A Generalized Nash Equilibrium Network Model for Post-Disaster Humanitarian Relief

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Abstract: We develop a Generalized Nash Equilibrium network model for post-disaster humanitarian relief by nongovernmental organizations (NGOs). NGOs derive utility from providing relief supplies to victims of the disaster at demand points in a supply chain context while competing with each other for financial funds provided by donations. The shared constraints consist of lower and upper bounds for demand for relief items at the demand points to reduce materiel convergence or congestion. This game theory problem is reformulated as an optimization problem and numerical examples and a theoretical case study on Hurricane Katrina given.

Keywords: disaster relief, humanitarian logistics, financial funds, supply chains, competition for funds, materiel convergence, NGOs, optimization, Generalized Nash Equilibrium, variational inequalities

1. Introduction

The number of disasters has doubled globally since the 1980s with the associated costs of the damage and losses estimated at an average $100 billion a year since the turn of the century (Watson et al. (2015)). Notable natural disasters have included Hurricane Katrina in 2005, the Haiti earthquake in 2010, the triple disaster in Fukushima, Japan in 2011, consisting of an earthquake, followed by a tsunami and a nuclear meltdown technological disaster, the second worst nuclear disaster since Chernobyl in 1986; Superstorm Sandy in 2012, tropical cyclone Haiyan in 2013, and the earthquake in Nepal in 2015 (cf. Nagurney, Masoumi, and Yu (2015)). Slow-onset, long-lasting disasters have included the political and humanitarian disaster of the Syrian refugee migration crisis, which escalated in 2015 and is continuing (The Economist (2015)). The Ebola crisis in West Africa, which peaked in 2015, and was the largest such outbreak in history, is an example of a health response disaster (cf. Boseley (2015)).

The number of people that are being affected by disasters is also increasing (see Nagurney and Qiang (2009)), posing challenges to disaster relief organizations, including nongovernmental organizations (NGOs). According to Kopinak (2013), humanitarian aid represents a commitment to support vulnerable populations that have experienced a sudden emergency and require assistance. She notes that distinct humanitarian relief organizations may have individual mandates, goals, strategies, and associated activities, but the majority operate under a single, common, humanitarian principle: to protect the vulnerable by decreasing morbidity and mortality, to reduce suffering, and support the quality of life. Nevertheless, Kopinak (2013) recognizes that many stakeholders believe that humanitarian aid has not been successful in delivering on these due to a lack of coordination, which results in duplication of services. Moreover, NGOs, such as the Red Cross, rely on donors for financial support and, hence, compete with one another for financial donations. Consequently, there are unique challenges in the important humanitarian aid for disaster relief domain. On the supply side, the main challenge is to secure the donations needed for disaster response and the sustainability of their organizations; whereas, on the demand side, the challenge presents itself as fulfilling the victims' needs while avoiding wasteful duplication and congestion in terms of logistics (cf. Nagurney, Yu, and Qiang (2011, 2012) and the references therein).

For example, Barrett and Novack (2015) in the compilation of the Forbes list of the 50 largest charitable organizations in the United States, noted that the Salvation Army, was the second largest charitable organization, and received private donations of $2.12 billion, in its last reported fiscal year, ending September 30, 2014. World Vision, number 11 on the Forbes list, received $832 million in the fiscal year prior, and the American Red Cross, number 16 on the list, received $736 million in private donations. According to Zhuang, Saxton, and Wu (2014), approximately $300 billion are donated to charities in the US each year.

Interestingly, and, as noted by Ortuño et al. (2013), although the importance of donations is a fundamental difference of humanitarian logistics with respect to commercial logistics, this topic has "not yet been sufficiently studied by academics and there is a wide field for future research in this context." Toyasaki and Wakolbinger (2014) developed perhaps the first models of financial flows that captured the strategic interaction between donors and humanitarian organizations using game theory and also included earmarked donations.

In this paper, we construct what we believe is the first Generalized Nash Equilibrium (GNE) model for post-disaster humanitarian relief, which contains both a financial component and a supply chain component. The Generalized Nash Equilibrium problem is a generalization of the Nash Equilibrium problem (cf. Nash (1950, 1951)) in that the players' strategies, as defined by the underlying constraints, depend also on their rivals' strategies.

We utilize a network perspective and focus on decentralized decision-making due to the competition among NGOs for financial funds from donations. Given that the number of registered US nonprofit organizations increased from 12,000 in 1940 to more than 1.5 million in 2012, competition for financial funds is clearly an important issue (cf. Muggy (2015)). Moreover, individual decisions of NGOs as to the delivery of resources affect the entire disaster relief system. For example, within three weeks following the 2010 Haiti earthquake there were 1,000 NGOs operating in that country (Schear (2015)).

At the same time, as noted in Arnett and Zobel (2015), research has shown that non-priority flows in the aftermath of disaster can exceed 50%. The flow of supplies, general donations (solicited or unsolicited), and equipment that travels to the site of the disaster is known as materiel convergence (Fritz and Mathewson (1957)). For example, weeks following the 2004 tsunami, the Colombo airport in Sri Lanka was flooded with humanitarian cargo, much of which was nonpriority cargo, which negatively impacted both airport operations and created pressures on warehousing storage. Months afterwards, many of the items remained unclaimed (Thomas and Fritz (2006)). Having superfluous supplies also poses hardship on relief workers (see Hechmann and Bunde-Birouste (2007)). Holguin-Veras, Jaller, and Wachtendorf (2012), in turn, noted that post the Haiti 2010 earthquake, news media attention of insufficient water supplies in Haiti resulted in massive donations of water to the Dominican Red Cross, resulting in a depreciation of such local goods. Port-au-Price was also saturated with both cargo and gifts-in-kind, so that shipments from the Dominican Republic had to be halted for multiple days. Following the once in 1,000 year flood that hit South Carolina in mid October 2015, resulting in more than 19 deaths, a call for bottled water, resulted in too much bottled water donated, shortly thereafter. As a consequence, rescue workers struggled with surplus pallets of it (GoUpstate.com (2015)). The flood of donated inappropriate materiel in response to a disaster is sometimes called the second disaster.

Importantly, our GNE model ensures that demands are met at demand points but not at the expense of overprovision, which results in waste, and an ineffective allocation of resources (Fritz and Mathewson (1957), Thomas and Fritz (2006)) and congestion (Haghani and Oh (1996), Nagurney, Masoumi, and Yu (2015)). We accomplish this through the imposition of common, that is, shared, constraints among the NGOs with respect to the amounts of the relief item needed at the demand points, in the form of lower and upper bounds on the demands. Such constraints may be imposed by a regulating body in the form of a government or a higher level coordinating humanitarian organization.

The novelty of our GNE network framework captures the following:

1. The nongovernmental (NGO) relief organizations compete for financial funds from donors who respond to the visibility of the organizations at the disaster points. Their utility functions consist of a financial component and a supply chain component.

2. The NGOs, in addition to their individual constraints, face common upper and lower bound demand constraints at each demand point so as to reduce both congestion and materiel convergence, while satisfying the demand.

3. We demonstrate that the GNE model, because of its special structure, can be transformed into an optimization problem, and one does not need to utilize, for example, quasivariational inequality theory (cf. Bensoussan (1974), von Heusinger (2009), and Fischer, Herrich, and Schonefeld (2014), and the references therein) for formulation, analysis, and computations.

4. We provide an easy to implement and effective computational procedure, which yields closed form expressions for the product flows and the Lagrange multipliers associated with the supply and demand bound constraints at each iteration.

5. We illustrate our framework with numerical examples that demonstrate the benefits of this framework vis a vis a purely Nash Equilibrium approach. In addition, we present a theoretical case study based on Hurricane Katrina, the costliest natural disaster to strike the United States.

The results in this paper contribute to both game theory frameworks for disaster relief as well as to the modeling of financial flow of funds in this public sector domain. Although there is now a rich literature on financial networks in the profit sector (see, e.g., Nagurney and Siokos (1997), Nagurney (2003), and Nagurney and Qiang (2009), and the references therein), the nonprofit sector, in terms of financial networks, remains only minimally explored research-wise to-date.

Our disaster relief game theory framework entails competition for donors as well as media exposure plus supply chain aspects. Hence, we now highlight some of the related literature on these topics. We begin first with the literature on donors and media. Natsios (1995) contends that the cheapest way for relief organizations to fundraise is to provide early relief in highly visible areas. Balcik et al. (2010) note that the media is a critical factor affecting relief operations with NGOs seeking visibility to attract more resources from donors. They also review the challenges in coordinating humanitarian relief chains and describe the current and emerging coordination practices in disaster relief. Olsen and Carstensen (2003) confirmed the frequently repeated argument that media coverage is critical in relation to emergency relief allocation in a number of cases that they analyzed. Van Wassenhove (2006) also emphasizes the role of the media in humanitarian logistics and states that following appeals in the media, humanitarian organizations are often flooded with unsolicited donations that can create bottlenecks in the supply chain. Zhuang, Saxton, and Wu (2014) develop a model that reveals the amount of charitable contributions made by donors is positively dependent on the amount of disclosure by the NGOs. They also emphasize that there is a dearth of existing game-theoretic research on nonprofit organizations. Our model attempts to help to fill this void. Toyasaki and Wakolbinger (2015) develop optimization models to analyze whether an NGO should establish a special fund after an emergency (in terms of earmarked donations) or allow only unearmarked donations.

