Quantifying Supply Chain Network Synergy for Humanitarian Organizations

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Abstract: The number of disasters is growing as well as the number of people affected, creating a great need for resilient disaster management. In this paper, we construct multiproduct supply chain network models for multiple humanitarian organizations. The models capture uncertainty associated with costs of their supply chain activities, including procurement, storage, and distribution, under multiple disaster scenarios, along with uncertainty associated with the demand for the disaster relief products at the demand points. The models reflect the organizations' operations, without and with cooperation, with the humanitarian organizations seeking to determine the disaster relief multiproduct flows that minimize their expected total cost and risk subject to expected demand satisfaction. We utilize a mean-variance approach to capture the risk associated with cost uncertainty and propose a synergy measure for the assessment of the potential strategic advantages of cooperation for resilient disaster management. We also identify the role of technology in helping to parameterize the models and illustrate the analytical framework with numerical examples, accompanied by managerial insights.

Key words: network synergy; horizontal cooperation; disaster relief; humanitarian logistics; humanitarian supply chain

1. Introduction

Climate change has made our societies more vulnerable to disasters. Moreover, with the ever increasing speed of urbanization, the impact of the disasters has been more severe, resulting in greater challenges for disaster management in all its phases of mitigation, preparedness, response, and recovery. For example, according to the Global Humanitarian Overview (United Nations (2019)), between 2014 and 2017, disasters due to natural hazards alone affected more than 870 million people per year in more than 160 countries and territories around the globe, resulting in loss of life, severely disrupting livelihoods, and displacing, annually, approximately 20 million people from their homes. 2017 was the costliest year in terms of natural (weather and climate-related) disasters in the United States, with the National Oceanographic and Atmospheric Administration estimating the financial cost at $306 billion, beating the previous record, set in 2005, with a cost of $205 billion due to such disasters (cf. Miller (2018)). The need for humanitarian assistance arises in both sudden-onset as well as slow-onset disasters, with the United Nations estimating that, in 2019, nearly 132 million people will need humanitarian assistance, many of those because of conflict. Given the severity and urgent nature of disaster response, humanitarian relief organizations are under increasing pressure to become more effective and cost-efficient (United Nations (2019)).

Whether disasters are sudden-onset, such as earthquakes, hurricanes, floods, etc., or slow-onset, such as droughts, famines, protracted conflicts, etc., they severely impact certain geographical areas and humanitarian organizations need to work together to enhance the response. However, as indicated in an ALNAP report regarding such organizations, "...coordination and collaboration among them are often limited at best. Failure to work together can lead to gaps in coverage and to duplications and inefficiencies in any given emergency response" (cf. page 5 of Saavedra and Knox-Clarke (2015)).

Recently, there has been growing impetus to explore the benefits of cooperation among humanitarian organizations in a quantifiable manner, with a goal including that of resiliency in highly uncertain and complex environments. Opportunities for cooperation among humanitarian organizations may exist along many different links in their supply chains from procurement to storage as well as transportation and distribution using, for example, shared freight services, and it is believed that cooperation may improve disaster preparedness and response (cf. Van Wassenhove (2006), Nagurney and Qiang (2009), Balcik et al. (2010), among others) as well as reduce materiel convergence (cf. Nagurney, Alvarez Flores, and Soylu (2016)). Nevertheless, as noted by Schulz and Blecken (2010), research on horizontal cooperation in the framework of disaster relief is only at the very early stages.

Given the above discussion, we propose models to study the cooperative synergies related to the multiproduct supply chains of multiple humanitarian organizations. In particular, these models capture the uncertainties associated with costs and demands. A mean-variance approach is used to capture risk associated with the uncertainties and we propose a synergy measure for the assessment of the potential strategic advantages of cooperation among humanitarian organizations for disaster management.

This paper is organized as follows. The literature review is conducted in Section 2. The pre-cooperation multiproduct humanitarian supply chain network model is developed in Section 3, followed by the construction of the cooperation multiproduct humanitarian supply chain model. The method of quantification of the synergistic gains, if any, is provided in Section 4, along with theoretical results. In Section 5 we present the algorithm, which we apply in Section 6 to compute solutions to numerical examples. The solutions not only illustrate the richness of the framework proposed in this paper, but also demonstrate quantitatively how various underlying model parameters associated with horizontal cooperation affect the possible synergies. We conclude the paper with Section 7, in which we summarize the results and present suggestions for future research.

2. Literature Review

Our perspective for the identification of potential synergy associated with horizontal cooperation between/among humanitarian organizations in disaster relief utilizes a supply chain network perspective in its full generality/complexity with associated activities of procurement, transportation, storage, and distribution. In the commercial space, the assessment of potential synergy associated with horizontal integration using a supply chain perspective, notably, in terms of mergers and acquisitions (M&As), is an important topic. Xu (2007) considered two firms and linear models. Nagurney (2009) also considered two firms, but proposed a nonlinear system-optimization approach, which was then adapted by Nagurney and Woolley (2010) to include not only costs but also environmental concerns in the form of emissions. Nagurney, Woolley, and Qiang (2010) also considered two cost-minimizing firms, but with the inclusion of multiple products, and assessed the synergy associated with a merger or an acquisition. Nagurney (2010), in turn, proposed a supply chain network perspective to evaluate the potential synergy associated with mergers/acquisitions among multiple profit-maximizing firms, but with a single product. Liu and Nagurney (2011) considered two firms engaged in a potential M&A and utilized a mean variance (MV) approach to minimize risk associated with cost uncertainty and proposed expected total cost and risk reduction synergy measurements but, unlike the models in this paper of ours, assumed known fixed demands.

In terms of the humanitarian space, Nagurney and Qiang (2009) discussed how the multiproduct supply chain network models of Nagurney, Woolley, and Qiang (2010) could be utilized to assess synergy associated with teaming in the form of horizontal cooperation between humanitarian organizations engaged in disaster relief. Masoumi, Yu, and Nagurney (2017), subsequently, proposed multiple synergy measures to evaluate the mergers or acquisitions associated with multiple (not just two) blood banks in the United States, which are, typically, nonprofits, and in an industry undergoing dramatic change due to economic and other pressures. The authors' blood supply chain network models pre- and post- the merger/acquisition were generalized, nonlinear networks in order to capture the perishability of blood and, unlike the previously noted models, had uncertain, rather, than fixed demands, along with penalties in the objective functions of the blood service organizations associated with shortages/surpluses at demand points. Toyasaki et al. (2017), inspired by horizontal cooperation for inventory management, as in practice done by the United Nations Humanitarian Response Depot (UNHRD) network, constructed an analytical framework to explore horizontal cooperation between humanitarian organizations for their inventory management. The authors proposed a single-product, two organization model and also discussed the relevance of system-optimization in their framework, which was also the foundation for many of the above models noted above. Our focus in this paper, in contrast, is on the full supply chain networks of humanitarian organizations, and the associated activities, which include also procurement as well as transportation and distribution, in addition to storage. However, we retain a system-optimization framework (see also Nagurney and Qiang (2009)), which is quite reasonable since humanitarian organizations are expected to report to their stakeholders as to the use of their resources, including the obtained financial donations (cf. Beamon and Kotleba (2006), Toyasaki and Wakolbinger (2014), Nagurney, Salarpour, and Daniele (2019), and the references therein). It is worth mentioning that Dafermos (1973) introduced multiclass system-optimization models in the context of transportation (see also, e.g., Nagurney (1999) for additional background). Although the methodologies used in the above M&A and humanitarian relief literature can be adapted to analyze synergies associated with humanitarian supply chain cooperation, there are, nevertheless, some gaps, which we address in the new models in this paper.

Specifically, here, we add to the existing literature by constructing "without cooperation" and "with horizontal cooperation" multiproduct supply chain network models of humanitarian organizations engaged in disaster relief. In the case of the former, each humanitarian organization optimizes just its own supply chain network, whereas, in the case of the latter, there is the possibility of the sharing of supply chain network resources. The models can handle as many humanitarian organizations, as need be. We utilize a mean-variance approach to capture risk associated with the uncertainty in the various link cost functions, associated with procurement, transportation, storage, and distribution. The use of a mean-variance approach for the measurement of risk originates in the celebrated work of the Nobel laureate Markowitz (1952, 1959) and remains (cf. Schneeweis, Crowder, and Kazemi (2010)) a powerful tool in finance to capture volatility. The MV approach has been increasingly used in the supply chain management literature to study decision-making under risk and uncertainty (cf. Liu and Nagurney (2011), Heckmann, Comes, and Nickel (2015), and the references therein). In this paper, our new supply chain network models for humanitarian organizations also include uncertainty associated with the demands for the relief items, along with appropriate penalties due to shortage or surplus. Moreover, the supply chain links in our models are subject to capacities.

We note that Nagurney and Nagurney (2016) also utilized a MV approach to construct an integrated supply chain network model for disaster relief with time targets, subject to cost and demand uncertainty, which captured decision-making in both disaster preparedness and response phases. However, that model was a single decision-maker, single product one and there were no upper bounds on the link flows in the form of capacities, which is very relevant since humanitarian organizations may not have unlimited resources in terms of freight access capacity, storage, etc. Hence, the new models in this paper integrate the models of Liu and Nagurney (2011), Nagurney and Nagurney (2016), and Masoumi, Yu, and Nagurney (2017) (in a pure network setting), and extend them to allow for multiple products, which is very reasonable in disaster relief since victims of a sudden-onset disaster may require food, water, medicines, and shelter essentially immediately, and within 72 hours (cf. Nagurney (2016) and the references therein). Victims of slow-onset disasters, on the other hand, may require regular deliveries of relief supplies over a longer time horizon. In our models in this paper, the products are critical needs products, which need to be delivered in a timely manner. As noted in Qiang and Nagurney (2012), critical needs products may be defined as those products and supplies that are essential to human health and life, with examples being: food, water, medicines, and vaccines.

