Food Security and Multicommodity Agricultural International Trade: Quantifying Optimal Consumer Subsidies for Nutritional Needs

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Abstract: Hundreds of millions of people are now facing food insecurity as challenges from climate change, the aftereffects of the COVID-19 pandemic, and strife and conflicts make availability of food at reasonable prices challenging. International trade has enabled the reallocation of agricultural products, essential for nutrition, from countries with supply markets to other points of demand and has been the subject of an increasing number of policy interventions by governments. In this paper, a multicommodity international trade network equilibrium model is constructed with the inclusion of nutritional minimal standards to support food security, accompanied by consumer subsidies, for which explicit formulae are provided. The theoretical and computational framework is based on variational inequalities. Numerical examples focusing on Ukraine and MENA (Middle Eastern and North African) countries and on a staple commodity of wheat demonstrate the modeling and policy framework.

Key words: international trade, networks, spatial price equilibrium, nutrition, food security, variational inequalities

1. Introduction

The 1996 World Food Summit defined food security as existing when "all people, at all times, have physical, social and economic access to sufficient, safe and nutritious food to meet dietary needs for a productive and healthy life" (World Food Summit (1996)). Hence, adequate nutrition is not only essential to the health and well-being of people around the globe but is also critical for food security. With shocks of climate change, the COVID-19 pandemic, strife, unrest, and wars, along with their negative ramifications and disruptions, hunger and food insecurity are rising globally on our planet. According to the World Food Programme (2023), based on available data, more than 345 million people in 2023 are faced with high levels of food insecurity, a number more than twice the number in 2020. The Food and Agricultural Organization (2023) of the United Nation estimates that between 691 and 783 million people faced hunger in 2022. Furthermore, the Food and Agricultural Organization (2021) reports that the prevalence of undernourishment (PoU) increased to about 9.9% in 2020, from 8.4% a year earlier, after being essentially unchanged from 2014 to 2019, and noted that the world is at a "critical juncture." The UN's second Sustainable Development Goal (SDG) aims for zero hunger and the achievement of food security and improved nutrition (United Nations (2023)).

Since agricultural products in the form of grains, fruits and vegetables, as well as dairy and meat products, can be important components of nutritious diets, their availability and affordability help to support food security. For example, according to the Food and Agricultural Organization (2016), the commodities of wheat, maize, and rice are foundational staple crops of global trade, and of high importance for food security. They comprise 43% of calories and 37% of protein consumed by humans. In countries with insufficient resources to produce the required volumes of commodities for nutritional minimum requirements locally, international trade of agricultural products can provide the necessary commodities through networks from points of production to points of demand and consumption. According to Zimmerman and Rapsomanikis (2023), over the past three decades, agricultural and food trade has increased more than twice in value, quantity, and calories. Furthermore, they argue that trade is essential to ensure food security in multiple dimensions stating that the availability and accessibility of foods and, hence, nutrients, would be more unevenly distributed without trade and more sensitive to any form of food disruptions domestically.

It is, therefore, understandable that agricultural trade has been the subject of multiple policy interventions by governments. Policy interventions well-known in this important economic sector include: tariffs, quotas, tariff-rate quotas (TRQs), and even subsidies. As discussed in Nagurney, Besik, and Nagurney (2019) regarding the relevance of trade policies in practice, the Uruguay Round in 1996 created more than 1,300 new TRQs and they are widely applied especially in agricultural trade for such products as: milk and dairy products, bananas, sugar, beef, eggs, poultry, potatoes, among other food products. The four most important food crops: wheat, corn, rice, and bananas have had this policy applied to them. According to Casey and Cimino-Isaacs (2021), between January 2020 and April 2021, countries took more than 220 actions banning or limiting the export of certain products for COVID-19-related reasons, with the products covered by these export curbs falling into two categories: medical items including pharmaceuticals and foodstuffs. For example, according to Reuters (2020), Egypt banned the export of legumes for three months during this period and both Vietnam and Cambodia as well as India instituted export controls on rice. India, the world's largest rice exporter, on July 20, 2023, due to heavy rains destroying paddy fields, banned the export of non-basmati white rice to "ensure adequate domestic availability at reasonable prices" (The Economist (2023)).

Subsidies for farmers have been applied in both developed and developing countries and have a long history. In the United States, according to O'Neill Hayes and Kerska (2023), subsidies for farmers by the federal government have averaged \$16 billion annually over the past decade. The Common Agricultural Policy (CAP), dating to the 1960s, provides subsidies to farmers in the European Union and is the EU's largest budget item, averaging 54 billion euros per year (Nicholas and Brady (2020)). China, in 2021, announced that it would give out 20 billion yuan (\$3.1 billion) in subsidies to grain farmers because of high fertilizer and diesel costs, with the goal of moderating the impact of rising commodity prices (Hua and Yao (2021)). And Russia's war on Ukraine, following the full-scale invasion of February 24, 2022, is reshaping the CAP for multiple members of the European Union (see Euroactive (2022)).

A major study by An (2013) that systematically reviewed evidence from field interventions on the effectiveness of monetary subsidies for consumers in promoting healthier food purchases and consumption revealed that subsidizing healthier foods tended to be effective in changing dietary behavior. World events, including Russia's war on Ukraine, are generating increased anxiety for certain governments. Barnes (2023) reports that Egypt depends on imported wheat, and, in recent years, the majority of wheat imports have come from Russia and Ukraine with the imported grain providing 50% of the flour for the subsidized bread program that about 72 million Egyptians depend on for their daily consumption. Van Berkum (2021), however, in an expansive study on trade, associated policies, and the food system, has noted that trade rules do not, typically, include goals for the provision of healthy diets. Furthermore, he recognizes that enhancing the nutritional impacts of food systems is a principal objective of "food system transformation." In this paper, we take on the challenge of quantifying subsidies for consumers in the context of international trade of agricultural products for the achievement of nutritional minimum standards, including calories and nutrients. Specifically, in this paper, a new international trade network equilibrium model is constructed consisting of multiple, spatially separated countries involved in the production and consumption of multiple commodities. Associated with each nutrient is a minimum amount required by the population in each country, with each commodity having its composition of nutrients and caloric content.

2. Literature Review, Contributions, and Organization of the Paper

Subsection 2.1 provides the literature review of the relevant papers, whereas Subsection 2.2 highlights the contributions in this paper, and Subsection 2.3 describes how this paper is organized.