We now highlight several references to the humanitarian supply chain literature, again, emphasizing that our framework integrates both financial funding aspects and supply chain aspects. We focus on optimization models in disaster relief since our model considers the maximization of each NGO's utility associated with the delivery of relief supplies. Haghani and Oh (1996) construct a multicommodity, multimodal network flow model for disaster relief and conclude that a nonlinear formulation that can handle congestion is needed. Our model cost functions are nonlinear and our model is also network-based. Sheu (2007) focuses on quick response to disasters and introduced a hybrid fuzzy clustering-optimization approach to emergency logistics operations co-distribution for urgent relief. Sheu (2010), subsequently, presents a novel dynamic relief demand management model for emergency logistics consisting of multiple steps and considers imperfect information. Tzeng, Cheng, and Huang

(2007) use multi-objective programming for the design of a real case relief delivery system with criteria consisting of total cost minimization, total travel time minimization, and the maximization of the minimal satisfaction during the planning period. Balcik, Beamon, and Smilowitz (2008) focus on last mile distribution and propose a mixed integer programming model for the equitable allocation of resources. Vitoriano et al. (2011) propose a multicriteria model for humanitarian aid distribution with criteria of time of response, equity, and security. Huang, Smilowitz, and Balcik (2012) also consider multiple objectives of efficiency, efficacy, and equity and impacts on vehicle routing and resource distribution of their model. Nagurney, Masoumi, and Yu (2015) construct a supply chain network optimization model for a disaster relief organization in charge of obtaining, storing, transporting, and distributing relief items to disaster-prone regions. The system-optimization approach minimizes the total operational costs on the links of the supply chain network subject to the uncertain demand for aid at the demand points being satisfied as closely as possible. Nagurney and Nagurney (2016), subsequently, develop a mean-variance disaster relief supply chain network model with stochastic link costs and time targets for the delivery of the relief supplies at the demand points, under demand uncertainty. Our model, in contrast to the above ones, integrates both financial donation aspects and supply chain aspects and utilizes game theory.

We now discuss some additional relevant literature, focusing on the Generalized Nash Equilibrium problem, which captures shared constraints among the players in a game, as is the case for our model. The Generalized Nash Equilibrium problem dates to Debreu (1952) and Arrow and Debreu (1954), although it was not termed as such. Rosen (1965) formalized the definition of a normalized Nash Equilibrium, provided qualitative properties, along with an algorithm. Bensoussan (1974), subsequently, formulated the GNE problem as a quasivariational inequality. Gabay and Moulin (1980), focusing on Nash equilibria (cf. Nash (1950, 1951)) in oligopolistic market settings, demonstrated that such problems, in which the feasible set underlying each player in the game depends only on his strategies, and not on the strategies of the other players, could be directly formulated as variational inequality problems, whereas GNE problems could not. Interestingly, the Generalized Nash Equilibrium problem did not gain attention until many years after its formulation, and this may be due to the challenges in solving such problems and perhaps due to a lack of pressing, relevant applications. In the late 1990s, GNE models were constructed for the energy sector, following an increasing interest in this sector (see, e.g., Contreras, Klusch, and Krawczyk (2004), Krawczyk (2005), and the references therein). For background on the GNE problem, see von Heusinger (2009) and the recent review by Fischer, Herrich, and Schonefeld (2014). It is worth noting that the state of the art for Generalized Nash Equilibrium problems in terms of both models and algorithms is not as advanced as that for Nash Equilibrium problems, which, especially in the commercial supply chain context, have a rich literature (see Nagurney (2006), Liu, Woolley, and Cruz (2012), Nagurney et al. (2013), and Toyasaki, Daniele, and Wakolbinger (2014)). Muggy and Stamm (2014), in turn, provide an excellent review of game theory in humanitarian operations and emphasize that there are many untapped research opportunities for modeling in this area.

The paper is organized as follows. In Section 2, we develop the GNE model for humanitarian relief post disasters. We assume that the NGOs compete and yet must satisfy the demand at the demand points, in terms of shared lower and upper bound constraints. We then show how the model can be transformed into an optimization problem. In addition, we derive a variational inequality formulation for the optimization problem, which enables the application of an effective and easy to implement algorithm. We also provide existence and uniqueness results. In Section 3, we recall the Euler method (cf. Dupuis and Nagurney (1993) and Nagurney and Zhang (1996)) and provide the closed form expressions of its realization for our model for the product flows and the Lagrange multipliers associated with the supply and demand constraints, at each iteration. In Section 4, we present numerical examples to illustrate our framework. For completeness, we also compare the solutions obtained to those for the Nash Equilibrium counterpart. In Section 5, we present a theoretical case study based on Hurricane Katrina. In Section 6, we summarize our results, present our conclusions, and provide suggestions for future research.

2. The Generalized Nash Equilibrium Model for Humanitarian Relief

In this section, we develop a Generalized Nash Equilibrium model for post-disaster humanitarian relief. We assume that there are m NGOs involved in providing disaster relief to n different locations as represented by the network in Figure 1. We denote a typical NGO by i; $i = 1, \ldots, m$, and a typical demand point by j; $j = 1, \ldots, n$. In the network in Figure 1, the downward links connecting the NGO nodes with the demand point nodes represent the relief item flows. These flows in our model correspond to a specific relief item, such as water, food, or medicines, etc. Since the NGOs are competing we expect them to be providing similar goods. In the case of heterogeneous items, we assume that the items can be approximated by that of a homogeneous item type. The links connecting the demand point nodes with node D represent the financial donations given by those who respond to the public visibility of the NGOs at various demand points through their relief efforts. The upward arrows from node D, which is a supersink node (see Nagurney and Dong (2002)), to the NGO nodes represent the financial flows obtained by the individual NGOs following the relief efforts. [1]

The relief item flow from NGO i to demand point j is denoted by q_{ij}. We group the nonnegative relief item flows from each i; $i = 1, \ldots, m$, into the vector $q_i \in R^n_+$ and then we group the relief item flows of all the NGOs to all the demand points into the vector $q \in R^{mn}_+$. Note that q_i is the vector of strategies of NGO i.

We assume that each NGO i has, at its disposal, an amount s_i of the relief item that it can allocate post-disaster. Hence, we have the following conservation of flow equation, which must hold for each i; $i = 1, \ldots, m$:

$$\sum_{j=1}^{n} q_{ij} \leq s_i. \tag{1}$$

In addition, we know that the product flows for each i; $i = 1, \ldots, m$, must be nonnegative, that is:

$$q_{ij} \geq 0, \quad j = 1, \ldots, n. \tag{2}$$

Each NGO i encumbers a cost, c_{ij}, associated with shipping the relief items to location

[1] Hence, in terms of the sequence of events, the NGOs first decide on the level of relief items to be provided at each demand point and deliver the amounts. Then they receive the corresponding financial flows. Therefore, the financial flows are received after the supplies arrive. Empirically, these funds are realized and made available quickly, and these two events are almost concurrent in many cases. Therefore, in our model, we assume that the discount factor to be zero. The justification of this assumption is also provided by the nature of the incentives of the decision-makers in our model, which is to provide humanitarian relief as quickly as possible whenever a need arises.

