Competitive Food Supply Chain Networks with Application to Fresh Produce

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Abstract: In this paper, we develop a network-based food supply chain model under oligopolistic competition and perishability, with a focus on fresh produce. The model incorporates food deterioration through the introduction of arc multipliers, with the inclusion of the discarding costs associated with the disposal of the spoiled food products. We allow for product differentiation due to product freshness and food safety concerns, as well as the evaluation of alternative technologies associated with various supply chain activities. We then propose an algorithm with elegant features for computation. A case study focused on the cantaloupe market is investigated within this modeling and computational framework, in which we analyze different scenarios prior/during/after a foodborne disease outbreak.

Key words: food supply chains, fresh produce, oligopolistic competition, food deterioration, product differentiation

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1. Introduction

Today, food supply chains are complex, global networks, creating pathways from farms to consumers, involving production, processing, distribution, and even the disposal of food (see Boehlje (1999), van der Vorst (2000), Aramyan et al. (2006), Monteiro (2007), Trienekens and Zuurbier (2008), and Ahumada and Villalobos (2009)). Consumers' expectation of year-around availability of fresh food products has encouraged the globalization of food markets (see Cook (2002), Monteiro (2007), Trienekens and Zuurbier (2008), and Ahumada and Villalobos (2009)). For instance, the United States is ranked number one as both importer and exporter in the international trade of horticultural commodities, accounting for about 18% of the $44 billion global horticultural trade even a decade ago (Cook (2002)). In the US alone, consumers now spend over 1.6 trillion dollars annually on food (Plunkett Research (2011)). With growing global competition (Ahumada and Villalobos (2009)), coupled with the associated greater distances between food production and consumption locations (Monteiro (2007)), there is increasing pressure for the integration of food production and distribution along the chain (Boehlje (1999) and Cook (2002)) and, hence, new challenges for food supply chain modeling and management, analysis, and solutions.

Food supply chains are distinct from other product supply chains. The fundamental difference between food supply chains and other supply chains is the continuous and significant change in the quality of food products throughout the entire supply chain until the points of final consumption (see Sloof, Tijskens, and Wilkinson (1996), van der Vorst (2000), Lowe and Preckel (2004), Ahumada and Villalobos (2009), Blackburn and Scudder (2009), Akkerman, Farahani, and Grunow (2010), and Aiello, La Scalia, and Micale (2012)). This is especially the case for fresh produce supply chains with increasing attention being placed on both freshness and safety. Clearly, many consumers prefer the freshest produce at a fair price (Cook (2002), Wilcock et al. (2004), and Lütke Entrup et al. (2005)). Moreover, statistics from the United States Department of Agriculture (USDA (2011)) suggest that the consumption of fresh vegetables has increased at a much faster pace than the demand for traditional crops such as wheat and other grains.

Given the thin profit margins in the food industries, product differentiation strategies are increasingly used in food markets (Lowe and Preckel (2004), Lusk and Hudson (2004), and Ahumada and Villalobos (2009)) with product freshness considered one of the differentiating factors (Kärkkäinen (2003) and Lütke Entrup et al. (2005)) and with a successful example being fresh-cut produce, including bagged salads, washed baby carrots, and fresh-cut melons (Cook (2002)). Retailers, such as Globus, a German retailer, are also now realizing that food freshness can be a competitive advantage (Lütke Entrup et al. (2005); see also Aiello, La

Scalia, and Micale (2012)).

Moreover, the high perishability of food products has resulted in immense food waste/loss, further stressing food supply chains and the associated quality and profitability. Some food wastage and losses are inevitable in food supply chain network links (Thompson (2002), Widodo et al. (2006), and Gustavsson et al. (2011)). However, it is estimated that approximately one third of the global food production is wasted or lost annually (Gustavsson et al. (2011)). In any country, 20% – 60% of the total amount of agricultural fresh products has been wasted or lost (Widodo et al. (2006)). Food products often require special handling, transportation, and storage technologies (Zhang, Habenicht, and Spieß (2003), Lowe and Preckel (2004), Trienekens and Zuurbier (2008), and Rong, Akkerman, and Grunow (2011)). Furthermore, the quality of food products is decreasing with time, even with the utilization of the most advanced facilities and conditions (Sloof, Tijskens, and Wilkinson (1996) and Zhang, Habenicht, and Spieß (2003)).

Such challenges have underlined the need for the efficient management of food supply chains, which is critical to profitability. Therefore, food supply chains have been receiving increasing attention. Nahmias (1982, 2011) and Silver, Pyke, and Peterson (1998) provided extensive reviews of the inventory management of perishable products. The reviews by Glen (1987) and Lowe and Preckel (2004) focused on farm planning. In addition, Lütke Entrup (2005) discussed thoroughly how to integrate shelf life into production planning within three sample food industries (yogurt, sausages, and poultry). Akkerman, Farahani, and Grunow (2010) outlined quantitative operations management applications in food distribution management. The survey by Lucas and Chhajed (2004) presented applications related to location problems in agriculture and recognized the challenges of strategic production-distribution planning problems in the agricultural industry. Due to the added complexity caused by food perishability, there are fewer articles related to perishable food products than those related to non-perishable ones, and even fewer models developed for fully integrated supply chain system approaches (Ahumada and Villalobos (2009)), which is the focus of this paper.

We now describe some of the contributions in the literature, which aim to integrate and synthesize two or more processes associated with food supply chains. Zhang, Habenicht, and Spieß (2003) studied a physical distribution system in order to minimize the total cost for storage and shipment with the product quality requirement fulfilled. Widodo et al. (2006) developed mathematical models dealing with flowering-harvesting and harvesting-delivering problems of agricultural fresh products by introducing a plant maturing curve and a loss function to address, respectively, the growing process and the decaying process of the fresh products. Ahumada and Villalobos (2011) discussed the packing and distribution problem of fresh produce, with the inclusion of perishability. They handled the perishability of the crops through storage constraints, and used a loss function in the objective function. In addition, Kopanos, Puigjaner, and Georgiadis (2012) studied the production-distribution planning problem of a multisite, multiproduct, semicontinuous food processing industry within the framework of mixed integer programming.

As noted by van der Vorst (2006), it is imperative to analyze food supply chains within the context of the full complexity of their network structure. Monteiro (2007) claimed that the theory of network economics (cf. Nagurney (1999)) provides a powerful mathematical framework in which the supply chain can be graphically represented and analyzed. He further adopted the theory of network economics to study the economics of traceability in food supply chains theoretically. Blackburn and Scudder (2009) suggested a cost minimization model for specific perishable product supply chain design, capturing the declining value of the product over time. They noticed that product value deteriorates significantly over time at rates that highly depend on temperature and humidity. Rong, Akkerman, and Grunow (2011), in turn, presented a mixed integer linear programming model for the planning of food production and distribution with a focus on product quality, which is strongly related to temperature control throughout the supply chain.

Liu and Nagurney (2012) developed a multiperiod supply chain network equilibrium model. That model can address perishability of products through changes in the underlying network topologies. Nagurney and Aronson (1989), Masoumi, Yu, and Nagurney (2012), Nagurney, Masoumi, and Yu (2012), Nagurney and Masoumi (2012), and Nagurney and Nagurney (2012) have adopted arc multipliers to capture the perishability/waste of product flows in a network. The latter three studies developed a system-optimization approach from a single firm/organization's perspective. Nagurney and Aronson (1989) constructed a general dynamic spatial price equilibrium model, which can handle perishable products through the use of arc multipliers. Masoumi, Yu, and Nagurney (2012) studied a generalized oligopoly model with particular relevance to the pharmaceutical industry.

