Human Migration Networks and Policy Interventions: Bringing Population Distributions in Line with System-Optimization

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January 2020; revised May 2020
International Transactions in Operational Research (2021), **28(1)**, pp 5-26.

Abstract: In this paper, we demonstrate that, through policy interventions, in the form of subsidies, a system-optimum for a multiclass human migration network can be achieved, despite the migrants, which can be refugees, behaving in a user-optimized manner. The formulation and analysis are conducted using variational inequality theory. The policy intervention allows governmental decision-makers to moderate the flow of migrants while enhancing societal welfare. An algorithm is proposed and applied to compute the solutions to a series of numerical examples, with changes in initial populations and utility functions, inspired by a pandemic, followed by a natural disaster.

Keywords: human migration, networks, variational inequalities, policy interventions, system-optimization, user-optimization, subsidies

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1. Introduction

Massive human migrations are posing major challenges to national governments across the globe. The reasons for recent migrations include: violence, wars, and persecution, climate change, a variety of disasters (earthquakes, hurricanes, floods, and tornadoes), and poverty and economic inequality, with the latter driving humans to seek better lives for their families and themselves. According to the UNHCR (2020), 70.8 million have fled their homes worldwide, the highest level of displacement on record. The United Nations (2017) is reporting that the number of international migrants was an estimated 258 million individuals in 2017, with the total number of international migrants increasing by almost 50% since the new millennium. The number of refugees and asylum seekers during this period has increased from 16 to 26 million, approximately 10% of the international migrants.

The media has been filled with news and images of migrants, including refugees, often undertaking dangerous journeys on land and sea to flee their compromised situations. The economic collapse of Venezuela (Kennedy (2019)) has resulted in a surge of migrants as has the violence in many parts of Central America (Bartenstein and McDonald (2019)). Syria continues to be in a state of war with many seeking to escape their suffering (United Nations Refugee Agency (2019)). And now Libya is attracting the attention of many governments, with Sakuma (2020) reporting that the largest flow of modern African migration is through Libya, which is serving as a gateway to Europe, via the Mediterranean Sea. Furthermore, natural disasters (cf. Nagurney and Qiang (2009)) are also generating new migratory flows even from Puerto Rico (Cardona (2020)).

According to Jones (2018), Italy has now become the main route into Europe for economic migrants and asylum seekers, with hundreds of thousands risking their lives in their journeys from North Africa each year and thousands dying at sea. Indeed, the United Nations Refugee Agency (2015) documented a maritime refugee crisis with 137,000 crossing the Mediterranean Sea to Europe in the first half of that year, using highly risky transportation modes. In April 2015, 800 people perished in the largest refugee shipwreck ever recorded. Kitsantonis (2019) states that the numbers of asylum seekers making the short but often treacherous journey from Turkey across the Aegean Sea to Greece has seen a rise again, with Greek officials looking towards replacing overcrowded migrant camps with centers and hoping to restrict the migrants' movements. And, recent data is showing that Cyprus is now hosting the most refugees per capita in the European Union. (Stevis-Gridneff (2020)). As emphasized in Nagurney and Daniele (2019), closer attention must be paid to human migration problems in economic, political, sociological, and even environmental dimensions.

Indeed, how to effectively manage human migration flows has become one of the major challenges of the new millennium. The governments of many nations, hence, are now faced with identifying suitable policies and regulations to address a variety of human migration flows. In managing international migration flows, governments usually focus on distinct classes of migrants, such as: highly skilled workers, dependents of migrant workers, irregular migrants, and refugees and asylum seekers (cf. Karagiannis (2016)). The United Nations, Department of Economic and Social Affairs, Population Division (2017) has compiled a list of high level policies of various countries regarding international migration. For example, policies associated with "irregular migration" include fines, detention or deportation of migrants in an irregular situation, as well as penalizing employers of such migrants. The International Organization of Migration (2020a) defines irregular migration as the movement of persons that takes place outside the laws, regulations, or international agreements governing the entry into or exit from the origin, transit or destination location; see also Karagiannis (2016). The United Nations (2013) emphasizes that migration policies in both origin and destination countries play an important role in determining the migratory flows.

Furthermore, the various human migration crises have even influenced the United Nation's 2030 Sustainable Development Goals (SDGs). In particular, the 2030 Agenda for Sustainable Development consists of a set of 17 SDGs and 169 targets (see United Nations Foundation (2020)). Migration is specifically referenced in seven of the SDGs and is also a cross-cutting issue, relevant to all 17 goals (cf. International Organization for Migration (2020b)).

2. Literature Review and Our Contributions

Clearly, the rigorous modeling of human migration networks with the inclusion of appropriate policies for the evaluation of outcomes merits serious attention. Although there have been efforts to construct computable models of human migration with those that are network-based garnering resurgent attention, the incorporation of associated policies has only been minimally explored. We first provide a literature review of the relevant models and then outline the contributions in this paper.

2.1 Literature Review

We now provide an overview of the relevant literature on human migration models, with a focus on networks. Specifically, it is important to mention that the network models proposed, to-date, have primarily been from the perspective of individual migrants making their decisions, which can be characterized as being that of "user-optimization" and leading to an equilibrium that includes the utilities associated with migration. Rahmati and Tularam

(2017) provide a critical review of a variety of theoretical frameworks for human migration models, which also reference foundational network equilibrium models. These authors highlight various mathematical theories of migration, focusing on macro-level, micro-level, and meso-level approaches. The classical network equilibrium model of human migration is that of Nagurney (1989). It is a multiclass model, and is isomorphic to a traffic network equilibrium with special structure (see also Nagurney (1999)). In that model, migrants of each class distribute themselves among the locations according to maximal utilities associated with the locations. The population of each class is assumed to be fixed and the utility functions are assumed to be concave and a function of the populations at the locations. Subsequently, Nagurney (1990) generalized that model to include migration costs that depend on the flows along with the governing equilibrium conditions.

Nagurney, Pan, and Zhao (1992a), continuing in the user-optimization vein of human migration modeling, constructed a multiclass human migration model, which they later extended to include class transformations (Nagurney, Pan, and Zhao (1992b)). Pan and Nagurney (1994) developed a multi-stage, Markov chain model and identified the connection between a sequence of variational inequalities and a non-homogeneous Markov chain. In all of the above papers, the specific governing equilibrium conditions were formulated as finite-dimensional variational inequality problems. Pan and Nagurney (2006) were the first to apply evolution variational inequalities to model the underlying dynamics of human migration and also to discuss associated algorithms (see also Daniele (2006)).

A conjectural variations equilibrium (CVE) for human migration was promulgated by Kalashnikov et al. (2008) (see, e.g., Isac, Bulavsky, and Kalashnikov (2002)). The authors reported results of numerical experiments based on population data in locations in Mexico. Cappello and Daniele (2019) developed a Nash equilibrium model of human migration with features of conjectural variations. In their model, the utility and migration cost functions for a given class depend on the populations and flows, respectively, of that class. The authors included a numerical example with sensitivity analysis inspired by the flow of migrants from Africa to Italy in 2018. And, as noted in Nagurney and Daniele (2019), Causa, Jadamba, and Raciti (2017) extended the Nagurney (1990) model to include uncertainty in the utility functions, the migration cost functions, and the populations. However, none of the above cited models of human migration included regulations and/or policies. Furthermore, they are all essentially user-optimized models.

As mentioned earlier, there has been very limited modeling work done on including policies and/or regulations in human migration networks. To-date, the only papers that we are aware of are those by Nagurney and Daniele (2019) and Nagurney, Daniele, and Nagurney

(2019). These models incorporated explicit constraints that could be imposed by governmental authorities to put capacities on the flows of different classes of migrants from certain origins to specific destinations. In these models it was found that the utility of those that were restricted decreased, whereas those that were allowed to migrate increased. The latter paper also allowed for routes of migration consisting of links that capture congestion and associated costs and can handle even migration through different nations. These models were inherently user-optimized ones.

