then propose formulae for integrated crop and cargo war risk insurance premiums that are supply market and commodity specific and incorporate them into the model, along with government subsidies for the premiums. A series of numerical examples, both illustrative and algorithmically solved, focusing on the ongoing war on Ukraine, reveal the critical role of integrated war risk insurance and government support in sustaining agricultural commodity trade flows and supply market prices as well as protecting the revenue of farmers.

Key words: agricultural trade, crop insurance, cargo insurance, wartime risk, food security, variational inequalities

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1. Introduction

The functionality of agricultural supply chain networks consisting of producers, transportation routes, and consumers is essential to global food security. Presently, 40% of the global caloric intake is provided by such crops as wheat, corn, and rice with more than 80% of the global trade in these staple crops plus oilseeds depending on several transportation routes (Denamiel et al. (2024)). Major geopolitical conflicts such as Russia's full-scale invasion of Ukraine on February 24, 2022, followed by the Houthis' attacks on ships in the Red Sea, beginning in November 2023, have created significant challenges to the trade of such essential agricultural commodities. Increases in prices due to heightened risk, transportation delays, and increasing production and transportation costs are affecting vulnerable populations, including those in the Middle East and Africa (cf. Baraniuk (2024)).

Insurance policies are an integral part of risk management in agricultural supply chains, especially on the production and transportation sides. On the supply side, government-backed insurance policies can mitigate farmers' losses by ensuring revenue for their crops (see, e.g., Ngo (2023)). On the transportation side, insurance for different modes of transport, including maritime transportation, can protect against losses or damages during transit (cf. Denamiel et al. (2024)). Such insurance policies cover a wide range of risks. Accurately estimating insurance premiums is crucial for protecting both insurance companies and policyholders. If premiums are set too low, the insurance company might not have enough money to cover claims, which can lead to financial instability. Conversely, if premiums are too high, customers may not be able to afford insurance, leaving them unprotected.

War risk insurance, according to Kagan (2021), is insurance that covers losses due to events such as war, invasions, strikes, and terrorism. Bourne (2024) notes that war risk insurance is a special type of insurance with typical insurance usually excluding war as an insurable "peril." Nevertheless, as noted therein, businesses involved in global operations may be able to buy war risk insurance that would also include transportation. For example, according to the Defense Intelligence Agency (2024), war risk insurance for container shipping through the Red Sea is roughly 1000% more, as of April 2024, due to Houthi attacks, than before the October 7, 2023 Hamas attacks on Israel. Interestingly, according to Morris (2018), insurance professionals consider marine insurance – hull and cargo – to be the oldest forms of insurance, going back to the Phoenicians trading in the Mediterranean, and dating back to 1200 BC, with the first formal policy being established in 1350.

Ukraine, often referred to, prior to the full-scale invasion, as the breadbasket of Europe, if not the world, provided about 10% of the global wheat exports, 15% of the global corn and barley exports, and 50% of global sunflower oil exports, with 90% of these agricultural commodities exported through Ukraine's deepwater Black Sea ports prior to the invasion (Denamiel et al. (2024)). In a series of studies, Nagurney et al. (2023, 2024a,b) quantified the effects of disruptions to production and transportation on commodity prices of wheat and corn being exported from Ukraine to the countries of Egypt and Lebanon, examples of MENA (Middle Eastern and North African) countries. These countries have relied heavily on Ukrainian agricultural commodity imports, under various transportation options including under the Black Sea Grain Initiative, which was pulled back in July 2023. Hassani et al. (2025), in turn, also considered possible disruptions to storage facilities in Ukraine. Kormych, Averochkina, and Kormych (2024) provide an overview of agreements on maritime shipping during the full-scale war on Ukraine in the form of two humanitarian corridors.

In this paper, we provide a framework for integrated crop and cargo war risk insurance. Our research is inspired by the ongoing war on Ukraine and the need there for war risk insurance associated with agricultural commodities that are foundations for food security. In such high risk environments as the Black Sea and the Red Sea, critical for the export of staple commodities, innovative insurance policies can play a prominent role in protecting producers as well as consumers in terms of agricultural commodity trade in wartime. We single out, as an example, and as noted in Denamiel et al. (2024), recently introduced discounted war risk insurance for agricultural product exporters via the Ukrainian corridor, with this corridor hugging the western coast of the Black Sea through the territorial waters of NATO countries of: Romania, Bulgaria, and Turkey. The war risk insurance is being referred to as the "Unity Facility." It consists, according to Denamiel et al. (2024), of a public-private partnership of the US- and UK-based insurance companies Marsh McLennan and Lloyd's of London working in conjunction with the Ukrainian government as well as the DZ Bank in Germany (see also Marsh McLennan (2024)). It has reduced by more than 50% the cost of existing available insurance policies. Additionally, the European Bank for Reconstruction and Development (EBRD) is considering establishing a war risk insurance system for domestic transportation and goods transported within Ukraine. This initiative, anticipated to be in place by the end of 2024 or early 2025, could be of great interest to Ukrainian farmers and agricultural Micro, Small, and Medium Enterprises (MSMEs) (Open4business (2024)). Similar insurance products offered by Ukrainian companies are quite costly, with premiums reaching up to 10 percent.

However, it is not only cargo insurance that is important in wartime but, also, crop insurance. According to Welsh and Glauber (2024), the U.S. Department of Agriculture estimates that the harvested area for wheat, corn (maize), and barley in Ukraine for 2023 is down 32%, 27%, and 37%, respectively, from 2021 levels. As noted in Nagurney et al. (2024a), the reasons for reduction of arable land in Ukraine in wartime are multifaceted, with lands mined, overtaken, and with destruction of farm machinery, storage, and port facilities, etc. Of course, shortages of farm labor in the war are also a significant problem. After the full-scale Russian invasion, Ukraine significantly increased various forms of state support for crop insurance. Following the 2024 amendments to the Law of Ukraine on Specifics of State-Supported Insurance of Agricultural Products, agricultural producers are now eligible for compensation of 60% of the insurance premiums they pay from the state budget (Verkhovna Rada (2024)). Also, under this law, the Ukrainian government limits maximum insurance premiums to ensure future harvests of sugar beet, grain crops grown during the spring-summer period, and winter crops. In 2023, the government-imposed limits on premiums for winter wheat insurance ranged from 8.34 to 11.67%, depending on the region (Cabinet of Ministers of Ukraine (2023)). In comparison, the average premium for crop insurance before the war was 3.4% (World Bank Group (2018)).

Against this background, it is important to highlight the initiative launched in 2023 by the United States Agency for International Development 'Investment for Business Resilience' Project (USAID IBR) (USAID IBR (2023)) in collaboration with the insurance company UNIVERSALNA and PrivatBank. With the support of the USAID IBR Project, a dedicated product for future crop insurance has been developed, resulting in the conclusion of 344 contracts valued at approximately 857 million Ukrainian hryvnia (UAH) (about 23,414,000 US$). The project compensated 95% of the cost of the insurance services. This co-financing program primarily targets micro and small businesses, representing the most vulnerable segment of agricultural producers. Additionally, it enables these businesses to access credit secured by future crop yields. So far, three enterprises that experienced crop losses have received compensation totaling 2.8 million UAH (76,500 US$) (UNIVERSALNA (2024)).

The research on war risk insurance is limited. We believe that, heretofore, integrated crop and cargo war risk insurance has not been considered, although, clearly, it is highly relevant given the geopolitical events on our planet, including the case of Russia's war on Ukraine. Specifically, farmers face risk in wartime, due to limits on the production capacity as well as transportation route capacities and possible losses or damages in transit of their agricultural products. Plus, supply price functions may be affected as well as the transportation cost functions due to higher freight rates. In addition, demand price functions may be altered under different wartime scenarios. Hence, providing a formula for such war risk insurance is very timely. Furthermore, we demonstrate how in our model the effect of the government paying for a portion or all of the war risk insurance, which is commodity and supply market dependent, is captured.

Our framework entails the solution of a series of variational inequality problems reflecting capacities at the supply markets and on the transportation routes as well as the commodity losses on the transportation routes under different wartime scenarios, plus the solution of a baseline variational inequality problem corresponding to prewar conditions. The variational inequality problems correspond to the equilibrium conditions associated with the perfectly competitive spatial price equilibrium problems, commonly used to model agricultural trade (cf. Nagurney et al. (2024a) and the references therein). The war risk insurance premium is then calculated using the baseline commodity supply market price and those under the scenarios, weighted by the probability of the scenario, to determine the expected loss in supply market price per ton (metric) of agricultural commodity. Subsequently, the insurance premiums are included in the next stage variational inequality problem, along with the percentage of the government subsidies, in order to assess the impact on the new equilibrium commodity flows and prices.

Historical wartime insurance models offer insights for our approach, notably the maritime schemes of World War II. The 20th century's major conflicts significantly altered war risk coverage, with insurance markets adopting uniform strategies to handle the immense losses due to armed conflicts (Atlas Magazine (2022)). In 1938, London insurers agreed to exclude war damages from standard policies via Clause NMA 464, addressing risks too vast to cover routinely (Malagon (2022)), paralleling our use of discrete scenarios for extreme risks. These historical adaptations shows the flexibility of insurance in conflict zones. Earlier, during World War I, the U.S. Bureau of War Risk Insurance, established in 1914, insured ships and cargo while expanding to offer government-subsidized benefits to servicemen, handling claims for over 4 million policies by 1918 (Department of Veterans Affairs (2022)). Similarly, recognizing private market limitations, the UK government in 1913 planned to reinsure 80% of war risks if war disrupted trade (Benzie (2004)). These precedents reinforce our methodology's relevance for modern conflict zones.

Furthermore, according to Hrazhdan (2024), Ukraine is working on establishing a State Agency for War Risk Insurance in order to create a common framework for risk assessment and to unify existing projects in one system. The war insurance would be financed by both Ukraine and donors. Such an insurance market would be helpful in stabilizing Ukraine's economy.

2. Related Literature, Contributions, and Organization of the Paper

We now provide a discussion of the related literature that provides some background and context to the contributions in this paper, which we also highlight. The organization of the paper then follows.

2.1 Related Literature

Operations research methods have been extensively applied in the insurance industry to address various challenges in risk management and actuarial science (Samson and Thomas (1985), von Lanzenauer and Wright (1991), and Brockett and Xia (1995)). Brockett and Xia (1995) provided a nice overview of various mathematical programming methodologies, from linear programming to network optimization, with even note of Data Envelopment Analysis (DEA) and game theory, that have been applied to various problems in insurance. The research in this paper is more closely aligned with that on insurance associated with catastrophic risks, since, clearly, an ongoing war is a catastrophe. Ermoliev et al. (2000) discussed the synergy between risk reduction measures and insurance mechanisms in managing rare but impactful events, emphasizing the importance of system approaches and decision support systems. Lodree Jr. and Taskin (2008) introduced an insurance risk management framework for disaster relief and supply chain disruption inventory planning, interpreting proactive inventory decisions as insurance premiums. Kalfin et al. (2022) provided a systematic literature review on using insurance as an alternative for sustainable economic recovery after natural disasters, highlighting its significance in funding post-disaster recovery. Fan et al. (2024b) developed a stochastic programming model to investigate the impact of purchasing catastrophe insurance on supply chain operational planning, demonstrating that insurance can influence production decisions and damage costs in catastrophe-prone environments. Zbib et al. (2024) introduced a mutual catastrophe insurance framework for horizontal collaboration in prepositioning strategic reserves, demonstrating the benefits of coordinated efforts in disaster preparedness. Our approach, however, makes use of the theory of variational inequalities (cf. Nagurney (1999) and the references therein) since we are interested in international trade of agricultural products in wartime, with multiple supply markets, demand markets, and multiple commodities as well as transportation routes on such networks. Our model captures also possible commodity losses in transportation under different scenarios. The model is most closely related to that of Nagurney et al. (2024b) but it is more general since it allows for losses in the form of generalized networks (see, also, Nagurney, Pour, and Samadi (2024) and the references therein) and the underlying functions can differ from scenario to scenario.

