A Multitiered Supply Chain Network Equilibrium Model for Disaster Relief with Capacitated Freight Service Provision

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Abstract: In this paper, a multitiered supply chain network equilibrium model is constructed, consisting of multiple humanitarian organizations, who seek to purchase services from multiple competing freight (logistic) service providers, for transportation of disaster relief supplies to multiple points of demand for distribution to victims. The freight service providers are faced with capacities associated with the volume of shipments that they can transport. We capture the behavior of the humanitarian organizations who individually minimize the total cost associated with payments for the freight transportation and their transaction costs. We also identify the profit-maximizing behavior of the freight service providers. The governing supply chain network equilibrium conditions are formulated as a variational inequality problem and conditions for existence given. We propose an algorithm for the computation of the equilibrium disaster relief item flows and Lagrange multipliers associated with the freight capacity constraints and provide conditions for convergence. The algorithm is then applied to several numerical examples comprising a case study focusing on an international healthcare crisis. In the case study, we explore the impacts of the addition of a freight service provider as well as that of a humanitarian organization on the profits of freight service providers and on the costs incurred by the humanitarian organizations. The theoretical and numerical results in this paper advance game theory frameworks for humanitarian operations and disaster relief, an area in which there is only a limited literature.

Keywords: humanitarian operations, disaster relief, game theory, freight service provision, logistics, supply chain networks, variational inequalities

1. Introduction

Freight service provision is an essential component of disaster relief since only with the effective transportation of the critical needs supplies can the suffering of victims be reduced and lives saved. At the same time, transportation portals and possible routes may be disrupted and severely compromised following a disaster, creating additional challenges for transportation services associated with disaster relief.

Although large humanitarian organizations may have acquired their own freight services and, hence, means of transportation of the needed supplies, which can include, for example, water, food, medicines, shelter items, etc., many humanitarian organizations do not have the financial resources to maintain freight fleets. Hence, they need to purchase such services. Freight service providers, in turn, are profit-maximizers, unlike humanitarian organizations and other nongovernmental organizations (NGOs), which are nonprofits. In addition, they compete among one another to acquire business. Hence, their behavior is distinct from that of humanitarian organizations, who not only must responsibly utilize the financial resources donated to them but also are under pressure to deliver a timely response post disasters. Given the fundamental importance of freight service provision post disasters, costs associated with transportation are second only to personnel for humanitarian organizations (see Pedraza Martinez, Stapleton, and Van Wassenhove (2011)).

In this paper, we construct what we believe is the first general multitiered supply chain network equilibrium model for disaster relief. The model can handle as many humanitarian organizations as needed by the disaster application under investigation; similarly, the number of freight service providers as well as the number of demand points for distribution of the supplies is not fixed but, rather, is as mandated by the disaster. The cost-minimizing behavior of the individual humanitarian organizations is captured and that of the profit-maximizing freight service providers, who are capacitated. The humanitarian organizations have a fixed amount of supplies that they need delivered to the various points of demand. The governing supply chain network equilibrium conditions are formulated as a variational inequality problem and conditions for existence provided. The solution of the model, for which an algorithm, is proposed, yields the equilibrium disaster relief item shipments from the humanitarian organizations via the freight service providers, along with the Lagrange multipliers associated with the freight service providers' capacity constraints. The algorithm decomposes the problem into specially structured network subproblems in the disaster relief item flows, and in the Lagrange multipliers. For the latter, we provide closed form expressions for the iterates. Convergence results are also given. We also demonstrate how to recover the prices the freight service providers charge the humanitarian organizations. In order to demonstrate the efficacy and applicability of the game theory framework, we apply the algorithm to several numerical examples comprising a case study inspired by the international healthcare crisis of the Ebola outbreak in 2014 and 2015.

This paper builds on the work of Nagurney (2016), who constructed a game theory model for disaster relief with a single humanitarian organization and multiple competing freight service providers, also commonly referred to as logistics service providers. The new model in this paper significantly extends the one therein by capturing the behavior of multiple competing humanitarian organizations and by including capacities associated with the freight service providers. Moreover, a distinct algorithm is proposed with less restrictive conditions for convergence, which, nevertheless, exploits, at each iteration, the specially structured underlying network structure associated with the required amounts of disaster relief item shipments between the humanitarian organization nodes and the points of demand. The lineage of supply chain network equilibrium models, which, nevertheless, assumed profit-maximizing decision-makers at each tier of the supply chain network, originated with the paper of Nagurney, Dong, and Zhang (2002). A spectrum of supply chain network equilibrium models, static as well as dynamic, can be found in the book by Nagurney (2006)). Supply chain network equilibrium models with a freight sector have recently incorporated price and quality competition among manufacturers and freight service providers (see Nagurney et al. (2015)) and time-based supply chain network competition (see Nagurney et al. (2014)). Here, in contrast, the model developed in this paper includes the humanitarian sector and the quantities demanded are no longer elastic and price-sensitive but, rather, fixed, since, post-disaster, the critically needed product supplies must be delivered.

The paper by Nagurney, Alvarez Flores, and Soylu (2016), in turn, considered multiple competing humanitarian organizations engaged in disaster relief, who competed for financial funds and provided needed supplies to the victims and demand points, which were subject to upper and lower bounds. The model was a Generalized Nash Equilibrium model, and, because of the structure of its financial donation functions, was amenable to reformulation as an optimization problem. That model was, subsequently, extended to capture competition on the logistics side and to handle more general financial donation functions using the concept of variational equilibrium by Nagurney et al. (2017). However, in both of these models there was not an explicit tier of freight service providers. Muggy and Heier Stamm (2014) provide a thorough review of game theory in humanitarian operations to that date and emphasize that there are many untapped research opportunities for modeling in this area. See also the dissertation of Muggy (2015). In these references, however, there are no multitiered supply chain network equilibrium models that include the essential freight service provisioning tier of decision-makers. The relevance of game theory to disaster relief provides new avenues for research, since, principally, centralized decision-making has been modeled using optimization techniques in a variety of settings, especially in the context of transportation, as in evacuation networks (cf. Sheffi, Mahmassani, and Powell (1982), Miller-Hooks and Sorrel (2008), Vogiatzis and Pardalos (2016), and the references therein), relief routing (cf. Huang, Smilowitz, and Balcik (2012)), and last mile distribution (see, e.g., Balcik, Beamon, and Smilowitz (2008) and the references therein). Nagurney, Masoumi, and Yu (2015), in turn, developed a supply chain network optimization model for disaster relief under demand uncertainty, whereas Nagurney and Nagurney (2016) also considered cost uncertainty. Both of these models were formulated and solved as variational inequality problems. For additional background on supply chain management and disaster relief, see Van Wassenhove (2006). Our focus in this paper, in contrast, is on noncooperative game theory (cf. Nash (1950a, 1951)). This framework can also serve in the future as the basis for further research on cooperative game theory as in the case of Nash bargaining solutions (cf. Nash (1950b, 1953)). For an application of noncooperative game theory and cooperative game theory to cyber security investments, see Nagurney and Shukla (2017).