It is also worth mentioning that the contributions in this paper add to the literature on the modeling of multiple, interacting humanitarian organizations engaged in disaster relief. Here, we are interested in cooperation among humanitarian organizations, but it is important to note that noncooperative game theory is also growing in prominence as a tool for the modeling of the behavior of multiple humanitarian organizations (cf. Muggy and Heier Stamm (2014)), since, for example, they engage in competition for financial donations (cf. Toyasaki and Wakolbinger (2014), Nagurney, Alvarez Flores, and Nagurney (2016), and

Nagurney, Salarpour, and Daniele (2019)) and also for freight service provision (cf. Nagurney (2018) and Gossler et al. (2019)).

3. Multiproduct Supply Chain Network Models

This Section develops the "without" and the "with horizontal cooperation" multiproduct supply chain network models for humanitarian organizations using a system-optimization approach with the inclusion of explicit capacities on the various links as well as the introduction of stochastic elements. Moreover, here, we provide variational inequality formulations of the multiproduct supply chains and their integration, which enable a computational approach that fully exploits the underlying network structure. We identify the supply chain network structures of both the with and the without cooperation models and construct a synergy measure.

Section 3.1 describes the underlying supply chain network associated with multiple, individual humanitarian organizations without horizontal cooperation and their associated economic activities of procurement, transportation, storage, and distribution. Section 3.2 then develops the supply chain network model with horizontal cooperation. The models are extensions of the Nagurney (2009), Nagurney, Woolley, and Qiang (2010), Nagurney and Nagurney (2016), and Masoumi, Yu, and Nagurney (2017) models to multiproduct supply chains of multiple humanitarian organizations, with uncertainties in both costs and demands, and upper bounds on links.

3.1 The Case without Horizontal Cooperation Multiproduct Supply Chain Network Model

We first formulate the multiproduct decision-making optimization problem faced by I humanitarian organizations without horizontal cooperation and we refer to this model as Case 0. We assume that each organization is represented as a network of its supply chain activities, as depicted in Figure 1. Each organization i; $i = 1, \ldots, I$, has available n^i_M procurement facilities; n^i_S storage facilities, and serves n^i_D disaster areas. Let $G_i = [N_i, L_i]$ denote the graph consisting of nodes $[N_i]$ and directed links $[L_i]$ representing the supply chain activities associated with each organization i; $i = 1, \ldots, I$. Let L^0 denote the links: $L_1 \cup L_2 \cup \cdots L_I$ as in Figure 1. We assume that each organization is involved in the procurement, transportation, storage, and distribution of J products, with a typical product denoted by j.

The links from the top-tiered nodes i; $i = 1, \ldots, I$, in each network in Figure 1 are connected to the procurement nodes of the respective humanitarian organization i, which

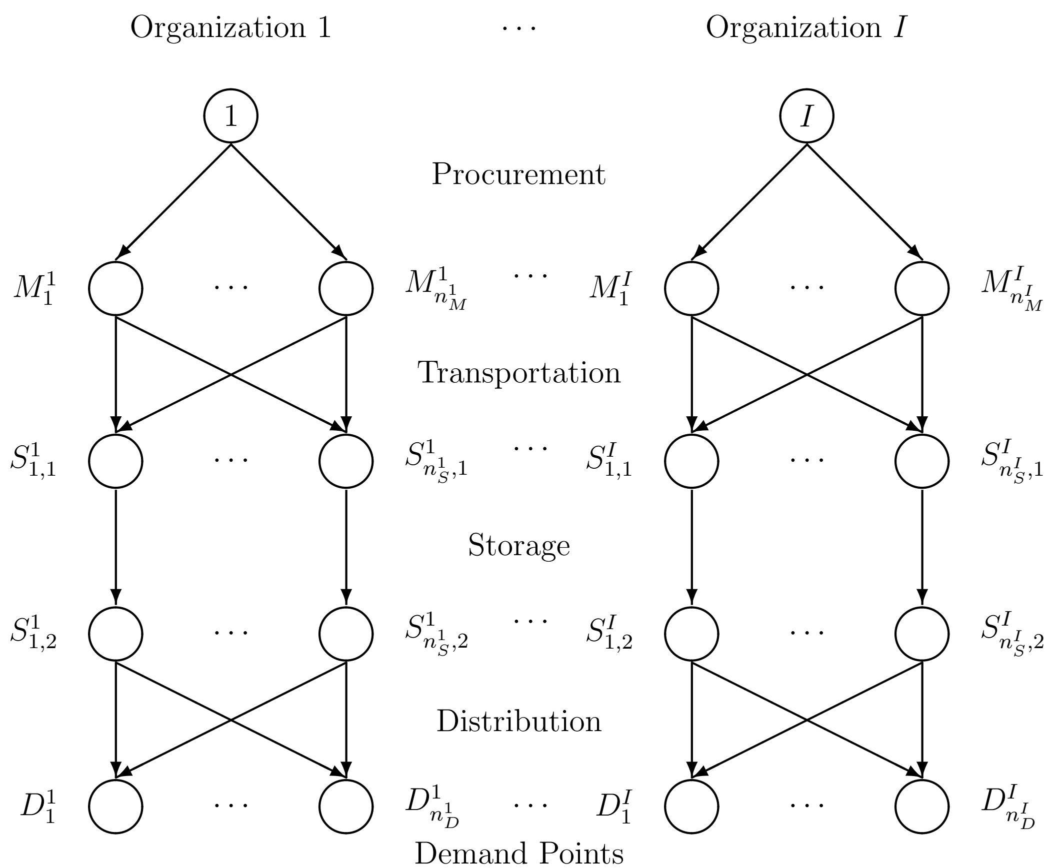

Figure 1: Supply Chains of Humanitarian Organizations 1 through I Prior to the Cooperation

are denoted, respectively, by: $M_1^i, \ldots, M^i_{n_M^i}$. These links represent the procurement links. The links from the procurement nodes, in turn, are connected to the storage center nodes of each humanitarian organization i; $i = 1, \ldots, I$, which are denoted by $S^i_{1,1}, \ldots, S^i_{n_S^i,1}$. These links correspond to the transportation links between the procurement facilities and the storage centers where the products are stored. The links joining nodes $S^i_{1,1}, \ldots, S^i_{n_S^i,1}$ with nodes $S^i_{1,2}, \ldots, S^i_{n_S^i,2}$ for $i = 1, \ldots, I$, correspond to the storage links for the products. Finally, there are the distribution links joining the nodes $S^i_{1,2}, \ldots, S^i_{n_S^i,2}$ for $i = 1, \ldots, I$ with the disaster area nodes: $D_1^i, \ldots, D^i_{n_D^i}$ for each humanitarian organization $i = 1, \ldots, I$. These nodes are also referred to as demand points. Each organization i is assumed to be responsible for delivery of the products to a set of disaster areas, as depicted in Figure 1, prior to the cooperation, for distribution to the victims.

The demands for the products are assumed to be random and are associated with each product, and each demand point. Let d^j_{ik} denote the random variable representing the actual demand for product j and let v^j_{ik} denote the projected random demand for product j; $j = 1, \ldots, J$, at demand point D^i_k for $i = 1, \ldots, I$; $k = 1, \ldots, n^i_D$. In addition, the probability density function of the actual demand for product j is $\mathcal{F}^j_{ik}(t)$ at disaster area D^i_k; $i = 1, \ldots, I$; $k = 1, \ldots, n^i_D$. Hence, we can define the cumulative probability distribution function of d^j_{ik} as $\mathcal{P}^j_{ik}(v^j_{ik}) = \mathcal{P}^j_{ik}(d^j_{ik} \leq v^j_{ik}) = \int_0^{v^j_{ik}} \mathcal{F}^j_{ik}(t)d(t)$. Following Masoumi, Yu, and Nagurney (2017) and Dong, Zhang, and Nagurney (2004), we also define the supply shortage and surplus for product j; $j = 1, \ldots, J$, at disaster area D^i_k; $i = 1, \ldots, I$; $k = 1, \ldots, n^i_D$, as

$$\Delta^{j-}_{ik} \equiv \Delta^{j-}_{ik}(v^j_{ik}) \equiv \max\{0, d^j_{ik} - v^j_{ik}\} \tag{1a}$$

$$\Delta^{j+}_{ik} \equiv \Delta^{j+}_{ik}(v^j_{ik}) \equiv \max\{0, v^j_{ik} - d^j_{ik}\}. \tag{1b}$$

The expected value of the shortage Δ^{j-}_{ik}, denoted by $E(\Delta^{j-}_{ik})$, and of the surplus Δ^{j+}_{ik}, denoted by $E(\Delta^{j+}_{ik})$, for $j = 1, \ldots, J$; D^i_k; $i = 1, \ldots, I$; $k = 1, \ldots, n^i_D$, are then given by

$$E(\Delta^{j-}_{ik}) = \int_{v^j_{ik}}^{\infty} (t - v^j_{ik})\mathcal{F}^j_{ik}(t)d(t), \quad E(\Delta^{j+}_{ik}) = \int_0^{v^j_{ik}} (v^j_{ik} - t)\mathcal{F}^j_{ik}(t)d(t). \tag{2}$$

Furthermore, we denote the penalty associated with the shortage and the surplus of the demand for product j; $j = 1, \ldots, J$, at the disaster area D^i_k by λ^{j-}_{ik} and λ^{j+}_{ik}, respectively, where $i = 1, \ldots, I$; $k = 1, \ldots, n^i_D$.