In this paper, we study the food supply chain management problem from a network perspective, with the inclusion of food deterioration. We focus on fresh produce items, such as vegetables and fruits, with simple or limited required processing, whose life cycle can be measured in days. The fresh produce supply chain network oligopoly model developed in this paper is distinct from other studies on perishable food products in several ways:

1. We capture the deterioration of fresh food along the entire supply chain from a network

perspective;

2. We handle the exponential time decay through the introduction of arc multipliers (more details are given in Section 2 as to how to determine the arc multipliers);

3. We study oligopolistic competition with product differentiation;

4. We include the disposal of the spoiled food products, along with the associated costs;

5. We allow for the assessment of alternative technologies involved in each supply chain activity.

We emphasize that with appropriate modifications, the model will be also applicable to supply chain management of other perishable products under oligopolistic competition, and even with quality competition.

This paper is organized as follows. In Section 2, we develop the new fresh produce supply chain network oligopoly model and derive variational inequality formulations. We also provide some qualitative properties. In Section 3, we present the computational algorithm, which we then apply to a case study focused on fresh produce in the case of cantaloupes in Section 4. We summarize our results and present our conclusions in Section 5.

2. The Fresh Produce Supply Chain Network Oligopoly Model

In this Section, we consider a finite number of I food firms, with a typical firm denoted by i. The food supply chain network activities include the production, processing, storage, distribution, and disposal of the food products. The food firms compete noncooperatively in an oligopolistic manner. We allow for product differentiation by consumers at the demand markets, due to product freshness and food safety concerns that may be associated with a particular firm. In other words, the fresh food products are not necessarily homogeneous. Each firm seeks to determine its optimal product flows throughout its entire supply chain network by using Figure 1 as a schematic.

Each food firm is represented as a network of its economic activities. Each food firm i; $i = 1, \ldots, I$ possesses n^i_M production facilities, n^i_C processors, and n^i_D distribution centers, in order to satisfy the demands at n_R demand markets. Let $G = [N, L]$ denote the graph consisting of the set of nodes N and the set of links L in Figure 1; and $L \equiv \cup_{i=1,\ldots,I} L^i$, where L^i denotes the set of directed links corresponding to the sequence of activities associated with firm i.

The first set of links connecting the top two tiers of nodes corresponds to the food production at each of the production units of firm i; $i = 1, \ldots, I$, which may involve such a sequence of seasonal operations as soil agitation, sowing, pest control, nutrient and water management, and harvesting. The multiple possible links connecting each top tier node i with its production facilities, $M_1^i, \ldots, M_{n_M^i}^i$, capture different possible production technologies that may be associated with a given facility.

The second set of links from the production facility nodes is connected to the processors of each firm i; $i = 1, \ldots, I$, which are denoted by $C_{1,1}^i, \ldots, C_{n_C^i,1}^i$. These links correspond to the shipment links between the production units and the processors. The alternative shipment links denote different possible modes of transportation, which represent the varying time durations and environmental conditions associated with the shipment links.

The third set of links connecting nodes $C_{1,1}^i, \ldots, C_{n_C^i,1}^i$ to $C_{1,2}^i, \ldots, C_{n_C^i,2}^i$; $i = 1, \ldots, I$ denotes the processing of fresh produce. In this paper, the major food processing activities are cleaning, sorting, labeling, and simple packaging. Different processing technologies may result in dissimilar levels of quality degradation associated with the processing activities.

The next set of nodes represents the distribution centers, and, thus, the fourth set of links connecting the processor nodes to the distribution centers is the set of shipment links. Such distribution nodes associated with firm i; $i = 1, \ldots, I$ are denoted by $D_{1,1}^i, \ldots, D_{n_D^i,1}^i$. There are also multiple shipment links, in order to capture different modes of transportation.

The fifth set of links, in turn, connects nodes $D_{1,1}^i, \ldots, D_{n_D^i,1}^i$ to $D_{1,2}^i, \ldots, D_{n_D^i,2}^i$; $i = 1, \ldots, I$, which represents the storage links. Since fresh produce items may require different storage conditions, we represent these alternatives through multiple links at this tier.

The last set of links connecting the two bottom tiers of the supply chain network corresponds to distribution links over which the stored fresh produce items are shipped from the distribution centers to the demand markets. Here we also allow for multiple modes of transportation.

In addition, the curved links joining the top-tiered nodes i with the processors, which are denoted by $C_{1,2}^i, \ldots, C_{n_C^i,2}^i$; $i = 1, \ldots, I$, capture the possibility of on-site production and processing.