From a governmental and societal perspective, it is worthwhile to consider also system-optimization for human migration networks. User-optimization (U-O) and system-optimization (S-O) are seminal concepts and constructs in transportation and so named by Dafermos and Sparrow (1969). They correspond, respectively, to Wardrop's two principles of travel behavior (Wardrop (1952)). In the case of system-optimization in congested urban transportation networks, a central controller routes the traffic between origin/destination pairs of nodes so that the total cost to society is minimized (and the demands are satisfied). In the case of user-optimization, travelers, behaving selfishly, seek their individual cost-minimal routes of travel between an origin/destination pair of nodes, also subject to the demand constraints being satisfied. Historically, the system-optimization problem has been easier to formulate and solve since it consists of a nonlinear programming problem with a single objective function. The user-optimized problem, with connections to Nash equilibria (1950, 1951), in contrast, has stimulated the development and application of novel mathematical methodologies, including variational inequality theory (see Dafermos (1980), Nagurney (1999), and the references therein). For novel applications of variational inequality theory to cybersecurity investments, see Colajanni et al. (2018); to competitive pharmaceutical supply chains and quality issues, see Nagurney, Li, and Nagurney (2018); to blood supply chains, see Nagurney and Dutta (2019), and to disaster relief - Gossler et al. (2019).

It is important to recognize that there are policies that, when imposed, alter the behavior of travelers, who are engaged in more short-term movement behavior than migrants. In the case of transportation, for example, the imposition of congestion tolls (see Dafermos and Sparrow (1969), Dafermos and Sparrow (1971), Dafermos (1973), Lawphongpanich, Hearn, and Smith (2006)), when done appropriately, results in system-optimized flows also being user-optimized. Hence, once the tolls are imposed, travelers, acting independently, will select routes of travel which result in a system optimum, that minimizes the total cost to the society. In this paper, we construct policies for human migration networks that maximize societal welfare.

Trapp et al. (2018) focused on the resettlement of refugees using optimization in a system perspective. Our work in this paper, in contrast, is focused on migration networks and demonstrating how, with appropriate policies, the system-optimized solution will, at the same time, be user-optimized. Hence, migrants of different classes will individually choose locations, while acting in their own self-interest, that are also optimal for society as a whole. Our approach explicitly recognizes individual behavior and choices. Papadaki et al. (2017) consider the growing stream of refugees from North Africa and the Middle East and emphasize that governmental policies are disjointed. They propose the development of a compact accommodation center comprised of shipping containers, to be utilized in one of the initial stages for refugee resettlement for the purposes of adaptation before full social integration of the displaced populations.

Robinson, Dilkina, and Moreno-Cruz (2020) focus on climate change and potential human migration in the United States due to sea level rise, using a data-driven model with implicit network concepts. However, in their interesting study, they note that because they utilize machine learning techniques, they give up the explanatory power of a migration model in favor of a more accurate prediction, with limits on being able to provide specific policy recommendations. Here, in contrast, we are able to provide explicit formulae for policy recommendations and our framework can handle international migration. Bier, Zhou, and Du (2019) also consider sea level rise due to climate change and propose a game theory model with the government as the first mover encourage pre-disaster relocation by residents living in areas at high risk of flooding due to sea-level rise. The authors observe that offering a subsidy (e.g., a partial buyout) can be effective if the government has a significantly lower discount rate than residents. The authors consider homogeneous residents and the timing of their relocations. Here, in contrast, we consider migrants of distinct classes and show how appropriate subsidies can bring population distributions in line with system-optimization. Moreover, our framework can capture the impact of disasters through changes in the utility functions associated with the classes at the various locations in the network economy as well as the changes in the initial fixed population of each class and, hence, also the final population distributions.

2.2 Our Contributions and Organization of the Paper

In this paper, we first propose a system-optimized multiclass network model of human migration. The model assumes no births and no deaths and there is a fixed population of each class of migrant. There are no costs associated with migration, since we are interested in the long term. There is a utility function associated with each class of migrant and each location in the network economy. Each such utility is a function of the populations of the classes at that location and at other locations, in general. The objective in the network economy is to optimize the system and the societal welfare by maximizing the total utility, subject to the conservation of flow. The total utility is equal to the sum for all classes and all locations of the product of the individual utility at the location and class times the population of that class at the location. We assume that the total utility function is concave and continuously differentiable and show that the system-optimized solution satisfies a variational inequality problem.

We then recall the user-optimized analogue of this human migration network, which was introduced in Nagurney (1989). Therein, the governing equilibrium conditions state that migrants of a class will keep on moving from location to location until the individual utility of each class at each location that is populated by that class is maximal and equalized. Hence, at the equilibrium, noone has any incentive to change his location. We also, for completeness, provide the variational inequality formulation of the user-optimized (equilibrium) solution.

We propose a ratio for assessing the societal welfare loss if migrants select their locations based on U-O rather than S-O behavior. The price of anarchy, introduced by Koutsoupias and Papadimitriou (1999), and later so named by Papadimitrou (2001), is an inspiration for the societal welfare loss ratio. The price of anarchy was originally constructed to measure the total cost evaluated at the U-O solution and divided by the total cost evaluated at the S-O solution (see also Roughgarden (2005)). Here, in contrast, we focus on total utility maximization in the network economy.

In this paper, we provide a procedure for computing subsidies that a government (or governments) can impose to guarantee that the system-optimized multiclass population distribution in the network economy is also user-optimized. These policies, when imposed, guarantee that individuals will choose their locations in a manner so that the societal welfare is maximized.

The paper is organized as follows. In Section 3, the multiclass system-optimized model of human migration is presented and its user-optimized analogue. The optimal solutions to both satisfy an appropriate variational inequality problem. The societal welfare loss ratio is proposed and an illustrative example presented in order to reinforce the basic concepts. In Section 4, we outline the procedure for determining the subsidies for the different migrant classes and locations, so that, when applied, the S-O multiclass population distribution in the network economy is, at the same time, U-O. We also discuss how the framework can be used post a disaster. An algorithm is proposed in Section 5 and convergence results provided. The algorithm is then applied in Section 6 to compute the solutions to a series of numerical examples, with changes in the demands, and in the utility functions, in order to also address the possible impacts of a pandemic, followed by a natural disaster. We summarize our results and present our conclusions in Section 7.

3. The System-Optimization Migration Network Model

In the network economy there are n locations at which the different classes in the population can locate. We assume freedom of movement between locations at zero cost, since here we are focusing on the long-term equilibrium population distributions at the various locations. We consider J classes of migrants with a typical class denoted by k. The network representation is given in Figure 1. Note that we associate locations with links (rather than nodes). Each link i; $i = 1, \ldots, n$, has an associated utility for each class denoted by U_i^k. The utility functions capture how attractive location i, $\forall i$, is for an individual of class k, $\forall k$. The relevant notation is given in Table 1. All vectors here are assumed to be column vectors.