2.1 Literature Review

The research in this paper builds on the work on spatial price equilibrium modeling (see Samuelson (1952), Takayama and Judge (1964, 1971)) but utilizing the theory of variational inequalities (cf. Florian and Los (1982), Dafermos and Nagurney (1984), Nagurney and Aronson (1989), Nagurney (1999)), with specific note on policies (Nagurney, Thore, and Pan (1996), Nagurney, Nicholson, and Bishop (1996), Nagurney, Li, and Nagurney (2014), Nagurney, Besik, and Dong (2019), Nagurney, Salarpour, and Dong (2022), Nagurney et al. (2023)). Spatial price equilibrium models have had wide applications to agricultural commodities and, therefore, this foundational framework is a reasonable one in which to also incorporate minimum nutritional standards for consumers in different countries. For example, as summarized by Nagurney, Li, and Nagurney (2014), spatial price equilibrium models have been applied in the agricultural sector by Thompson (1989); specifically, to eggs by Judge (1956), to potatoes by Howard (1984), to beef by Sohn (1970), and to cereal grains by Ruijs et al. (2001) and Nagurney et al. (2023). Spatial price equilibrium models have also been applied to the important agricultural commodity of soybeans by Barraza De La Cruz, Pizzolato, and Barrazade La Cruz (2010), and to dairy by Bishop, Pratt, and Novakovic (1994) and by Nagurney, Besik, and Dong (2019). Spatial equilibrium models have also been applied to energy markets. For a recent such paper that includes transportation capacities and impacts, see Birge et al. (2022).

The achievement of nutritional goals has, in addition, been the target of various studies in operations research. In fact, what is known as "The Diet Problem," has a long history dating to World War II, with luminaries such as Stigler and Dantzig making significant contributions and with Linear Programming being a fundamental tool for formulation, analysis, and solution. See van Dooren (2018) for a review. Agriculture has also been a topic of great interest to operations researchers with the recent special issue of the International Transactions in Operational Research on OR and big data in agriculture edited by Albornoz et al. (2023) demonstrating the breadth and depth of issues as well as applications. Relevant references on operations research and agriculture include the earlier works of Weintraub and Romero (2006) and Pla, Sanders, and Higgins (2014). Food security has also drawn the attention of operations researchers; see Schweigman (2008) for an overview of food security studies and potential in several countries in Africa.

2.2 Contributions

This is the first work that integrates spatial price equilibrium modeling and minimum nutritional standards, coupled with consumer subsidies in an international trade context. In particular, the results in this paper include formulae for the consumer subsidies that guarantee that minimum nutritional standards are met and, thus, are supportive of food security. The derived equilibrium conditions are novel and consist of the multicommodity trade flows and the Lagrange multipliers (shadow prices) associated with the nutrient minimum standards. The equilibrium conditions are formulated as alternative variational inequalities and qualitative properties of existence and uniqueness discussed. Illustrative examples are presented, followed by an algorithmic scheme, with nice features for implementation. The algorithm is applied to compute the equilibrium solutions in a series of numerical examples focusing on the commodity of wheat and on the countries of Ukraine, Egypt, and Lebanon.

The paper adds to the literature on the incorporation of trade policies in computable network equilibrium models for international trade to quantify and assess the impacts on volumes of commodities as well as prices. The model in this paper, for the first time, combines international trade, multiple commodities, and nutritional minimum standards in a general network equilibrium framework in which consumer subsidies are also quantified via explicit formulae with a goal of achieving food security.

2.3 Organization of the Paper

The paper is organized as follows. In addition to Sections 1 and 2 above, in Section 3, the international trade model with consumer subsidies for food security is constructed, the necessary notation provided, and the bipartite network underlying the problem depicted. The model can handle nonlinear and asymmetric multicommodity supply price functions, unit transportation cost functions, as well as demand price functions. The equilibrium conditions expand upon spatial price equilibrium conditions to include minimum nutritional standards based on the populations in the countries and the nutrient composition of the agricultural commodities. Alternative variational inequality formulations of the governing equilibrium conditions are also derived. Drawing upon the equilibrium conditions which include both multicommodity trade flows as well as Lagrange multipliers associated with the minimum nutritional amounts, explicit formulae for consumer subsidies are provided to support food security. Illustrative examples are solved analytically in order to illustrate the framework. Results for existence and uniqueness of a multicommodity equilibrium trade flow pattern are also given. Section 4 then describes an algorithm that, when applied to the variational inequality in multicommodity trade flows and Lagrange multipliers, results in closed form expressions for these variables at each iteration. Conditions for convergence are also provided. The algorithm is then applied in Section 5 to solve numerical examples focused on the commodity of wheat and the countries of Ukraine, Egypt, and Lebanon under different scenarios. Section 6 summarizes the results and presents the conclusions.

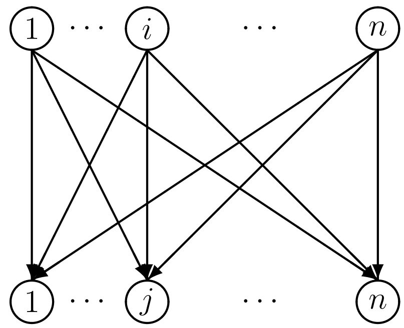

Figure 1: The International Trade Network

3. The International Trade Model with Consumer Subsidies for Food Security

The international trade model with consumer subsidies (if needed) for food security consists of n country supply markets and n country demand markets. The supply markets and the demand markets are involved in the production and consumption, repectively, of H commodities with a typical commodity denoted by h. A typical country supply market is denoted by i and a typical country demand market by j. The underlying structure of the model can be represented as a bipartite network as illustrated in Figure 1.

The country supply and demand markets are spatially separated and, hence, there is a unit transportation cost associated with each commodity between supply and demand markets. Associated with each country supply market and commodity is a supply price function and with each country demand market and commodity a demand price function. The focus in this paper is on food security and that entails that the population(s) receive the necessary nutrients, which also means that caloric needs are achieved. A representative human is considered here for the nutritional requirements. In the model a typical nutrient is denoted by l, with there being L nutrients. The notation for the parameters in the model, and that for the variables and the functions is given in Table 1. All vectors are column vectors.

