Hospital Competition in Prices and Quality:
A Variational Inequality Framework

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March 2017; revised August 2017 and October 2017

Operations Research for Health Care (2017), **15**, pp 91-101.

Abstract

In this paper, we construct a game theory model to capture competition among hospitals for patients for their medical procedures. The utility functions of the hospitals contain a revenue component and a component due to altruism benefit. The hospitals compete in prices charged to paying patients as well as in the quality levels of their procedures. Both prices and quality levels are subject to lower and upper bounds. We state the governing Nash equilibrium conditions and provide the variational inequality formulation. We establish existence of an equilibrium price and quality pattern and also present a Lagrange analysis of the equilibrium solutions. An algorithm is proposed and then applied to numerical examples comprising a case study focusing on four major hospitals in Massachusetts.

Keywords: game theory, hospitals, competition, quality, healthcare, variational inequalities

1. Introduction

Hospitals are essential institutions for the provision of healthcare to society, providing medical diagnostics, surgeries, treatments, deliveries of babies, and emergency care. They are complex ecosystems, whose existence depends on delivering quality care to their patients. At the same time, hospitals in the United States are under increasing pressure and stresses with many consolidations in the industry, driven, in part, by needs to reduce costs, as well as to be perceived as being value-based (see Commins (2016)). In 2015, there were over 100 hospital and health system consolidations in the United States among over 5,500 registered hospitals (American Hospital Association (2017)). Hospitals are also, often, regulated and have been subject to reforms internationally to enhance competition (see Brekke et al. (2010)).

Given the importance of competition as a salient feature of hospitals today, there is a large empirical literature on the relationship between quality and hospital competition (Gaynor and Town (2011), Gravelle, Santos, and Siciliani (2014), Colla et al. (2016)). Other studies have examined the relationships between competition and health care system costs (Rivers and Glover (2008)), and between competition and patient satisfaction (Miller (1996) and Brook and Kosecoff (1988)). The majority of the empirical literature has been on the US experience, with more recent studies focusing on the United Kingdom and other European countries (see, e.g., Kessler and McLellan (2000), Kessler and Geppert (2005), Cooper et al. (2011)). Of course, it is important to quantify quality in this setting. Specifically, as noted by Gravelle, Santos, and Siciliani (2014), although quality is often measured by hospital mortality, they itemize sixteen different measures of hospital quality, with six of the sixteen quality measures based on standardized mortality rates, seven on standardized readmission, revisions, and redo rates, and three constructed from surveys of patients' experiences.

However, the literature on theoretical frameworks for hospital competition is not as advanced and is primarily the purview of economists rather than operations researchers. For example, Gravelle, Santos, and Siciliani (2014) construct a hospital quality competition model under fixed prices, building on the work of Ma and Burges (1993), Gaynor (2007), and Brekke, Siciliani, and Straum (2011). The model in this paper differs in several significant ways; notably, we have competition in both prices and quality and we consider multiple procedures for each hospital. Plus, our prices and quality levels must satisfy lower and upper bounds. Longo et al. (2016) present a simple, yet elegant, two hospital model of quality and efficiency competition. Brekke et al. (2010) develop a competitive hospital model in quality with regulated prices in a Hotelling (1929) framework using a differential game methodology. Rivers and Glover (2008) provide an excellent review of competition and healthcare and emphasize the importance of being able to identify and understand the mechanism of competition in this industry in order to provide higher quality of care and patient satisfaction.

Interestingly, a survey on operations research and healthcare (cf. Rais and Viana (2010)) does not mention the term *game theory*, although it does acknowledge the seminal contributions of Roth, Sönmez, and Ünver (2004) on kidney exchanges, which, as the latter authors remark, resemble some of the "housing" problems considered in the mechanism design literature for indivisible products. In addition, we note the survey of Moretti (2013), which reviews recent applications of coalition games in medical research, along with an identification of some open problems.

We believe that a rigorous game theory framework for hospital competition that can handle price and quality regulations in the form of lower and upper bounds and also enables the computation of equilibrium solutions is valuable. Here we construct such a framework, through the use of the theory of variational inequalities, for the formulation of the governing Nash equilibrium conditions, the qualitative analysis, and the computation of the equilibrium quality and price patterns. For background on the methodology of variational inequalities, but applied to supply chain competition in quality, see the book by Nagurney and Li (2016).

This paper is organized as follows. In Section 2, we present the hospital competition model, in which the hospitals compete in both prices and quality for patients for the procedures that they offer. The utility function of each hospital consists of a revenue component and also a component associated with altruism benefit since hospitals are decision-makers in healthcare. Each hospital's benefit function captures the total benefit of the patients from receiving treatment at the hospital (see, e.g., Brekke, Siciliani, and Straume (2011)), weighted by a factor reflecting the monetized value of altruism of the hospital. The demands for procedures at different hospitals are elastic and depend on prices charged as well as the quality levels, whereas the costs of different procedures depend on the quality levels. The prices and quality levels are subject to lower and upper bounds, which allow us to capture different regulations, such as minimum quality standards. Also, if, as in the case of a price for a procedure, one sets the lower bound equal to the upper bound, then one has, in effect, a fixed price, which is useful in modeling such pricing schemes that may occur in different country health systems. We define the Nash equilibrium governing the noncooperative game and present the variational inequality formulation. We also prove that an equilibrium solution is guaranteed to exist.

In Section 3, we construct an alternative formulation of the variational inequality through the use of Lagrange multipliers and give an analysis of the marginal utilities of the hospitals when the prices and quality levels of the hospital procedures lie within or at one of the bounds. Such an analysis enables both hospitals as well as policymakers to assess the impacts of loosening or tightening certain regulations. We note that there are several papers that have contributed to the analysis of the behavior of the solutions to a variational inequality, which models equilibrium problems through the use of Lagrange multipliers. For example, in operations research, the papers by Barbagallo et al. (2014) and Daniele, Giuffré, and Lorino (2016) have done so for the financial equilibrium problem, and the paper by Daniele and Giuffré (2015) for the random traffic equilibrium problem. Also, recently, Daniele, Maugeri, and Nagurney (2017) analyzed a cybersecurity investment supply chain game theory model with nonlinear budget constraints by means of Lagrange multipliers.

In Section 4, we first describe the algorithm that we use in our case study. The case study consists of four hospitals in eastern Massachusetts and three major procedures that they all provide. In the case study we report, for different scenarios, the computed equilibrium prices and quality levels of the hospital procedures, the demand for these procedures, as well as the incurred net revenues and utilities. We conclude the paper with Section 5, where we summarize our results and provide suggestions for future research.

2. The Hospital Competition Model

We now present the hospital competition model consisting of m hospitals with a typical hospital denoted by i and with each being able to carry out n medical procedures with a typical medical procedure denoted by k. Let p_{ik} denote the price charged by hospital i for procedure k. We group the prices associated with hospital i into the vector $p_i \in R^n_+$ and we then group the vectors of prices of all the hospitals into the vector $p \in R^{mn}_+$. In addition, we let Q_{ik} denote the quality associated with hospital i carrying out procedure k. We group the quality levels of hospital i into the vector $Q_i \in R^n_+$ and the quality levels of all hospitals into the vector $Q \in R^{mn}_+$. The strategic variables of each hospital i; $i = 1, \ldots, m$, are its vector of prices charged and its vector of quality levels for the procedures, which, at the equilibrium, are denoted, respectively, by p_i^* and Q_i^*. All vectors are column vectors.

We assume that there are lower and upper bounds on the price charged by hospital i for procedure k, denoted by $\underline{p}_{ik}$ and $\bar{p}_{ik}$, respectively, so that the prices p_{ik}; $i = 1, \ldots, m$, must satisfy the constraints:

$$\underline{p}_{ik} \leq p_{ik} \leq \bar{p}_{ik}, \quad k = 1, \ldots, n. \tag{1}$$

Observe that, if, because of regulations, there is a fixed price imposed for a hospital i and procedure k then we set: $\underline{p}_{ik} = \bar{p}_{ik}$. This is standard, for example, in England (cf.

Gravelle, Santos, and Siciliani (2014)). We assume that patients undergoing the procedures are responsible for the payments, which may come out of pocket, through insurance, and/or a government subsidy.

In addition, there are bounds associated with the quality levels. Regulatory bodies often impose minimum quality standards, which we denote by $\underline{Q}_{ik}$ for $i = 1, \ldots, m$; $k = 1, \ldots, n$, to ensure a minimum level of quality. At the same time, hospitals may be limited by the maximum level of quality that they can achieve for different procedures with $\bar{Q}_{ik}$ representing the maximum for hospital i and procedure k with $i = 1, \ldots, m$; $k = 1, \ldots, n$. Hence, the following constraints must also hold for each i; $i = 1, \ldots, m$:

$$\underline{Q}_{ik} \leq Q_{ik} \leq \bar{Q}_{ik}, \quad k = 1, \ldots, n. \tag{2}$$

We let K_i denote the feasible set corresponding to hospital i; $i = 1, \ldots, m$, where $K_i \equiv \{(p_i, q_i) | (1) \text{ and } (2) \text{ hold}\}$. These feasible sets are closed and convex.