This paper is organized as follows. In Section 2, we present the multitiered supply chain network equilibrium model for disaster relief with capacitated, competing freight service providers. The behavior of both the humanitarian organizations and that of the freight service providers is detailed and the governing supply chain network equilibrium conditions defined. The variational inequality formulation is then derived and conditions for existence of an equilibrium solution given. Section 3 presents the algorithm and identifies the special network structure of the induced subproblems, along with conditions for convergence. In Section 4 the case study is described. The case study is inspired by the recent Ebola healthcare crisis and focuses on the delivery of personal protective equipment (PPEs) needed by the medical professionals who cared for those infected by this highly contagious disease in western Africa. The case study builds on a dataset formulated by Nagurney (2016), to which freight service provision capacities are added, as well as another freight service provider and, subsequently, an additional humanitarian organization. Complete results are reported in terms of the equilibrium solutions as well as the total costs incurred by the humanitarian organizations, under the three distinct scenarios, and the profits of the freight service providers. Prices that the humanitarian organizations are charged by the freight service providers are also reported. We discuss the impacts of additional freight service providers and humanitarian organizations as to who wins and who loses. In Section 5, we summarize our results and present our conclusions.

2. The Multitiered Supply Chain Network Equilibrium Model for Disaster Relief

We consider m humanitarian organizations involved in delivering relief supplies post a disaster, with a typical organization denoted by i. The relief items can be food, water, medicines, shelter supplies, as well as supplies needed by the emergency and healthcare professionals responding to the disaster, etc. There are n competing freight service providers that the organizations can avail themselves of for transporting the relief items, with a typical freight service provider denoted by j. The humanitarian organizations are interested in having the relief items delivered to o points of demand for distribution to the victims, with a typical demand point denoted by k. The multitiered structure of the disaster relief supply chain network is depicted in Figure 1.

The humanitarian organizations compete among themselves for the freight service provision and the freight service providers compete for their business. The humanitarian organizations, which are non-profits, seek to individually minimize the total costs associated with having their disaster relief supplies delivered by the freight service providers to the victims of the disaster at the demand points. The freight service providers, in turn, seek to maximize their profits and compete among one another for the transport of the relief items for the humanitarian organizations.

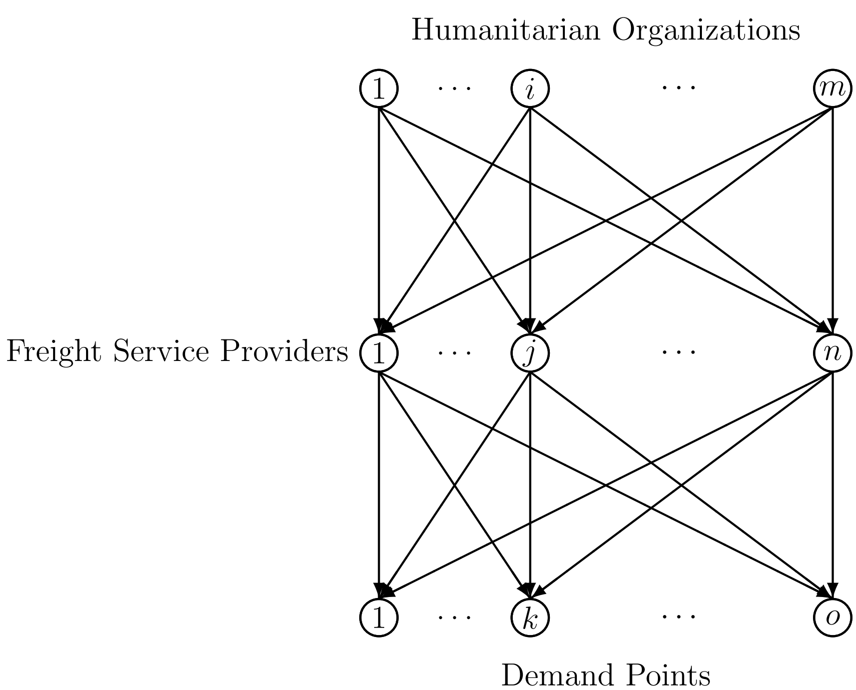

Figure 1: The Multitiered Disaster Relief Humanitarian Organization and Freight Service Provision Supply Chain Network

We first describe the behavior of the humanitarian organizations and then that of the freight service providers. We, subsequently, state the network equilibrium conditions for the disaster relief supply chain network and derive the variational inequality formulation. Qualitative properties of the equilibrium pattern are also given. We assume that all vectors are column vectors.

2.1 Behavior of the Humanitarian Organizations

Each humanitarian organization i; $i = 1, \ldots, m$, wishes to have an amount s^i_k of the relief item, which it has in stock and has prepositioned, transported to demand points: $k = 1, \ldots, o$. Let Q^i_{jk} denote the amount of the relief item that i contracts with freight service provider j to have delivered to demand point k. We group the relief item shipments of each humanitarian organization i into the vector $Q^i \in R^{no}_+$.

The per unit price that freight service provider j charges i for transport to k is denoted by ρ^{i*}_{jk}. These prices are revealed once the supply chain network equilibrium model for disaster relief is solved. We demonstrate the procedure of how to recover these prices following the derivation of the variational inequality of the governing supply chain network equilibrium conditions.

Each humanitarian organization i is faced with a total cost $\hat{c}^i_j$ associated with transacting with freight service provider j. This cost includes the cost associated with handling the product until pickup by provider j and interacting with provider j. The total cost $\hat{c}^i_j$; $i = 1, \ldots, m$; $j = 1, \ldots, n$, hence, includes all the costs associated with i contracting with a respective freight service provider j. Observe that the cost associated with a humanitarian organization in transacting with a freight service provider, can, in general, depend not only on its own shipments associated with the freight service provider but also on those of other humanitarian organizations and the same or other freight service providers. The freight service providers guarantee delivery of the disaster relief items in a timely fashion, given what is known about the disaster landscape, and charge accordingly.

The optimization problem faced by humanitarian organization i; $i = 1, \ldots, m$, with the objective function representing total cost to be minimized, is:

$$\text{Minimize} \quad \sum_{j=1}^{n} \sum_{k=1}^{o} \rho^{i*}_{jk} Q^i_{jk} + \sum_{j=1}^{n} \hat{c}^i_j(Q) \tag{1}$$

subject to:

$$\sum_{j=1}^{n} Q^i_{jk} = s^i_k, \quad k = 1, \ldots, o, \tag{2}$$

$$Q^i_{jk} \geq 0, \quad j = 1, \ldots, n; k = 1, \ldots, o. \tag{3}$$

The first term preceding the plus sign in the objective function (1) corresponds to the amount that i must pay to the freight service providers whereas the term following the plus sign is the total costs associated with transacting with the freight service providers. Equation (2) guarantees that the relief supplies are delivered to the points of demand. Equation (3) is the nonnegativity assumption for the relief item flows. We define the feasible set K^i; $i = 1, \dots, m$, where $K^i \equiv \{Q^i | Q^i \geq 0 \,\text{and satisfies}\,(2)\}$. We then define the feasible set $K \equiv \prod_{i=1}^m K_i$ for all the humanitarian organizations.

Remark

In the case that humanitarian organization i; $i = 1, \dots, m$, has to purchase some or all of the disaster relief supplies then the total cost functions $\hat{c}^i_j$; $j = 1, \dots, n$, would include the purchasing cost, in addition to the freight service provision transaction costs.

