A Game Theory Model for Freight Service Provision Security Investments for
High-Value Cargo

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Abstract:

In this paper, we develop a game theory model in which freight service providers seek to maximize their expected utility by competing for business from shippers and also investing in security. The focus is on high-value cargo, which has been the target of attacks globally. Shippers reflect their preferences for freight service providers through the prices they are willing to pay which depend on quantities shipped and security levels invested in. The Nash Equilibrium is formulated as a variational inequality problem for which existence is guaranteed. Numerical examples illustrate the framework and give essential freight security investment policy related information.

Key words: security, freight, networks, high-value cargo, game theory, variational inequalities, security policy
1. Introduction

Effective freight services, as critical service components of supply chains, are essential to the transportation and delivery of products from points of origin to destinations. Shippers expect their goods to arrive in their entirety, in good condition, and in a timely manner. Nevertheless, according to Heyn (2014), the US Federal Bureau of Investigation reports that, each year, approximately $30 billion worth of cargo is lost, with estimates of cargo theft reaching record highs in 2012. Cargo theft is not limited to the continental United States, however, and, in Europe, cargo theft increased 24 percent in 2012, and rose in Asia as well (Terry (2014)). The greatest risk of cargo theft currently exists in Brazil, Mexico, and South Africa, often via hijacking.

High-value products, in particular, which can range from high tech equipment to precious metals and jewelry and certain fashion and other luxury items, alcohol and high-end food products, as well as pharmaceuticals, are especially attractive targets for theft while in transit. High-value goods have always required extra security, but recently several factors have increased the risk. As the world’s appetite for luxury goods grows, sourcing and marketing locations have become more spread out and diverse with longer supply chains adding touch points and, hence, increasing vulnerability (Terry (2014)). Indeed, as global trade expands, companies are faced with greater security challenges, and to illustrate the scope of issues, observe that more than 200 million containers are shipped between the world’s seaports annually (Closs and McGarrell (2004)), with the United States receiving approximately 19.6 million containers in 2014, which corresponds to about 53,700 per day (see World Shipping Council (2016)). In addition, crime organizations are increasingly focused on goods in transit. Furthermore, localized disruptions such as severe weather, political unrest, and natural disasters can also increase risk by idling high-value cargo. Holiday seasons, such as the period of Christmas and New Year’s, and even the July 4th holiday, pose additional challenges because of increased cargo thefts in the US (cf. Kilcarr (2015)). In 2016, incidents involving theft of full truckload continued as the most prevalent method of theft during the third quarter in the United States, with 78% of all reported thefts, recording an average loss value of over $120,000, according to FreightWatch (see Cole (2016)).

According to Weiss (2016), cargo thefts in Europe, the Middle East, and Africa have almost tripled in the past five years, based on data reported by the Transport Asset Protection Association (TAPA). To illustrate the breadth of high-value goods that have been targets of thefts, according to TAPA, and, as reported in Weiss (2016), in recent months, criminals have absconded with salmon worth 100,000 euros ($112,000) from a trailer in Norway, 80 cases of whiskey from a vehicle near London, and truckloads of nuts worth $10 million in more than...
30 incidents. 85% of all major cargo theft involves trucks, according to TAPA, with such thefts costing businesses more than $10 billion annually worldwide (Brown (2013)). TAPA was initially established to protect shipments of electronic goods but criminals are also diversifying as to the products that they steal and the Internet is making it easier to fence the pilfered items. In addition, some freight thieves are becoming cybercriminals, impersonating companies, and engaging in fictitious pickups and redirected deliveries (cf. Morris (2015)).

Shippers, as noted by Meixell and Norbis (2012) (see also Rinehart, Myers, and Eckert (2004)), can reduce security-related negative impacts by selecting security-conscious carriers, which we refer to here as freight service providers (FSPs), who, in turn, must decide on the best mode and route choices from the origin node where the goods are picked up from to the destination points, where the goods are delivered to. Voss et al. (2006) also argue that security practices are an important criterion in carrier selection. Of course, shippers also consider price in making a decision as to the carrier or freight service provider (cf. Meixell and Norbis (2008), Nagurney et al. (2015), and the references therein). Moreover, to keep cargo safe, freight service provider companies may use teams of drivers, GPS tracking technology, and remote vehicle disabling in transit, as examples of possible security measures (Heyn (2014)).

Investing in security is, nevertheless, costly (see, e.g., Russell and Saldanha (2003)). Hence, FSPs must take their security investments into consideration when pricing for their services. Peleg-Gillai, Bhat, and Sept (2006), in their investigations of 11 manufacturers from a variety of industries and 3 freight service providers, considered to be innovators in the area of supply chain security, determined that the vast majority of companies were able to realize many benefits from their security investments, with some of them reaching very significant levels. Based on these inputs, the authors concluded that investments in supply chain security can help organizations to improve internal operations, strengthen relationships with their customers, and increase, in general, their profitability.

Shippers, these days, including those dealing with high-value products, have a spectrum of FSPs that they can select from, with examples in the United States including UPS, FedEx, DHL, Ryder, Schneider, Brink’s, etc. FSPs, in turn, compete for the shippers’ business and security is increasingly an essential requirement to shippers. Thefts of high-value products may result in incurred damages and insurance may be insufficient to cover the losses. Plus, a freight service provider’s reputation may be at stake with negative publicity associated with thefts which can affect future business prospects. An overview of cargo theft and supply chain security, from a practitioner standpoint, can be found in the book by Burges (2013). Ekwall (2012) provides an excellent overview of supply chain security issues and possible
solutions. The edited volume of Wagner and Bode (2009) contains interesting contributions to security and risk with a focus on logistics service providers.