We now highlight literature on insurance associated with agriculture but, first, we note that interest in operations research approaches to agriculture have been growing as evidenced by the recent special issue of the International Transactions in Operational Research edited by Albornoz et al. (2023). Ahsan et al. (1982) developed a theory of crop insurance, showing its risk-spreading role and discussed the challenges of imperfect information in competitive markets. Myers (1988) evaluated the value of ideal contingency markets, such as futures and crop insurance, finding that, while they increase economic efficiency, they may not always benefit farmers and consumers simultaneously. Mahul and Wright (2003) analyzed the design of optimal crop revenue insurance, considering the effects of basis risk on indemnity schedules. Anderson and Monjardino (2019) investigated contract designs in agricultural supply chains with random yields, showing how input discounts in exchange for reduced crop prices can benefit risk-averse growers. Fan et al. (2024a) examined different types of agricultural subsidies and their implications on output and wealth distribution among farmers. They found that, while harvesting subsidies lead to efficient resource use and higher social welfare, planting subsidies are more effective in balancing farmer income but may perform poorly under aggressive output targets. Huh and Lall (2013) considered optimal crop choice and irrigation allocation under contract farming, addressing how contracts can mitigate climate and price risks. Alizamir et al. (2019) analyzed U.S. government subsidy programs in agriculture, comparing price versus revenue protection schemes and their impacts on farmers, consumers, and social welfare. Petsakos and Rozakis (2015) discussed the calibration of agricultural risk programming models, proposing a nonlinear mean–variance specification to better capture farmers' risk behavior. Tang et al. (2016) analyzed partially-guaranteed-price contracts between farmers and agri-food companies, demonstrating how such contracts create mutual benefits by ensuring a stable supply and sharing price risks. Zhang et al. (2017), in turn, explored supply chain coordination of fresh agricultural products under agricultural insurance, finding that insurance can optimize the supply chain by managing natural and market risks. Wu and Shi (2017) analyzed the optimization of a three-echelon agricultural supply chain under crop revenue insurance, demonstrating that insurance can stabilize and improve supply chain coordination. Shi et al. (2021) studied optimal strategies for capital-constrained contract-farming supply chains with yield insurance, highlighting how insurance affects farm size, yield, and wholesale pricing decisions. Assa et al. (2021) examined the role of price insurance products in stimulating investment in agri-food supply chains, demonstrating that managing price risk through insurance can encourage higher investment levels.

Cargo insurance, however, has received less attention from an operations research perspective. We, nevertheless, highlight several relevant, interesting papers. Ksciuk et al. (2023)

provided a literature review on uncertainty in maritime ship routing and scheduling, noting the role of operations research in addressing uncertainties that can be mitigated through insurance mechanisms. Ellili et al. (2023) conducted a bibliometric analysis of marine insurance literature, identifying key trends, influential authors, and suggesting areas for future research in risk assessment and the insurance industry. Gallagher et al. (2005) examined corn pricing, testing theories for corn prices under different CIF (Cost, Insurance, and Freight) pricing models. Goerlandt and Montewka (2015) analyzed issues in maritime transportation risk analysis, emphasizing the need for systematic approaches to risk definitions. Kang et al. (2020) developed a dual-channel supply chain disruption model under cargo transportation insurance, demonstrating that purchasing insurance can effectively reduce losses from transportation disruptions and impact supply chain profits. Kotenko et al. (2022) showed the expected losses from cargo transportation risks in maritime transport, focusing on dynamic risks such as those arising from military conflicts. Jalal et al. (2022) addressed distribution planning in pharmaceutical networks, transporting decisions for high-value cargo and considering freight shipping types, fleet sizing, and escorting services, highlighting the importance of cargo insurance for valuable goods.

There is, clearly, a notable gap regarding war insurance modeling, in general, and one that is focused on the agricultural sector, in particular. Gupta et al. (2023) identified opportunities in farming research from an operations management perspective but noted that war insurance remains underexplored. Li et al. (2023) focused on agricultural insurance and power structures in capital-constrained supply chains, providing insights that could be extended to war-impacted regions. Plus, as noted earlier, in this paper, we construct premiums for integrated crop and cargo insurance, since in wartime, as vividly illustrated now in Ukraine, both production at supply markets as well as transportation to demand markets may be severely impacted in conflict scenarios. We note that crop insurance (cf. Hawker and Smart (2018)) at the federal level can also provide coverage for storage of the harvested agricultural commodities up to 20 days after the crop is harvested and even further out.

Since in this paper we also explore governmental subsidies for war risk insurance for farmers, we now note some related literature on subsidies and spatial price equilibrium models using variational inequality theory. Nagurney (2023) introduced an agricultural trade network model with minimal nutritional standards for purposes of food security and provided formulae for consumer subsidies (see, also, Schweigman (2008)). Nagurney and Besedina (2023), earlier, constructed a model with non-tariff measures, including subsidies, and with quality of agricultural commodities. Nagurney et al. (2023) included subsidies, along with exchange rates calculated for routes, which could go through different countries, between supply markets and demand markets. Nagurney, Salarpour, and Dong (2022), in turn, developed a network modeling framework to assess different policies in the Covid-19 pandemic on essential products. None of these models, however, considered insurance, either crop insurance or cargo insurance. Nagurney, Daniele, and Cappello (2021) focused on another societal problem - that of human migration and proposed subsidies that, when imposed, would achieve a system optimum, despite migrants behaving is a user-optimizing manner. Additional societal problems that have been modeled using variational inequality theory have also included, among other applications: environmental problems (Daniele and Sciacca (2021)), issues of cybercrime and cybersecurity (Colajanni et al. (2018)), challenges associated with pharmaceutical supply chains (see Nagurney, Li, and Nagurney (2013)), and issues associated with labor in supply chain networks (cf. Nagurney (2022)).

2.2 Contributions

The contributions in this paper are manifold. Foremost, this is the first time that an integrated crop and cargo war risk insurance framework has been constructed. Furthermore, its features capture such realities associated with war scenarios such as reduced agricultural production capacity because of destruction of land, reduced availability of resources, including agricultural inputs, machinery, liquidity, labor, etc., as well as reduced capacity for the transport of the agricultural products because of attacks, destruction of critical infrastructure, blockades, compromised routes because of mining, restricted access, and possible losses in transportation to the demand markets of the commodities, coupled with heightened risk and uncertainty. Plus, the supply price, transportation cost, and demand price functions can be scenario-dependent. Unlike previous studies such as that of Fan et al. (2024b), which considered transportation networks without incorporating supply and demand functions, and Dong et al. (2018), which focused primarily on the supply side and analyzed production chain disruptions, our work integrates both the supply markets and the demand markets with a general transportation network, under different wartime scenarios, with associated probabilities. War risk insurance is also a topic of great interest as well as discussion in Ukraine, so having a quantitative framework, such as the one we develop here, can be very relevant to practice (see Yasko (2024)).

The contributions herein are also interesting from a methodological standpoint. A series of variational inequalities is proposed under different wartime scenarios to formulate the underlying governing spatial price equilibrium conditions relevant to international trade of agricultural commodities. The solution of these variational inequality problems, which provide the multicommodity trade flows and the incurred commodity supply market and demand market prices under the commodity losses, allows for the calculation of the insurance premiums, which are commodity and supply market specific, based on associated probabilities of the scenarios. In addition, we allow for the quantification of the effects of the subsidization of the war risk insurance by governments, which is increasingly happening in practice, on the new equilibrium flows and commodity prices.

The numerical examples are drawn from an ongoing conflict, which continues to impact food security globally.

2.3 Organization of this Paper

The paper is organized as follows. In addition to the Introduction and the above overview of the related literature and our contributions, in Section 3, we develop the international trade network equilibrium model under uncertainty and possible commodity losses. Each wartime scenario is accorded its corresponding set of equilibrium conditions, with a derived specific variational inequality problem. A series of such variational inequality problems is associated with the spectrum of scenarios. In Section 4, we then construct the formulae for the war risk insurance premiums, followed by the equilibrium conditions that include them, and the associated variational inequality formulation. A numerical example is provided for illustrative purposes, accompanied by sensitivity analysis on the government subsidy on the insurance premium. In Section 5, we present larger-scale numerical examples that are solved algorithmically, along with insights, in order to demonstrate the applicability of the modeling framework and the insurance premiums. Section 6 summarizes the results and presents suggestions for future research.

3. The International Trade Model Under Uncertainty and Commodity Losses

In this Section, we develop the international trade network equilibrium model under uncertainty due to wartime disruptions and possible commodity losses. The timeline is a year; that is, the insurance premiums that are calculated are for a year and then would need to be updated and renewed. The year time horizon is reasonable due to planting, harvesting, and transportation of the agricultural commodities over the seasons. The model extends the model developed in Nagurney et al. (2024b) to capture commodity losses in transportation due to wartime risk such as attacks, the mining of transportation routes, etc. Furthermore, because of the challenges associated with agriculture in supply chains in wartime and decisions that need to be made by farmers, in the model in this paper, unlike the model in Nagurney et al. (2024b), we have capacities over all commodities associated with a

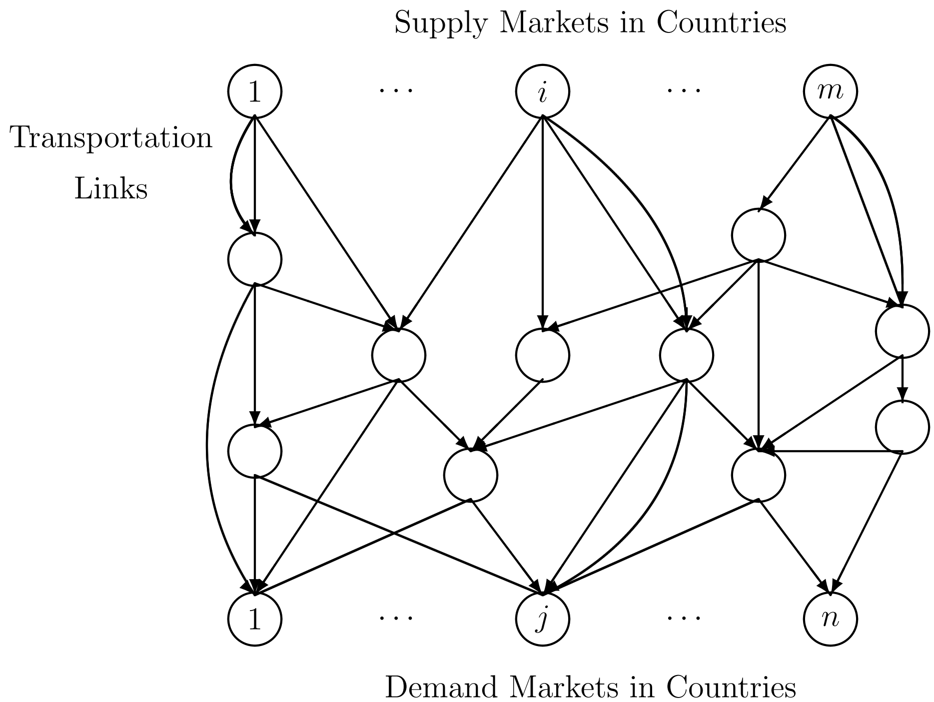

Figure 1: An International Trade Network Topology

supply market and the same for a route. In addition, the underlying functions can differ from scenario to scenario. The model is a perfectly competitive one in a spatial price equilibrium setting and includes exchange rates as in Nagurney et al. (2024a). There are m supply markets involved in the production of K commodities and n demand markets. We denote a typical supply market by i, a typical commodity by k, and a typical demand market by j. There are assumed to be n_{ij} routes joining the pair of supply and demand markets (i, j), with a typical route denoted by r. There are P routes in the international trade network. The network representation of the model is depicted in Figure 1. The model introduced in this Section, will be utilized as the one for the construction of insurance premiums (integrated crop and cargo premiums) in Section 4.

The basic model notation is given in Table 1. All vectors are column vectors.