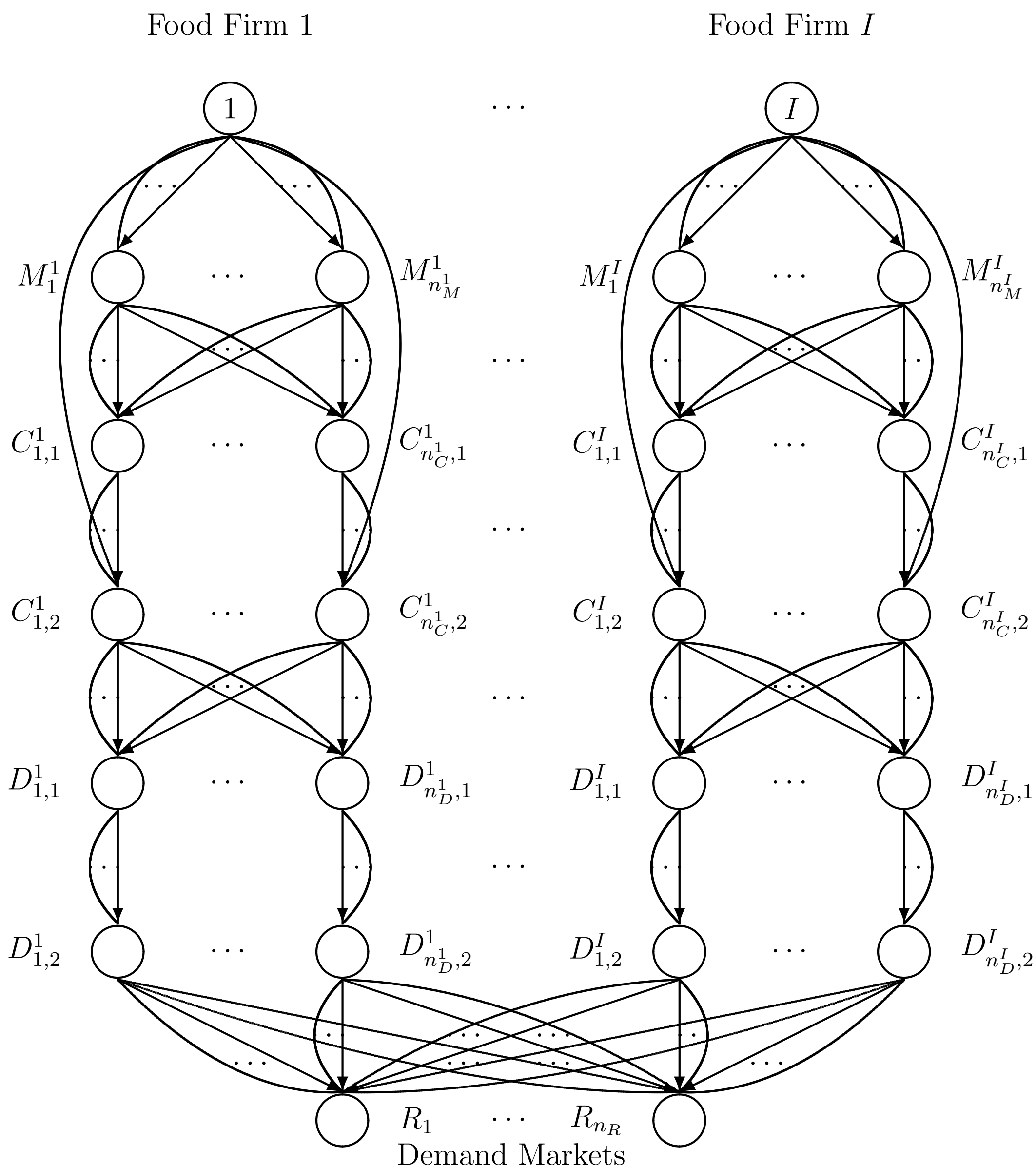

Figure 1: The Fresh Produce Supply Chain Network Topology

Most of fresh produce items reach their peak quality at the time of production, and then deteriorate substantially over time (Blackburn and Scudder (2009)). Microbiological decay is one of the major causes of the food quality degradation, especially for the fresh produce (Fu and Labuza (1993)). Therefore, food deterioration usually follows the first-order reactions with exponential time decay (see Ghare and Schrader (1963), Labuza (1982), Nahmias (1982), Tijskens and Polderdijk (1996), Blackburn and Scudder (2009), Nga (2010), and Rong, Akkerman, and Grunow (2011)). In contrast to fixed lifetime perishability, exponential time decay is a special case of random lifetime perishability, which means that the time to spoilage is uncertain (Nahmias (1982); see also van Zyl (1964)). It also has been recognized that the decay rate varies significantly with different temperatures and under other environmental conditions (Blackburn and Scudder (2009) and Rong, Akkerman, and Grunow (2011)). Hence, based on various temperature requirements, food supply chains can be grouped into three types: frozen, chilled, and ambient. The normal temperature of the frozen chain is $-18°$C, while temperatures range from $0°$C for fresh fish to $15°$C for, e.g., potatoes and bananas for the chilled chain (Smith and Sparks (2004) and Akkerman, Farahani, and Grunow (2010)). There is no required temperature control in an ambient chain (Akkerman, Farahani, and Grunow (2010)).

In the existing literature on perishability, exponential time decay has been utilized, in order to describe either the decrease in quantity or the degradation in quality. The decrease in quantity, which has been discussed in studies on perishable inventory (see Nahmias (1982)), represents the number of units of decayed products (e.g. vegetables and fruits), while the degradation in quality emphasizes that all the products deteriorate at the same rate simultaneously (see Tijskens and Polderdijk (1996), Blackburn and Scudder (2009), and Rong, Akkerman, and Grunow (2011)), which is more relevant to meat, dairy, and bakery products. With a focus on such fresh produce items as vegetables and fruits, our model adopts exponential time decay so as to capture the discarding of spoiled products associated with all the post-production supply chain activities (see Thompson (2002) and Gustavsson et al. (2011)).

As mentioned in Section 1, the food products deteriorate over time even under optimal conditions. We assume that the temperature and other environmental conditions associated with each post-production activity/link are given and fixed. Following Nahmias (1982), we assume that each unit has a probability of $e^{-\lambda t}$ to survive another t units of time, where λ is the decay rate, which is given and fixed. Let N_0 denote the quantity at the beginning of the time interval (link). Then, the quantity surviving at the end of the time interval (which is reflected in each link in our network) follows a binomial distribution with parameters $n = N_0$ and $p = e^{-\lambda t}$. Hence, the expected quantity surviving at the end of the time interval (specific link), denoted by $N(t)$, can be expressed as:

$$N(t) = N_0 e^{-\lambda t}. \tag{1}$$

As in Nagurney, Masoumi, and Yu (2012) (see also Nagurney and Masoumi (2012), Masoumi, Yu, and Nagurney (2012), and Nagurney and Nagurney (2012)), we can assign a multiplier to each post-production link in the supply chain network, be it a processing link, a shipment/distribution link, or a storage link, in order to capture the decay in number of units. Let α_a denote the throughput factor associate with every link a in the supply chain network, which lies in the range of $(0, 1]$. Therefore, only $\alpha_a \times 100\%$ of the initial flow of product on link a reaches the successor node of that link.

Hence, we can represent the throughput factor α_a for a post-production link a as:

$$\alpha_a = e^{-\lambda_a t_a}, \tag{2}$$

where λ_a and t_a are the decay rate and the time duration associated with the link a, respectively, which are given and fixed. We assume that the value of α_a for a production link is equal to 1. In rare cases, food deterioration follows the zero order reactions with linear decay (see Tijskens and Polderdijk (1996) and Rong, Akkerman, and Grunow (2011)). Then, $\alpha_a = 1 - \lambda_a t_a$ for a post-production link.

Let f_a denote the (initial) flow of product on link a; and f'_a denote the final flow on link a; i.e., the flow that reaches the successor node of the link after deterioration has taken place. Therefore,

$$f'_a = \alpha_a f_a, \quad \forall a \in L. \tag{3}$$

Consequently, the number of units of the spoiled fresh produce on link a is the difference between the initial and the final flow, $f_a - f'_a$, where

$$f_a - f'_a = (1 - \alpha_a) f_a, \quad \forall a \in L. \tag{4}$$

Associated with the food deterioration is a total discarding cost function, $\hat{z}_a$, which, in view of (4), is a function of flow on the link, f_a, that is,

$$\hat{z}_a = \hat{z}_a(f_a), \quad \forall a \in L, \tag{5}$$

which is assumed to be convex and continuously differentiable. In developed countries, the overall average losses of fruits and vegetables during post-production supply chain activities are approximately 12% of the initial production (Gustavsson et al. (2011) and Aiello, La Scalia, and Micale (2012)). The losses in developing regions are even severer (Gustavsson et al. (2011)). It is imperative to remove the spoiled fresh food products from the supply chain network. For instance, fungi are the common post-production diseases of fresh fruits and vegetables, which can colonize the fruits and vegetables rapidly (Sommer, Fortlage, and Edwards (2002)). Here, we mainly focus on the disposal of the decayed food products at the processing, storage, and distribution stages (see also Thompson (2002)).

Here x_p represents the (initial) flow of product on path p joining an origin node, i, with a destination node, R_k. The path flows must be nonnegative:

$$x_p \geq 0, \quad \forall p \in P^i_k;\ i = 1, \ldots, I;\ k = 1, \ldots, n_R, \tag{6}$$

where P_k^i is the set of all paths joining the origin node i; $i = 1, \ldots, I$ with destination node R_k.

We define the multiplier, α_{ap}, which is the product of the multipliers of the links on path p that precede link a in that path, as follows:

$$\alpha_{ap} \equiv \begin{cases} \delta_{ap} \prod_{b \in \{a' < a\}_p} \alpha_b, & \text{if } \{a' < a\}_p \neq \emptyset, \\ \\ \delta_{ap}, & \text{if } \{a' < a\}_p = \emptyset, \end{cases} \tag{7}$$

where $\{a' < a\}_p$ denotes the set of the links preceding link a in path p, and $\emptyset$ denotes the null set. In addition, δ_{ap} is defined as equal to 1 if link a is contained in path p, and 0, otherwise. If link a is not contained in path p, then α_{ap} is set to zero. Hence, the relationship between the link flow, f_a, and the path flows can be expressed as:

$$f_a = \sum_{i=1}^{I} \sum_{k=1}^{n_R} \sum_{p \in P_k^i} x_p \alpha_{ap}, \quad \forall a \in L. \tag{8}$$

Let μ_p denote the multiplier corresponding to the throughput on path p, defined as the product of all link multipliers on links comprising that path:

$$\mu_p \equiv \prod_{a \in p} \alpha_a, \quad \forall p \in P_k^i; \ i = 1, \ldots, I; \ k = 1, \ldots, n_R. \tag{9}$$

The demand for food firm i's fresh food product; $i = 1, \ldots, I$, at demand market R_k; $k = 1, \ldots, n_R$, denoted by d_{ik}, is equal to the sum of all the final flows – subject to perishability – on paths joining (i, R_k):

$$\sum_{p \in P_k^i} x_p \mu_p = d_{ik}, \quad i = 1, \ldots, I; \ k = 1, \ldots, n_R. \tag{10}$$

The consumers may differentiate the fresh food products, due to food safety and health concerns. We group the demands d_{ik}; $i = 1, \ldots, I$; $k = 1, \ldots, n_R$ into the $I \times n_R$-dimensional vector d.