According to the Federal Bureau of Investigation, cargo theft is a $15 billion to $30 billion a year problem (FleetOwner (2016)). Such thefts are indication of a much larger and a more pressing issue of inconsistencies in perception of cargo theft along a supply chain. There are multiple stakeholders in this complex network that involves multiple modes as well. In addition to coordination and collaboration, these stakeholders also need to communicate their issues, best practices, and security related activities to withstand the decentralized, diffused, and evolving threats they are facing. While transportation security administration and related authorities have been building standards and regulations to improve security, much of the onus lies on the FSPs. Since in our paper we focus on the security measures adopted by freight service providers and the willingness to pay for their services being sensitive to those measures, we focus on how the investments into security can affect their overall security policies.

Generally, FSPs employ experienced investigators and surveillance personnel to monitor shipments closely at all times. They also use technology with bar-code and internet-based tracking systems to follow the shipment as long as it is in their possession. These measures would determine the security levels we discuss in this paper. Investments into these security measures ultimately determine the individual FSPs and, thus, the entire networks vulnerability. The policies and mandates set by the governmental authorities can get complemented by smart security investments at the operational level by stakeholders in the supply chain. In this paper, we discuss the implications of security investments by FSPs.

Gould, Macharis, and Haasis (2010), in their review of the literature on security and supply chain management to that date, note that topics such as passing on the costs of security to customers as well as understanding security performance, and supply chain benefits, is an area for future research. Meixell and Norbis (2012), in their review of carrier selection and supplier selection, and in motivating their integrated supplier and carrier selection model with a focus on security, observe that not much research has been devoted to including supply chain security measures into either carrier or supplier choice decision models. Their work provides an elegant multi-objective optimization framework based on a single buyer.

We emphasize that several frameworks expanding the breadth of the freight modeling literature, but not focused on security, have been utilized to-date. A summarized literature review of the work until 1985 is provided in Harker (1985). The majority of the literature until then focused on one or two shipper or carrier problems in an intercity freight transportation
system with subsequent advances including the contributions of Harker and Friesz (1986a,b), Dafermos and Nagurney (1987), Harker (1988), Miller, Tobin, and Friesz (1991), Hurley and Petersen (1994), Forkenbrock (1999), Fernandez, De Cea, and Soto (2003), Agrawal and Ziliaskopoulos (2006), Xiao and Yang (2007), Xu and Holguin-Veras (2009), and Lin and Huang (2017), with freight network equilibrium utilized by most of these authors. Spatial price equilibrium, in turn, has been employed by Florian and Los (1982), Friesz, Tobin, and Harker (1983), and Dafermos and Nagurney (1984), among others. In addition, as noted by Lee, Boile, and Theofanis (2014), Stackelberg games have been used for evaluating sequential decision-making (see, e.g., Miller, Tobin, and Friesz (1991), Xiao and Yang (2007)). There has also been work in the coopetition (competition and cooperation) space of freight network equilibrium (Lin, Huang, and Ng (2017)) as well as in the context of supply chain network equilibrium (cf. Nagurney (2006), Saberi et al. (2018), and the references therein).

The model that we develop in this paper fills gaps in the literature in several ways. We develop a game theory model consisting of FSPs who compete with one another as to the quantity of the high-value product that they will transport from origin locations to destinations. The shippers, in turn, reflect their preferences for transport of the high-value cargo through the prices that they are willing to pay, which depend on the quantities carried as well as the investment in security by the FSPs. We posit security investment cost functions, which the FSPs encumber, if they invest in security, and include the probability of an attack on the logistics/transport links, and the associated damages. Each FSP seeks to maximize his expected utility associated with the quantities that he transports as well as his investment in security, which may differ for different links. The governing Nash Equilibrium (1950, 1951) conditions are then shown to satisfy a variational inequality problem for which existence is guaranteed. Conditions for uniqueness are provided and an algorithmic scheme proposed, which yields closed form expressions at each iteration in the quantity shipments as well as the security levels to be invested in. The fact that we demonstrate, for a computable model, sensitivity to capturing security in a freight network of high value cargo, show how shippers reflect their preferences, and how investments by FSPs can affect their businesses in an equilibrium setting (and not optimization), is among our contributions.

Although there is a rich body of literature on game theory models for homeland security (cf. Kardes (2007) for a review), the modeling of security in supply chain contexts, as already noted earlier, is limited, and, even more so, for security associated with freight service provision investments. Bakir (2011) considers a defender and attacker engaged in a game regarding cargo container transportation. Gkonis and Psaraftis (2010), earlier, developed a game theory model with discrete choices (whether to invest or not) for container shipping.
transportation, which was inspired by the work of Kunreuther and Heal (2003) and Heal and Kunreuther (2005). The novelty of our framework lies in that the shippers respond to the security investments of the freight service providers, who compete for business, through the prices that they are willing to pay and we also capture risk in that the level of security affects the probability of attack and the expected damages. The security levels in our model are continuous and have upper bounds. Furthermore, our work is motivated by freight service provision of high-value goods and not terrorism. For examples of innovative game theory models for counter-terrorism, see the work of Bier (2006) and Wein et al. (2006). Our work is not on what terrorists can put into cargo and cause disruptions with but, rather, on outright thefts of cargo, with specific relevance to truck cargo.