A path consists of a sequence of links originating at a node i; $i = 1, \ldots, I$ and denotes supply chain activities comprising procurement, transportation, storage, and distribution of the products to the disaster area nodes. Let x^j_p denote the nonnegative flow of product j on path p. Let $P^0_{D^i_k}$ denote the set of all paths joining an origin node i with (destination) disaster area node D^i_k. Clearly, since we are first considering the organizations prior to any cooperation, the paths associated with a given organization have no links in common with paths of the other organization. This changes (see also Nagurney (2009) and Masoumi, Yu, and Nagurney (2017)) when the cooperation occurs, in which case the number of paths and the sets of paths also change, as do the number of links and the sets of links, as described in Section 3.2. The following conservation of flow equations must hold for each organization $i; i = 1, \ldots, I$, each product $j; j = 1, \ldots, J$, and each disaster area $D^i_k; k = 1, \ldots, n^i_D$:

$$\sum_{p \in P^0_{D^i_k}} x^j_p = v^j_{ik}, \quad i = 1, ..., I; \quad j = 1, \ldots, J; \quad k = 1, \ldots, n^i_D, \tag{3}$$

that is, the projected demand for each product associated with a humanitarian organization at a demand point must be satisfied by the sum of the product path flows of the organization's supply chain network.

Links are denoted by a, b, etc. Let f_a^j denote the flow of product j on link a. We also must have the following conservation of flow equations satisfied:

$$f_a^j = \sum_{p \in P^0} x_p^j \delta_{ap}, \quad j = 1 \ldots, J; \quad \forall a \in L^0, \tag{4}$$

where $\delta_{ap} = 1$ if link a is contained in path p and $\delta_{ap} = 0$, otherwise. Here P^0 denotes the set of *all* paths in Figure 1, that is, $P^0 = \cup_{i=1,\ldots,I;k=1,\ldots,n_D^i} P^0_{D_k^i}$. The path flows must be nonnegative, that is,

$$x_p^j \geq 0, \quad j = 1, \ldots, J; \quad \forall p \in P^0. \tag{5}$$

We group the path flows into the vector x. All vectors are assumed to be column vectors.

Note that the different products flow on the supply chain networks depicted in Figure 1 and share resources with one another. To capture the costs, we proceed as follows. There is a total cost associated with each product j; $j = 1, \ldots, J$, and each link (cf. Figure 1) of the network corresponding to each humanitarian organization i; $i = 1, \ldots, I$. We denote the total cost on a link a associated with product j by $\hat{c}_a^j$. The total cost of a link associated with a product, be it a procurement link, a transportation/distribution link, or a storage link is assumed, in general, to be a function of the flow of all the products on the link; see, for example, Dafermos (1973). In addition, given the uncertain nature of disasters, we allow the total costs to be influenced by uncertainty factors. Hence, the total cost on link a, $\hat{c}_a^j$, takes the form:

$$\hat{c}_a^j = \hat{c}_a^j(f_a^1, \ldots, f_a^J, \omega_a^j), \quad j = 1, \ldots, J; \quad \forall a \in L^i, \forall i. \tag{6}$$

In equation (6), ω_a^j is a random variable associated with various disaster events, which have an impact on the total cost of link a, $\forall a$, and product j; $j = 1, \ldots, J$. We assume that the distribution of the ω_a^js is known.

The top tier links in Figure 1 have multiproduct total cost functions associated with them that capture the procurement costs of the products; the second tier links have multiproduct total cost functions associated with them that correspond to the total costs associated with the subsequent transportation/shipment to the storage facilities, and the third tier links, since they are the storage links, have associated with them multiproduct total cost functions that correspond to storage. Finally, the bottom-tiered links, since they correspond to the distribution links to the disaster areas, have total cost functions associated with them that capture the costs of distribution of the products.

We remark that the supply chain networks of the humanitarian organizations, as depicted in Figure 1, capture the prepositioning of the supplies in the preparedness phase of disaster management, through the storage links, as well as the distribution of the supplies through the distribution links in the response phase.

The humanitarian organizations consider both costs and risks in their operations using a mean-variance framework and each organization seeks to minimize its expected total cost and the valuation of its risk. In addition, since the organizations' supply chain networks without horizontal cooperation have no links or costs in common (cf. Figure 1). The optimization problems of the organizations are independent pre-cooperation. Hence, each organization i; $i = 1, \ldots, I$, seeks to find the values of the link flows and the projected random demands that solve the following optimization problem:

$$\text{Minimize} \quad \Bigg[E\Big(\sum_{j=1}^{J} \sum_{a \in L_i} \hat{c}_a^j(f_a^1, \ldots, f_a^J, \omega_a^j)\Big) + \xi_i\Big(V\Big(\sum_{j=1}^{J} \sum_{a \in L_i} \hat{c}_a^j(f_a^1, \ldots, f_a^J, \omega_a^j)\Big)\Big)$$

$$+ \sum_{j=1}^{J} \sum_{k=1}^{n_D^i} \left(\lambda_{ik}^{j-} E(\Delta_{ik}^{j-}) + \lambda_{ik}^{j+} E(\Delta_{ik}^{j+})\right) \Bigg] \tag{7}$$

subject to: constraints (3) – (5) and the following capacity constraints:

$$\sum_{j=1}^{J} \alpha_j f_a^j \leq u_a, \quad \forall a \in L_i. \tag{8}$$

in (8), the term α_j denotes the volume taken up by product j, whereas u_a denotes the nonnegative capacity of link a.

In equation (7), the first and the second terms denote the expected total cost of humanitarian organization i and the variance of the total cost, respectively, with the term ξ_i representing the risk aversion factor of organization i. $V(\sum_{j=1}^{J} \sum_{a \in L_i} \hat{c}_a^j(f_a^1, \ldots, f_a^J, \omega_a^j))$ denotes the variance of the total cost of organization i. In addition, the third term in (7) represents the total costs related to the shortage and/or surplus of the humanitarian products at the disaster areas associated with i. We assume that the total operational costs and the variances in (7) are convex. Furthermore, we know that $\sum_{k=1}^{n_D^i}(\lambda_{ik}^{j-} E(\Delta_{ik}^{j-}) + \lambda_{ik}^{j+} E(\Delta_{ik}^{j+}))$ is also convex, as established in Nagurney, Masoumi, and Yu (2012) for the single product case. Hence, the objective function (7) is convex for each i; $i = 1, \ldots, I$. Furthermore, the individual terms in (7) are continuously differentiable. Under the above imposed assumptions, the optimization problem is a convex optimization problem and, clearly, the feasible set underlying the problem represented by the constraints (3) – (5) and (8) is non-empty, so it follows from the standard theory of nonlinear programming (cf. Weierstrass theorem in Bazaraa, Sherali, and Shetty (1993)) that an optimal solution exists.

We refer to objective function (7) as the total generalized cost TGC_i^0 for $i = 1, \ldots, I$.

We associate the Lagrange multiplier β_a (please refer to the KKT conditions in Chapter 4 of Bazaraa, Sherali, and Shetty (1993)) with constraint (8) for each $a \in L^0$ with $\beta_a \geq 0, \forall a \in L^0$. We denote the associated optimal Lagrange multiplier by $\beta_a^*, \forall a \in L^0$. This term may be interpreted as the price or value of an additional unit of capacity on link a; it is also sometimes referred to as the *shadow price*. We group the link flows into the vector f; the projected demands into the vector v, and the Lagrange multipliers into the vector β.

Let $\mathcal{K}^0$ denote the set where $\mathcal{K}^0 \equiv \{(f, v, \beta) | \exists x \, \text{such that} \, (3) - (5) \, \text{and} \, \beta \geq 0 \, \text{hold}\}$. We now provide the variational inequality formulation of the problem (7) for all humanitarian organizations i; $i = 1, \ldots, I$, simultaneously. For convenience, and, since we are considering Case 0, we denote the solution of variational inequality (VI) (9) below as $(f^{0*}, v^{0*}, \beta^{0*})$ and we refer to the corresponding vectors of variables with superscripts of 0.

Theorem 1: Variational Inequality Formulation of Case 0: No Cooperation

The vector of link flows, projected demands, and Lagrange multipliers $(f^{0*}, v^{0*}, \beta^{0*}) \in \mathcal{K}^0$ *is an optimal solution to (7), for all humanitarian organizations* i*;* $i = 1, \ldots, I$*, subject to their constraints (3)–(5) and (8), if and only if it satisfies the following variational inequality problem:*

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{a\in L_i}[\frac{\partial E(\sum_{l=1}^{J}\sum_{a\in L_i}\hat{c}_a^l(f_a^{1*},\ldots,f_a^{J*},\omega_a^l))}{\partial f_a^j}+\xi_i\frac{\partial V(\sum_{l=1}^{J}\sum_{a\in L_i}\hat{c}_a^l(f_a^{1*},\ldots,f_a^{J*},\omega_a^l))}{\partial f_a^j}+\alpha_j\beta_a^*]$$

$$\times[f_a^j - f_a^{j*}] + \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{n_D^i}\left[\lambda_{ik}^{j+}\mathcal{P}_{ik}^j(v_{ik}^{j*}) - \lambda_{ik}^{j-}(1-\mathcal{P}_{ik}^j(v_{ik}^{j*})\right] \times [v_{ik}^j - v_{ik}^{j*}]$$

$$+\sum_{a\in L^0}[u_a - \sum_{j=1}^{J}\alpha_j f_a^{j*}] \times [\beta_a - \beta_a^*] \geq 0, \quad \forall (f^0, v^0, \beta^0) \in \mathcal{K}^0. \tag{9}$$

Proof: See Bertsekas and Tsitsiklis (1989), Nagurney (1999), Dong, Zhang, and Nagurney (2004), Liu and Nagurney (2011), and Masoumi, Yu, and Nagurney (2017).

3.2 The Case with Horizontal Cooperation Multiproduct Humanitarian Supply Chain Network Model

We now formulate the case with horizontal cooperation multiproduct humanitarian supply chain network model, referred to as Case 1. Figure 2 depicts the humanitarian supply chain network topology under Case 1. Note that there is now a *supersource* node 0 which represents the "joining" in terms of cooperation of the organizations in terms of their supply chain networks with additional links connecting node 0 to nodes 1 through I.

