Supply Chain Network Competition in Price and Quality with Multiple Manufacturers and Freight Service Providers

Anna Nagurney, Sara Saberi, and Shivani Shukla
Department of Operations and Information Management
Isenberg School of Management
University of Massachusetts
Amherst, Massachusetts 01003
and
Jonas Floden
School of Business, Economics and Law
University of Gothenburg
Gothenburg, Sweden

December 2014; revised January 2015
Transportation Research E (2015), **77**, pp 248-267.

Abstract:

In this paper, we develop both static and dynamic supply chain network models with multiple manufacturers and freight service providers competing on price and quality. The manufacturers compete with one another in terms of price and quality of the product manufactured, whereas the freight service providers compete on price and quality of the transportation service they provide for multiple modes. Both manufacturers and freight service providers maximize their utilities (profits) while considering the consequences of the competitors' prices and quality levels. Bounds on prices and quality levels are included that have relevant policy-related implications. The governing equilibrium conditions of the static model are formulated as a variational inequality problem. The underlying dynamics are then described, with the stationary point corresponding to the variational inequality solution. An algorithm which provides a discrete-time adjustment process and tracks the evolution of the quality levels and prices over time is proposed, and convergence results given. Numerical examples illustrate how such a supply chain network framework, which is relevant to products ranging from high value to low value ones, can be applied in practice.

Keywords: supply chains, game theory, Nash equilibrium, manufacturing, freight services, quality, competition, variational inequalities, projected dynamical systems

1. Introduction

Manufacturers and freight service providers are fundamental decision-makers in today's globalized supply chain networks as products are produced and distributed to businesses and consumers, often traveling great distances via multiple modes of transportation. The decisions that the firms make affect the prices and quality of products as well as that of the freight services provided, which, in turn, impact their own profitability. It is well-known today that success is determined by how well the entire supply chain performs, rather than the performance of its individual entities. Quality and price have been identified empirically as critical factors in transport mode selection for product/goods delivery (cf. Floden, Barthel, and Sorkina (2010), Saxin, Lammgard, and Floden (2005), and the references therein). Quality has also become one of the most essential factors in the success of supply chains of various products, including food and agro-based products, other perishable products such as blood, pharmaceuticals, medical nuclear supply chains, durable manufactured products, including automobiles, high tech products, such as microprocessors, and even services associated with the Internet. Although the term quality in many freight studies suffers from a somewhat vague definition (cf. Meixell and Norbis (2008 for a discussion)), it, typically, encompasses factors such as on-time deliveries, reliability, frequency, and risk of damage (see also Danielis, Marcucci, and Rotaris (2005) and Zamparini, Layaa, and Dullaert (2011)).

The growth of intercontinental multi-channel distribution, containerization, and direct to business and direct to customer shipping has led to fierce competition among freight service providers who are subjected to pricing pressures and increased expectations to handle more complex services (Hakim (2014) and DHL (2014)). To maintain their competitive edge, freight service providers are increasingly focused on positioning themselves as more than just a commodity business. The providers may offer flexibility to meet customer needs of safety, and/or traceability and, furthermore, differentiate themselves from the rest of the competition, thereby migrating towards being more value-oriented than cost-oriented (Bowman (2014) and Glave, Joerss, and Saxon (2014)). The quality of service is driving logistics performance in both developed and emerging economies (Arvis et al. (2014)). Clearly, quality in freight service is gaining in importance.

Increasingly, tough customer demands are also putting the transport system under pressure. The online retailer Amazon.com recently submitted a patent (United States patent (2013)) for anticipatory shipping and speculative shipping, meaning that, based on advanced forecasts of customer behavior (previous purchases, behavior during homepage visits, demographics, etc.) they actually ship the products before the customer orders it! The product is shipped towards a region where a purchase is expected and is redirected during transport when the order is placed, thus, allowing almost instant deliveries (Bensinger (2014)). Transport owners that cannot offer the desired level of quality are forced to leave the market, as was the case when the intermodal company CargoNet withdrew from the Swedish rail market, claiming unreliable infrastructure as one of the main reasons (Floden and Woxenius (2013)).

Some of the pioneers in the study of product quality competition include: Akerlof (1970), Spence (1975), Sheshinski (1976), and Mussa and Rosen (1978), who discussed firms decisions on price and quality in a quality differentiated monopoly market with heterogeneous customers. Dixit (1979) and Gal-or (1983) initiated the study of quantity and quality competition in an oligopolistic market with multiple firms, where several symmetric cases of oligopolistic equilibria were considered. Brekke, Siciliani, and Straume (2010) investigated the relationship between competition and quality via a spatial price-quality competition model. Nagurney and Li (2014a) developed a dynamic model of Cournot-Nash oligopolistic competition with product differentiation and quality competition in a network framework. Further contributing to this work, Nagurney, Li, and Nagurney (2014) proposed a spatial price equilibrium model with information asymmetry in quality in both static and dynamic versions. Others who have added to research on the topic of quality competition in general include: Ronnen (1991), Banker, Khosla, and Sinha (1998), Johnson and Myatt (2003), and Acharyya (2005).

In this paper, we focus on the development of game theory models in both equilibrium and dynamic settings. We consider a supply chain network with multiple manufacturers and multiple freight service providers handling freight transportation. The decision-makers at each echelon compete in prices. Quality of the product is traced along the supply chain with consumers differentiating among the products offered by the manufacturers. Also, quality of freight service providers is accounted for in the model and the providers are shown to be competing on both price and quality. Heretofore, the integration of price and quality competitive behavior with both manufacturers and freight service providers has not been examined in a rigorous theoretical and computationally tractable framework.

Our framework is inspired, in part, by the work of Nagurney et al. (2013) and Saberi, Nagurney, and Wolf (2014). The latter proposed network economic game theory models of service-oriented Internet architectures with price and quality competition occurring between content and network providers. Here, we go further in that we allow for multiple modes of transportation and each freight service provider can have a different number of mode options. In addition, we consider a mode in a general way in that it can correspond to intermodal transportation. The former studied a network economic game theory model of a service-oriented Internet with choices and quality competition. For background on freight transportation modelling, we refer the reader to the books by Tavasszy and De Jong (2013) and Ben-Akiva, Meersman, and Can de Voorde (2013) and the references therein.

The new static and dynamic models in this paper also build on the work of Nagurney, Dong, and Zhang (2002), which introduced supply chain network equilibrium models but here the competition is in price and quality and not in quantities. See, also, the dynamic multilevel financial/informational/logistical framework of Nagurney et al. (2002), the supernetwork model with freight carriers in Yamada et al. (2011), and the maritime chain model with carriers, ports and shippers of Talley and Ng (2013). For a plethora of supply chain network equilibrium models, along with the underlying dynamics, see the book by Nagurney (2006). For an overview of projected dynamical systems, which is the methodology that we utilize to describe the underlying competitive dynamics and the evolution of prices and quality, see Nagurney and Zhang (1996). However, none of the above multitiered competitive supply chain network equilibrium models with freight service provider behavior captured quality in transportation as well as in production. An extensive review of the overall supply chain network design literature has been provided by Farahani et al. (2014). Our framework is not in the context of supply chain network design; for an extensive review of the overall supply chain network design literature, see Farahani et al. (2014).

Our contributions to the existing literature are:

- We model explicit competition among manufacturing firms and freight service providers (carriers) in terms of prices and quality of the products that the firms offer and the prices and quality of the freight services provided. This multi-faceted inclusion of competition from price and quality dimensions leads to results that not just quantify quality at the product and service ends, but also helps to assess the trade-offs between quality and costs at each echelon of the supply chain that ultimately influences the demand. A model that considers oligopolistic competition among manufacturers and freight service providers under price and quality with multiple modes of transportation and non-separable, nonlinear, and asymmetric demand and cost functions is constructed for the first time with this paper.

- The analysis for freight service providers contains price and quality evaluations for multiple modes of transportation. The transportation costs, resultantly, differ by mode, leading to a pertinent evaluation of quality vs. costs for the freight service providers and the modes of transportation that they offer to the customers. In our frame of reference, modes can also imply intermodal transportation of products.

- We handle heterogeneity in the providers' cost functions and in the consumers' demands and do not limit ourselves to specific functional forms. Utility of each manufacturing firm considers price and quality for not just his own products, but that of other manufacturing firms as well. Similarly, the utility of each freight service provider includes the implications of other providers' prices and quality for various modes in addition to his own. Also, we impose bounds on the prices and quality levels with positive minimum quality levels corresponding to minimum quality standards, relevant for policy-making.

- We provide qualitative properties of the equilibrium price and quality pattern and also present the underlying dynamics associated with the evolution of the prices and quality levels over time until the equilibrium is achieved.

- The theoretical framework is supported by a rigorous algorithm that is well-suited for implementation.

- The computational scheme is applied to a spectrum of numerical examples in order to illustrate the generality of the framework. Specifically, we provide complete imput and output data for 5 examples and 3 variants, for a total of 8 examples.

The structure of the paper is as follows. Section 2 presents the multitiered supply chain network game theory model with manufacturers and freight service providers. We capture the firms' behavior that accounts for the prices and quality levels of the products at the demand markets. In parallel, we model freight service providers' behavior that deals with the prices and quality levels of their services for various modes. The freight service providers compete in terms of price and quality that differ by mode. A variational inequality formulation is derived, which unifies the firms' and freight service providers' behaviors. An existence result for a solution to the unified variational inequality formulation (cf. Nagurney (1999)) is also given. A projected dynamical systems model is, subsequently, constructed in Section 3 to capture the underlying dynamics of the competitive behavior. In Section 4, we present an algorithm for solving the proposed variational inequality formulation, accompanied by convergence results. At each iteration, the algorithm yields closed form expressions for the prices and qualities of the firms and freight service providers. It also serves as a time-discretization of the continuous time adjustment processes in prices and quality levels. Section 5 illustrates the model and the computational algorithm through several numerical examples in order to gain managerial insights. In Section 6, we summarize our results and present our conclusions.

2. The Supply Chain Network Model with Price and Quality Competition

In the supply chain network there are N manufacturing firms involved in the production of substitutable products that are transported by O freight service providers or carriers to Q demand markets. We denote a typical manufacturing firm by F_i; $i = 1, \ldots, N$, a typical freight service provider by C_j; $j = 1, \ldots, O$, and a typical demand market by k; $k = 1, \ldots, Q$. Each freight service provider C_j; $j = 1, \ldots, O$ has M_j possible modes of transport/shipment, associated with which is also a distinct quality. The modes of shipment may include rail, air, truck, sea, or even bicycles for last mile deliveries, etc. Moreover, for the sake of modeling flexibility and generality, a *mode* in our framework may represent a composition of modes as in the case of intermodal transportation. The freight service providers are responsible for picking up the products at the manufacturers and delivering them to consumers at the demand markets. Note that each freight service provider may have a different number of modes available to him based on vehicle ownership and access, contracts, prior relationships, geographical issues, etc. The supply chain network representation of our game theory model is depicted in Figure 1. The manufacturing firms compete with one another as do the freight service providers.

Firm F_i manufactures a product of quality q_i at the price p_i. As in Nagurney and Li (2014c), we define and quantify quality as the quality conformance level, that is, the degree to which a specific product conforms to a design or specification (Gilmore (1974), Juran and Gryna (1988)). We group the prices of all firms' products into the vector $p_F \in R^N_+$, and their quality levels into the vector $q_F \in R^N_+$.

The quality and price associated with freight service provider C_j retrieving the product from firm F_i and delivering it to demand market k via mode m are denoted, respectively, by q^m_{ijk} and p^m_{ijk}; $i = 1, \ldots, N$; $j = 1, \ldots, O$; $k = 1, \ldots, Q$; $m = 1, \ldots, M_j$. Quality with respect to freight in our model corresponds to level of service as emphasized by Mancera, Bruckmann, and Weidmann (2013). We group these quality levels and prices into the vectors $q_C \in R_+^{NOQ\sum_{j=1}^{O} M_j}$ and $p_C \in R_+^{NOQ\sum_{j=1}^{O} M_j}$.

The consumers at demand market k; $k = 1, \ldots, Q$, reveal their preferences for firm F_i's product transported by freight service provider C_j via mode m through a demand function d^m_{ijk}. The demand d^m_{ijk} depends not only on the price and quality of firm F_i's product, but also, in general, on the prices and quality levels of all other substitutable products as well as on the prices and quality levels associated with transportation:

$$d^m_{ijk} = d^m_{ijk}(p_F, q_F, p_C, q_C), \quad i = 1, \ldots, N; j = 1, \ldots, O; k = 1, \ldots, Q; m = 1, \ldots, M_j. \quad (1)$$

The generality of the demand functions allows for the modeling of competition on the demand

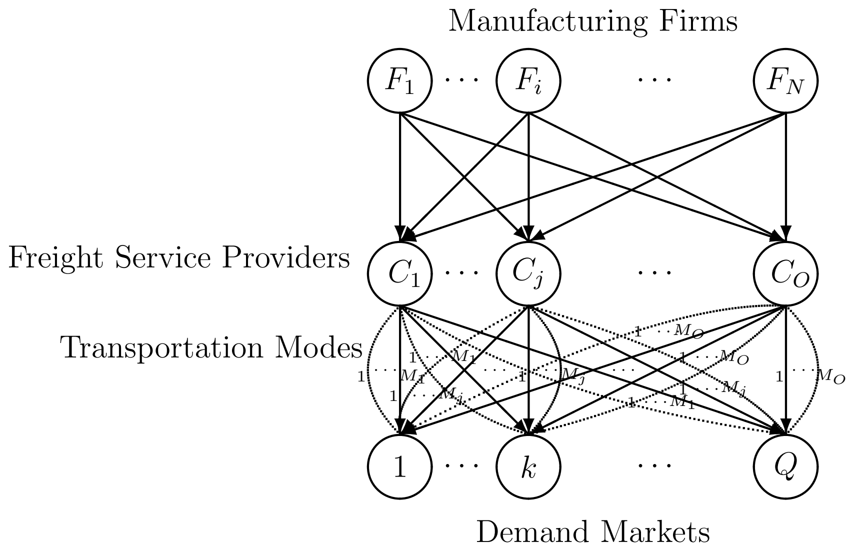

Figure 1: The Supply Chain Network Structure of the Game Theory Model

side for the products and freight service provision. We expect that the demand d_{ijk}^m will increase (decrease) as the price (quality) of firm F_i's product or the shipment price (quality) of freight service provider C_j decreases. We group the demands into the $NOQ\sum_{j=1}^{O} M_j$-dimensional vector $d(p_F, q_F, p_C, q_C)$.