We denote the demand price of food firm i's product at demand market R_k by ρ_{ik} and assume that

$$\rho_{ik} = \rho_{ik}(d), \quad i = 1, \ldots, I; \ k = 1, \ldots, n_R. \tag{11}$$

Note that the price of food firm i's product at a particular demand market may depend not only on the demands for its product at the other demand markets, but also on the demands for the other substitutable food products at all the demand markets. These demand price functions are assumed to be continuous, continuously differentiable, and monotone decreasing.

In order to address the competition among various food firms for resources used in the production, processing, storage, and distribution of the fresh produce, we assume that the total operational cost on link a, in general, depend upon the product flows on all the links, that is,

$$\hat{c}_a = \hat{c}_a(f), \quad \forall a \in L, \tag{12}$$

where f is the vector of all the link flows. The total cost on each link is assumed to be convex and continuously differentiable.

Let X_i denote the vector of path flows associated with firm i; $i = 1, \ldots, I$, where $X_i \equiv \{\{x_p\}|p \in P^i\}\} \in R_+^{n_{P^i}}$, $P^i \equiv \cup_{k=1,\ldots,n_R} P_k^i$, and n_{P^i} denotes the number of paths from firm i to the demand markets. Thus, X is the vector of all the food firms' strategies, that is, $X \equiv \{\{X_i\}|i = 1, \ldots, I\}$.

The profit function of a food firm is defined as the difference between its revenue and it total costs, where the total costs are composed of the total operational costs as well as the total discarding costs of spoiled food products over the post-production links in the supply chain network. Hence, the profit function of firm i, denoted by U_i, is expressed as:

$$U_i = \sum_{k=1}^{n_R} \rho_{ik}(d) d_{ik} - \sum_{a \in L^i} \Big(\hat{c}_a(f) + \hat{z}_a(f_a) \Big). \tag{13}$$

Of course, depending on the fresh food product, as well as on the firm, discarding may be done on only certain links of the firm's supply chain network. The inclusion of both the operational costs and the discarding costs in (13) allows for the optimal selection of alternative technologies associated with various supply chain activities since particular links in Figure 1 represent distinct technologies.

In view of (10), we may write:

$$\hat{\rho}_{ik}(x) = \rho_{ik}(d), \quad i = 1, \ldots, I; \; k = 1, \ldots, n_R. \tag{14}$$

In lieu of the conservation of flow expressions (8) and (10), and the functional expressions (5), (12), and (14), we may define $\hat{U}_i(X) = U_i$ for all firms i; $i = 1, \ldots, I$, with the I-dimensional vector $\hat{U}$ being the vector of the profits of all the firms:

$$\hat{U} = \hat{U}(X). \tag{15}$$

In the Cournot-Nash oligopolistic market framework, each firm selects its product path flows in a noncooperative manner, seeking to maximize its own profit, until an equilibrium is achieved, according to the definition below.

Definition 1: Supply Chain Network Cournot-Nash Equilibrium

A path flow pattern $X^* \in K = \prod_{i=1}^{I} K_i$ constitutes a supply chain network Cournot-Nash equilibrium if for each firm i; $i = 1, \dots, I$:

$$\hat{U}_i(X_i^*, \hat{X}_i^*) \geq \hat{U}_i(X_i, \hat{X}_i^*), \quad \forall X_i \in K_i, \tag{16}$$

where $\hat{X}_i^* \equiv (X_1^*, \dots, X_{i-1}^*, X_{i+1}^*, \dots, X_I^*)$ and $K_i \equiv \{X_i | X_i \in R_+^{n_{P^i}}\}$.

In other words, an equilibrium is established if no firm can unilaterally improve its profit by changing its product flows throughout its supply chain network, given the product flow decisions of the other firms.

Next, we derive the variational inequality formulations of the Cournot-Nash equilibrium for the fresh produce supply chain network under oligopolistic competition satisfying Definition 1, in terms of both path flows and link flows (see Cournot (1838), Nash (1950, 1951), Gabay and Moulin (1980), and Nagurney (2006)).

Theorem 1

Assume that, for each food firm i; $i = 1, \dots, I$, the profit function $\hat{U}_i(X)$ is concave with respect to the variables in X_i, and is continuously differentiable. Then $X^* \in K$ is a supply chain network Cournot-Nash equilibrium according to Definition 1 if and only if it satisfies the variational inequality:

$$-\sum_{i=1}^{I} \langle \nabla_{X_i} \hat{U}_i(X^*)^T, X_i - X_i^* \rangle \geq 0, \quad \forall X \in K, \tag{17}$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in the corresponding Euclidean space and $\nabla_{X_i} \hat{U}_i(X)$ denotes the gradient of $\hat{U}_i(X)$ with respect to X_i. Variational inequality (17), in turn, for our model, is equivalent to the variational inequality: determine the vector of equilibrium path flows $x^* \in K^1$ such that:

$$\sum_{i=1}^{I} \sum_{k=1}^{n_R} \sum_{p \in P_k^i} \left[\frac{\partial \hat{C}_p(x^*)}{\partial x_p} + \frac{\partial \hat{Z}_p(x^*)}{\partial x_p} - \hat{\rho}_{ik}(x^*)\mu_p - \sum_{l=1}^{n_R} \frac{\partial \hat{\rho}_{il}(x^*)}{\partial x_p} \sum_{q \in P_l^i} \mu_q x_q^* \right] \times [x_p - x_p^*] \geq 0,$$

$$\forall x \in K^1, \tag{18}$$

where $K^1 \equiv \{x | x \in R_+^{n_P}\}$, *and for each path* p; $p \in P_k^i$; $i = 1, \ldots, I$; $k = 1, \ldots, n_R$,

$$\frac{\partial \hat{C}_p(x)}{\partial x_p} \equiv \sum_{a \in L^i} \sum_{b \in L^i} \frac{\partial \hat{c}_b(f)}{\partial f_a} \alpha_{ap}, \; \frac{\partial \hat{Z}_p(x)}{\partial x_p} \equiv \sum_{a \in L^i} \frac{\partial \hat{z}_a(f_a)}{\partial f_a} \alpha_{ap}, \; and \; \frac{\partial \hat{\rho}_{il}(x)}{\partial x_p} \equiv \frac{\partial \rho_{il}(d)}{\partial d_{ik}} \mu_p. \quad (19)$$

Variational inequality (18) can also be re-expressed in terms of link flows as: determine the vector of equilibrium link flows and the vector of equilibrium demands $(f^*, d^*) \in K^2$, *such that:*

$$\sum_{i=1}^{I} \sum_{a \in L^i} \left[\sum_{b \in L^i} \frac{\partial \hat{c}_b(f^*)}{\partial f_a} + \frac{\partial \hat{z}_a(f_a^*)}{\partial f_a} \right] \times [f_a - f_a^*]$$

$$+ \sum_{i=1}^{I} \sum_{k=1}^{n_R} \left[-\rho_{ik}(d^*) - \sum_{l=1}^{n_R} \frac{\partial \rho_{il}(d^*)}{\partial d_{ik}} d_{il}^* \right] \times [d_{ik} - d_{ik}^*] \geq 0, \quad \forall (f, d) \in K^2, \quad (20)$$

where $K^2 \equiv \{(f, d) | x \geq 0, \mathit{and} \, (8) \, \mathit{and} \, (10) \, \mathit{hold}\}$.

Proof: See Masoumi, Yu, and Nagurney (2012).