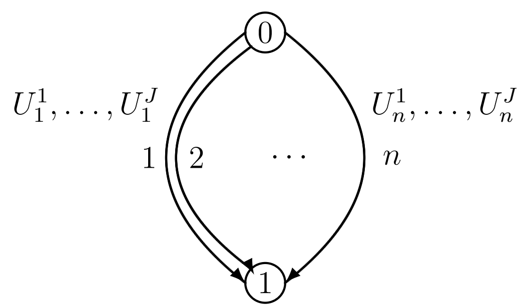

Figure 1: Network Structure of the Human Migration Model

Table 1: Notation for the Human Migration Models

Notation	**Definition**
p_i^k	the population of class k at location i. The $\{p_i^k\}$ elements are grouped into the vector $p^k \in R_+^n$. We then further group the p^k vectors; $k = 1, \ldots, J$, into the vector $p \in R_+^{Jn}$.
P^k	the population of class k in the network economy; $k = 1, \ldots, J$.
$U_i^k(p)$	the utility perceived by individuals of class k at location i; $i = 1, \ldots, n$. We group the utility functions for each k into the vector $U^k \in R^n$ and then group all such vectors for all k into the vector $U \in R^{Jn}$.
$\hat{U}_i^k(p)$	the total utility perceived by class k at location i; $i = 1, \ldots, n$. The total utility $\hat{U}_i^k(p) = U_i^k(p) \times p_i^k$; $k = 1, \ldots, J; i = 1, \ldots, n$.

The conservation of flow equation for each class k; $k = 1, \ldots, J$, is:

$$\sum_{i=1}^{n} p_i^k = P^k. \tag{1}$$

Hence, the population distribution of each class among the various locations must sum up to the population of that class in the network economy.

Moreover, the populations must be nonnegative, that is,

$$p_i^k \geq 0, \quad \forall i; \forall k. \tag{2}$$

Note that, according to Table 1, the total utility associated with a location and class, may, in general, depend upon the distribution of not only that particular class but also on that of the other classes. This is very reasonable: migrants may wish to locate where there is a certain concentration of those similar to themselves at the same location and, in proximity. At the same time, because of cultural and even economic factors they may wish to distance themselves from other classes.

We define the feasible set $K^1 \equiv \{p|\, (1)\, \text{and}\, (2)\, \text{hold.}\}$.

3.1 The System-Optimization (S-O) Problem

In the system-optimization (S-O) problem, the goal is to maximize the total utility in the network economy. This is achieved when the following optimization problem is solved:

$$\text{Maximize} \quad \sum_{k=1}^{J} \sum_{i=1}^{n} \hat{U}_i^k(p) = \sum_{k=1}^{J} \sum_{i=1}^{n} U_i^k(p) \times p_i^k \tag{3}$$

subject to the conservation of flow constraints (1) for all k and the nonnegativity ones (2).

Observe that the objective function (3) captures the total utility of the society/economy under study. Locations i can correspond to different countries; to different regions in different countries, or to regions within a country, if the focus is on within country movements/migrations.

Under the assumption that the total utility functions for all the classes at all the locations are concave, and are continuously differentiable, we know that the optimal solution, denoted by p', satisfies the following variational inequality (VI): determine $p' \in K^1$, such that

$$-\sum_{k=1}^{J} \sum_{i=1}^{n} \left[\sum_{l=1}^{J} \sum_{j=1}^{n} \frac{\partial \hat{U}_j^l(p')}{\partial p_i^k} \right] \times (p_i^k - p_i^{k'}) \geq 0, \quad \forall p \in K^1. \tag{4}$$

Furthermore, if all the utility functions are strictly concave, it follows that the optimal population distribution p' is unique.

It is interesting to remark that, since $\hat{U}_i^k(p) = U_i^k(p) \times p_i^k$, then variational inequality (4) is equivalent to

$$-\sum_{k=1}^{J}\sum_{i=1}^{n}\sum_{l=1}^{J}\sum_{j=1}^{n}\frac{\partial U_j^l(p')}{\partial p_i^k} \times (p_i^k - p_i^{k'}) - \sum_{k=1}^{J}\sum_{i=1}^{n} U_i^k(p') \times (p_i^k - p_i^{k'}) \geq 0, \quad \forall [\in K^1$$

and the second term coincides with (6).

3.2 The User-Optimization (U-O) Problem

As mentioned in the Introduction, it may be challenging, if nearly impossible, to reallocate the various class populations so that the system-optimal solution is achieved (unless one is living in a totalitarian state). This is especially the case, since migrants may not individually care about such a solution but may act selfishly in order to achieve an optimal solution for themselves "individually." In the case of user-optimization (U-O), it is assumed that the migrants are rational and that migration will continue until no individual of any class has any incentive to move since a unilateral decision will no longer yield an increase in the utility. The governing solution will satisfy the following migration equilibrium conditions proposed by Nagurney (1989); see also Nagurney (1999): Mathematically, a multiclass population vector $p^* \in K^1$ is said to be in equilibrium if for each class k; $k = 1, \ldots, J$; $i = 1, \ldots, n$:

$$U_i^k(p^*) \begin{cases} = \lambda^k, & \text{if} \quad p_i^{k^*} > 0 \\ \leq \lambda^k, & \text{if} \quad p_i^{k^*} = 0. \end{cases} \tag{5}$$

Equilibrium conditions (5) state that, for a given class k, only those locations i with maximal utility equal to an indicator λ^k will have a positive population of the class. Furthermore, the utilities for a given class at populated locations by that class are equalized, that is, equilibrated across the locations. We note that λ^k is, in actuality, the Lagrange multiplier associated with constraint (1) for k. Indeed, from the duality theory (see Barbagallo et al. (2014) and Colajanni et al. (2018)), we have that the multipliers $\bar{\lambda}$ and $\bar{\mu}$ associated with constraints (1) and (2) respectively satisfy the following conditions:

$$U_i^k(p^*) = -\bar{\lambda}^k + \bar{\mu}_i^k, \quad i = 1, \ldots, n, k = 1, \ldots, J; \quad \bar{\mu}_i^k p_i^{k*} = 0, \quad \bar{\mu}_i^k \geq 0$$

and, hence, we obtain (5).

As shown in the above noted references, the equilibrium p^* satisfies the variational inequality problem: determine $p^* \in K^1$ such that

$$-\sum_{k=1}^{J}\sum_{i=1}^{n} U_i^k(p^*) \times (p_i^k - p_i^{k*}\rangle \geq 0, \quad \forall p \in K^1. \tag{6}$$

Clearly, the solution p^* to VI (6) can be expected to be distinct from the solution p' to VI (4).

3.3 A Simple Example

In order to reinforce the above concepts we present an example for which both the system-optimized and the user-optimized solutions are provided. We consider a network economy consisting of two locations and a single class; hence, we suppress the superscript notation. The total population is: $P = 100$ and the utility functions at the two locations are:

$$U_1(p) = -p_1 + 200, \quad U_2(p) = -p_2 + 220.$$

The user-optimized solution is:

$$p_1^* = 40, \quad p_2^* = 60,$$

yielding $\lambda = 160$, since

$$U_1(p^*) = U_2(p^*) = 160.$$

Observe that VI (6) is satisfied by this p^*. Note that, at this population distribution, the total utility: $\hat{U}_1 + \hat{U}_2 = 16,000$.

On the other hand, the system-optimized solution is:

$$p_1' = 45, \quad p_2' = 55,$$

and VI (4) is satisfied since

$$-\frac{\partial \hat{U}_1(p')}{\partial p_1} = -\frac{\partial \hat{U}_2(p')}{\partial p_2} = -110.$$

The corresponding total utility is: $16,050$. Clearly, $16,050 > 16,000$. Furthermore, at the system-optimized solution, we have that:

$$U_1(p') = 155, \quad U_2(p') = 165$$

and, clearly, the S-O solution is not U-O. Hence, without appropriate policy interventions, and, if humans are "free" to move/migrate, the S-O solution would be difficult to sustain. Indeed, one can expect that some of those at location 1 will migrate to location 2 until the U-O solution is achieved since the utility in location 2 is higher than in location 1 and people will move; so the initial S-O solution $(45, 55)$ would become $(40, 60)$.