The demand for procedure k over the time horizon of interest at hospital i, which is denoted by d_{ik}, is given by the function

$$d_{ik} = d_{ik}(Q, p, \alpha_{ik}), \quad i = 1, \ldots, m; k = 1, \ldots, n, \tag{3}$$

where α_{ik} is a vector of demand parameters that capture the location of patients and other hospitals relative to hospital i, patient preferences over distance and quality, and other factors that can influence a patient's choice sets. Gravelle, Santos, and Siciliani (2014) proposed demand parameter vectors in the context of hospital quality competition; here we refine the vectors from the hospital to the hospital-procedure level. Furthermore, we allow for the demand at i for k to depend on the prices of the procedure not only at i but also at the other hospitals as well as on the prices of other procedures. Moreover, the demand functions can also, in general, depend on the quality levels of all procedures at all hospitals, as well as on the vector of additional demand parameters associated with each hospital and procedure. We assume that d_{ik} is increasing in Q_{ik} and is decreasing in p_{ik} for all hospitals i; $i = 1, \ldots, m$ and all procedures k; $k = 1, \ldots, n$.

It is important to emphasize that, in the case of elective procedures, patients may have more flexibility as to the hospital selected than in the case of emergency procedures. Hence, for the latter, distance to the hospital would be a bigger factor and the demand would be more inelastic.

The cost associated with procedure k at hospital i is denoted by c_{ik} and this function takes the form:

$$c_{ik} = c_{ik}(Q, \beta_{ik}), \quad i = 1, \ldots, m; k = 1, \ldots, n, \tag{4}$$

where β_{ik} is a vector of cost parameters associated with hospital i and procedure k. Gravelle, Santos, and Siciliani (2014) utilized cost parameter vectors associated with hospitals that capture exogenous factors, such as input prices, which can correspond to supplies needed, etc.; here, we further refine these to the hospital-procedure level.

For simplicity, we group the demand parameters α_{ik} for each i, $\forall k$, into the vector α_i and then we group these vectors, $\forall i$, into the vector α. Similarly, we group the cost parameters β_{ik} for each i, $\forall k$, into the vector β_i and then we group all such vectors for all the hospitals into the vector β.

Hospitals, since they are in healthcare, cen be expected to have utility functions that incorporate aspects of altruism. Hence, the component u_i of the utility function of hospital i, U_i, which we will soon construct fully, is as follows:

$$u_i = \omega_i B_i(Q, p), \quad i = 1, \ldots, m, \tag{5a}$$

where ω_i is a monetized weight, which reflects the degree of altruism of hospital i, and $B_i(Q, p)$ is a function representing the total benefit of the patients from receiving treatment at hospital i at the price and quality levels. Altruism functions have been utilized in Brekke, Siciliani, and Straume (2011) in the case of hospital competition and by Nagurney, Alvarez Flores, and Soylu (2016) in disaster relief, where additional references can be found.

Specifically, we consider benefit functions $B_i(Q, p) = \sum_{k=1}^{n} d_{ik}(Q, p, \alpha_{ik}) Q_{ik}$, so that

$$u_i = \omega_i \sum_{k=1}^{n} d_{ik}(Q, p, \alpha_{ik}) Q_{ik}, \quad i = 1, \ldots, m. \tag{5b}$$

Therefore, the total benefit to the patients treated at hospital i is equal to the sum over all procedures of the demand for a given procedure times the quality level for that procedure at the hospital.

We assume that all the above functions are continuously differentiable.

The utility function of hospital i, U_i; $i = 1, \ldots, m$, is:

$$U_i(p, Q, \alpha_i, \beta_i) = \left[\sum_{k=1}^{n} [p_{ik} - c_{ik}(Q, \beta_{ik})] d_{ik}(Q, p, \alpha_{ik}) \right] + \omega_i \sum_{k=1}^{n} d_{ik}(Q, p, \alpha_{ik}) Q_{ik}. \tag{6}$$

Note that each hospital's utility function corresponds to its net revenue, since hospitals must be financial sustainable, plus its monetized weighted benefit function.

The network structure of the problem is depicted in Figure 1 with the flows on links joining each pair of nodes (i, k) corresponding to the strategic variables p_{ik} and Q_{ik}.

Hence, in terms of the language of game theory (see, e.g., Gabay and Moulin (1980)), the players in this noncooperative game are the hospitals with a hospital i; $i = 1, \ldots, m$ being faced with the utility given by (6) which it seeks to maximize. The strategies of a hospital i, in turn, correspond to its vector of prices p_i charged for its medical procedures and its vector of quality levels Q_i associated with his medical procedures. The price and quality level strategies for each hospital i must lie in the feasible set K_i as defined following (2).

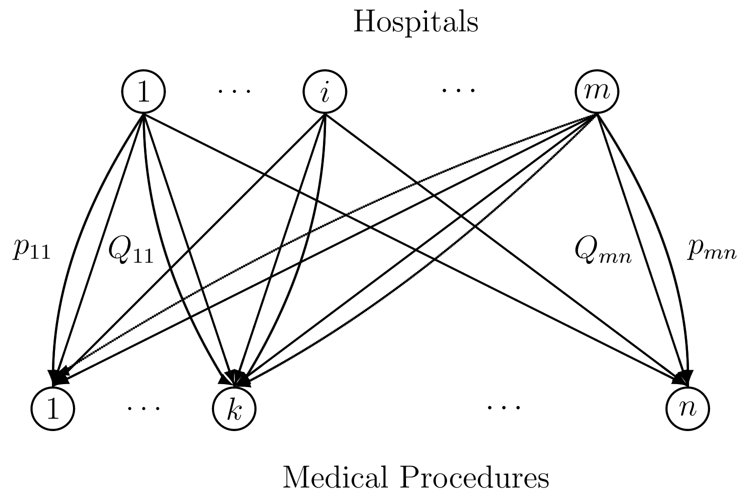

Figure 1: The Network Structure of the Game Theory Model for Hospital Competition

We assume that the hospitals compete noncooperatively and that they must treat all patients that enter their system. The governing concept in the game theory model is that of a Nash equilibrium (cf. Nash (1950, 1951)) as defined below.

Definition 1: Nash Equilibrium in Prices and Quality Levels

A price and quality level pattern $(p^*, Q^*) \in K \equiv \prod_{i=1}^m K_i$, *is said to constitute a Nash equilibrium if for each hospital* i*;* $i = 1, \ldots, m$*:*

$$U_i(p_i^*, \hat{p_i^*}, Q_i^*, \hat{Q_i^*}, \alpha_i, \beta_i) \geq U_i(p_i, \hat{p_i^*}, Q_i, \hat{Q_i^*}, \alpha_i, \beta_i), \quad \forall (p_i, Q_i) \in K_i, \tag{7}$$

where

$$\hat{p_i^*} \equiv (p_1^*, \ldots, p_{i-1}^*, p_{i+1}^*, \ldots, p_m^*) \text{ and } \hat{Q_i^*} \equiv (Q_1^*, \ldots, Q_{i-1}^*, Q_{i+1}^*, \ldots, Q_m^*). \tag{8}$$

According to (7), a Nash equilibrium is established if no hospital can unilaterally improve upon its utility by selecting an alternative vector of prices and quality levels for its procedures.

We now derive the variational inequality formulation of the governing equilibrium conditions.

Theorem 1: Variational Inequality Formulations of Nash Equilibrium in Prices and Quality

Assume that each hospital's utility function is concave with respect to its strategic variables, and is continuously differentiable. Then $(p^*, Q^*) \in K$ *is a Nash equilibrium according to Definition 1 if and only if it satisfies the variational inequality:*

$$-\sum_{i=1}^{m}\sum_{k=1}^{n}\frac{\partial U_i(p^*,Q^*,\alpha_i,\beta_i)}{\partial p_{ik}}\times(p_{ik}-p^*_{ik})-\sum_{i=1}^{m}\sum_{k=1}^{n}\frac{\partial U_i(p^*,Q^*,\alpha_i,\beta_i)}{\partial Q_{ik}}\times(Q_{ik}-Q^*_{ik})\geq 0,$$
$$\forall (p,Q)\in K, \tag{9}$$

or, equivalently,

$$-\sum_{i=1}^{m}\sum_{k=1}^{n}\Bigg[d_{ik}(Q^*,p^*,\alpha_{ik})+\sum_{j=1}^{n}p^*_{ij}\frac{\partial d_{ij}(Q^*,p^*,\alpha_{ij})}{\partial p_{ik}}-\sum_{j=1}^{n}c_{ij}(Q^*,\beta_{ij})\frac{\partial d_{ij}(Q^*,p^*,\alpha_{ij})}{\partial p_{ik}}$$
$$+\omega_i\sum_{j=1}^{n}\frac{\partial d_{ij}(Q^*,p^*,\alpha_{ij})}{\partial p_{ik}}Q^*_{ij}\Bigg]\times[p_{ik}-p^*_{ik}]$$
$$-\sum_{i=1}^{m}\sum_{k=1}^{n}\Bigg[\sum_{j=1}^{n}\Bigg[p^*_{ij}\frac{\partial d_{ij}(Q^*,p^*,\alpha_{ij})}{\partial Q_{ik}}-c_{ij}(Q^*,\beta_{ij})\frac{\partial d_{ij}(Q^*,p^*,\alpha_{ij})}{\partial Q_{ik}}$$
$$-\frac{\partial c_{ij}(Q^*,\beta_{ij})}{\partial Q_{ik}}d_{ij}(Q^*,p^*,\alpha_{ij})\Bigg]+\omega_i\Bigg[d_{ik}(Q^*,p^*,\alpha_{ik})+\sum_{j=1}^{n}\frac{\partial d_{ij}(Q^*,p^*,\alpha_{ij})}{\partial Q_{ik}}Q^*_{ij}\Bigg]\Bigg]\times[Q_{ik}-Q^*_{ik}]\geq 0,$$
$$\forall (p,Q)\in K. \tag{10}$$