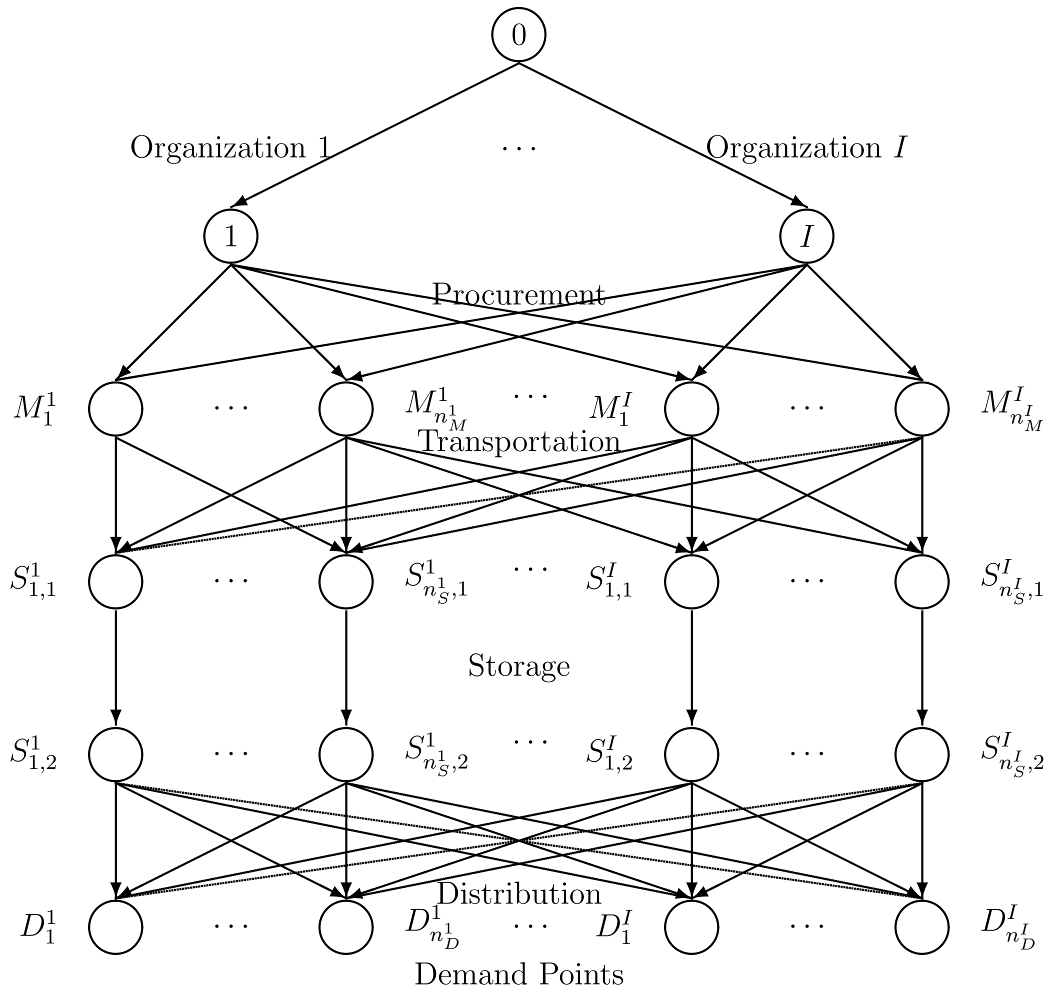

Figure 2: Humanitarian Supply Chain Network after Cooperation

As in Case 0, the optimization problem in Case 1 is also concerned with cost and risk minimization. Specifically, we retain the nodes and links associated with the multiorganization supply chain network depicted in Figure 1 but now we add the additional links connecting the procurement facilities of each organization and the distribution centers of the other organization as well as the links connecting the distribution centers of each organization and the disaster areas of the other organization. We refer to the network in Figure 2, underlying this integration, as $G^1 = [N^1, L^1]$ where $N^1 \equiv N^0 \cup$ node 0 and $L^1 \equiv L^0 \cup$ the additional links as in Figure 2. We associate total cost functions as in (6) with the new links, for each product j. Note that if the total cost functions associated with the cooperation links connecting node 0 to node 1 through node I are set equal to zero, this means that the cooperation is *costless* in terms of the integrated supply chain network of the humanitarian organizations.

A path p now (cf. Figure 2) originates at the node 0 and is destined for one of the bottom disaster nodes. Let x_p^j, under the cooperation network configuration given in Figure

2, denote the flow of product j on path p joining (origin) node 0 with a (destination) disaster area (demand) node. Then, the following conservation of flow equations must hold for each i, j, k:

$$\sum_{p \in P^1_{D^i_k}} x^j_p = v^j_{ik}, \tag{10}$$

where $P^1_{D^i_k}$ denotes the set of paths connecting node 0 with disaster area node D^i_k in Figure 2. Due to the cooperation, the disaster areas can obtain each product j from any procurement facility, and any storage facility. The set of paths $P^1 \equiv \cup_{i=1,I;k=1,\dots,n^i_D} P^1_{D^i_k}$.

In addition, as before, let f^j_a denote the flow of product j on link a. Hence, we must also have the following conservation of flow equations satisfied:

$$f^j_a = \sum_{p \in P^1} x^j_p \delta_{ap}, \quad j = 1, \dots, J; \quad \forall a \in L^1. \tag{11}$$

Of course, we also have that the path flows must be nonnegative for each product j, that is,

$$x^j_p \geq 0, \quad j = 1, \dots, J; \quad \forall p \in P^1. \tag{12}$$

We assume, again, that the supply chain network activities have nonnegative capacities, denoted as u_a, $\forall a \in L^1$, with α_j representing the volume factor for product j. Hence, the following constraints must be satisfied:

$$\sum_{j=1}^{J} \alpha_j f^j_a \leq u_a, \quad \forall a \in L^1. \tag{13}$$

The term ξ denotes the associated risk aversion factor of the integrated organizations under cooperation.

Consequently, the optimization problem for the integrated humanitarian supply chain network is:

$$\text{Minimize} \quad E(\sum_{j=1}^{J} \sum_{a \in L^1} \hat{c}^j_a(f^1_a, \dots, f^J_a, \omega^j_a)) + \xi \left[V(\sum_{j=1}^{J} \sum_{a \in L^1} \hat{c}^j_a(f^1_a, \dots, f^J_a, \omega^j_a)) \right]$$

$$+ \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{n^i_D} (\lambda^{j-}_{ik} E(\Delta^{j-}_{ik}) + \lambda^{j+}_{ik} E(\Delta^{j+}_{ik})) \tag{14}$$

subject to constraints: (10) – (13).

Similar to equation (7), the terms in (14) represent the total operational costs, the risks, and the shortage/surplus costs related to the uncertain demand. The term ξ is the associated risk aversion factor of the integrated organizations under cooperation. The solution to the optimization problem (14) subject to constraints (10), for all i, j, k, through (13) can also be obtained as a solution to a variational inequality problem akin to (9) where now links $a \in L^1$. The vectors f, v, and β have identical definitions as before, but are re-dimensioned/expanded accordingly and superscripted with 1. Finally, instead of the feasible set $\mathcal{K}^0$ we now have $\mathcal{K}^1 \equiv \{(f, v, \beta) | \exists x \, \text{such that} \, (10) - (12) \, \text{hold and} \, \beta \geq 0\}$.

We refer to objective function (14) as the total generalized cost TGC^1.

We denote the solution to the variational inequality (VI) problem (15) below governing Case 1 by $(f^{1*}, v^{1^*}, \beta^{1*})$ and denote the vectors of corresponding variables as (f^1, v^1, β^1). We now, for completeness, provide the variational inequality formulation of the Case 1 problem. The proof is immediate.

Theorem 2: Variational Inequality Formulation of Case 1: Cooperation

The vector of link flows, projected demands, and Lagrange multipliers $(f^{1*}, v^{1*}, \beta^{1*}) \in \mathcal{K}^1$ *is an optimal solution to (14), subject to constraints (10)–(13), if and only if it satisfies the following variational inequality problem:*

$$\sum_{j=1}^{J} \sum_{a \in L^1} [\frac{\partial E(\sum_{l=1}^{J} \sum_{a \in L^1} \hat{c}_a^l(f_a^1, \ldots, f_a^J, \omega_a^l))}{\partial f_a^j} + \xi \frac{\partial V(\sum_{l=1}^{J} \sum_{a \in L^1} \hat{c}_a^l(f_a^{1*}, \ldots, f_a^{J*}, \omega_a^l))}{\partial f_a^j} + \alpha_j \beta_a^*]$$

$$\times [f_a^j - f_a^{j*}] + \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{n_D^i} \left[\lambda_{ik}^{j+} \mathcal{P}_{ik}^j(v_{ik}^{j*}) - \lambda_{ik}^{j-}(1 - \mathcal{P}_{ik}^j(v_{ik}^{j*}) \right] \times [v_{ik}^j - v_{ik}^{j*}]$$

$$+ \sum_{a \in L^1} [u_a - \sum_{j=1}^{J} \alpha_j f_a^{j*}] \times [\beta_a - \beta_a^*] \geq 0, \quad \forall (f^1, v^1, \beta^1) \in \mathcal{K}^1. \tag{15}$$

Theorem 2 states that the solution to (15) coincides with the solution to the optimization problem (14). Therefore, we can utilize the existing theories and algorithms for VIs to explore the problem further and to generate managerial insights through numerical computations.