Variational inequalities (18) and (20) can be put into standard form (see Nagurney (1999)): determine $X^* \in \mathcal{K}$ such that:

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \quad (21)$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in n-dimensional Euclidean space. Let $X \equiv x$ and

$$F(X) \equiv \left[\frac{\partial \hat{C}_p(x)}{\partial x_p} + \frac{\partial \hat{Z}_p(x)}{\partial x_p} - \hat{\rho}_{ik}(x) \mu_p - \sum_{l=1}^{n_R} \frac{\partial \hat{\rho}_{il}(x)}{\partial x_p} \sum_{q \in P_l^i} \mu_q x_q; \right.$$

$$\left. p \in P_k^i; \; i = 1, \ldots, I; \; k = 1, \ldots, n_R \right], \quad (22)$$

and $\mathcal{K} \equiv K^1$, then (18) can be re-expressed as (21). Similarly, for the variational inequality in terms of link flows, if we define the column vectors: $X \equiv (f, d)$ and $F(X) \equiv (F_1(X), F_2(X))$, where

$$F_1(X) = \left[\sum_{b \in L^i} \frac{\partial \hat{c}_b(f)}{\partial f_a} + \frac{\partial \hat{z}_a(f_a)}{\partial f_a}; \; a \in L^i; \; i = 1, \ldots, I \right],$$

$$F_2(X) = \left[-\rho_{ik}(d) - \sum_{l=1}^{n_R} \frac{\partial \rho_{il}(d)}{\partial d_{ik}} d_{il}; \; i = 1, \ldots, I; \; k = 1, \ldots, n_R \right], \quad (23)$$

and $\mathcal{K} \equiv K^2$, then (20) can be re-written as (21).

Since the feasible set K^1 is not compact, and the same holds for K^2, we cannot obtain the existence of a solution simply based on the assumption of the continuity of F. However, the demand d_{ik} for each food firm i's product; $i = 1, \ldots, I$ at every demand market R_k; $k = 1, \ldots, n_R$, may be assumed to be bounded, since the population requiring these products is finite (although it may be large). Consequently, in light of (10), we have that:

$$\mathcal{K}_b \equiv \{x| \, 0 \leq x \leq b, \}, \tag{24}$$

where $b > 0$ and $x \leq b$ means that $x_p \leq b$ for all $p \in P^i_k$; $i = 1, \ldots, I$, and $k = 1, \ldots, n_R$. Then $\mathcal{K}_b$ is a bounded, closed, and convex subset of K^1. Thus, the following variational inequality

$$\langle F(X^b)^T, X - X^b \rangle \geq 0, \quad \forall X \in \mathcal{K}_b, \tag{25}$$

admits at least one solution $X^b \in \mathcal{K}_b$, since $\mathcal{K}_b$ is compact and F is continuous. Therefore, following Kinderlehrer and Stampacchia (1980) (see also Theorem 1.5 in Nagurney (1999)), we have the following theorem:

Theorem 2: Existence

There exists at least one solution to variational inequality (18) (equivalently, to (20)), since there exists a $b > 0$, such that variational inequality (25) admits a solution in $\mathcal{K}_b$ with

$$x^b \leq b. \tag{26}$$

In addition, we now provide a uniqueness result.

Theorem 3: Uniqueness

With Theorem 2, variational inequality (25) and, hence, variational inequality (20) admits at least one solution. Moreover, if the function $F(X)$ of variational inequality (20), as defined in (23), is strictly monotone on $\mathcal{K} \equiv K^2$, that is,

$$\langle (F(X^1) - F(X^2))^T, X^1 - X^2 \rangle > 0, \quad \forall X^1, X^2 \in \mathcal{K}, \, X^1 \neq X^2, \tag{27}$$

then the solution to variational inequality (20) is unique, that is, the equilibrium link flow pattern and the equilibrium demand pattern are unique.

Our proposed supply chain network model can also be applied to other fresh food supply chain oligopoly problems under quality competition, which is highly relevant to meat, dairy, and bakery products. In such cases, all the food products get delivered to the demand markets eventually, with distinct levels of quality degradation. Thus, the arc multiplier for a post-production link, α_a, captures the corresponding food quality degradation associated with that link, instead of the number of the spoiled products. We refer to Labuza (1982) and Man and Jones (1994) for thorough discussions about food quality deterioration.

3. The Algorithm

We now recall the Euler method, which is induced by the general iterative scheme of Dupuis and Nagurney (1993). Specifically, at an iteration τ of the Euler method (see also Nagurney and Zhang (1996)) one computes:

$$X^{\tau+1} = P_{\mathcal{K}}(X^\tau - a_\tau F(X^\tau)), \tag{28}$$

where $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$ and F is the function that enters the variational inequality problem (21).

As shown in Dupuis and Nagurney (1993) and Nagurney and Zhang (1996), for convergence of the general iterative scheme, which induces the Euler method, the sequence $\{a_\tau\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \to 0$, as $\tau \to \infty$. Specific conditions for convergence of this scheme as well as various applications to the solutions of network oligopolies can be found in Nagurney and Zhang (1996), Nagurney, Dupuis, and Zhang (1994), Nagurney (2010), Nagurney and Yu (2012), and Masoumi, Yu, and Nagurney (2012).

Explicit Formulae for the Euler Method Applied to the Fresh Produce Supply Chain Network Oligopoly Variational Inequality (18)

The elegance of this procedure for the computation of solutions to the fresh produce supply chain network oligopoly problem can be seen in the following explicit formulae. In particular, we have the following closed form expressions for the fresh produce path flows:

$$x_p^{\tau+1} = \max\{0, x_p^\tau + a_\tau(\hat{\rho}_{ik}(x^\tau)\mu_p + \sum_{l=1}^{n_R} \frac{\partial \hat{\rho}_{il}(x^\tau)}{\partial x_p} \sum_{q \in P_l^i} \mu_q x_q^\tau - \frac{\partial \hat{C}_p(x^\tau)}{\partial x_p} - \frac{\partial \hat{Z}_p(x^\tau)}{\partial x_p})\},$$

$$\forall p \in P_k^i;\ i = 1, \ldots, I;\ k = 1, \ldots, n_R. \tag{29}$$

The number of strategic variables x_p, as well as the number of the paths, in the supply chain network grows linearly in terms of the number of nodes in the supply chain network, be it a production facility, a processor, a distribution center, or a demand market. Therefore, even a fresh produce supply chain network with hundreds of demand markets is still tractable within our proposed modeling and computational framework.

In the next Section, we solve fresh produce supply chain network oligopoly problems using the above algorithmic scheme.

4. Case Study

In this Section, we focus on the cantaloupe market in the United States. Most of cantaloupes consumed in the United States are originally produced in California, Mexico, and in some countries in Central America. In our case study, there are two firms, Firm 1 and Firm 2, which may represent, for example, one food firm in California and one food firm in Central America, respectively. Each firm has two production sites, one processor, two distribution centers, and serves two geographically separated demand markets, as depicted in Figure 2. The production sites and the processor of Firm 1 are located in California, whereas the production sites and the processor of Firm 2 are located in Central America, with lower operational costs. However, all the distribution centers are located in the United States as are the demand markets. The distribution centers D_1^1 and D_1^2 are located closer to their respective production sites than the distribution centers D_2^1 and D_2^2 are. The demand market R_1 is located closer to the distribution centers D_1^1 and D_1^2, whereas the demand market R_2 is located closer to the distribution centers D_2^1 and D_2^2, that is, the demand market R_1 is located closer to the production sites.

Typically, cantaloupes can be stored for 12–15 days at 2.2° to 5°C (36° to 41°F) (Suslow, Cantwell, and Mitchell (1997)). It has been noticed that the decay of cantaloupes may result from such post-production disease as Rhizopus, Fusarium, Geotrichum, etc., depending on the season, the region, and the handling technologies utilized between production and consumption (see Suslow, Cantwell, and Mitchell (1997) and Sommer, Fortlage, and Edwards (2002)). As discussed in Section 2, we captured the food deterioration through the arc multipliers. The values of the decay rates and the time durations, although hypothetical, were selected so as to reflect the different technologies associated with the various supply chain activities. The values of the arc multipliers were, in turn, calculated using equation (2), which captured the percentage of the spoiled fresh food products at the post-production supply chain stages (see, e.g., Gustavsson et a l. (2011)). For instance, Firm 1 utilizes more effective cleaning and sanitizing equipment for its processing activities of the cantaloupes, which results in relatively higher operational costs, but lower decay rates associated with the successive supply chain activities.