Proof: Since the feasible set K is closed and convex and the utility functions are concave and continuously differentiable, the variational inequality (9) follows from Gabay and Moulin (1980). Variational inequality (10) then follows by expanding the marginal utility functions for each hospital i with respect to the strategic variables p_{ik} and Q_{ik}, for all procedures k, and summing up the resultants. □

We now put variational inequality (10) into standard variational inequality form (see Nagurney (1999)), that is: determine $X^* \in \mathcal{K}$ where X is a vector in R^N, $F(X)$ is a continuous function such that $F(X): X \mapsto \mathcal{K} \subset R^N$, and

$$\langle F(X^*), X - X^*\rangle \geq 0, \qquad \forall X \in \mathcal{K}, \tag{11}$$

where $\langle \cdot,\cdot \rangle$ denotes the inner product in N-dimensional Euclidean space. We set $\mathcal{K} \equiv K$, which is a closed and convex set, and $N = 2mn$. We define the vector $X \equiv (p, Q)$ and

$F(X) \equiv (F^1, F^2)$ with the (i,k)-th component of F^1 and F^2 given, respectively, for $i = 1, \ldots, m$; $k = 1, \ldots, n$, by:

$$F^1_{ik} = -\frac{\partial U_i(p, Q, \alpha_i, \beta_i)}{\partial p_{ik}}$$

$$= -d_{ik}(Q, p, \alpha_{ik}) - \sum_{j=1}^{n} p_{ij} \frac{\partial d_{ij}(Q, p, \alpha_{ij})}{\partial p_{ik}} + \sum_{j=1}^{n} c_{ij}(Q, \beta_{ij}) \frac{\partial d_{ij}(Q, p, \alpha_{ij})}{\partial p_{ik}} - \omega_i \sum_{j=1}^{n} \frac{\partial d_{ij}(Q, p, \alpha_{ij})}{\partial p_{ik}} Q_{ij}, \tag{12}$$

$$F^2_{ik} = -\frac{\partial U_i(p, Q, \alpha_i, \beta_i)}{\partial Q_{ik}}$$

$$= \sum_{j=1}^{n} \left[-p_{ij} \frac{\partial d_{ij}(Q, p, \alpha_{ij})}{\partial Q_{ik}} + c_{ij}(Q, \beta_{ij}) \frac{\partial d_{ij}(Q, p, \alpha_{ij})}{\partial Q_{ik}} + \frac{\partial c_{ij}(Q, \beta_{ij})}{\partial Q_{ik}} d_{ij}(Q, p, \alpha_{ij}) \right]$$

$$-\omega_i \left[d_{ik}(Q, p, \alpha_{ik}) + \sum_{j=1}^{n} \frac{\partial d_{ij}(Q, p, \alpha_{ij})}{\partial Q_{ik}} Q_{ij} \right]. \tag{13}$$

Also, we let $\mathcal{K} \equiv K$. Then, clearly, variational inequality (10) (and (9)) can be put into standard form (11).

Theorem 2: Existence of a Solution

A solution $(p^*, Q^*) \in K$ to variational inequality (9); equivalently, variational inequality (10), is guaranteed to exist.

Proof: Follows from the classical theory of variational inequalities (cf. Kinderlehrer and Stampacchia (1980)) since the feasible set K is compact, due to the price and quantity bounds, and the marginal utilities are all continuous. □

3. Alternative Formulation of the Variational Inequality and Analysis of Marginal Utilities

In this section, we provide an alternative formulation to variational inequalities (8) and (9) governing the competitive hospital Nash equilibrium, which then allows us to further analyze the marginal utilities.

Observe that the feasible set K can be expressed as follows:

$$K = \{(p, Q) \in R^{2mn} : -p_{ik} + \underline{p}_{ik} \leq 0, p_{ik} - \bar{p}_{ik} \leq 0,$$

$$-Q_{ik} - \underline{Q}_{ik} \leq 0, Q_{ik} - \bar{Q}_{ik} \leq 0, \; i = 1, \ldots, m; k = 1, \ldots, n\}. \tag{14}$$

Also, variational inequality (8) can be equivalently rewritten as a minimization problem. By letting:

$$V(p,Q) = -\sum_{i=1}^{m}\sum_{k=1}^{n} \frac{\partial U_i(p^*,Q^*,\alpha_i,\beta_i)}{\partial p_{ik}} \times (p_{ik} - p^*_{ik})$$
$$-\sum_{i=1}^{m}\sum_{k=1}^{n} \frac{\partial U_i(p^*,Q^*,\alpha_i,\beta_i)}{\partial Q_{ik}} \times (Q_{ik} - Q^*_{ik}),$$

we have that

$$V(p,Q) \geq 0 \text{ in } K \text{ and } \min_K V(p,Q) = V(p^*,Q^*) = 0. \tag{15}$$

We now construct the Lagrange function $\mathcal{L}$ such that

$$\mathcal{L}(p,Q,\lambda^1,\lambda^2,\mu^1,\mu^2) = -\sum_{i=1}^{m}\sum_{k=1}^{n} \frac{\partial U_i(p^*,Q^*,\alpha_i,\beta_i)}{\partial p_{ik}} \times (p_{ik} - p^*_{ik})$$
$$-\sum_{i=1}^{m}\sum_{k=1}^{n} \frac{\partial U_i(p^*,Q^*,\alpha_i,\beta_i)}{\partial Q_{ik}} \times (Q_{ik} - Q^*_{ik})$$
$$+\sum_{i=1}^{m}\sum_{k=1}^{n} \lambda^1_{ik}(-p_{ik} + \underline{p}_{ik}) + \sum_{i=1}^{m}\sum_{k=1}^{n} \lambda^2_{ik}(p_{ik} - \bar{p}_{ik})$$
$$+\sum_{i=1}^{m}\sum_{k=1}^{n} \mu^1_{ik}(-Q_{ik} + \underline{Q}_{ik}) + \sum_{i=1}^{m}\sum_{k=1}^{n} \mu^2_{ik}(Q_{ik} - \bar{Q}_{ik}), \tag{16}$$

where $(p,Q) \in R^{2mn}$, $\lambda^1,\lambda^2 \in R^{mn}_+$, $\mu^1,\mu^2 \in R^{mn}_+$, and λ^1 consists of all the λ^1_{ik} elements; λ^2 consists of all the λ^2_{ik} elements, μ^1 consists of all the μ^1_{ik} elements, and μ^2 consists of all the μ^2_{ik}; $i = 1,\ldots,m$; $k = 1,\ldots,n$.