This paper is organized as follows. In Section 2, the game theory model for freight security investments for high-value cargo is constructed and qualitative results provided. In Section 3, an algorithm is proposed and then applied to compute solutions to numerical examples that illustrate the practicality of the framework. In Section 4, we summarize the results and present suggestions for future research.
2. The Game Theory Model for Freight Service Provision Security Investments

In this section, we develop the game theory model, define the governing Nash Equilibrium conditions, and present the variational inequality formulation, for which existence results are then provided, along with conditions for uniqueness of the equilibrium quantity flow and security investment pattern.

We consider \( m \) FSPs, with a typical provider denoted by \( i \); \( n \) shipper “origin” nodes from which the high-value products are to be picked up from for delivery (and corresponding to distinct shippers), with a typical such node denoted by \( j \), and \( o \) destination nodes for delivery of the high-value products, with a typical such node denoted by \( k \). The network structure of the problem is depicted in Figure 1.

![Diagram](https://via.placeholder.com/150)

**Figure 1:** The Network Structure of the Freight Security Investment Game Theory Model

We now introduce the necessary notation. Let \( q_{ijk} \) denote the quantity of the high-value product that FSP \( i; i = 1, \ldots, m \), transports from \( j \) to \( k \), where \( j = 1, \ldots, n \), and \( k = 1, \ldots, o \). The vector \( q_i \) is then the \( no \)-dimensional vector consisting of all the high-value cargo shipments of FSP \( i \). Associated with each FSP \( i \) and cargo shipment from shipper node \( j \) to destination node \( k \) are the following bounds:

\[
0 \leq q_{ijk} \leq \bar{q}_{ijk}, \quad \forall j, \forall k,
\]

where \( \bar{q}_{ijk} \) denotes the upper bound of the high-value cargo shipment between \( j \) and \( k \) that freight service provider \( i \) can carry. We group the cargo shipments of all the freight service providers into the vector \( q \in \mathbb{R}^{mno}_+ \).
Also, let $s_{ijk}$ denote the security level that FSP $i; i = 1, \ldots, m$, invests in from $j$ to $k$, with $s_i$ denoting the $no$-dimensional vector consisting of all the security levels of FSP $i$. The security level for each FSP $i$ must lie in the range:

$$0 \leq s_{ijk} \leq \bar{s}_{ijk}, \quad \forall j, \forall k,$$  

(2)

where $\bar{s}_{ijk}$ denotes the upper bound on the security level between $j$ and $k$ of FSP $i$ and this upper bound is less than 1, since here 1 represents perfect security, which, in practice, is not realizable. We further group the security levels of all the freight service providers into the vector $s \in \mathbb{R}_+^{mno}$.

Associated with acquiring a security level $s_{ijk}$ is an investment cost function $h_{ijk}; i = 1, \ldots, m; j = 1, \ldots, n; k = 1, \ldots, o$, with the function assumed to be continuously differentiable and convex. We assume that, for a given FSP $i$, $h_{ijk}(0) = 0$ denotes an entirely insecure route/mode choice between $j$ and $k$ and $h_{ijk}(1) = \infty$ is the investment cost associated with complete security. An example of an $h_{ijk}(s_{ijk})$ function that satisfies these properties and that we utilize in our model as

$$h_{ijk}(s_{ijk}) = \alpha_{ijk} \left( \frac{1}{\sqrt{(1 - s_{ijk})}} - 1 \right) \text{ with } \alpha_{ijk} > 0, \quad \forall i, \forall j, \forall k.$$  

(3)

The term $\alpha_{ijk}$ allows distinct freight service providers to have different investment cost functions based on their needs and expert knowledge associated with transport between different origin and destination nodes. Related security investment cost functions have been used in the context of cybersecurity, but those in (3) are more general since that apply at the link level through $\alpha_{ijk}$ and $s_{ijk}$ (see, e.g., Nagurney, Nagurney, and Shukla (2015), Nagurney, Daniele, and Shukla (2017)). It is critical to note that the above investment cost function is an illustration. Our model is general enough to accommodate other forms of this function, provided it is convex.

The interpretation of the investment cost function (3) is that the lower the investment, the higher the vulnerability $(1 - s_{ijk})$. The costs for security increase as the security increases and improving security level by a unit will impose an increased marginal cost on the user.

The probability of successful theft of the high-value cargo from $i$ going from $j$ to $k$, $p_{ijk}$, is given by

$$p_{ijk} = (1 - s_{ijk}), \quad \forall i, \forall j, \forall k.$$  

(4)

According to (4), if there is no investment in security by $i$ along transport link $(j, k)$ and, hence, $s_{ijk} = 0$, then the probability of an attack against $i$, transporting the high-value cargo from $j$ to $k$, $p_{ijk}$, is precisely equal to 1.
Each FSP $i; i = 1, \ldots, m$, charges a price $\rho_{ijk}$ to shipper $j$ for transporting a unit of the high-value product from $j$ to $k$, where we assume that, in general,

$$\rho_{ijk} = \rho_{ijk}(q, s), \quad \forall j, \forall k.$$  \hspace{1cm} (5)

The price $\rho_{ijk}$ reflects how much shipper $j$ is willing to pay $i$ for having the high-value product be transported from $j$ to $k$. Note that the price depends not only on the quantities transported but also on the security levels associated with the links joining the mid-tier nodes to the bottom-tier nodes in the network in Figure 1. We assume that the prices are continuously differentiable and are decreasing in the corresponding quantity but increasing in the corresponding security level.

In addition, each FSP $i; i = 1, \ldots, m$, is faced with a total cost associated with transporting the high-value cargo items from $j$ to $k$ given by $\hat{c}_{ijk}$, where

$$\hat{c}_{ijk} = \hat{c}_{ijk}(q), \quad \forall j, \forall k.$$ \hspace{1cm} (6)

According to (6), the total cost associated with transporting the high-value cargo may depend, in general, on the vector of quantities transported. We assume that these total cost functions are continuously differentiable and convex. Hence, the freight service providers are affected by the quantities transported by the other freight service providers through the total costs incurred as well as through the prices associated with transporting the high-value cargo.