We implemented the Euler method (cf. (29)) for the solution of variational inequality (18), using Matlab. We set the sequence $a_\tau = .1(1, \frac{1}{2}, \frac{1}{2}, \cdots)$, and the convergence tolerance was 10^{-6}. In other words, the absolute value of the difference between each path flow in two consecutive iterations was less than or equal to this tolerance. We initialized the algorithm by setting the path flows equal to 20.

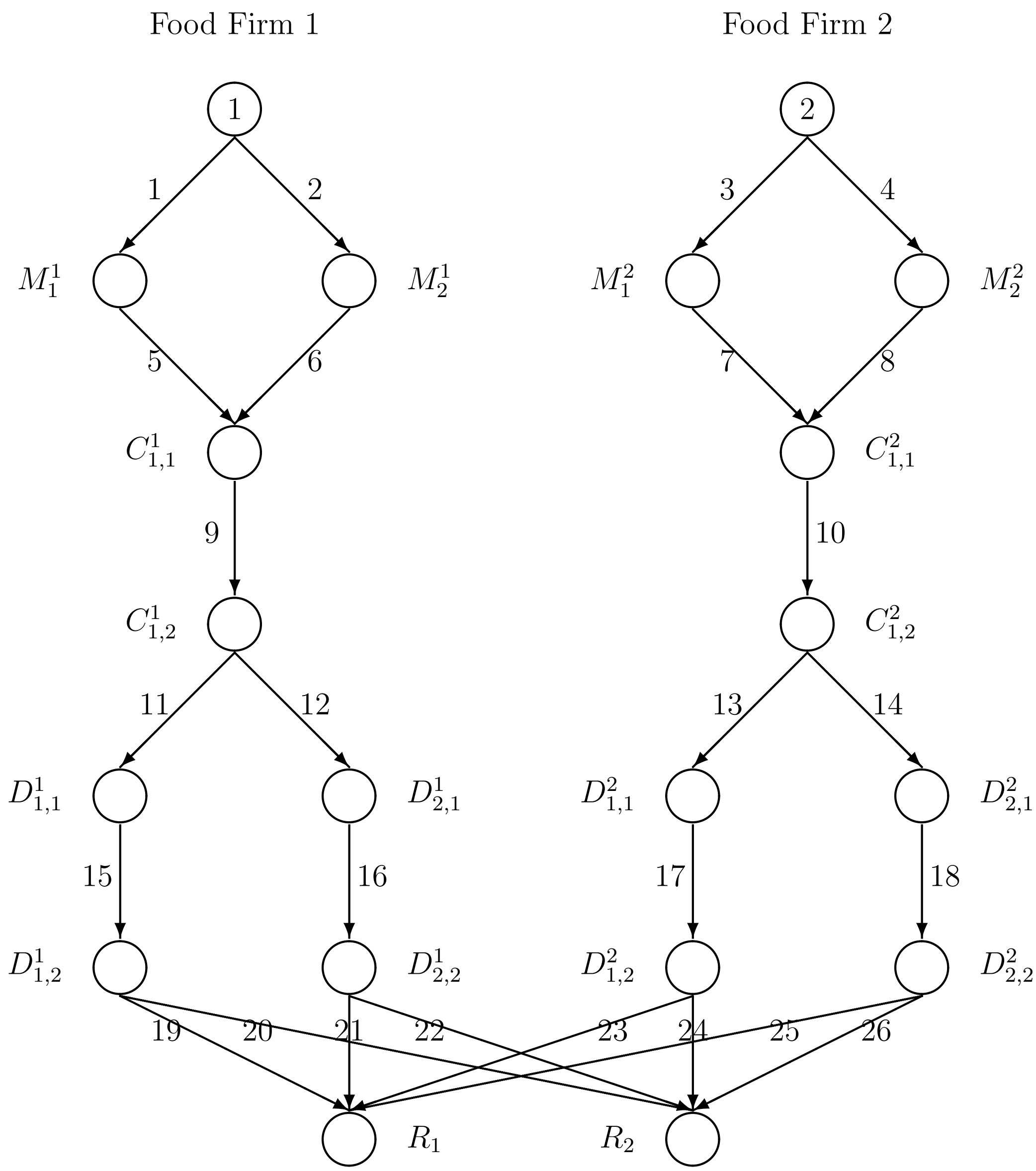

Figure 2: The Fresh Produce Supply Chain Network Topology for the Case Study

Case 1

In Case 1, we assumed that consumers at the demand markets were indifferent between cantaloupes of Firm 1 and Firm 2. Furthermore, consumers at demand market R_2 were willing to pay relatively more as compared to those at demand market R_1. The corresponding demand price functions were as follows:

$$\text{Firm 1:} \quad \rho_{11} = -.0001d_{11} - .0001d_{21} + 4, \quad \rho_{12} = -.0001d_{12} - .0001d_{22} + 6;$$

$$\text{Firm 2:} \quad \rho_{21} = -.0001d_{21} - .0001d_{11} + 4, \quad \rho_{22} = -.0001d_{22} - .0001d_{12} + 6.$$

The arc multipliers, the total operational cost functions, and the total discarding cost functions are reported in Table 1, as well as the decay rates (/day) and the time durations (days) associated with all the links. These cost functions have been constructed based on the data of the average costs available on the web (see, e.g., Meister (2004a, 2004b)).

Table 1 also provides the computed equilibrium product flows on all the links in Figure 2. The computed equilibrium demands for cantaloupes were:

$$d_{11}^* = 7.86, \quad d_{12}^* = 123.62, \quad d_{21}^* = 27.19, \quad \text{and} \quad d_{22}^* = 139.38.$$

The incurred equilibrium prices at each demand market were as follows:

$$\rho_{11} = 4.00, \quad \rho_{12} = 5.97, \quad \rho_{21} = 4.00, \quad \text{and} \quad \rho_{22} = 5.97.$$

Furthermore, the profits of two firms were:

$$U_1 = 370.46, \quad \text{and} \quad U_2 = 454.72.$$

Since consumers do not differentiate the cantaloupes produced by these two firms, the prices of these two firms' cantaloupes at each demand market are identical. Due to the difference in consumers' willingness to pay, the price at demand market R_1 is relatively lower than the price at demand market R_2. Consequently, the distribution links: 21 and 25, connecting Firm 1 and Firm 2 to demand market R_1, respectively, have zero product flows. In other words, there is no shipment from distribution centers D_2^1 and D_2^2 to demand market R_1. In addition, the volume of product flows on distribution link 22 (or link 26) is higher than that of distribution link 20 (or link 24), which indicates that it is more cost-effective to provide fresh fruits from the nearby distribution centers. As a result of its lower operational costs, Firm 2 dominates both of these two demand markets, leading to a substantially higher profit.