Since for the convex set K the Slater condition is verified and (p^*,Q^*) is a minimal solution to problem (15), due to well-known theorems (see Jahn (1994)), there exist $\bar{\lambda}^1$, $\bar{\lambda}^2 \in R^{mn}_+$, $\bar{\mu}^1,\bar{\mu}^2 \in R^{mn}_+$ such that the vector $(p^*,Q^*,\bar{\lambda}^1,\bar{\lambda}^2,\bar{\mu}^1,\bar{\mu}^2)$ is a saddle point of the Lagrange function (16); that is,

$$\mathcal{L}(p^*,Q^*,\lambda^1,\lambda^2,\mu^1,\mu^2) \leq \mathcal{L}(p^*,Q^*,\bar{\lambda}^1,\bar{\lambda}^2,\bar{\mu}^1,\bar{\mu}^2) \leq \mathcal{L}(p,Q,\bar{\lambda}^1,\bar{\lambda}^2,\bar{\mu}^1,\bar{\mu}^2) \tag{17}$$

for all $(p,Q) \in K$, for all $\lambda^1,\lambda^2 \in R^{mn}_+$, for all $\mu^1,\mu^2 \in R^{mn}_+$, and

$$\bar{\lambda}^1_{ik}(\underline{p}_{ik} - p^*_{ik}) = 0, \quad \bar{\lambda}^2_{ik}(p^*_{ik} - \bar{p}_{ik}) = 0, \quad i = 1,\ldots,m; k = 1,\ldots,n,$$
$$\bar{\mu}^1_{ik}(\underline{Q}_{ik} - Q^*_{ik}) = 0, \quad \bar{\mu}^2_{ik}(Q^*_{ik} - \bar{Q}_{ik}) = 0, \quad i = 1,\ldots,m; k = 1,\ldots,n. \tag{18}$$

From the right-hand side of (17) it follows that $(p^*, Q^*) \in R_+^{2mn}$ is a minimal point of $\mathcal{L}(p, Q, \bar{\lambda}^1, \bar{\lambda}^2, \bar{\mu}^1, \bar{\mu}^2)$ in the entire space R^{2mn} and, thus, for all $i = 1, \dots, m$ and $k = 1, \dots, n$, we have that:

$$\frac{\partial \mathcal{L}(p^*, Q^*, \bar{\lambda}^1, \bar{\lambda}^2, \bar{\mu}^1, \bar{\mu}^2)}{\partial p_{ik}} = -\frac{\partial U_i(p^*, Q^*, \alpha_i, \beta_i)}{\partial p_{ik}} - \bar{\lambda}^1_{ik} + \bar{\lambda}^2_{ik} = 0 \tag{19}$$

$$\frac{\partial \mathcal{L}(p^*, Q^*, \bar{\lambda}^1, \bar{\lambda}^2, \bar{\mu}^1, \bar{\mu}^2)}{\partial Q_{ik}} = -\frac{\partial U_i(p^*, Q^*, \alpha_i, \beta_i)}{\partial Q_{ik}} - \bar{\mu}^1_{ik} + \bar{\mu}^2_{ik} = 0, \tag{20}$$

along with conditions (18).

Conditions (18) – (20) correspond to an equivalent formulation of variational inequality (9).

Indeed, multiplying (19) by $(p_{ik} - p^*_{ik})$ we obtain:

$$-\frac{\partial U_i(p^*, Q^*, \alpha_i, \beta_i)}{\partial p_{ik}}(p_{ik} - p^*_{ik}) - \bar{\lambda}^1_{ik}(p_{ik} - p^*_{ik}) + \bar{\lambda}^2_{ik}(p_{ik} - p^*_{ik}) = 0.$$

Utilizing now (18), we obtain:

$$-\frac{\partial U_i(p^*, Q^*, \alpha_i, \beta_i)}{\partial p_{ik}}(p_{ik} - p^*_{ik}) = \bar{\lambda}^1_{ik}(p_{ik} - \underline{p}_{ik}) + \bar{\lambda}^2_{ik}(p_{ik} - \bar{p}_{ik}) \geq 0. \tag{21}$$

Similarly, multiplying (20) by $(Q_{ik} - Q^*_{ik})$ we obtain:

$$\frac{\partial U_i(p^*, Q^*, \alpha_i, \beta_i)}{\partial Q_{ik}}(Q_{ik} - Q^*_{ik}) - \bar{\mu}^1_{ik}(Q_{ik} - Q^*_{ik}) + \bar{\mu}^2_{ik}(Q_{ik} - Q^*_{ik}) = 0.$$

Using (18), we get:

$$\frac{\partial U_i(p^*, Q^*, \alpha_i, \beta_i)}{\partial Q_{ik}}(Q_{ik} - Q^*_{ik}) = \bar{\mu}^1_{ik}(Q_{ik} - \underline{Q}_{ik}) + \bar{\mu}^2_{ik}(Q_{ik} - \bar{Q}_{ik}) \geq 0. \tag{22}$$

Summation of (21) and (22) over all i and over all k yields variational inequality (8).

3.1 Analysis of the Marginal Utilities

We now proceed, using the above framework, to provide a deeper analysis of the marginal utilities of the hospitals. We first consider the marginal utilities with respect to the prices of the hospital procedures and then turn to the marginal utilities associated with the hospitals' quality levels.

From (19) we have that:

$$-\frac{\partial U_i(p^*, Q^*, \alpha_i, \beta_i)}{\partial p_{ik}} - \bar{\lambda}^1_{ik} + \bar{\lambda}^2_{ik} = 0.$$

Hence, if $\underline{p}_{ik} < p^*_{ik} < \bar{p}_{ik}$, then, also making use of (10), it follows that:

$$-\frac{\partial U_i(p^*, Q^*, \alpha_i, \beta_i)}{\partial p_{ik}}$$

$$= \Bigg[-d_{ik}(Q^*, p^*, \alpha_{ik}) - \sum_{j=1}^{n} p^*_{ij} \frac{\partial d_{ij}(Q^*, p^*, \alpha_{ij})}{\partial p_{ik}} + \sum_{j=1}^{n} c_{ij}(Q^*, \beta_{ij}) \frac{\partial d_{ij}(Q^*, p^*, \alpha_{ij})}{\partial p_{ik}}$$

$$-\omega_i \sum_{j=1}^{n} \frac{\partial d_{ij}(Q^*, p^*, \alpha_{ij})}{\partial p_{ik}} Q^*_{ij} \Bigg] = 0. \tag{23}$$

However, if $\bar{\lambda}^1_{ik} > 0$, and, therefore, $p^*_{ik} = \underline{p}_{ik}$ and $\bar{\lambda}^2_{ik} = 0$, we get

$$-\frac{\partial U_i(p^*, Q^*, \alpha_i, \beta_i)}{\partial p_{ik}}$$

$$= \Bigg[-d_{ik}(Q^*, p^*, \alpha_{ik}) - \sum_{j=1}^{n} p^*_{ij} \frac{\partial d_{ij}(Q^*, p^*, \alpha_{ij})}{\partial p_{ik}} + \sum_{j=1}^{n} c_{ij}(Q^*, \beta_{ij}) \frac{\partial d_{ij}(Q^*, p^*, \alpha_{ij})}{\partial p_{ik}}$$

$$-\omega_i \sum_{j=1}^{n} \frac{\partial d_{ij}(Q^*, p^*, \alpha_{ij})}{\partial p_{ik}} Q^*_{ij} \Bigg] = \bar{\lambda}^1_{ik}, \tag{24}$$

and, if $\bar{\lambda}^2_{ik} > 0$, and, hence, $p^*_{ik} = \bar{p}_{ik}$ and $\bar{\lambda}^1_{ik} = 0$, we have that

$$-\frac{\partial U_i(p^*, Q^*, \alpha_i, \beta_i)}{\partial p_{ik}}$$

$$= \Bigg[-d_{ik}(Q^*, p^*, \alpha_{ik}) - \sum_{j=1}^{n} p^*_{ij} \frac{\partial d_{ij}(Q^*, p^*, \alpha_{ij})}{\partial p_{ik}} + \sum_{j=1}^{n} c_{ij}(Q^*, \beta_{ij}) \frac{\partial d_{ij}(Q^*, p^*, \alpha_{ij})}{\partial p_{ik}}$$

$$-\omega_i \sum_{j=1}^{n} \frac{\partial d_{ij}(Q^*, p^*, \alpha_{ij})}{\partial p_{ik}} Q^*_{ij} \Bigg] = -\bar{\lambda}^2_{ik}. \tag{25}$$

From (23), which holds if $\underline{p}_{ik} < p^*_{ik} < \bar{p}_{ik}$, we see that for hospital i performing procedure k, the marginal utility with respect to the price charged is equal to zero; that is, the marginal revenue associated with the price hospital i charges for procedure k, $d_{ik}(Q^*, p^*, \alpha_{ik}) + \sum_{j=1}^{n} p^*_{ij} \frac{\partial d_{ij}(Q^*, p^*, \alpha_{ij})}{\partial p_{ik}}$, plus the marginal utility due to altruism with respect to price, $\omega_i \sum_{j=1}^{n} \frac{\partial d_{ij}(Q^*, p^*, \alpha_{ij})}{\partial p_{ik}} Q^*_{ij}$, is equal to the associated marginal cost $\sum_{j=1}^{n} c_{ij}(Q^*, \beta_{ij}) \frac{\partial d_{ij}(Q^*, p^*, \alpha_{ij})}{\partial p_{ik}}$.

In (24), minus the marginal utility with respect to the price of the procedure is equal to $\bar{\lambda}^1_{ik}$; namely, the marginal cost associated with the price charged by the hospital for the procedure is greater than the associated marginal revenue plus the marginal altruism utility with respect to price. Hospital i has a marginal loss with respect to the price charged for procedure k given by $\bar{\lambda}^1_{ik}$.

On the other hand, in the case of (25), where $p^*_{ik} = \bar{p}_{ik}$, and $\bar{\lambda}^2_{ik} > 0$, the marginal revenue with respect to the price charged for the procedure is greater than the associated marginal cost plus marginal utility due to altruism and with respect to price. Hospital i has a marginal gain given by $\bar{\lambda}^2_{ik}$.