The damage in case of an attack on $i$ traveling between $j$ and $k$ is denoted by $D_{ijk}$ and the value is positive for all $i, j, k$. In the case of a successful attack on FSP $i$ traveling from $j$ to $k$, the expected damage is given by: $p_{ijk}D_{ijk}$ so that his total expected damages correspond to:

$$\sum_{j=1}^{n} \sum_{k=1}^{o} p_{ijk}D_{ijk}.$$ \hspace{1cm} (7)

The above is in line with the security risk equation as given by Transportation Planning division of the Department of Transportation which is Security risk = Probability of incident attempt $\times$ Vulnerability $\times$ Damage (Department of Transportation (2003)).

Each FSP $i; i = 1, \ldots, m$, seeks to maximize his expected profit, $E(U_i)$, given by:

$$E(U_i) = \sum_{j=1}^{n} \sum_{k=1}^{o} (1 - p_{ijk})(\rho_{ijk}(q, s)q_{ijk} - \hat{c}_{ijk}(q))$$

$$+ \sum_{j=1}^{n} \sum_{k=1}^{o} p_{ijk}(\rho_{ijk}(q, s)q_{ijk} - \hat{c}_{ijk}(q) - D_{ijk}) - \sum_{j=1}^{n} \sum_{k=1}^{o} h_{ijk}(s_{ijk}).$$ \hspace{1cm} (8)
The first term in (8) after the equal sign represents the expected profit of FSP $i$ in the absence of an attack on links joining a shipper origin node and destination node. The second term in (8) following the equal sign represents the expected profit in the case of a successful attack on each link and the last term represents the expenditures associated with security investments of FSP $i$ on each of the transport links $(j, k)$ in the network in Figure 1. Different route/mode combinations may be more or less susceptible to attacks, and, hence, having security investments associated with links is very reasonable since destination nodes can correspond to more or less safe transit.

Hence, each FSP $i; i = 1, \ldots, m$, seeks to maximize his expected profit $E(U_i)$ given by (8), subject to the constraints: (1) and (2). Observe that the decisions of each freight service provider in terms of the quantities he agrees to transport and the level of security he invests in for the various links affects not only his expected utility but also those of the other freight service providers that he is in competition with.

Let $K_i$ denote the feasible set corresponding to FSP $i$, where $K_i \equiv \{(q_i, s_i)|0 \leq q_{ijk} \leq \bar{q}_{ijk}, \forall j, k$ and $0 \leq s_{ijk} \leq \bar{s}_{ijk}, \forall j, k\}$. We also define the feasible set corresponding to all the freight service providers: $K \equiv \prod_{i=1}^{m} K_i$.

The $m$ FSPs compete noncooperatively in delivering the high-value cargo and invest in security, with each one trying to maximize his own expected profit. We seek to determine a nonnegative high-value cargo shipment and security level pattern $(q^*, s^*)$ for which the $m$ freight service providers will be in a state of equilibrium as defined below. Nash (1950, 1951) generalized Cournot’s concept (see Cournot (1838)) of an equilibrium for a model of several players, that is, decision-makers, each of which acts in his/her own self-interest, in what has been come to be called a noncooperative game.

Definition 1: A Nash Equilibrium in High-Value Product Shipments and Security Levels

A high-value product shipment and security level pattern $(q^*, s^*) \in K$ is said to constitute a Nash equilibrium if for each freight service provider $i; i = 1, \ldots, m$,

$$E(U_i(q_i, s_i, \hat{q}_i^*, \hat{s}_i^*)) \geq E(U_i(q_i, s_i, \hat{q}_i, \hat{s}_i^*)), \quad \forall (q_i, s_i) \in K_i,$$

(9)

where

$$\hat{q}_i^* \equiv (q_1^*, \ldots, q_{i-1}^*, q_{i+1}^*, \ldots, q_m^*); \quad \text{and} \quad \hat{s}_i^* \equiv (s_1^*, \ldots, s_{i-1}^*, s_{i+1}^*, \ldots, s_m^*).$$

(10)
According to (9), an equilibrium is established if no freight service provider can unilaterally improve upon his expected profits by selecting an alternative vector of high-value product shipments and security levels.

We now present alternative variational inequality formulations of the above Nash Equilibrium in high value product shipments and security levels.

**Theorem 1: Variational Inequality Formulations**

Assume that, for each freight service provider \( i; i = 1, \ldots, m \), the expected profit function \( E(U_i(q, s)) \) is concave with respect to the variables \( \{q_{i11}, \ldots, q_{ino}\} \) and \( \{s_{i11}, \ldots, s_{ino}\} \), and is continuously differentiable. Then \((q^*, s^*) \in K\) is a Nash Equilibrium according to Definition 1 if and only if it satisfies the variational inequality

\[
-\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{o} \frac{\partial E(U_i(q^*, s^*))}{\partial q_{ijk}} (q_{ijk} - q^*_{ijk}) - \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{o} \frac{\partial E(U_i(q^*, s^*))}{\partial s_{ijk}} (s_{ijk} - s^*_{ijk}) \geq 0, \quad \forall (q, s) \in K,
\]

or, equivalently, \((q^*, s^*) \in K\) is a Nash Equilibrium high-value product shipment and security level pattern if and only if it satisfies the variational inequality