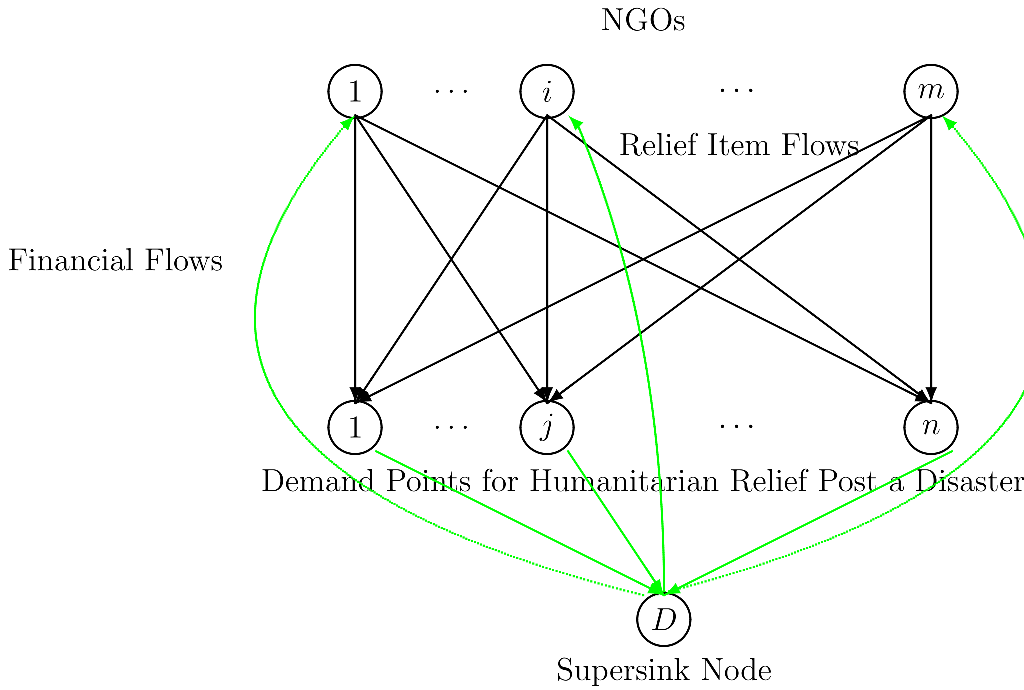

Figure 1: The Network Structure of the Game Theory Model

j, denoted by c_{ij}, where we assume that

$$c_{ij} = c_{ij}(q_{ij}), \quad j = 1, \dots n, \tag{3}$$

with these cost functions being strictly convex and continuously differentiable. These costs also include transaction costs. Since we expect there to be congestion at the points of demand, such assumptions are not unreasonable (see also Nagurney (1999)).

In addition, each NGO i; $i = 1, \dots, m$, derives utility associated with providing the relief items to j; $j = 1, \dots, n$, with its utility over all demand points given by $\sum_{j=1}^{n} \gamma_{ij} q_{ij}$. Here γ_{ij} is a positive factor representing a measure of satisfaction/utility that NGO i acquires through its supply chain activities to demand point j. Each NGO i; $i = 1, \dots, m$, associates a positive weight ω_i with $\sum_{j=1}^{n} \gamma_{ij} q_{ij}$, which provides a monetization of, in effect, this component of the objective function. Such weight concepts are used in multicriteria decision-making; see, e.g., Fishburn (1970), Chankong and Haimes (1983), Yu (1985), Keeney and Raiffa (1993), and Nagurney and Toyasaki (2003) for an application to supply chains.

Finally, each NGO i; $i = 1, \dots, m$, based on the media attention and the visibility of NGOs at location j; $j = 1, \dots, n$, acquires funds from donors given by the expression

$$\beta_i \sum_{j=1}^{n} P_j(q), \tag{4}$$

where $P_j(q)$ represents the financial funds in donation dollars due to visibility of all NGOs at location j. Hence, β_i is a parameter that reflects the proportion of total donations collected for the disaster at demand point j that is received by NGO i. Expression (4), therefore, represents the financial flow on the link joining node D with node NGO i. Note that, according to (4), there is competition among all the NGOs for financial donations. Also, according to (4), an NGO may benefit from donations even through visibility of other NGOs providing the product because of, for example, loyalty and support for a specific NGO. We assume that the P_j functions are increasing, concave, and continuously differentiable. Hence, we have positive but decreasing marginal utility of providing aid (in terms of the NGOs effect on attracting donations).

We are now ready to formulate the optimization problem faced by each NGO. For additional background on utility functions for nonprofit and charitable organizations, see Rose-Ackerman (1982) and Malani, Philipson, and David (2003). We will follow with the common constraints.

Each NGO seeks to maximize its utility with the utility corresponding to the financial gains associated with the visibility through media of the relief item flow allocations, $\beta_i \sum_{j=1}^{n} P_j(q)$, plus the utility associated with the supply chain aspect of delivery of the relief items, $\sum_{j=1}^{n} \gamma_{ij} q_{ij} - \sum_{j=1}^{n} c_{ij}(q_{ij})$. The optimization problem faced by NGO i; $i = 1, \ldots, m$, is, hence,

$$\text{Maximize} \quad \beta_i \sum_{j=1}^{n} P_j(q) + \omega_i \sum_{j=1}^{n} \gamma_{ij} q_{ij} - \sum_{j=1}^{n} c_{ij}(q_{ij}) \tag{5}$$

subject to constraints (1) and (2).

Note that the above model, in the absence of any common constraints, is a Nash Equilibrium problem, which we know can be formulated and solved as a variational inequality problem (cf. Gabay and Moulin (1980) and Nagurney (1999)). However, the NGOs are faced with several common constraints, which complexifies the problem and which also poses further challenges in terms of formulation and computations. Later in this section, we establish that, because of the special structure of the model, we can transform our Generalized Nash Equilibrium model into an optimization problem and need not make use of quasivariational inequalities (see von Heusinger (2009)).

The common constraints, which are imposed by an authority, such as a governmental one or a higher level humanitarian coordination agency, ensure that the needs of the disaster victims are met, while recognizing the negative effects of waste and material convergence. These constraints are assumed to be common knowledge for the NGOs. The model is prescriptive and the subsequent numerical examples and case study illustrate the benefits of the common constraints.

Specifically, we also have that, at each demand point j; $j = 1, \ldots, n$:

$$\sum_{i=1}^{m} q_{ij} \geq \underline{d}_j, \tag{6}$$

and

$$\sum_{i=1}^{m} q_{ij} \leq \bar{d}_j, \tag{7}$$

where $\underline{d}_j$ denotes a lower bound for the amount of the relief items needed at demand point j and $\bar{d}_j$ denotes an upper bound on the amount of the relief items needed post the disaster at demand point j.

Expressions (6) and (7) provide flexibility for a regulatory or coordinating body, as it is unlikely that the demand for the relief items will be precisely known in a disaster situation. It is, however, reasonable to assume that, as represented in these equations, estimations for total demand for relief items will be available at the local level.

We assume that

$$\sum_{i=1}^{m} s_i \geq \sum_{j=1}^{n} \underline{d}_j, \tag{8}$$

so that the supply resources of the NGOs are sufficient to meet the minimum financial resource needs at all the demand points following the disaster. Hence, we assume prepositioning of supplies so that they are in stock and available. As noted in Roopanarine (2013), propositioning of supplies can make emergency relief more effective and this is a strategy followed not only by the UNHRD (United Nations Humanitarian Response Depot) but also by the Red Cross and even some smaller relief organizations such as AmeriCares. Indeed, Gatignon, Van Wassenhove, and Charles (2010) also reported on the benefits of proper prepositioning of supplies in the case of the International Federation of the Red Cross (IFRC) in terms of cost reduction and a more timely response.

Hence, each NGO i; $i = 1, \ldots, m$, seeks to determine its optimal vector of relief items or strategies, q_i^*, that maximizes objective function (5), subject to constraints (1), (2), and (6), (7). This is the Generalized Nash Equilibrium problem for our humanitarian relief post disaster problem.

Theorem 1: Optimization Formulation of the Generalized Nash Equilibrium Model of Financial Flow of Funds

The above Generalized Nash Equilibrium problem, with each NGO's objective function (5), can be rewritten as:

$$\textit{Minimize} \quad -\beta_i \sum_{j=1}^{n} P_j(q) - \omega_i \sum_{j=1}^{n} \gamma_{ij} q_{ij} + \sum_{j=1}^{n} c_{ij}(q_{ij}) \tag{9}$$

and subject to constraints (1) and (2), with common constraints (6) and (7), is equivalent to the solution of the following optimization problem:

$$\textit{Minimize} \quad -\sum_{j=1}^{n} P_j(q) - \sum_{i=1}^{m}\sum_{j=1}^{n} \frac{\omega_i \gamma_{ij}}{\beta_i} q_{ij} + \sum_{i=1}^{m}\sum_{j=1}^{n} \frac{1}{\beta_i} c_{ij}(q_{ij}) \tag{10}$$

subject to constraints: (1), (2), (6), and (7).

Proof: We observe that the above Generalized Nash Equilibrium problem consists of a separable term for each decision-maker or player, NGO i in the game, given by: $-\omega_i \sum_{j=1}^{n} \gamma_{ij} q_{ij} + \sum_{j=1}^{n} c_{ij}(q_{ij})$, as well as a common parameterized term across all decision-makers or NGOs, given by: $\sum_{j=1}^{n} P_j(q)$, with the parameter β_i being distinct for each NGO i; $i = 1, \ldots, m$. Hence, it is precisely of the general structure considered in Li and Lin (2013). An application of Theorem 2.3 therein, which established that the normalized stationary point of the GNE problem coincides with the stationary point of the optimization problem, of the form above for our model, yields the result. $\square$

Following the proof of Theorem 1, we have added the following paragraph: We know that (5) is equivalent to (9). Moreover, the structure of (9), which has a separability feature in that the second and third terms are specific for each NGO, but the first term for each NGO i; $i = 1, \ldots, m$, consists of a common term $P_j(q)$ and differs from one NGO to another only by its coefficient β_i is precisely of the structure of the Generalized Nash Equilibrium problems considered by Li and Lin (2013), which can be transformed into optimization problems by extending the normalized equilibrium concept to the stationarity sense, as they have done.