We assume that the total cost functions $\hat{c}^i_j$; $i = 1, \dots, m$; $j = 1, \dots, n$, are continuously differentiable and convex. Under these assumptions, and the fact that K is convex, we know that a solution to the above optimization problems for the m humanitarian organizations, who compete for freight service provision, simultaneously, coincides with a solution to the variational inequality problem: determine $Q^* \in K$, such that

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{o}\left[\sum_{l=1}^{n}\frac{\partial \hat{c}^i_l(Q^*)}{\partial Q^i_{jk}} + \rho^{i*}_{jk}\right] \times \left[Q^i_{jk} - Q^{i*}_{jk}\right] \geq 0, \quad \forall Q \in K. \tag{4}$$

This result follows from the connection between Nash equilibria (cf. Nash (1950a, 1951)) and variational inequalities (cf. Gabay and Moulin (1980) and Nagurney (1999)).

2.2 Behavior of the Freight Service Providers

Since the freight service providers are profit-maximizers, they must cover their costs. The cost associated with freight service provider j delivering the relief items from i to demand point k is denoted by c^j_{ik}, where, here we assume, for the sake of generality, and in order to effectively capture competition, that

$$c^j_{ik} = c^j_{ik}(Q), \quad j = 1, \dots, m, \tag{5}$$

with the freight service provider cost functions assumed to be continuously differentiable and convex. Note that the cost functions in (5) depend, in general, not only on the freight service provider's shipment quantities but also on those of the other freight service providers, since there may be congestion, competition for labor, etc.

In our model each humanitarian organization is providing a similarly-sized relief item. Also, each freight service provider can consolidate the shipments from the various humanitarian organizations, if need be, and then transport to points of demand, as inferred by the topology of the network in Figure 1. Each freight service provider j; $j = 1, \ldots, n$, has an associated capacity, denoted by u_j. Hence, the following constraint must hold for each provider j:

$$\sum_{i=1}^{m} \sum_{k=1}^{o} Q^i_{jk} \le u_j. \tag{6}$$

We make the assumption that the total shipment capacity availability is sufficient to meet the total demand, that is,

$$\sum_{i=1}^{m} \sum_{k=1}^{o} s^i_k \le \sum_{j=1}^{n} u_j.$$

The optimization problem faced by freight service provider j; $j = 1, \ldots, n$, with the objective function corresponding to the profits to be maximized, is:

$$\text{Maximize} \quad \sum_{i=1}^{m} \sum_{k=1}^{n} \rho^{i*}_{jk} Q^i_{jk} - \sum_{i=1}^{m} \sum_{k=1}^{n} c^j_{ik}(Q) \tag{7}$$

subject to (6) and:

$$Q^i_{jk} \ge 0, \quad k = 1, \ldots, n. \tag{8}$$

As in Nagurney (2016), but in a simpler, single humanitarian organization competitive freight service provider supply chain, and without capacities, we assume that the freight service providers j; $j = 1, \ldots, n$, compete noncooperatively for the disaster relief items, each one seeking to maximize its profits. We associate a nonnegative Lagrange multiplier λ_j with capacity constraint (6) for each freight service provider j; $j = 1, \ldots, n$, and we group the Lagrange multipliers for all freight service providers into the vector $\lambda \in R^n_+$.

The optimality conditions of all freight service providers holding simultaneously, which correspond to a Nash equilibrium, must satisfy the variational inequality problem (cf. Gabay and Moulin (1980), Nagurney (1999, 2006)): determine $Q^* \in R^{mn}_+$ and $\lambda^* \in R^n_+$, such that:

$$\sum_{j=1}^{n} \left[\sum_{i=1}^{m} \sum_{k=1}^{o} \left[\sum_{h=1}^{m} \sum_{l=1}^{n} \frac{\partial c^j_{hl}(Q^*)}{\partial Q^i_{jk}} - \rho^{i*}_{jk} + \lambda^*_j \right] \right] \times \left[Q^i_{jk} - Q^{i*}_{jk} \right]$$

$$+ \sum_{j=1}^{n} \left[u_j - \sum_{i=1}^{m} \sum_{k=1}^{o} Q^{i*}_{jk} \right] \times \left[\lambda_j - \lambda^*_j \right] \ge 0, \quad \forall Q \in R^{mn}_+, \forall \lambda \in R^n_+. \tag{9}$$

The network equilibrium conditions for the multitiered disaster relief supply chain network model are given below.

Definition 1: Supply Chain Network Equilibrium for Disaster Relief

A supply chain network equilibrium for disaster relief is said to be established if the disaster relief flows between the two tiers of decision-makers coincide and the flows, prices, and Lagrange multipliers satisfy the sum of variational inequalities (4) and (9).

According to Definition 1, the humanitarian organization and the freight service providers must agree on the amounts of the relief items that they deliver to the demand points. This agreement is accomplished through the prices ρ_{jk}^{i*}; $i = 1, \ldots, m$; $j = 1, \ldots, n$; $k = 1, \ldots, o$. We first present the variational inequality formulation of the supply chain network equilibrium conditions and then discuss how to recover the equilibrium prices.

Theorem 1: Variational Inequality Formulation of Supply Chain Network Equilibrium for Disaster Relief

A disaster relief item shipment pattern $Q^* \in K$ and Lagrange multiplier vector $\lambda^* \in R^n_+$ is a supply chain network equilibrium for disaster relief with capacitated freight service provision if and only if it satisfies the variational inequality problem:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{o}\left[\sum_{l=1}^{n}\frac{\partial \hat{c}_l^i(Q^*)}{\partial Q_{jk}^i}+\sum_{h=1}^{m}\sum_{l=1}^{n}\frac{\partial c_{hl}^j(Q^*)}{\partial Q_{jk}^i}+\lambda_j^*\right]\times\left[Q_{jk}^i-Q_{jk}^{i*}\right]$$
$$+\sum_{j=1}^{n}\left[u_j-\sum_{i=1}^{m}\sum_{k=1}^{o}Q_{jk}^{i*}\right]\times\left[\lambda_j-\lambda_j^*\right]\geq 0, \quad \forall Q\in K, \forall \lambda \in R^n_+. \tag{10}$$

Proof: We first establish necessity, that is, if $Q^* \in K$, $\lambda^* \in R^n_+$, is a supply chain network equilibrium according to Definition 1 then it also satisfies variational inequality (10). Indeed, summation of (4) and (9) yields variational inequality (10) with the shipment flows coinciding.