Table 1: Notation for the International Trade Network Model for Food Security

Notation	**Parameter Definition**
t_j^l	the minimum amount required of nutrient l; $l = 1, \ldots, L$, for the population in country demand market j; $i = 1, \ldots, n$.
$\alpha^{l,h}$	the amount of nutrient l; $l = 1, \ldots, L$, in a unit of commodity $h; h = 1, \ldots, H$.
Notation	**Variable Definition**
s_i^h	the supply of commodity h; $h = 1, \ldots, H$, at country supply market i; $i = 1, \ldots, n$. We group all the commodity supplies into the vector $s \in R_+^{Hn}$.
d_j^h	the demand for commodity h; $h = 1, \ldots, H$, at country demand market j; $j = 1, \ldots, n$. We group all the commodity demands into the vector $d \in R_+^{Hn}$.
Q_{ij}^h	the shipment of commodity h; $h = 1, \ldots, H$, from country supply market i; $i = 1, \ldots, n$, to country demand market j; $j = 1, \ldots, n$. We group all the commodity shipments into the vector $Q \in R_+^{H2n}$.
λ_j^l	the Lagrange multiplier (shadow price) associated with the minimum requirement of nutrient l in country demand market j, with $l = 1, \ldots, L$ and $j = 1, \ldots, n$. We group all such Lagrange multipliers into the vector $\lambda \in R_+^{Ln}$.
Notation	**Function Definition**
$\pi_i^h(s)$	the supply price function for commodity h; $h = 1, \ldots, H$, at country supply market i; $i = 1, \ldots, n$. We group all the commodity supply price functions into the vector $\pi(s) \in R^{Hn}$.
$\rho_j^h(d)$	the demand price function for commodity h; $h = 1, \ldots, H$, at country demand market j; $j = 1, \ldots, n$. We group all the commodity demand price functions into the vector $\rho(d) \in R^{Hn}$.
$c_{ij}^h(Q)$	the unit transportation cost associated with shipping commodity h; $h = 1, \ldots, H$, from country supply market i; $i = 1, \ldots, n$, to country demand market j; $i = 1, \ldots, n$. We group the commodity unit transportation costs for all supply/demand market pairs into the vector $c(Q) \in R^{H2n}$.

The Conservation of Flow Equations

The supply, demand, and nonnegativity constraints are now presented. The conservation of flow equations are as follows:

$$s_i^h = \sum_{j=1}^{n} Q_{ij}^h, \quad h = 1, \ldots, H; i = 1, \ldots, n; \tag{1}$$

$$d_j^h = \sum_{i=1}^{n} Q_{ij}^h, \quad h = 1, \ldots, H; j = 1, \ldots, n; \tag{2}$$

$$Q_{ij}^h \geq 0, \quad h = 1, \ldots, H; i = 1, \ldots, n; j = 1, \ldots, n. \tag{3}$$

Equation (1) states that the amount of each commodity produced at a country supply market is equal to the sum of the shipments of the commodity out of the country supply market to all the country demand markets. Equation (2), on the other hand, states that the amount of the commodity consumed at a country demand market equals the amount of the commodity transported to it from all the country supply markets. Equation (3) guarantees that the commodity shipments are all nonnegative

The commodity supply price, demand price, and unit transportation cost functions are assumed to be continuous. All the functions are in a common currency; typically, in US dollars. While the commodity supply price functions and the unit transportation cost functions are monotone increasing, the commodity demand price functions are monotonically decreasing (cf. Nagurney (1999)). Note that according to these functions, as delineated in Table 1, the supply price of a commodity at a country supply market can, in general, depend not only on the supply of the commodity at the country supply market but also on the supplies of the other commodities, as well as on the supplies of the commodities at other country supply markets. A similar level of generality is possible for the demand price functions but in terms of country demand markets and also for the commodity transportation costs with the latter functions also, in general, depending not only on the particular commodity shipment between a pair of country supply and demand markets. The generality of these functions allows one to capture different levels of competition.

The Food Security Nutritional Constraints

The food security nutritional constraints guarantee that the nutritional needs are met for each country's population and are as follows

$$\sum_{h=1}^{H} \alpha^{l,h} d_j^h \geq t_j^l, \quad l = 1, \ldots, L; j = 1, \ldots, n. \tag{4a}$$

In view of (2), we may rewrite the constraints in (4a) as:

$$\sum_{h=1}^{H} \alpha^{l,h} \sum_{i=1}^{n} Q_{ij}^{h} \geq t_j^l, \quad l = 1, \ldots, L; j = 1, \ldots, n. \tag{4b}$$

According to (4a) (and also (4b)), the minimum amount needed for nutrition from each nutrient (including calories) derived from all the commodities for the population in each country must be met. We refer to the t_j^ls as the minimum standards for nutrient l in country demand market j.

In the statement of the international trade equilibrium conditions, we associate a Lagrange multiplier λ_j^l with the constraint in (4b) corresponding to nutrient l and country demand market j.

New multicommodity supply price functions $\tilde{\pi}_i^h(Q)$ for $h = 1, \ldots, H$; $i = 1, \ldots, n$, and new demand price functions $\tilde{\rho}_j^h(Q)$ for $h = 1, \ldots, H$; $j = 1, \ldots, n$, that are functions of the commodity shipments are now defined. This is doable because of conservation of flow equations (1) and (2). Hence, we have that:

$$\tilde{\pi}_i^h = \tilde{\pi}_i^h(Q) \equiv \pi_i^h(s), \quad h = 1, \ldots, H; i = 1, \ldots, n, \tag{5}$$

and

$$\tilde{\rho}_j^h = \tilde{\rho}_j^h(Q) \equiv \rho_j^h(d), \quad h = 1, \ldots, H; j = 1, \ldots, n. \tag{6}$$

The feasible set $\mathcal{K}^1 \equiv \{(Q, \lambda) | (Q, \lambda) \in R_+^{H2n+Ln}\}$.

3.1 The International Trade Equilibrium Conditions with Food Security Minimum Standards and Variational Inequality Formulations

The international trade equilibrium conditions with food security nutritional minimum standards are presented in this Subsection. The variational inequality formulation of the governing equilibrium conditions is then derived and an alternative variational inequality formulation provided. Conditions for existence and uniqueness of a solution are also given. In the absence of the minimum nutritional standards that must be met, the model collapses to a multicommodity spatial price equilibrium model. The equilibrium conditions below expand the classical spatial price equilibrium conditions of Samuelson (1952) and Takayama and Judge (1971) to include food security in the form of minimum nutritional standards. The spatial price equilibrium framework, in a variational inequality context (cf. Florian and Los (1982), Dafermos and Nagurney (1984)), has yielded rich modeling advances and applications, especially to agricultural products (see, e.g., Nagurney and Aronson (1989), Nagurney, Dong, and Nagurney (2014), Nagurney (2021), Nagurney, Salarpour, and Dong (2022), Nagurney and Besik (2022), Nagurney et al. (2023)).