\[
\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{o} \left[ -\sum_{h=1}^{n} \sum_{l=1}^{o} \frac{\partial \hat{c}_{ihl}(q^*)}{\partial q_{ijk}} - \rho_{ijk}(q^*, s^*) - \sum_{h=1}^{n} \sum_{l=1}^{o} \frac{\partial \rho_{ihl}(q^*, s^*)}{\partial q_{ijk}} q^*_{ihl} \right] (q_{ijk} - q^*_{ijk}) \\
\hfill + \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{o} \left[ -D_{ijkl} + \frac{\partial h_{ijkl}(s^*_{ijkl})}{\partial s_{ijkl}} - \sum_{h=1}^{n} \sum_{l=1}^{o} \frac{\partial \rho_{ihl}(q^*, s^*)}{\partial s_{ijkl}} q^*_{ihl} \right] (s_{ijkl} - s^*_{ijkl}) \geq 0, \quad \forall (q, s) \in K.
\]

**Proof:** (11) follows directly from Gabay and Moulin (1980) and Dafermos and Nagurney (1987).

In order to obtain variational inequality (12) from variational inequality (11), recall (4) and note that, at the equilibrium, for \( i = 1, \ldots, m; j = 1, \ldots, n; k = 1, \ldots, o \):

\[
-\frac{\partial E(U_i)}{\partial q_{ijk}} = \left[ -\sum_{h=1}^{n} \sum_{l=1}^{o} \frac{\partial \hat{c}_{ihl}(q^*)}{\partial q_{ijk}} - \rho_{ijk}(q^*, s^*) - \sum_{h=1}^{n} \sum_{l=1}^{o} \frac{\partial \rho_{ihl}(q^*, s^*)}{\partial q_{ijk}} q^*_{ihl} \right];
\]

and

\[
-\frac{\partial E(U_i)}{\partial s_{ijkl}} = \left[ -D_{ijkl} + \frac{\partial h_{ijkl}(s^*_{ijkl})}{\partial s_{ijkl}} - \sum_{h=1}^{n} \sum_{l=1}^{o} \frac{\partial \rho_{ihl}(q^*, s^*)}{\partial s_{ijkl}} q^*_{ihl} \right].
\]
Substitution of (13) and (14) into (11) yields (12) \[\Box\]

We now put the above variational inequality formulation (12) of the Nash Equilibrium
problem into standard variational inequality form (see Nagurney (1999)), that is: determine
\(X^* \in K \subset \mathbb{R}^N\), such that

\[
\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in K,
\tag{15}
\]

where \(F\) is a given continuous function from \(K\) to \(\mathbb{R}^N\) and \(K\) is a closed and convex set.

We define the \((2mno)\)-dimensional vector \(X \equiv (q, s)\) and the \((2mno)\)-dimensional vector
\(F(X) = (F^1(X), F^2(X))\) with the \((i, j, k)\)-th component, \(F^1_{ijk}\), of \(F^1(X)\) given by

\[
F^1_{ijk}(X) \equiv -\frac{\partial E(U_i(q, s))}{\partial q_{ijk}},
\tag{16}
\]

the \((i, j, k)\)-th component, \(F^2_{ijk}\), of \(F^2(X)\) given by

\[
F^2_{ijk}(X) \equiv -\frac{\partial E(U_i(q, s))}{\partial s_{ijk}},
\tag{17}
\]

and with the feasible set \(K \equiv K\) and \(N = 2mno\). Then, clearly, variational inequality (12)
can be put into standard form (15).

Existence of a solution to variational inequality (11) and to its equivalence (12) is guaran-
teed to exist from the standard theory of variational inequalities (cf. Kinderlehrer and
Stampacchia (1980)) since the feasible set underlying them is compact and the functions
that enter the variational inequalities are assumed to be continuous.

Moreover, if the function that enters the variational inequality, as in its standard form
(15) is strictly monotone, that is,

\[
\langle F(X^1) - F(X^2), X^1 - X^2 \rangle \geq 0, \quad \forall X^1, X^2 \in K, \quad X^1 \neq X^2,
\tag{18}
\]

then the solution \(X^*\) to (15) is unique and, hence, the solution \((q^*, s^*)\) to both (11) and (12)
is also unique.
3. The Algorithm and Numerical Examples

For the solution of numerical examples of the model, we utilize the Euler method, which is induced by the general iterative scheme of Dupuis and Nagurney (1993). Specifically, iteration $\tau$ of the Euler method (see also Nagurney and Zhang (1996)) is given by:

$$X^{\tau+1} = P_K(X^\tau - a_\tau F(X^\tau)),$$

where $P_K$ is the projection on the feasible set $K$ and $F$ is the function that enters the variational inequality problem (15).

As established in Dupuis and Nagurney (1993), for convergence of the general iterative scheme, which induces the Euler method, the sequence $\{a_\tau\}$ must satisfy: $\sum_{\tau=0}^\infty a_\tau = \infty$, $a_\tau > 0$, $a_\tau \to 0$, as $\tau \to \infty$. Specific conditions for convergence of this scheme as well as various applications to the solutions of other network-based game theory models can be found in Nagurney (2006) and the references therein.