In the model, there are assumed to be ω distinct war (disruption) scenarios that can affect the upper bounds (capacities) of the supply of the multiple commodities, their transportation on the routes, and the route flow multipliers, which range from greater than 0 through 1. In this setting, 1 means that there is no loss in the commodities on the route r; a value of .5 would mean that only 50% of the commodities make it to the demand market that are transported on that route, and so on. The functions can differ according to the scenario. The war scenario set $\Xi^1 \equiv \{\xi_1, \xi_2, \ldots, \xi_\omega\}$ with ξ_0 denoting the scenario with no disruption

Table 1: Notation for the International Trade Network Model

Notation	Parameter Definition
$u_i^{s\xi_l}$	upper bound on the supply of the commodities at supply market i; $i = 1, \ldots, m$ under war scenario ξ_l; $l = 1, \ldots, \omega$.
$u_{ijr}^{Q\xi_l}$	upper bound on the transportation of all the commodities from supply market i; $i = 1, \ldots, m$ to demand market j; $j = 1, \ldots, n$ on route r; $r = 1, \ldots, n_{ij}$ under war scenario ξ_l; $l = 1, \ldots, \omega$.
$\alpha_{ijr}^{\xi_l}$	the route r flow multiplier which quantifies how much of all the commodities remain after being transported on route r; $r = 1, \ldots, n_{ij}$ under war scenario ξ_l; $l = 1, \ldots, \omega$.
$e_{ij}^{\xi_l}$	the exchange rate from supply market i; $i = 1, \ldots, m$ to demand market j; $j = 1, \ldots, n$ under scenario ξ_l; $l = 1, \ldots, \omega$.
σ_i^k	fraction of the premium for supply market i; $i = 1, \ldots, m$, and commodity k; $k = 1, \ldots, K$ covered by an authority with the values lying between 0 and 1.
Notation	**Variable Definition**
$s_i^{k\xi_l}$	the supply of the commodity k; $k = 1, \ldots, K$ at supply market i; $i = 1, \ldots, m$ under war scenario ξ_l; $l = 1, \ldots, \omega$. Group all the supplies at war scenario ξ_l; $l = 1, \ldots, \omega$ into the vector $s^{\xi_l} \in R_+^{Km}$.
$d_j^{k\xi_l}$	the demand for the commodity k; $k = 1, \ldots, K$ at demand market j; $j = 1, \ldots, n$ under war scenario ξ_l; $l = 1, \ldots, \omega$. Group all the demands at scenario ξ_l; $l = 1, \ldots, \omega$ into the vector $d^{\xi_l} \in R_+^{Kn}$.
$Q_{ijr}^{k\xi_l}$	the shipment of the commodity k; $k = 1, \ldots, K$ from supply market i; $i = 1, \ldots, m$ to demand market j; $j = 1, \ldots, n$ on route r; $r = 1, \ldots, n_{ij}$ under war scenario ξ_l; $l = 1, \ldots, \omega$. Group all the commodity shipments at scenario ξ_l; $l = 1, \ldots, \omega$ into the vector $Q^{\xi_l} \in R_+^{KP}$.
$\lambda_i^{s\xi_l}$	the Lagrange multiplier associated with the production capacity constraint at supply market i; $i = 1, \ldots, m$ under war scenario ξ_l; $l = 1, \ldots, \omega$. Group all these Lagrange multipliers at scenario ξ_l; $l = 1, \ldots, \omega$ into the vector $\lambda^{s\xi_l} \in R_+^m$.
$\lambda_{ijr}^{Q\xi_l}$	the Lagrange multiplier associated with the transportation capacity constraint on route r; $r = 1, \ldots, n_{ij}$ joining supply market i; $i = 1, \ldots, m$ and demand market j; $j = 1, \ldots, n$ under war scenario ξ_l; $l = 1, \ldots, \omega$. Group all these Lagrange multipliers at scenario ξ_l; $l = 1, \ldots, \omega$ into the vector $\lambda^{Q\xi_l} \in R_+^P$.
Notation	**Function Definition**
$\pi_i^{k\xi_l}(s^{\xi_l})$	the supply price function for commodity k; $k = 1, \ldots, K$ at supply market i; $i = 1, \ldots, m$ under war scenario ξ_l; $l = 1, \ldots, \omega$.
$\rho_j^{k\xi_l}(d^{\xi_l})$	the demand price function for commodity k; $k = 1, \ldots, K$ at demand market j; $j = 1, \ldots, n$ under war scenario ξ_l; $l = 1, \ldots, \omega$.
$c_{ijr}^{k\xi_l}(Q^{\xi_l})$	the unit transportation cost associated with transporting the commodity k; $k = 1, \ldots, K$ from supply market i; $i = 1, \ldots, m$ to demand market j; $j = 1, \ldots, n$ via route r; $r = 1, \ldots, n_{ij}$ under war scenario ξ_l; $l = 1, \ldots, \omega$.

(the initial case). The full network in Figure 1 is one where no disruptions happen, i.e., ξ_0. The upper bounds (capacities) are updated according to each war scenario in Ξ^1, along with the route multipliers, as are the functions and the exchange rates. If transportation is impossible along a route, then the corresponding upper bound can be set to 0. Furthermore, we associate a probability with each war scenario defined, respectively, as: $p_{\xi_1}, p_{\xi_2}, \ldots, p_{\xi_\omega}$. These war scenarios are assumed to be independent, as in Nagurney et al. (2024b). In addition, as in that paper, and as in Nagurney and Qiang (2012), we make use of discrete probabilities. Determining probabilities in wartime associated with disruption scenarios may be challenging because of lack of historical data. Nevertheless, discrete probabilities, with, possibly, the use of experts' subjective judgment, enables a decent framework.

Indeed, although data scarcity remains an issue in conflict zones, a discrete scenario approach can incorporate partial or fragmented information and the war scenario probabilities can be updated as new intelligence or situational changes occur. This flexibility allows the model to be applied even with limited data. In our numerical examples, we assume equal probabilities for low damage and high damage scenarios for illustrative purposes, but, in practice, these could be adjusted based on real-time data or expert judgment.

The commodity shipment variables, since they correspond to agricultural products, are in metric tons. The supply market, demand market, and unit transportation costs on routes are for a metric ton of commodity. Note that the supply markets may be in the same or in different countries.

The Conservation of Flow Equations

The conservation of flow equations are as follows.

The commodity shipments must be nonnegative in all the war scenarios; that is:

$$Q^{k\xi_l}_{ijr} \geq 0, \quad k = 1, \ldots, K; i = 1, \ldots, m; j = 1, \ldots, n; r = 1, \ldots, n_{ij}; l = 1, \ldots, \omega. \tag{1}$$

The supply of commodity k produced at supply market i under each war scenario ξ_l must be equal to the sum of the commodity shipments from supply market i to all the demand markets in the disaster scenario over all the routes; that is:

$$s^{k\xi_l}_i = \sum_{j=1}^{n} \sum_{r=1}^{n_{ij}} Q^{k\xi_l}_{ijr}, \quad k = 1, \ldots, K; i = 1, \ldots, m; l = 1, \ldots, \omega. \tag{2}$$

The demand for each commodity k at each demand market j must be equal to the amount of the commodity that actually arrives, after losses, to demand market j in each war scenario

ξ_l. Hence, the following must hold:

$$d_j^{k\xi_l} = \sum_{i=1}^{m} \sum_{r=1}^{n_{ij}} \alpha_{ijr}^{\xi_l} Q_{ijr}^{k\xi_l}, \quad k = 1, \ldots, K; j = 1, \ldots, n; l = 1, \ldots, \omega. \tag{3}$$

We now describe the constraints that capture disruptions in the form of capacity reductions under the war scenarios.

The supply of the commodities produced at supply market i under war scenario ξ_l cannot exceed the supply capacity at the supply market; that is:

$$\sum_{k=1}^{K} s_i^{k\xi_l} \leq u_i^{s\xi_l}, \quad i = 1, \ldots, m; l = 1, \ldots, \omega. \tag{4a}$$

In view of (2), we may rewrite the constraints in (4a) as:

$$\sum_{k=1}^{K} \sum_{j=1}^{n} \sum_{r=1}^{n_{ij}} Q_{ijr}^{k\xi_l} \leq u_i^{s\xi_l}, \quad i = 1, \ldots, m; l = 1, \ldots, \omega. \tag{4b}$$

Note that, according to (4a) (and (4b)), farmers must decide how much of each commodity to produce under the production capacity at each supply market. The supplies of the commodities are all in metric tons.

The Lagrange multiplier $\lambda_i^{s\xi_l}$ is associated with constraint (4b) for $i = 1, \ldots, m; l = 1, \ldots, \omega$.

The amount of the commodities that can be transported from supply market i to demand market j on route r under war scenario ξ_l cannot exceed the capacity of that route, so that:

$$\sum_{k=1}^{K} Q_{ijr}^{k\xi_l} \leq u_{ijr}^{Q\xi_l}, \quad i = 1, \ldots, m; j = 1, \ldots, n; r = 1, \ldots, n_{ij}; l = 1, \ldots, \omega. \tag{5}$$

The Lagrange multiplier $\lambda_{ijr}^{Q\xi_l}$ for each $i = 1, \ldots, m$; $j = 1, \ldots, n$; $r = 1, \ldots, n_{ij}; l = 1, \ldots, \omega$ is associated with constraint (5).

According to (5), there is a limited capacity on each transportation route for the transport of the commodities.

Because of (2) and (3), we can redefine the supply price functions and the demand price functions solely in terms of the commodity shipment variables Q^{ξ_l}. This will also allow us to construct a variational inequality formulation of the governing international trade equilibrium conditions in only commodity shipments and Lagrange multipliers as variables. Specifically, we define new supply price functions $\tilde{\pi}_i^{k\xi_l}(Q^{\xi_l}) \equiv \pi_i^{k\xi_l}(s^{\xi_l})$ for $k = 1, \ldots, K$; $i = 1, \ldots, m$, and $l = 1, \ldots, \omega$ and new demand price functions $\tilde{\rho}_j^{k\xi_l}(Q^{\xi_l}) \equiv \rho_j^{k\xi_l}(d^{\xi_l})$ for $k = 1, \ldots, K$; $j = 1, \ldots, n$, and $l = 1, \ldots, \omega$. In the model, the focus is on the capacity reductions and the commodity losses in wartime for purposes of determination of integrated premiums for crop and cargo insurance.

Definition 1: The International Trade Network Equilibrium Conditions Under Capacity Reductions and Commodity Losses

A multicommodity shipment and Lagrange multiplier pattern $(Q^{\xi_l*}, \lambda^{s\xi_l*}, \lambda^{Q\xi_l*}) \in \mathcal{K}^{\xi_l}$*, where*

$$\mathcal{K}^{\xi_l} \equiv \{(Q^{\xi_l}, \lambda^{s\xi_l}, \lambda^{Q\xi_l})|(Q^{\xi_l}, \lambda^{s\xi_l}, \lambda^{Q\xi_l}) \in R_+^{KP+m+P}\}$$

is a multicommodity international trade network equilibrium under capacity reductions and commodity losses in war scenario ξ_l*;* $l = 1, \ldots, \omega$*, if the following conditions hold: for all commodities* k*;* $k = 1, \ldots, K$*; for all supply and demand market pairs:* (i, j)*;* $i = 1, \ldots, m$*;* $j = 1, \ldots, n$*, and for all routes* r*;* $r = 1, \ldots, n_{ij}$*:*

$$(\tilde{\pi}_i^{k\xi_l}(Q^{\xi_l*}) + c_{ijr}^{k\xi_l}(Q^{\xi_l*}))e_{ij}^{\xi_l} + \lambda_i^{s\xi_l*} + \lambda_{ijr}^{Q\xi_l*} \begin{cases} = \alpha_{ijr}^{\xi_l}\tilde{\rho}_j^{k\xi_l}(Q^{\xi_l*}), & \text{if } Q_{ijr}^{k\xi_l*} > 0, \\ \geq \alpha_{ijr}^{\xi_l}\tilde{\rho}_j^{k\xi_l}(Q^{\xi_l*}), & \text{if } Q_{ijr}^{k\xi_l*} = 0; \end{cases} \tag{6}$$

for all commodities k*;* $k = 1, \ldots, K$*, and for all supply markets* i*;* $i = 1, \ldots, m$*:*

$$u_i^{s\xi_l} \begin{cases} = \sum_{k=1}^K \sum_{j=1}^n \sum_{r=1}^{n_{ij}} Q_{ijr}^{k\xi_l*}, & \text{if } \lambda_i^{s\xi_l*} > 0, \\ \geq \sum_{k=1}^K \sum_{j=1}^n \sum_{r=1}^{n_{ij}} Q_{ijr}^{k\xi_l*}, & \text{if } \lambda_i^{s\xi_l*} = 0; \end{cases} \tag{7}$$

for all commodities k*;* $k = 1, \ldots, K$*, and for all supply and demand markets* (i, j)*;* $i = 1, \ldots, m$*;* $j = 1, \ldots, n$*, and for all routes* r*;* $r = 1, \ldots, n_{ij}$*:*

$$u_{ijr}^{Q\xi_l} \begin{cases} = \sum_{k=1}^K Q_{ijr}^{k\xi_l*}, & \text{if } \lambda_{ijr}^{Q\xi_l*} > 0, \\ \geq \sum_{k=1}^K Q_{ijr}^{k\xi_l*}, & \text{if } \lambda_{ijr}^{Q\xi_l*} = 0. \end{cases} \tag{8}$$

According to the equilibrium conditions (6) through (8): for a given war scenario ξ_l; $l = 1, \ldots, \omega$, if there is a positive shipment of a commodity on a route between a pair of supply and demand markets, and the route is not at its capacity, and the production at the country supply market is not at its capacity, then the supply price of the commodity at the supply market plus the unit transportation cost associated with transporting the commodity on the route multiplied by the exchange rate is equal to the effective demand price of the commodity at the demand market. If the route is at its capacity, and/or the supply is at its capacity at the supply market, and the shipment of the commodity on a route is positive, then the effective demand price of the commodity at the demand market is greater than or equal to the sum of commodity supply price and its unit transportation cost on the route multiplied by the exchange rate, with the sum of the corresponding Lagrange multipliers equal to the nonnegative difference. If the flow of a commodity is zero on a route, then the effective demand market price of the commodity is less than or equal to the supply market price plus the unit transportation cost on the route times the exchange rate plus the Lagrange multipliers.