We now establish sufficiency. We rewrite variational inequality (10) as:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{o}\left[\sum_{l=1}^{n}\frac{\partial \hat{c}_i^j(Q^*)}{\partial Q_{jk}^i}+\sum_{h=1}^{m}\sum_{l=1}^{n}\frac{\partial c_{hl}^j(Q^*)}{\partial Q_{jk}^i}-\rho_{jk}^{i*}+\rho_{jk}^{i*}+\lambda_j^*\right]\times\left[Q_{jk}^i-Q_{jk}^{i*}\right]$$
$$\sum_{j=1}^{n}\left[u_j-\sum_{i=1}^{m}\sum_{k=1}^{o}Q_{jk}^{i*}\right]\times\left[\lambda_j-\lambda_j^*\right]\geq 0, \quad \forall Q\in K, \lambda \in R^n_+. \tag{11}$$

But (11) may be expressed as:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{o}\left[\sum_{l=1}^{n}\frac{\partial \hat{c}_l^i(Q^*)}{\partial Q_{jk}^i}+\rho_{jk}^{i*}\right]\times\left[Q_{jk}^i-Q_{jk}^{i*}\right]$$

$$+\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{o}\left[\sum_{h=1}^{m}\sum_{l=1}^{n}\frac{\partial c_{hl}^{j}(Q^*)}{\partial Q_{jk}^{i}}-\rho_{jk}^{i*}+\lambda_j^*\right]\times\left[Q_{jk}^{i}-Q_{jk}^{i*}\right]$$

$$+\sum_{j=1}^{n}\left[u_j-\sum_{i=1}^{m}\sum_{k=1}^{o}Q_{jk}^{i*}\right]\times\left[\lambda_j-\lambda_j^*\right]\geq 0,\quad \forall Q\in K,\forall\lambda\in R_+^n. \tag{12}$$

(12) corresponds to Definition 1 holding for the prices and shipment pattern $Q^* \in K$ and the vector of Lagrange multipliers $\lambda^* \in R_+^n$. $\square$

Note that in order to recover the equilibrium prices ρ_{jk}^{i*}, $\forall i, j, k$, one sets, according to (9): $\rho_{jk}^{i*} = \sum_{h=1}^{m}\sum_{l=1}^{n}\frac{\partial c_{hl}^{j}(Q^*)}{\partial Q_{jk}^{i}} + \lambda_j^*$, $\forall i, j, k$ with $Q_{jk}^{i*} > 0$. By setting the freight delivery prices thus, variational inequality (9) holds, so each freight service provider has maximized his profits. Furthermore, we know that the variational inequality (4) governing the humanitarian organizations' noncooperative behavior also holds under these prices.

We now put variational inequality (10) into standard form (cf. Nagurney (1999)): determine $X^* \in \mathcal{K}$, such that

$$\langle F(X^*), X - X^*\rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{13}$$

where $F(X)$ is an N-dimensional vector which is a continuous function from $\mathcal{K}$ to R^N, X is an N-dimensional vector, $\mathcal{K}$ is closed and convex, and $\langle\cdot,\cdot\rangle$ denotes the inner product in N-dimensional Euclidean space. We define $\mathcal{K} \equiv K \times R_+^n$, $X \equiv (Q, \lambda)$. Also, we define $F(X) \equiv (F^1(X), F^2(X))$ where $F^1(X)$ consists of components F_{jk}^i with $F_{jk}^i(X) \equiv \left[\sum_{l=1}^{n}\frac{\partial \hat{c}_l^j(Q)}{\partial Q_{jk}^i} + \sum_{h=1}^{m}\sum_{l=1}^{n}\frac{\partial c_{hl}^j(Q)}{\partial Q_{jk}^i} + \lambda_j\right]$; $i = 1,\ldots,m$; $j = 1,\ldots,n$; $k = 1,\ldots,o$. Also, $F^2(X)$ consists of components: $F_j^2(X) \equiv \left[u_j - \sum_{i=1}^{m}\sum_{k=1}^{o}Q_{jk}^i\right]$; $j = 1,\ldots,n$. Here $N = mno + n$. Then variational inequality (10) takes on the standard form (13).

We emphasize that, in this paper, we sometimes will express $\langle x, y\rangle$ as $x^T \cdot y$, where the superscript T denoted transpose.

2.3 Qualitative Properties of the Equilibrium Pattern for Disaster Relief

We now turn to the examination of qualitative properties of the equilibrium pattern, that is, the solution to variational inequality (10), equivalently, (13).

Since the feasible set $\mathcal{K}$ for our model is unbounded, due to the presence of the Lagrange multipliers, we impose a coercivity condition.

Theorem 2: Existence of an Equilibrium Pattern

If the function $F(X)$ in (13) is coercive, that is,

$$\lim_{\substack{X \in \mathcal{K} \\ \|X\| \to \infty}} \frac{\langle F(X), X \rangle}{\|X\|} = \infty, \tag{14}$$

then variational inequality (13) has a solution.

Proof: Follows from the classical theory of variational inequalities (Kinderlehrer and Stampacchia (1980) and Nagurney (1999)).

3. The Computational Procedure

Before we proceed to our case study, which is on Ebola in western Africa, we discuss the computational procedure that we will utilize to solve the numerical examples. The algorithm that we will apply in the next section to compute the solution to variational inequality (10), using the standard form (13), is the modified projection method of Korpelevich (1977).

The requirements for convergence are that $F(X)$ is monotone and Lipschitz continuous, since we know that a solution to our model exists. Below we provide their definitions, for completeness.

Definition 2: Monotonicity

The function $F(X)$ as in (13) is said to be monotone on $\mathcal{K}$ if the following property holds:

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle \geq 0, \quad \forall X^1, X^2 \in \mathcal{K}. \tag{15}$$

Definition 2: Lipschitz Continuity

The function $F(X)$ in (13) is said to be Lipschitz continuous on $\mathcal{K}$ if the following property holds:

$$\|F(X^1) - F(X^2)\| \leq L\|X^1 - X^2\|, \quad \forall X^1, X^2 \in \mathcal{K}. \tag{16}$$

Specifically, the statement of the modified projection method is as follows.

The Modified Projection Method

Step 0: Initialization

Initialize with an $X^0 \in \mathcal{K}$. Set $\tau = 1$ and select α, such that $0 < \alpha < \frac{1}{L}$, where L is the Lipschitz constant for the function $F(X)$ in the variational inequality problem (13).

Step 1: Construction and Computation

Compute $\bar{X}^{\tau-1}$ by solving the variational inequality subproblem:

$$\left[\bar{X}^{\tau-1} + (\alpha F(X^{\tau-1}) - X^{\tau-1})\right]^T \cdot \left[X - \bar{X}^{\tau-1}\right] \geq 0, \quad \forall X \in \mathcal{K}. \tag{17}$$

Step 2: Adaptation

Compute X^τ by solving the variational inequality subproblem:

$$\left[X^\tau + (\alpha F(\bar{X}^{\tau-1}) - X^{\tau-1})\right]^T \cdot [X - X^\tau] \geq 0, \quad \forall X \in \mathcal{K}. \tag{18}$$

Step 3: Convergence Verification

If $|X^\tau - X^{\tau-1}| \leq \epsilon$, for $\epsilon > 0$, a prespecified tolerance, then, stop; else, set $\tau = \tau + 1$ and go to Step 1.

Note that the iterate $\bar{X}^{\tau-1}$ in (17) is actually the solution to the following quadratic programming problem:

$$\text{Minimize} \quad \frac{1}{2} X^T \cdot X + (\alpha F(X^{\tau-1}) - X^{\tau-1})^T \cdot X, \tag{19}$$

subject to: $X \in \mathcal{K}$.

It is straightforward, given the above, to also construct the quadratic programming formulation that will yield the solution $\bar{X}^{\tau-1}$ to variational inequality subproblem (18).