A similar analysis to the above can be obtained for the marginal utilities of hospital i with respect to the quality level of procedure k and the associated Lagrange multipliers $\bar{\mu}^1_{ik}$ and $\bar{\mu}^2_{ik}$. We now provide such an analysis, for completeness, and since the hospital altruism functions also yield interesting insights.

From (20) we know that:

$$-\frac{\partial U_i(p^*, Q^*, \alpha_i, \beta_i)}{\partial Q_{ik}} - \bar{\mu}^1_{ik} + \bar{\mu}^2_{ik} = 0.$$

Therefore, if $\underline{Q}_{ik} < Q_{ik} < \bar{Q}_{ik}$, using also (10), we have that:

$$-\frac{\partial U_i(p^*, Q^*, \alpha_i, \beta_i)}{\partial Q_{ik}}$$

$$= \left[\sum_{j=1}^{n} \left[-p^*_{ij}\frac{\partial d_{ij}(Q^*, p^*, \alpha_{ij})}{\partial Q_{ik}} + c_{ij}(Q^*, \beta_{ij})\frac{\partial d_{ij}(Q^*, p^*, \alpha_{ij})}{\partial Q_{ik}} + \frac{\partial c_{ij}(Q^*, \beta_{ij})}{\partial Q_{ik}} d_{ij}(Q^*, p^*, \alpha_{ij})\right]\right.$$
$$\left. -\omega_i \left[d_{ik}(Q^*, p^*, \alpha_{ik}) + \sum_{j=1}^{n} \frac{\partial d_{ij}(Q^*, p^*, \alpha_{ij})}{\partial Q_{ik}} Q^*_{ij}\right]\right] = 0. \tag{26}$$

If, on the other hand, $\bar{\mu}^1_{ik} > 0$, and, hence, $Q^*_{ik} = \underline{Q}_{ik}$ and $\bar{\mu}^2_{ik} = 0$, then

$$-\frac{\partial U_i(p^*, Q^*, \alpha_i, \beta_i)}{\partial Q_{ik}}$$

$$= \left[\sum_{j=1}^{n} \left[-p^*_{ij}\frac{\partial d_{ij}(Q^*, p^*, \alpha_{ij})}{\partial Q_{ik}} + c_{ij}(Q^*, \beta_{ij})\frac{\partial d_{ij}(Q^*, p^*, \alpha_{ij})}{\partial Q_{ik}} + \frac{\partial c_{ij}(Q^*, \beta_{ij})}{\partial Q_{ik}} d_{ij}(Q^*, p^*, \alpha_{ij})\right]\right.$$
$$\left. -\omega_i \left[d_{ik}(Q^*, p^*, \alpha_{ik}) + \sum_{j=1}^{n} \frac{\partial d_{ij}(Q^*, p^*, \alpha_{ij})}{\partial Q_{ik}} Q^*_{ij}\right]\right] = \bar{\mu}^1_{ik}. \tag{27}$$

If, however, $\bar{\mu}^2_{ik} > 0$, and, therefore, $Q^*_{ik} = \bar{Q}_{ik}$ and $\bar{\mu}^1_{ik} = 0$, we have that:

$$-\frac{\partial U_i(p^*, Q^*, \alpha_i, \beta_i)}{\partial Q_{ik}}$$

$$= \left[\sum_{j=1}^{n}\left[-p_{ij}^{*}\frac{\partial d_{ij}(Q^*,p^*,\alpha_{ij})}{\partial Q_{ik}} + c_{ij}(Q^*,\beta_{ij})\frac{\partial d_{ij}(Q^*,p^*,\alpha_{ij})}{\partial Q_{ik}} + \frac{\partial c_{ij}(Q^*,\beta_{ij})}{\partial Q_{ik}}d_{ij}(Q^*,p^*,\alpha_{ij})\right]\right.$$

$$\left. -\omega_i\left[d_{ik}(Q^*,p^*,\alpha_{ik}) + \sum_{j=1}^{n}\frac{\partial d_{ij}(Q^*,p^*,\alpha_{ij})}{\partial Q_{ik}}Q_{ij}^{*}\right]\right] = -\bar{\mu}_{ik}^{2}. \tag{28}$$

From (26), which is the case when $\underline{Q}_{ik} < Q_{ik}^* < \bar{Q}_{ik}$, we observe that, for hospital i and procedure k, the hospital's marginal utility with respect to the quality level of that procedure at the hospital is equal to zero. Hence, the marginal revenue associated with the procedure's quality level plus the marginal utility due to altruism with respect to quality is equal to the marginal cost for that procedure at the hospital with respect to the quality level.

In (27), minus the marginal utility of the hospital i with respect to the quality level of the procedure k is equal to $\bar{\mu}_{ik}^{1}$. Hence, the marginal cost associated with the quality level for the procedure at the hospital is greater than the marginal revenue associated with the procedure's quality level plus the marginal utility due to altruism with respect to quality. Hospital i experiences a marginal loss with respect to the quality level of $\bar{\mu}_{ik}^{1}$.

Finally, according to (28), when $Q_{ik}^* = \bar{Q}_{ik}$, and $\mu_{ik}^{2} > 0$, the marginal revenue associated with the procedure's quality level plus the marginal utility due to altruism with respect to quality exceeds the associated marginal cost with respect to the quality level of the procedure at the hospital. Hospital i, hence, experiences a marginal gain of $\bar{\mu}_{ik}^{2}$.

4. The Computational Procedure and a Case Study

Before we present a case study we outline our computational procedure. The computational procedure that we use to solve variational inequalities (9) and (10) is the Euler method of Dupuis and Nagurney (1993), which, at each iteration, results in closed form expressions for the hospital procedure prices and quality levels.

Specifically, iteration τ of the Euler method, where the variational inequality is expressed in standard form (11), is given by:

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - a_{\tau}F(X^{\tau})), \tag{29}$$

where $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$ and F is the function that enters the variational inequality problem (10), where recall that $X \equiv Q$ and $F(X)$ consists of the components given in (12) and (13).

As proven in Dupuis and Nagurney (1993), for convergence of the general iterative scheme, which induces the Euler method, the sequence $\{a_{\tau}\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_{\tau} = \infty$, $a_{\tau} > 0$,

$a_\tau \to 0$, as $\tau \to \infty$. Conditions for convergence for a variety of network-based problems can be found in Nagurney and Zhang (1996) and Nagurney (2006).

Explicit Formulae for the Euler Method Applied to the Hospital Competition Game Theory Model

The elegance of this algorithm for our variational inequality (10) for the computation of solutions to our model is apparent from the following explicit formulae.

We have the following closed form expression for the hospital procedure prices for $i = 1, \ldots, m; k = 1, \ldots, n$, at iteration $\tau + 1$:

$$p_{ik}^{\tau+1} = \max\{\underline{p}_{ik}, \min\{\bar{p}_{ik}, p_{ik}^\tau + a_\tau(d_{ik}(Q^\tau, p^\tau, \alpha_{ik}) + \sum_{j=1}^{n} p_{ij}^\tau \frac{\partial d_{ij}(Q^\tau, p^\tau, \alpha_{ij})}{\partial p_{ik}}$$

$$-\sum_{j=1}^{n} c_{ij}(Q^\tau, \beta_{ij}) \frac{\partial d_{ij}(Q^\tau, p^\tau, \alpha_{ij})}{\partial p_{ik}} + \omega_i \sum_{j=1}^{n} \frac{\partial d_{ij}(Q^\tau, p^\tau, \alpha_{ij})}{\partial p_{ik}} Q_{ij}^\tau)\}\}. \quad (30)$$

Also, we have the following closed form expression for the hospital quality service levels for $i = 1, \ldots, m; k = 1, \ldots, n$, at iteration $\tau + 1$:

$$Q_{ik}^{\tau+1} = \max\{\underline{Q}_{ik}, \min\{\bar{Q}_{ik}, Q_{ik}^\tau + a_\tau(\sum_{j=1}^{n} \left[p_{ij}^\tau \frac{\partial d_{ij}(Q^\tau, p^\tau, \alpha_{ij})}{\partial Q_{ik}} - c_{ij}(Q^\tau, \beta_{ij}) \frac{\partial d_{ij}(Q^\tau, p^\tau, \alpha_{ij})}{\partial Q_{ik}}\right.$$

$$\left. - \frac{\partial c_{ij}(Q^\tau, \beta_{ij})}{\partial Q_{ik}} d_{ij}(Q^\tau, p^\tau, \alpha_{ij})\right] + \omega_i \left[d_{ik}(Q^\tau, p^\tau, \alpha_{ik}) + \sum_{j=1}^{n} \frac{\partial d_{ij}(Q^\tau, p^\tau, \alpha_{ij})}{\partial Q_{ik}} Q_{ij}^\tau\right])\}\}. \quad (31)$$

We now present the case study. The case study is inspired by four major hospitals in eastern Massachusetts, which are in relative proximity, and based in the Cambridge-Boston area. Specifically, the four hospitals, which we refer to, henceforth, as Hospital 1, Hospital 2, Hospital 3, and Hospital 4, are modeled after: Beth Israel, Brigham and Women's, Massachusetts General Hospital (MGH), and Mount Auburn, respectively. All four hospitals are considered acute care hospitals, providing a variety of inpatient and outpatient services. The three procedures that we consider are common procedures. They are denoted by Procedure 1, Procedure 2, and Procedure 3, and correspond, respectively, to: appendectomy, knee joint replacement, and Cesarean delivery. All four hospitals conduct all three procedures. The time horizon is a year. In order to construct the demand functions at the hospitals for the three procedures we used data from 2009. We used the private payer data to estimate the demand functions. In addition, we obtained prices charged for the procedures by these hospitals, both lower and upper bounds. The cost functions we parameterized using the average price over the year for each hospital's procedure under study, noting that, according to Fuller (2014), hospitals in Massachusetts have an average markup of 221%. The quality level here is a composite of "patient experience," "process of care," and "patient outcomes," which measures mortality and readmission rates. Our resources for the above included documents by the Massachusetts Division of Health Care Finance and Policy (2011a, b, c).