Case 2

The Centers for Disease Control (CDC) has reported 23 cantaloupe-associated outbreaks between 1984 and 2002, which resulted in $1,434$ people falling ill, 42 hospitalizations, and 2 deaths (Bowen et al. (2006)). In Case 2, we considered the scenario that the CDC reported a multi-state cantaloupe-associated outbreak. Due to food safety and health concerns, the regular consumers of cantaloupes switched to other fresh fruits. The demand price functions were no longer as in Case 1 and were given by:

$$\text{Firm 1:} \quad \rho_{11} = -.001d_{11} - .001d_{21} + .5, \quad \rho_{12} = -.001d_{12} - .001d_{22} + .5;$$

Table 1: Arc Multipliers, Total Operational Cost and Total Discarding Cost Functions, and Equilibrium Link Flow Solution for Case 1

Link a	λ_a	t_a	α_a	$\hat{c}_a(f)$	$\hat{z}_a(f_a)$	f_a^*
1	–	–	1.00	$.005f_1^2 + .03f_1$	0.00	76.32
2	–	–	1.00	$.006f_2^2 + .02f_2$	0.00	75.73
3	–	–	1.00	$.001f_3^2 + .02f_3$	0.00	103.74
4	–	–	1.00	$.001f_4^2 + .02f_4$	0.00	105.62
5	.150	0.20	.970	$.003f_5^2 + .01f_5$	0.00	76.32
6	.150	0.25	.963	$.002f_6^2 + .02f_6$	0.00	75.73
7	.150	0.30	.956	$.001f_7^2 + .02f_7$	0.00	103.74
8	.150	0.30	.956	$.001f_8^2 + .01f_8$	0.00	105.62
9	.040	0.50	.980	$.002f_9^2 + .05f_9$	$.001f_9^2 + 0.02f_9$	147.01
10	.060	0.50	.970	$.001f_{10}^2 + .02f_{10}$	$.001f_{10}^2 + 0.02f_{10}$	200.14
11	.015	1.50	.978	$.005f_{11}^2 + .01f_{11}$	0.00	65.98
12	.015	3.00	.956	$.01f_{12}^2 + .01f_{12}$	0.00	78.12
13	.025	2.00	.951	$.005f_{13}^2 + .02f_{13}$	0.00	96.47
14	.025	4.00	.905	$.01f_{14}^2 + .01f_{14}$	0.00	97.76
15	.010	3.00	.970	$.004f_{15}^2 + .01f_{15}$	$.001f_{15}^2 + 0.02f_{15}$	64.51
16	.010	3.00	.970	$.004f_{16}^2 + .01f_{16}$	$.001f_{16}^2 + 0.02f_{16}$	74.68
17	.015	3.00	.956	$.004f_{17}^2 + .01f_{17}$	$.001f_{17}^2 + 0.02f_{17}$	91.77
18	.015	3.00	.956	$.004f_{18}^2 + .01f_{18}$	$.001f_{18}^2 + 0.02f_{18}$	88.45
19	.015	1.00	.985	$.005f_{19}^2 + .01f_{19}$	$.001f_{19}^2 + 0.02f_{19}$	7.98
20	.015	3.00	.956	$.015f_{20}^2 + .1f_{20}$	$.001f_{20}^2 + 0.02f_{20}$	54.62
21	.015	3.00	.956	$.015f_{21}^2 + .1f_{21}$	$.001f_{21}^2 + 0.02f_{21}$	0.00
22	.015	1.00	.985	$.005f_{22}^2 + .01f_{22}$	$.001f_{22}^2 + 0.02f_{22}$	72.48
23	.020	1.00	.980	$.005f_{23}^2 + .01f_{23}$	$.001f_{23}^2 + 0.02f_{23}$	27.74
24	.020	3.00	.942	$.015f_{24}^2 + .1f_{24}$	$.001f_{24}^2 + 0.02f_{24}$	59.99
25	.020	3.00	.942	$.015f_{25}^2 + .1f_{25}$	$.001f_{25}^2 + 0.02f_{25}$	0.00
26	.020	1.00	.980	$.005f_{26}^2 + .01f_{26}$	$.001f_{26}^2 + 0.02f_{26}$	84.56

$$\text{Firm 2:} \quad \rho_{21} = -.001d_{21} - .001d_{11} + .5, \quad \rho_{22} = -.001d_{22} - .001d_{12} + .5.$$

The longer time durations associated with shipment links 13 and 14 in Table 2, represent the prolonged transportation from Firm 2's processor to its distribution centers in the United States, because of more imported food inspections by the U.S. Food and Drug Administration (FDA). Therefore, the values of arc multipliers associated with links 13 and 14 in Table 2 are lower than those in Table 1, which implies more perished cantaloupes generated at this stage. The other arc multipliers, the total operational and the total discarding cost functions are the same as in Case 1, as shown in Table 2. The new computed equilibrium link flows are also reported in Table 2.

The computed equilibrium demands for cantaloupes were:

$$d^*_{11} = 4.51, \quad d^*_{12} = 3.24, \quad d^*_{21} = 5.96, \quad \text{and} \quad d^*_{22} = 4.21.$$

The incurred equilibrium prices at each demand market were as follows:

$$\rho_{11} = 0.49, \quad \rho_{12} = 0.49, \quad \rho_{21} = 0.49, \quad \text{and} \quad \rho_{22} = 0.49.$$

Furthermore, the profits of two firms were:

$$U_1 = 1.16, \quad \text{and} \quad U_2 = 1.63.$$

The demand for cantaloupes is battered by the cantaloupe-associated outbreak, with significant decreases in demand prices at demand markets R_1 and R_2. Both Firm 1 and Firm 2, in turn, experience dramatic declines in their profits. In addition, additional distribution links: 20, 21, 24, and 25, have zero product flows (as compared to Case 1), since the extremely low demand price cannot cover the costs associated with long-distance distribution.

Case 3

Given the severe shrinkage in the demand for cantaloupes, Firm 1 has realized the importance of regaining consumers' confidence in its own product after the cantaloupe-associated outbreak. Thus, Firm 1 had its label of cantaloupes redesigned in order to incorporate the guarantee of food safety, with additional expenditures associated with its processing activities. The demand price functions corresponding to the two demand markets for cantaloupes from these two firms were given by:

$$\text{Firm 1:} \quad \rho_{11} = -.001d_{11} - .0005d_{21} + 2.5, \quad \rho_{12} = -.0003d_{12} - .0002d_{22} + 3;$$

$$\text{Firm 2:} \quad \rho_{21} = -.001d_{21} - .001d_{11} + .5, \quad \rho_{22} = -.001d_{22} - .001d_{12} + .5.$$

Table 2: Arc Multipliers, Total Operational Cost and Total Discarding Cost Functions, and Equilibrium Link Flow Solution for Case 2

Link a	λ_a	t_a	α_a	$\hat{c}_a(f)$	$\hat{z}_a(f_a)$	f_a^*
1	–	–	1.00	$.005f_1^2 + .03f_1$	0.00	4.43
2	–	–	1.00	$.006f_2^2 + .02f_2$	0.00	4.40
3	–	–	1.00	$.001f_3^2 + .02f_3$	0.00	5.94
4	–	–	1.00	$.001f_4^2 + .02f_4$	0.00	6.94
5	.150	0.20	.970	$.003f_5^2 + .01f_5$	0.00	4.43
6	.150	0.25	.963	$.002f_6^2 + .02f_6$	0.00	4.40
7	.150	0.30	.956	$.001f_7^2 + .02f_7$	0.00	5.94
8	.150	0.30	.956	$.001f_8^2 + .01f_8$	0.00	6.94
9	.040	0.50	.980	$.002f_9^2 + .05f_9$	$.001f_9^2 + 0.02f_9$	8.53
10	.060	0.50	.970	$.001f_{10}^2 + .02f_{10}$	$.001f_{10}^2 + 0.02f_{10}$	12.31
11	.015	1.50	.978	$.005f_{11}^2 + .01f_{11}$	0.00	4.82
12	.015	3.00	.956	$.01f_{12}^2 + .01f_{12}$	0.00	3.54
13	.025	3.00	.928	$.005f_{13}^2 + .02f_{13}$	0.00	6.86
14	.025	5.00	.882	$.01f_{14}^2 + .01f_{14}$	0.00	5.09
15	.010	3.00	.970	$.004f_{15}^2 + .01f_{15}$	$.001f_{15}^2 + 0.02f_{15}$	4.72
16	.010	3.00	.970	$.004f_{16}^2 + .01f_{16}$	$.001f_{16}^2 + 0.02f_{16}$	3.38
17	.015	3.00	.956	$.004f_{17}^2 + .01f_{17}$	$.001f_{17}^2 + 0.02f_{17}$	6.36
18	.015	3.00	.956	$.004f_{18}^2 + .01f_{18}$	$.001f_{18}^2 + 0.02f_{18}$	4.49
19	.015	1.00	.985	$.005f_{19}^2 + .01f_{19}$	$.001f_{19}^2 + 0.02f_{19}$	4.58
20	.015	3.00	.956	$.015f_{20}^2 + .1f_{20}$	$.001f_{20}^2 + 0.02f_{20}$	0.00
21	.015	3.00	.956	$.015f_{21}^2 + .1f_{21}$	$.001f_{21}^2 + 0.02f_{21}$	0.00
22	.015	1.00	.985	$.005f_{22}^2 + .01f_{22}$	$.001f_{22}^2 + 0.02f_{22}$	3.28
23	.020	1.00	.980	$.005f_{23}^2 + .01f_{23}$	$.001f_{23}^2 + 0.02f_{23}$	6.08
24	.020	3.00	.942	$.015f_{24}^2 + .1f_{24}$	$.001f_{24}^2 + 0.02f_{24}$	0.00
25	.020	3.00	.942	$.015f_{25}^2 + .1f_{25}$	$.001f_{25}^2 + 0.02f_{25}$	0.00
26	.020	1.00	.980	$.005f_{26}^2 + .01f_{26}$	$.001f_{26}^2 + 0.02f_{26}$	4.29