3.4 The Societal Welfare Loss Ratio

We now construct the societal welfare loss ratio. Note that the classical price of anarchy focused on total cost minimization in the congested transportation/telecommunications network context. Here, in contrast, we are concerned with total utility maximization in a network economy and the associated societal welfare loss under U-O as opposed to S-O behavior.

Our societal welfare loss ratio π is defined as follows:

$$\pi = \frac{TU(p^*)}{TU(p')}, \tag{7}$$

where TU denotes the total utility such that

$$TU = \sum_{k=1}^{J} \sum_{i=1}^{n} \hat{U}_i^k; \tag{8}$$

and recall that p^* is the population distribution pattern under the U-O solution for a given migration network problem and p' is that for the S-O solution for the problem. The ratio quantifies the societal welfare loss if migrants select their U-O destinations rather than being allocated to locations under S-O. Note that here we consider multiple classes of migrants in our ratio.

For the above numerical example, we have that:

$$\pi = 16,000/16,050 = .997.$$

The value of π in (7) can never exceed 1, since the highest societal total utility TU is achieved at p', the S-O solution. The smaller the value of the societal welfare loss ratio π, the greater the welfare loss to society under the U-O solution.

4. Policy Intervention in the Form of Subsidies

We now proceed to ask the following question. Is there a migration policy that, when applied, can make the system-optimized solution also a user-optimized one? If so, an application of such a policy would result in no users having any incentive to switch their locations, and the population distribution would be one that is also optimal for the society (system-optimized).

The answer is: Yes! The derivation and construction of such a migration policy is as follows. We first solve for the system-optimized solution p'. For each class k, we denote those locations with a positive population by $k_1, \ldots, k_{n_k}$, where n_k is the number of locations in the network economy with a positive population of class k. We also introduce notation for subsidies associated with the different locations for each class denoted by class k by: $(subsidy)_{k_1}$, $(subsidy)_{k_2}$, ..., $(subsidy)_{k_{n_k}}$. We can then list those location as:

$$U^k_{k_1}(p') + subsidy^k_{k_1} = \mu^k,$$

$$U^k_{k_2}(p') + subsidy^k_{k_2} = \mu^k, \tag{9a}$$

and so on until

$$U^k_{k_{n_k}}(p') + subsidy^k_{k_{n_k}} = \mu^k,$$

where μ^k is the incurred utility for class k after the subsidies are distributed for the class at the locations with positive populations of that class. Also, we can number those locations for that class with zero populations of that class (if there are any) as follows:

$$U^k_{k_{n_{k+1}}}(p') + subsidy^k_{k_{n_k+1}} \leq \mu^k,$$

and so on until

$$U^k_{k_n}(p') + subsidy^k_{k_n} \leq \mu^k. \tag{9b}$$

According to (9a) and (9b), the cognizant government authority selects the μ^k for each class k, and then the subsidy for each location for that class is easily determined via subtraction.

The question now arises as to what value is reasonable for μ^k? We propose that μ^k be set as: $\max_{k_l; l=1,\ldots,n_k} U^k_{k_l}(p')$. This procedure guarantees that all the subsidies will be nonnegative and that all enjoy the maximal utility for each class at all the populated locations. Also, for the subsidies associated with locations with no populations of a class k (see (9b)), those subsidies are set to zero.

Returning to the above simple example and, again suppressing superscripts since there is a single class, the above subsidy formulae simplify to:

$$U_1(p') + subsidy_1 = \mu,$$

$$U_2(p') + subsidy_2 = \mu,$$

or

$$155 + subsidy_1 = 165,$$

$$165 + subsidy_2 = 165.$$

Location 2 has a subsidy of zero, whereas those at location 1 receive a subsidy of 10. It is always the case that, with the above procedure, the location(s) with the highest utility at the system-optimized solution are not subsidized, since it is wasteful to do so. Of course, according to (9a) the government is "free" to set μ^k as high as it is willing to and the budget allows.

In terms of practical implementation, we note that congestion pricing in the form of tolls has achieved success around the world with notable examples including the cities of Gothenburg and Stockholm in Sweden, London in the United Kingdom, as well as Singapore, with tolls even on the horizon in New York City (see Smolens (2019), Short (2019)).

We view the provision of the above subsidies as investments by government(s) that might help to alleviate various migrant and refugee crises around the globe. As for the budgets, if an individual government falls short, provision should be provided by a supra authority such as the World Bank, the United Nations, or if in Europe, the European Union.

4.1 Applying the Framework Post a Disaster

Disasters, as mentioned in the Introduction, may lead to migrations of humans. Of course, it is important to emphasize that there are both slow-onset disasters (certain wars, droughts, famine, pestilence, etc.), as well as sudden-onset disasters (earthquakes, hurricanes, tsunamis, floods, etc.). For an edited volume on dynamics of disasters, see Kotsireas, Nagurney, and Pardalos (2016).

In the case of a disaster, one would expect to encounter changes in the utility functions associated with locations that the disaster has impacted and such changes may affect certain classes more or less. In particular, one would expect that, in general, the location(s) would become less attractive because of compromised infrastructure, loss of resources and amenities, and even perhaps dangerous conditions. The above system-optimization model can then be adapted to incorporate the modified utility functions and appropriate subsidies provided.

Similarly, once a disaster strikes, there may be major loss of life and that would then affect the population in the network economy of one or more classes of migrants. The system-optimization model can then be rerun to identify the new optimal multiclass population distribution and assign the subsidies accordingly.

The above system-optimization model and policy intervention are relevant to a network economy even prior to a disaster since individuals may wish to seek better lives for themselves by changing their locations within a country or across countries and governments may wish to intervene to enhance the societal welfare in terms of the population distribution and associated total utility.

5. Computation of the S-O Multiclass Population Distribution Pattern

In order to determine the subsidies to make the S-O behavior, after imposition, also U-O, we first must compute the system-optimizing flow pattern.

For purposes of standardizing the mechanism, we put variational inequality (4) into standard form (Nagurney (1999)): determine $X^{**} \in \mathcal{K} \subset R^N$ such that:

$$\langle F(X^{**}), X - X^{**} \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{10}$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in N-dimensional Euclidean space. $F(X)$ is a given continuous function such that $F(X) : X \to \mathcal{K} \subset R^N$. $\mathcal{K}$ is a closed, bounded, and convex set.

We define the vector $X \equiv p$ and the vector $F(X)$ with elements: $F_{k,i}(p) \equiv \sum_{l=1}^{J} \sum_{j=1}^{n} -\frac{\partial \hat{U}_j^l(p)}{\partial p_i^k}$; $k = 1, \ldots, J$; $i = 1, \ldots, n$. The feasible set $\mathcal{K} \equiv K^1$ and $N = Jn$. Then, clearly, VI (4) can be put into the standard form (10) with $X^{**} = p'$. Similarly, VI (6) can also be put into standard form with X and $\mathcal{K}$ as above and with the components of its $F(X)$ given by $-U_i^k(p)$; $\forall k$, $\forall i$, and with $X^{**} = p^*$.

We emphasize that there exists a solution to both VI (4) and VI (6) since the underlying feasible set is compact and the corresponding function that enters the variational inequality, $F(X)$ is continuous under our imposed assumptions (cf. Kinderlehrer and Stampacchia (1980)).

We apply the Euler method to compute the solutions to the numerical examples in the next section. The Euler method is induced by the general iterative scheme of Dupuis and Nagurney (1993). Specifically, iteration τ of the Euler method (see also Nagurney and Zhang (1996)) is given by:

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - a_{\tau} F(X^{\tau})), \tag{11}$$

where $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$ and F is the function that enters the variational inequality problem (10).