Explicit Formulae for the Euler Method Applied to the Freight Service Provision Game Theory Model with Security Investments

The elegance of this procedure for the computation of solutions to our model is illustrated by the following explicit formulae. Specifically, we have the following closed form expression for the high-value cargo shipments $i = 1, \ldots, m; j = 1, \ldots, n; k = 1, \ldots, o$:

$$q_{ijk}^{\tau+1} = \max\{0, \min\{\bar{q}_{ijk}, q_{ij}^{\tau} + a_\tau (\rho_{ijk}(q_{ij}^{\tau}, s_{ijk}) + \sum_{h=1}^n \sum_{l=1}^o \partial \rho_{ihl}(q_{ij}^{\tau}, s_{ijk}) \partial q_{ijkl}^{\tau} - \sum_{h=1}^n \sum_{l=1}^o \partial \hat{c}_{ihl}(q_{ij}^{\tau}) \partial q_{ijkl}^{\tau})\}\},$$

and the following closed form expression for the security levels $i = 1, \ldots, m; j = 1, \ldots, n; k = 1, \ldots, o$:

$$s_{ijk}^{\tau+1} = \max\{0, \min\{\bar{s}_{ijk}, s_{ijk}^{\tau} + a_\tau (\sum_{h=1}^n \sum_{l=1}^o \partial \rho_{ihl}(q_{ij}^{\tau}, s_{ijk}) \partial s_{ijkl}^{\tau} - \partial h_{ijk}(s_{ijk}^{\tau}) \partial s_{ijkl}^{\tau} + D_{ijkl})\}\}. $$

We now provide the convergence result. The proof is direct from Theorem 5.8 in Nagurney and Zhang (1996).

Theorem 2: Convergence

In the freight service provision game theory model developed above let $F(X) = -\nabla E(U(Q, s))$ be strictly monotone at any equilibrium pattern. Also, assume that $F$ is uniformly Lipschitz continuous. Then there exists a unique equilibrium high-value cargo shipment and security
level pattern \((q^*, s^*) \in K\) and any sequence generated by the Euler method as given by (19), with \(\{a_\tau\}\) satisfies \(\sum_{\tau=0}^\infty a_\tau = \infty, a_\tau > 0, a_\tau \to 0, \text{ as } \tau \to \infty\) converges to \((q^*, s^*)\).

We now apply the above Euler method to compute the high-value product shipments and security level investments in a series of numerical examples and sensitivity analysis. We implemented the algorithm in FORTRAN and used a LINUX system at the University of Massachusetts Amherst for the computations. The convergence criterion was that the absolute value of the difference of the cargo shipment and security level iterates at two successive iterations was less than or equal to \(10^{-5}\). All the variables (shipments and security levels) were initialized to 0.00. The sequence \(\{a_\tau\} = \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \ldots\}\).

**Example 1: One Freight Service Provider, One Shipper, and One Destination Node**

The first example consists of a single FSP (FSP 1), a single shipper, and a single destination, as in the network in Figure 2. The high-value cargo consists of precious metals, in units of pounds.

![Figure 2: Example 1: One Freight Service Provider, One Shipper, and One Destination Node](image.png)

The data are as follows. The total cost function is:

\[
\hat{c}_{111} = q_{111}^2 + 5q_{111},
\]

the demand price function is:

\[
\rho_{111} = -2q_{111} + 10s_{111} + 100,
\]
the upper bound on the security level is:

\[ \hat{s}_{111} = .99, \]

the upper bound on the cargo shipment is:

\[ \bar{q}_{111} = 100. \]

The damages, in order to reflect the high value of the cargo are:

\[ \$50,000, \]

so that, at a unit price of 500 and a maximum capacity of 100 for the shipment, we obtain $50,000.

The security investment cost function is as in (3), with \( \alpha_{111} = 10 \). This reflects that the freight service provider does not have much security to begin with and, hence, the \( \alpha_{111} \) is rather large.

The Euler method yields the equilibrium solution: \( q^*_{111} = 17.48 \) and \( s^*_{111} = .99 \). The demand price for shipping one unit, \( \rho_{111} \), evaluated at the equilibrium pattern, is 74.93. The expected utility of Freight Service Provider 1, \( E(U_1) \), is 327. FSP 1 invests in the maximum security level possible and still garners a positive expected utility.

**Example 2: Two Freight Service Providers, One Shipper, and One Destination Node**

Example 2 introduces a competitor to the market in the form of a second FSP, as depicted in Figure 3.

The data for FSP 1 remain as in Example 1 except that there is now a new demand price function due to competition.

The demand price functions for the FSPs are:

\[ \rho_{111} = -2q_{111} - q_{211} + 10s_{111} + 100, \quad \rho_{211} = -3q_{211} - 2q_{111} + 10s_{211} + 110. \]

Also, the total cost function for the second, new, FSP is:

\[ \hat{c}_{211} = .5q_{211}^2 + 5q_{211}. \]

The security investment cost function for FSP 2 is of the form (3) with \( \alpha_{211} = 10 \) and the upper bound on the cargo shipment \( \bar{q}_{211} = 120 \). The damage \( D_{211} = 40,000 \).
Figure 3: Example 2: Two Freight Service Providers, One Shipper, and One Destination

The Euler method converges to the following equilibrium shipment and security level pattern:

\[ q_{111}^* = 15.49, \quad q_{211}^* = 11.99, \quad s_{111}^* = .99, \quad s_{211}^* = .99. \]

The demand prices at the equilibrium solution are:

\[ \rho_{111} = 66.94, \quad \rho_{211} = 52.96. \]

FSP 1 now has an expected utility, \( E(U_1) = 129.36 \), whereas FSP 2 has an expected utility \( E(U_2) = 58.16 \). With increased competition, FSP 1 now has a lower expected utility than in Example 1. Moreover, FSP 1 now charges a lower price for high-value cargo shipment than he did in Example 1, when there was no competition. The total volume of shipments from the shipper origin node to the destination node increases. This may be viewed as the shipper diversifying his risk.