Remark: Existence and Uniqueness

A solution q^* of product flows to the above optimization problem for humanitarian relief is guaranteed to exist since the objective function consists of continuous functions and the feasible set comprised of the constraints is compact. Moreover, under the assumptions that the functions P_j, $\forall j$, are concave, and the functions $c_{ij}(q_{ij})$ are strictly convex for all i, j, since we know that $-\frac{\omega_i \gamma_{ij}}{\beta_i} q_{ij}$ are convex for all i, j, the objective function (10) is strictly convex, and, hence, uniqueness of the product flow pattern q^* is also guaranteed.

In order to solve this problem, we can use the following variational inequality formulation of the above optimization problem, since this will allow us to apply the Euler method (cf. Dupuis and Nagurney (1993) and Nagurney and Zhang (1996)), which yields closed form expressions in the financial flows and Lagrange multipliers associated with the supply and the lower bound and upper bound demand constraints.

Variational Inequality Formulation

Specifically, it is straightforward to verify (see also Kinderlehrer and Stampacchia (1980) and Nagurney (1999, 2006)) that the solution q^* with associated Lagrange multipliers λ_k^*; $k=1,\ldots,m$, for the supply constraints, the Lagrange multipliers: $\lambda_l^{1^*}$; $l=1,\ldots,n$, for the lower bound demand constraints, and the Lagrange multipliers: $\lambda_l^{2^*}$; $l=1,\ldots,n$, for the upper bound demand constraints, can be obtained by solving the variational inequality problem: determine $(q^*,\lambda^*,\lambda^{1^*},\lambda^{2^*})\in R_+^{mn+m+2n}$ such that

$$\sum_{k=1}^{m}\sum_{l=1}^{n}\left[-\sum_{j=1}^{n}(\frac{\partial P_j(q^*)}{\partial q_{kl}})-\frac{\omega_k\gamma_{kl}}{\beta_k}+\frac{1}{\beta_k}\frac{\partial c_{kl}(q_{kl}^*)}{\partial q_{kl}}+\lambda_k^*-\lambda_l^{1^*}+\lambda_l^{2^*}\right]\times[q_{kl}-q_{kl}^*]$$

$$+\sum_{k=1}^{m}(s_k-\sum_{l=1}^{n}q_{kl}^*)\times(\lambda_k-\lambda_k^*)+\sum_{l=1}^{n}(\sum_{k=1}^{n}q_{kl}^*-\underline{d}_l)\times(\lambda_l-\lambda_l^{1^*})$$

$$+\sum_{l=1}^{n}(\bar{d}_l-\sum_{k=1}^{m}q_{kl}^*)\times(\lambda_l^2-\lambda_l^{2^*})\geq 0,\quad \forall(q,\lambda,\lambda^1,\lambda^2)\in R_+^{mn+m+2n}, \tag{11}$$

where λ is the vector of Lagrange multipliers: $(\lambda_1,\ldots,\lambda_m)$, λ^1 is the vector of Lagrange multipliers: $(\lambda_1^1,\ldots,\lambda_n^1)$, and λ^2 is the vector of Lagrange multipliers: $(\lambda_1^2,\ldots,\lambda_n^2)$.

We now put variational inequality (11) into standard variational inequality form (see Nagurney (1999)), that is: determine $X^*\in\mathcal{K}\subset R^N$, such that

$$\langle F(X^*),X-X^*\rangle\geq 0,\quad \forall X\in\mathcal{K}, \tag{12}$$

where F is a given continuous function from $\mathcal{K}$ to R^N, $\mathcal{K}$ is a closed and convex set, with both the vectors $F(X)$ and X being column vectors, and $N=mn+m+2n$.

We define the $(mn+m+2n)$-dimensional column vector $X\equiv(q,\lambda,\lambda^1,\lambda^2)$ and the $(mn+m+2n)$-dimensional column vector $F(X)\equiv(F^1(X),F^2(X),F^3(X),F^4(X))$ with the (k,l)-th component, F_{kl}^1, of $F^1(X)$ given by

$$F_{kl}^1(X)\equiv-\sum_{j=1}^{n}(\frac{\partial P_j(q)}{\partial q_{kl}})-\frac{\omega_k\gamma_{kl}}{\beta_k}+\frac{1}{\beta_k}\frac{\partial c_{kl}(q_{kl})}{\partial q_{kl}}+\lambda_k-\lambda_l^1+\lambda_l^2, \tag{13}$$

the k-th component, F_k^2, of $F^2(X)$ given by

$$F_k^2(X) \equiv s_k - \sum_{l=1}^{n} q_{kl}, \tag{14}$$

the l-th component, F_l^3, of $F^3(X)$ given by

$$F_l^3(X) \equiv \sum_{k=1}^{n} q_{kl} - \underline{d}_l, \tag{15}$$

and the l-th component, F_l^4, of $F^4(X)$ given by

$$F_l^4(X) \equiv \bar{d}_l - \sum_{k=1}^{m} q_{kl}, \tag{16}$$

with the feasible set $\mathcal{K} \equiv R_+^{mn+m+2n}$. Then, clearly, variational inequality (11) can be put into standard form (12).

3. The Algorithm

Variational inequality (12); equivalently, variational inequality (11), can be solved via the Euler method of Dupuis and Nagurney (1993), which, at each iteration, yields closed form expressions for the financial flows and the Lagrange multipliers.

Specifically, iteration τ of the Euler method, where the variational inequality (11) is expressed in standard form (12), is given by:

$$X^{\tau+1} = P_{\mathcal{K}}(X^\tau - a_\tau F(X^\tau)), \tag{17}$$

where $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$ and F is the function that enters the variational inequality problem (12), where recall that $X \equiv (q, \lambda, \lambda^1, \lambda^2)$ and $F(X)$ consists of the components as defined in (13) – (16).

As established in Dupuis and Nagurney (1993), for convergence of the general iterative scheme, which induces the Euler method, the sequence $\{a_\tau\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \to 0$, as $\tau \to \infty$. Conditions for convergence for a variety of network-based problems can be found in Nagurney and Zhang (1996) and Nagurney (2006).

Explicit Formulae for the Euler Method Applied to the Game Theory Model

The elegance of this algorithm for our variational inequality (17) for the computation of solutions to our game theory model lies in the following explicit formulae. In particular, we have the following closed form expression for the product flows $k = 1, \ldots, m; l = 1, \ldots, n$, at each iteration:

$$q_{kl}^{\tau+1} = \max\{0, \{q_{kl}^\tau + a_\tau(\sum_{j=1}^n (\frac{\partial P_j(q^\tau)}{\partial q_{kl}}) + \frac{\omega_k \gamma_{kl}}{\beta_{kl}} - \frac{1}{\beta_k}\frac{\partial c_{kl}(q_{kl}^\tau)}{\partial q_{kl}} - \lambda_k^\tau + \lambda_l^{1^\tau} - \lambda_l^{2^\tau})\}, \quad (18)$$

the following closed form expressions for the Lagrange multipliers associated with the supply constraints, respectively, for $k = 1, \ldots, m$:

$$\lambda_k^{\tau+1} = \max\{0, \lambda_k^\tau + a_\tau(-s_k + \sum_{l=1}^n q_{kl}^\tau)\}, \quad (19)$$

the following closed form expressions for the Lagrange multipliers associated with the lower bound demand constraints, respectively, for $l = 1, \ldots, n$:

$$\lambda_l^{1^{\tau+1}} = \max\{0, \lambda_l^{1^\tau} + a_\tau(-\sum_{k=1}^n q_{kl}^\tau + \underline{d}_l)\}, \quad (20)$$

andthe following closed form expressions for the Lagrange multipliers associated with the upper bound demand constraints, respectively, for $l = 1, \ldots, n$:

$$\lambda_l^{2^{\tau+1}} = \max\{0, \lambda_l^{2^\tau} + a_\tau(-\bar{d}_l + \sum_{k=1}^m q_{kl}^\tau)\}, \quad (21)$$

In the next Section, we apply the Euler method to computer solutions to numerical examples.