Definition 1: Total Generalized Costs at the Optimal Solutions to the Supply Chain Network Problems without and with Cooperation

Let TGC^{0*} *denote the total generalized cost equal to* $\sum_i^I TGC_i^0 = E(\sum_{j=1}^J \sum_{a \in L^0} \hat{c}_a^j(f_a^1, \ldots, f_a^J, \omega_a^j)) + \sum_{i=1}^I \xi_i \left[V(\sum_{j=1}^J \sum_{a \in L_i} \hat{c}_a^j(f_a^1, \ldots, f_a^J, \omega_a^j)) \right] + \sum_{i=1}^I \sum_{j=1}^J \sum_{k=1}^{n_D^i} (\lambda_{ik}^{j-} E(\Delta_{ik}^{j-}) + \lambda_{ik}^{j+} E(\Delta_{ik}^{j+}))$, *evaluated at the optimal solution* $(f^{0*}, v^{0*}, \beta^{0*})$ *to (9).*

Also, let $TGC^{1*} = E(\sum_{j=1}^{J}\sum_{a\in L^1}\hat{c}_a^j(f_a^1,\ldots,f_a^J,\omega_a^j))$
$+\,\xi\left[V(\sum_{j=1}^{J}\sum_{a\in L^1}\hat{c}_a^j(f_a^1,\ldots,f_a^J,\omega_a^j))\right]+\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{n_D^i}(\lambda_{ik}^{j-}E(\Delta_{ik}^{j-})+\lambda_{ik}^{j+}E(\Delta_{ik}^{j+}))$*, denote the total generalized cost evaluated at the solution* $(f^{1*}, v^{1*}, \beta^{1*})$ *to (15).*

4. Quantifying Synergy of Horizontal Cooperation of the Multiproduct Humanitarian Supply Chain Networks

We now quantify the synergy associated with cooperation of the humanitarian organizations by analyzing the generalized costs under the cases with and without humanitarian supply chain network cooperation (cf. Eccles, Lanes, and Wilson (1999) and Nagurney (2009)).

We denote the synergy here by $\mathcal{S}^{TGC}$. It is calculated as the percentage difference between the total generalized cost without *vs.* with the horizontal cooperation (evaluated at the respective optimal solutions):

$$\mathcal{S}^{TGC} \equiv [\frac{TGC^{0*} - TGC^{1*}}{TGC^{0*}}] \times 100\%. \tag{16}$$

From (16), one can see that the lower the total generalized cost TGC^{1*}, the higher the synergy associated with the humanitarian supply chain network cooperation and, therefore, the greater the total cost savings resulting from the cooperation. It is important to further emphasize that the general costs include not only the monetary costs, but also the risks and uncertainties involved in the humanitarian supply chain as well as the associated penalties of shortages and surpluses. Of course, in specific operations one may wish to evaluate the integration of humanitarian supply chain networks with only a subset of the links joining the original supply chain networks. In that case, Figure 2 would be modified accordingly and the synergy as in (16) computed with TGC^{1*} corresponding to that supply chain network topology.

We now provide a theorem which shows that, under certain assumptions related to the total operational costs associated with the supply chain integration and the risk factors, the associated synergy can never be negative.

Theorem 3

If the total generalized cost functions associated with the cooperation links from node 0 *to nodes* 1 *through* I *for each product are identically equal to zero; and the risk aversion factors* ξ_i*;* $i = 1,\ldots,I$*, are all equal and set to* ξ*, then the associated synergy,* $\mathcal{S}^{TGC}$*, can never be negative.*

Proof: The result follows using the same arguments as the proof of Theorem 3 in Nagurney, Woolley, and Qiang (2010). □

Another interpretation of this theorem is that, in the system-optimization context (assuming that the total cost functions remain the same as do the demands), the addition of new links can never make the total cost increase. This is in contrast to what may occur in the context of user-optimized networks, where the addition of a new link may make everyone worse-off in terms of user cost, which is what occurs in the case of the well-known Braess paradox (1968); see, also, Braess, Nagurney, and Wakolbinger (2005). More specifically, in the classical Braess paradox, the addition of a link, which results in a new path available for travelers from an origin to a destination, results in an increase in travel cost (travel time) for all travelers in the network. Hence, users are better off without the network addition. The Braess paradox can occur not only in congested urban transportation networks but also on the Internet (cf. Korilis, Lazar, and Orda (1999)).

5. The Computational Scheme

In view of the conservation of flow equations (10) and (11), and constraints (12) and (13), we can also construct a variational inequality formulation akin to (15), but in path flows, rather than in links flows (the same holds for a path flow analogue of variational inequality (9)). We now present the path flow variational inequality for the cooperation supply chain network problem, with the accompanying computational scheme (which can easily be adapted to also solve the pre-cooperation VI problem in path flows).

We group the path flows into the vector $x \in R^{n_{P^1}}$, where n_{P^1} is the number of paths in P^1. Also, n_{L^1} denotes the number of links in L^1.

We define the feasible set $\mathcal{K}^2 \equiv \{(x, \beta) | x \geq 0, \beta \geq 0\}$. Then the VI (17) below follows directly from the relationships between variational inequalities and nonlinear programming problems (cf. Nagurney (1999) and the references therein) (or, equivalently, by utilizing the conservation of flow expressions and embedding them into the link flow VI analogue (15), along with algebraic simplification). A vector of path flows and Lagrange multipliers $(x^*, \beta^*) \in \mathcal{K}^2$ is an optimal solution to problem (14) subject to (10) – (13) if and only if it satisfies the variational inequality:

$$\sum_{j=1}^{J} \sum_{p \in P^1} \left[\frac{\partial TGC^1(x^*)}{\partial x_p^j} + \alpha_j \sum_{a \in L^1} \beta_a^* \delta_{ap} \right] \times \left[x_p^j - x_p^{j*} \right]$$

$$+ \sum_{a \in L^1} \left[u_a - \sum_{j=1}^{J} \alpha_j \sum_{p \in P^1} x_p^{j*} \delta_{ap} \right] \times [\beta_a - \beta_a^*] \geq 0, \quad \forall (x, \beta) \in \mathcal{K}^2. \tag{17}$$

We now put VI (17) into standard form (cf. Nagurney (1999)): determine $X^* \in \mathcal{L} \subset R^{\mathcal{N}}$, such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{L}, \tag{18}$$

where F is a given continuous function from $\mathcal{L}$ to $R^{\mathcal{N}}$, $\mathcal{L}$ is a closed, convex set, and $\langle \cdot, \cdot \rangle$ denotes the inner product in $\mathcal{N}$-dimensional Euclidean space.

Let $X \equiv (x, \beta)$ and $F(X) \equiv (F_1(X), F_2(X))$, where $F_1(X)$ consists of elements: $\left[\frac{\partial TGC^1(x^*)}{\partial x_p^j} + \alpha_j \sum_{a \in L^1} \beta_a^* \delta_{ap}\right]$, $\forall j, \forall p \in P^1$, and $F_2(X)$ of elements: $\left[u_a - \sum_{j=1}^J \alpha_j \sum_{p \in P^1} x_p^{j*} \delta_{ap}\right]$, $\forall a \in L^1$, and all vectors are column vectors. Then, clearly, (17) can be put into the form (18), where $\mathcal{N} = n_{P^1} + n_{L^1}$.

The algorithm that we apply for the computation of the optimal product path flow and Lagrange multiplier patterns for both supply chain network problems is the modified projection method (see Korpelevich (1977)). The algorithm is guaranteed to converge, provided that the function $F(X)$ is monotone and Lipschitz continuous (cf. the Appendix), and that a solution exists.

5.1 The Modified Projection Method

The steps of the modified projection method are:

Step 0: Initialization

Initialize with $X^0 \in \mathcal{L}$. Set $t := 1$ and select η, such that $0 < \eta \leq \frac{1}{\bar{L}}$, where $\bar{L}$ is the Lipschitz constant (cf. (20)) for the function F in the variational inequality problem.

Step 1: Construction and Computation

Compute $\bar{X}^t \in \mathcal{L}$ by solving the variational inequality subproblem:

$$\langle \bar{X}^t + \eta F(X^{t-1}) - X^{t-1}, X - \bar{X}^t \rangle \geq 0, \quad \forall X \in \mathcal{L}. \tag{19}$$

Step 2: Adaptation

Compute $X^t \in \mathcal{L}$ by solving the variational inequality subproblem:

$$\langle X^t + \eta F(\bar{X}^t) - X^{t-1}, X - X^t \rangle \geq 0, \quad \forall X \in \mathcal{L}. \tag{20}$$

Step 3: Convergence Verification

If $|X^t - X^{t-1}| \leq \epsilon$, for $\epsilon > 0$, a specified tolerance, then, stop; otherwise, set $t := t + 1$ and go to Step 1.

For both the pre-cooperation model and the cooperation model, Steps 1 and 2 of the modified projection method above (cf. (21) and (22)) result in closed form expressions for the product path flows as well as the Lagrange multipliers at each iteration. Below we provide the associated explicit formulae for Step 1 for the solution of VI (17). Analogous ones are easily obtained for Step 2.

Closed Form Expressions for the Product Path Flows and the Lagrange Multipliers at Step 1 of Iteration t

We now present the closed form expressions for the solution variational inequality subproblem (19) associated with VI (17).

The closed form expression for the product path flow $\bar{x}_p^{j\,t}$ for each $p \in P^1$; $j = 1, \ldots, J$, at iteration t is:

$$\bar{x}_p^{j\,t} = \max\{0, \eta\left[-\frac{\partial TGC^1(x^{t-1})}{\partial x_p^j} - \alpha_j \sum_{a \in L^1} \beta_a^{t-1} \delta_{ap}\right] + x_p^{j\,t-1}\}. \tag{21}$$

The closed form expression for the Lagrange multiplier $\bar{\beta}_a^t$ for $a \in L^1$ is:

$$\bar{\beta}_a^t = \max\{0, \eta\left[\sum_{j=1}^{J} \alpha_j \sum_{p \in P^1} x_p^{j\,t-1} \delta_{ap} - u_a\right] + \beta_a^{t-1}\}. \tag{22}$$

Theorem 4: Convergence of the Algorithm

Assume that the function $F(X)$ that enters the variational inequality (18) is monotone and Lipschitz continuous and that a solution exists. Then, the modified projection method outlined above converges to a solution.