Example 3: Two Freight Service Providers, One Shipper, and Two Destination Nodes

Example 3 now introduces another destination node. Hence, in Example 3 there are two FSPs, one shipper, and two destination nodes, as depicted in Figure 4.

The data remain as in Example 2 but with new data added as per below.

The total cost functions that are added are:

\[ \hat{c}_{112} = 1.5q_{112}^2 + 5q_{112}, \quad \hat{c}_{212} = q_{212}^2 + 5q_{212}. \]
The added demand price functions are:

\[ \rho_{112} = -3q_{112} - q_{212} + 5s_{112} + 270, \quad \rho_{212} = -2q_{212} - q_{112} + 5s_{212} + 200. \]

At the new Destination Node 2, shippers are willing to pay more per unit of freight service provision, given the distance to Destination Node 2 and the more challenging transport environment.

The damages associate with transport to destination node 2 are:

\[ D_{112} = 5600, \quad D_{212} = 10000. \]

FSP 1 has purchased some insurance, as has FSP 2, so possible damages are lower for Destination Node 2 than for Destination Node 1.

The form of the investment cost functions is, again, as in (3) with

\[ \alpha_{112} = 12, \quad \alpha_{212} = 10. \]

The upper bounds on the high-value cargo shipments on the new links are:

\[ \bar{q}_{112} = 80, \quad \bar{q}_{212} = 100. \]

The Euler method converges to the equilibrium solution:

\[ q_{111}^* = 15.49, \quad q_{112}^* = 26.64, \quad q_{211}^* = 11.99, \quad q_{212}^* = 28.89, \]
\[ s_{111}^* = .99, \quad s_{112}^* = .46, \quad s_{211}^* = s_{212}^* = .99. \]

The demand prices at the computed equilibrium pattern are:

\[ \rho_{111} = 66.94, \quad \rho_{112} = 163.48, \quad \rho_{211} = 52.96, \quad \rho_{212} = 120.54. \]

The expected utilities of the freight service providers are now:

\[ E(U_1) = 237.83, \quad E(U_2) = 2371.25. \]

With a new destination node to ship the high-value cargo to, both FSPs garner enhanced expected utilities in comparison to their values in Example 2. FSP 2 especially benefits from the new destination node requiring freight service provision. The prices that are paid for the freight service provision at Destination Node 2 are more than double those paid for at Destination Node 1 to a given FSP. This is due to the fact that the fixed components (intercepts) of the demand price functions to the new destination are higher than to Destination Node 1, demonstrating that shippers are willing to pay a higher price for delivery to Destination Node 2. The quantities of the high-value cargo reaching Destination Node 2 are, thus, higher as well, and this is due to both the demand price functions and the total cost functions, which are lower to Destination Node 2 than to Destination Node 1.

FSP 2 provides maximum security levels for transportation for both destinations and earns a higher expected utility than does FSP 1 who has a security level about one half that at Destination Node 2 than at Destination Node 1. This is due, in part, to FSP 1’s lower damages as compared to those that would be accrued for FSP 2, given an attack, at Destination Node 2.

**Example 4: Two Freight Service Providers, Two Shippers, and Two Destination Nodes**

Example 4 is constructed from Example 3 and has the same data except that now we have an additional shipper who wishes to explore freight service provision from the two freight service providers. The underlying network is as in Figure 5.

The added data for Example 4 are below.

The total cost functions associated with the second shipper are:

\[ \hat{c}_{121} = q_{121}^2 + q_{121}, \quad \hat{c}_{122} = .5q_{122}^2 + q_{122}, \quad \hat{c}_{221} = q_{221}^2 + 2q_{221}, \quad \hat{c}_{222} = 1.5q_{222}^2 + 3q_{222}. \]
Figure 5: Example 4: Two Freight Service Providers, Two Shippers, and Two Destination Nodes

The demand price functions associated with transacting with the second shipper are:

\[ \rho_{121} = -2q_{121} - q_{221} + s_{121} + 150, \quad \rho_{122} = -3q_{122} - q_{222} + 2s_{122} + 130, \]
\[ \rho_{221} = -4q_{221} - q_{121} + 5s_{221} + 120, \quad \rho_{222} = -5q_{222} - 2q_{112} + 3s_{222} + 140. \]

As in all the previous examples, the security investment functions are as in (3) with the following coefficients for the new possible investments:

\[ \alpha_{121} = 5, \quad \alpha_{122} = 4, \quad \alpha_{221} = 3, \quad \alpha_{222} = 12. \]

The additional damage terms are:

\[ D_{121} = 20000, \quad D_{122} = 15000, \quad D_{221} = 25000, \quad D_{222} = 2000. \]

The upper bounds on the cargo shipments from the second shipper to the two destinations are:

\[ \bar{q}_{121} = 100, \quad \bar{q}_{122} = 80, \quad \bar{q}_{221} = 70, \quad \bar{q}_{222} = 60. \]

The Euler method converges to the following equilibrium shipment and security level pattern:

\[ q_{111}^* = 15.71, \quad q_{112}^* = 26.64, \quad q_{121}^* = 23.34, \quad q_{122}^* = 17.78, \]
\[ q_{211}^* = 10.65, \quad q_{212}^* = 28.89, \quad q_{221}^* = 9.96, \quad q_{222}^* = 6.56. \]
\[ s_{11}^* = .99, \quad s_{112}^* = .46, \quad s_{121}^* = .99, \quad s_{122}^* = .99, \]
\[ s_{211}^* = .99, \quad s_{212}^* = .99, \quad s_{221}^* = .99, \quad s_{222}^* = .00. \]