The equilibrium conditions (6) through (8) expand the classical spatial price equilibrium conditions of Samuelson (1952) and Takayama and Judge (1971) to include commodity losses, limited production and transportation capacities, exchange rates plus functions, and all of these are for different scenarios. Furthermore, the underlying supply price, demand price, and unit route transportation cost functions in our model can be nonlinear and asymmetric, and the unit transportation cost functions are not fixed but are flow-dependent, which is important for capturing congestion. We assume that all the functions are continuous.

Theorem 1: Variational Inequality Formulation of the International Trade Network Equilibrium Conditions Under Capacity Reductions and Commodity Losses

A multicommodity shipment and Lagrange multiplier pattern $(Q^{\xi_l *}, \lambda^{s\xi_l *}, \lambda^{Q\xi_l *}) \in \mathcal{K}^{\xi_l}$ for each ξ_l; $l = 1, \ldots, \omega$, is an international trade network equilibrium under capacity disruptions and commodity losses, according to Definition 1, if and only if it satisfies the variational inequality:

$$\sum_{k=1}^{K}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{n_{ij}}\Big[(\tilde{\pi}_i^{k\xi_l}(Q^{\xi_l *}) + c_{ijr}^{k\xi_l}(Q^{\xi_l *}))e_{ij}^{\xi_l} + \lambda_i^{s\xi_l *} + \lambda_{ijr}^{Q\xi_l *} - \alpha_{ijr}^{\xi_l}\tilde{\rho}_j^{k\xi_l}(Q^{\xi_l *})\Big] \times (Q_{ijr}^{k\xi_l} - Q_{ijr}^{k\xi_l *})$$

$$+\sum_{i=1}^{m}\Big[u_i^{s\xi_l} - \sum_{k=1}^{K}\sum_{j=1}^{n}\sum_{r=1}^{n_{ij}} Q_{ijr}^{k\xi_l *}\Big]\times(\lambda_i^{s\xi_l} - \lambda_i^{s\xi_l *}) + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{n_{ij}}\Big[u_{ijr}^{Q\xi_l} - \sum_{k=1}^{K} Q_{ijr}^{k\xi_l *}\Big]\times(\lambda_{ijr}^{Q\xi_l} - \lambda_{ijr}^{Q\xi_l *}) \geq 0,$$

$$\forall (Q^{\xi_l}, \lambda^{s\xi_l}, \lambda^{Q\xi_l}) \in \mathcal{K}^{\xi_l}. \tag{9}$$

Proof: Necessity is, first, established; that is, if $(Q^{\xi_l *}, \lambda^{s\xi_l *}, \lambda^{Q\xi_l *}) \in \mathcal{K}^{\xi_l}$, for $l = 1, \ldots, \omega$, satisfies equilibrium conditions (6) through (8), then it also satisfies variational inequality (9). From the equilibrium conditions, for an equilibrium commodity shipment and Lagrange multiplier pattern, and for fixed k, i, j, r, we have that:

$$\Big[(\tilde{\pi}_i^{k\xi_l}(Q^{\xi_l *}) + c_{ijr}^{k\xi_l}(Q^{\xi_l *}))e_{ij}^{\xi_l} + \lambda_i^{s\xi_l *} + \lambda_{ijr}^{Q\xi_l *} - \alpha_{ijr}^{\xi_l}\tilde{\rho}_j^{k\xi_l}(Q^{\xi_l *})\Big] \times (Q_{ijr}^{k\xi_l} - Q_{ijr}^{k\xi_l *}) \geq 0, \forall Q_{ijr}^{k\xi_l} \geq 0, \tag{10}$$

since if $Q_{ijr}^{k\xi_l*} > 0$, then the left-hand side in (10) is zero, so (10) holds. Since $Q_{ijr}^{k\xi_l} \geq Q_{ijr}^{k\xi_l*}$, if $Q_{ijr}^{k\xi_l*} = 0$, then the left-hand side expression is nonnegative, and (10) also holds. Since (10) is true for any k, i, j, r, its summation over all these indices results in:

$$\sum_{k=1}^{K}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{n_{ij}}\left[(\tilde{\pi}_i^{k\xi_l}(Q^{\xi_l*})+c_{ijr}^{k\xi_l}(Q^{\xi_l*}))e_{ij}^{\xi_l}+\lambda_i^{s\xi_l*}+\lambda_{ijr}^{Q\xi_l*}-\alpha_{ijr}^{\xi_l}\tilde{\rho}_j^{k\xi_l}(Q^{\xi_l*})\right]\times(Q_{ijr}^{k\xi_l}-Q_{ijr}^{k\xi_l*}) \geq 0,$$

$$\forall Q^{\xi_l} \in R_+^{KP}. \tag{11}$$

In addition, from equilibrium conditions (7), for a fixed i, we know that:

$$\left[u_i^{s\xi_l} - \sum_{k=1}^{K}\sum_{j=1}^{n}\sum_{r=1}^{n_{ij}} Q_{ijr}^{k\xi_l*}\right] \times (\lambda_i^{s\xi_l} - \lambda_i^{s\xi_l*}) \geq 0, \quad \forall \lambda_i^{s\xi_l} \geq 0. \tag{12}$$

Summing (12) over all indices i, yields:

$$\sum_{i=1}^{m}\left[u_i^{s\xi_l} - \sum_{k=1}^{K}\sum_{j=1}^{n}\sum_{r=1}^{n_{ij}} Q_{ijr}^{k\xi_l*}\right] \times (\lambda_i^{s\xi_l} - \lambda_i^{s\xi_l*}) \geq 0, \quad \forall \lambda^{s\xi_l} \in R_+^m. \tag{13}$$

Also, from equilibrium conditions (8), for fixed i, j, r, it follows that:

$$\left[u_{ijr}^{Q\xi_l} - \sum_{k=1}^{K} Q_{ijr}^{k\xi_l*}\right] \times (\lambda_{ijr}^{Q\xi_l} - \lambda_{ijr}^{Q\xi_l*}) \geq 0, \quad \forall \lambda_{ijr}^{Q\xi_l} \geq 0. \tag{14}$$

Summing (14) over all indices i, j, r, results in:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{n_{ij}}\left[u_{ijr}^{Q\xi_l} - \sum_{k=1}^{K} Q_{ijr}^{k\xi_l*}\right] \times (\lambda_{ijr}^{Q\xi_l} - \lambda_{ijr}^{Q\xi_l*}) \geq 0, \quad \forall \lambda^{Q\xi_l} \in R_+^P. \tag{15}$$

Adding (11), (13), and (15), gives us variational inequality (9). Necessity has been established.

Sufficiency is now established. Setting $\lambda_i^{s\xi_l} = \lambda_i^{s\xi_l*}$ for all i; $\lambda_{ijr}^{Q\xi_l} = \lambda_{ijr}^{Q\xi_l*}$ for all i, j, r, and $Q_{ijr}^{k\xi_l} = Q_{ijr}^{k\xi_l*}$ for all k, i, j, r, except for $k = \tilde{k}$, $i = \tilde{i}$, $j = \tilde{j}$, and $r = \tilde{r}$, and substituting thsee resultants into (9), reduces the variational inequality (9) to:

$$\left[(\tilde{\pi}_{\tilde{i}}^{\tilde{k}\xi_l}(Q^{\xi_l*})+c_{\tilde{i}\tilde{j}\tilde{r}}^{\tilde{k}\xi_l}(Q^{\xi_l*}))e_{\tilde{i}\tilde{j}}^{\xi_l}+\lambda_{\tilde{i}}^{s\xi_l*}+\lambda_{\tilde{i}\tilde{j}\tilde{r}}^{Q\xi_l*}-\alpha_{\tilde{i}\tilde{j}\tilde{r}}^{\xi_l}\tilde{\rho}_{\tilde{j}}^{\tilde{k}\xi_l}(Q^{\xi_l*})\right]\times(Q_{\tilde{i}\tilde{j}\tilde{r}}^{\tilde{k}\xi_l}-Q_{\tilde{i}\tilde{j}\tilde{r}}^{\tilde{k}\xi_l*}) \geq 0, \forall Q_{\tilde{i}\tilde{j}\tilde{r}}^{\tilde{k}\xi_l} \geq 0, \tag{16}$$

from which it follows that the equilibrium conditions (6) hold.

Now, setting $Q_{ijr}^{k\xi_l} = Q_{ijr}^{k\xi_l *}$ for all k, i, j, r; $\lambda_{ijr}^{Q\xi_l} = \lambda_{ijr}^{Q\xi_l *}$ for all i, j, r, and $\lambda_i^{s\xi_l} = \lambda_i^{s\xi_l *}$ for all i, except for $i = \tilde{i}$, and substituting the resultants into (9), reduces the variational inequality (9) to:

$$\left[u_{\tilde{i}}^{s\xi_l} - \sum_{k=1}^{K} \sum_{j=1}^{n} \sum_{r=1}^{n_{ij}} Q_{\tilde{i}jr}^{k\xi_l *} \right] \times (\lambda_{\tilde{i}}^{s\xi_l} - \lambda_{\tilde{i}}^{s\xi_l *}) \geq 0, \quad \forall \lambda_{\tilde{i}}^{s\xi_l} \geq 0, \tag{17}$$

from which it follows that the equilibrium conditions (7) must hold.

And, setting $Q_{ijr}^{k\xi_l} = Q_{ijr}^{k\xi_l *}$ for all k, i, j, r; $\lambda_i^{s\xi_l} = \lambda_i^{s\xi_l *}$ for all i, and $\lambda_{ijr}^{Q\xi_l} = \lambda_{ijr}^{Q\xi_l *}$ for all i, j, r, except for $i = \tilde{i}$, $j = \tilde{j}$, and $r = \tilde{r}$, and substituting the resultants into (9), reduces the variational inequality (9) to:

$$\left[u_{\tilde{i}\tilde{j}\tilde{r}}^{Q\xi_l} - \sum_{k=1}^{K} Q_{\tilde{i}\tilde{j}\tilde{r}}^{k\xi_l *} \right] \times (\lambda_{\tilde{i}\tilde{j}\tilde{r}}^{Q\xi_l} - \lambda_{\tilde{i}\tilde{j}\tilde{r}}^{\xi_l *}) \geq 0, \quad \forall \lambda_{\tilde{i}\tilde{j}\tilde{r}}^{Q\xi_l} \geq 0, \tag{18}$$

from which it follows that the equilibrium conditions (8) must hold.

The above results hold for all ξ_l; $l = 1, \dots, \omega$.

□

Remark 1

Note that both Definition 1 and variational inequality (9) hold for the baseline case of ξ_0 by setting $l = 0$ throughout the statement of Definition 1 and also the statement of Theorem 1 and its proof. We will need to solve the baseline scenario variational inequality (VI) as well as the VIs for the war scenarios $\xi_1, \dots, \xi_\omega$ to construct the integrated crop and cargo war insurance for each commodity, which we provide the formulae for in the next Section.

4. Calculation of the Integrated Crop and Cargo War Insurance Premiums

In this Section, we construct a formula for the war insurance premium which integrates crop and cargo insurance and is for a given supply market and commodity. The formula focuses on the expected losses to farmers in terms of the supply price drop under the war scenarios. We employ the Net Premium Principle (cf. Bowers et al. (1997)) because of the discrete scenarios with associated probabilities, which enables us to compute the expected economic loss in terms of supply prices of the commodities in the absence of extensive historical claims data typical of traditional insurance settings. By defining a finite set of war scenarios ξ_l; $l = 1, \dots, \omega$, with probabilities p_{ξ_l}, as discussed in Section 3, we calculate the premium as the expected supply price drop across these scenarios. This approach simplifies the application of insurance principles to an agricultural supply chain framework under wartime conditions, offering a practical approach to calculate the insurance premia.