Proof: According to Korpelevich (1977), the modified projection method converges to the solution of the variational inequality problem of the form (18), if the function F that enters the variational inequality is monotone and Lipschitz continuous and a solution exists. $\square$

6. Numerical Examples

In this Section, we compute solutions to numerical examples illustrating the modeling and algorithmic framework. The numerical examples are inspired, in part, by ongoing refugee/migrant crises as in Central America and Mexico (cf. Stemple (2019)), Yemen (see Palmer (2018)), Syria (Ki-Moon (2018)), among many others. Such slow-onset, ongoing disasters are providing immense challenges for humanitarian organizations (in addition to governments) to provide the necessary food, water, medicines, etc., to the needy in a variety of shelters. Our numerical examples are stylized but reflect real-world features. Moreover, as in the case of the refugee/migrant crisis emanating from Central America, numerous humanitarian organizations are involved in providing assistance and, hence, it is valuable to be able to assess possible synergies since the demand is so great. In particular, with carefully calibrated historical data and information, our models can be used to assist the humanitarian organizations on how to cooperate in terms of the delivery of humanitarian relief products in a cost-effective fashion.

The pre-cooperation supply chain network for the numerical examples is depicted in Figure 3 and the cooperation one in Figure 4.

According to Figure 3, there are two humanitarian organizations, HO 1 and HO 2, each of which is to provide relief items to disaster victims at two demand points. The demand points associated with HO 1, D^1_1 and D^1_2, differ from those of HO 2, that is, D^2_1 and D^2_2.

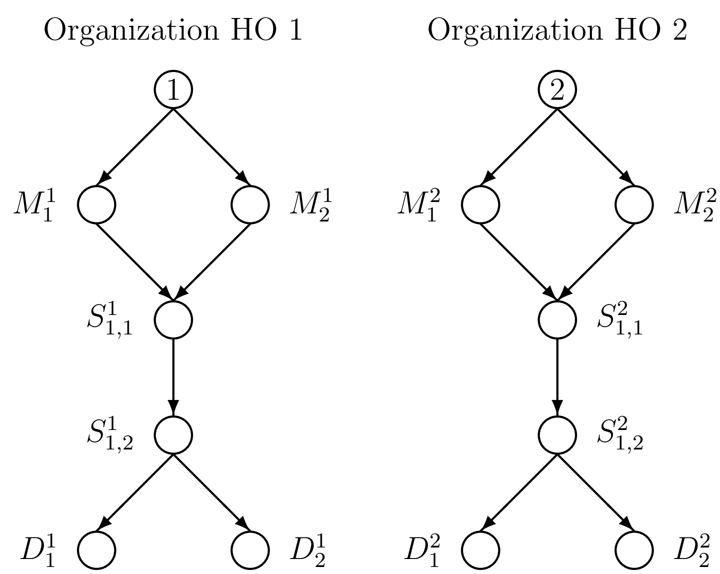

Figure 3: Pre-Cooperation Supply Chain Network Topology for the Numerical Examples

Pre-cooperation, each organization can procure the relief items from two possible locations (distinct for each organization) and then have the items transported for storage to a separate storage facility, from which the relief items are ultimately transported to the points of demand. On the other hand, under cooperation, as the supply chain network figure in Figure 4 reveals, the demand points can be serviced by either humanitarian organization (or both), and they can make use of one another's storage facilities as well as freight services for transportation and distribution, and can also avail themselves of all the procurement location options.

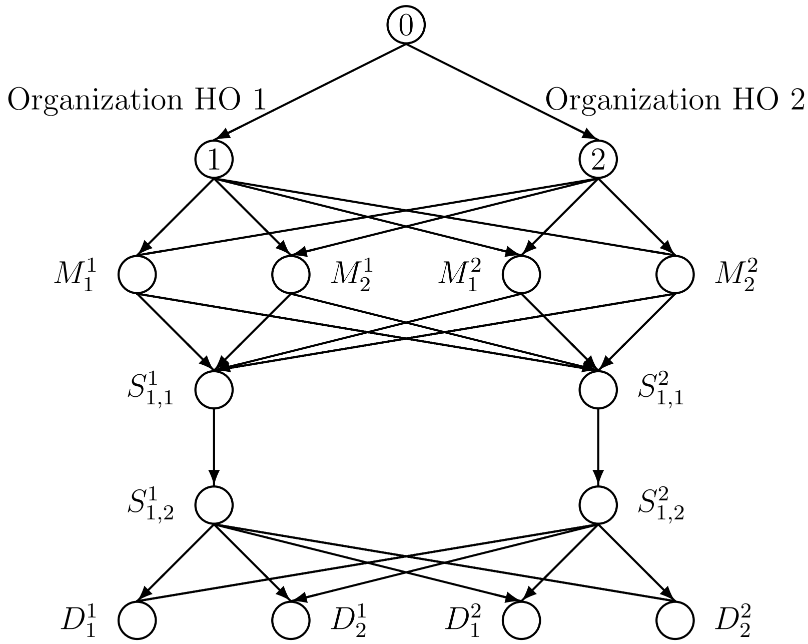

Figure 4: Cooperation Supply Chain Network Topology for the Examples

In the numerical examples, we consider a single product and, hence, we suppress the superscripts associated with products in our notation of Section 2 and 3.

The total link cost functions are of the form:

$$\hat{c}_a = c_a(f_a, \omega_a) = \omega_a \hat{g}_a f_a + g_a f_a, \quad \forall a \in L^1. \tag{23}$$

The objective function (14) then becomes:

$$\text{Minimize} \quad \sum_{a \in L^1} E(\omega_a)\hat{g}_a f_a + \sum_{a \in L^1} g_a f_a + \xi V(\sum_{a \in L^1} \omega_a \hat{g}_a f_a) + \sum_{i=1}^{I} \sum_{k=1}^{n_D^i} (\lambda_{ik}^- E(\Delta_{ik}^-) + \lambda_{ik}^+ E(\Delta_{ik}^+)), \tag{24}$$

where in our examples $I = 2$.

The covariance matrix associated with the $\hat{c}_a(f_a, \omega_a)$, $\forall a \in L^1$, is the 28×28 matrix $\sigma^2 I$, since there are 28 links in the network in Figure 4.

Following Nagurney and Nagurney (2016), we know that:

$$\sum_{a\in L^1} \sigma^2 \hat{g}_a^2 f_a^2 = V(\sum_{a\in L^1} \omega_a \hat{g}_a f_a) = V(\sum_{a\in L^1} \omega_a \hat{g}_a \sum_{q\in \mathcal{P}} x_q \delta_{aq}); \tag{25}$$

hence,

$$\frac{\partial V(\sum_{a\in L^1} \omega_a \hat{g}_a \sum_{q\in\mathcal{P}} x_q \delta_{aq})}{\partial x_p} = 2\sigma^2 \sum_{a\in L^1} \hat{g}_a^2 f_a \delta_{ap}. \tag{26}$$

We define the following marginal cost on a link a, $\hat{Gc}'_a$, as:

$$G\hat{c}'_a \equiv E(\omega_a)\hat{g}_a + g_a + \xi 2\sigma^2 \hat{g}_a^2 f_a, \tag{27}$$

and the following marginal cost on a path:

$$G\hat{C}'_p \equiv \sum_{a\in L^1} G\hat{c}'_a \delta_{ap}, \quad \forall p \in P^1, \tag{28}$$

so that $\frac{\partial TGC^1(x^{t-1})}{\partial x_p}$ in the algorithmic statement (21) would have the form (cf. Masoumi, Yu, and Nagurney (2017)), $\forall i$, $\forall k$, $\forall p \in P^1_{D^i_k}$:

$$\frac{\partial TGC^1(x^{t-1})}{\partial x_p} = G\hat{C}'_p(x^{t-1}) - \lambda^-_{ik}(1 - \mathcal{P}_{ik}(\sum_{q\in P^1_{D^i_k}} x_q^{t-1})) + \lambda^+_{ik}\mathcal{P}_{ik}(\sum_{q\in P^1_{D^i_k}} x_q^{t-1}). \tag{29}$$

We implemented the algorithm in FORTRAN and utilized a Unix system at the University of Massachusetts Amherst for the computations. The algorithm was initialized with the projected demand for each demand point set to 100 and equally distributed among the paths. The convergence tolerance ϵ was set to 10^{-5}; that is, the algorithm was terminated when the absolute value of the difference of successive path flows at two iterations as well as that of successively computed Lagrange multipliers were all less than or equal to this ϵ value.

Example 1

The definition of the links, the upper bounds on the links, the associated total links cost functions are given in Table 1 for Example 1. Example 2 is a variant of Example 1 and has the same data except for the probability distribution functions at the demand points. The time horizon under consideration is one week.

Since we assume one type of relief item, we set (cf. (8)), $\alpha_1 = 1$. The product to be delivered to the shelters is that of relief item kits, so our costs/prices associated with the procurement links (cf. Table 1) are reasonable (cf. Nagurney, Salarpour, and Daniele

(2019)). The weights $\xi_1 = \xi_2 = 1$ for the without cooperation supply chain network problem (cf. Figure 3).

The demand at each of the four demand points in Figures 3 and 4 for Example 1 is assumed to follow a continuous uniform distribution on the intervals: $[150, 400]$, $[150, 250]$, $[150, 500]$, and $[100, 200]$, respectively. Hence, the demand at the second demand point of each HO is lower than at its first demand location. We then have that, for HO 1:

$$\mathcal{P}_{11}(\sum_{p \in P^0_{D^1_1}}) = \frac{\sum_{p \in P^0_{D^1_1}} x_p - 150}{400 - 150}, \quad \mathcal{P}_{12}(\sum_{p \in P^0_{D^1_2}}) = \frac{\sum_{p \in P^0_{D^1_2}} x_p - 150}{250 - 150},$$

and, for HO 2:

$$\mathcal{P}_{21}(\sum_{p \in P^0_{D^2_1}}) = \frac{\sum_{p \in P^0_{D^2_1}} x_p - 150}{500 - 150}, \quad \mathcal{P}_{22}(\sum_{p \in P^0_{D^2_2}}) = \frac{\sum_{p \in P^0_{D^2_2}} x_p - 100}{200 - 100}.$$

The demand points associated with HO 1 are in the western part of a region, whereas those associated with HO2 are in the eastern part. Their respective storage centers are located centrally.