The demand prices incurred at the equilibrium pattern are:

\[ \rho_{111}^* = 67.83, \quad \rho_{112}^* = 163.48, \quad \rho_{121}^* = 94.35, \quad \rho_{122}^* = 72.10, \]
\[ \rho_{211}^* = 47.61, \quad \rho_{212}^* = 120.54, \quad \rho_{221}^* = 61.77, \quad \rho_{222}^* = 53.94. \]

The expected utilities of the freight service providers are: \( E(U^1) = 2567.49 \) and \( E(U^2) = 708.97. \)

With a second shipper node added, there is the potential for increased business for the two FSPs. Although FSP 1 now enjoys an expected utility that is more than tenfold higher than that in Example 3, FSP 2 experiences a high security investment cost function associated with Destination Node 2 and his security level associated with shipping from Shipper 2 to Destination Node 2 is .00 at the equilibrium. FSP 1 handles three times the volume of cargo from the two shippers to Destination Node 2. The lowest cargo shipment is \( q_{222}^* \) with security level \( s_{222}^* = .00. \)

### 3.1 Further Discussion

Our model and results capture the effects of interdependent security on the utilities of the FSPs with the underlying functions of the FSPs including general demand price functions and total cost functions associated with transportation. In Example 3, observe that the damages expected on the FSP 1, Shipper 1, and Destination Node 2 combination, \( D_{112} = 5600. \) Hence, the FSP is not keen in investing highly in security. Thus, \( s_{112}^* = 0.46. \) In spite of the link security not being as high as others, FSP 1 encounters a decent quantity flow of \( q_{112}^* = 26.64 \) (the highest being \( q_{212}^* = 28.89. \) The overall perception of the network’s security is on the higher side due to information asymmetry, leading to revenue benefits for FSP 1. The FSP is essentially free-riding. In Example 4, we observe a similar phenomenon. \( D_{222} = 2000 \) and perceptive benefits lead to absolutely no security investments in the case of FSP 2, Shipper 2, and Destination Node 2, \( s_{222}^* = 0. \) Yet, there is flow of cargo (although the lowest), \( q_{222}^* = 6.56, \) resulting in a revenue benefit of 353.85 from the link for FSP 2.

In the model, the damages cannot be modified by changing the investment costs; however, investments in security affect the expected damages, and the expected utility. This has an externality effect on FSPs since the benefit of investing in the Shipper 1, Destination Node 2 combination by FSP 1 can be reaped by Shipper 1, Destination Node 2 of FSP 2. This
is true since we define prices as $\rho_{ijk}(q, s)$. The prices encountered by FSP $i$ depend not just on its own security investments but on those of others as well. Even though $\alpha_{112} = 12$; in other words, this investment cost parameter is high and damages are low, $D_{112} = 5600$, giving no reason for FSP 1 to invest in the security of this combination, a security level of $s_{112}^* = 0.46$ is observed. While FSP 1 does free-ride here, there is overall market benefit since $\rho_{112} = 163.48$. Note that $q_{112}^* = 26.64$. The competitor, FSP 2, is only slightly more preferred with $\rho_{212} = 120.54$ and $q_{212}^* = 28.89$. The benefits from the lower investment cost factor of FSP 2, $\alpha_{212} = 10$, is reaped by FSP 1. In spite of not investing in security, it gets a high price for its goods.

To conclude, we suggest that heterogeneity and externality effects can contribute immensely to the dynamics of the equilibrium solutions with or without the influences of parameters such as $\alpha$ and $D$. Our model is multifaceted and, hence, realistic enough to capture these phenomena. When applied to specific situations, it can lead to additional significant managerial insights. The numerical results are for the demonstration of the computability of the model and allow for transparency and reproducibility of results since all the input and output data are provided. Especially since the framework is aimed at an audience that is free to alter the functional forms to serve their purposes, or policy makers that would like to see the implications of security investments in a freight network, our results are more implicative and directional than penetrative.

4. Summary and Conclusions

In this paper, we developed a game theory model in which freight service providers compete for business and also invest in security. The focus is on high-value cargo, which has been the target of attacks globally, from luxury items of clothing and jewelry to food and high tech products. Although there is a rich literature on supply chain risk and vulnerability, our focus is on freight security investment and competition and this paper fills the gap in the literature in several ways, which we itemize below.

1. We quantify security investment cost functions which may differ for distinct freight service provider/shipper/destination node combinations.

2. Shippers reveal their preferences and sensitivity to investments in security through the prices that they are will to pay for freight service provision and these also can be distinct for different freight service provider/shipper/destination node combinations.

3. The freight service providers seek to maximize their expected utilities, which capture the probability of an attack associated with different links and are a function of the security
level associated with that link. Hence, risk is also captured in the competitors’ objective functions.

4. The model is not limited to the number of freight service providers, shippers, and/or destination nodes.

5. The equilibrium conditions, which correspond to a Nash Equilibrium, are formulated as a variational inequality problem for which a solution is guaranteed to exist.

6. The model is computable and numerical examples reveal the equilibrium high-value cargo shipments plus security levels that the freight service providers deliver and invest in, respectively.

7. The model gives significant investment policy results that can help FSPs to adhere to their freight security initiatives while considering their competition, the shippers, and the demand markets in a high-value cargo context.

There is potential to extend the research in several directions. One may include multiple links or pathways from shipper nodes to destination nodes. One could also introduce other tiers in a supply chain network context such as manufacturers and also consider whether their investments in security may be worthwhile. Finally, the issue of security and freight service provision in disaster relief is also a timely topic. We leave such research for the future.

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