The modified projection method has nice features for our model. In particular, the relief item flows can be computed using an exact equilibration algorithm highlighted in Nagurney (2016), wherein, however, only one humanitarian organization was modeled and there were no capacities associated with the freight service providers. Indeed, the structure of the induced network subproblems for the relief item flows, in both Steps 1 and 2 of the modified projection method, is as depicted in Figure 2. These are equivalent to fixed demand transportation network equilibrium problems of special structure (cf. Nagurney

(1999)) in that the paths connecting each origin/destination pair of nodes corresponding to the humanitarian organization and demand point pairs have no links in common with any other path.

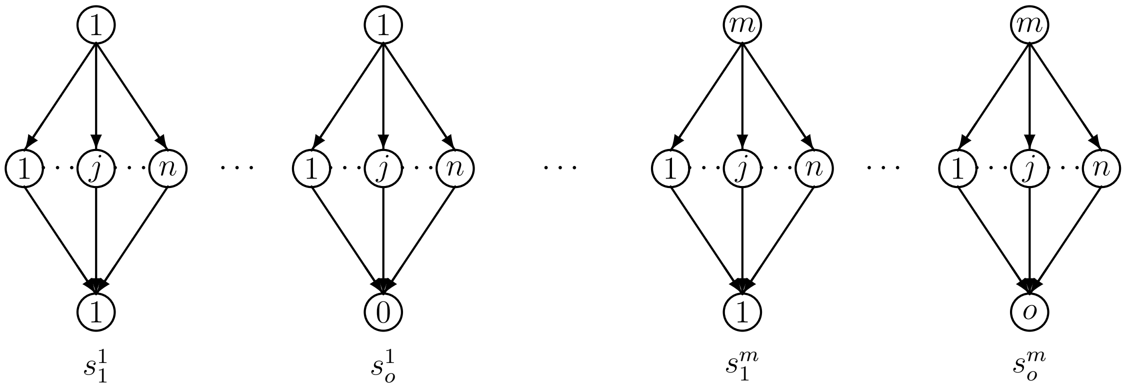

Figure 2: The Special Network Structure of the Relief Item Subproblems at Each Iteration of the Modified Projection Method

The Lagrange multipliers, at each iteration, can be solved exactly and in closed form, as detailed below for subproblem (17). An analogous expression can be obtained also for (18).

Explicit Formulae for the Modified Projection Method for the Lagrange Multipliers

The elegance of this algorithm for our variational inequality (10) for the computation of solutions to our model is apparent also from the following explicit formulae, which provide exact solutions for the Lagrange multipliers in subproblem (17). Indeed, we have the following closed form expression for the Lagrange multipliers for $j = 1, \ldots, n$, at iteration $\tau + 1$:

$$\bar{\lambda}_j^{\tau+1} = \max\{0, \lambda_j^{\tau-1} + \alpha(\sum_{i=1}^{m}\sum_{k=1}^{o} Q_{jk}^{i\tau-1} - u_j)\}, \quad j = 1, \ldots, n. \tag{20}$$

For results on the linear convergence rate of the modified projection method, see Tseng (1995), where references on variants of this algorithm can also be found.

We apply the above modified projection method, with the embedded equilibration algorithm in the next section.

4. An Ebola Case Study

For our case study, we revisit the Ebola crisis which impacted western Africa in 2014 and 2015. It captured the world's attention because of the suffering of those with the disease and the fear of this highly contagious disease (cf. Ap (2015)). This was the worst outbreak of Ebola since it was first identified in 1976. 21 months after the first reported case in March 2014, 11,315 people were reported as having died from Ebola, out of 28,637 cases, in the countries of: Liberia, Sierra Leone, and Guinea, as well as in Nigeria, Mali, and even the US (see BBC.com (2016)). There were eight cases in Nigeria, six in Mali, and one in the US.

In August 2014, according to Agence France-Presse (2014), the World Health Organization declared the Ebola epidemic ravaging parts of west Africa an international health emergency. The World Health Organization (2015) reported that over 800 healthcare workers contracted Ebola during this crisis. There were numerous logistical challenges, as well, including that many healthcare facilities had shortages of needed supplies in addition to their workers contracting the disease (see O'Byrne (2014)).

Wilson (2015) provided a medical professional perspective on this crisis from the frontlines. Wilson (2016), in turn, emphasized the importance of logistics and logisticians in battling this disease. Essential items needed by the healthcare workers caring for those stricken with Ebola included personal protective equipment (PPEs), which is the relief item in our case study (see also Fischer, Hynes, and Perl (2014)).

This disease even affected commercial shipping because of the fear of contagion of freight crews (cf. Saul (2014)) and, hence, freight provision was under added stress as well as added risk.

Scenario 1: Single Humanitarian Organization, Two Freight Service Providers (Without Capacities and With Capacities), and Three Demand Points

In scenario 1, we consider the supply chain network in Figure 3. There is a single humanitarian organization considering two freight service providers and requiring shipment of the PPEs to each of the three major Ebola impacted countries, that is, to Liberia, Sierra Leone, and Guinea. These countries correspond to demand points 1, 2, and 3, respectively. We first recall the results for the analogous example in Nagurney (2016), which is uncapacitated, but serves as the baseline for our case study, and then we investigate the impacts of the imposition of capacities on the freight service providers.

We utilize the data constructed by Nagurney (2016), but here we update our notation to

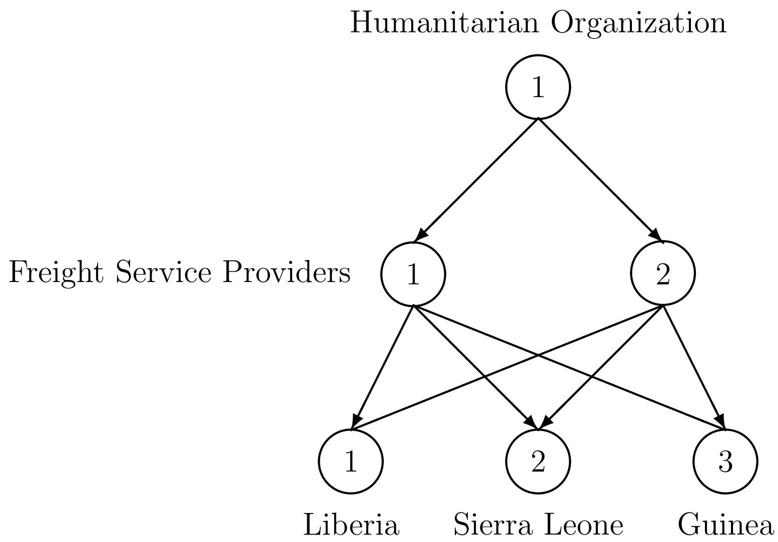

Figure 3: Supply Chain Network Topology for the Ebola Case Study Scenario 1

conform to that in this paper. Therein, The World Bank (2016) data was used to identify the cost of transport of a container of 20 feet, which can hold 1360 cubic feet of supplies, via ship from the US to these countries. The cost was then multiplied by 14, as per the United States Department of Commerce (2016), to obtain an estimated cost for air freight since speed of delivery was essential, given all the existing challenges.

The demands are 10,000 PPE items to each of the three destinations; hence, $s_1^1 = s_2^1 = s_3^1 = 10,000$.