4. Numerical Examples

In this Section, we present numerical examples to illustrate our Generalized Nash Equilibrium framework. The Euler method was implemented in FORTRAN and the system utilized for the computations was a Linux system at the University of Massachusetts Amherst. The sequence $\{a_\tau\}$ that we used and which satisfies the requirements for convergence was: $.1(1, \frac{1}{2}, \frac{1}{2}. \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, ...\}$. We initialized the Euler method with all the initial relief item flows equal to 1 and all the Lagrange multipliers equal to zero. The Euler method was deemed to converge when the absolute value of all the relief item flows and Lagrange multipliers differed from one iteration to the next by no more than 10^{-5}.

Example 1

The first example consists of two NGOs and a single demand point, as depicted in Figure 2

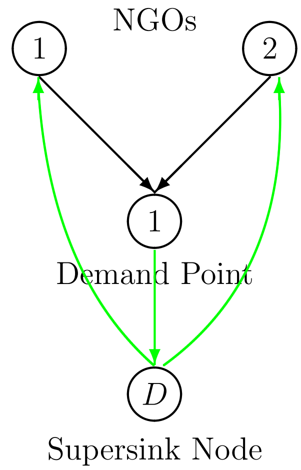

Figure 2: The Network Topology for Example 1

The data are as follows:

$$\omega_1 = 1, \quad \omega_2 = 2,$$

$$\gamma_{11} = 1000, \quad \gamma_{21} = 400,$$

$$c_{11}(q_{11}) = q_{11}^2 + 10q_{11}, \quad c_{21}(q_{21}) = q_{21}^2 + 20q_{21}.$$

Hence, NGO 2 is located further from the demand point for the relief items than NGO 1. Also, these functions are nonlinear to capture congestion. Also, NGO 1 derives greater utility from providing for the victims than NGO 2.

In this example,

$$P_1(q) = 5\sqrt{\sum_{i=1}^{2} q_{i1}},$$

with

$$\beta_1 = .5, \quad \beta_2 = .5.$$

Therefore, the donors split the donations equally between the two NGOs.

NGO 1 has a supply $s_1 = 10000$ whereas NGO 2 has a supply $s_2 = 20000$.

The lower bound for the demand is $\underline{d}_1 = 500$ and the upper bound is $\bar{d}_1 = 600$.

The computed equilibrium solution is:

$$q_{11}^* = 352.5, \quad q_{21}^* = 247.5,$$

so that total amount of supplies shipped to the demand point is 600.0. Note that this amount is at the upper bound of the demand.

Also, the computed equilibrium Lagrange multipliers are:

$$\lambda_1^* = 0.00, \quad \lambda_2^* = 0.00,$$

and

$$\lambda_1^{1^*} = 0.00, \quad \lambda_1^{2^*} = 570.0.$$

NGO 1 receives a utility of: 224786.4 and NGO 2 receives a utility of: 131860.6.

The total donations, that is, the financial flows from the demand points to node D in Figure 2 are 122.48, with 61.24 of these flowing back to NGO 1 and the same amount flowing back to NGO 2.

We then proceeded to compute the solution to the Nash Equilibrium counterpart of this model, which has the same utility functions, supply constraints, and nonnegativity constraints on the relief item flows but no upper bound and lower bound demand constraints (which recall are the shared constraints imposed by a higher regulatory or coordinating body to reduce materiel convergence, for example, and to minimize waste). This model can also be solved using the Euler method. Indeed, one just eliminates Steps (20) and (21) and removes the Lagrange multipliers associated with the demand bounds in expression (18). The so-modified Step (18) and Step (19) then can be applied to compute the Nash Equilibrium solution as we have done.

The Nash Equilibrium solution is:

$$q_{11} = 495.0, \quad q_{21} = 390.0,$$

with the total amount of the relief item shipped to the demand point equal to 885.0. Note that, without the demand bounds, each NGO provides more than 140% of the relief item under the Nash Equilibrium solution than it does under the Generalized Nash Equilibrium solution. There is an excess of 285 units of the relief item, which demonstrates the benefit of the Generalized Nash Equilibrium approach. Not surprisingly, the NGOs enjoy a higher utility with NGO 1 having a utility of 2450899.4 under the Nash Equilibrium concept and NGO 2 having a utility of 152174.4. The total amount of financial donations is now 148.75 with 74.37 flowing to NGO 1 and the same amount to NGO 2.

Example 2

Example 2 is constructed from Example 1 and has the same data except for the added data corresponding to Demand Point 2, which represents a new location that the two NGOs have found out also needs supplies for relief. The network topology is as depicted in Figure 3.

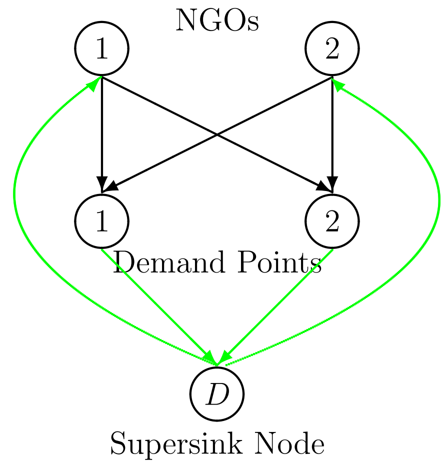

Figure 3: The Network Topology for Example 2

The new data are:

$$\gamma_{12} = 1000, \quad \gamma_{22} = 400,$$

$$c_{12}(q_{12}) = q_{12}^2 + 20, \quad c_{22}(q_{22}) = q_{22}^2 + 30.$$

Note that Demand Point 2 is further away from both NGOs than is Demand Point 1. Also, we have that

$$P_2(q) = 6\sqrt{\sum_{i=1}^{2} q_{i2}}$$

and

$$\underline{d}_2 = 700, \quad \bar{d}_2 = 800.$$

Hence, the lower and upper bounds for the demand at Demand Point 2 are higher than those for Demand Point 1.

The Euler method yields the following equilibrium solution:

$$q_{11}^* = 352.5, \quad q_{12}^* = 452.5, \quad q_{21}^* = 247.5, \quad q_{22}^* = 347.5,$$

with equilibrium Lagrange multipliers:

$$\lambda_1^* = 0.00, \quad \lambda_2^* = 0.00,$$

$$\lambda_1^{1^*} = 0.00, \quad \lambda_2^{1^*} = 0.00, \quad \lambda_1^{2^*} = 570.0, \quad \lambda_2^{2^*} = 150.1.$$

The total amount of relief item shipments to Demand Point 1 is: 600 and to Demand Point 2: 800. Both these demands are at the respective upper bounds for the demand at those demand points.

The utility of NGO 1 is: 463565.4 and that of NGO 2: 278764.6.

The total financial donations that flow from the demand point nodes to node D in Figure 3 are: 292.19 with half flowing back to NGO 1 and half to NGO 2.

As in Example 1, we then proceeded to compute the solution to the Nash Equilibrium counterpart for this example.

The Nash Equilibrium flow pattern is:

$$q_{11} = 495.0, \quad q_{12} = 490.0, \quad q_{21} = 390.0, \quad q_{22} = 385.0.$$

The amount of supplies shipped to Demand Point 1 is: 885.0 and the amount shipped to Demand Point 2 is: 875.0. These shipments exceed the demand upper bounds by a large margin, demonstrating, again, that NGOs may oversupply materiel to victims post a disaster if interventions such as regulations or enhanced coordination are lacking. The total amount of donations is: 326.24, with half going to NGO 1 and half to NGO 2.

Example 3

The third example has the topology given in Figure 4. The data are as in Example 2 but with new data added for the third demand point. Hence, there are two NGOs in this example and three demand points.

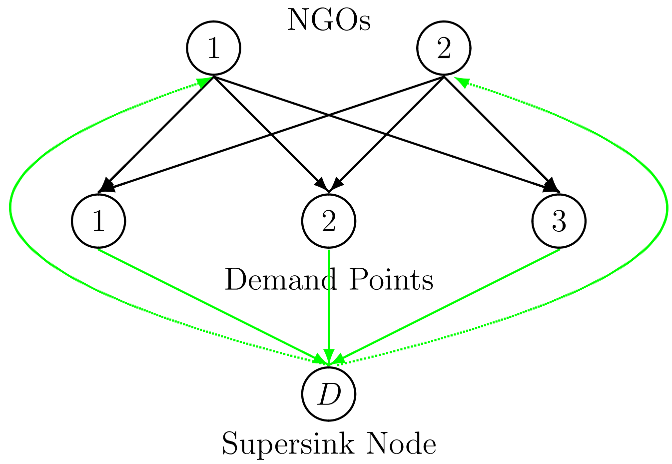

Figure 4: The Network Topology for Example 3

The new data are:

$$\gamma_{13} = 700, \quad \gamma_{23} = 300,$$

and since Demand Point 3 is closer to the two NGOs than the other two demand points, the transportation cost functions to the third demand point are:

$$c_{13}(q_{13}) = q_{13}^2 + 5, \quad c_{23}(q_{23}) = q_{23}^2 + 10.$$

Also, we have that:

$$P_3(q) = 10\sqrt{\sum_{i=1}^{2} q_{i2}}$$

and

$$\underline{d}_2 = 1000, \quad \bar{d}_2 = 1500.$$

Hence, the demand range is larger at Demand Point 3 than at the other two demand points and the values of the bounds are also higher signifying a greater need for relief supplies there.