Th network for the case study is given in Figure 2.

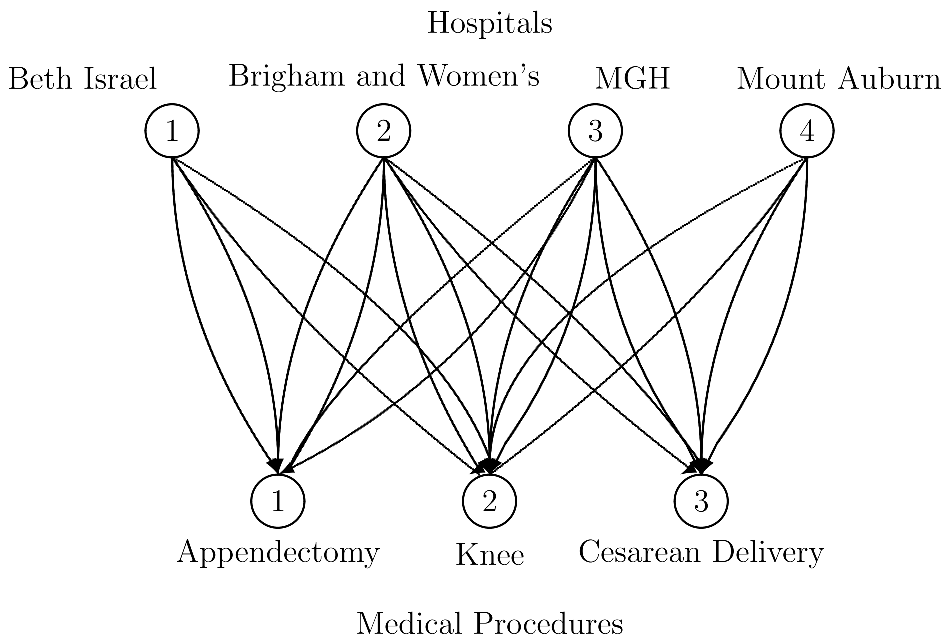

Figure 2: The Network Structure of the Case Study of Four Hospitals in Massachusetts

We set the quality bounds for all procedures as: $\underline{Q}_{ik} = 50$ and $\bar{Q}_{ik} = 98$, $\forall i, \forall k$, with an upper bound of quality of 100 equal to perfect quality, which is not achievable.

The algorithm was implemented in FORTRAN and a Linux system at the University of Massachusetts Amherst was used for the computations. The algorithm was initialized as follows: all price and quality variables were set to their lower bounds. Also, the convergence tolerance was .0001, which means that the absolute value of the difference of two successive computed variable iterates of prices and quality levels differed by no more than that amount.

In the case study we report on two sets of examples. For each set of examples, we consider first weights $w_i = 1$; $i = 1, 2, 3, 4$, and then set the weights $w_i = 0$; $i = 1, 2, 3, 4$, in order to investigate the differences in computed equilibrium prices and quality levels, as well as the net revenues and utilities of the hospitals.

Example Set 1

The demand data for Example Sets 1 and 2 are as follows. We took both proximity of hospitals as well as annual demand data for each procedure and hospital into consideration in constructing the below functions. We emphasize that here we are interested in "proof of concept" and demonstrating the kinds of questions and issues that can be addressed via our computable framework. In reporting the various functions, we suppress the parameter vectors α_i and β_i, $\forall i$, for simplicity.

Demand functions for Procedure 1 at the four hospitals are:

$$d_{11}(Q,p) = -.01p_{11}+Q_{11}+.02p_{41}-.5Q_{41}+90, \quad d_{21}(Q,p) = -.01p_{21}+Q_{21}+.03p_{11}-.5Q_{11}+65,$$

$$d_{31}(Q,p) = -.02p_{31}+Q_{31}+.02p_{21}-Q_{21}+65, \quad d_{41}(Q,p) = -.03p_{41}+1.5Q_{41}+.03p_{21}-Q_{21}+100.$$

Demand functions for Procedure 2 at the four hospitals are:

$$d_{12}(Q,p) = -.04p_{12}+1.5Q_{12}+.03p_{22}-2Q_{22}+60, \quad d_{22}(Q,p) = -.02p_{22}+2Q_{22}+.01p_{21}-Q_{21}+225.$$

$$d_{32}(Q,p) = -.02p_{32}+4.5Q_{32}+.02p_{22}-.5Q_{22}+475, \quad d_{42}(Q,p) = -.04p_{42}+2Q_{42}+.02p_{22}-2Q_{22}+60.$$

Demand functions for Procedure 3 at the four hospitals are:

$$d_{13}(Q,p) = -.03p_{13}+3Q_{1}+.02p_{23}-3Q_{23}+800, \quad d_{23}(Q,p) = -.01p_{23}+4Q_{23}+.03p_{13}-2Q_{13}+1080.$$

$$d_{33}(Q,p) = -.02p_{33}+2Q_{33}+.04p_{32}-3Q_{32}+400, \quad d_{43}(Q,p) = -.04p_{43}+2Q_{43}+.03p_{23}-4Q_{33}+190.$$

The price bounds for Example Sets 1 and 2 are:

$$\underline{p}_{11} = 2412.26, \quad \bar{p}_{11} = 49149.54,$$

$$\underline{p}_{12} = 9118.80, \quad \bar{p}_{12} = 80282.20,$$

$$\underline{p}_{13} = 3365.32, \quad \bar{p}_{13} = 24304.56,$$

$$\underline{p}_{21} = 3091.44, \quad \bar{p}_{21} = 62987.76,$$

$$\underline{p}_{22} = 9726.72, \quad \bar{p}_{22} = 85633.28,$$

$$\underline{p}_{23} = 3535.00, \quad \bar{p}_{23} = 25530.00,$$

$$\underline{p}_{31} = 3068.02, \quad \bar{p}_{31} = 62510.58,$$

$$\underline{p}_{32} = 9574.74, \quad \bar{p}_{32} = 84295.26,$$

$$\underline{p}_{33} = 4213.72, \quad \bar{p}_{33} = 30431.76,$$

$$\underline{p}_{41} = 2178.06, \quad \bar{p}_{41} = 44377.74,$$

$$\underline{p}_{42} = 9574.74, \quad \bar{p}_{42} = 84295.26,$$

$$\underline{p}_{43} = 2686.60, \quad \bar{p}_{43} = 19402.80.$$

Example 1 in Example Set 1

The demand for Example 1 in Example Set 1 is as above and so are the quality and price bounds.

The cost functions for Example 1 in Example Set 1 for each hospital and procedure are given below.

Cost functions for Hospital 1 for the three procedures are:

$$c_{11}(Q) = .2Q_{11}^2+10Q_{11}+2810.29, \quad c_{12}(Q) = .3Q_{12}^2+8Q_{12}+791.52, \quad c_{13}(Q) = .4Q_{13}^2+9Q_{12}+2969.84.$$

Cost functions for Hospital 2 for the three procedures are:

$$c_{21}(Q) = .2Q_{21}^2+10Q_{21}+3261.10, \quad c_{22}(Q) = .3Q_{22}^2+9Q_{22}+8318.39, \quad c_{23}(Q) = .5Q_{23}^2+11Q_{23}+3265.51.$$

Cost functions for Hospital 3 for the three procedures are:

$$c_{31}(Q) = .3Q_{31}^2+10Q_{31}+3328.99, \quad c_{32}(Q) = .2Q_{32}^2+11Q_{32}+7855.20, \quad c_{33}(Q) = .4Q_{33}^2+11Q_{33}+3485.22.$$

Cost functions for Hospital 4 for the three procedures are:

$$c_{41}(Q) = .3Q_{41}^2+9Q_{41}+2167.06, \quad c_{42}(Q) = .4Q_{42}^2+9Q_{42}+6358.03, \quad c_{43}(Q) = .3Q_{43}^2+8Q_{43}+2192.34.$$

We now report the computed equilibrium prices of the hospital procedures and the equilibrium quality levels as well as the incurred demands.