2.1 The Firms' Behavior

The supply of firm F_i's product, s_i, is equal to the demand, that is,

$$s_i(p_F, q_F, p_C, q_C) = \sum_{k=1}^{Q}\sum_{j=1}^{O}\sum_{m=1}^{M_j} d_{ijk}^m(p_F, q_F, p_C, q_C), \qquad i = 1, \ldots, N, \tag{2}$$

since we expect the markets to clear.

The production cost of firm F_i, PC_i, depends, in general, upon the entire production (supply) pattern, as well as on the product quality levels, that is:

$$PC_i = PC_i\big(s_F(p_F, q_F, p_C, q_C), q_F\big), \qquad i = 1, \ldots, N, \tag{3}$$

where $s_F(p_F, q_F, p_C, q_C) \in R_+^N$ is the vector of all the supplies of the products. The generality of the production cost functions allows us to capture competition for resources in manufacturing, whether natural, human, and/or capital.

The utility of firm F_i, U_{F_i}; $i = 1, \ldots, N$, represents his profit, and is the difference between the firm's revenue and the production cost:

$$U_{F_i}(p_F, q_F, p_C, q_C) = p_i\Big[\sum_{k=1}^{Q}\sum_{j=1}^{O}\sum_{m=1}^{M_j} d_{ijk}^m(p_F, q_F, p_C, q_C)\Big] - PC_i\big(s_F(p_F, q_F, p_C, q_C), q_F\big). \tag{4}$$

Each firm F_i is faced with a nonnegative lower bound $\underline{q}_i$ on the quality of his product as well as an upper bound $\bar{q}_i$, so that

$$\underline{q}_i \leq q_i \leq \bar{q}_i, \quad i = 1, \ldots, N. \tag{5}$$

Typically, $\bar{q}_i = 100$ corresponds to perfect quality conformance as discussed in Nagurney and Li (2014b). If that is not achievable by a firm, then the upper bound would be set to a lower value. Also, a positive lower bound $\underline{q}_i$ corresponds to a minimum quality standard as discussed in Nagurney and Li (2014c).

In addition, each firm F_i is faced with an upper bound on the price that he charges for his product, that is,

$$0 \leq p_i \leq \bar{p}_i, \quad i = 1, \ldots, N. \tag{6}$$

The price that firm F_i charges and his quality level correspond to his strategic variables in the competitive game.

Let K_i^1 denote the feasible set corresponding to F_i, where $K_i^1 \equiv \{(p_{F_i}, q_{F_i}) \mid (5) \text{ and } (6) \text{ hold}\}$. We define: $K^1 \equiv \prod_{i=1}^N K_i^1$. We assume that all the above functions are continuous and continuously differentiable.

The manufacturers compete in a noncooperative manner which we formalize in Section 2.3.

2.2 The Freight Service Providers' Behavior

Recall that freight service provider C_j transports a product from firm F_i to demand market k via mode m at a quality level q_{ijk}^m at a unit price of p_{ijk}^m. We group the quality levels of freight service provider C_j into the vector $q_{C_j} \in R_+^{NQM_j}$ and his prices into the vector $p_{C_j} \in R_+^{NQM_j}$. These are his strategic variables.

We denote the transportation cost between firm F_i and demand market k via mode m of freight service provider C_j by TC_{ijk}^m and assume that:

$$TC_{ijk}^m = TC_{ijk}^m\big(d(p_F, q_F, p_C, q_C), q_C\big), \; i = 1, \ldots, N; j = 1, \ldots, O; k = 1, \ldots, Q; m = 1, \ldots, M_j, \tag{7}$$

that is, the transportation cost may depend, in general, on the vector of demands and the vector of quality levels of the freight service providers. In the transportation costs we also include handling costs associated with, for example, loading and unloading and, perhaps, also, storage of the products over a period of time.

The utility or profit function of freight service provider C_j, U_{C_j}, is the difference between his revenue and his transportation costs:

$$U_{C_j}(p_F, q_F, p_C, q_C) = \sum_{i=1}^{N}\sum_{k=1}^{O}\sum_{m=1}^{M_j}\left[p^m_{ijk} d^m_{ijk}(p_F, q_F, p_C, q_C)\right] - \sum_{i=1}^{N}\sum_{k=1}^{Q}\sum_{m=1}^{M_j} TC^m_{ijk}\big(d(p_F, q_F, p_C, q_C), q_C\big). \tag{8}$$

Each C_j; $j = 1, \ldots, O$, is faced with a lower and upper bound on the quality of transport shipment $\underline{q}^m_{ijk}$, $\bar{q}^m_{ijk}$, respectively, and an upper bound for price, $\bar{p}^m_{ijk}$, between i and k so that

$$\underline{q}^m_{ijk} \leq q^m_{ijk} \leq \bar{q}^m_{ijk}, \quad i = 1, \ldots, N; k = 1, \ldots, Q; m = 1, \ldots, M_j, \tag{9}$$

$$0 \leq p^m_{ijk} \leq \bar{p}^m_{ijk}, \quad i = 1, \ldots, N; k = 1, \ldots, Q; m = 1, \ldots, M_j. \tag{10}$$

The freight service provider lower bounds are assumed to be nonnegative as in the case of product quality with a positive value corresponding to a minimum quality standard.

Let K^2_j denote the feasible set corresponding to C_j, where $K^2_j \equiv \{(p_{C_j}, q_{C_j}) \mid (9) \text{ and } (10) \text{ hold}\}$. We then define $K^2 \equiv \prod_{j=1}^{O} K^2_j$. We assume that all the above functions associated with the freight service providers are continuous and continuously differentiable.

The freight service providers also compete in a noncooperative manner, as per below.

2.3 The Nash Equilibrium Conditions and Variational Inequality Formulation

We now present the Nash (1950, 1951) equilibrium definition that captures the decision-makers' competitive behavior in our model.

Definition 1: Nash Equilibrium in Prices and Quality Levels

A price and quality level pattern $(p^*_F, q^*_F, p^*_C, q^*_C) \in K^3 \equiv \prod_{i=1}^{N} K^1_i \times \prod_{j=1}^{O} K^2_j$, is said to constitute a Nash equilibrium if for each firm F_i; $i = 1, \ldots, N$:

$$U_{F_i}(p^*_i, \hat{p^*_i}, q^*_i, \hat{q^*_i}, p^*_C, q^*_C) \geq U_{F_i}(p_i, \hat{p^*_i}, q_i, \hat{q^*_i}, p^*_C, q^*_C), \quad \forall (p_i, q_i) \in K^1_i, \tag{11}$$

where

$$\hat{p^*_i} \equiv (p^*_1, \ldots, p^*_{i-1}, p^*_{i+1}, \ldots, p^*_N) \text{ and } \hat{q^*_i} \equiv (q^*_1, \ldots, q^*_{i-1}, q^*_{i+1}, \ldots, q^*_N), \tag{12}$$

and if for each freight service provider C_j; $j = 1, \ldots, O$:

$$U_{C_j}(p^*_F, q^*_F, p^*_{C_j}, \hat{p^*_{C_j}}, q^*_{C_j}, \hat{q^*_{C_j}}) \geq U_{C_j}(p^*_F, q^*_F, p_{C_j}, \hat{p^*_{C_j}}, q_{C_j}, \hat{q^*_{C_j}}), \quad \forall (p_{C_j}, q_{C_j}) \in K^2_j, \tag{13}$$

where

$$\hat{p^*_{C_j}} \equiv (p^*_{C_1}, \ldots, p^*_{C_{j-1}}, p^*_{C_{j+1}}, \ldots, p^*_{C_O}) \text{ and } \hat{q^*_{C_j}} \equiv (q^*_{C_1}, \ldots, q^*_{C_{j-1}}, q^*_{C_{j+1}}, \ldots, q^*_{C_O}). \tag{14}$$

According to (11) and (13), a Nash equilibrium is established if no decision-maker, whether a manufacturing firm or freight service provider, can unilaterally improve upon his profits by selecting an alternative vector of prices and quality levels.

We assume that the above utility functions are concave. Under our previously imposed assumptions on the production cost, transportation cost, and demand functions, we know that the utility functions are continuous and continuously differentiable. We now derive the variational inequality formulation of the governing equilibrium conditions.

Theorem 1: Variational Inequality Formulations of Nash Equilibrium in Prices and Quality

Assume that the manufacturing firms' and freight service providers' utility functions are concave, continuous, and continuously differentiable. Then $(p_F^*, q_F^*, p_C^*, q_C^*) \in \mathcal{K}^3$ *is a Nash equilibrium according to Definition 1 if and only if it satisfies the variational inequality:*

$$-\sum_{i=1}^{N} \frac{\partial U_{F_i}(p_F^*, q_F^*, p_C^*, q_C^*)}{\partial p_i} \times (p_i - p_i^*) - \sum_{i=1}^{N} \frac{\partial U_{F_i}(p_F^*, q_F^*, p_C^*, q_C^*)}{\partial q_i} \times (q_i - q_i^*)$$

$$-\sum_{j=1}^{O}\sum_{i=1}^{N}\sum_{k=1}^{Q}\sum_{m=1}^{M_j} \frac{\partial U_{C_j}(p_F^*, q_F^*, p_C^*, q_C^*)}{\partial p_{ijk}^m} \times (p_{ijk}^m - p_{ijk}^{m*})$$

$$-\sum_{j=1}^{O}\sum_{i=1}^{N}\sum_{k=1}^{Q}\sum_{m=1}^{M_j} \frac{\partial U_{C_j}(p_F^*, q_F^*, p_C^*, q_C^*)}{\partial q_{ijk}^m} \times (q_{ijk}^m - q_{ijk}^{m*}) \geq 0, \quad \forall (p_F, q_F, p_C, q_C) \in \mathcal{K}^3, \tag{15}$$

or, equivalently,

$$\sum_{i=1}^{N}\Bigg[\sum_{l=1}^{N} \frac{\partial PC_i\big(s_F(p_F^*, q_F^*, p_C^*, q_C^*), q_F^*\big)}{\partial s_l} \times \frac{\partial s_l(p_F^*, q_F^*, p_C^*, q_C^*)}{\partial p_i}$$

$$-\sum_{j=1}^{O}\sum_{k=1}^{Q}\sum_{m=1}^{M_j} d_{ijk}^m(p_F^*, q_F^*, p_C^*, q_C^*) - p_i^* \sum_{j=1}^{O}\sum_{k=1}^{Q}\sum_{m=1}^{M_j} \frac{\partial d_{ijk}^m(p_F^*, q_F^*, p_C^*, q_C^*)}{\partial p_i}\Bigg] \times (p_i - p_i^*)$$

$$+\sum_{i=1}^{N}\Bigg[\sum_{l=1}^{N} \frac{\partial PC_i\big(s_F(p_F^*, q_F^*, p_C^*, q_C^*), q_F^*\big)}{\partial s_l} \times \frac{\partial s_l(p_F^*, q_F^*, p_C^*, q_C^*)}{\partial q_i}$$

$$+\frac{\partial PC_i\big(s_F^*, q_F^*\big)}{\partial q_i} - p_i^* \sum_{j=1}^{O}\sum_{k=1}^{Q}\sum_{m=1}^{M_j} \frac{\partial d_{ijk}^m(p_F^*, q_F^*, p_C^*, q_C^*)}{\partial q_i}\Bigg] \times (q_i - q_i^*)$$

$$+\sum_{j=1}^{O}\sum_{i=1}^{N}\sum_{k=1}^{Q}\sum_{m=1}^{M_j}\Bigg[\sum_{l=1}^{N}\sum_{s=1}^{Q}\sum_{t=1}^{M_j}\Bigg[\sum_{r=1}^{N}\sum_{v=1}^{O}\sum_{w=1}^{Q}\sum_{z=1}^{M_v} \frac{\partial TC_{ljs}^t(d(p_F^*, q_F^*, p_C^*, q_C^*), q_C^*)}{\partial d_{rvw}^z} \times \frac{\partial d_{rvw}^z(p_F^*, q_F^*, p_C^*, q_C^*)}{\partial p_{ijk}^m}\Bigg]$$

$$-d_{ijk}^m(p_F^*,q_F^*,p_C^*,q_C^*)-\sum_{l=1}^N\sum_{s=1}^Q\sum_{t=1}^{M_j}\frac{\partial d_{ljs}^t(p_F^*,q_F^*,p_C^*,q_C^*)}{\partial p_{ijk}^m}\times p_{ljs}^{t*}\Bigg]\times(p_{ijk}^m-p_{ijk}^{m*})$$

$$+\sum_{j=1}^O\sum_{i=1}^N\sum_{k=1}^Q\sum_{m=1}^{M_j}\Bigg[\sum_{l=1}^N\sum_{s=1}^Q\sum_{t=1}^{M_j}\Bigg[\sum_{r=1}^N\sum_{v=1}^O\sum_{w=1}^Q\sum_{z=1}^{M_v}\frac{\partial TC_{ljs}^t(d(p_F^*,q_F^*,p_C^*,q_C^*),q_C^*)}{\partial d_{rvw}^z}\times\frac{\partial d_{rvw}^z(p_F^*,q_F^*,p_C^*,q_C^*)}{\partial q_{ijk}^m}\Bigg]$$

$$+\sum_{l=1}^N\sum_{s=1}^Q\sum_{t=1}^{M_j}\frac{\partial TC_{ljs}^t(d^*,q_C^*)}{\partial q_{ijk}^m}-\sum_{l=1}^N\sum_{s=1}^Q\sum_{t=1}^{M_j}\frac{\partial d_{ljs}^t(p_F^*,q_F^*,p_C^*,q_C^*)}{\partial q_{ijk}^m}\times p_{ljs}^{t*}\Bigg]\times(q_{ijk}^m-q_{ijk}^{m*})\geq 0,$$

$$\forall(p_F,q_F,p_C,q_C)\in\mathcal{K}^3, \qquad (16)$$

where $s_F^*\equiv s_F(p_F^*,q_F^*,p_C^*,q_C^*)$ *and* $d^*\equiv d(p_F^*,q_F^*,p_C^*,q_C^*)$.