Definition 2: Integrated Crop and Cargo War Insurance Premiums

The insurance premium IP_i^k for commodity k; $k = 1,\ldots,K$ and supply market i; $i = 1,\ldots,m$ is given by the following formula:

$$IP_i^k \equiv \sum_{l=1}^{\omega} \left[\tilde{\pi}_i^{k\xi_0}(Q^{\xi_0 *}) - \tilde{\pi}_i^{k\xi_l}(Q^{\xi_l *}) \right] \times p_{\xi_l}. \tag{19}$$

According to (19), each expression in the summand is the difference between the first supply price in the expression for the commodity and supply market, which is evaluated at the equilibrium solution for the baseline scenario ξ_0, and the second supply price for the commodity and supply market, which is evaluated at the war scenario ξ_l equilibrium solution, multiplied by the probability of the war scenario ξ_l. Formula (19) captures the expected loss per (metric) ton of each commodity, in terms of the supply price for the commodity at the supply market under the war scenarios.

We now let σ_i^k denote the fraction of the premium for supply market i; $i = 1,\ldots,m$ and commodity k; $k = 1,\ldots,K$ that will be covered by an authority, which, in many cases, we expect to be the government. Each parameter σ_i^k, hence, lies in the range of 0 through 1. Note that the supply markets may be in different countries, so the respective governments would have to be responsible if they wish to reduce the burden on farmers and pay, fully, or, in part, their insurance premiums.

The premiums are paid before the war scenarios take place and, therefore, the data for the baseline scenario in terms of upper bounds and commodity losses are relevant. We now present the definition of the equilibrium under the war insurance premiums.

Definition 3: The International Trade Network Equilibrium Conditions Under the War Insurance Premiums

A multicommodity shipment and Lagrange multiplier pattern $(Q^{\xi_0 **}, \lambda^{s\xi_0 **}, \lambda^{Q\xi_0 **}) \in \mathcal{K}^{\xi_0}$, where

$$\mathcal{K}^{\xi_0} \equiv \{(Q^{\xi_0}, \lambda^{s\xi_0}, \lambda^{Q\xi_0}) | (Q^{\xi_0}, \lambda^{s\xi_0}, \lambda^{Q\xi_0}) \in R_+^{KP+m+P}\}$$

is a multicommodity international trade network equilibrium under the war insurance premiums, if the following conditions hold: for all commodities k; $k = 1,\ldots,K$; for all supply and demand market pairs: (i,j); $i = 1,\ldots,m$; $j = 1,\ldots,n$, and for all routes r; $r = 1,\ldots,n_{ij}$:

$$(\tilde{\pi}_i^{k\xi_0}(Q^{\xi_0 **}) + c_{ijr}^{k\xi_0}(Q^{\xi_0 **}))e_{ij}^{\xi_0} + IP_i^k(1-\sigma_i^k) + \lambda_i^{s\xi_0 **} + \lambda_{ijr}^{Q\xi_0 **} \begin{cases} = \alpha_{ijr}^{\xi_0} \tilde{\rho}_j^{k\xi_0}(Q^{\xi_0 **}), & \text{if } Q_{ijr}^{k\xi_0 **} > 0, \\ \geq \alpha_{ijr}^{\xi_0} \tilde{\rho}_j^{k\xi_0}(Q^{\xi_0 **}), & \text{if } Q_{ijr}^{k\xi_0 **} = 0; \end{cases} \tag{20}$$

for all commodities k; $k = 1, \ldots, K$, and for all supply markets i; $i = 1, \ldots, m$:

$$u_i^{s\xi_0} \begin{cases} = \sum_{k=1}^{K} \sum_{j=1}^{n} \sum_{r=1}^{n_{ij}} Q_{ijr}^{k\xi_0**}, & \textit{if } \lambda_i^{s\xi_0**} > 0, \\ \geq \sum_{k=1}^{K} \sum_{j=1}^{n} \sum_{r=1}^{n_{ij}} Q_{ijr}^{k\xi_0**}, & \textit{if } \lambda_i^{s\xi_0**} = 0; \end{cases} \tag{21}$$

for all commodities k; $k = 1, \ldots, K$, and for all supply and demand markets (i, j); $i = 1, \ldots, m$; $j = 1, \ldots, n$, and for all routes r; $r = 1, \ldots, n_{ij}$:

$$u_{ijr}^{Q\xi_0} \begin{cases} = \sum_{k=1}^{K} Q_{ijr}^{k\xi_0**}, & \textit{if } \lambda_{ijr}^{Q\xi_0**} > 0, \\ \geq \sum_{k=1}^{K} Q_{ijr}^{k\xi_0**}, & \textit{if } \lambda_{ijr}^{Q\xi_0**} = 0. \end{cases} \tag{22}$$

Remark 2

Note that, if all the σ_i^k; $k = 1, \ldots, K$; $i = 1, \ldots, m$ are equal to 1; that is, all the insurance premiums are paid for by an outside authority then the equilibrium conditions and solution in Definition 2 collapse to the corresponding ones in Definition 1 for ξ_0.

The below result follows using similar arguments to those in the proof of Theorem 1.

Theorem 2: Variational Inequality Formulation of the International Trade Network Equilibrium Conditions Under the War Insurance Premiums

A multicommodity shipment and Lagrange multiplier pattern $(Q^{\xi_0**}, \lambda^{s\xi_0**}, \lambda^{Q\xi_0**}) \in \mathcal{K}^{\xi_0}$ is an international trade network equilibrium under war insurance premiums, according to Definition 2, if and only if it satisfies the variational inequality:

$$\sum_{k=1}^{K} \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{r=1}^{n_{ij}} \left[(\tilde{\pi}_i^{k\xi_0}(Q^{\xi_0**}) + c_{ijr}^{k\xi_0}(Q^{\xi_0**}) + IP_i^k(1 - \sigma_i^k) + \lambda_i^{s\xi_0**} + \lambda_{ijr}^{Q\xi_0**} - \alpha_{ijr}^{\xi_l} \tilde{\rho}_j^{k\xi_0}(Q^{\xi_0**}) \right]$$
$$\times (Q_{ijr}^{k\xi_0} - Q_{ijr}^{k\xi_0**})$$
$$+ \sum_{i=1}^{m} \left[u_i^{s\xi_0} - \sum_{k=1}^{K} \sum_{j=1}^{n} \sum_{r=1}^{n_{ij}} Q_{ijr}^{k\xi_0**} \right] \times (\lambda_i^{s\xi_0} - \lambda_i^{s\xi_0**}) + \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{r=1}^{n_{ij}} \left[u_{ijr}^{Q\xi_0} - \sum_{k=1}^{K} Q_{ijr}^{k\xi_0**} \right]$$
$$\times (\lambda_{ijr}^{Q\xi_0} - \lambda_{ijr}^{Q\xi_0**}) \geq 0, \quad \forall (Q^{\xi_0}, \lambda^{s\xi_0}, \lambda^{Q\xi_0}) \in \mathcal{K}^{\xi_0}. \tag{23}$$

4.1 An Illustrative Example and Sensitivity Analysis

In order to fix ideas, we now present an illustrative example. It consists of a single commodity, that is wheat, a single supply market, say, Ukraine, and a single demand market - that of Lebanon. There is one route connecting the supply market with the demand market which includes a maritime link on the Black Sea. For simplicity, the functions are in US dollars. The baseline ξ_0 nondisrupted scenario is prior to the full-scale invasion of February 24, 2022. The supply price function is:

$$\pi_1^{1\xi_0}(s^{\xi_0}) = .0002s_1^{1\xi_0} + 170.$$

The unit transportation cost on the route is:

$$c_{111}^{1\xi_0}(Q^{\xi_0}) = .0001Q_{111}^{1\xi_0} + 30$$

and the demand price function is:

$$\rho_1^{1\xi_0}(d^{\xi_0}) = -.0001d_1^{1\xi_0} + 400.$$

The upper bounds are:

$$u_1^{s\xi_0} = u_{111}^{Q\xi_0} = 1,000,000,$$

the route flow multiplier $\alpha_{111}^{\xi_0} = 1$, and the exchange rate $e_{11}^{\xi_0} = 1$. We assume that the supply price function, the transportation cost function, and the demand price function remain as in ξ_0 for scenarios: ξ_1 and ξ_2.

Solving variational inequality (9) for scenario ξ_0 yields:

$$s_1^{1\xi_0} = Q_{111}^{1\xi_0 *} = d_1^{1\xi_0} = 500,000$$

with both Lagrange multipliers being equal to 0 and with $\pi_1^1 = 270$, $c_{111}^1 = 80$, and $\rho_1^1 = 350$ at the equilibrium pattern with $\alpha_{111}^{\xi_0}\rho_1^1 = 350$.

We now consider two war scenarios: ξ_1 represents the scenario with low damage whereas ξ_2 represents the scenario with high damage. Accordingly, we have the following data for these scenarios:

$$u_1^{s\xi_1} = 500,000, \quad u_{111}^{Q\xi_1} = 500,000.$$
$$u_1^{s\xi_2} = 300,000, \quad u_{111}^{Q\xi_2} = 300,000,$$
$$\alpha_{111}^{\xi_1} = 0.9, \quad \alpha_{111}^{\xi_2} = 0.8,$$

and $p_{\xi_1} = 0.5, p_{\xi_2} = 0.5$.

The equilibrium solution for war scenario ξ_1 is:

$$s_1^{1\xi_1 *} = Q_{111}^{1\xi_1 *} = 419,947.51, \quad d_1^{1\xi_1 *} = 377,952.76$$

with both Lagrange multipliers being equal to 0 and with $\pi_1^1 = 253.99$, $c_{111}^1 = 71.99$, and $\rho_1^1 = 362.20$ evaluated at the equilibrium pattern and with $\alpha_{111}^{\xi_1}\rho_1^1 = 325.98$.

The equilibrium solution for war scenario ξ_2 is:

$$s_1^{1\xi_2*} = Q_{111}^{1\xi_2*} = 300,000.00, \quad d_1^{1\xi_2*} = 240,000.00$$

with supply Lagrange multiplier being equal to 10.8, the other Lagrange multiplier being equal to 0 and with $\pi_1^1 = 230.00$, $c_{111}^1 = 60.00$, and $\rho_1^1 = 376.00$ evaluated at the equilibrium pattern and with $\alpha_{111}^{\xi_2}\rho_1^1 = 300.8$. In scenario ξ_2, the upper bound capacities are binding.

The insurance premium IP_1^1, calculated using (19), is, therefore:

$$IP_1^1 = (270 - 253.99) \times 0.5 + (270 - 230) \times 0.5 = 28.005.$$

The insurance premium IP_1^1 is 28.005, and represents the expected loss in the supply price per metric ton of the commodity due to the uncertainty associated with production and transportation capacities and losses associated with transportation from the supply market to the demand market.

The results illustrate the impact of the war scenarios and insurance premium coverage on the equilibrium patterns in the trade network. The war scenarios ξ_1 and ξ_2 lead to reduced equilibrium flow quantities compared to the baseline scenario ξ_0. Specifically, in scenario ξ_1, the equilibrium supply quantity decreases from $500,000$ to $419,947.51$, and, in scenario ξ_2, it further decreases to $300,000$. Furthermore, due to commodity losses in transportation, the commodity flow that actually arrives at the demand market is 500,000 under scenario ξ_0 (when there are no losses), $377,952.76$ in scenario ξ_1, and only 240,000 in scenario ξ_2.

We now calculate the new equilibrium patterns with the insurance premium for $\sigma_1^1 = 0$, $\sigma_1^1 = .25$, $\sigma_1^1 = .5$, and $\sigma_1^1 = .75$ with notice that, if $\sigma_1^1 = 1$, then the equilibrium solution remains as in scenario ξ_0 above with $s_1^{1\xi_0*} = Q_{111}^{1\xi_0*} = d_1^{1\xi_0*} = 500,000$.

For $\sigma_1^1 = 0$, the equilibrium solution is:

$$s_1^{1\xi_0**} = Q_{111}^{1\xi_0**} = d_1^{1\xi_0**} = 429,986.88,$$

with $\pi_1^1 = 255.997$, $c_{111}^1 = 72.999$, $IP_1^1(1 - \sigma_1^1) = 28.005 \times (1 - 0) = 28.005$, $\rho_{111}^1 = 357.001$, and $\alpha_{111}^{\xi_0}\rho_{111}^1 = 357.001$.