The data are as follows.

The humanitarian organization is faced with the following total costs associated with transacting with the two freight service providers, respectively:

$$\hat{c}_1^1 = 4.50 \times (Q_{11}^1 + Q_{12}^1 + Q_{13}^1), \quad \hat{c}_2^1 = 4.25 \times (Q_{21}^1 + Q_{22}^1 + Q_{23}^1).$$

The humanitarian organization has to purchase the PPE items, so that the $\hat{c}_j^1$; $j = 1, 2$, cost functions include the purchase cost. The total cost associated with freight service provider 1, $\hat{c}_1^1$, is higher than that for freight service provider 2, $\hat{c}_2^1$, since it does not have as much experience with the former provider and the transfer cost is higher per unit.

The freight service provider total costs are as follows:

For freight service provider 1:

$$c^1_{11} = .0001{Q^1_{11}}^2 + 18.48Q^1_{11}, \quad c^1_{12} = .001{Q^1_{12}}^2 + 16.59Q^1_{12}, \quad c^1_{13} = .001{Q^1_{13}}^2 + 12.81Q^1_{13};$$

For freight service provider 2:

$$c^2_{11} = .001{Q^1_{21}}^2 + 18.48Q^1_{21}, \quad c^2_{12} = .0001{Q^1_{22}}^2 + 16.59Q^1_{22}. \quad c^2_{13} = .01{Q^1_{23}}^2 + 12.81Q^1_{23}.$$

As noted in Nagurney (2016), the nonlinear terms in the cost functions faced by the freight service provider capture the risk associated with transporting the supplies to the points of demand.

The computed equilibrium solution via the projection method, as reported in Nagurney (2016), but adapted here to our new notation, which can handle multiple humanitarian organizations, is:

$$Q^{1*}_{11} = 8,976.31, \quad Q^{1*}_{12} = 796.43, \quad Q^{1*}_{13} = 9,079.99,$$

$$Q^{1*}_{21} = 1,023.69, \quad Q^{1*}_{22} = 9,203.57, \quad Q^{1*}_{23} = 920.01.$$

The prices charged by the freight service providers are:

$$\rho^{1*}_{11} = 20.28, \quad \rho^{1*}_{12} = 18.18, \quad \rho^{1*}_{13} = 30.97,$$

$$\rho^{1*}_{21} = 20.53, \quad \rho^{1*}_{22} = 18.43, \quad \rho^{1*}_{23} = 31.23.$$

The value of the objective function of the humanitarian organization (cf. (1)) is: 829,254.38. The humanitarian organization pays the freight service providers an amount: 697,041.25, which, as noted in Nagurney (2016), corresponds to 84% for transport. This is reasonable since, as also noted in the Introduction, approximately 80% of humanitarian organizations' budgets are towards transportation in disasters. The value of freight service provider 1's objective function (cf. (6)), which coincides with its profits, is: 91,137.94 and that of freight service provider 2 is: 17,982.72. From the results, we see that freight service provider 1 delivers the bulk (the majority) of the PPE supplies to Liberia and Guinea, whereas freight service provider 2 delivers the majority of the supplies to Sierra Leone. Freight service provider 1 carries a total of 18,852.73 of the PPEs whereas freight service provider 2 carries an amount: 11,147.27.

We now assume that upper bounds are imposed on freight service provision with

$$u_1 = 10,000, \quad u_2 = 20,000.$$

In particular, freight service provider 1 has suffered a major disruption in terms of its freight provision in that certain crew members are refusing to deliver the supplies to the Ebola-stricken countries.

Note that the total supply of the PPEs to be delivered is still 30,000 and that is the combined capacity of the two freight service providers. In the original, uncapacitated example, freight service provider 1 delivered almost 19,000 of the PPE items with the remainder being delivered by freight service provider 2.

The modified projection method, as described above, was implemented in FORTRAN and a Linux system at the University of Massachusetts Amherst used for this and the subsequent numerical examples. The algorithm was initialized with s_k^1; $k = 1, 2, 3$, equally divided between the two freight service providers, for each demand point k, to construct the initial disaster relief item shipments. Also, the two Lagrange multipliers associated with the freight service provider capacity constraints were initialized to zero. The convergence tolerance was 10^{-5}, that is, the absolute value of two successive iterates of each of the shipments and each of the Lagrange multipliers differed by no more than this value. We set $\alpha = .3$ in the modified projection method for this scenario.

The modified projection method yielded the following equilibrium shipment and Lagrange multiplier vector solution:

$$Q_{11}^{1*} = 1652.60, \quad Q_{12}^{1*} = 0.00, \quad Q_{13}^{1*} = 8347.40,$$

$$Q_{21}^{1*} = 8347.40, \quad Q_{22}^{1*} = 10000.00, \quad Q_{23}^{1*} = 1652.60,$$

$$\lambda_1^* = 1616.76, \quad \lambda_2^* = 1600.64.$$

The prices charged by the freight service providers are now:

$$\rho_{11}^{1*} = 1635.57, \quad \rho_{12}^{1*} = 1633.35, \quad \rho_{13}^{1*} = 1646.26,$$

$$\rho_{21}^{1*} = 1635.82, \quad \rho_{22}^{1*} = 1619.23, \quad \rho_{23}^{1*} = 1646.50.$$

The humanitarian organization now pays an amount: 49,013,128.00 to the freight service providers. It encumbers a total cost of 49,143,128, which includes its transaction costs. The profit of freight service provider 1 now is: 16,237,542.00 and that of freight service provider 2: 32,119,844.00.

Note that both freight service providers are operating at their respective capacity with freight service provider 1 transporting a total of 10,000 PPEs and freight service provider

2 transporting a total of 20,000 PPEs. Hence, their associated Lagrange multipliers are positive.

Interestingly, the amounts of the PPEs shipped to Liberia have essentially flipped between the two freight service providers as compared to the respective shipment values in the uncapacitated version. Also, interestingly, freight service provider 2 now satisfies the entire demand for PPEs in Sierra Leone, with freight service provider 1 not even servicing this affected country. The prices charged now escalate tremendously because the freight service providers are both at their physical capacities.

Scenario 2: Single Humanitarian Organization, Three Freight Service Providers With Capacities, and Three Demand Points

In scenario 2, the data are as in the capacitated example in scenario 1 except that we add one more freight service provider. Hence, the supply chain network topology is now as in Figure 4. We investigate the impact of enhanced competition among the freight service providers on the humanitarian organization as well as on the original freight service providers.