Proof: The feasible set $\mathcal{K}^3$, underlying both variational inequalities (15) and (16) is convex since it consists of the box-type constraints (5), (6), and (9), (10). (15) then follows from Gabay and Moulin (1980) and Dafermos and Nagurney (1987). In order to obtain (16) from (15), for each i we have:

$$-\frac{\partial U_{F_i}}{\partial p_i}=\sum_{l=1}^N\frac{\partial PC_i}{\partial s_l}\times\frac{\partial s_l}{\partial p_i}-\sum_{j=1}^O\sum_{k=1}^Q\sum_{m=1}^{M_j}d_{ijk}^m-p_i\sum_{j=1}^O\sum_{k=1}^Q\sum_{m=1}^{M_j}\frac{\partial d_{ijk}^m}{\partial p_i}, \qquad (17)$$

$$-\frac{\partial U_{F_i}}{\partial q_i}=\sum_{l=1}^N\frac{\partial PC_i}{\partial s_l}\times\frac{\partial s_l}{\partial q_i}+\frac{\partial PC_i}{\partial q_i}-p_i\sum_{j=1}^O\sum_{k=1}^Q\sum_{m=1}^{M_j}\frac{\partial d_{ijk}^m}{\partial q_i}, \qquad (18)$$

and, for each i,j,k and m, we have:

$$-\frac{\partial U_{C_j}}{\partial p_{ijk}^m}=\sum_{l=1}^N\sum_{s=1}^Q\sum_{t=1}^{M_j}\Big[\sum_{r=1}^N\sum_{v=1}^O\sum_{w=1}^Q\sum_{z=1}^{M_v}\frac{\partial TC_{ljs}^t}{\partial d_{rvw}^z}\times\frac{\partial d_{rvw}^z}{\partial p_{ijk}^m}\Big]$$

$$-d_{ijk}^m-\sum_{l=1}^N\sum_{s=1}^Q\sum_{t=1}^{M_j}\frac{\partial d_{ljs}^t}{\partial p_{ijk}^m}\times p_{ljs}^t, \qquad (19)$$

$$-\frac{\partial U_{C_j}}{\partial q_{ijk}^m}=\sum_{l=1}^N\sum_{s=1}^Q\sum_{t=1}^{M_j}\Big[\sum_{r=1}^N\sum_{v=1}^O\sum_{w=1}^Q\sum_{z=1}^{M_v}\frac{\partial TC_{ljs}^t}{\partial d_{rvw}^z}\times\frac{\partial d_{rvw}^z}{\partial q_{ijk}^m}\Big]$$

$$+\sum_{l=1}^N\sum_{s=1}^Q\sum_{t=1}^{M_j}\frac{\partial TC_{ljs}^t}{\partial q_{ijk}^m}-\sum_{l=1}^N\sum_{s=1}^Q\sum_{t=1}^{M_j}\frac{\partial d_{ljs}^t}{\partial q_{ijk}^m}\times p_{ljs}^t. \qquad (20)$$

Substituting expressions (17) – (20) into (15) yields variational inequality (16). □

We now put the above Nash equilibrium problem into standard variational inequality form (see Nagurney (1999)) that is: determine $X^*\in\mathcal{K}$ where X is a vector in R^n , $F(X)$ is a continuous function such that $F(X): X\mapsto\mathcal{K}\subset R^n$, and

$$\langle F(X^*),X-X^*\rangle\geq 0, \qquad \forall X\in\mathcal{K}, \qquad (21)$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in n-dimensional Euclidean space. We set $\mathcal{K} \equiv \mathcal{K}^3$, which is a closed and convex set, and $n = 2N + 2(NOQ\sum_{j=1}^{O} M_j)$. We define the vector $X \equiv (p_F, q_F, p_C, q_C)$ and $F(X) \equiv (F_{p_F}, F_{q_F}, F_{p_C}, F_{q_C})$ with the i-th component of F_{p_F} and F_{q_F} given, respectively, by:

$$F_{p_i} = -\frac{\partial U_{F_i}}{\partial p_i}, \tag{22}$$

$$F_{q_i} = -\frac{\partial U_{F_i}}{\partial q_i}, \tag{23}$$

and the (i, j, k, m)-th component of F_{p_C} and F_{q_C}, respectively, given by:

$$F_{p^m_{ijk}} = -\frac{\partial U_{C_j}}{\partial p^m_{ijk}}, \tag{24}$$

$$F_{q^m_{ijk}} = -\frac{\partial U_{C_j}}{\partial q^m_{ijk}}. \tag{25}$$

Then, clearly, variational inequality (16) can be put into standard form (21).

Theorem 2: Existence of a Solution

A solution to variational inequality (15), equivalently, (16), exists.

Proof: The feasible set $\mathcal{K}^3$ is convex and compact since it consists of box-type constraints (5), (6), and (9), (10), which are bounded below and above, resulting in bounded prices and quality levels for both manufacturers and freight service providers. Existence of a solution to variational inequality (15), equivalently, variational inequality (16), is, thus, guaranteed since the feasible set $\mathcal{K}$ is compact and the function $F(X)$ (cf. (21)) in our model is continuous, under the assumptions made on the underlying functions (see Kinderlehrer and Stampacchia (1980)). $\square$

3. The Dynamics

We now propose dynamic adjustment processes for the evolution of the firms' product prices and quality levels and those of the freight service providers (carriers). Each manufacturing firm adjusts the prices and quality of his products in a direction that maximizes his utility while maintaining the price and quality bounds. Also, each freight service provider adjusts his prices and quality levels in order to maximize his utility while keeping the prices and quality levels within their minimum and maximum levels. This kind of behavior, as we show below, yields a *projected dynamical system.* We, also, demonstrate that the stationary point of the projected dynamical system coincides with the solution of the variational inequality governing the Nash equilibrium of the supply chain network model introduced in

Section 2. Hence, the adjustment processes provide a reasonable economic and behavioral description of the underlying competitive interactions.

For a current price and quality level pattern at time t, $X(t) = \big(p_F(t), q_F(t), p_C(t), q_C(t)\big)$, $-F_{p_i}(X(t)) = \frac{\partial U_{F_i}\left(p_F(t), q_F(t), p_C(t), q_C(t)\right)}{\partial p_i}$, given by (22), is the marginal utility (profit) of firm F_i with respect to the price that he charges for his product, $-F_{q_i}(X(t)) = \frac{\partial U_{F_i}\left(p_F(t), q_F(t), p_C(t), q_C(t)\right)}{\partial q_i}$, defined in (23), is the marginal utility of firm F_i with respect to the quality of his product, and $-F_{p^m_{ijk}}(X(t)) = \frac{\partial U_{C_j}\left(p_F(t), q_F(t), p_C(t), q_C(t)\right)}{\partial p^m_{ijk}}$, given by (24), and $-F_{q^m_{ijk}}(X(t)) = \frac{\partial U_{C_j}\left(p_F(t), q_F(t), p_C(t), q_C(t)\right)}{\partial q^m_{ijk}}$, defined in (25), are, respectively, the marginal utility of freight service provider C_j with respect to price and with respect to quality of shipment, from manufacturing firm F_i to demand market k by mode m. In this framework, the rate of change of the price that firm F_i charges is in proportion to $-F_{p_i}(X)$, as long as the price p_i is positive and less than $\bar{p}_i$. Namely, when $0 < p_i < \bar{p}_i$, then

$$\dot{p}_i = \frac{\partial U_{F_i}(p_F, q_F, p_C, q_C)}{\partial p_i}, \tag{26}$$

where $\dot{p}_i$ denotes the rate of change of p_i. However, when $\frac{\partial U_{F_i}(p_F, q_F, p_C, q_C)}{\partial p_i} \leq 0$ or $\frac{\partial U_{F_i}(p_F, q_F, p_C, q_C)}{\partial p_i} \geq \bar{p}_i$, constraint (6) forces the price to remain zero or equal to $\bar{p}_i$, hence

$$\dot{p}_i = \max\big\{0, \min\{\frac{\partial U_{F_i}(p_F, q_F, p_C, q_C)}{\partial p_i}, \bar{p}_i\}\big\}. \tag{27}$$

We may write (26) and (27) concisely as:

$$\dot{p}_i = \begin{cases} \frac{\partial U_{F_i}(p_F, q_F, p_C, q_C)}{\partial p_i}, & \text{if} \quad 0 < p_i < \bar{p}_i \\ \max\big\{0, \min\{\frac{\partial U_{F_i}(p_F, q_F, p_C, q_C)}{\partial p_i}, \bar{p}_i\}\big\}, & \text{if} \quad p_i = 0 \text{ or } p_i = \bar{p}_i. \end{cases} \tag{28}$$

The rate of change of the product quality of firm F_i, in turn, is in proportion to $-F_{q_i}(X)$, if $\underline{q}_i < q_i < \bar{q}_i$, so that

$$\dot{q}_i = \frac{\partial U_{F_i}(p_F, q_F, p_C, q_C)}{\partial q_i}, \tag{29}$$

where $\dot{q}_i$ denotes the rate of change of q_i. However, when $\frac{\partial U_{F_i}(p_F, q_F, p_C, q_C)}{\partial q_i} \leq \underline{q}_i$ or $\frac{\partial U_{F_i}(p_F, q_F, p_C, q_C)}{\partial q_i} \geq \bar{q}_i$, constraint (5) forces the quality level to remain at least $\underline{q}_i$ or at most $\bar{q}_i$, respectively. Therefore,

$$\dot{q}_i = \max\big\{\underline{q}_i, \min\{\frac{\partial U_{F_i}(p_F, q_F, p_C, q_C)}{\partial q_i}, \bar{q}_i\}\big\}. \tag{30}$$

Combining (29) and (30), we may write:

$$\dot{q}_i = \begin{cases} \frac{\partial U_{F_i}(p_F, q_F, p_C, q_C)}{\partial q_i}, & \text{if} \quad \underline{q}_i < q_i < \bar{q}_i \\ \max\big\{\underline{q}_i, \min\{\frac{\partial U_{F_i}(p_F, q_F, p_C, q_C)}{\partial q_i}, \bar{q}_i\}\big\}, & \text{if} \quad q_i = \underline{q}_i \text{ or } q_i = \bar{q}_i. \end{cases} \tag{31}$$

The rate of change of price p^m_{ijk}, in turn, that freight service provider C_j charges demand market k to ship the product from firm F_i via mode m, is in proportion to $-F_{p^m_{ijk}}$, as long as $0 < p^m_{ijk} < \bar{p}^m_{ijk}$, so that

$$\dot{p}^m_{ijk} = \frac{\partial U_{C_j}(p_F, q_F, p_C, q_C)}{\partial p^m_{ijk}}, \tag{32}$$

where $\dot{p}^m_{ijk}$ is the rate of change of p^m_{ijk}. Otherwise, constraint (10) forces the price to be zero or at most equal to $\bar{p}^m_{ijk}$. Thus,

$$\dot{p}^m_{ijk} = \max\{0, \min\{\frac{\partial U_{C_j}(p_F, q_F, p_C, q_C)}{\partial p^m_{ijk}}, \bar{p}^m_{ijk}\}\}. \tag{33}$$

We can write (32) and (33) compactly as:

$$\dot{p}^m_{ijk} = \begin{cases} \frac{\partial U_{C_j}(p_F, q_F, p_C, q_C)}{\partial p^m_{ijk}}, & \text{if} \quad 0 < p^m_{ijk} < \bar{p}^m_{ijk} \\ \max\{0, \min\{\frac{\partial U_{C_j}(p_F, q_F, p_C, q_C)}{\partial p^m_{ijk}}, \bar{p}^m_{ijk}\}\}, & \text{if} \quad p^m_{ijk} = 0 \text{ or } p^m_{ijk} = \bar{p}^m_{ijk}. \end{cases} \tag{34}$$