The arc multipliers, the total operational and the total discarding cost functions are the same as in Case 2, except for the total operational cost function associated with the processing link 9. The computed values of the equilibrium link flows are given in Table 3.

The computed equilibrium demands for cantaloupes were:

$$d^*_{11} = 17.52, \quad d^*_{12} = 46.46, \quad d^*_{21} = 5.81, \quad \text{and} \quad d^*_{22} = 3.58.$$

The incurred equilibrium prices at each demand market were as follows:

$$\rho_{11} = 2.48, \quad \rho_{12} = 2.99, \quad \rho_{21} = 0.48, \quad \text{and} \quad \rho_{22} = 0.45.$$

Furthermore, the profits of two firms were:

$$U_1 = 84.20, \quad \text{and} \quad U_2 = 1.38.$$

Based on the above results, consumers differentiate cantaloupes due to food safety and health concerns in Case 3. With the newly designed label, Firm 1 has managed to encourage the consumption of its cantaloupes at both of these two demand markets, whereas the demands for Firm 2's cantaloupes are even lower than those in Case 2. Considering the cantaloupe-associated outbreak, it is certainly not easy to reclaim the same profit level as in Case 1. A comparison of the results in Case 2 and Case 3 suggests that practicing product differentiation may be an effective strategy for a food firm to maintain its profit at an acceptable level. It is also interesting to note that the demand for Firm 1's product at demand market R_1 in Case 3 is even higher than that of Case 1, which is probably caused by the remarkable decrease in the price as well as the introduced guarantee of food safety.

Table 3: Arc Multipliers, Total Operational Cost and Total Discarding Cost Functions, and Equilibrium Link Flow Solution for Case 3

Link a	λ_a	t_a	α_a	$\hat{c}_a(f)$	$\hat{z}_a(f_a)$	f_a^*
1	–	–	1.00	$.005f_1^2 + .03f_1$	0.00	36.92
2	–	–	1.00	$.006f_2^2 + .02f_2$	0.00	36.64
3	–	–	1.00	$.001f_3^2 + .02f_3$	0.00	5.43
4	–	–	1.00	$.001f_4^2 + .02f_4$	0.00	6.44
5	.150	0.20	.970	$.003f_5^2 + .01f_5$	0.00	36.92
6	.150	0.25	.963	$.002f_6^2 + .02f_6$	0.00	36.64
7	.150	0.30	.956	$.001f_7^2 + .02f_7$	0.00	5.43
8	.150	0.30	.956	$.001f_8^2 + .01f_8$	0.00	6.44
9	.040	0.50	.980	$.003f_9^2 + .06f_9$	$.001f_9^2 + 0.02f_9$	71.11
10	.060	0.50	.970	$.001f_{10}^2 + .02f_{10}$	$.001f_{10}^2 + 0.02f_{10}$	11.35
11	.015	1.50	.978	$.005f_{11}^2 + .01f_{11}$	0.00	36.33
12	.015	3.00	.956	$.01f_{12}^2 + .01f_{12}$	0.00	33.38
13	.025	3.00	.928	$.005f_{13}^2 + .02f_{13}$	0.00	6.68
14	.025	5.00	.882	$.01f_{14}^2 + .01f_{14}$	0.00	4.33
15	.010	3.00	.970	$.004f_{15}^2 + .01f_{15}$	$.001f_{15}^2 + 0.02f_{15}$	35.52
16	.010	3.00	.970	$.004f_{16}^2 + .01f_{16}$	$.001f_{16}^2 + 0.02f_{16}$	31.91
17	.015	3.00	.956	$.004f_{17}^2 + .01f_{17}$	$.001f_{17}^2 + 0.02f_{17}$	6.20
18	.015	3.00	.956	$.004f_{18}^2 + .01f_{18}$	$.001f_{18}^2 + 0.02f_{18}$	3.82
19	.015	1.00	.985	$.005f_{19}^2 + .01f_{19}$	$.001f_{19}^2 + 0.02f_{19}$	17.78
20	.015	3.00	.956	$.015f_{20}^2 + .1f_{20}$	$.001f_{20}^2 + 0.02f_{20}$	16.69
21	.015	3.00	.956	$.015f_{21}^2 + .1f_{21}$	$.001f_{21}^2 + 0.02f_{21}$	0.00
22	.015	1.00	.985	$.005f_{22}^2 + .01f_{22}$	$.001f_{22}^2 + 0.02f_{22}$	30.96
23	.020	1.00	.980	$.005f_{23}^2 + .01f_{23}$	$.001f_{23}^2 + 0.02f_{23}$	5.93
24	.020	3.00	.942	$.015f_{24}^2 + .1f_{24}$	$.001f_{24}^2 + 0.02f_{24}$	0.00
25	.020	3.00	.942	$.015f_{25}^2 + .1f_{25}$	$.001f_{25}^2 + 0.02f_{25}$	0.00
26	.020	1.00	.980	$.005f_{26}^2 + .01f_{26}$	$.001f_{26}^2 + 0.02f_{26}$	3.65

5. Summary and Conclusions

This paper focused on food deterioration between production and consumption locations, which poses unique challenges to food supply chain management. In particular, we developed a network-based food supply chain model under oligopolistic competition and perishability, with a concentration in fresh produce, such as vegetables and fruits. Each food firm is involved in such supply chain activities as the production, processing, storage, distribution, and even the disposal of the food products, and seeks to determine its optimal product flows throughout its supply chain, in order to maximize its own profit.

We captured the food exponential time decay in number of units through the introduction of arc multipliers, which depend on the time duration and environmental conditions associated with each post-production supply chain activity. We also incorporated the discarding costs associated with the disposal of the spoiled food products at the processing, storage, and distribution stages. Moreover, the competitive model allows consumers to differentiate food products at the demand markets due to product freshness and food safety concerns. In addition, the flexibility of the supply chain network topology allows decision-makers to evaluate alternative technologies involved in various supply chain activities.

We derived the variational inequality formulations of the food supply chain network Cournot-Nash equilibrium conditions, and studied the qualitative properties of the equilibrium pattern. We also adopted an algorithm which yields subproblems at each iteration with nice features for computation. We then illustrated the proposed model as well as the algorithm by presenting several numerical cases, which focused on the cantaloupe market in the United States. The results of the case study suggested that product differentiation may be an effective strategy for a firm to keep itself financially resilient, especially in times of outbreaks of foodborne diseases.

We emphasized that our model can be applied – albeit after appropriate modifications – to other perishable product supply chain problems under oligopolistic competition, and even with quality competition. Possible extensions could include the incorporation of supply side as well as demand side variability, such as demand price volatility and delivery reliability. In addition, the development of food supply chain network design models with capacities associated with various supply chain activities being strategic variables (see Nagurney (2010)) is an interesting future research direction.

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