Definition 1: The International Trade Equilibrium Conditions with Food Security Nutritional Minimum Standards

A multicommodity shipment and Lagrange multiplier pattern $(Q^*, \lambda^*) \in \mathcal{K}^1$ is an international trade equilibrium with food security nutritional minimum standards if it satisfies the following conditions: for each commodity h; $h = 1, \ldots, H$, and for each pair of country supply and demand markets (i, j); $i = 1, \ldots, n$; $j = 1, \ldots, n$:

$$\tilde{\pi}_i^h(Q^*) + c_{ij}^h(Q^*) - \sum_{l=1}^{L} \alpha^{l,h} \lambda_j^{l*} \begin{cases} = \tilde{\rho}_j^h(Q^*), & \textit{if} \;\; Q_{ij}^{h*} > 0, \\ \geq \tilde{\rho}_j^h(Q^*), & \textit{if} \;\; Q_{ij}^{h*} = 0, \end{cases} \tag{7}$$

and for each nutrient l; $l = 1, \ldots, L$, and for each country demand market j; $j = 1, \ldots, n$:

$$\sum_{h=1}^{H} \alpha^{l,h} \sum_{i=1}^{n} Q_{ij}^{h*} - t_j^l \begin{cases} = 0, & \textit{if} \;\; \lambda_j^{l*} > 0, \\ \geq 0, & \textit{if} \;\; \lambda_j^{l*} = 0. \end{cases} \tag{8}$$

According to the above equilibrium conditions, if the minimum nutritional target is exceeded for a nutrient in a country demand market, there is a positive shipment of the commodity between a pair of country supply and demand markets if the supply price of the commodity at the country supply market plus the unit transportation cost associated with shipping the commodity is equal to the demand price of the commodity at the country demand market. There will not be a positive shipment of the commodity in equilibrium between the pair of supply and demand markets if the supply price plus the unit transportation cost exceeds the demand price. On the other hand, if the Lagrange multiplier associated with a country demand market and nutrient is positive in equilibrium and the commodity shipment is positive, then the demand price for the commodity at the country demand market will be lower than the respective supply price plus the unit transportation cost. The amount by which it is lower is equal to the value of the contribution by the commodity to achieving the nutrient targets at the country demand market, which is quantified by $\sum_{l=1}^{L} \alpha^{l,h} \lambda_j^{l*}$ for commodity h and country demand market j. Subsequently, we will show that this term is precisely the consumer subsidy that should be applied to guarantee that the nutritional needs for country demand market j are met for nutrient l.

The variational inequality formulation of the above governing equilibrium conditions is now established

Theorem 1: Variational Inequality Formulation of the International Trade Equilibrium Conditions with Food Security Minimum Nutritional Standards

A multicommodity shipment and Lagrange multiplier pattern $(Q^*, \lambda^*) \in \mathcal{K}^1$ is an international trade equilibrium with food security minimum nutritional standards according to Definition 1 if and only if it satisfies the variational inequality problem:

$$\sum_{h=1}^{H}\sum_{i=1}^{n}\sum_{j=1}^{n}(\tilde{\pi}_i^h(Q^*) + c_{ij}^h(Q^*) - \sum_{l=1}^{L}\alpha^{l,h}\lambda_j^{l*} - \tilde{\rho}_j^h(Q^*)) \times (Q_{ij}^h - Q_{ij}^{h*})$$

$$+\sum_{l=1}^{L}\sum_{j=1}^{n}(\sum_{h=1}^{H}\alpha^{l,h}\sum_{i=1}^{n}Q_{ij}^{h*} - t_j^l) \times (\lambda_j^l - \lambda_j^{l*}) \geq 0, \quad \forall (Q, \lambda) \in \mathcal{K}^1. \tag{9}$$

Proof: Necessity is first established; that is, if $(Q^*, \lambda^*) \in \mathcal{K}^1$ satisfies equilibrium conditions (7) and (8) according to Definition 1, then it also satisfies variational inequality (9).

Note that, for a fixed pair of country supply and demand markets (i, j), and commodity h, (7) implies that

$$(\tilde{\pi}_i^h(Q^*) + c_{ij}^h(Q^*) - \sum_{l=1}^{L}\alpha^{l,h}\lambda_j^{l*} - \tilde{\rho}_j^h(Q^*)) \times (Q_{ij}^h - Q_{ij}^{h*}) \geq 0, \quad \forall Q_{ij}^h \geq 0. \tag{10}$$

Indeed, since, if $Q_{ij}^{h*} > 0$, we know, from (7), that the expression to the left of the multiplication sign in (10) is equal to zero, so (10) holds true. On the other hand, if $Q_{ij}^{h*} = 0$, then the expression before and after the multiplication sign in (10) will be nonnegative and, thus, the product is also nonnegative and (10) holds. Since (10) holds for all h, i, j, summation of (10) yields:

$$\sum_{h=1}^{H}\sum_{i=1}^{n}\sum_{j=1}^{n}(\tilde{\pi}_i^h(Q^*) + c_{ij}^h(Q^*) - \sum_{l=1}^{L}\alpha^{l,h}\lambda_j^{l*} - \tilde{\rho}_j^h(Q^*)) \times (Q_{ij}^h - Q_{ij}^{h*}) \geq 0, \quad \forall Q \in R_+^{H2n}. \tag{11}$$

From equilibrium conditions (8), in turn, one knows that the following inequality must hold for a fixed nutrient l and country demand market j:

$$(\sum_{h=1}^{H}\alpha^{l,h}\sum_{i=1}^{n}Q_{ij}^{h*} - t_j^l) \times (\lambda_j^l - \lambda_j^{l*}) \geq 0, \quad \forall \lambda_j^l \geq 0. \tag{12}$$

Inequality (12) holds for any l and j and, therefore, we have the following inequality:

$$\sum_{l=1}^{L}\sum_{j=1}^{n}(\sum_{h=1}^{H}\alpha^{l,h}\sum_{i=1}^{n}Q_{ij}^{h*} - t_j^l) \times (\lambda_j^l - \lambda_j^{l*}) \geq 0, \quad \forall \lambda \in R_+^{Ln}. \tag{13}$$

Summation of (11) and (13) yields variational inequality (9).