Finally, the rate of change of q^m_{ijk}, which is given by $\dot{q}^m_{ijk}$, is in proportion to $-F_{q^m_{ijk}}$, while the quality of mode m of freight service provider C_j for shipping the product from firm F_i to demand market k, q^m_{ijk}, is more than his lower bound and less than his upper bound. In other words, when $\underline{q}^m_{ijk} < q^m_{ijk} < \bar{q}^m_{ijk}$, we have

$$\dot{q}^m_{ijk} = \frac{\partial U_{C_j}(p_F, q_F, p_C, q_C)}{\partial q^m_{ijk}}, \tag{35}$$

otherwise:

$$\dot{q}^m_{ijk} = \max\{\underline{q}^m_{ijk}, \min\{\frac{\partial U_{C_j}(p_F, q_F, p_C, q_C)}{\partial q^m_{ijk}}, \bar{q}^m_{ijk}\}\}. \tag{36}$$

Combining (35) and (36), the quality level q^m_{ijk} evolves according to

$$\dot{q}^m_{ijk} = \begin{cases} \frac{\partial U_{C_j}(p_F, q_F, p_C, q_C)}{\partial q^m_{ijk}}, & \text{if} \quad \underline{q}^m_{ijk} < q^m_{ijk} < \bar{q}^m_{ijk} \\ \max\{\underline{q}^m_{ijk}, \min\{\frac{\partial U_{C_j}(p_F, q_F, p_C, q_C)}{\partial q^m_{ijk}}, \bar{q}^m_{ijk}\}\}, & \text{if} \quad q^m_{ijk} = \underline{q}^m_{ijk} \text{ or } q^m_{ijk} = \bar{q}^m_{ijk}. \end{cases} \tag{37}$$

Applying (28) and (31) to all manufacturing firms F_i; $i = 1, \ldots, N$, and applying (34) and (37) to all modes $m = 1, \ldots, M_j$ of freight service providers C_j; $j = 1, \ldots, O$ used in shipping the product from firm F_i; $i = 1, \ldots, N$ to all demand markets k; $k = 1, \ldots, Q$, and combining the resultants, yields the following pertinent Ordinary Differential Equation (ODE) for the adjustment processes of the prices and quality levels of firms and freight service providers, in vector form:

$$\dot{X} = \Pi_{\mathcal{K}}(X, -F(X)), \quad X(0) = X^0. \tag{38}$$

Note that $\Pi_{\mathcal{K}}$ is the projection operator of $-F(X)$ onto $\mathcal{K}$ and X^0 is the initial point $(p_F^0, q_F^0, p_C^0, q_C^0)$ corresponding to the initial price and quality levels of the manufacturing firms and freight service providers. Specifically, according to Dupuis and Nagurney (1993), $\Pi_{\mathcal{K}}$ with respect to $\mathcal{K}$, with $\mathcal{K} \equiv K^3$ being a convex polyhedron, of the $2N + 2(NOQ\sum_{j=1}^{O} M_j)$-dimensional vector $-F(X)$ at X, is defined as:

$$\Pi_{\mathcal{K}}(X, -F(X)) = \lim_{\delta \to 0} \frac{P_{\mathcal{K}}(X - \delta F(X)) - X}{\delta}, \tag{39}$$

with $P_{\mathcal{K}}$ denoting the projection map:

$$P_{\mathcal{K}}(X) = \text{argmin}_{z \in \mathcal{K}} \|X - z\|, \tag{40}$$

and where $\|\cdot\| = \langle x, x \rangle$. Recall that, for our model, $F(X)$ is the vector of minus the marginal utilities of the manufacturing firms and the freight service providers with respect to their strategic variables of prices and quality levels, with the individual components of $F(X)$ given by (22) through (25).

The dynamical system (38) is a *Projected Dynamical System* (PDS) termed by Zhang and Nagurney (1995) and is nonclassical due to the discontinuity of the projection map $\Pi_{\mathcal{K}}$ at the boundary of $\mathcal{K}$ according to Dupuis and Nagurney (1993). The trajectory provides the dynamic evolution of the prices charged and the quality levels of the manufacturing firms' products and carriers' freight services and the dynamic interactions among them. We note that ODE (38) ensures that the prices and quality levels of all firms and carriers are always within their lower and upper bounds.

The following theorem from Dupuis and Nagurney (1993) holds true in our framework since the feasible set is convex.

Theorem 3

X^* solves the variational inequality problem (21) (equivalently, (15) and (16)) if and only if it is a stationary point of the ODE (42), that is,

$$\dot{X} = 0 = \Pi_{\mathcal{K}}(X^*, -F(X^*)). \tag{41}$$

This theorem demonstrates that the necessary and sufficient condition for a product and freight service price and quality level pattern $X^* = (p_F^*, q_F^*, p_C^*, q_C^*)$ to be a Nash equilibrium, according to Definition 1, is that $X^* = (p_F^*, q_F^*, p_C^*, q_C^*)$ is a stationary point of the adjustment processes defined by ODE (38), that is, X^* is the point at which $\dot{X} = 0$.

4. The Computational Procedure

The feasible set underlying variational inequality (16) consists of box-type constraints, a feature that we exploit for computational purposes. Specifically, PDS (38) yields continuous-time adjustment processes in prices and quality levels of firms and freight service providers. However, for computational purposes, a discrete-time algorithm, which can serve as an approximation to the continuous-time trajectories is needed. Such an algorithm is the Euler method, which is induced by the general iterative scheme of Dupuis and Nagurney (1993), and which, at iteration τ (see also Nagurney and Zhang (1996)), results in the following problem:

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - a_{\tau}F(X^{\tau})), \tag{42}$$

where $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$ and F is the function that enters the variational inequality problem.

The nice feature of this algorithm is that, in the context of our new supply chain game theory models, the firms' product prices and quality levels and those of the freight service roviders', can be determined explicitly, at each iteration, using simple formulae, because of the structure of the feasible set. As shown in Dupuis and Nagurney (1993) and Nagurney and Zhang (1996), for convergence of the general iterative scheme, which induces the Euler method, the sequence $\{a_{\tau}\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_{\tau} = \infty$, $a_{\tau} > 0$, $a_{\tau} \to 0$, as $\tau \to \infty$.

Explicit Formulae for the Euler Method Applied to the Multitiered Supply Chain Network Problem

Observe that, at each iteration τ, $X^{\tau+1}$ in (46) is actually the solution to the strictly convex quadratic programming problem:

$$X^{\tau+1} = \text{Minimize}_{X \in \mathcal{K}} \quad \frac{1}{2}\langle X, X\rangle - \langle X^{\tau} - a_{\tau}F(X^{\tau}), X\rangle. \tag{43}$$

In particular, we have the following closed form expressions for all firms' product price $p_i; i = 1, \ldots, N$ and product quality $q_i; i = 1, \ldots, N$, respectively:

$$p_i^{\tau+1} = \max\Bigg\{0\,, \min\Bigg\{\bar{p}_i\,, p_i^{\tau} + a_{\tau}\Big[\sum_{j=1}^{O}\sum_{k=1}^{Q}\sum_{m=1}^{M_j} d_{ijk}^m(p_F^{\tau}, q_F^{\tau}, p_C^{\tau}, q_C^{\tau})$$

$$+p_i^{\tau}\sum_{j=1}^{O}\sum_{k=1}^{Q}\sum_{m=1}^{M_j}\frac{\partial d_{ijk}^m(p_F^{\tau}, q_F^{\tau}, p_C^{\tau}, q_C^{\tau})}{\partial p_i}$$

$$-\sum_{l=1}^{N}\frac{\partial PC_i(s_F(p_F^{\tau}, q_F^{\tau}, p_C^{\tau}, q_C^{\tau}), q_F^{\tau})}{\partial s_l} \times \frac{\partial s_l(p_F^{\tau}, q_F^{\tau}, p_C^{\tau}, q_C^{\tau})}{\partial p_i}\Big]\Bigg\}\Bigg\}, \tag{44}$$

$$q_i^{\tau+1} = \max\Big\{\underline{q}_i\,, \min\Big\{\bar{q}_i\,, q_i^\tau + a_\tau\,[p_i^\tau \sum_{j=1}^{O}\sum_{k=1}^{Q}\sum_{m=1}^{M_j} \frac{\partial d_{ijk}^m(p_F^\tau, q_F^\tau, p_C^\tau, q_C^\tau)}{\partial q_i}$$

$$-\sum_{l=1}^{N} \frac{\partial PC_i(s_F(p_F^\tau, q_F^\tau, p_C^\tau, q_C^\tau), q_F^\tau)}{\partial s_l} \times \frac{\partial s_l(p_F^\tau, q_F^\tau, p_C^\tau, q_C^\tau)}{\partial q_i} - \frac{\partial PC_i(s_F^\tau, q_F^\tau)}{\partial q_i}]\Big\}\Big\}. \tag{45}$$

Also, we have the following closed form expressions for the prices, p_{ijk}^m, and the quality levels, q_{ijk}^m, of the freight service providers: $i = 1, \ldots, N$; $j = 1, \ldots, O$; $k = 1, \ldots, Q$; $m = 1, \ldots, M_j$), respectively:

$$p_{ijk}^{m(\tau+1)} = \max\Big\{0\,, \min\Big\{\bar{p}_{ijk}^m\,, p_{ijk}^{m\tau} + a_\tau\,[d_{ijk}^m(p_F^\tau, q_F^\tau, p_C^\tau, q_C^\tau)$$

$$+\sum_{l=1}^{N}\sum_{s=1}^{Q}\sum_{t=1}^{M_j} \frac{\partial d_{ljs}^t(p_F^\tau, q_F^\tau, p_C^\tau, q_C^\tau)}{\partial p_{ijk}^m} \times p_{ljs}^{t\tau}$$

$$-\sum_{l=1}^{N}\sum_{s=1}^{Q}\sum_{t=1}^{M_j}(\sum_{r=1}^{N}\sum_{v=1}^{O}\sum_{w=1}^{Q}\sum_{z=1}^{M_v} \frac{\partial TC_{ljs}^t(d(p_F^\tau, q_F^\tau, p_C^\tau, q_C^\tau), q_C^\tau)}{\partial d_{rvw}^z} \times \frac{\partial d_{rvw}^z(p_F^\tau, q_F^\tau, p_C^\tau, q_C^\tau)}{\partial p_{ijk}^m})]\Big\}\Big\}, \tag{46}$$

$$q_{ijk}^{m(\tau+1)} = \max\Big\{\underline{q}_{ijk}^m\,, \min\Big\{\bar{q}_{ijk}^m\,, q_{ijk}^{m\tau} + a_\tau\,[\sum_{l=1}^{N}\sum_{s=1}^{Q}\sum_{t=1}^{M_j} \frac{\partial d_{ljs}^t(p_F^\tau, q_F^\tau, p_C^\tau, q_C^\tau)}{\partial q_{ijk}^m} \times p_{ljs}^{t\tau}$$

$$-\sum_{l=1}^{N}\sum_{s=1}^{Q}\sum_{t=1}^{M_j}(\sum_{r=1}^{N}\sum_{v=1}^{O}\sum_{w=1}^{Q}\sum_{z=1}^{M_v} \frac{\partial TC_{ljs}^t(d(p_F^\tau, q_F^\tau, p_C^\tau, q_C^\tau), q_C^\tau)}{\partial d_{rvw}^z} \times \frac{\partial d_{rvw}^z(p_F^\tau, q_F^\tau, p_C^\tau, q_C^\tau)}{\partial q_{ijk}^m})$$

$$-\sum_{l=1}^{N}\sum_{s=1}^{Q}\sum_{t=1}^{M_j} \frac{\partial TC_{ljs}^t(d^\tau, q_C^\tau)}{\partial q_{ijk}^m}]\Big\}\Big\}. \tag{47}$$

Note that all the functions to the left of the equal signs in (44) - (47) are evaluated at their respective variables computed at the τ-th iteration.

Also, the below convergence result is immediate following Nagurney and Zhang (1996) since the feasible set $\mathcal{K}$ is compact.

Theorem 4: Convergence

In our multitiered supply chain network game theory model, assume that $F(X) = -\nabla U(p_F, q_F, p_C, q_C)$ is strictly monotone. Also, assume that F is uniformly Lipschitz continuous. Then, there exists a unique equilibrium price and quality pattern $(p_F^*, q_F^*, p_C^*, q_C^*) \in \mathcal{K}$ and any sequence generated by the Euler method as given by (44) – (47), where $\{a_\tau\}$ satisfies $\sum_{\tau=0}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \to 0$, as $\tau \to \infty$ converges to $(p_F^*, q_F^*, p_C^*, q_C^*)$.

5. Numerical Examples

In this Section, we present numerical examples illustrating the multitiered supply chain network game theory framework developed in Sections 2 and 3. The equilibrium solutions of the model are computed by applying the Euler method as outlined in Section 4. Specifically, we present a spectrum of examples with various combinations of manufacturing firms, freight service providers, and modes. The supply chain network topology for each numerical example is described before the data and solution are presented.

The computations via the Euler method are carried out using Matlab. The algorithm was implemented on a VAIO S Series laptop with an Intel Core i7 processor and 12 GB RAM. The convergence tolerance is 10^{-6}, so that the algorithm is deemed to have converged when the absolute value of the difference between each successive price and quality level is less than or equal to 10^{-6}. The sequence $\{\alpha_\tau\}$ is set to: $.1\{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, ...\}$. We initialize the algorithm by setting the prices and quality levels at their lower bounds. The ranges in which the prices and quality levels vary are noted for each example.

The first two examples are simple examples, for exposition purposes and clarity. The subsequent examples, along with their variants, reveal various aspects of the underlying competition. For the first two examples, we also provide the trajectories of the evolution of the prices and quality.

Our framework can be applied to both high value and low value products with appropriate modifications in the underlying functions. For example, valuable goods would require greater quality in freight service provision, but at a higher associated cost; also, their production/manufacturing costs, given the components, we would also expect to be higher.