Also, we modified the β_i parameters so that now the NGOs do not get an equal allocation of the donations:

$$\beta_1 = .25, \quad \beta_2 = .75.$$

The Euler method yields the following equilibrium relief item flow solution:

$$q_{11}^* = 423.8, \quad q_{12}^* = 471.3, \quad q_{13}^* = 436.9,$$

$$q_{21}^* = 176.3, \quad q_{22}^* = 328.8, \quad q_{23}^* = 563.1,$$

and the following Lagrange multipliers:

$$\lambda_1^* = 0.00, \quad \lambda_2^* = 0.00,$$

$$\lambda_1^{1*} = 0.00, \quad \lambda_2^{1*} = 0.00, \quad \lambda_3^{1*} = 714.8,$$

$$\lambda_1^{2*} = 570.0, \quad \lambda_2^{2*} = 150.1, \quad \lambda_3^{2*} = 0.00.$$

Interestingly, the shipments to Demand Point 1 are at the demand upper bound as are those to Demand Point 2. However, the relief item shipments to Demand Point 3 are at the demand lower bound for that demand point.

NGO 1 has a utility of 592620.44 whereas NGO 2 has a utility of 267093.16. The total amount of financial funds given by donors is: 608.41, with NGO 1 receiving 152.1 and NGO 2 receiving 456.3.

The Nash Equilibrium solution is the following:

$$q_{11}^* = 495.0, \quad q_{12}^* = 490.0, \quad q_{13}^* = 347.5,$$

$$q_{21}^* = 390.0, \quad q_{22}^* = 385.0, \quad q_{23}^* = 295.1.$$

Interestingly, there is an oversupply of relief items at Demand Points 1 and 2. However, there is an undersupply of relief items at Demand Point 3 since the lower bound there was 1000 and only 642.6 were delivered. Also, interestingly, the total donations drop and are now: 579.7, with NGO 1 receiving 144.9 and NGO 2 receiving 434.8. This example illustrates that imposed shared constraints clearly benefits victims of a disaster and can even help the NGOs in terms of financial donations, as this example reveals.

Example 4

Example 4 was constructed from Example 3 to show that the framework can be used in the case that the demands at all the demand points are known. The algorithm (and model) can handle such a situation by just having each demand point lower bound equal to the upper bound, which is equal to the fixed demand.

Hence, for Example 4, we doubled the upper bounds for each demand point and set the demand lower bounds to the midpoint of the respective demand bounds in Example 3 so that:

$$\underline{d}_1 = 550, \quad \bar{d}_1 = 550, \quad \underline{d}_2 = 750, \quad \bar{d}_2 = 750, \quad \underline{d}_3 = 1250, \quad \bar{d}_3 = 1250.$$

The Euler method yields the following equilibrium relief item flow solution:

$$q^*_{11} = 411.3, \quad q^*_{12} = 458.8, \quad q^*_{13} = 499.4,$$

$$q^*_{21} = 138.8, \quad q^*_{22} = 291.3, \quad q^*_{23} = 750.6.$$

The total donations are now: 635.12 with NGO 1 receiving 158.78 and NGO 2 receiving 476.34. The utility of NGO 1 is 574991.2 and that of NGO 2: 108392.6.

5. A Case Study on Hurricane Katrina

In this section, we demonstrate further the model by presenting a theoretical case study on Hurricane Katrina, the costliest disaster in the history of the United States (U.S. House of Representatives (2006)). Making landfall in August of 2005, Katrina caused extensive damages to property and infrastructure, left 450,000 people homeless, and took 1,833 lives in Florida, Texas, Mississippi, Alabama, and Louisiana (Louisiana Geographic Information Center (2005)). Given the hurricane's trajectory, most of the damage was concentrated in Louisiana and Mississippi. In fact, 63% of all insurance claims were in Louisiana, a trend that is also reflected in FEMA's post-hurricane damage assessment of the region (FEMA (2006)). The total damage estimates range from $105 billion (Louisiana Geographic Information Center (2005)) to $150 billion (White (2015)), making Hurricane Katrina not only a far-reaching and costly disaster, but also a very challenging environment for providing humanitarian assistance.

In Louisiana alone, over 1.3 million people were affected, with Katrina being responsible for 300,000 jobs lost, 200,000 people left homeless, and over 1,500 fatalities (Louisiana Geographic Information Center (2005)). For comprehensive studies on the impacts of Hurricane Katrina in Louisiana, we refer the reader to Louisiana Geographic Information Center (2005), U.S. House of Representatives (2006), FEMA (2006), and Logan (2006). In addition, Erikson and Peek (2011) provide a bibliography on Hurricane Katrina research that includes topics ranging from its effects on children and schools to healthcare and even media.

Unfortunately, the U.S. government's response to Hurricane Katrina was insufficient and was widely criticized by media outlets (cf. Moynihan (2009)). In fact, the criticism towards the government's, and, in particular, FEMA's response, was so strong, that Congress created the Select Bipartisan Committee to Investigate the Preparation for and Response to Hurricane Katrina. In its final 2006 report (see U.S. House of Representatives (2006)), the Committee goes as far as saying that "if 9/11 was a failure of imagination, then Katrina was a failure of initiative. It was a failure of leadership." Interestingly enough, that same report provides a list of failures including inadequate infrastructure and planning, inefficient and late execution of the National Response Plan, a general lack of coordination, and even that charitable organizations faced challenges due to the scale of the disaster, inadequate logistics, and disorganized processes. Indeed, the Congressional report points out what our game theoretical model for post-disaster humanitarian relief predicts: that a lack of coordination in humanitarian operations can lead to inefficient supply chain operations, which have a direct effect on the vulnerable population. For this reason, the rest of this section analyzes the nongovernmental sector's response to Hurricane Katrina through the model developed in Section 2.

Before describing the implementation of the model to Hurricane Katrina, we note that governmental analyses and qualitative accounts published by the media suggest that the actual behavior of decision-makers can be described with good precision based on the proposed framework, which includes both a Generalized Nash Equilibrium model and, in the absence of common constraints, a Nash Equilbrium model. For example, The New York Times reported that the Red Cross mismatched supplies with the victim's needs; thereby, leading to obsolete inventory (Strom (2006)). Similarly, there was extensive media coverage during the response phase, propelling donations to NGOs providing aid to record numbers, with estimates ranging from three to four billion dollars (U.S. House of Representatives (2006)).

The Generalized Nash Equilibrium formulation for the humanitarian response to Hurricane Katrina follows the same general functional form as in the examples provided in Section 4, though it requires a few additional assumptions. Given the limited amount of data on the donations received by NGOs for their response to Hurricane Katrina or the cash that they had available at the time, we assume that NGOs had an initial supply equal to the donations that they received. For example, the Red Cross and the Salvation Army received \$2.2 billion (American Red Cross (2010)) and \$382 million (The Salvation Army International (2015)), respectively. We group the rest of the donations as a third player, termed, 'Others', that plays the role of all other NGOs for which data is not readily available. While we previously stated that the donations are made available after the NGOs decide on their strategic variables q_i; $i = 1, \ldots, m$, the fact that donations are received in a short period of time allows for, the purpose of exposition, that nonprofits had the capacity to provide at least that amount in aid.

As far as demand is concerned, there are some limitations in the data that was made public through governmental agencies and other reports. The most obvious of these limitations is the lack of a temporal component to the estimates; that is to say, that the reported figures of \$150 billion and \$105 billion do not specify how much of that need corresponds to emergency relief in the immediate aftermath of Hurricane Katrina. Hence, in order to estimate the upper and lower bounds on the demand one must first estimate the proportion of total needs corresponding to humanitarian emergency relief. According to an analysis conducted by the Greater New Orleans Community Center (cf. Plyer (2013)), the U.S. government allocated roughly \$75 billion to emergency relief and \$45 billion to the areas affected by Hurricane Katrina. Under the assumption that the proportion the government allocated to emergency relief is representative of the total of emergency relief needs, we estimate that the total amount of emergency relief needed for Hurricane Katrina is approximately \$94 billion dollars. Given that the government allocated \$75 billion for emergency relief, nonprofit organizations are left with a gap of \$18.75 billion dollars in needed emergency relief.