As discussed in Nagurney and Zhang (1996), the Euler method is, in fact, a discrete-time approximation to the continuous-time trajectories associated with a projected dynamical system, whose set of stationary points coincides with the set of solutions to the corresponding variational inequality problem. In the multiclass human migration network context here, this means that there is an associated projected dynamical system to both VI (4) and to VI (6). Projected dynamical systems, as noted by Dupuis and Nagurney (1993)), are nonclassical in that the right-hand side is discontinuous, but capture the underlying feasible set corresponding to the constraints of the given problem.

Dupuis and Nagurney (1993) proved that, for convergence of the general iterative scheme, which induces the Euler method, the sequence $\{a_\tau\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \to 0$, as $\tau \to \infty$. Specific conditions for convergence of the Euler method within many network-based models can be found in Nagurney and Zhang (1996) and in Nagurney (2006) and the references therein.

We now provide the convergence result.

Theorem: Convergence

In the S-O model of human migration constructed above let $F(X)$ be strictly monotone at any equilibrium pattern. Also, assume that F is uniformly Lipschitz continuous. Then there exists a unique S-O population distribution pattern $p' \in \mathcal{K}$ and any sequence generated by the Euler method as given by (11), with $\{a_\tau\}$ satisfies $\sum_{\tau=0}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \to 0$, as $\tau \to \infty$ converges to p', satisfying VI (4).

The proof follows from Theorem 5.8 in Nagurney and Zhang (1996).

The analogue of the above Theorem for VI (6) easily follows.

In view of the special network structure underlying our multiclass model (cf. Figure 1), it makes sense to use an algorithm for the solution of the encountered separable quadratic programming problems at each iteration of the above Euler method. Note that (11), because of the network structure of the feasible set $\mathcal{K}$, consists of J separable quadratic programming problems, one for each class k, and subject to the conservation of flow constraint (1) and the nonnegativity constraints (2) for each class k. Specifically, for these network subproblems of special structure, we propose the use of the exact equilibration algorithm (cf. Dafermos and Sparrow (1969) and Nagurney (1999)). This algorithm yields the exact solution at each iteration and guarantees that the conservation of equations (1) and (2) are satisfied.

6. Numerical Examples

We now present several numerical examples, which are solved using the Euler method outlined in the preceding section. The algorithm was implemented in FORTRAN and the system used was a Unix system at the University of Massachusetts Amherst. The series

$\{a_\tau\}$ in the algorithm was set to: $.1(1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \ldots$. the convergence tolerance ϵ was 10^{-5}, that is, the algorithm was deemed to have converged when the absolute value of each of the computed population values at two successive iterations was less than or equal to .00001.

There are two classes of migrants and five locations in all the numerical examples. We report the computed U-O and the S-O solutions, as well as the subsidies, for each of them.

Numerical Example 1

This example serves as the baseline for all the other examples in this section.

The fixed populations in the network economy of the two classes are, respectively:

$$P^1 = 1,000.00 \quad P^2 = 2,000.00.$$

The utility functions and the total utility functions for class 1 are:

$$U_1^1(p) = -2p_1^1 - .2p_1^2 + 2,000, \quad \hat{U}_1^1(p) = -2(p_1^1)^2 - .2p_1^2p_1^1 + 2,000p_1^1,$$
$$U_2^1(p) = -3p_2^1 - .1p_2^2 + 1,500, \quad \hat{U}_2^1(p) = -3(p_2^1)^2 - .1p_2^2p_2^1 + 1,500p_2^1,$$
$$U_3^1(p) = -p_3^1 - .3p_3^2 + 3,000, \quad \hat{U}_3^1(p) = -(p_3^1)^2 - .3p_3^2p_3^1 + 3,000p_3^1,$$
$$U_4^1(p) = -p_4^1 - .2p_4^2 + 2,500, \quad \hat{U}_4^1(p) = -(p_4^1)^2 - .2p_4^2p_4^1 + 2,500p_4^1,$$
$$U_5^1(p) = -2p_5^1 - .3p_5^2 + 4,000, \quad \hat{U}_5^1(p) = -2(p_5^1)^2 - .3p_5^2p_5^1 + 4,000p_5^1.$$

The utility functions and the total utility functions for class 2 are:

$$U_1^2(p) = -p_1^2 - .4p_1^1 + 4,000, \quad \hat{U}_1^2(p) = -(p_1^2)^2 - .4p_1^1p_1^2 + 4,000p_1^2,$$
$$U_2^2(p) = -2p_2^2 - .6p_2^1 + 3,000, \quad \hat{U}_2^2(p) = -2(p_2^2)^2 - .6p_2^1p_2^2 + 3,000p_2^2,$$
$$U_3^2(p) = -p_3^2 - .2p_3^1 + 5,000, \quad \hat{U}_3^2(p) = -(p_3^2)^2 - .2p_3^1p_3^2 + 5,000p_3^2,$$
$$U_4^2(p) = -2p_4^2 - .3p_4^1 + 4,000, \quad \hat{U}_4^2(p) = -2(p_4^2)^2 - .3p_4^1p_4^2 + 4,000p_4^2,$$
$$U_5^2(p) = -p_5^2 - .4p_5^1 + 3,000, \quad \hat{U}_5^2(p) = -(p_5^2)^2 - .4p_5^1p_5^2 + 3,000p_5^2.$$

We first computed the U-O solution since it is interesting to compare it with the S-O solution. If the migrants are free to move between locations (and no subsidies are provided), the U-O solution satisfying VI (6) is:

Class 1 U-O Population Distribution

$$p_1^{1*} = 0.00, \quad p_2^{1*} = 0.00, \quad p_3^{1*} = 167.31, \quad p_4^{1*} = 41.68, \quad p_5^{1*} = 791.01.$$

Class 2 **U-O Population Distribution**

$$p_1^{2*} = 415.89, \quad p_2^{2*} = 0.00, \quad p_3^{2*} = 1,382.41, \quad p_4^{2*} = 201.69, \quad p_5^{2*} = 0.00.$$

For class 1, the incurred utility at equilibrium at the populated locations 3, 4, and 5 is $2,417.98$ and it is lower at locations 1 and 2 - $1,916.82$ and $1,500.00$, respectively. For class 2, the incurred utility at equilibrium at the populated locations 1, 3, and 4 is $3,584.11$. At location 2, the utility of class 2 is $3,000.00$ and at location 5 it is $2,683.60$.

Neither class, under U-O behavior, elects to migrate to and locate at location 2. Class 1 migrants only locate at locations 3 through 5. Only those of class 2 locate at location 1, in equilibrium, whereas only those of class 1 locate at location 5.

Now we present the S-O solution for this problem, which satisfies VI (4).

Class 1 **S-O Population Distribution**

$$p_1^{1'} = 0.00, \quad p_2^{1'} = 0.00, \quad p_3^{1'} = 120.43, \quad p_4^{1'} = 314.39, \quad p_5^{1'} = 565.19.$$

Class 2 **S-O Population Distribution**

$$p_1^{2'} = 606.48, \quad p_2^{2'} = 53.23, \quad p_3^{2'} = 1,076.35, \quad p_4^{2'} = 263.94, \quad p_5^{2'} = 0.00.$$

We see that in the S-O solution, location 2 is now populated by class 2. Locations 1 and 2 remain unpopulated under S-O for class 1 as does location 5 for class 2.

Using the procedure outlined in Section 4 (cf. (9a) and (9b)), we set $\mu^1 = 2,869.63$ since the highest utility under the S-O flow pattern for class 1 is that at location 5 and it is equal to the above value. Also, we set $\mu^2 = 3,899.56$ since the highest utility for class 2 under the S-O flow pattern is achieved at location 3 and it is equal to $3,899.56$.