For Hospital 1:

$$p_{11}^* = 16382.16, \quad Q_{11}^* = 87.32, \quad d_{11}(Q^*, p^*) = 256.34,$$

$$p_{12}^* = 10868.41, \quad Q_{12}^* = 50.00, \quad d_{12}(Q^*, p^*) = 111.53,$$

$$p_{13}^* = 24304.56, \quad Q_{13}^* = 86.11, \quad d_{13}(Q^*, p^*) = 689.80.$$

For Hospital 2:

$$p_{21}^* = 24782.72, \quad Q_{21}^* = 98.00, \quad d_{21}(Q^*, p^*) = 362.98,$$

$$p_{22}^* = 19843.56, \quad Q_{22}^* = 92.02, \quad d_{22}(Q^*, p^*) = 260.98,$$

$$p_{23}^* = 25530.00, \quad Q_{23}^* = 50.00, \quad d_{23}(Q^*, p^*) = 1581.61.$$

For Hospital 3:

$$p_{31}^* = 12426.94, \quad Q_{31}^* = 50.00, \quad d_{31}(Q^*, p^*) = 264.12,$$

$$p_{32}^* = 59649.25, \quad Q_{32}^* = 98.00, \quad d_{32}(Q^*, p^*) = 73.88,$$

$$p_{33}^* = 30431.76, \quad Q_{33}^* = 50.00, \quad d_{33}(Q^*, p^*) = 1983.33.$$

For Hospital 4:

$$p_{41}^* = 13392.20, \quad Q_{41}^* = 50.00, \quad d_{41}(Q^*, p^*) = 418.72,$$

$$p_{42}^* = 9797.56, \quad Q_{42}^* = 71.80, \quad d_{42}(Q^*, p^*) = 24.54,$$

$$p_{43}^* = 13224.16, \quad Q_{43}^* = 70.39, \quad d_{43}(Q^*, p^*) = 367.72.$$

The net revenue of Hospital 1 in this example for the three procedures for the year is: 15,226,406.00 and its utility, which recall contains the altruism benefit component, is: 15,313,767.00 The net revenue for Hospital 2, in turn, is: 41,254,516.00, and its utility is: 41,393,184.00. The net revenue for Hospital 3 is: 56,047,404.00 and its utility: 56,167,016.00. Finally, the net revenue of Hospital 4 in this example is: 7,518,522.00 and its utility: 7,567,104.50.

Observe that several of the hospital procedure quality levels are at their lower bounds, specifically, Q_{12}^*, Q_{23}^*, Q_{33}^*, and Q_{41}^*, whereas only two are at their upper bounds: Q_{21}^* and Q_{32}^*. Provision of quality care is costly and it may be difficult for hospitals to provide higher levels for certain procedures at their establishments. Also, note that three hospitals (Hospitals 1, 2 and 3) charge at their respective upper bounds for Procedure 3, which, recall is a Cesarean delivery. This makes sense given that multiple lives are involved and the stakes (and even malpractice possibilities) also high.

For procedure 2, knee joint replacement, Hospitals 1 and 4 charged a 19.19% and 2.3% increase from their lower price bounds, respectively. In contrast, Hospitals 2 and 3 charged a 104.01% and 522.99% increase from their lower price bounds, respectively. Overall, Procedure 2 shows the greatest variation in equilibrium price levels based on the percent increase from their lower price bounds. Reasons for such a high price variation may be due to the increasing demand for knee joint replacements in the country as the US population ages and the percent population requiring knee surgeries increases. Studies have shown that knee joint replacements vary greatly by region, with a 313% difference in price between the least and most expensive price range in the Boston-Worcester market in Massachusetts (Melton (2015)). This supports our findings for Procedure 2, knee joint replacement, which has the highest equilibrium price variation among the procedures on our study among the four hospitals.

Example 2 in Example Set 1

Example 2 in Example Set 1 has the same data as Example 1 in this set except that now we investigate the impact of the hospitals not having an altruism benefit component in their utility functions so that: $w_i = 0$, for $i = 1, 2, 3, 4$.

The computed equilibrium prices of the hospital procedures and the equilibrium quality levels as well as the incurred demands are reported below.

For Hospital 1:

$$p_{11}^* = 16367.35, \quad Q_{11}^* = 85.18, \quad d_{11}(Q^*, p^*) = 255.34,$$

$$p_{12}^* = 10913.13, \quad Q_{12}^* = 50.00, \quad d_{12}(Q^*, p^*) = 110.90,$$

$$p_{13}^* = 24304.56, \quad Q_{13}^* = 85.18, \quad d_{13}(Q^*, p^*) = 687.00.$$

For Hospital 2:

$$p_{21}^* = 24765.24, \quad Q_{21}^* = 98.00, \quad d_{21}(Q^*, p^*) = 363.78,$$

$$p_{22}^* = 19775.65, \quad Q_{22}^* = 90.42, \quad d_{22}(Q^*, p^*) = 258.97,$$

$$p_{23}^* = 25530.00, \quad Q_{23}^* = 50.00, \quad d_{23}(Q^*, p^*) = 1583.48.$$

For Hospital 3:

$$p_{31}^* = 12485.10, \quad Q_{31}^* = 50.00, \quad d_{31}(Q^*, p^*) = 262.60,$$

$$p_{32}^* = 59629.71, \quad Q_{32}^* = 98.00, \quad d_{32}(Q^*, p^*) = 73.71,$$

$$p_{33}^* = 30431.76, \quad Q_{33}^* = 50.00, \quad d_{33}(Q^*, p^*) = 1982.55.$$

For Hospital 4:

$$p_{41}^* = 13441.43, \quad Q_{41}^* = 50.00, \quad d_{41}(Q^*, p^*) = 416.71,$$

$$p_{42}^* = 9816.37, \quad Q_{42}^* = 71.09, \quad d_{42}(Q^*, p^*) = 24.20,$$

$$p_{43}^* = 13180.91, \quad Q_{43}^* = 68.77, \quad d_{43}(Q^*, p^*) = 366.20.$$

The net revenue of Hospital 1 in Example 2 of Set 1 for the three procedures for the year is: 15,239,808.00. The net revenue for Hospital 2 is: 41,293,612.00, that for Hospital 3: 56,021,276.00, and that for Hospital 4: 7,520,433.50. The utilities of the hospitals coincide with their net revenues since the component of the utility associated with altruism benefit, with an associated weight of zero, is not included in this example.

We note that the same quality levels as in Example 1 are at their bounds.

Observe that, in the case of the elimination of altruism, which benefits the consumers/patients, the quality levels of all three procedures at all four hospitals remain the same or are lower ($Q^*_{11}, Q^*_{13}, Q^*_{42}, Q^*_{43}$) than for their counterpart in Example 1, except for one. Only Q^*_{22} is now greater, although, not substantially. Moreover, the net revenues for Hospitals 1, 2, and 4 are now higher than in Example 1 but the net revenue of Hospital 3 is lower, although not significantly.

Reasons for Hospitals 1 and 4 suffering a reduction in quality without altruism may be due to their smaller sizes as acute hospitals and availability of resources compared to Hospitals 2 and 3. Compared to Brigham and Womens and MGH (Hospitals 2 and 3), Beth Israel and Mount Auburn (Hospitals 1 and 4) have fewer staffed beds and resources. As a result, altruism benefit is more important for Hospitals 1 and 4 to remain competitive and to maintain a high quality of care. By comparing situations with and without altruism, we can further understand how hospitals in our case study compete in both price and quality.

Example Set 2

In Example Set 2 we consider the following scenario. We assume that the costs associated with quality have been reduced, through, for example, enhanced education, the application of operations research techniques for processes, innovations in surgical procedures, etc. Specifically, we assume that the coefficient in each hospital procedure cost function in Example 1, Set 1, associated with the second power quality level term, has now been reduced by a factor of 10.

Example 1 in Example Set 2

Hence, the cost functions for Example 1 in Example Set 2 for each hospital and procedure are as given below.