Example 1

In the first example, we have a single manufacturing firm, F_1, a single freight service provider, C_1, with one mode of transport, and a single demand market, as depicted in the supply chain network in Figure 2.

The demand function for demand market 1 is:

$$d^1_{111} = 43 - 1.62p^1_{111} + 1.6q^1_{111} - 1.45p_1 + 1.78q_1.$$

The supply of F_1 is:

$$s_1 = d^1_{111}.$$

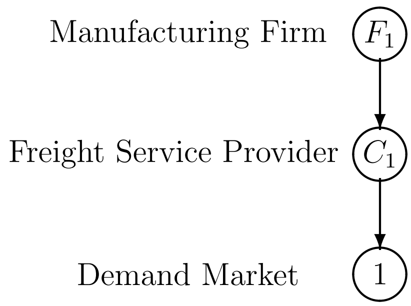

Figure 2: The Supply Chain Network Topology for Example 1

The production cost of manufacturing firm F_1 is:

$$PC_1 = 1.55(s_1 + 1.15q_1^2).$$

The utility of manufacturing firm F_1 is:

$$U_{F_1} = p_1 s_1 - PC_1.$$

The quality and price of the firm are bounded as per the following constraints:

$$0 \leq p_1 \leq 80, \qquad 10 \leq q_1 \leq 100.$$

The transportation cost of freight service provider C_1 is:

$$TC_{111}^1 = .5d_{111}^1 + (q_{111}^1)^2.$$

The utility of freight service provider C_1 is:

$$U_{C_1} = p_{111}^1 d_{111}^1 - TC_{111}^1,$$

with the following limitations on his price and quality:

$$0 \leq p_{111}^1 \leq 70, \quad 9 \leq q_{111}^1 \leq 100.$$

The Jacobian of -$\nabla U(p_{111}^1, p_1, q_{111}^1, q_1)$, denoted by $J(p_{111}^1, p_1, q_{111}^1, q_1)$, is

$$J = \begin{pmatrix} 3.24 & 1.45 & -1.60 & -1.78 \\ 1.62 & 2.90 & -1.60 & -1.78 \\ -1.60 & 0 & 2.00 & 0 \\ 0 & -1.78 & 0 & 3.57 \end{pmatrix}.$$

The eigenvalues of the symmetric part of J, $(J + J^T)/2$, are all positive and they are: 0.79, 1.14, 3.28, and 6.47. The equilibrium result, after 60 iterations, is:

$$p_{111}^{1*} = 16.63, \qquad p_1^* = 19.57, \qquad q_{111}^{1*} = 12.90, \qquad q_1^* = 10.00.$$

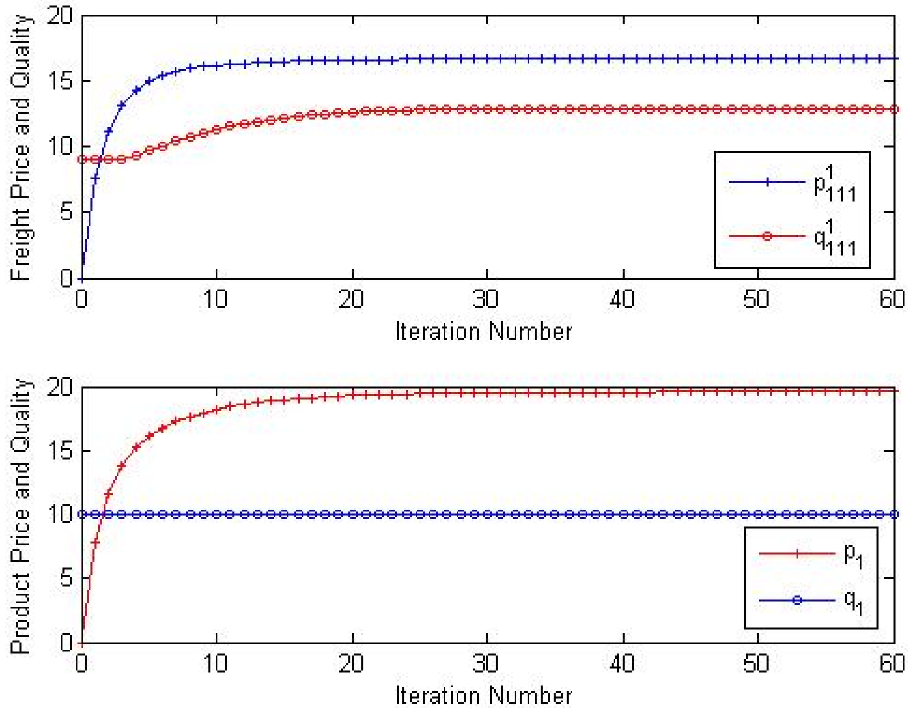

Figure 3: Prices and Quality Levels for the Product and Freight of Example 1

The iterates displayed in Figure 3 provide a discrete-time evolution of the prices and quality levels of the manufacturer and freight service provider as they respond through the time periods to the demands for the product and service. We observe that the prices move much above the quality levels and reach significantly higher values than their points of initiation, while the quality levels do not gain as much. This can be attributed to a lack of competition and enough scope at the demand market for gaining revenues. The manufacturer and freight service provider would try to extract the maximum price out of the market while offering a low quality product and services.

Indeed, in the absence of competition, the manufacturing firm and the freight service provider produce and transport at low quality levels. This explains the low equilibrium values of q_1^* and q_{111}^{1*}. The utility of firm F_1 is 292.60 and that of freight service provider C_1 is 254.95. Also, the demand d_{111}^1 at equilibrium is 26.13. The demand function is assumed so that more weight is given to the quality of the product than of the freight service provision and the price of the service provider than the product price. Since there is no competition, the manufacturing firm ends up with a higher utility by selling a low quality product, while the service provider gains but not as much as the manufacturer.

Example 2

In Example 2, we extend Example 1 by adding another mode of shipment for freight service provider C_1. The supply chain network topology is now as depicted in Figure 4.

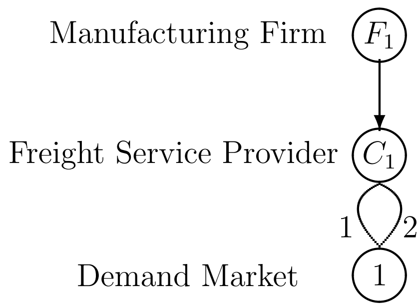

Figure 4: The Supply Chain Network Topology for Example 2

The demand functions are:

$$d^1_{111} = 43 - 1.62p^1_{111} + 1.6q^1_{111} - 1.45p_1 + 1.78q_1 + .03p^2_{111} - .2q^2_{111},$$

$$d^2_{111} = 52 - 1.75p^2_{111} + 1.21q^2_{111} - 1.45p_1 + 1.78q_1 + .03p^1_{111} - .2q^1_{111}.$$

The contribution of quality of the product is higher in the demand functions than its price. Also, the contribution of price of the service provider is higher in the demand functions than the quality he offers. Here, the freight service providers are striving to position themselves as a value added service.

The supply of manufacturing firm F_1 is changed to:

$$s_1 = d^1_{111} + d^2_{111}$$

since there are two modes of shipment available now.

The production cost function of F_1 is the same as Example 1. The transportation costs of the freight service provider C_1 for modes 1 and 2 are:

$$TC^1_{111} = .5d^1_{111} + (q^1_{111})^2,$$

$$TC^2_{111} = .45d^2_{111} + .54(q^2_{111})^2 + .0035d^2_{111}q^2_{111}.$$

Note that mode 1's cost remains as in Example 1.

The utility of freight service provider C_1 is:

$$U_{C_1} = p^1_{111} d^1_{111} + p^2_{111} d^2_{111} - TC^1_{111} - TC^2_{111},$$

with the constraints on the price and quality of shipment kept for the first mode as in Example 1 and for the added second mode as below:

$$0 \leq p^2_{111} \leq 70, \quad 9 \leq q^2_{111} \leq 100.$$

The symmetric part of J, $(J+J^T)/2$, has positive eigenvalues, which guarantees the strict monotonicity of $F(X)$. The equilibrium solution, after 166 iterations, is:

$$p^{1*}_{111} = 21.68, \qquad p^{2*}_{111} = 24.16, \qquad p^*_1 = 27.18,$$

$$q^{1*}_{111} = 14.58, \qquad q^{2*}_{111} = 22.43, \qquad q^*_1 = 25.59.$$

The trajectories in Figure 5 provide a discrete-time evolution of the prices and quality levels of the manufacturer and freight service provider. As compared to Figure 3, the quality levels, and, therefore, the prices, of both manufacturer and freight service provider increase. This would be a result of the competing modes. We observe that the quality of mode 2 is much better than that of mode 1. Hence, the freight service provider quotes a higher price for mode 2. At the manufacturer's level, we continue to obtain a higher price in comparison to the quality level. However, we see the difference between the prices and quality levels to be much less than Figure 3 (the trajectories move along more closely in Figure 5 than in Figure 3 for the manufacturer).

At equilibrium, the utility of manufacturing firm F_1 is 737.29 and that of freight service provider C_1 is 1190.05. The amount shipped via mode 1, d^1_{111}, is 33.59 and that shipped via mode 2, d^2_{111}, is 40.73. Interestingly, even though the price offered by service provider C_1 for mode 2 is slightly higher, the quality level of mode 2 is much better than that of mode 1, which increases the demand satisfied by mode 2 as compared to mode 1. Also, the fixed component of the demand function, d^2_{111} is higher than that of d^1_{111}. This also contributes to the higher demand shipped by mode 2 to demand market 1.

The differences in the utilities of the manufacturer (737.29) and the service provider (1190.05) are explained mainly by the production costs and transportation costs, respectively. It is judicious to assume that the production costs of a manufacturing firm would be higher than the transportation costs incurred by a freight service provider. This difference gets aptly captured in the (comparatively) higher coefficients of the production cost function.

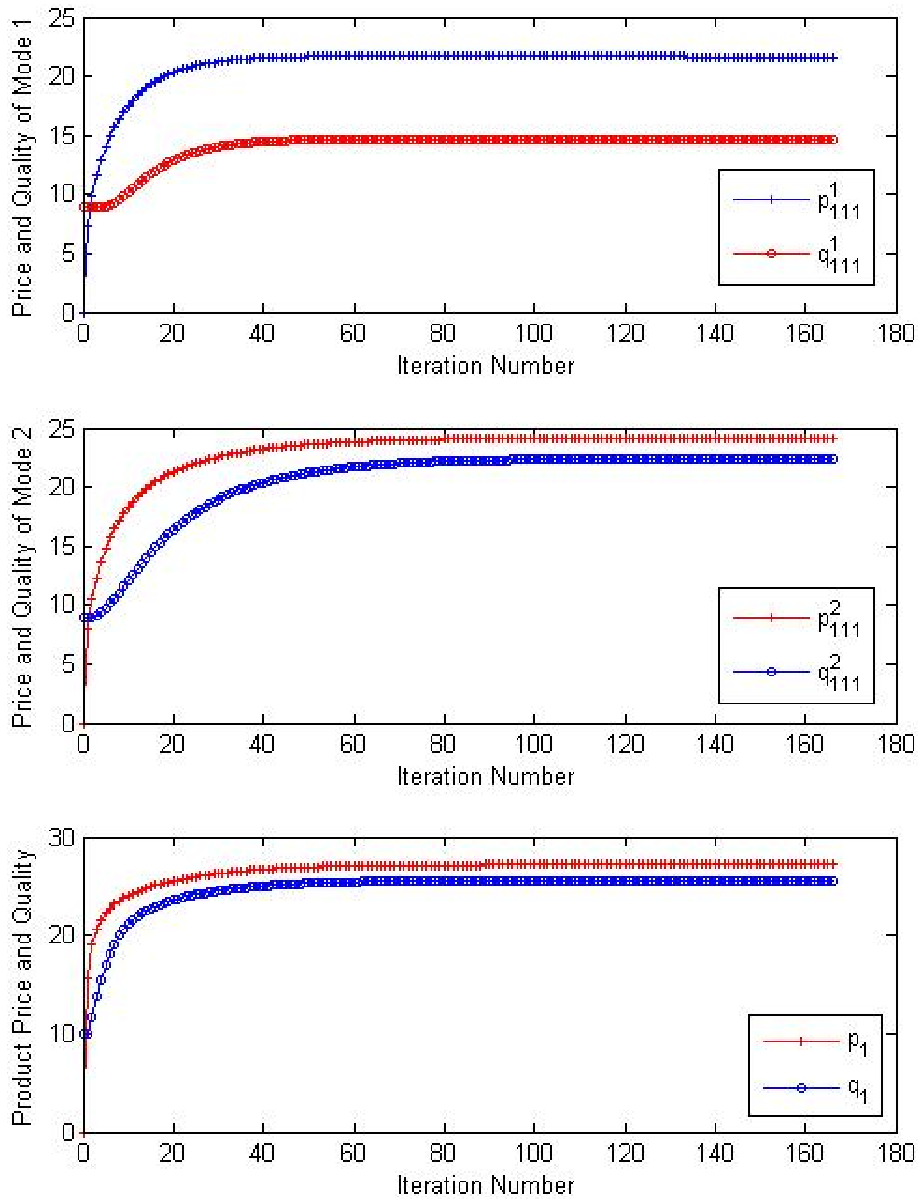

Figure 5: Prices and Quality Levels for Products and Modes 1 and 2 of Example 2

Example 3 and Variant

In Example 3 and its variant, we extend Example 2 by including another freight service provider with one mode of shipment as illustrated in Figure 6.