For $\sigma_1^1 = 0.25$, the equilibrium solution is:

$$s_1^{1\xi_0**} = Q_{111}^{1\xi_0**} = d_1^{1\xi_0**} = 447,490.16,$$

with $\pi_1^1 = 259.498$, $c_{111}^1 = 74.749$, $IP_1^1(1 - \sigma_1^1) = 28.005 \times (1 - 0.25) = 21.004$, $\rho_1^1 = 355.251$, and $\alpha_{111}^{\xi_0}\rho_1^1 = 355.251$.

For $\sigma_1^1 = 0.5$, the equilibrium solution is:

$$s_1^{1\xi_0**} = Q_{111}^{1\xi_0**} = d_1^{1\xi_0**} = 464,993.44,$$

with $\pi_1^1 = 262.999$, $c_{111}^1 = 76.499$, $IP_1^1(1 - \sigma_1^1) = 28.005 \times (1 - 0.5) = 14.003$, $\rho_1^1 = 353.501$, and $\alpha_{111}^{\xi_0}\rho_1^1 = 353.501$.

And for $\sigma_1^1 = 0.75$, the equilibrium solution is:

$$s_1^{1\xi_0**} = Q_{111}^{1\xi_0**} = d_1^{1\xi_0**} = 482,496.72,$$

with $\pi_1^1 = 266.499$, $c_{111}^1 = 78.250$, $IP_1^1(1 - \sigma_1^1) = 28.005 \times (1 - 0.75) = 7.001$, $\rho_1^1 = 351.750$, and $\alpha_{111}^{\xi_0}\rho_1^1 = 351.750$.

As σ_1^1 increases from 0 to 0.75, the equilibrium quantity increases from 429,986.88 to 482,496.72. This indicates that, with greater coverage of the insurance premium by the government, the supply of the agricultural commodity increases. The supply price π_1^1 also increases as σ_1^1 increases, reflecting the reduction in the payment of the insurance premium by the farmers. At $\sigma_1^1 = 0$, with the farmers responsible for the full insurance premium, the supply price is 255.997, whereas at $\sigma_1^1 = 0.75$, the supply price increases to 266.499. For $\sigma_1^1 = 0$, the insurance premium IP_1^1 is 28.005, which is approximately 10.94% of the supply price; for $\sigma_1^1 = 0.25$, the effective premium paid by the farmers is reduced to 21.004, about 8.09% of the supply price; for $\sigma_1^1 = 0.5$, the effective premium is 14.003, about 5.32% of the supply price; and for $\sigma_1^1 = 0.75$, the effective premium is 7.001, about 2.63% of the supply price. These percentages show that, as a larger fraction of the insurance premium is covered, the farmers' share decreases significantly. This reduction in the effective insurance premium encourages higher commodity supply levels; thereby, decreasing the negative impact of the war scenarios on the agricultural supply chain network and, also, on food security, by providing a higher level of commodity flow.

The average crop insurance premium for wheat in Ukraine ranged from 2.0% to 4.9% between 2005 and 2017 (Chvertko et al. (2019)). This system of agricultural insurance was originally introduced with government support and subsidies, aimed at mitigating economic risks for agricultural producers. However, economic challenges, including limited funding and insufficient stakeholder cooperation, have hindered its development over time, leading to reduced adoption and inconsistent implementation. Over the past two decades, the proportion of insured agricultural land in Ukraine has consistently remained below 5% of the total sown area, demonstrating a significant gap in crop insurance coverage (Skydan et al. (2023)). The cargo insurance rate before Russia's invasion of Ukraine for transportation on the Black Sea was at the pre-war level of 1% of the cargo's value (Grigorenko (2024)). According to the World Trade Institute's report on Ukrainian grain shipments in 2022, there was a \$15 per metric ton insurance premium due to Ukraine being classified as a risk zone (Häberli and Kostetsky (2023)), which accounted for approximately 8% of the average wheat price of \$188.24 per metric ton in 2022 (USDA (2024)), while at the reported 2024 wheat price of \$238 per metric ton (UkrAgroConsult (2024)), the insurance share decreased to 6.3%. Our results, depending on the coverage level of the premium, range from 2.6% to 10.9%, with the case of $\sigma_1^1 = 0.25$ closely aligning with the reported figures.

5. Algorithmically Solved Numerical Examples

In this Section, we present insurance premiums results for numerical examples, all of which are solved using the modified projection method (see Korpelevich (1977)). The algorithm was implemented in FORTRAN on a Linux system at the University of Massachusetts Amherst. If the absolute value of the difference between each computed variable at two successive iterations is less than or equal to .01, the algorithm is deemed to have converged. The first set of examples assumes no commodity losses, whereas the second set does. The first set of examples is drawn from examples in Nagurney et al. (2024a) for the purposes of continuity. The examples consider the commodities of wheat (commodity 1) and corn (commodity 2) with the supply market in Ukraine (supply market 1) and the demand markets in Lebanon (demand market 1) and Egypt (demand market 2). As in Nagurney et al. (2024a), there are two routes from Ukraine to each of the country demand markets. The first route corresponds to the transportation through a Black Sea port in Ukraine, such as the port of Odesa, and the second route corresponds to the transportation of the commodities via barge, rail, or truck through the western borders of Ukraine to Romania, followed by transport from a Romanian port on the Black Sea, such as the port of Costanta.

5.1 Numerical Example Set 1 - Examples Without Commodity Losses in Transportation

We first provide the data for the baseline example, corresponding to scenario ξ_0. The example makes use of the situation prior to the full-scale invasion of Ukraine by Russia on February 24, 2022. We then provide the data for two wartime scenarios ξ_1 and ξ_2. All the input and output data are reported for each numerical example for completeness and easy reference. We follow with the calculation of the insurance premium for each commodity and provide sensitivity analysis results for government subsidies on the insurance premiums.

Example 1 - Scenario ξ_0

The exchange rates are from early January 2022:

$$e_{11}^{\xi_0} = 55.0581, \quad e_{12}^{\xi_0} = .5714,$$

$$USD/UAH = 27.4619, \quad USD/LBP = 1,512.0000, \quad USD/EGP = 15.7300,$$

with USD being the currency code for the US dollar, UAH being the currency code for Ukrainian hryvnia, LBP being that for the Lebanese pound, and EGP for the Egyptian pound.

The supply price functions in Ukrainian hryvnia for wheat and corn per metric ton are:

$$\pi_1^{1\xi_0}(s^{\xi_0}) = .000136s_1^{1\xi_0} + .000068s_1^{2\xi_0} + 7,001.60, \quad \pi_1^{2\xi_0}(s^{\xi_0}) = .000073s_1^{1\xi_0} + .000142s_1^{2\xi_0} + 6,728.20.$$

The unit transportation cost functions for wheat and corn per metric ton in Ukrainian hryvnia are:

$$c_{111}^{1\xi_0}(Q^{\xi_0}) = .000556Q_{111}^{1\xi_0} + 2,046.80, \; c_{112}^{1\xi_0}(Q^{\xi_0}) = .007512Q_{112}^{1\xi_0} + 10,984.60,$$

$$c_{121}^{1\xi_0}(Q^{\xi_0}) = .000185Q_{121}^{1\xi_0} + 2,046.80, \; c_{122}^{1\xi_0}(Q^{\xi_0}) = .007312Q_{122}^{1\xi_0} + 10,984.60,$$

$$c_{111}^{2\xi_0}(Q^{\xi_0}) = .005566Q_{111}^{2\xi_0} + 2,046.80, \; c_{112}^{2\xi_0}(Q^{\xi_0}) = .006812Q_{112}^{2\xi_0} + 10,984.60,$$

$$c_{121}^{2\xi_0}(Q^{\xi_0}) = .001259Q_{121}^{2\xi_0} + 2,046.80, \; c_{122}^{2\xi_0}(Q^{\xi_0}) = .007012Q_{122}^{2\xi_0} + 10,984.60.$$

The demand price functions for wheat and corn in the demand market local currencies per metric ton are:

$$\rho_1^{1\xi_0}(d^{\xi_0}) = -.15d_1^{1\xi_0} + 602,344.00, \quad \rho_1^{2\xi_0}(d^{\xi_0}) = -.68d_1^{2\xi_0} + 574,560.00,$$

$$\rho_2^{1\xi_0}(d^{\xi_0}) = -.000475d_2^{1\xi_0} + 6,290.00, \quad \rho_2^{2\xi_0}(d^{\xi_0}) = -.000758d_2^{2\xi_0} + 5,980.00.$$

The supply capacity, in metric tons, in Ukraine is: $u_1^{s\xi_0} = 5,000,000.00$.

The capacities, in metric tons, on the transportation routes are:

$$u_{111}^{Q\xi_0} = 5,000,000.00, \quad u_{112}^{Q\xi_0} = 500,000.00, \quad u_{121}^{Q\xi_0} = 5,000,000.00, \quad u_{122}^{Q\xi_0} = 500,000.00.$$

Justification for the functions and capacities can be found in Nagurney et al. (2024a), Example 6 - Pre-War Scenario, where a slightly different notation has been used.

The equilibrium commodity shipments and the incurred commodity supply market and demand market prices in US dollars are reported in Table 2. All the Lagrange multipliers at the equilibrium are equal to 0.00.

For completeness, we now also provide the equilibrium commodity supply market and demand market prices below in the local currencies. The incurred equilibrium supply market prices in Ukraine in hryvnia are:

$$\pi_1^{1\xi_0}(s^{\xi_0 *}) = 7,328.3252, \quad \pi_1^{2\xi_0}(s^{\xi_0 *}) = 6,971.0166,$$

which, as noted in Nagurney et al. (2024a), is quite reasonable, since prior to the full-scale invasion, Ukrainian farmers could earn about $270 per ton for wheat and corn, which is very close to the computed supply prices.

The incurred equilibrium demand market prices in Lebanon in Lebanese pounds are:

$$\rho_1^{1\xi_0}(d^{\xi_0 *}) = 530,781.1875, \quad \rho_1^{2\xi_0}(d^{\xi_0 *}) = 520,752.9063$$

and those in Egypt in Egyptian pounds are:

$$\rho_2^{1\xi_0}(d^{\xi_0 *}) = 5,527.3057, \quad \rho_2^{2\xi_0}(d^{\xi_0 *}) = 5,555.4214.$$

The demand market prices in Lebanon and Egypt are close to the pre-war prices.

Example 2 - Scenario ξ_1

This scenario considers a war situation, which actually occurred reflecting a full-scale invasion (as on February 24, 2022). As a consequence, given the importance of maritime shipping on the Black Sea in Ukraine, due to a blockade and mining of the maritime routes, the capacity of the two maritime routes in ξ_0 was reduced to 0.00 but the remainder of the data stays as in Example 1. This example is drawn from Example 7 in Nagurney et al. (2024a). The equilibrium commodity shipment pattern is reported in Table 2, along with the incurred equilibrium supply market and demand market prices in US dollars.

Below we provide the prices at the equilibrium in the local currencies.

All Lagrange multipliers are equal to 0.0000 with the exception that $\lambda_{12}^{2\xi_1 *} = 468.4277$.

The incurred equilibrium supply market prices in hryvnia in Ukraine are:

$$\pi_1^{1\xi_1}(s^{\xi_1 *}) = 7,099.0347, \quad \pi_1^{2\xi_1}(s^{\xi_1 *}) = 6,780.4995.$$

The incurred equilibrium demand market prices in Lebanon in Lebanese pounds are:

$$\rho_1^{1\xi_1}(d^{\xi_1 *}) = 569,879.0000, \quad \rho_1^{2\xi_1}(d^{\xi_1 *}) = 6,052.5005,$$

and those in Egypt in Egyptian pounds are:

$$\rho_2^{1\xi_1}(d^{\xi_1 *}) = 574,560.0000, \quad \rho_2^{2\xi_1}(d^{\xi_1 *}) = 5,980.0000.$$

Example 3 - Scenario ξ_2

This wartime scenario considers a reduction in production capacity because of bombs, mining of land, shortages of labor, but capacities of the original maritime routes as in scenario ξ_0. The supply market supply capacity is now: $u_1^{s\xi_2} = 1,000,000.00$. In addition, the supply price functions, the transportation cost functions are modified to reflect a worsening possible war scenario and the demand price functions are also modified to capture a worsening economic situation in the MENA countries. The exchange rates are also changed. This example is drawn from Example 8 in Nagurney et al. (2024a). The data for scenario ξ_2, for completeness, are given below.