Sufficiency is now proven; that is, if $(Q^*, \lambda^*) \in \mathcal{K}^1$ satisfies variational inequality (9) then it also satisfies equilibrium conditions (7) and (8).

Let $\lambda_j^l = \lambda_j^{l*}$, $\forall l, j$. Substitution into variational inequality (9) yields:

$$\sum_{h=1}^{H}\sum_{i=1}^{n}\sum_{j=1}^{n}(\tilde{\pi}_i^h(Q^*) + c_{ij}^h(Q^*) - \sum_{l=1}^{L}\alpha^{l,h}\lambda_j^{l*} - \tilde{\rho}_j^h(Q^*)) \times (Q_{ij}^h - Q_{ij}^{h*}) \geq 0, \quad \forall Q \in R_+^{H2n}, \quad (14)$$

and, with further substitution of $Q_{op}^r = Q_{op}^{r*}$, for $r \neq h$; $o \neq i$, and $p \neq j$, into (14), gives us:

$$(\tilde{\pi}_i^h(Q^*) + c_{ij}^h(Q^*) - \sum_{l=1}^{L}\alpha^{l,h}\lambda_j^{l*} - \tilde{\rho}_j^h(Q^*)) \times (Q_{ij}^h - Q_{ij}^{h*}) \geq 0, \quad \forall Q_{ij}^h \geq 0, \quad (15)$$

from which it follows that equilibrium conditions (7) must hold.

Letting now $Q_{ij}^h = Q_{ij}^{h*}$ for all h, i, j, and substituting these resultants into variational inequality (9) gives us:

$$\sum_{l=1}^{L}\sum_{j=1}^{n}(\sum_{h=1}^{H}\alpha^{l,h}\sum_{i=1}^{n}Q_{ij}^{h*} - t_j^l) \times (\lambda_j^l - \lambda_j^{l*}) \geq 0, \quad \forall \lambda \in R_+^{Ln}. \quad (16)$$

Letting now $\lambda_g^r = \lambda_g^{r*}$, for all $r \neq l$ and $g \neq j$, and substituting the resultants into (15) yields:

$$(\sum_{h=1}^{H}\alpha^{l,h}\sum_{i=1}^{n}Q_{ij}^{h*} - t_j^l) \times (\lambda_j^l - \lambda_j^{l*}) \geq 0, \quad \forall \lambda_j^l \geq 0, \quad (17)$$

from which equilibrium conditions (8) follow.

The proof is complete. □

Variational inequality (9) is now put into standard form (cf. Nagurney (1999)): determine $X^* \in \mathcal{K}$, where

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \quad (18)$$

where $\mathcal{K}$ denotes the feasible set, which must be closed and convex. The vector X is $\mathcal{N}$-dimensional, as is $F(X)$, with $F(X)$ being continuous and given, and maps X from $\mathcal{K}$ into $R^{\mathcal{N}}$. $\langle \cdot, \cdot \rangle$ denotes the inner product in $\mathcal{N}$-dimensional Euclidean space. Define $X \equiv (Q, \lambda)$ and $F(X)$ with components $F_{ij}^h(X) = (\tilde{\pi}_i^h(Q) + c_{ij}^h(Q) - \sum_{l=1}^{L}\alpha^{l,h}\lambda_j^l - \tilde{\rho}_j^h(Q))$; $h = 1, \ldots, H$; $i = 1, \ldots, n$; $j = 1, \ldots, n$ and $F_j^l(X) = (\sum_{h=1}^{H}\alpha^{l,h}\sum_{i=1}^{n}Q_{ij}^h - t_j^l)$; $l = 1, \ldots, L$; $j = 1, \ldots, n$. Here, $\mathcal{N} = H2n + Ln$. Also, the feasible set $\mathcal{K} \equiv \mathcal{K}^1$. Clearly, (9) is, thus, of the form (18).

An alternative variational inequality formulation to (9) of the above equilibrium conditions is now provided. Distinct variational inequality formulations may suggest different algorithms for the computation of the equilibrium pattern. Examples of algorithms for variational inequality problems and convergence conditions can be found in Dafermos (1983) and in Dupuis and Nagurney (1992) with specific applications highlighted also in Nagurney (1999). For example, the general iterative scheme of Dafermos (1983) induces projection and relaxation methods whereas the general iterative scheme of Dupuis and Nagurney (1992) induces such algorithms as the Euler and Heun methods. Pang (1985) provides iterative methods for variational inequality problems defined over Cartesian products of sets. Typically, a variational inequality algorithm resolves the original variational inequality problem into subproblems that are easier to solve and sometimes the subproblems can be of the form of optimization problems.

Furthermore, theoretical results may also be possible because of alternative formulations as well as interpretations, which we elaborate on further below in terms of the Lagrange multipliers.

We define the feasible set $\mathcal{K}^2$, where $\mathcal{K}^2 \equiv \{Q \in R_+^{H2n} | (4b) \text{ holds}\}$.

Theorem 2: Alternative Variational Inequality Formulation

A multicommodity shipment flow pattern $Q^* \in \mathcal{K}^2$ is an equilibrium according to (7) and (8) if and only if it satisfies the variational inequality problem:

$$\sum_{h=1}^{H}\sum_{i=1}^{n}\sum_{j=1}^{n}(\tilde{\pi}_i^h(Q^*) + c_{ij}^h(Q^*) - \tilde{\rho}_j^h(Q^*)) \times \left[Q_{ij}^h - Q_{ij}^{h*}\right] \geq 0, \quad \forall Q \in \mathcal{K}^2 \tag{19}$$

with $\lambda^* \in R_+^{Ln}$ being the optimal Lagrange multiplier vector associated with the constraints in (4b).

Proof: We construct the function $\Phi : R_+^{H2n} \mapsto R$ such that

$$\Phi(Q) = \sum_{h=1}^{H}\sum_{i=1}^{n}\sum_{j=1}^{n}(\tilde{\pi}_i^h(Q^*) + c_{ij}^h(Q^*) - \tilde{\rho}_j^h(Q^*)) \times Q_{ij}^h. \tag{20}$$

Q^* solves (19) if and only if Q^* is a global minimum point of Φ in $\mathcal{K}^2$.

In addition, we construct the Lagrangian function $\mathcal{L}$, such that

$$\mathcal{L}(Q,\lambda) = \Phi(Q) + \sum_{l=1}^{L}\sum_{j=1}^{n}\lambda_j^l(-\sum_{h=1}^{H}\alpha^{l,h}\sum_{i=1}^{n}Q_{ij}^h + t_j^l), \tag{21}$$

where recall that λ is the Lagrange multiplier vector associated with the nutritional minimum standards (4b).