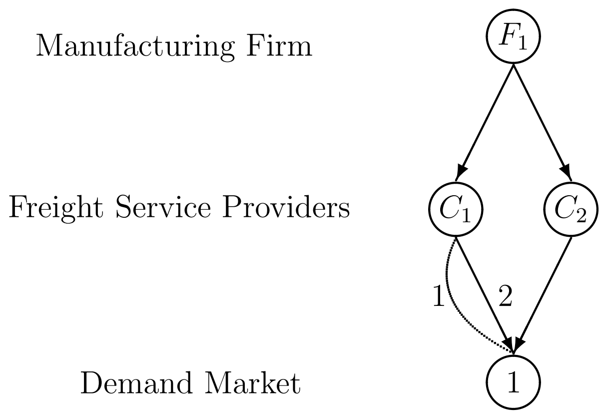

Figure 6: The Supply Chain Network Topology for Example 3 and Variant

The demand functions are:

$$d^1_{111} = 43 - 1.62p^1_{111} + 1.6q^1_{111} - 1.45p_1 + 1.78q_1 + .03p^2_{111} - .2q^2_{111} + .04p^1_{121} - .1q^1_{121},$$

$$d^2_{111} = 52 - 1.75p^2_{111} + 1.21q^2_{111} - 1.45p_1 + 1.78q_1 + .03p^1_{111} - .2q^1_{111} + .04p^1_{121} - .1q^1_{121},$$

$$d^1_{121} = 47 - 1.79p^1_{121} + 1.41q^1_{121} - 1.45p_1 + 1.78q_1 + .03p^1_{111} - .2q^1_{111} + .04p^2_{111} - .1q^2_{111}.$$

The supply of F_1 is:

$$s_1 = d^1_{111} + d^2_{111} + d^1_{121}.$$

The production cost of F_1 is the same as in Example 2. Therefore, the utility function of F_1 has not changed. The transportation costs of freight service provider C_1 are:

$$TC^1_{111} = .5d^1_{111} + (q^1_{111})^2 + .045d^1_{121},$$

$$TC^2_{111} = .45d^2_{111} + .54(q^2_{111})^2 + .005d^2_{111}q^2_{111},$$

and that of freight service provider C_2 is:

$$TC^1_{121} = .64d^1_{121} + .76(q^1_{121})^2.$$

The utility function of C_1 and his price and quality constraints have not changed. The utility of C_2 is:

$$U_{C_2} = p^1_{121}d^1_{121} - TC^1_{121}.$$

The maximum and minimum levels of price and quality of C_2 are:

$$0 \leq p^1_{121} \leq 65, \qquad 12 \leq q^1_{121} \leq 100.$$

The Jacobian of $F(X)$ for this example is also positive-definite. The new equilibrium solution, computed after 218 iterations, is:

$$p^{1*}_{111} = 45.69, \qquad p^{2*}_{111} = 45.32, \qquad p^{1*}_{121} = 44.82, \qquad p^*_1 = 53.91,$$

$$q^{1*}_{111} = 31.69, \qquad q^{2*}_{111} = 41.32, \qquad q^{1*}_{121} = 41.24, \qquad q^*_1 = 78.43.$$

In addition to the competition between modes captured in Example 2, in Example 3, we capture the competition among freight service providers. This adds pragmatism and generality. The assumption regarding the demand functions being more inclined towards the quality of the product manufactured and the prices of the service providers remains valid in this instance as well. This supposition induced by the assumed coefficients of the demand and cost functions gets clearly reflected in the equilibrium solution ($p^*_1 = 53.91; q^*_1 = 78.43$).

At equilibrium, the utility of manufacturing firm F_1 is 961.39 and that of freight service providers C_1 and C_2 are 4753.06 and 2208.92, respectively. Demand market 1 receives amounts of 71.88 and 76.81 via modes 1 and 2 from C_1, and 79.07 from C_2. The inclusion of an additional freight service provider helps to increase the total demand as compared to Example 2. The increasing demand provides an incentive for manufacturing firm F_1 to increase his quality level and, consequently, his price. This surge in demand also has a positive effect on the utilities of the manufacturing firm and both freight service providers. Higher demand gets satisfied by C_2 since his price is lower and the quality level is at par with the quality provided by C_1 for both modes. Clearly, mode 1 of C_1 carries the lowest amount of the total demand due to the higher price and lower quality combination he offers.

Variant of Example 3

We consider a variant of Example 3 wherein the demand function is more sensitive to the price of the product manufactured and the quality offered by the service providers. Keeping the other data consistent, the demand functions are, hence, modified to the following:

$$d^1_{111} = 43 - 1.44p^1_{111} + 1.53q^1_{111} - 1.82p_1 + 1.21q_1 + .03p^2_{111} - .2q^2_{111} + .04p^1_{121} - .1q^1_{121},$$

$$d^2_{111} = 52 - 1.49p^2_{111} + 1.65q^2_{111} - 1.82p_1 + 1.21q_1 + .03p^1_{111} - .2q^1_{111} + .04p^1_{121} - .1q^1_{121},$$

$$d^1_{121} = 47 - 1.57p^1_{121} + 1.64q^1_{121} - 1.82p_1 + 1.21q_1 + .03p^1_{111} - .2q^1_{111} + .04p^2_{111} - .1q^2_{111}.$$

The equilibrium solution, computed after 553 iterations, is:

$$p_{111}^{1*} = 8.71, \qquad p_{111}^{2*} = 63.17, \qquad p_{121}^{1*} = 16.22, \qquad p_1^* = 24.80,$$

$$q_{111}^{1*} = 9.00, \qquad q_{111}^{2*} = 93.15, \qquad q_{121}^{1*} = 16.92, \qquad q_1^* = 23.67.$$

It should be noted that the quality levels offered by the freight service providers take on higher values than their prices as opposed to a vice versa situation in the case of Example 3. At equilibrium, the utility of manufacturing firm F_1 is 1952.19 and that of service providers C_1 and C_2 are 1073.86 and 164.99, respectively. The transportation costs increase to ensure high quality transportation. Thus, the utility of the manufacturing firm is higher than the utilities of both freight service providers. This can be explained by the fact that, apart from the price and quality level of the second mode of service provider C_1, the prices and quality levels of the other mode and the other service provider take on much smaller values than in the equilibrium solution of the previous assumption. Since the emphasis is given to the quality of the service provider in the demand functions, the low quality levels result in lower demand. Demand market 1 receives amounts of 9.96 and 92.51 via modes 1 and 2 of freight service provider C_1, and 24.46 via freight service provider C_2. The low demand further reduces the utilities.

Example 4 and Variant

Example 4 and its variant extend the previous numerical examples through the addition of another manufacturing firm, as shown in Figure 7. These manufacturers offer substitutable products to the demand markets.

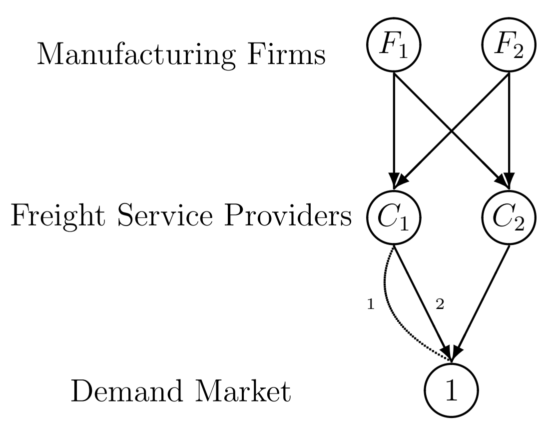

Figure 7: The Supply Chain Network Topology for Example 4 and Variant

The demand functions for manufacturing firm F_1 are:

$$d_{111}^1 = 43 - 1.62p_{111}^1 + 1.6q_{111}^1 - 1.45p_1 + 1.78q_1 + .08p_2 - .04q_2 + .03p_{111}^2 - .2q_{111}^2 + .04p_{121}^1 - .1q_{121}^1,$$

$$d_{111}^2 = 52 - 1.75p_{111}^2 + 1.21q_{111}^2 - 1.45p_1 + 1.78q_1 + .08p_2 - .04q_2 + .03p_{111}^1 - .2q_{111}^1 + .04p_{121}^1 - .1q_{121}^1,$$

$$d_{121}^1 = 47 - 1.79p_{121}^1 + 1.41q_{121}^1 - 1.45p_1 + 1.78q_1 + .08p_2 - .04q_2 + .03p_{111}^1 - .2q_{111}^1 + .04p_{111}^2 - .1q_{111}^2,$$

and that of manufacturing firm F_2 are:

$$d_{211}^1 = 51 - 1.57p_{211}^1 + 1.26q_{211}^1 - 1.65p_2 + 1.98q_2 + .08p_1 - .04q_1 + .04p_{211}^2 - .1q_{211}^2 + .02p_{221}^1 - .12q_{221}^1,$$

$$d_{211}^2 = 44 - 1.63p_{211}^2 + 1.21q_{211}^2 - 1.65p_2 + 1.98q_2 + .08p_1 - .04q_1 + .04p_{211}^1 - .1q_{211}^1 + .02p_{221}^1 - .12q_{221}^1,$$

$$d_{221}^1 = 56 - 1.46p_{221}^1 + 1.41q_{221}^1 - 1.65p_2 + 1.98q_2 + .08p_1 - .04q_1 + .04p_{211}^1 - .1q_{211}^1 + .02p_{211}^2 - .12q_{211}^2.$$

The supply of F_1 is similar to that in Example 3 and that of manufacturing firm F_2 is:

$$s_2 = d_{211}^1 + d_{211}^2 + d_{221}^1.$$

The production cost functions of F_1 and F_2 are:

$$PC_1 = 1.55s_1 + 1.88q_1^2 + .02s_2 + .06q_2,$$

$$PC_2 = 1.47s_2 + 1.94q_2^2 + .041s_1 + .032q_1.$$

Manufacturing firm F_1 has the same utility function and price and quality bounds as in Example 3. The utility of manufacturing firm F_2 is:

$$U_{F_2} = p_2 s_2 - PC_2,$$

and the price and quality of his product are constrained in the following manner:

$$0 \leq p_2 \leq 95, \qquad 8 \leq q_2 \leq 100.$$

The transportation cost functions of freight service provider C_1 are changed to:

$$TC_{111}^1 = .5d_{111}^1 + (q_{111}^1)^2 + .0045d_{121}^1 + .0045d_{221}^1 + .0045d_{211}^1,$$

$$TC_{111}^2 = .45d_{111}^2 + .54(q_{111}^2)^2 + .0011d_{211}^2,$$

$$TC_{211}^1 = .68d_{211}^1 + .79(q_{211}^1)^2 + .002d_{211}^1 + .002d_{221}^1,$$

$$TC_{211}^2 = .57d_{211}^2 + .74(q_{211}^2)^2 + .005d_{111}^2,$$

and the functions of freight service provider C_2 are changed to:

$$TC_{121}^1 = .64d_{121}^1 + .76(q_{121}^1)^2 + .0015d_{221}^1,$$

$$TC^1_{221} = .59d^1_{221} + .80(q^1_{221})^2 + .01d^1_{121} + .01d^1_{111} + .01d^1_{211}.$$

The utility of C_1 is:

$$U_{C_1} = p^1_{111}d^1_{111} + p^2_{111}d^2_{111} + p^1_{211}d^1_{211} + p^2_{211}d^2_{211} - TC^1_{111} - TC^2_{111} - TC^1_{211} - TC^2_{211},$$

and that of C_2 is:

$$U_{C_2} = p^1_{121}d^1_{121} + p^1_{221}d^1_{221} - TC^1_{121} - TC^1_{221}.$$

The lower and upper bounds of the prices for service providers are now:

$$0 \le p^{M_1}_{i1k} \le 90, \qquad \forall i, k, M_1, \text{ for } M_1 = 2,$$

$$0 \le p^{M_2}_{i2k} \le 85, \qquad \forall i, k, M_2, \text{ for } M_2 = 1.$$

The equilibrium solution, computed after 231 iterations, is:

$$p^{1*}_{111} = 40.20, \qquad p^{2*}_{111} = 40.72, \qquad p^{1*}_{121} = 39.79, \qquad p^*_1 = 48.08,$$

$$p^{1*}_{211} = 51.17, \qquad p^{2*}_{211} = 42.88, \qquad p^{1*}_{221} = 69.18, \qquad p^*_2 = 50.89,$$

$$q^{1*}_{111} = 27.73, \qquad q^{2*}_{111} = 37.76, \qquad q^{1*}_{121} = 36.53, \qquad q^*_1 = 66.25,$$

$$q^{1*}_{211} = 37.64, \qquad q^{2*}_{211} = 29.42, \qquad q^{1*}_{221} = 63.97. \qquad q^*_2 = 75.65.$$

In this example, we consider competition at the manufacturers' level, the freight service providers' level, and between modes of a particular service provider. This, further, increases the generality, as well as the complexity, of the problem when compared with Example 3. The assumption regarding the demand functions being more inclined towards the quality of the product manufactured and the prices of the service providers remains valid in this instance as well. The equilibrium solution ($p^*_1 = 48.08; q^*_1 = 66.25; p^*_2 = 50.89; q^*_2 = 75.65$) supports this assumption.

The utilities of manufacturing firms F_1 and F_2 are 1179.39 and 976.85, respectively. Moreover, the utilities of service providers C_1 and C_2 are 8743.66 and 5340.84, respectively. The demand market receives an amount of 132.37 of the product manufactured by F_1 from service provider C_1 and an amount of 70.05 from C_2. Firm F_2 sends 144.51 units via C_1 and 100.14 units by C_2.