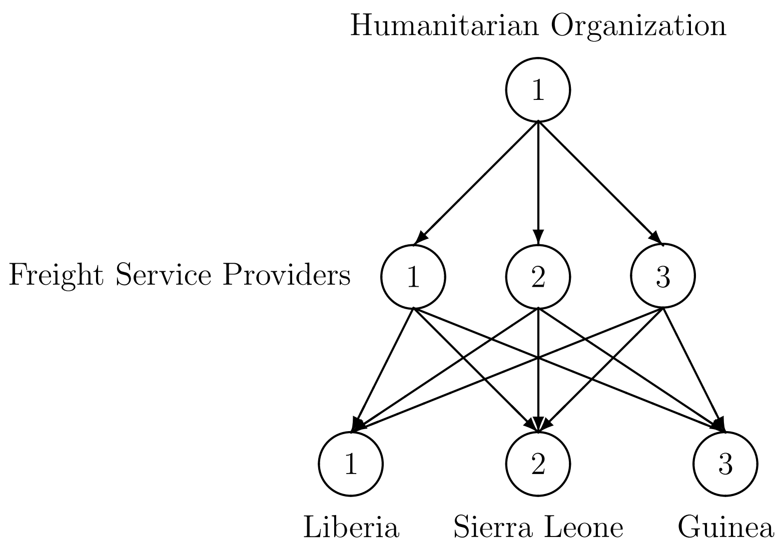

Figure 4: Supply Chain Network Topology for the Ebola Case Study Scenario 2

The added data are as follows.

The cost associated with the humanitarian organization transacting with freight service provider 3 is:

$$\hat{c}_3^1 = 4.75(Q_{31}^1 + Q_{32}^1 + Q_{33}^2)$$

and the costs associated with freight service provider 3 and the three demand points are:

$$c^3_{11} = .0001{Q^1_{31}}^2 + 12Q_{31}, \quad c^3_{12} + .0001{Q^1_{32}}^2 + 12.5Q^1_{32}, \quad c^3_{13} = .0001{Q^1_{33}}^2 + 11.5Q^1_{33}.$$

According to the above data, the humanitarian organization has higher transaction costs in dealing with the new freight service provider, since it has not done business with it in the past. However, freight service provider 3 is more cost efficient in terms of the three demand points as compared to the original two freight service providers, since it has experience in the western part of Africa.

Also, the capacity of freight service provider 3, $u_3 = 10000$.

We set $\alpha = .1$ in the modified projection method for this example. In order to construct the initial disaster relief item shipments, we divided the supplies needed at each demand point equally among the freight service providers. All three Lagrange multipliers were initialized to zero.

The modified projection method yielded the following equilibrium shipment and Lagrange multiplier vector solution for scenario 2:

$$Q^{1*}_{11} = 5,571.19, \quad Q^{1*}_{12} = 796.68, \quad Q^{1*}_{13} = 3,395.15,$$

$$Q^{1*}_{21} = 682.25, \quad Q^{1*}_{22} = 9,203.32, \quad Q^{1*}_{23} = 351.42,$$

$$Q^{1*}_{31} = 3,746.56, \quad Q^{1*}_{32} = 0.00, \quad Q^{1*}_{33} = 6,253.44.$$

$$\lambda^*_1 = 0.00, \quad \lambda^*_2 = 0.00, \quad \lambda^*_3 = 6.60.$$

The prices charged by the freight service providers are:

$$\rho^{1*}_{11} = 19.59, \quad \rho^{1*}_{12} = 18.18, \quad \rho^{1*}_{13} = 19.60,$$

$$\rho^{1*}_{21} = 19.84, \quad \rho^{1*}_{22} = 18.43, \quad \rho^{1*}_{23} = 19.84,$$

$$\rho^{1*}_{31} = 24.09, \quad \rho^{1*}_{32} = 23.85, \quad \rho^{1*}_{33} = 24.10.$$

Freight service provider 1 transports 9,763.02 PPEs; freight service provider 2, in turn, transports 10,236.98 PPEs, whereas freight service provider 3 transports 10,000.00 PPEs, which is its capacity. Observe that freight service provider 3 charges the highest prices.

The humanitarian organization pays out 621,281.88 to the freight service providers for transportation. It is now faced with a total cost of 756,222.63, which includes its transaction costs. The percentage of total cost for freight is 82%, which is, again, in line with what one sees in practice.

The profit of freight service provider 1 is now: 15,265.55; that of freight service provider 2: 10,170.52, and that of freight service provider 3: 118,765.33. Both original freight service providers suffer financially from enhanced competition. However, the humanitarian organization greatly reduces its total cost. Also, it is interesting to see that freight service provider 3 only transports the PPEs to Liberia and Guinea and delivers no PPE shipments to Sierra Leone. Clearly, humanitarian organizations benefit (and implicitly so do the donors) by having additional freight service providers interested in transporting their relief item cargos.

Scenario 3: Two Humanitarian Organization, Three Freight Service Providers With Capacities, and Three Demand Points

Scenario 3 consists of two humanitarian organization, three freight service providers, and three demand points as illustrated in Figure 5.

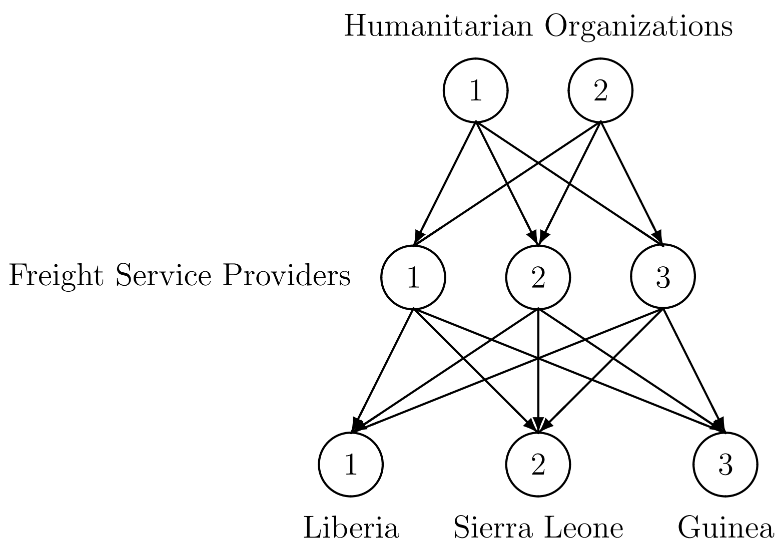

Figure 5: Supply Chain Network Topology for the Ebola Case Study Scenario 3

In particular, this example has the same data as Scenario 2 but now we add data associated with the second humanitarian organization as detailed below. There is now increased demand for additional PPEs, which the second humanitarian organization is willing to provide.

The second humanitarian organization has worked closely with all the freight service providers in previous disasters and, hence, its transaction costs are lower than those for humanitarian organization 1. The costs associated with the second humanitarian organization transacting with the three freight service providers are:

$$\hat{c}_1^2 = 3(Q_{11}^2 + Q_{12}^2 + Q_{13}^2), \quad \hat{c}_2^2 = 3.5(Q_{21}^2 + Q_{22}^2 + Q_{23}^2), \quad \hat{c}_3^2 = 3(Q_{31}^2 + Q_{32}^2 + Q_{33}^2).$$

The freight service providers, in turn, incur the following costs associated with transporting the disaster relief supplies from humanitarian organization 2:

$$c_{21}^1 = .0002{Q_{11}^2}^2 + 10Q_{11}^2, \quad c_{22}^1 = .0001{Q_{12}^2}^2 + 8Q_{12}^2, \quad c_{23}^1 = .0002{Q_{13}^2}^2 + 9Q_{13}^2.$$

$$c_{21}^2 = .0001{Q_{21}^2}^2 + 8Q_{21}^2, \quad c_{22}2 = .0002{Q_{22}^2}^2 + 7Q_{22}^2, \quad c_{23}^2 = .0001{Q_{23}^2}^2 + 6Q_{23}^2,$$

$$c_{21}^3 = .0002{Q_{31}^2}^2 + 9Q_{31}^2, \quad c_{22}^3 = .0001{Q_{32}^2}^2 + 7Q_{32}^2, \quad c_{23}^3 = .0001{Q_{33}^2}^2 + 6Q_{33}^2.$$

Also, the amount of the supplies that humanitarian organization 2 wishes to have delivered are:

$$s_1^2 = 3000, \quad s_2^2 = 3000, \quad s_3^2 = 4000.$$

Observe that, now, the total demand for shipments is exactly equal to the total capacity of the three freight service providers.