The KKT conditions are then:

$$\frac{\partial \mathcal{L}(Q^*, \lambda^*)}{\partial Q_{ij}^h} \geq 0, \quad Q_{ij}^{h*} \geq 0, \quad \frac{\partial \mathcal{L}(Q^*, \lambda^*)}{\partial Q_{ij}^h} Q_{ij}^{h*} = 0, \quad h = 1, \ldots, H; \forall i, j, \tag{22}$$

$$\lambda_j^{l*}(\sum_{l=1}^{L}\sum_{j=1}^{n}(-\sum_{h=1}^{H}\alpha^{l,h}\sum_{i=1}^{n}Q_{ij}^{h*} + t_j^l) = 0, \quad \lambda_j^{l*} \geq 0, \sum_{l=1}^{L}\sum_{j=1}^{n}(-\sum_{h=1}^{H}\alpha^{l,h}\sum_{i=1}^{n}Q_{ij}^{h*} + t_j^l) \leq 0,$$
$$l = 1, \ldots, L; j = 1, \ldots, n. \tag{23}$$

Because the objective function Φ and the constraints (4b) are linear in Q, the KKT conditions are both necessary and sufficient for Q^* to be a minimum.

Also, since $\forall h, i, j$:

$$\frac{\partial \mathcal{L}(Q^*, \lambda^*)}{\partial Q_{ij}^h} = \tilde{\pi}_i^h(Q^*) + c_{ij}^h(Q^*) - \sum_{l=1}^{L}\alpha^{l,h}\lambda_j^{l*} - \tilde{\rho}_j(Q^*), \tag{24}$$

one can re-express (22) as: for $h = 1, \ldots, H$, and for all $i = 1, \ldots, n$ and all $j = 1, \ldots, n$:

$$\tilde{\pi}_i^h(Q^*) + c_{ij}^h(Q^*) - \sum_{l=1}^{L}\alpha^{l,h}\lambda_j^{l*} \begin{cases} = \tilde{\rho}_j^h(Q^*), & \text{if } Q_{ij}^{h*} > 0, \\ \geq \tilde{\rho}_j^h(Q^*), & \text{if } Q_{ij}^{h*} = 0, \end{cases} \tag{25}$$

which corresponds to equilibrium conditions (7).

Furthermore, conditions (23) coincide with equilibrium conditions (8). □

Remark

It follows from the classical theory of variational inequalities (cf. Kinderlehrer and Stampacchia (1982) and Nagurney (1999), that if the supply price functions are strongly monotone is s; the unit transportation cost functions are strongly monotone in Q, and the minus demand price functions are strongly monotone in d, then both existence of a solution Q^* to variational inequality (19) as well as uniqueness of the solution Q^* are guaranteed.

In the absence of the minimum nutritional standards as represented by the constraints in (4b), the above model collapses to a special case - that of a multicommodity spatial price equilibrium model as delineated below.

Corollary 1

In the absence of the nutritional minimum standards (4b), the above multicommodity international trade model collapses to the special case multicommodity spatial price equilibrium model, where $\mathcal{K}^3 \equiv \{Q|Q \in R_+^{H2n}\}$ *with variational inequality formulation: determine* $Q^* \in \mathcal{K}^3$ *such that*

$$\sum_{h=1}^{H}\sum_{i=1}^{n}\sum_{j=1}^{n}(\tilde{\pi}_i^h(Q^*)+c_{ij}^h(Q^*)-\tilde{\rho}_j^h(Q^*))\times\left[Q_{ij}^h-Q_{ij}^{h*}\right]\geq 0, \quad \forall Q\in\mathcal{K}^3. \tag{26}$$

Proof: See Nagurney (1999) and Nagurney and Besik (2022).

3.2 Consumer Subsidies

Observe that equilibrium conditions (7) may be rewritten as: for each commodity h; $h = 1,\dots,H$, and for each pair of country supply and demand markets (i,j); $i = 1,\dots,n$; $j = 1,\dots,n$:

$$\tilde{\pi}_i^h(Q^*)+c_{ij}^h(Q^*)\begin{cases} = \tilde{\rho}_j^h(Q^*)+\sum_{l=1}^{L}\alpha^{l,h}\lambda_j^{l*}, & \text{if} \quad Q_{ij}^{h*}>0, \\ \geq \tilde{\rho}_j^h(Q^*)+\sum_{l=1}^{L}\alpha^{l,h}\lambda_j^{l*}, & \text{if} \quad Q_{ij}^{h*}=0. \end{cases} \tag{27}$$

Hence, according to (27), the expression $\sum_{l=1}^{L}\alpha^{l,h}\lambda_j^{l*}$ for each commodity h and country demand market j provides the additional "value" associated with the commodity h at country demand market j and, in effect, is the additional price of the commodity that will guarentee that the commodity shipments will be such that the minimum nutritional standards are met in the country for nutrients needed by its population. Without such a subsidy with respect to each commodity h, the volume of commodities to its country demand market may not be sufficient to guarantee that the nutritional minimal standards are achieved since consumers would only be willing to pay a price of $\tilde{\rho}_j^h(Q^*)$ for commodity h at j without the subsidy.

Also, this consumer subsidy is for a "unit" of the commodity and, in the case of agricultural commodities in international trade, the unit of measure is, typically, a ton. The total financial outlay, hence, for the government of country j then would be, for commodity h:

$$\sum_{l=1}^{L}\alpha^{l,h}\lambda_j^*\sum_{i=1}^{n}Q_{ij}^{h*},$$

with the financial outlay covering subsidies for all commodities:

$$\sum_{h=1}^{H}\sum_{l=1}^{L}\alpha^{l,h}\lambda_j^*\sum_{i=1}^{n}Q_{ij}^{h*}.$$

2.3 Illustrative Examples

To further amplify the importance of the modeling framework, simple illustrative examples are now presented.

First, an example is presented without nutritional minimum standards and then the results given for the same example, but with an added nutritional minimum standard. The commodity is wheat, which is essential since it is used in flour for the production of bread, a staple in MENA countries, among other countries. Wheat, in addition to providing calories, is also a source of protein.

The country supply market is Ukraine, often called the bread basket of Europe. The period of time is 1 year and prior to the full-scale invasion of Ukraine by Russia on February 24, 2022. The country demand market is Lebanon. Without loss of generality, the country supply market of Ukraine is labeled by 1 and the country demand market of Lebanon by 1. We suppress the superscript h since there is only a single commodity.