We set $\sigma^2 = 1$. For the construction of the $G\hat{c}'_a$, $\forall a \in L^1$, please refer to equation (27) and Table 1.

Also, we set: $\lambda^-_{ik} = 10000$ and $\lambda^+_{ik} = 100$ for both HOs and all demand points since shortages are penalized more than surpluses.

The computed optimal link flows and Lagrange multipliers for this example, prior to cooperation, are reported in Table 2.

The component of the total generalized cost TGC^{0*} not including the penalized expected shortages and surpluses is equal to 1,415,963, whereas the total generalized cost $TGC^{0*}=$ 1,024,443,264.

As can be seen from the results in Table 2, the volume of relief item flows into each demand point is above the minimum amount of the corresponding interval of the associated probability distribution. Interestingly, the relief item flows on the procurement links of both HOs are at their respective link capacities and, hence, the corresponding optimal Lagrange multipliers are positive. The HOs may wish to discuss with their suppliers the possibility of procuring additional items in the future.

In Table 3, we report the computed optimal solution for the cooperation supply chain network for Example 1. We set $\xi = 1$.

Table 1: Definition of Links, the Link Upper Bounds, and Associated Total Cost and Other Functions for Examples 1 and 2

Link a	From Node	To Node	u_a	$\hat{c}_a(f_a,\omega_a)=\omega_a\hat{g}_af_a+g_af_a$	$E(\omega_a)$	$G\hat{c}'_a$
1	1	M_1^1	200	$\omega_1 2f_1+60f_1$	1	$8f_1+62$
2	1	M_2^1	175	$\omega_2 f_2+55f_2$	1	$2f_2+56$
3	M_1^1	$S_{1,1}^1$	250	$\omega_3 f_3+4f_3$	1	$2f_3+5$
4	M_2^1	$S_{1,1}^1$	200	$\omega_4 f_4+5f_4$	1	$2f_4+6$
5	$S_{1,1}^1$	$S_{1,2}^1$	400	$\omega_5 f_5+2f_5$	1	$2f_5+3$
6	$S_{1,2}^1$	D_1^1	300	$\omega_6 2f_6+2f_6$	1	$8f_6+4$
7	$S_{1,2}^1$	D_2^1	300	$\omega_7 2f_7+2f_7$	1	$8f_7+4$
8	2	M_1^2	175	$\omega_8 f_8+50f_8$	1	$2f_8+51$
9	2	M_2^2	175	$\omega_9 f_9+45f_9$	1	$2f_9+46$
10	M_1^2	$S_{1,1}^2$	300	$\omega_{10} f_{10}+2f_{10}$	1	$2f_{10}+3$
11	M_2^2	$S_{1,1}^2$	300	$\omega_{11} f_{11}+6f_{11}$	1	$2f_{11}+7$
12	$S_{1,1}^2$	$S_{1,2}^2$	450	$\omega_{12} 2f_{12}+2f_{12}$	1	$8f_{12}+4$
13	$S_{1,2}^2$	D_1^2	350	$\omega_{13} f_{13}+7f_{13}$	1	$2f_{13}+8$
14	$S_{1,2}^2$	D_2^2	200	$\omega_{14} f_{14}+8f_{14}$	1	$2f_{14}+9$
15	1	M_1^2	150	$\omega_{15} f_{15}+50f_{15}$	1	$2f_{15}+51$
16	1	M_2^1	175	$\omega_{16} f_{16}+45f_{16}$	1	$2f_{16}+46$
17	2	M_1^1	175	$\omega_{17} 2f_{17}+60f_{17}$	1	$8f_{17}+62$
18	2	M_2^1	150	$\omega_{18} f_{18}+55f_{18}$	1	$2f_{18}+56$
19	M_1^1	$S_{1,1}^2$	200	$\omega_{19} f_{19}+5f_{19}$	1	$2f_{19}+6$
20	M_2^1	$S_{1,1}^2$	200	$\omega_{20} f_{20}+6f_{20}$	1	$2f_{20}+7$
21	M_1^2	$S_{1,1}^1$	200	$\omega_{21} f_{21}+3f_{21}$	1	$2f_{21}+4$
22	M_2^2	$S_{1,1}^1$	200	$\omega_{22} f_{22}+7f_{22}$	1	$2f_{22}+8$
23	$S_{1,2}^1$	D_1^2	200	$\omega_{23} 2f_{23}+3f_{23}$	1	$8f_{23}+5$
24	$S_{1,2}^1$	D_2^2	200	$\omega_{24} 2f_{24}+3f_{24}$	1	$8f_{24}+5$
25	$S_{1,2}^2$	D_1^1	150	$\omega_{25} f_{25}+8f_{25}$	1	$2f_{25}+9$
26	$S_{1,2}^2$	D_2^1	150	$\omega_{26} f_{26}+9f_{26}$	1	$2f_{26}+10$
27	0	1	large	0	–	0
28	0	2	large	0	–	0

Again, the relief item flows to the demand points are all greater than the lower value of the interval of the respective probability distribution. Moreover, whereas in the case without cooperation, a total of 725 relief items were delivered across all demand points, now 850 have been delivered under cooperation. Hence, victims benefit from the cooperation of HOs.

In the optimal solution to the supply chain network with cooperation, as reported in Table 3, the relief item flows at the two storage locations are now at capacity levels, as is the flow on the shipment link from the second storage facility to the fourth (last) demand

Table 2: Optimal Link Flows and Lagrange Multipliers for Examples 1 and 2 without Co-operation

			Example 1		Example 2	
Link a	From Node	To Node	f_a^*	β_a^*	f_a^*	β_a^*
1	1	M_1^1	200	3448	200	1878
2	1	M_2^1	175	4753	175	3183
3	M_1^1	$S_{1,1}^1$	200	0	200	0
4	M_2^1	$S_{1,1}^1$	175	0	175	0
5	$S_{1,1}^1$	$S_{1,2}^1$	375	0	375	0
6	$S_{1,2}^1$	D_1^1	202	0	187.5	0
7	$S_{1,2}^1$	D_2^1	173	0	187.5	0
8	2	M_1^2	175	3774	175	1026
9	2	M_2^2	175	3775	175	1027
10	M_1^2	$S_{1,1}^2$	175	0	175	0
11	M_2^2	$S_{1,1}^2$	175	0	175	0
12	$S_{1,1}^2$	$S_{1,2}^2$	350	0	350	0
13	$S_{1,2}^2$	D_1^2	226	0	200	0
14	$S_{1,2}^2$	D_2^2	124	0	150	0

point. Hence, the Lagrange multipliers associated with these links (links 5, 12, and 26) are now positive.

The component of the total generalized cost TGC^{1*} not including the penalized expected shortages and surpluses is equal to 1,480,565, whereas the total generalized cost TGC^{1*}=466,333,824.

The resulting synergy for Example 1, associated with cooperation, is, hence, $\mathcal{S}^{TGC} = 54\%$. The HOs also gain under cooperation, in addition to the refugees.

Example 2

Example 2 has the identical data to that in Example 1 except that we assume that there are now better estimates of the demand ranges for the first and third demand points. Hence, we now have that, for HO 1:

$$\mathcal{P}_{11}(\sum_{p \in P^0_{D_1^1}}) = \frac{\sum_{p \in P^0_{D_1^1}} x_p - 150}{250 - 150},$$

Table 3: Optimal Link Flows and Lagrange Multipliers for Examples 1 and 2 with Cooperation

			Example 1		Example 2	
Link a	From Node	To Node	f_a^*	β_a^*	f_a^*	β_a^*
1	1	M_1^1	86	0	106	0
2	1	M_2^1	112	0	106	0
3	M_1^1	$S_{1,1}^1$	78.5	0	100	0
4	M_2^1	$S_{1,1}^1$	106	0	100	0
5	$S_{1,1}^1$	$S_{1,2}^1$	400	7291	400	11305
6	$S_{1,2}^1$	D_1^1	79.5	0	110	0
7	$S_{1,2}^1$	D_2^1	140	0	110	0
8	2	M_1^2	114	0	106	0
9	2	M_2^2	114	0	106	0
10	M_1^2	$S_{1,1}^2$	120	0	113	0
11	M_2^2	$S_{1,1}^2$	120	0	113	0
12	$S_{1,1}^2$	$S_{1,2}^2$	450	4917	450	9737
13	$S_{1,2}^2$	D_1^2	96	0	123	0
14	$S_{1,2}^2$	D_2^2	115	0	82	0
15	1	M_1^2	114	0	106	0
16	1	M_2^1	114	0	106	0
17	2	M_1^1	86	0	106	0
18	2	M_2^1	112	0	106	0
19	M_1^1	$S_{1,1}^2$	93.5	0	113	0
20	M_2^1	$S_{1,1}^2$	117	0	113	0
21	M_1^2	$S_{1,1}^1$	108	0	100	0
22	M_2^2	$S_{1,1}^1$	108	0	100	0
23	$S_{1,2}^1$	D_1^2	81	0	100	0
24	$S_{1,2}^1$	D_2^2	100	0	70	0
25	$S_{1,2}^2$	D_1^1	90	0	123	0
26	$S_{1,2}^2$	D_2^1	150	703	123	0
27	0	1	425	0	425	0
28	0	2	425	0	425	0

and for HO 2:

$$\mathcal{P}_{21}(\sum_{p \in P^0_{D_1^2}}) = \frac{\sum_{p \in P^0_{D_1^2}} x_p - 150}{250 - 150}.$$

The computed optimal solution for the supply chain network for Example 2 without cooperation is reported in Table 2 and that for the supply chain network with cooperation is reported in Table 3.

In Example 2, the same links in the without cooperation supply chain network are at their capacities, in terms of the link flows, as in Example 1; that is, the procurement links. Also, in the case of cooperation, the storage links are at their capacities in both Examples 1 and 2, whereas link 26, corresponding to a shipment/distribution link is only at its capacity in Example 1 and not in Example 2.

The component of the total generalized cost TGC^{0*} not including the penalized expected shortages and surpluses is equal to 1,409,139, whereas the total generalized cost TGC^{0*}=494,335,328.