We, again, set $\alpha = .1$ in the modified projection method. The shipments and Lagrange multipliers were initialized as in Scenario 2 with the former being equally distributed, given the supply/demand, for each humanitarian organization and demand point, among the freight service providers. The modified projection method yielded the following equilibrium shipment and Lagrange multiplier patterns:

$$Q_{11}^{1*} = 5385.70, \quad Q_{12}^{1*} = 689.91, \quad Q_{13}^{1*} = 3415.49,$$

$$Q_{21}^{1*} = 830.67, \quad Q_{22}^{1*} = 9310.09, \quad Q_{23}^{1*} = 368.14,$$

$$Q_{31}^{1*} = 3783.63, \quad Q_{32}^{1*} = 0.00, \quad Q_{33}^{1*} = 6216.37,$$

$$Q_{11}^{2*} = 0.00, \quad Q_{12}^{2*} = 508.91, \quad Q_{13}^{2*} = 0.00,$$

$$Q_{21}^{2*} = 3000.00, \quad Q_{22}^{2*} = 2491.09, \quad Q_{23}^{2*} = 4000.00,$$

$$Q_{31}^{2*} = 0.00, \quad Q_{32}^{2*} = 0.00, \quad Q_{33}^{2*} = 0.00,$$

$$\lambda_1^* = 1569.02, \quad \lambda_2^* = 1568.71, \quad \lambda_3^* = 1575.62.$$

The prices charged by the freight service providers are:

$$\rho_{11}^{1*} = 1588.58, \quad \rho_{12}^{1*} = 1586.99, \quad \rho_{13}^{1*} = 1588.66,$$

$$\rho_{21}^{1*} = 1588.85, \quad \rho_{22}^{1*} = 1587.16, \quad \rho_{23}^{1*} = 1588.88,$$

$$\rho_{31}^{1*} = 1593.13, \quad \rho_{32}^{1*} = 1592.87, \quad \rho_{33}^{1*} = 1593.11,$$

$$\rho_{11}^{2*} = 1579.02, \quad \rho_{12}^{2*} = 1577.12, \quad \rho_{13}^{2*} = 1578.02,$$

$$\rho_{21}^{2*} = 1577.31, \quad \rho_{22}^{2*} = 1576.71, \quad \rho_{23}^{2*} = 1575.51,$$

$$\rho_{31}^{2*} = 1584.62, \quad \rho_{32}^{2*} = 1584.62, \quad \rho_{33}^{2*} = 1581.62.$$

Humanitarian organization 1 pays out 47,689,076.00 to the freight service providers and encumbers a total cost of 47,823,948.00. Humanitarian organization 2 pays out 15,764,292.00 to the freight service providers and encumbers a total cost (which recall includes the transaction costs) of 15,799,038.00. The total disaster relief volume transported by freight service provider 1 is: 10,000.00; the total amount transported by freight service provider 2 is: 20,000.00, and the total amount by freight service provider 3 is: 10,000.00. Hence, all freight service providers are at their respective capacity and, therefore, the Lagrange multipliers are all positive. Freight service provider 1 now enjoys a profit of 15,705,270.00, whereas freight service provider has a profit of 31,388,628.00, and freight service provider a profit of 15,809,006.00.

It is interesting that humanitarian organization 2 does not utilize the services of freight service provider 3 at all and that the majority of its shipments go via freight service provider 2. Humanitarian organization 1, on the other hand, relies primarily on the services of freight service provider 1 for shipments to Liberia and Guinea and the services of freight service provider 2 for shipments to Sierra Leone. With increased demand for their services, because of the needs of humanitarian organization 2, all freight service providers have higher profits than in Scenario 2. Because of the increased competition for freight service provision from the added humanitarian organization, humanitarian organization 1 now has a substantially higher total cost than in Scenario 2.

This example vividly illustrates that the humanitarian organizations might benefit from cooperating rather than competing.

5. Summary and Conclusions

In this paper, a multitiered supply chain network equilibrium model for disaster relief was constructed, which can handle as many humanitarian organizations as well as freight (logistic) service providers engaged in the delivery of disaster relief supplies to multiple demand points for distribution to the victims, as needed by the specific disaster relief application. In addition, the model incorporates capacities associated with the freight service providers' transportation of the relief items. Previous freight service competitive modeling in disaster relief considered only a single humanitarian organization and was uncapacitated (cf. Nagurney (2016)). Hence, the new model is a significant extension on prior work and also a contribution to the still very limited literature on game theory and disaster relief. Also, although supply chain network equilibrium models have generated a rich literature, beginning with the first model of Nagurney, Dong, and Zhang (2002), all, except for the above-noted work and this paper, have focused on profit-maximization as the primary objective of the various decision-makers associated with the supply chain network tiers. Here, in contrast, since we are dealing with humanitarian organizations, which are nonprofit entities, their objective functions are comprised of cost minimization. Of course, the cost can also represent a generalized cost, since these functions are nonlinear, and can capture risk, for example, or even time, weighted accordingly.

In this paper, in addition to the new model, existence results were provided, as well as an algorithm, which has less restrictive conditions for convergence than the previously proposed projection method for the single humanitarian organization case and also results in a decomposition of the disaster relief item flows at each step into network subproblems of special structure and closed form expressions for the Lagrange multipliers associated with the capacity constraints. The algorithm was then applied to a case study inspired by a major international healthcare crisis - that of the Ebola outbreak, which devastated multiple western African countries in 2014 and 2015. In particular, we first included capacities on freight service provision from a dataset constructed in Nagurney (2016) associated with the transportation of personal protective equipment needed by the medical professionals battling Ebola. We then investigated the impact of the addition of a new freight service provider and, subsequently, also the addition of a second humanitarian organization. We reported the equilibrium disaster relief shipments for the three scenarios along with the Lagrange multipliers as well as the payouts of the humanitarian organizations to the freight service providers and their total costs, plus the profits of the freight service providers and the prices that they charge the humanitarian organizations for freight service provision.

The numerical examples making up the case study demonstrate that humanitarian organizations benefit from the availability of a larger number of competitive freight service providers (although this affects freight service providers negatively in terms of profits). Also, the addition of humanitarian organizations competing for services from the freight service providers results in higher prices since the capacities may be achieved. Hence, the case study illustrates that cooperation may be a fruitful avenue for future research on game theory and freight service provision for humanitarian organizations in disaster relief. In addition, it would be interesting to compute solutions to large-scale numerical examples and to explore alternative algorithms for computational purposes. We leave such research for the future.

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