The unit of commodity shipment is a ton, which is, as noted earlier, quite standard for agricultural products in the context of international trade. Also, the economic functions are in a common currency - the US dollar.

The supply price function in Ukraine is:

$$\pi_1(s) = .0001s_1 + 200.$$

The demand price function in Lebanon is:

$$\rho_1(d) = -.0001d_1 + 411,$$

and the unit transportation cost associated with shipping the wheat from Ukraine to Lebanon, which would be via rail and then the Black Sea, is:

$$c_{11} = .00002Q_{11} + 90.$$

Note that, in this example, one has that

$$s_1^* = d_1^* = Q_{11}^*.$$

Solving for Q_{11}^* in the equilibrium condition, since $Q_{11}^* > 0$:

$$\pi_1 + c_{11} = \rho_1,$$

one obtains, after making use of the conservation of flow equations:

$$.0001Q_{11} + 200 + .00002Q_{11} + 90 = -.0001Q_{11} + 411,$$

which simplifies to:

$$.00022Q_{11}^* = 121,$$

or

$$Q_{11}^* = 550,000.$$

Furthermore, $\pi_1 = 255$, $c_{11} = 101$, and $\rho_1 = 356$. Clearly, the equilibrium conditions hold. Furthermore, the supply price, unit transportation cost, and the demand price are very reasonable (cf. Nagurney et al. (2023)) and correspond to similar values achieved in practice prior to the war.

This example is now expanded to include a nutritional minimum standard. Specifically, wheat (which is processed into flour) has 3,300,000 calories per ton. Plus, assuming 3,000 calories per day per individual (which is generous), and a population in Lebanon of 5.5 million, since there are 365 days in a year, $t_1 = 6.02 \times 10^{12}$.

Assuming that $\lambda_1^* > 0$, according to equilibrium condition (8), we obtain

$$3.3 \times 10^6 Q_{11}^* = 6.02 \times 10^{12},$$

and, therefore, $Q_{11}^* = 1.825 \times 10^6$.

Under this commodity shipment pattern, the supply price is:

$$\pi_1 = 382.50,$$

the unit transportation cost is:

$$c_{11} = 126.50,$$

and the demand price is:

$$\rho_1 = 228.50.$$

Making use of the equality in equilibrium conditions (7) yield:

$$\lambda_1^* = 8.5 \times 10^{-5},$$

with

$$\alpha^1 \lambda_1^* = 280.50.$$

Hence, the consumer subsidy that the government of Lebanon should pay out is 280.50 per ton of wheat. Without the subsidy, the total calories obtained by the Lebanese from wheat would be 18.15×10^{11}, which is an order of magnitude less than 6.02×10^{12}, the amount obtained with the consumer subsidy. The total financial payout in terms of consumer subsidies (for all the tons) is, therefore, $\alpha^1 \lambda_a^* Q_{11}^* = 280.50 \times 1.825 \times 10^6 = 511.91 \times 10^6$.

4. The Computational Procedure

The algorithm that is applied to compute the equilibrium multicommodity shipment and Lagrange multiplier patterns in numerical examples of relevance to practice in Section 4 is the modified projection method of Korpelevich (1977). Its convergence is guaranteed if the $F(X)$ as in (18) is monotone and Lipschitz continuous.

Recall that function $F(X)$ is said to be monotone if

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle \geq 0, \quad \forall X^1, X^2 \in \mathcal{K}. \tag{28}$$

Also, $F(X)$ is Lipschitz continuous, if there exists an $\eta > 0$, known as the Lipschitz constant, such that

$$\|F(X^1) - F(X^2)\| \leq \eta \|X^1 - X^2\|, \quad \forall X^1, X^2 \in \mathcal{K}. \tag{29}$$

Note that if the π, c, and $-\rho$ functions are strongly or strictly monotone or just monotone then $F(X)$ for the international trade model is monotone.

For completeness and easy reference, the steps of the modified projection method are delineated below. Here, τ corresponds to an iteration. The explicit form of these steps for the solution of the international trade equilibrum model with minimum nutritional standards is then given. Because the feasible set $\mathcal{K}$ for the model is the nonnegative orthant, the steps consist of closed form expressions for the multicommodity shipments and for the Lagrange multipliers at each iteration. The algorithm is, hence, easy to implement.

The Modified Projection Method

Step 0: Initialization

Initialize with $X^0 \in \mathcal{K}$. Set the iteration counter $\tau = 1$ and let ζ be a scalar such that $0 < \zeta \leq \frac{1}{\eta}$, where η is the Lipschitz constant.

Step 1: Computation

Compute $\bar{X}^\tau$ by solving the variational inequality subproblem:

$$\langle \bar{X}^\tau + \zeta F(X^{\tau-1}) - X^{\tau-1}, X - \bar{X}^\tau \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{30}$$

Step 2: Adaptation

Compute X^τ by solving the variational inequality subproblem:

$$\langle X^\tau + \zeta F(\bar{X}^\tau) - X^{\tau-1}, X - X^\tau \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{31}$$

Step 3: Convergence Verification

If $|X^\tau - X^{\tau-1}| \leq \epsilon$, with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $\tau := \tau + 1$ and go to Step 1.

The explicit formulae for Step 1 above are now given.

Explicit Formulae at Iteration τ for the Multicommodity Shipments in Step 1

The modified projection method results in the following closed form expressions for (30) for the multicommodity shipments in Step 1 for the solution of variational inequality (9):

$$\bar{Q}_{ij}^{h\tau} = \max\{0, Q_{ij}^{h\tau-1} + \zeta(\tilde{\rho}_j^h(Q^{\tau-1}) - \tilde{\pi}_i^h(Q^{\tau-1}) - c_{ij}^h(Q^{\tau-1}) + \sum_{l=1}^{L} \alpha^{l,h} \lambda_j^{l\tau-1})\}, \quad \forall h, i, j. \tag{32}$$

Explicit Formulae at Iteration τ for the Lagrange Multipliers in Step 1

The closed form expressions for the Lagrange multipliers in (27) for our variational inequality are:

$$\bar{\lambda}_j^{l\tau} = \max\{0, \lambda_j^{l\tau-1} + \zeta(-\sum_{h=1}^{H} \alpha^{l,h} \sum_{i=1}^{n} Q_{ij}^{h\tau-1} + t_j^l)\}, \quad \forall l, j. \tag{33}$$

The analogous explicit formulae for the multicommodity shipment variables and for the Lagrange multiplier variables in (31) of Step 2 readily follow.