Due to the added competition at the manufacturers' level, the quality and price of the product manufactured at firm F_1 have declined as compared to Example 3. This was expected since to attain more market share, the prices would be lowered, which would result in a lowering of quality levels. The utility of F_1 is higher than that of F_2. A product with reduced prices and quality levels would require cheaper prices (and, hence, quality) of the transporters. Resultantly, prices and quality levels of freight service provider C_1 carrying products from F_1 have also been reduced. It is interesting to note that even though the price and quality level of C_2 transporting the product manufactured by F_2 are the highest of all $(p^{1*}_{221}; q^{1*}_{221})$, more demand for F_2 is satisfied by service provider C_2 (100.14) than that of F_1 (70.05). The prices and quality levels of service provider C_2 transporting goods of manufacturer F_1 are at par with that of service provider C_1. Clearly, both manufacturers prefer service provider C_1 to service provider C_2.

Variant of Example 4

We now construct a variant of Example 4 wherein the demand function is more sensitive to the price of the product manufactured and the quality offered by the service providers. Keeping the other data consistent, the demand functions are, hence, modified to the following:

$$d^1_{111} = 43-1.44p^1_{111}+1.53q^1_{111}-1.82p_1+1.21q_1+.08p_2-.04q_2+.03p^2_{111}-.2q^2_{111}+.04p^1_{121}-.1q^1_{121},$$

$$d^2_{111} = 52-1.49p^2_{111}+1.65q^2_{111}-1.82p_1+1.21q_1+.08p_2-.04q_2+.03p^1_{111}-.2q^1_{111}+.04p^1_{121}-.1q^1_{121},$$

$$d^1_{121} = 47-1.57p^1_{121}+1.64q^1_{121}-1.82p_1+1.21q_1+.08p_2-.04q_2+.03p^1_{111}-.2q^1_{111}+.04p^2_{111}-.1q^2_{111},$$

$$d^1_{211} = 51-1.39p^1_{211}+1.66q^1_{211}-1.88p_2+1.25q_2+.08p_1-.04q_1+.04p^2_{211}-.1q^2_{211}+.02p^1_{221}-.12q^1_{221},$$

$$d^2_{211} = 44-1.42p^2_{211}+1.58q^2_{211}-1.88p_2+1.25q_2+.08p_1-.04q_1+.04p^1_{211}-.1q^1_{211}+.02p^1_{221}-.12q^1_{221},$$

$$d^1_{221} = 56-1.40p^1_{221}+1.63q^1_{221}-1.88p_2+1.25q_2+.08p_1-.04q_1+.04p^1_{211}-.1q^1_{211}+.02p^2_{211}-.12q^2_{211}.$$

The equilibrium solution, computed after 568 iterations, is:

$$p^{1*}_{111} = 8.30, \quad p^{2*}_{111} = 64.70, \quad p^{1*}_{121} = 15.54, \quad p^*_1 = 25.02,$$

$$p^{1*}_{211} = 28.70, \quad p^{2*}_{211} = 18.47, \quad p^{1*}_{221} = 36.15, \quad p^*_2 = 21.38,$$

$$q^{1*}_{111} = 9.00, \quad q^{2*}_{111} = 96.71, \quad q^{1*}_{121} = 16.16, \quad q^*_1 = 22.71,$$

$$q^{1*}_{211} = 28.34, \quad q^{2*}_{211} = 17.19, \quad q^{1*}_{221} = 38.55. \quad q^*_2 = 19.24.$$

At equilibrium, the utilities of manufacturing firms F_1 and F_2 are 2037.45 and 1511.87, and that of freight service providers C_1 and C_2 are 1729.44 and 737.02. It is important to note that, based on the previous equilibrium solution, the utilities of the freight service providers were higher than those of the manufacturers. However, based on the variant's solution, the utilities of the freight service providers (focus on quality) are lower than the utilities of the manufacturers (focus on price). This is directly connected to the transportation costs which increase in order to ensure high quality transportation. Demand market 1 receives 104.81 units of F_1's product from service provider C_1 and 23.37 units from C_2. Also, the demand market receives 62.52 units of F_2's product via C_1 and 49.79 via C_2.

Example 5 and Variant

In this example and its variant, we extend the previous ones by adding another demand market to the supply chain network; see Figure 8. The manufacturers and freight service providers compete to serve two demand markets now.

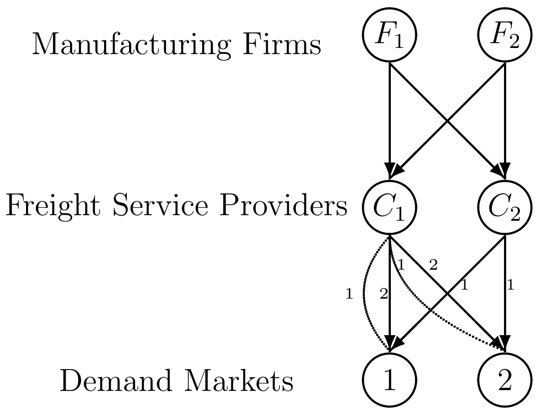

Figure 8: The Supply Chain Network Topology for Example 5 and Variant

The demand functions at demand market 2 for manufacturing firm F_1 are:

$$d^1_{112} = 50-1.63p^1_{112}+1.55q^1_{112}-1.48p_1+1.74q_1+.06p_2-.05q_2+.05p^2_{112}-.23q^2_{112}+.02p^1_{122}-.13q^1_{122},$$

$$d^2_{112} = 39-1.78p^2_{112}+1.21q^2_{112}-1.48p_1+1.74q_1+.06p_2-.05q_2+.05p^1_{112}-.23q^1_{112}+.02p^1_{122}-.13q^1_{122},$$

$$d^1_{122} = 42-1.66p^1_{122}+1.41q^1_{122}-1.48p_1+1.74q_1+.06p_2-.05q_2+.05p^1_{112}-.23q^1_{112}+.02p^2_{112}-.13q^2_{112},$$

and for manufacturing firm F_2:

$$d^1_{212} = 38-1.49p^1_{212}+1.34q^1_{212}-1.61p_2+1.86q_2+.06p_1-.05q_1+.05p^2_{212}-.09q^2_{212}+.03p^1_{222}-.08q^1_{222},$$

$$d^2_{212} = 43-1.57p^2_{212}+1.26q^2_{212}-1.61p_2+1.86q_2+.06p_1-.05q_1+.05p^1_{212}-.09q^1_{212}+.03p^1_{222}-.08q^1_{222},$$

$$d^1_{222} = 58-1.53p^1_{222}+1.31q^1_{222}-1.61p_2+1.86q_2+.06p_1-.05q_1+.05p^1_{212}-.09q^1_{212}+.03p^2_{212}-.08q^2_{212}.$$

The supply functions for both manufacturers are changed in the following manner:

$$s_1 = d^1_{111} + d^2_{111} + d^1_{121} + d^1_{112} + d^2_{112} + d^1_{122},$$

$$s_2 = d^1_{211} + d^2_{211} + d^1_{221} + d^1_{212} + d^2_{212} + d^1_{222}.$$

There is no change to the utility functions of the manufacturing firms. However, the transportation functions of freight service provider C_1 have been changed to:

$$TC^1_{111} = .5d^1_{111} + (q^1_{111})^2 + .0045d^1_{121} + .0045d^1_{221} + .0045d^1_{211} + .0045d^1_{112},$$

$$TC^2_{111} = .45d^2_{111} + .54(q^2_{111})^2 + .0011d^2_{211} + .0011d^2_{212},$$

$$TC^1_{211} = .68d^1_{211} + .79(q^1_{211})^2 + .002d^1_{111} + .002d^1_{121} + 0.002d^1_{212},$$

$$TC^2_{211} = .57d^2_{211} + .74(q^2_{211})^2 + .005d^2_{111} + .005d^2_{212},$$

$$TC^1_{112} = .61d^1_{112} + .7(q^1_{112})^2 + .0037d^1_{111} + .0037d^1_{122} + 0.0037d^1_{212},$$

$$TC^2_{112} = .52d^2_{112} + .58(q^2_{112})^2 + .0024d^2_{212},$$

$$TC^1_{212} = .49d^1_{212} + .59(q^1_{212})^2 + .0017d^1_{112} + .0017d^1_{122},$$

$$TC^2_{212} = .43d^2_{212} + .55(q^2_{212})^2 + .0023d^2_{112},$$

and that of freight service provider C_2 to:

$$TC^1_{121} = .64d^1_{121} + .76(q^1_{121})^2 + .0015d^1_{221},$$

$$TC^1_{221} = .59d^1_{221} + .80(q^1_{221})^2 + .014d^1_{121} + .014d^1_{111} + .014d^1_{211},$$

$$TC^1_{122} = .67d^1_{122} + .73(q^1_{122})^2 + .0031d^1_{222} + .0031d^1_{212},$$

$$TC^1_{222} = .45d^1_{222} + .58(q^1_{222})^2 + .012d^1_{122} + .012d^1_{112} + .012d^1_{212}.$$

With the same constraints on the prices and quality levels, the utilities of freight service providers become:

$$U_{C_1} = p^1_{111}d^1_{111} + p^2_{111}d^2_{111} + p^1_{211}d^1_{211} + p^2_{211}d^2_{211} + p^1_{112}d^1_{112} + p^2_{112}d^2_{112} + p^1_{212}d^1_{212} + p^2_{212}d^2_{212}$$

$$-TC^1_{111} - TC^2_{111} - TC^1_{211} - TC^2_{211} - TC^1_{112} - TC^2_{112} - TC^1_{212} - TC^2_{212},$$

$$U_{C_2} = p^1_{121}d^1_{121} + p^1_{221}d^1_{221} + p^1_{122}d^1_{122} + p^1_{222}d^1_{222} - TC^1_{121} - TC^1_{221} - TC^1_{122} - TC^1_{222}.$$

The equilibrium solution, after 254 iterations, is:

$$p^{1*}_{111} = 56.79, \qquad p^{2*}_{111} = 55.45, \qquad p^{1*}_{112} = 72.96, \qquad p^{2*}_{112} = 36.93,$$

$$p^{1*}_{121} = 55.19, \quad p^{1*}_{122} = 53.55, \quad p^{1*}_{211} = 62.77, \quad p^{2*}_{211} = 53.28,$$

$$p^{1*}_{212} = 72.94, \quad p^{2*}_{212} = 65.91, \quad p^{1*}_{221} = 76.15, \quad p^{1*}_{222} = 83.73,$$

$$p^{*}_{1} = 63.76, \quad p^{*}_{2} = 64.90, \quad q^{*}_{1} = 100.00, \quad q^{*}_{2} = 100.00,$$

$$q^{1*}_{111} = 39.53, \quad q^{2*}_{111} = 51.20, \quad q^{1*}_{112} = 74.61, \quad q^{2*}_{112} = 23.54,$$

$$q^{1*}_{121} = 50.93, \quad q^{1*}_{122} = 51.05, \quad q^{1*}_{211} = 46.25, \quad q^{2*}_{211} = 36.72,$$

$$q^{1*}_{212} = 76.89, \quad q^{2*}_{212} = 69.56, \quad q^{1*}_{221} = 61.18, \quad q^{1*}_{222} = 94.70.$$

In this example, we consider competition at the manufacturers' level, the freight service providers' level, and between modes of a particular service provider, wherein all these players are competing to satisfy the demands at two different demand markets. This makes the problem quite complex. The assumption regarding the demand functions being more sensitive to the quality of the product manufactured and the prices of the service providers remains valid in this example as well. The equilibrium solution ($p^*_1 = 63.76; q^*_1 = 100.00; p^*_2 = 64.90; q^*_2 = 100.00$) supports this assumption. The price and quality levels have gone up as compared to Example 4 since there are two demand markets to be satisfied now as opposed to one.

The utilities of manufacturers F_1 and F_2 have increased to 15244.22 and 19922.55, respectively. Also, the freight service providers C_1 and C_2 are now witnessing higher utilities of 29256.82 and 16905.45, respectively. Since more demand from multiple demand markets has increased the prices and quality levels of products, the utilities have increased. The results indicate that service provider C_1 transports an amount of 279.46 to demand market 1 and an amount of 381.13 to demand market 2. Also, service provider C_2 carries an amount of 207.96 to demand market 1 and 215.20 to demand market 2.

As there is enough demand for products of both manufacturers F_1 and F_2, the prices of the products are high and the quality levels are at their upper bounds of 100. This happens since the emphasis is on quality rather than price for manufacturers. Resultantly, the overall prices and quality levels of the two service providers also go up as compared to Example 4.