The next consideration that needs to be made is concerning the allocation of these needs to specific geographic locations. One way to do this is to calculate the per-capita need and then use population densities to estimate the demand at each location. According to the National Academy of the Sciences, a total of 1.7 million people were affected by Hurricane Katrina, which computes to a demand of 11,029.41 in emergency relief per capita (Goldman and Coussens (2007)). However, due to the fact that people are evacuated from compromised areas, one cannot assume that the populations are the same in the immediate aftermath of the disaster. Instead, a better estimate of the demand can be made by calculating the amount of people who stay and multiplying it by the per-capita demand. Recall that the demand levels are disseminated by the coordinating authority, which implies that they can only use data on historical evacuation rates to estimate the demand at different geographies for this disaster. At this point, the analysis is limited by the availability of data. Nevertheless, the state of Louisiana, where most of the damage was sustained by Hurricane Katrina, conducted a study after Hurricane Ivan, roughly 1 year before Katrina. While this satisfies the logical requirements outlined above, complete data is only available for 10 Parishes in Louisiana and no data is available for the other affected states (Howell and Bonner (2005)). Despite the limitations due to the lack of granular data, using these data still provides a reasonable use-case for the model, considering that 63% of the total damages were in Louisiana.

Under the above stated conditions, one can construct upper and lower demand estimates. Since the evacuation rates were estimated by sampling a population and the report provides the number of people interviewed, one can construct a confidence interval to estimate the amount of people who evacuated the region. Thus, upper bound demand is calculated by multiplying the per-capita demand by the base population density, obtained through the Census Bureau, at a demand point minus the lower tail of a 99% confidence interval. The lower bound demand estimate is then calculated in a similar manner.

We assume that the $P_j(q)$ functions for $j = 1, \ldots, 10$, take on the same form as they did in the previous numerical examples; that is, $P_j(q) = k_j\sqrt{\sum_i^m q_{ij}}$. Such functions are concave and continuously differentiable. Although there is no clear way in which to estimate these coefficients due to the lack of granular data, certain assumptions can be made for these coefficients. First, we wish to select k_j such that the total donations made available are approximately $4 billion and such that the NGOs decide to provide flows q_{ij} that result in the donations observed in actuality. The parameters k_j; $j = 1, \ldots, n$, have to be chosen in a way that the demand points that were likely to receive more media attention have higher coefficients. These coefficients were changed until this behavior was observed, while ensuring that demand point with higher damages and populations had the largest values. A summary of these estimates is given in Table 1 with how the parameters were obtained described further below. The p_j term in Table 1 for $j = 1, \ldots, 10$, denotes the percentage of homes with major or severe damage, as given by FEMA estimates, for demand point j. Also, the network representation for the Hurricane Katrina case study, consisting of three NGOs and ten demand points, is depicted in Figure 5.

The Generalized Nash Equilibrium problem for humanitarian relief; therefore, has upper

Hurricane Katrina Demand Point Parameters					
Parish	Demand Node j	k_j	$\underline{d}_j$	$\bar{d}_j$	p_j (in %)
St. Charles	1	8	16.45	50.57	2.4
Terrebonne	2	16	752.26	883.82	6.7
Assumption	3	7	106.36	139.24	1.9
Jefferson	4	29	742.86	1,254.89	19.5
Lafourche	5	6	525.53	653.82	1.7
Orleans	6	42	1,303.99	1,906.80	55.9
Plaquemines	7	30	33.28	62.57	57.5
St. Barnard	8	42	133.61	212.43	78.4
St. James	9	9	127.53	166.39	1.2
St. John the Baptist	10	7	19.05	52.59	6.7

Table 1: Demand Point Data for the Generalized Nash Equilibrium Problem for Hurricane Katrina

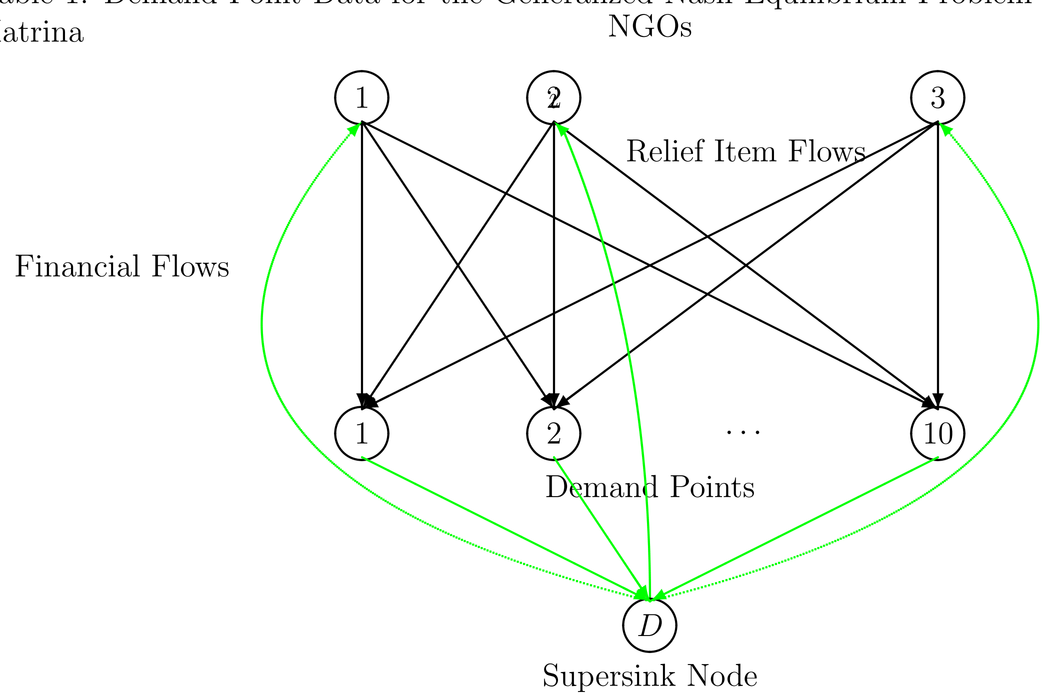

Figure 5: Hurricane Katrina Relief Network Structure

and lower bounds on the demand points for which the State of Louisiana could have reasonably had estimates at the time of Katrina, as in Table 1, in millions of aid-units. In addition, we assume that the three NGOs in the game, the Red Cross, the Salvation Army, and Others, have supply levels equal to the donations that they actually received for Hurricane Katrina in millions of dollars. In addition, we assume that all three players derive the same utility of $\gamma_{ij} = 950$ from providing aid to the different Parishes and that $\omega_1 = \omega_2 = \omega_3 = 1$. Since there is limited data, there is no concrete way in which to estimate differences in these parameters and it is best to assume that the NGOs value helping people equally. For this example, the coefficients for β_i are calculated as the percentage of total donations received, as summarized in Table 2.

Data Parameters for NGOs Providing Aid					
NGO Name	NGO Node i	π_i	γ_{ij}	β_i	s_i
Others	1	.82	950	.355	1,418
Red Cross	2	.83	950	.55	2,200
Salvation Army	3	.81	950	.095	382

Table 2: NGO Data for the Generalized Nash Equilibrium Problem for Hurricane Katrina

We then estimate the cost of providing aid to the Parishes as a function of the total damage in the area and the supply chain efficiency of each NGO. We assume that these costs follow the structures observed by Van Wassenhove (2006) and randomly generate a number based on his research with a mean of $\hat{p} = .8$ and standard deviation of $s = \sqrt{\frac{.8(.2)}{3}}$. We denote the corresponding coefficients by π_i. Thus, each NGO i; $i = 1, 2, 3$, incurs costs according the the following functional form:

$$c_{ij}(q_{ij}) = \left(\pi_i q_{ij} + \frac{1}{1 - p_j}\right)^2.$$

The models in this section were solved using SAS Studio to demonstrate that optimization software can also be used, given our reformulation. The tolerance for convergence of the nonlinear programming solver was set to 10^{-7}.

The computed flow pattern solution of the Generalized Nash Equilibrium problem for humanitarian relief for Hurricane Katrina is given in Table 3.