The component of the total generalized cost TGC^{1*} not including the penalized expected shortages and surpluses is equal to 1,498,029, whereas the total generalized cost $TGC^{*}1$=1,536,779.

The resulting synergy associated with cooperation for Example 2, $\mathcal{S}^{TGC} = 99\%$. With tighter estimates of the projected demand, a higher generalized total cost synergy is achieved. Furthermore, the needy now receive volumes of relief kits closer to the higher bound of the respective interval over which the probability distribution function is defined.

Example 3

In Example 3, we considered the situation where HO 1 is in a developed country with access to more resources, whereas HO 2 is in a developing country with fewer resources, and is also more susceptible/exposed to natural disasters and strife, with a greater number of victims requiring shelters.

The data for Example 3 was as in Example 2 except for the following: the capacities on certain procurement links were increased so that:

$$u_1 = 400, u_2 = 350, \quad u_7 = 350, u_8 = 350.$$

Also, in order to reflect that HO 1 has access to greater resources, the capacity on its storage link (link 5) was increased, so that now

$$u_5 = 600.$$

$\mathcal{P}_{11}$ and $\mathcal{P}_{12}$ remained as in Example 2, but, in order to reflect higher demand at demand points originally associated with HO 2 (cf. Figure 3) in Example 3 we had that

$$\mathcal{P}_{21}(\sum_{p \in P^0_{D^2_1}}) = \frac{\sum_{p \in P^0_{D^2_1}} x_p - 400}{500 - 400},$$

and

$$\mathcal{P}_{22}(\sum_{p\in P^0_{D^2_2}}) = \frac{\sum_{p\in P^0_{D^2_2}} x_p - 300}{400-300}.$$

The computed optimal solution for this example without cooperation is reported in Table 4, and that for this example with cooperation, in Table 5.

Table 4: Optimal Link Flows and Lagrange Multipliers for Example 3 without Cooperation

			Example 3	
Link a	From Node	To Node	f_a^*	β_a^*
1	1	M_1^1	207	0
2	1	M_2^1	200	0
3	M_1^1	$S_{1,1}^1$	207	0
4	M_2^1	$S_{1,1}^1$	200	1276
5	$S_{1,1}^1$	$S_{1,2}^1$	407	0
6	$S_{1,2}^1$	D_1^1	204	0
7	$S_{1,2}^1$	D_2^1	204	0
8	2	M_1^2	225.4	0
9	2	M_2^2	225.6	0
10	M_1^2	$S_{1,1}^2$	225	0
11	M_2^2	$S_{1,1}^2$	225	0
12	$S_{1,1}^2$	$S_{1,2}^2$	450	17549
13	$S_{1,2}^2$	D_1^2	274	0
14	$S_{1,2}^2$	D_2^2	176	0

The component of the total generalized cost TGC^{0*} not including the penalized expected shortages and surpluses is equal to 1,974,112 whereas the total generalized cost TGC^{0*}=2,574,611,712.

Whereas in Example 2 the total volume of delivered relief items was 850; in Example 3 the total volume is 1,050.

In Example 3, under cooperation, both HOs utilize the storage facilities to their capacities.

The component of the total generalized cost TGC^{1*} not including the penalized expected shortages and surpluses is equal to 2,108,016 whereas the total generalized cost TGC^{1*}=2,255,516.

The synergy $\mathcal{S}^{TGC}$ is again 99%, demonstrating the benefits of cooperation between HOs for disaster relief.

Table 5: Optimal Link Flows and Lagrange Multipliers for Example 3 with Cooperation

			Example 3	
Link a	From Node	To Node	f_a^*	β_a^*
1	1	M_1^1	131	0
2	1	M_2^1	131	0
3	M_1^1	$S_{1,1}^1$	150	0
4	M_2^1	$S_{1,1}^1$	150	0
5	$S_{1,1}^1$	$S_{1,2}^1$	600	12652
6	$S_{1,2}^1$	D_1^1	99	0
7	$S_{1,2}^1$	D_2^1	99	0
8	2	M_1^2	131	0
9	2	M_2^2	131	0
10	M_1^2	$S_{1,1}^2$	113	0
11	M_2^2	$S_{1,1}^2$	113	0
12	$S_{1,1}^2$	$S_{1,2}^2$	450	21084
13	$S_{1,2}^2$	D_1^2	234	0
14	$S_{1,2}^2$	D_2^2	153	0
15	1	M_1^2	131	0
16	1	M_2^1	131	0
17	2	M_1^1	131	0
18	2	M_2^1	131	0
19	M_1^1	$S_{1,1}^2$	113	0
20	M_2^1	$S_{1,1}^2$	113	0
21	M_1^2	$S_{1,1}^1$	150	0
22	M_2^2	$S_{1,1}^1$	150	0
23	$S_{1,2}^1$	D_1^2	200	12619
24	$S_{1,2}^1$	D_2^2	200	2565
25	$S_{1,2}^2$	D_1^1	32	0
26	$S_{1,2}^2$	D_2^1	32	0
27	0	1	525	0
28	0	2	525	0

Remark

The above examples are stylized but, nevertheless, yield managerial insights into the benefits of cooperation among humanitarian organizations for both them and for the needy that they serve. Of course, the models can be parameterized to particular disaster settings and scenarios. This would require obtaining the requisite data for the various supply chain network link cost functions as well as data associated with the demands at different points of demand for disaster relief item delivery/distribution.

In the supply chain disruption literature, catastrophic events with low probability of occurrence but high impact are receiving more and more attention due to their devastating and long-lasting effects (see, for example, Sheffi (2005) and Kleindorfer and Saad (2005)). Therefore, the estimation of the probability occurrence as well as the corresponding impacts of these events has attracted more research attention. Information such as historical data, expert opinion as well as simulation is often used in estimating the frequency, impact as well as the resulting costs of disaster events (Knemeyer et al. (2009)). As discussed by Van Wassenhove and Pedraza Martinez (2012), humanitarian logistics can achieve significant improvement by adopting successful supply chain management techniques and best practices. Hence, estimation methodologies used in commercial supply chain disruptions can be readily used in the planning of humanitarian logistics.

In terms of the demand uncertainty in humanitarian relief, as described by Van Wassenhove (2006), "Unlike logisticians in the private sector, humanitarians are always faced with the unknown. They do not know when, where, what, how much, where from and how many." Hence, recognizing and measuring the uncertainties in demand for the relief supplies is critical. Several studies have shown major advantages using certain methodologies in estimating demand. For example, by using the data from Center for Research and Epidemiology of Disasters, and historical disaster information, Cort et al. (2009) assess the affected area demand after accounting for the population growth over the years. The demand data is then used to fit into a distribution to be utilized for the humanitarian planning. Furthermore, in the case of slow-onset disasters and ensuing refugee crises, which were the focus of our numerical examples, data obtained from the tracking of movements of refugees towards shelter locations, such as that from social media or even the use of drones, may be utilized to assess prospective demand.

7. Summary and Conclusions and Suggestions for Future Research

In this Section, we present the summary and conclusions of the paper, which can lead to a few possible future research directions.

7.1 Summary and Conclusions

Natural catastrophes as well as large man-made disasters have been occurring at a record-breaking pace and scale in terms of the size of populations affected and the loss of assets, creating significant challenges for humanitarian organizations in their relief efforts. At the same time, humanitarian logistics practice is in dire need of operating more efficiently in order to have the relief products delivered to the affected population in a timely manner.

However, due, in part, to the lack of collaboration among many humanitarian organizations, it's been reported that service gaps and duplication occur frequently in emergency response. In this paper, based on previous research, but with a multiproduct extension of earlier work (cf. Nagurney (2009), Nagurney, Woolley, and Qiang (2010), Nagurney and Nagurney (2016), and Masoumi, Yu, and Nagurney (2017)), to include also stochastic components of uncertain supply chain link costs as well as demands, and link capacities, we construct pre-cooperation and horizontal cooperation multiproduct supply chain network models of humanitarian organizations. By recognizing the complexities in the relief efforts, the supply chain network models make use of a MV approach to capture risks and uncertainties in both costs and demands, which are typically fluctuating in the case of disasters.

We also propose a measure to capture the synergy resulting from the aforementioned cooperation. In addition, we propose a computational method that is used to compute solutions to the numerical examples in Section 5. For the numerical examples, we report the optimal link flows of relief items, the Lagrange multipliers associated with the links, as well as the total generalized cost without and with cooperation. The corresponding synergy for each example is also calculated. The numerical results support the theory, with positive synergy obtained in each of our examples. Moreover, the results demonstrate that victims also gain in that, with cooperation, a greater total number of relief items is delivered. Hence, both humanitarian organizations as well as the needy benefit when the former cooperate horizontally from a supply chain network operational perspective.

The parameters of the model can be calibrated with information/data specific to the region(s) of concern to provide guidance to humanitarian organizations as well as to cognizant governmental agencies.

7.2. Suggestions for Future Research

There are many possibilities for future research, including having the link capacities be uncertain and developing multiperiod models that consider disaster events such that the size of the affected population changes over time. This is particularly relevant in situations where refugees transverse borders to other countries for shelter and assistance. In addition, it would be interesting to incorporate explicit budget constraints of the HOs and to construct associated synergy measures. We leave such work for future research.

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Appendix

We here, for easy reference, recall the definitions of monotonicity and Lipschitz continuity of $F(X)$ as in (18).

Definition A1: Monotonicity

$F(X)$ is monotone if

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle \geq 0, \quad \forall X^1, X^2 \in \mathcal{L}. \tag{A1}$$

Definition A2: Lipschitz Continuity

$F(X)$ is Lipschitz continuous on $\mathcal{L}$ if the following condition holds:

$$\|F(X') - F(X'')\| \leq \bar{L}\|X' - X''\|, \quad \forall X', X'' \in \mathcal{L}, \tag{A2}$$

where $\bar{L} > 0$ is known as the Lipschitz constant.