We then, via subtraction of the particular location and class utility evaluated at the S-O population pattern, obtain the following subsidies:

Class 1 Subsidies

$$subsidy_1^1 = 0.00, \quad subsidy_2^1 = 0.00, \quad subsidy_3^1 = 312.96, \quad subsidy_4^1 = 736.80, \quad subsidy_5^1 = 0.00,$$

Class 2 Subsidies

$$subsidy_1^2 = 506.04, \quad subsidy_2^2 = 1,006.03, \quad subsidy_3^2 = 0.00, \quad subsidy_4^2 = 521.75, \quad subsidy_5^2 = 0.00.$$

In order to verify the above theory we modified the original utility functions by adding the above subsidies and solved for the U-O pattern, and the answer, as expected, was identical to the above reported S-O pattern. Hence, the policy of subsidies accomplishes what it was designed for.

Numerical Example 2

Numerical Example 2 has the same data as Numerical Example 1, except that the demands are now switched so that:

$$P^1 = 2,000.00 \quad P^2 = 1,000.00.$$

The U-O solution is now:

Class 1 **U-O Population Distribution**

$$p_1^{1*} = 0.00, \quad p_2^{1*} = 0.00, \quad p_3^{1*} = 631.39, \quad p_4^{1*} = 412.41, \quad p_5^{1*} = 956.21.$$

Class 2 **U-O Population Distribution**

$$p_1^{2*} = 63.16, \quad p_2^{2*} = 0.00, \quad p_3^{2*} = 936.84, \quad p_4^{2*} = 0.00, \quad p_5^{2*} = 0.00.$$

For class 1, the incurred utility at equilibrium at the populated locations 3, 4, and 5 is $2,087.59$ and it is lower at locations 1 and 2 - $1,987.37$ and $1,500.00$, respectively. For class 2, the incurred utility at equilibrium at the populated locations 1 and 3 is $3,936.89$. At location 2, the utility of class 2 is $3,000.00$, at location 4 it is $3,876.28$, and at location 5 it is $2,617.52$.

The computed S-O solution, which is also utilized to determine the subsidies is:

Class 1 **S-O Population Distribution**

$$p_1^{1'} = 205.08, \quad p_2^{1'} = 75.92, \quad p_3^{1'} = 265.89, \quad p_4^{1'} = 714.23, \quad p_5^{1'} = 738.89.$$

Class 2 **S-O Population Distribution**

$$p_1^{2'} = 225.39, \quad p_2^{2'} = 0.00, \quad p_3^{2'} = 720.43, \quad p_4^{2'} = 54.18, \quad p_5^{2'} = 0.00.$$

Again, we see in this example that the S-O population pattern is quite different from the U-O one. In fact, at the U-O solution, class 1 only located at three locations: $3 - 5$, whereas, at the S-O solution, there was a positive population of this class at the S-O solution at all locations. Hence, both quantitatively and qualitatively, we can expect the U-O and the S-O solutions to differ, demonstrating the need for appropriate migration policies.

We now report the subsidies, noting that in this example, we set $\mu^1 = 2,522.23$ since the highest utility under the S-O flow pattern for class 1 is that at location 5 and it is equal to the above value. Also, we set $\mu^2 = 4,226.39$ since the highest utility for class 2 under the S-O flow pattern is achieved, again, at location 3.

The subsidies are now:

Class 1 Subsidies

$$subsidy_1^1 = 977.46, \quad subsidy_2^1 = 1,250.00, \quad subsidy_3^1 = 4.25, \quad subsidy_4^1 = 747.29, \quad subsidy_5^1 = 0.00,$$

Class 2 Subsidies

$$subsidy_1^2 = 533.81, \quad subsidy_2^2 = 0.00, \quad subsidy_3^2 = 0.00, \quad subsidy_4^2 = 549.01, \quad subsidy_5^2 = 1,521.94.$$

Numerical Example 3

In this example we consider a healthcare disaster hitting the network economy in the form of a pandemic. This example is inspired, in part, by the coronavirus outbreak emanating from Wuhan, China (Shih, Denyer, and Taylor (2020)). The data in this example was as in Numerical Example 1, except that now we assumed that 50% of the population of each class has perished, so that:

$$P^1 = 500.00 \quad P^2 = 1,000.00.$$

Note that in this example the utility functions remain unchanged since the disaster does not affect infrastructure, per se, and involves "only" loss of life.

Below we report the complete results for this example, as we have done for the others in this section. The U-O population distribution is now:

Class 1 **U-O Population Distribution**

$$p_1^{1*} = 0.00, \quad p_2^{1*} = 0.00, \quad p_3^{1*} = 0.00, \quad p_4^{1*} = 0.00, \quad p_5^{1*} = 500.00.$$

Class 2 **U-O Population Distribution**

$$p_1^{2*} = 0.00, \quad p_2^{2*} = 0.00, \quad p_3^{2*} = 1,000.00, \quad p_4^{2*} = 0.00, \quad p_5^{2*} = 0.00.$$

This solution is quite interesting. Note that all migrants of class 1 choose to locate exclusively at location 5 whereas those of class 2 all migrate, post the disaster, to location 3. There is a complete separation of these two classes under U-O behavior in the network economy.

Those of class 1 have a utility of $3,000.00$ at location 5, whereas those of class 1 have a utility of $4,000.00$ at location 3.

The computed S-O solution is:

Class 1 S-O Population Distribution

$$p_1^{1'} = 0.00, \quad p_2^{1'} = 0.00, \quad p_3^{1'} = 47.98, \quad p_4^{1'} = 43.17, \quad p_5^{1'} = 408.85.$$

Class 2 S-O Population Distribution

$$p_1^{2'} = 206.96, \quad p_2^{2'} = 0.00, \quad p_3^{2'} = 694.96, \quad p_4^{2'} = 98.08, \quad p_5^{2'} = 0.00.$$

In the S-O solution, there is more "spreading out" of the classes among the locations than in the U-O solution.

For this example, we set $\mu^1 = 3,182.31$ since the highest utility under the S-O flow pattern for class 1 is that at location 5 and it is equal to the above value. Also, we set $\mu^2 = 4,295.44$ since the highest utility for class 2 under the S-O flow pattern is achieved, again, at location 3.

The subsidies (see, again, (9a) and (9b)) are:

Class 1 Subsidies

$$subsidy_1^1 = 0.00, \quad subsidy_2^1 = 0.00, \quad subsidy_3^1 = 438.78, \quad subsidy_4^1 = 745.10, \quad subsidy_5^1 = 0.00,$$

Class 2 Subsidies

$$subsidy_1^2 = 502.40, \quad subsidy_2^2 = 0.00, \quad subsidy_3^2 = 0.00, \quad subsidy_4^2 = 504.56, \quad subsidy_5^2 = 0.00.$$

Numerical Example 4

Numerical Example 4 had the same data as that in Numerical Example 3 except that we now consider the impact of a natural disaster following the healthcare disaster. We assume that locations 3 and 5 are impacted so that the utility functions of both classes of migrants associated with these locations are modified as follows. The fixed term in each of the noted utility functions in Examples 1 through 3 is reduced by 50% yielding new associated utility functions for the classes at those locations of:

$$U_3^1(p) = -p_3^1 - .3p_3^2 + 1,500, \quad U_5^1(p) = -2p_5^1 - .3p_5^2 + 2,000,$$

$$U_3^2(p) = -p_3^2 - .2p_3^1 + 2,500, \quad U_5^2(p) = -p_5^2 - .4p_5^1 + 1,500.$$

The computed U-O solution is:

Class 1 **U-O Population Distribution**

$$p_1^{1*} = 0.00, \quad p_2^{1*} = 0.00, \quad p_3^{1*} = 0.00, \quad p_4^{1*} = 480.97, \quad p_5^{1*} = 19.03.$$

Class 2 **U-O Population Distribution**

$$p_1^{2*} = 714.75, \quad p_2^{2*} = 0.00, \quad p_3^{2*} = 0.00, \quad p_4^{2*} = 285.25, \quad p_5^{2*} = 0.00.$$

With the negatively impacted by the disaster locations 3 and 5, the majority of class 1 migrates from location 5 to location 4; whereas those of class 2 completely leave location 3 (as do those of class 1). Those of class 2 leave location 3 for locations 1 and 4.