Generalized Nash Equilibrium Product Flows			
Demand Point	Others	Red Cross	Salvation Army
St. Charles	17.48	28.89	4.192
Terrebonne	267.023	411.67	73.57
Assumption	49.02	77.26	12.97
Jefferson	263.69	406.68	72.45
Lafourche	186.39	287.96	51.18
Orleans	463.33	713.56	127.1
Plaquemines	21.89	36.54	4.23
St. Barnard	72.31	115.39	16.22
St. James	58.67	92.06	15.66
St. John the Baptist	18.2	29.99	4.40

Table 3: Flows to Demand Points under Generalized Nash Equilibrium

The total utility obtained through the above flows for the Generalized Nash Equilibrium for Hurricane Katrina is $9,257,899$, with the Red Cross capturing 3,022,705, the Salvation Army 3,600,442.54, and Others 2,590,973. It is interesting to see that, despite having the lowest available supplies, the Salvation Army is able to capture the largest part of the total utility. This is due to the fact that the costs of providing aid grow at a nonlinear rate, so even if the Salvation Army was less efficient and used all of its available supplies, it will not be capable of providing the most expensive supplies. In addition, we have that the Red Cross, the Salvation Army, and Others receive 2,200.24, 1418.01, and 382.31 million in donations, respectively. Also, notice how the flows meet at least the lower bound, even if doing so is very expensive due to the damages to the infrastructure in the region. Furthermore, the above flow pattern behaves in a way that, after the minimum requirements are met, any additional supplies are allocated in the most efficient way. For example, only the minimum requirements are met in New Orleans Parish, while the upper bound is met for St. James Parish.

If we then remove the shared constraints from the optimization problem for Hurricane Katrina, we can compare the outcomes of the humanitarian relief efforts for Hurricane Katrina, as we have also done for the previous numerical examples. The results are reported in in Table 4.

Under the Nash Equilibrium, the NGOs also obtain a higher utility than under the Generalized Nash Equilibrium. Specifically, of the total utility $10,346,005.44$, 2,804,650 units are received by the Red Cross, 5,198,685 by the Salvation Army, and 3,218,505 are captured by all other NGOs. Under this product flow pattern, there are total donations of 3,760.73, of which 2,068.4 are donated to the Red Cross, 357.27 to the Salvation Army, and 1,355 to the other players. It is immediately clear that there is a large contrast between the flow patterns under the Generalized Nash and Nash Equilibria. For example, the Nash Equilibrium flow pattern results in about \$500 million less in donations. While this has strong implications about how collaboration between NGOs can be beneficial for their fundraising efforts, the differences in the general flow pattern highlights a much stronger point.

Under the Nash Equilibrium, NGOs successfully maximize their utility. Overall, the Nash Equilibrium solution leads to an increase of utility of roughly 21% when compared to the flow patterns under the Generalized Nash Equilibrium. But they do so at the expense of those in need. In the Nash Equilibrium, each NGO chooses to supply relief items such that costs can be minimized. On the surface, this might be a good thing, but recall that, given the nature of disasters, it is usually more expensive to provide aid to demand points with the greatest needs. With this in mind, one can expect oversupply to the demand points

Nash Equilibrium Product Flows			
Demand Point	Others	Red Cross	Salvation Army
St. Charles	142.51	220.66	38.97
Terrebonne	142.50	220.68	38.93
Assumption	142.51	220.66	38.98
Jefferson	142.38	220.61	38.74
Lafourche	142.50	220.65	38.98
Orleans	141.21	219.59	37.498
Plaquemines	141.032	219.28	37.37
St. Barnard	138.34	216.66	34.59
St. James	142.51	220.65	38.58
St. John the Baptist	145.51	220.66	38.98

Table 4: Flows to Demand Points under Nash Equilibrium

with lower demand levels, and undersupply to the most affected under a purely competitive scheme. This behavior can be seen explicitly in the results summarized in Table 4. For example, St. Charles Parish receives roughly 795% of its upper demand, while Orleans Parish only receives about 30.5% of its minimum requirements. That means that much of the 21% in 'increased' utility is in the form of waste. In contrast, the flows under the Generalized Nash Equilibrium guarantee that minimum requirements will be met and that there will be no waste; that is to say, *as long as there is a coordinating authority that can enforce the upper and lower bound constraints, the humanitarian relief flow patterns under this bounded competition will be significantly better than under untethered competition.*

In addition, changes to the values in the functional form result in changes in the product flows, but the general behavioral differences are robust to changes in the coefficients: β_i, γ_{ij}, k_j, $\forall i, j$, and the bounds on upper and lower demand estimates.

In this section, we implemented the Generalized Nash Equilibrium Model for humanitarian relief developed in Section 2 to a case study on Hurricane Katrina. Through the earlier examples as well as the case study, we were able to observe the behavioral and numerical differences between the Generalized and Nash Equilibrium models. In addition, these examples provided both quantitative and qualitative reasons to conclude that the Generalized Nash Equilibrium flow patterns, although still operating under competition, are socially preferable to those under a Nash Equilibrium.

6. Summary, Conclusions, and Suggestions for Future Research

In this paper, we presented a network model consisting of multiple NGOs who seek to supply multiple demand points with relief items post a disaster. Each NGO is faced with a utility function that has a supply chain component and a financial funds component associated with donations post the disaster. The NGOs compete for financial funds and visibility at the demand points and are subject to common, that is, shared, constraints in the form of upper and lower bounds for the demands at the demand points. Such constraints can reduce materiel convergence and even congestion associated with the demand points. The model is a Generalized Nash Equilibrium model. We demonstrate that the equilibrium problem can be transformed into an optimization problem because of its special structure, which means that we do not need to make use of quasivariational inequality theory. Existence of a solution is guaranteed and conditions for uniqueness given.

We provide an algorithm for the solution of the model, which yields closed form expressions for the relief item flows and the Lagrange multipliers associated with the supply constraints and the demand bound constraints at each iteration. The algorithm is then applied to compute solutions to numerical examples and the solutions compared to those obtained in a pure Nash Equilibrium, in which the demand bound constraints are removed. Through numerical examples and a case study on Hurricane Katrina, we show how the Generalized Nash Equilibrium, which includes demand lower and upper bounds at demand points, is capable of improving the results under a Nash Equilibrium, by reducing congestion and eliminating the possibility of undersupply and material convergence.

This is the first Generalized Nash Equilibrium model developed for humanitarian relief and, hence, this work adds to the growing literature on game theory in humanitarian operations. Another novelty of our framework is that both supply chain aspects and financial funding aspects are explicitly included.

When comparing the outcomes under the Generalized Nash and Nash Equilibria, it becomes clear that coordination is critical to achieving better outcomes in humanitarian relief operations. The Generalized Nash solution is not only capable of eliminating the possibility of having under or oversupply, it guarantees - through competition- the efficient allocation of resources once the minimum requirements are met. Therefore, it has benefits both for the population in need and the NGOs. In addition, we also demonstrated that, under certain circumstances, the Generalized Nash solution is capable of attracting more donations than an unrestricted, competitive solution.

This research has a host of implications for both coordinating authorities, that is, governments and organizing bodies such as the United Nations, and managers in the public sector. As far as coordinating authorities are concerned, this research provides a strong argument for their importance in having successful humanitarian relief efforts. In fact, our research suggests that, if authorities can impose the constraints on upper and lower demand levels, they can provide an effective mechanism for improving responses to disasters. In particular, it is imperative that we stress the importance of having reliable statistics on the population that can be used to create the estimates on the upper and lower bounds on the demand. If NGOs do not believe that these estimates are reliable, it is highly unlikely that a coordinating authority will be capable of imposing the associated constraints in practice. In addition, governments and other authorities should include collaboration with and between NGOs as part of their preparation before a disaster. It is easy to imagine that attempts to perform large-scale coordination efforts during the response phase are likely to go awry or to be ineffective. Coordinating authorities that are successful in doing so should be capable of improving the outcomes of their relief efforts under the assumption that NGOs have the goodwill to be part of this mechanism.

In addition, the Generalized Nash Equilibrium model has implications for managers of nongovernmental organizations. First of all, the proposed model provides managers with a strategic framework to analyze its interactions with other NGOs in the same sector, while also providing insights into its operations. Furthermore, the framework we propose answers two of the most important questions any NGO must answer: (1) How and where should we provide aid? and (2) How can we fund those operations? A model that is capable of answering these questions not only advances the economic theory of not-for-profits, but it also provides an actionable framework for decision-makers within NGOs. But, perhaps most importantly, this analysis suggests that, despite the competition between NGOs for fundraising, there are strong reasons for them to collaborate and, thereby, achieve better results for those they set out to help. In fact, our analysis suggests that cooperation may, in fact, increase the amount of donations received by all NGOs in the game.

While these findings are significant and novel, they are only the initial steps of research that could improve humanitarian responses to disasters. The model is formulated in a way that allows for many functional forms (as long as it results in separable and shared parameterized terms) and would benefit from exploring these possibilities. Another extension of this research would be to find a solution methodology for functions that cannot be solved as optimization problems. This would include, for example, cases in which the β_is are extended to β_{ij}s or may even depend on the product flows. Further developing the theory to account for multiple time periods and pre- and post-disaster components would also be significant improvements to the research presented herein. Finally, another interesting extension of this research would be to include the possibility of coalition formation, and to explore how this may have an effect on the relief item flows, as well as on the coordinating authority's ability to impose the shared constraints.

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