Cost functions for Hospital 1 for the three procedures are:

$$c_{11}(Q) = .02Q_{11}^2{+}10Q_{11}{+}2810.29, \quad c_{12}(Q) = .03Q_{12}^2{+}8Q_{12}{+}791.52, \quad c_{13}(Q) = .04Q_{13}^2{+}9Q_{12}{+}2969.84.$$

Cost functions for Hospital 2 for the three procedures are:

$$c_{21}(Q) = .02Q_{21}^2 + 10Q_{21} + 3261.10, \quad c_{22}(Q) = .03Q_{22}^2 + 9Q_{22} + 8318.39, \quad c_{23}(Q) = .05Q_{23}^2 + 11Q_{23} + 3265.51.$$

Cost functions for Hospital 3 for the three procedures are:

$$c_{31}(Q) = .03Q_{31}^2 + 10Q_{31} + 3328.99, \quad c_{32}(Q) = .02Q_{32}^2 + 11Q_{32} + 7855.20, \quad c_{33}(Q) = .04Q_{33}^2 + 11Q_{33} + 3485.22.$$

Cost functions for Hospital 4 for the three procedures are:

$$c_{41}(Q) = .03Q_{41}^2 + 9Q_{41} + 2167.06, \quad c_{42}(Q) = .04Q_{42}^2 + 9Q_{42} + 6358.03, \quad c_{43}(Q) = .03Q_{43}^2 + 8Q_{43} + 2192.34.$$

The remainder of the data, that is, the bounds, the weights, $w_i; i = 1, 2, 3, 4$, which, recall were set to 1, as well as the demand functions, are as in Example 1 of Set 1. Hence, here we wish to evaluate the impact of a major cost reduction associated with quality delivery of all procedures in all the hospitals under study.

We now report the computed equilibrium prices of the hospital procedures and the equilibrium quality levels as well as the incurred demands.

For Hospital 1:

$$p_{11}^* = 17201.04, \quad Q_{11}^* = 98.00, \quad d_{11}(Q^*, p^*) = 272.08,$$

$$p_{12}^* = 11713.31, \quad Q_{12}^* = 98.00, \quad d_{12}(Q^*, p^*) = 150.37,$$

$$p_{13}^* = 23849.75, \quad Q_{13}^* = 98.00, \quad d_{13}(Q^*, p^*) = 595.11.$$

For Hospital 2:

$$p_{21}^* = 26687.76, \quad Q_{21}^* = 98.00, \quad d_{21}(Q^*, p^*) = 363.15,$$

$$p_{22}^* = 20263.27, \quad Q_{22}^* = 98.00, \quad d_{22}(Q^*, p^*) = 283.59,$$

$$p_{23}^* = 25530.00, \quad Q_{23}^* = 98.00, \quad d_{23}(Q^*, p^*) = 1736.19.$$

For Hospital 3:

$$p_{31}^* = 13951.58, \quad Q_{31}^* = 98.00, \quad d_{31}(Q^*, p^*) = 319.72,$$

$$p_{32}^* = 59331.55, \quad Q_{32}^* = 98.00, \quad d_{32}(Q^*, p^*) = 85.63,$$

$$p_{33}^* = 30431.76, \quad Q_{33}^* = 98.00, \quad d_{33}(Q^*, p^*) = 2066.63.$$

For Hospital 4:

$$p_{41}^* = 15254.30, \quad Q_{41}^* = 98.00, \quad d_{41}(Q^*, p^*) = 492.00,$$

$$p_{42}^* = 9574.74, \quad Q_{42}^* = 98.00, \quad d_{42}(Q^*, p^*) = 82.28,$$

$$p_{43}^* = 11078.17, \quad Q_{43}^* = 98.00, \quad d_{43}(Q^*, p^*) = 316.77.$$

The net revenue of Hospital 1 in this example for the three procedures for the year is: 15,712,297.00 and its utility, which contains the altruism benefit component, is: 15,812,017.00. The net revenue for Hospital 2, in turn, is: 47,087,584.00, and its utility is: 47,321,112.00. The net revenue for Hospital 3 is: 59,956,932.00 and its utility: 60,199,188.00. Lastly, the net revenue of Hospital 4 in this example is: 8,498,926.00 and its utility: 8,586,249.00. Each hospital now enjoys a significantly higher net revenue than it did in Example 1 in Set 1, and a higher overall utility.

With the reduction in cost associated with quality, all hospitals achieve the upper bound of quality for each of the three procedures. Also as compared to the results for Example 1 in Set 1, p_{13}^* and p_{23}^* are, again, at their respective upper bounds, but p_{33}^* is no longer at its upper bound.

In addition, the demand for all procedures at all hospitals increases, except for two: the demand for Procedure 3 at Hospital 1 and the demand for Procedure 3 at Hospital 4.

This example illustrates the situation that, by reducing costs associated with quality of procedures, which can occur, for example, through innovations in surgical procedures (see, e.g., Costa-Navarro, Jimenez-Fuertes, and Illan-Riquelme (2013)), hospitals, as well as patients can gain, creating a win-win situation.

Example 2 in Example Set 2

Example 2 has the same data as Example 1 except that now we set all the weights w_1; $i = 1, 2, 3, 4$, equal to zero.

The computed equilibrium prices of the hospital procedures, the equilibrium quality levels, as well as the incurred demands are reported below.

For Hospital 1:

$$p_{11}^* = 17252.40, \quad Q_{11}^* = 98.00, \quad d_{11}(Q^*, p^*) = 272.50,$$

$$p_{12}^* = 11797.87, \quad Q_{12}^* = 98.00, \quad d_{12}(Q^*, p^*) = 148.19,$$

$$p_{13}^* = 23898.58, \quad Q_{13}^* = 98.00, \quad d_{13}(Q^*, p^*) = 593.64.$$

For Hospital 2:

$$p_{21}^* = 26697.24, \quad Q_{21}^* = 98.00, \quad d_{21}(Q^*, p^*) = 364.60,$$

$$p_{22}^* = 20303.56, \quad Q_{22}^* = 98.00, \quad d_{22}(Q^*, p^*) = 282.88,$$

$$p_{23}^* = 25530.00, \quad Q_{23}^* = 98.00, \quad d_{23}(Q^*, p^*) = 1737.66.$$

For Hospital 3:

$$p_{31}^* = 13970.94, \quad Q_{31}^* = 98.00, \quad d_{31}(Q^*, p^*) = 319.53,$$

$$p_{32}^* = 59284.91, \quad Q_{32}^* = 98.00, \quad d_{32}(Q^*, p^*) = 87.37,$$

$$p_{33}^* = 30431.76, \quad Q_{33}^* = 98.00, \quad d_{33}(Q^*, p^*) = 2064.76.$$

For Hospital 4:

$$p_{41}^* = 15301.00, \quad Q_{41}^* = 98.00, \quad d_{41}(Q^*, p^*) = 490.89,$$

$$p_{42}^* = 9575.59, \quad Q_{42}^* = 98.00, \quad d_{42}(Q^*, p^*) = 83.05,$$

$$p_{43}^* = 11125.95, \quad Q_{43}^* = 98.00, \quad d_{43}(Q^*, p^*) = 314.86.$$

The net revenue of Hospital 1 in this example for the three procedures for the year is: 15,738,224.00. The net revenue for Hospital 2, in turn, is: 47,157,288.00. The net revenue for Hospital 3 is: 59,996,940.00, and that for Hospital 4 is: 8,510,231.00. Each hospital now enjoys a significantly higher net revenue than it did in Example 1 in Set 1, and a higher overall utility.

The equilibrium prices of all procedures charged by all hospitals in this example, as compared to the previous one, are all higher or the same (as in the case of p_{23}^*, which is at its upper bound) as in Example 1 in Set 2, except for p_{32}^*, which is lower, but only slightly. Again, the equilibrium quality levels remain at the upper bounds, which was also the case for the previous example, due to the much reduced cost coefficients.

5. Summary and Conclusions

In this paper, we constructed a game theory model for hospitals that provide various medical procedures and compete for patients through the quality levels as well as the prices that they charge to paying patients. The price and quality levels are subject to both lower and upper bounds. The governing equilibrium concept for the model is that of Nash equilibrium. The utility functions of the hospitals consist of a revenue component as well as a component capturing altruism benefit, since hospitals are in healthcare, with a monetized weight for the latter.

We formulated the equilibrium conditions as a variational inequality problem, established existence of an equilibrium price and quality level pattern, and also provided a Lagrange analysis for the equilibrium solutions when the variables achieve upper or lower bounds or do not lie at the extremes. A computational procedure was proposed, which resolves the game theory problem into subproblems in prices and quality levels, each of which can be solved, at a given iteration, using a closed form expression, which we delineated. The algorithm was then applied to compute the equilibria in a case study, inspired by four major hospitals in Massachusetts, each of which offers three major medical procedures, which we focused on. We report the incurred demands at the equilibrium patterns, the net revenues, as well as the values of the utilities obtained by the hospitals. We find that the inclusion of altruism benefit in the hospital utility functions can yield higher procedure quality levels as well as lower prices.

This paper is the first to capture at this level of detail competition among hospitals in both prices and quality of procedures from an operations research computable game theory perspective. Future research might consider the incorporation of different payment schemes for procedures, which may include, for example, fixed prices. In addition, the study of demand elasticity for different procedures, in the context of sensitivity analysis, would be worthwhile.

Acknowledgments

The authors thank the two anonymous reviewers and the Editor for their helpful comments and suggestions on the original version of this paper and on a revision.

The first author acknowledges support from the Radcliffe Institute for Advanced Study at Harvard University where she was a 2017 Summer Fellow and from the John F. Smith Memorial Fund at the University of Massachusetts Amherst.

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