The computed S-O solution is:

Class 1 **S-O Population Distribution**

$$p_1^{1'} = 6.85, \quad p_2^{1'} = 0.00, \quad p_3^{1'} = 0.00, \quad p_4^{1'} = 389.05, \quad p_5^{1'} = 104.10.$$

Class 2 **S-O Population Distribution**

$$p_1^{2'} = 648.28, \quad p_2^{2'} = 75.17, \quad p_3^{2'} = 0.00, \quad p_4^{2'} = 276.54, \quad p_5^{2'} = 0.00.$$

The S-O population distribution is also affected by the disaster with neither class locating at location 3. No-one of class 2 remains at location 5, whereas those of class 1 have a higher population at location 5 under the S-O solution, than under the U-O one.

In Numerical Example 4, $\mu^1 = 2,055.64$ and $\mu^2 = 3,348.98$.

The subsidies are now:

Class 1 Subsidies

$$subsidy_1^1 = 199.00, \quad subsidy_2^1 = 0.00, \quad subsidy_3^1 = 0.00, \quad subsidy_4^1 = 0.00, \quad subsidy_5^1 = 263.83,$$

Class 2 Subsidies

$$subsidy^2_1 = 0.00, \quad subsidy^2_2 = 499.33, \quad subsidy^2_3 = 848.98, \quad subsidy^2_4 = 18.78, \quad subsidy^2_5 = 1,890.62.$$

6. Summary and Conclusions

Humans throughout history have sought to migrate to locations where they could enjoy a greater quality of life with enhanced safety, security, prosperity, and access to resources. With climate change, the increasing number of natural disasters, and their major impact, as well as wars, violence, and persecution in many countries around the globe, the world is witnessing some of the largest human migrations. Governments, hence, are feeling increasing pressure and stress to respond to the challenges of migratory flows through appropriate policies and regulations.

Mathematical models of human migration networks have advanced over the past three decades to include, among others, multiple classes of migrants, costs associated with migration between locations, the incorporation of dynamics, etc. However, essentially all of the rigorous operations research based migration modeling work has focused on selfish, that is, user-optimizing, behavior. Such a perspective is rich in theory and scope but such behavior may lead to migratory flows and the resulting population distributions among regions and countries that are far from optimal from a societal perspective.

In this paper, we introduce a new multiclass network model of human migration that assumes system-optimizing behavior. The model fills a research gap in the literature. We then, using its classical user-optimizing analogue, demonstrate how governments can provide subsidies in order to make the system-optimizing population distribution pattern across multiple locations, also user-optimizing. Hence, once the subsidies are provided, migrants will independently locate themselves where it is also best from a societal perspective.

We provide, for completeness, the variational inequality formulations of both models and draw analogues to traffic network models and policies of tolls that alter travelers' behavior to make drivers selects routes of transport that are system-optimizing. We also propose a societal welfare loss ratio, inspired by the price of anarchy.

An algorithm is proposed that is a time-discretization of the underlying dynamics until the optimal population distribution is achieved/computed. The algorithm is then applied to compute solutions to a series of multiclass numerical examples and the population distributions reported under user-optimization, under system-optimization, along with the subsidies for the different classes at the different locations.

Acknowledgments

The authors appreciate very much the feedback and comments received on an earlier version of this paper, including those of the Editor.

This paper was revised during the Covid-19 pandemic. It is dedicated to all essential workers, including healthcare workers, farmers, grocery store workers, freight service providers, and educators across the globe. We thank you for your extraordinary efforts and are indebted to you.

References

Barbagallo, A., Daniele, P., Giuffre, S., Maugeri, A.,2014. Variational approach for a general financial equilibrium problem: The Deficit Formula, the Balance Law and the Liability Formula. A path to the economy recovery. European Journal of Operational Research 237(1), 231-244.

Bartenstein, B., McDonald, M.D., 2019. Exporting people: How Central America encourages mass migration. Bloomberg, July 19; available at:
https://www.bloomberg.com/news/articles/2019-06-19/migrant-crisis-at-border-how-central-america-encourages-exodus

Bier, V.M., Zhou, Y., Du, H., 2019. Game-theoretic modeling of pre-disaster relocation. The Engineering Economist. DOI: 10.1080/0013791X.2019.1677837

Cappello, G., Daniele, P., 2019. A variational formulation for a human migration model. In: Paolucci M., Sciomachen A., Uberti P., Editors, Advances in Optimization and Decision Science for Society, Services and Enterprises. AIRO Springer Series, vol 3. Springer Nature Switzerland, pp. 185-195.

Cardona, A.C., 2020. Earthquakes could further traumatize Puerto Ricans, UM Professor says. Miami New Times, January 8.

Causa, A., Jadamba, B., Raciti, F., 2017. A migration equilibrium model with uncertain data and movement costs. Decisions in Economics and Finance 40, 159-175.

Colajanni, G., Daniele, P., Giuffre, S., Nagurney, A., 2018. Cybersecurity investments with nonlinear budget constraints and conservation laws: Variational equilibrium, marginal expected utilities, and Lagrange multipliers. International Transactions in Operational Research 25(5), 1443-1464.

Dafermos, S., 1980. Traffic equilibrium and variational inequalities. Transportation Science 14(1), 42-54.

Dafermos, S.C., 1973. Toll patterns for multiclass-user transportation networks. Transportation Science 7(3), 211-223.

Dafermos, S., Sparrow, F.T., 1971. Optimal resource allocation and toll patterns in user-optimized transport networks. Journal of Transport Economics and Policy 5(2), 184-200.

Dafermos S.C., Sparrow, F.T., 1969. The traffic assignment problem for a general network. Journal of Research of the National Bureau of Standards 73B, 91-118.

Daniele, P., 2006. Dynamic Networks and Evolutionary Variational Inequalities. Edward Elgar Publishing, Cheltenham, England.

Dupuis, P., Nagurney, A., 1993. Dynamical systems and variational inequalities. Annals of Operations Research 44, 9-42.

Gossler, T., Wakolbinger, T., Nagurney, A., Daniele, P., 2019. How to increase the impact of disaster relief: A study of transportation rates, framework agreements and product distribution. European Journal of Operational Research 274(1), 126-141.

International Organization of Migration, 2020a. Key migration terms. Available at: https://www.iom.int/key-migration-terms

International Organization of Migration, 2020b. Migration data portal. Available at: https://migrationdataportal.org/themes/sustainable-development-goals-sdgs

Isac, G., Bulavsky, V.A., Kalashnikov, V.V., 2002. Complementarity, Equilibrium, Efficiency and Economics. Kluwer Academic Publishers, Dordrecht, The Netherlands.

Jones, G., 2018. Italy will no longer be 'Europe's refugee camp', vows new government. World News June 4.

Kalashnikov, V., Kalashnykova, N., Rojas, R.L. Munos, M.M., Uranga, C., Rojas, A.L., 2008. Numerical experimentation with a human migration model. European Journal of Operational Research 189, 208-229.