The exchange rates are:

$$e_{11}^{\xi_2} = 41.3469, \quad e_{12}^{\xi_2} = .5236,$$

$$USD/UAH = 36.5686, \quad USD/LBP = 1,512.0000, \quad USD/EGP = 19.1500.$$

The supply price functions for wheat and corn per ton in Ukrainian hryvnia are now:

$$\pi_1^{1\xi_2}(s^{\xi_2}) = .000136 s_1^{1\xi_2} + .000068 s_1^{2\xi_2} + 3,364.60, \quad \pi_1^{2\xi_2}(s^{\xi_2}) = .000073 s_1^{1\xi_2} + .000142 s_1^{2\xi_2} + 4,022.50.$$

The unit transportation cost functions for wheat and corn per ton in Ukrainian hryvnia are now:

$$c_{111}^{1\xi_2}(Q^{\xi_2}) = .000556 Q_{111}^{1\xi_2} + 13,867.90, \quad c_{112}^{1\xi_2}(Q^{\xi_2}) = .007512 Q_{112}^{1\xi_2} + 15,591.10,$$

$$c_{121}^{1\xi_2}(Q^{\xi_2}) = .000185 Q_{121}^{1\xi_2} + 13,867.90, \quad c_{122}^{1\xi_2}(Q^{\xi_2}) = .007312 Q_{122}^{1\xi_2} + 15,591.10,$$

$$c_{111}^{2\xi_2}(Q^{\xi_2}) = .005566 Q_{111}^{2\xi_2} + 13,867.90, \quad c_{112}^{2\xi_2}(Q^{\xi_2}) = .006812 Q_{112}^{2\xi_2} + 15,591.10,$$

$$c_{121}^{2\xi_2}(Q^{\xi_2}) = .001259 Q_{121}^{2\xi_2} + 13,867.90, \quad c_{122}^{2\xi_2}(Q^{\xi_2}) = .007012 Q_{122}^{2\xi_2} + 15,591.10.$$

The demand price functions for wheat and corn per ton in local currencies are:

$$\rho_1^{1\xi_2}(d^{\xi_2}) = -.15 d_1^{1\xi_2} + 796,162.50, \quad \rho_1^{2\xi_2}(d^{\xi_2}) = -.68 d_1^{2\xi_2} + 718,256.40,$$

$$\rho_2^{1\xi_2}(d^{\xi_2}) = -.000475 d_2^{1\xi_2} + 10,000.60, \quad \rho_2^{2\xi_2}(d^{\xi_2}) = -.000758 d_2^{2\xi_2} + 9,900.50.$$

The equilibrium commodity shipment pattern is given in Table 2. All equilibrium Lagrange multipliers are equal to 0.0000, except that $\lambda_1^{s\xi_2 *} = 591.6817$.

The incurred supply prices in Ukraine in hryvnia at the equilibrium are:

$$\pi_1^{1\xi_2}(s^{\xi_2 *}) = 3,500.6001, \quad \pi_1^{2\xi_2}(s^{\xi_2 *}) = 4,095.5000.$$

The incurred demand prices at the equilibrium in Lebanon in Lebanese pounds are:

$$\rho_1^{1\xi_2}(d^{\xi_2 *}) = 729,014.8125, \quad \rho_1^{2\xi_2}(d^{\xi_2 *}) = 718,256.3750,$$

whereas the demand prices in Egypt in Egyptian pounds are:

$$\rho_2^{1\xi_2}(d^{\xi_2 *}) = 9,738.2344, \quad \rho_2^{2\xi_2}(d^{\xi_2 *}) = 9,900.5000.$$

Table 2: Equilibrium Commodity Shipments for Numerical Examples in Set 1

	Scenario		
Equilibrium Commodity Flows	ξ_0	ξ_1	ξ_2
$Q_{111}^{1\xi_l *}$	477,085.5938	–	477,651.1563
$Q_{112}^{1\xi_l *}$	0.0000	216,433.1406	0.0000
$Q_{121}^{1\xi_l *}$	1,605,672.50000	–	552,348.4375
$Q_{122}^{1\xi_l *}$	0.0000	500,000.00	0.0000
$Q_{111}^{2\xi_l *}$	79,128.0781	–	0.0000
$Q_{112}^{2\xi_l *}$	0.0000	0.0000	0.0000
$Q_{121}^{2\xi_l *}$	560,130.3750	–	0.0000
$Q_{122}^{2\xi_l *}$	0.0000	0.0000	0.0000
Equilibrium Supply Prices in USD	ξ_0	ξ_1	ξ_2
$\pi_1^{1\xi_l}(s^{\xi_l *})$	266.8542	258.5048	95.7269
$\pi_1^{2\xi_l}(s^{\xi_l *})$	253.8432	246.9056	111.9949
Equilibrium Demand Prices in USD	ξ_0	ξ_1	ξ_2
$\rho_1^{1\xi_l}(d^{\xi_l *})$	351.0457	376.9041	482.1526
$\rho_2^{1\xi_l}(d^{\xi_l *})$	351.3862	380.0000	508.5239
$\rho_1^{2\xi_l}(d^{\xi_l *})$	344.4132	384.7743	475.0372
$\rho_2^{2\xi_l}(d^{\xi_l *})$	353.1436	380.1653	516.9973

Insurance Premiums

We assume that the probabilities associated with the two wartime scenarios are: $p_{\xi_1} = .5$ and $p_{\xi_2} = .5$. We now calculate the insurance premiums for the two commodities, using the supply market prices for the different scenarios in US dollars, as follows:

$$IP_1^1 = [266.8542 - 258.5048] \times .5 + [266.8542 - 95.7269] \times .5 = 89.7384$$

$$IP_1^2 = [253.8432 - 246.9056] \times .5 + [253.8432 - 111.9949] \times .5 = 74.3930.$$

Of course, one can also do the calculations in the Ukrainian hryvnia currency, which would result in: $IP_1^1 = 2,464.43$ and $IP_1^2 = 2,042.89$.

Interestingly, we find that, without any government subsidies (cf. VI (23) with $\sigma_1^1 = 0$ and $\sigma_1^2 = 0$), the above insurance premiums would result in all commodity shipments being equal to 0.0000.

If the government, on the other hand, subsidizes the integrated insurance premiums for both commodities of wheat and corn at 50%, then the following equilibrium trade flow pattern is obtained:

$$Q_{111}^{1\xi_0**} = 164,251.4531,\ Q_{112}^{1\xi_0**} = 0.0000,\ Q_{121}^{1\xi_0**} = 596,442.7500,\ Q_{122}^{1\xi_0**} = 0.0000,$$

$$Q_{111}^{2\xi_0**} = 30,557.1270,\ Q_{112}^{2\xi_0**} = 0.0000,\ Q_{121}^{2\xi_0**} = 223,400.5625,\ Q_{122}^{2\xi_0**} = 0.0000.$$

The incurred equilibrium supply market prices in hryvnia and US dollars are then:

$$\pi_1^{1\xi_0}(s^{\xi_0**}) = 7,122.3239 = \$259.3529, \quad \pi_1^{2\xi_0}(s^{\xi_0**}) = 6,819.793 = \$248.3365.$$

Plus, if the government subsidizes at 75% so that $\sigma_1^1 = \sigma_2^2 = .75$, then the new equilibrium commodity trade flow pattern is:

$$Q_{111}^{1\xi_0**} = 320,679.4063,\ Q_{112}^{1\xi_0**} = 0.0000,\ Q_{121}^{1\xi_0**} = 1,100,821.7500,\ Q_{122}^{1\xi_0**} = 0.0000,$$

$$Q_{111}^{2\xi_0**} = 54,843.8047,\ Q_{112}^{2\xi_0**} = 0.0000,\ Q_{121}^{2\xi_0**} = 391,747.4375,\ Q_{122}^{2\xi_0**} = 0.0000.$$

The incurred equilibrium supply market prices in hryvnia and US dollars are now:

$$\pi_1^{1\xi_0}(s^{\xi_0**}) = 7,235.2927 = \$263.4666, \quad \pi_1^{2\xi_0}(s^{\xi_0**}) = 6,895.3857 = \$251.0892.$$

A higher subsidization rate for the insurance premiums results in higher commodity supply market prices for wheat and corn for the farmers and brings the supply market prices closer to those obtained pre-war as in scenario ξ_0. In addition, the volumes of the commodity trade flows increase, which is of benefit to those in the demand markets. The commodities of wheat and corn are very important for food security.

In Numerical Example Set 1, with 50% subsidization, wheat shipments reach 760,694 metric tons, including 164,251 metric tons to Lebanon and 596,443 metric tons to Egypt. Corn shipments total 253,958 metric tons, with 30,557 metric tons to Lebanon and 223,401 metric tons to Egypt. These shipments generate revenues for the farmers of $197.29 million for wheat and $63.07 million for corn. At 75% subsidization, wheat shipments grow to 1,421,501 metric tons, with 320,679 metric tons to Lebanon and 1,100,822 metric tons to Egypt. Corn shipments increase to 446,591 metric tons, split into 54,844 metric tons to Lebanon and 391,747 metric tons to Egypt. Revenues for farmers also improve, reaching $374.52 million for wheat and $112.13 million for corn. This means that an additional 25% subsidy results in an increase of 660,807 metric tons in wheat shipments and 192,633 metric tons in corn shipments, translating into an additional $177.23 million in revenue for wheat and $49.06 million for corn. These results reinforce the findings of Hazell and Varangis (2020), confirming how subsidies directly benefit farmers and promote trade continuity in high risk environments. These results strongly suggest the importance of subsidization of insurance for farmers, as those in Ukraine, during wartime.

5.2 Numerical Example Set 2 - Examples With Commodity Losses in Transportation

We now consider scenarios with additional destruction in wartime in terms of commodity losses on the transportation routes. In particular, we retain the scenario ξ_0 representing the status in Ukraine prior to the full-scale invasion but now both wartime scenarios ξ_1 and ξ_2 have all the route multiplier α_{ijr} terms set to .9. These changes yield scenarios ξ_3 and ξ_4, respectively; that is, Example 4 corresponding to scenario ξ_3 has the same data as that in Example 2 representing scenario ξ_1 but with the commodity losses. Also, Example 5 has the same data as that in Example 3 corresponding to scenario ξ_2 but with the added commodity losses on the routes.

The computed equilibrium commodity trade flows and incurred prices in US dollars are reported in Table 3.

Insurance Premiums

We assume that the probabilities of scenarios ξ_3 and ξ_4 are each equal to .5. We, as in the calculation of the previous premiums, assume the baseline scenario of ξ_0. Using the

Table 3: Equilibrium Commodity Shipments for Numerical Examples in Set 2

	Scenario	
Equilibrium Commodity Flows	ξ_3	ξ_4
$Q_{111}^{1\xi_l*}$	–	26,877.5488
$Q_{112}^{1\xi_l*}$	93,835.6094	0.0000
$Q_{121}^{1\xi_l*}$	–	0.0000
$Q_{122}^{1\xi_l*}$	408,930.5938	0.0000
$Q_{111}^{2\xi_l*}$	–	0.0000
$Q_{112}^{2\xi_l*}$	0.0000	0.0000
$Q_{121}^{2\xi_l*}$	–	0.0000
$Q_{122}^{2\xi_l*}$	0.0000	0.0000
Equilibrium Supply Prices in USD	ξ_3	ξ_4
$\pi_1^{1\xi_l}(s^{\xi_l*})$	257.4467	92.1079
$\pi_1^{2\xi_l}(s^{\xi_l*})$	246.3377	110.0523
Equilibrium Demand Prices in USD	ξ_3	ξ_4
$\rho_1^{1\xi_l}(d^{\xi_l*})$	389.9975	524.1627
$\rho_2^{1\xi_l}(d^{\xi_l*})$	388.7592	522.2245
$\rho_1^{2\xi_l}(d^{\xi_l*})$	380.0000	475.0373
$\rho_2^{2\xi_l}(d^{\xi_l*})$	380.1653	516.9974

commodity supply market prices for wheat and corn in US dollars we have that:

$$IP_1^1 = [266.8542 - 257.4467] \times .5 + [266.8524 - 92.1079] \times .5 = 92.1210$$

$$IP_1^2 = [253.8432 - 246.3377] \times .5 + [253.8432 - 110.0523] \times .5 = 75.6482$$

or, equivalently, in hryvnia: 2,529.8170 and 2,077.4433, respectively.

As one would expect, these insurance premiums are higher than those calculated in Section 5.1, due to the commodity losses in transportation.

We now investigate the impacts of subsidization, as we did in Section 5.1. When there is no subsidization, then the new equilibrium commodity flow, once the above insurance premiums are imposed, are all equal to 0.0000, which is expected, given the higher insurance premiums.