Variant of Example 5

Once again, we consider a variant wherein the demand function is more sensitive to the price of the product manufactured and the quality offered by the service providers. Keeping the other data consistent, the demand functions are, hence, modified to the following:

$$d^1_{112} = 50 - 1.37p^1_{112} + 1.67q^1_{112} - 1.91p_1 + 1.33q_1 + .06p_2 - .05q_2 + .05p^2_{112} - .23q^2_{112} + .02p^1_{122} - .13q^1_{122},$$

$$d^2_{112} = 39 - 1.41p^2_{112} + 1.65q^2_{112} - 1.91p_1 + 1.33q_1 + .06p_2 - .05q_2 + .05p^1_{112} - .23q^1_{112} + .02p^1_{122} - .13q^1_{122},$$

$$d^1_{122} = 42-1.35p^1_{122}+1.70q^1_{122}-1.91p_1+1.33q_1+.06p_2-.05q_2+.05p^1_{112}-.23q^1_{112}+.02p^2_{112}-.13q^2_{112},$$

$$d^1_{212} = 38-1.33p^1_{212}+1.59q^1_{212}-1.87p_2+1.29q_2+.06p_1-.05q_1+.05p^2_{212}-.09q^2_{212}+.03p^1_{222}-.08q^1_{222},$$

$$d^2_{212} = 43-1.36p^2_{212}+1.67q^2_{212}-1.87p_2+1.29q_2+.06p_1-.05q_1+.05p^1_{212}-.09q^1_{212}+.03p^1_{222}-.08q^1_{222},$$

$$d^1_{222} = 58-1.42p^1_{222}+1.68q^1_{222}-1.87p_2+1.29q_2+.06p_1-.05q_1+.05p^1_{212}-.09q^1_{212}+.03p^2_{212}-.08q^2_{212}.$$

The equilibrium solution, after 769 iterations, is:

$$p^{1*}_{111} = 22.05, \quad p^{2*}_{111} = 80.01, \quad p^{1*}_{112} = 44.02, \quad p^{2*}_{112} = 77.79,$$

$$p^{1*}_{121} = 46.56, \quad p^{1*}_{122} = 71.98, \quad p^{1*}_{211} = 62.01, \quad p^{2*}_{211} = 47.77,$$

$$p^{1*}_{212} = 82.80, \quad p^{2*}_{212} = 85.62, \quad p^{1*}_{221} = 64.72, \quad p^{1*}_{222} = 85.00,$$

$$p^*_1 = 43.78, \quad p^*_2 = 52.86, \quad q^*_1 = 85.79, \quad q^*_2 = 100.00,$$

$$q^{1*}_{111} = 9.00, \quad q^{2*}_{111} = 100.00, \quad q^{1*}_{112} = 39.34, \quad q^{2*}_{112} = 100.00,$$

$$q^{1*}_{121} = 49.85, \quad q^{1*}_{122} = 82.99, \quad q^{1*}_{211} = 61.55, \quad q^{2*}_{211} = 46.18,$$

$$q^{1*}_{212} = 100.00, \quad q^{2*}_{212} = 100.00, \quad q^{1*}_{221} = 65.62, \quad q^{1*}_{222} = 100.00.$$

The utilities of firms F_1 and F_2 are 6333.31 and 10285.25, respectively. The utilities of freight service providers C_1 and C_2 are 18654.58 and 10277.76, respectively. As expected, the utilities are increasing from those in Example 3 onwards. This particular variant registers the highest. Since the focus of the freight service providers is on quality, there are multiple cases wherein the quality levels of the providers are at their upper bounds. The demand markets have grown which lets the manufacturers and service providers increase their prices and quality levels. Higher quality levels, however, ensure that the transportation costs go up which, in turn, reduces the utilities of the freight service providers.

6. Summary and Conclusions

In this paper, we developed a game theory supply chain network model in both static and dynamic versions with multiple manufacturers and freight service providers competing on price and quality. This multi-faceted inclusion of competition in the model assesses the quality conformance level of the product and the level of service of freight service providers along with the prices at which the products and the transportation services were offered. The model handles multiple modes of transportation for delivery of shipments. The utility of each manufacturer (or service provider) depends on the prices and on the quality levels offered by its competitors as well as those of the others.

Variational inequality theory was employed in the formulation of the equilibrium governing the manufacturers' and freight service providers' behaviors with respect to price and quality followed by the rigorous description of the underlying dynamic interactions until a stationary point; equivalently, an equilibrium is achieved. The dynamics were shown to satisfy a projected dynamical system. The computational procedure utilized was the Euler method. The discrete-time algorithm, also serving as an approximation to the continuous-time trajectories, yields an equilibrium price and quality patterns for the manufacturers and the freight service providers.

In order to demonstrate the generality of the framework and the computational scheme, we then provided solutions to a series of numerical examples, beginning with smaller scale examples. In the larger examples, a scenario and its variant were explored while computing and analyzing the solutions for various combinations of manufacturing firms, freight service providers, and modes of transportation. The competition within echelons of the different examples altered the price and quality levels, and, thereby, the utilities, of the entities. We considered a scenario wherein the demand functions were more sensitive to the quality of the product manufactured and the price charged by the freight service providers. The variant took a contrasting position, whereby the demand markets were giving more importance to the price of the product manufactured and the quality levels offered by the freight service providers. These contradictory situations brought about interesting comparisons between the utilities of the manufacturers and the freight service providers, and how they changed when the emphasis on price and quality levels changed.

There are many aspects to our proposed framework that are worthy of further discussion and investigation. For instance, additional tiers of supply chain decision-makers could be included. The quality levels might be explicitly modeled for the freight service providers in terms of time-conformance of delivery, reliability of the service, emission standards (to compare the environmental viability of various modes), the quality of in-house transportation infrastructure, and so on. It is interesting to note from the results of this paper that in order to capture a higher market share, manufacturers or freight service providers might try to quote a lower price and offer a lower quality level (leading to a lower cost). However, a lower quality product/service might not be able to sustain the market share.

Our work fills the gap in the existing literature by capturing quality in transportation as well as production in a multitiered competitive supply chain network, along with prices as strategic variables. It provides a critical foundation for future research in this area.

Acknowledgments

The research of the first two authors was supported, in part, by the National Science Foundation (NSF) grant CISE #1111276, for the NeTS: Large: Collaborative Research: Network Innovation Through Choice project awarded to the University of Massachusetts Amherst. The first author also acknowledges support from the School of Business, Economics and Law at the University of Gothenburg under its Visiting Professorship Programme. The second author acknowledges support from the 2014 Isenberg Scholar Award. All the support is gratefully acknowledged.

We thank the two anonymous reviewers and the Editor for helpful comments and suggestions on an earlier version of this paper.

References

Acharyya, R., 2005. Consumer targeting under quality competition in a liberalized vertically differentiated market. Journal of Economic Development, 30(1), 129-150.

Akerlof, G.A., 1970. The market for lemons: Quality uncertainty and the market mechanism. Quarterly Journal of Economics, 84(3), 488-500.

Arvis, J.-F., Saslavsky, D., Ojala, L., Shepherd, B., Busch, C.. Raj, A., 2014. Connecting to Compete 2014, Trade Logistics in the Global Economy, The Logistics Performance Index and Its Indicators, The World Bank, Washington.

Banker, R.D., Khosla, I., Sinha, K.K., 1998. Quality and competition. Management Science, 44(9), 1179-1192.

Ben-Akiva, M., Meersman, H., Can de Voorde, E., Editors, 2013. Freight Transport Modelling. Emerald Group Publishing, Bingley, UK.

Bensinger, G., 2014. Amazon wants to ship your package before you buy it. The Wall Street Journal, January 17.

Bowman, R., 2014. Third-party logistics providers are shrinking in number, growing in size. Forbes, June 3.

Brekke, K.R., Siciliani, L., Straume, O.R., 2010. Price and quality in spatial competition. Regional Science and Urban Economics, 40, 471-480.

Dafermos, S., Nagurney, A., 1987. Oligopolistic and competitive behavior of spatially separated markets. Regional Science and Urban Economics, 17(2), 245-254.

Danielis, R., Marcucci, E., Rotaris, L., 2005. Logistics managers' stated preferences for freight service attributes. Transportation Research E, 41(3), 201-215.

DHL, 2014. Logistics Trend Radar, DHL Customer Solutions & Innovation, Troisdorf, Germany.

Dixit, A., 1979. Quality and quantity competition. Review of Economic Studies, 46(4), 587-599.

Dupuis, P., Nagurney, A., 1993. Dynamical systems and variational inequalities. Annals of Operations Research, 44, 9-42.

Farahani, R. Z., Rezapour, S., Drezner, T., Fallah, S., 2014. Competitive supply chain network design: An overview of classifications, models, solution techniques and applications. Omega, 45, 92-118.

Floden, J., Barthel, F., Sorkina, E., 2010. Factors influencing transport buyers' choice of transport service: A European literature review. Proceedings of the 12th WCTR Conference, July 11-15, Lisbon, Portugal.

Floden, J., Woxenius, J., 2013. Agility in the Swedish intermodal freight market - the effects of the withdrawal of the main provider. Paper presented at 13th World Conference on Transport Research, WCTR 2013, Rio de Janeiro, Brazil.

Gabay, D., Moulin, H., 1980. On the uniqueness and stability of Nash equilibria in noncooperative games. In: Bensoussan, A., Kleindorfer, P., Tapiero, C.S. (Eds.), Applied Stochastic Control in Econometrics and Management Science. North-Holland, Amsterdam, The Netherlands, pp 271-294.

Gal-or, E., 1983. Quality and quantity competition. Bell Journal of Economics, 14, 590-600.

Gilmore, H.L., 1974. Product conformance cost. Quality Progress, 7(5), 16-19.

Glave, T., Joerss, M., Saxon, S., 2014, The hidden opportunity in container shipping, McKinsey & Company, Travel, Transport & Logistics, New York.

Hakim, D., 2014. Aboard a cargo colossus. The New York Times, October 3.

Johnson, J.P., Myatt, D.P., 2003. Multiproduct quality competition: Fighting brands and product line pruning. American Economic Review, 93(3), 748-774.

Juran, J.M., Gryna, F.M., 1988. Quality Control Handbook, 4th edition. McGraw-Hill, New York.

Kinderlehrer, D., Stampacchia, G., 1980. An Introduction to Variational Inequalities and Their Applications. Academic Press, New York.

Mancera, A., Bruckmann, D., Weidmann, A., 2013. Level-of-Service based evaluation of freight networks. Presentation, European Transport Conference, Frankfurt, Germany, October.

Meixell, M.J., Norbis, M., 2008. A review of the transportation mode choice and carrier selection literature. The International Journal of Logistics Management, 19(2), 183-211.

Mussa, M., Rosen, S., 1978. Monopoly and product quality. Journal of Economic Theory, 18, 301-317.

Nagurney, A., 1999. Network Economics: A Variational Inequality Approach. Kluwer Academic Publishers, Dordrecht, The Netherlands.

Nagurney, A., 2006. Supply Chain Network Economics: Dynamics of Prices, Flows and Profits. Edward Elgar Publishing Inc., Cheltenham, UK.

Nagurney, A., Dong, J., Zhang, D., 2002. A supply chain network equilibrium model. Transportation Research E, 38, 281-303.

Nagurney, A., Ke, K., Cruz, J., Hancock, K., Southworth, F., 2002. Dynamics of supply chains: A multilevel (logistical/informational/financial) network perspective. Environment & Planning B, 29, 795- 818.

Nagurney, A., Li, D., 2014a. A dynamic network oligopoly model with transportation costs, product differentiation, and quality competition. Computational Economics, 44(2), 201-229.

Nagurney, A., Li, D., 2014b. A supply chain network game theory model with product differentiation, outsourcing of production and distribution, and quality and price competition. Annals of Operations Research, in press.

Nagurney, A., Li, D., 2014c. Equilibria and dynamics of supply chain network competition with information asymmetry in quality and minimum quality standards. Computational Management Science, 11(3), 285-315.

Nagurney, A., Li, D., Nagurney, L.S., 2014. Spatial price equilibrium with information asymmetry in quality and minimum quality standards. International Journal of Production Economics, 158, 300-313.

Nagurney, A., Li, D., Wolf, T., Saberi, S., 2013. A network economic game theory model of a service-oriented Internet with choices and quality competition. Netnomics, 14(1-2), 1-25.

Nagurney, A., Zhang, D., 1996. Projected Dynamical Systems and Variational Inequalities with Applications. Kluwer Academic Publishers, Boston, Massachusetts.

Nash, J.F., 1950. Equilibrium points in n-person games. Proceedings of the National Academy of Sciences of the United States of America (PNAS), 36, 48-49.

Nash, J.F., 1951. Noncooperative games. Annals of Mathematics, 54, 286-298.

Ronnen, U., 1991. Minimum quality standards, fixed costs, and competition. Rand Journal of Economics, 22(4), 490-504.

Saberi, S., Nagurney, A., Wolf, T., 2014. A network economic game theory model of a service-oriented Internet with price and quality competition in both content and network provision. Service Science, 6(4), 229-250.

Saxin, B., Lammgard, C., Floden, J., 2005. Meeting the demand for goods transports - Identification of flows and needs among Swedish companies, NOFOMA 2005, Copenhagen, Denmark.

Sheshinski, E., 1976. Price quality and quantity regulation in monopoly situation. Economica, 43, 127-137.

Spence, M., 1975. Monopoly, quality, and regulation. The Bell Journal of Economics, 6(2), 417-429.

Talley, W.K., Ng, M., 2013. Maritime transport chain choice by carriers, ports and shippers. International Journal of Production Economics, 142, 311-316.

Tavasszy, L., De Jong, G., 2013. Modelling Freight Transport. Elsevier, Amsterdam, The Netherlands.

United States Patent, 2013. Patent no. US 8,615,473 B2, Method and System for Anticipatory Package Shipping, Amazon Technologies Inc, 24 December, USA.

Yamada, T., Imai, K., Nakamura, T., Taniguchi, E., 2011. A supply chain-transport supernetwork equilibrium model with the behaviour of freight carriers. Transportation Research E, 47(6), 887-907.

Zamparini, L., Layaa, J., Dullaert, W., 2011. Monetary values of freight transport quality attributes: A sample of Tanzanian firms. Journal of Transport Geography, 19(6), 1222-1234.

Zhang, D., Nagurney, A., 1995. On the stability of projected dynamical systems. Journal of Optimization Theory and Applications, 85(1), 97-124.