Karagiannis, G.M., 2016. Emergency management perspective of the European migrant crisis. Newsletter - special edition, The International Emergency Management Society, issue 4, October, 12-24, Brussels, Belgium.

Kennedy, M., 2019. U.N. says more than 4 million people have left Venezuela. NPR; available at: https://www.npr.org/2019/06/07/730687807/u-n-says-more-than-4-million-people-have-left-venezuela

Kinderlehrer, D., Stampacchia, G., 1980. An Introduction to Variational Inequalities and Their Applications. Academic Press, New York.

Kitsantonis, N., 2019. Greek refugee camps are near catastrophe, Rights Chief warns. The New York Times, October 31.

Kotsireas, I.S., Nagurney, A., Pardalos, P.M., Editors, 2019. Dynamics of Disasters: Key Concepts, Models, Algorithms, and Insights. Springer International Publishing Switzerland.

Koutsoupias, E., Papadimitriou, C.H., 1999. Worst-case equilibria. In: Theoretical Aspects of Computer Science: Proc. 16th Annual Symp. on Theoretical Aspects of Computer Science (STACS '99), Meinel, G., Tison, S., Editors, Lecture Notes in Computer Science, Vol. 1563, Springer-Verlag, Berlin, Germany, pp. 404-413.

Lawphongpanich, S., Hearn, D.W., Smith, M.J., Editors, 2006. Mathematical and Computational Models for Congestion Charging. Springer Science + Business Media, New York.

Nagurney, A., 1989. Migration equilibrium and variational inequalities. Economics Letters 31, 109-112.

Nagurney, A., 1990. A network model of migration equilibrium with movement costs. Mathematical and Computer Modelling 13, 79-88.

Nagurney, A., 1999. Network Economics: A Variational Inequality Approach, second and revised edition. Kluwer Academic Publishers, Dordrecht, The Netherlands.

Nagurney, A., 2006. Supply Chain Network Economics: Dynamics of Prices, Flows and Profits. Edward Elgar Publishing, Cheltenham, England.

Nagurney, A., Daniele, P., 2020. International human migration networks under regulations. European Journal of Operational Research, in press.

Nagurney, A., Daniele, P., Nagurney, L.S., 2019. Refugee migration networks and regulations: A multiclass, multipath variational inequality framework. Isenberg School of Management, University of Massachusetts Amherst.

Nagurney, A., Dutta, P., 2019. Competition for blood donations. Omega 212, 103-114.

Nagurney, A., Li, D., Nagurney, L.S., 2013. Pharmaceutical supply chain networks with outsourcing under price and quality competition. International Transactions in Operational Research 20(6), 859-888.

Nagurney, A., Pan, J., Zhao, L., 1992a. Human migration networks. European Journal of Operational Research 59, 262-274.

Nagurney, A., Pan, J., Zhao, L., 1992b. Human migration networks with class transformations. In: Structure and Change in the Space Economy. Lakshmanan, T.R., Nijkamp, P., Editors, Springer-Verlag, Berlin, Germany, pp. 239-258.

Nagurney, A., Qiang, Q., 2009. Fragile networks: identifying vulnerabilities and synergies in an uncertain age. International Transactions in Operational Research 19, 123-160.

Nagurney, A., Zhang, D., 1996. Projected Dynamical Systems and Variational Inequalities with Applications. Kluwer Academic Publishers, Boston, Massachusetts.

Nash, J.F., 1950. Equilibrium points in n-person games. Proceedings of the National Academy of Sciences USA 36, 48-49.

Nash, J.F., 1951. Noncooperative games. Annals of Mathematics 54, 286-298.

Pan, J., Nagurney, A., 1994. Using Markov chains to model human migration in a network equilibrium framework. Mathematical and Computer Modelling 19, 31-39.

Pan, J., Nagurney, A., 2006. Evolution variational inequalities and projected dynamical systems with application to human migration. Mathematical and Computer Modelling 43(5-6), 646-657.

Papadaki, S., Banias, G., Achillas, C., Aidonis, D., Folinas, D., Bochtis, D., Papangelou, S., 2017. A humanitarian logistics case study for the intermediary phase accommodation center for refugees and other humanitarian disaster victims. In: Dynamics of Disasters: Algorithmic Approaches and Applications, Kotsireas, I.S., Nagurney, A., Pardalos, P.M., Editors, Springer International Publishing Switzerland, pp. 157-202.

Papadimitriou, C.H., 2001. Algorithms, games, and the Internet. In: Automata, Languages and Programming: Proc. 28th Intl. Colloquium (ICALP 2001), Orejas, F., Spirakis, S.G., van Leeuwen, J., Editors, Lecture Notes in Computer Science, Vol. 2076, Springer-Verlag, Berlin, Germany, pp. 1-3.

Rahmati, H.S., Tularam, G.A., 2017. A critical review of human migration models. Climate Change 3(12), 924-952.

Robinson, C., Dilkina, B., Moreno-Cruz, J., 2020. Modeling migration patterns in the USA under sea level rise. PLoS ONE 15(1): e0227436. https://doi.org/10.1371/journal.pone.0227436

Roughgarden, T., 2005. Selfish Routing and the Price of Anarchy. MIT Press, Cambridge, Massachusetts.

Sakuma, A., 2020. Damned for trying. Available at: http://www.msnbc.com/specials/migrant-crisis/libya

Shih, G., Denyer, S., Taylor, A., 2020. Worries grow that quarantine in China not enough to stem increasingly virulent coronavirus. Washington Post, January 27.

Short, A., 2019. Europe's congestion pricing lessons for America. StreetsBlogNYC, October 31; available at:
https://nyc.streetsblog.org/2019/10/31/europes-congestion-pricing-lessons-for-america/

Skodo, A., 2018. Sweden: By turns welcoming and restrictive in its immigration policy. Migration Policy Institute, December 6; available at:
https://www.migrationpolicy.org/article/sweden-turns-welcoming-and-restrictive-its-immigration-policy

Smolens, M., 2019. Column: Where 'congestion pricing' has worked, and why. The San Diego Union-Tribune. April 26.

Stevis-Gridneff, M., 2020. Asylum seekers find a new route to Europe, flowing into a divided Cyprus. The New York Times, January 28.

Trapp, A.C. Teytelboym, A., Martinello, A., Andersson, T., Ahani, N., 2018. Placement optimization in refugee resettlement. Working papers 2018:23, Lund University, Department of Economics, Sweden.

UNHCR, 2020. Figures at a glance. Available at: https://www.unhcr.org/en-us/figures-at-a-glance.html

United Nations, 2013. International migration policies: Government views and priorities. Department of Economic and Social Affairs, Population Division, New York.

United Nations, 2017. Population facts. Department of Economic and Social Affairs, Popu-

lation Division, New York, December; available at:
https://www.un.org/en/development/desa/population/publications/pdf/popfacts/PopFacts_2017-5.pdf

United Nations, Department of Economic and Social Affairs, Population Division, 2017. International Migration Policies: Data Booklet (ST/ESA/ SER.A/395).

United Nations Foundation, 2020. United Nations Sustainable Development Goals. Available at: https://unfoundation.org/what-we-do/issues/sustainable-development-goals/

United Nations Refugee Agency, 2015. The sea route to Europe: The Mediterranean passage in the age of refugees. July 1; available at:
https://www.unhcr.org/en-us/protection/operations/5592bd059/sea-route-europe-mediterranean-passage-age-refugees.html

United Nations Refugee Agency, 2019. Syria refugee crisis explained. March 7; available at:
https://www.unrefugees.org/news/syria-refugee-crisis-explained/

Wardrop, J.G., 1952. Some theoretical aspects of road traffic research. Proceedings of the Institution of Civil Engineers, Part II 1: pp. 325-378.