If the government subsidizes the insurance premiums for both commodities at 50%, then the new equilibrium commodity trade flow pattern is:

$$Q_{111}^{1\xi_0**} = 155,808.5469,\ Q_{112}^{1\xi_0**} = 0.0000,\ Q_{121}^{1\xi_0**} = 569,190.2250,\ Q_{122}^{1\xi_0**} = 0.0000,$$

$$Q_{111}^{2\xi_0**} = 30,312.0859,\ Q_{112}^{2\xi_0**} = 0.0000,\ Q_{121}^{2\xi_0**} = 221,703.2188,\ Q_{122}^{2\xi_0**} = 0.0000.$$

The incurred equilibrium supply market prices in hryvnia in Ukraine and in US dollars are:

$$\pi_1^{1\xi_0}(s^{\xi_0**}) = 7,117.3367 = \$259.1713, \quad \pi_1^{2\xi_0}(s^{\xi_0**}) = 6,806.9112 = \$247.8675.$$

If the government subsidizes at 75%, the equilibrium commodity trade flow pattern is:

$$Q_{111}^{1\xi_0**} = 316,493.1250,\ Q_{112}^{1\xi_0**} = 0.0000,\ Q_{121}^{1\xi_0**} = 1,087,311.50000,\ Q_{122}^{1\xi_0**} = 0.0000,$$

$$Q_{111}^{2\xi_0**} = 54,458.0430,\ Q_{112}^{2\xi_0**} = 0.0000,\ Q_{121}^{2\xi_0**} = 389,074.3750,\ Q_{122}^{2\xi_0**} = 0.0000.$$

The incurred equilibrium supply market prices in hryvnia and US dollars are now:

$$\pi_1^{1\xi_0}(s^{\xi_0**}) = 7,222.6776 = \$263.0072, \quad \pi_1^{2\xi_0}(s^{\xi_0**}) = 6,893.6592 = \$251.0263.$$

We see that, in this set of numerical examples with explicit commodity losses, higher subsidization rates of the insurance premiums result in higher commodity supply market prices, which is of benefit to the farmers, and, also, in higher commodity flows, which is important from a food security standpoint. These results also reveal that, without subsidization, the insurance premiums in wartime may be so high that they, if paid exclusively by farmers, would result in no commodity production and, hence, trade. Note that the computed insurance premiums would be paid per metric ton of commodity.

In Numerical Example Set 2, at 50% subsidization, wheat shipments are 724,999 metric tons, including 155,809 metric tons to Lebanon and 569,190 metric tons to Egypt. Corn shipments total 252,015 metric tons, with 30,312 metric tons going to Lebanon and 221,703 metric tons to Egypt. Wheat revenues for farmers reach \$187.90 million, and corn revenues total \$62.47 million. At 75% subsidization, wheat shipments increase to 1,403,805 metric tons, with 316,493 metric tons going to Lebanon and 1,087,312 metric tons to Egypt. Corn shipments rise to 443,533 metric tons, with 54,458 metric tons going to Lebanon and 389,074 metric tons to Egypt. Revenues obtained by farmers for wheat and corn rise to \$369.21 million and \$111.34 million, respectively. Here, an additional 25% subsidy results in an increase of 678,806 metric tons in wheat shipments and 191,518 metric tons in corn shipments, adding \$181.31 million to wheat revenues and \$48.87 million to corn revenues. Combined revenue results for corn and wheat under subsidization levels of 50%, 75%, and 100% for Numerical Example Set 1 and Numerical Example Set 2 are shown in Figure 2.

These findings also align with Hazell and Varangis (2020), who argue that subsidies are critical for addressing market failures and ensuring food availability despite possible catastrophic losses. In the United States, as of 2016, farmers paid only 38% of their crop insurance

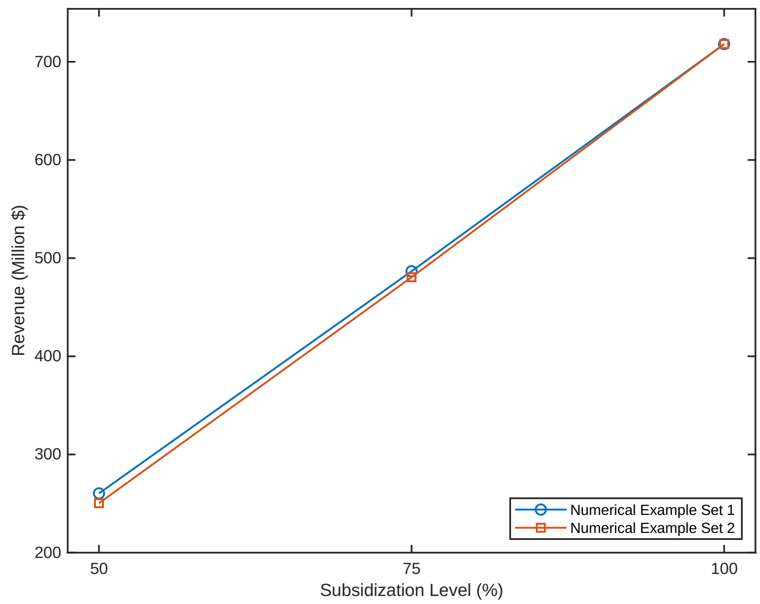

Figure 2: Combined Revenue for Corn and Wheat Under Subsidization Levels of 50%, 75%, and 100%

premiums, a significant reduction from 74% in the early 1990s, highlighting the importance of government subsidies in farming (Zulauf (2016)). However, Edwards (2023) cautions against federal farm subsidies in the US that could distort market behavior, such as inducing overproduction or inflating land values, and notes that US federal farm subsidies reached $35 billion in 2022, which shows the scale of government involvement to stabilize agricultural sectors. Due to the high costs of maintaining subsidies, the Government Accountability Office's analysis suggests that savings with minimal effects on the supply side are achievable by decreasing premium subsidy rates for high-income policyholders (GAO (2023)). This contrasting view emphasizes the need for a careful analysis to support the needs of farmers. It is important to recognize that the majority of farmers in Ukraine are small-scale farmers (see Mykhalchuk (2023)).

From a food security standpoint, subsidization helps ensure that more wheat and corn reach demand markets, reducing the chance of shortages. In Numerical Example Set 1, wheat shipments increase from 760,694 metric tons under 50% subsidization to 1,421,501 metric tons under 75% subsidization, a rise of 86.8%. For corn, shipments increase from 253,958 metric tons to 446,591 metric tons, a growth of 75.8%. Similarly, in Numerical Example Set 2, wheat shipments rise from 724,999 metric tons at 50% subsidization to 1,403,805 metric tons at 75% subsidization, an increase of 93.6%. Corn shipments grow from 252,015 metric tons to 443,533 metric tons, a rise of 76%. These increases highlight that, for both sets, an additional 25% subsidy results in approximately a doubling of the commodity shipment volumes, ensuring food availability at the demand markets during wartime. Hazell and Varangis (2020) emphasize the role of subsidies in overcoming market barriers and in ensuring agricultural trade stability despite high risk scenarios, which is also reflected in our findings.

6. Summary and Suggestions for Future Reseach

Wars such as the Russian invasion of Ukraine and the conflict around the Red Sea have greatly affected global agricultural supply chains, resulting in increased costs, uncertainty, and food insecurity. The challenges in affected countries are further exacerbated because of their dependence on staple food imports, such as those of wheat and corn. War risk insurance, which is different from regular insurance, has been gaining increasing attention in practice as a means to protect, among others, farmers in terms of agricultural commodity prices as well as revenues.

In this paper, we focus on the construction of integrated crop and cargo war risk insurance, which is inspired by Russia's war on Ukraine. We first develop a multicommodity international trade network equilibrium model with exchange rates on a general transportation network that incorporates capacities on production and transportation, and possible commodity losses in transportation. The model includes a baseline (pre-war) scenario as well as multiple war scenarios, with associated probabilities, and impacts on the exchange rates, the commodity supply market production capacities, the transportation route capacities, and losses in transportation represented by multipliers on the routes. Each scenario is represented by equilibrium conditions and an associated variational inequality formulation. Using the model as a foundational framework, we then construct integrated war risk insurance premiums that are specific for supply markets and commodities. These insurance premiums are then incorporated into a variational inequality model that also includes a subsidization rate for the insurance premiums, as by a government or governments, for example.

A series of numerical examples, both illustrative and algorithmically solved, are presented to demonstrate how our model can be applied. Focusing on the ongoing war in Ukraine, we considered scenarios with reduced production and transportation capacities and commodity losses to analyze the effects of disruptions under wartime scenarios on the flow of commodities to countries dependent on Ukrainian agricultural exports, such as the MENA countries. In Numerical Example Set 1, we assume wartime scenarios with reduced production and transportation capacities but without explicit commodity losses during transportation. In

Numerical Example Set 2, we consider more severe disruptions, with commodity losses during transportation. In both sets of algorithmically solved numerical examples, our results show that an additional 25% subsidy nearly doubles the revenues for farmers.

Our numerical examples and sensitivity analysis results have important implications for policymakers in the agricultural sector, for farmers, and also for consumers at the demand markets. The results highlight the critical role of integrated war risk insurance and government subsidies of the premiums in sustaining agricultural commodity trade flows and supply market prices during war situations. By subsidizing insurance premiums, governments can reduce the negative effects of war on supply market prices and help to maintain the flow of agricultural commodities, including of such important staples as wheat and corn, to demand markets, thus enhancing food security. Notably, higher subsidization of the war risk insurance premiums leads to increased agricultural commodity shipments and higher supply market prices for farmers as well as revenues. Without subsidies, insurance premiums can become so high that production and trade stop. These results align with the existing literature that emphasizes the importance of government subsidization in war conditions.

Specifically, the insurance premium $IP_i^k(1-\sigma_i^k)$ associated with commodity k at supply market i affects supply prices, demand prices, and transportation costs by altering equilibrium shipments and Lagrange multipliers. Our numerical examples, both illustrative ones and algorithmically solved ones, suggest that subsidies increase commodity shipments, supply prices, and transportation costs, while decreasing demand prices. Although these numerical example results suggest correlations of an inverse relation between insurance premium cost and supply prices, and a direct relation between insurance premium cost and demand prices, the variational inequality can be comprised of nonlinear functions, which, along with capacities and exchange rates, can make generalizations challenging across all scenarios. However, with the algorithmic framework computations are readily possible along with results.

We note that our modeling and computational framework is not limited to that of Ukraine. Policy mechanisms in other war-affected regions, such as Syria and Yemen, further illustrate the adaptability of subsidized insurance models. In Syria, the FAO and UK Aid's Building Local Resilience in Syria (BLRS) project targets 54,600 unique households with climate-smart, agriculture-oriented actions to boost production and improve food security (FAO (2023)), a framework that could be extended to subsidize crop insurance premiums amidst conflict. In Yemen, the European Commission is providing 125 million euros in humanitarian funding through UN agencies and NGOs to support food security and assistance for vulnerable communities (Stur (2024)), suggesting potential for insurance-linked support to mitigate losses from disrupted trade routes. These examples highlight the role of international organizations in funding and structuring assistance. Our model's focus on discrete scenarios could be applied in such settings with adjusting probabilities to local conflicts. Globally, government subsidies for agricultural insurance are common across high-income, upper middle-income, lower middle-income, and low-income countries, ranging from 0 to 80 percent for fixed premiums and up to 100 percent for variable premiums, supporting farmers against various risks (Mahul and Stutley (2010)). Such widespread government subsidizing shows the viability of our subsidy-based approach in conflict zones.

There are multiple promising possible extensions for future research. Conducting additional sensitivity analysis on probabilities of wartime scenarios and associated impacts on capacities as well as sensitivity analysis on exchange rates would be worthwhile. In addition, deriving theoretical results for various functional and parameter changes as well as for network topological changes would be interesting. Some early research on sensitivity analysis and spatial price equilibrium problems can be found in Nagurney (1983) and Dafermos and Nagurney (1984). A possible extension to the model would be to introduce multiple time periods, while retaining commodity losses, in order to include storage and capacity on storage under uncertainty, since storage facilities have also be targeted in Russia's war on Ukraine. Further disaggregating the insurance risk premiums to cover risk on specific transportation routes may be worthwhile. In addition, here the framework has been one of perfect competition. Introducing war risk insurance, along with subsidization, in an oligopoly with large-scale farmers, would also be quite interesting. Given that war risk insurance is a nascent topic, we expect the area to grow in interest because of its relevance to practice.

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