5. Numerical Examples

The numerical examples consist of three countries: Ukraine, Egypt, and Lebanon. The examples consist of Examples 1 through 3 and consider the commodity of wheat. The

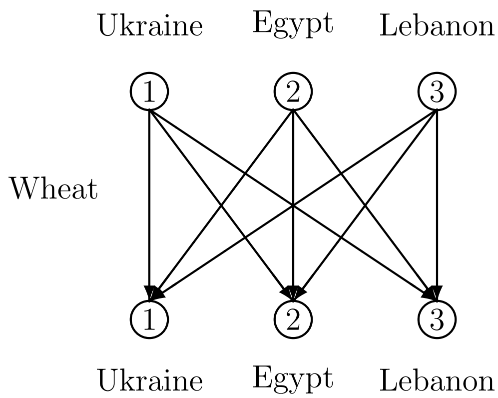

Figure 2: The International Trade Network for Examples 1 Through 3

network topology for these examples is, hence, as depicted in Figure 2. Ukraine corresponds to supply and demand market node 1; Egypt to such nodes 2, and Lebanon to supply and demand market node 3.

The modified projection method was implemented in Fortran and a Linux sytem at the University of Massachusetts used for the computations. The complete input and output data are reported for all the numerical examples below. The algorithm was initialized with all variables set to 0.00. The ζ was set to .01. The algorithm was deemed to have converged if the absolute value of all successive variable iterates was less than or equal to $\epsilon = 10^{-2}$.

Example 1: Baseline

Example 1 is situated (as the Illustrative Examples were) prior to Russia's invasion of Ukraine on February 24, 2022.

Example 1 has no minimum nutritional standards for wheat and serves as the baseline for subsequent examples. The nutrient considered is that of calories.

The data, in the US dollar currency, are as follows. Wheat is denoted by the superscript 1.

The country supply price functions for wheat are:

$$\pi_1^1(s) = .000002s_1^1 + 230, \quad \pi_2^1(s) = .000002s_2^1 + 265, \quad \pi_3^1(s) = .000155s_3^1 + 275.$$

The commodity unit transportation cost functions for wheat are:

$$c_{11}^1(Q) = .000001Q_{11}^1 + 35, \quad c_{12}^1(Q) = .000005Q_{12}^1 + 80, \quad c_{13}^1(Q) = .000025Q_{13}^1 + 80,$$

$$c_{21}^1(Q) = .001565Q_{21}^1 + 80, \quad c_{22}^1(Q) = .000001Q_{22}^1 + 40, \quad c_{23}^1(Q) = .001175Q_{23}^1 + 50,$$

$$c_{31}^1(Q) = .001375Q_{31}^1 + 80, \quad c_{32}^1(Q) = .001195Q_{32}^1 + 50, \quad c_{33}^1(Q) = .000095Q_{33}^1 + 30.$$

The country demand price functions for wheat are:

$$\rho_1^1(d) = -.000001d_1^1 + 320, \quad \rho_2^1(d) = -.000001d_2^1 + 355, \quad \rho_3^1(d) = -.000017d_3^1 + 355.$$

The computed equilibrium commodity shipments are reported in Table 2.

Based on the equilibrium volumes, the caloric needs of the populations in Egypt and Lebanon are not satisfied but they are in Ukraine. Therefore, the impacts of imposing minimum nutritional caloric amounts in each of the three countries, based on the population of each country, are now explored in the next two examples.

Example 2

Example 2 has the identical data to that in Example 1 except that now minimum caloric requirements for the populations in Ukraine, Egypt, and Lebanon are included. A population of 43 millionis assumed for Ukraine, a population of 109 million for Egypt, and a population of 5.5 million for Lebanon. As in the second Illustrative Example, a caloric minimum of 3,000 per individual is assumed and 365 days to the year. Wheat has 3,300,000 calories per ton. Since here a single nutrient (calories) is considered and note that wheat can have a substantial amount of protein, the l in the notation is suppressed. Hence, $\alpha^1 = 3,300,000$, and $t_1 = 4.71 \times 10^{13}$, $t_2 = 1.19 \times 10^{14}$, and $t_3 = 6.02 \times 10^{12}$. The computed equilibrium commodity shipments are given in Table 2.

The consumer subsidies are as follows. The subsidy in Ukraine is: 19.02. The subsidy in Egypt is: 78.00, and the subsidy in Lebanon is: 71.26.

Example 3

Example 3 has the identical data to that in Example 2 except that now each individual in each country has a minimum caloric requirement of only 2,000 calories. α^1 remains at $3,300,000$, but now $t_1 = 3.14 \times 10^{13}$, $t_2 = 7.96 \times 10^{13}$, and $t_3 = 4.02 \times 10^{12}$. The computed equilibrium commodity shipments are reported in Table 2.

The consumer subsidies are now as follows. The consumer subsidy is 0.00 in Ukraine. The consumer subsidy is 38.40 in Egypt, and the consumer subsidy is 32.79 in Lebanon. Ukraine is able to meet its nutritional caloric requirements by itself and no imports are needed, which is quite reasonable. Note that the subsidies for consumers in all three countries are lower than in Example 2 and that is because the caloric minimum is set at 2,000 rather than at

Table 2: Equilibrium Solution for Examples 1, 2, and 3

Equilibrium Wheat Commodity Flows	**Ex. 1**	**Ex. 2**	**Ex. 3**
Q_{11}^{1*}	11650770.00	14200000.00	9520001.00
Q_{12}^{1*}	1335727.75	8664518.00	5833827.00
Q_{13}^{1*}	362736.38	1444176.25	961641.25
Q_{21}^{1*}	0.00	0.00	0.00
Q_{22}^{1*}	10912198.00	27382862.00	18261452.00
Q_{23}^{1*}	7732.52	21733.81	13132.48
Q_{31}^{1*}	0.00	0.00	0.00
Q_{32}^{1*}	0.00	12619.76	4720.99
Q_{33}^{1*}	163465.44	354090.00	245226.22
Wheat Supply Prices at Equilibrium	**Ex. 1**	**Ex. 2**	**Ex. 3**
$\pi_1^1(s^*)$	256.70	278.62	262.63
$\pi_2^1(s^*)$	286.84	319.81	301.55
$\pi_3^1(s^*)$	300.34	331.84	313.55
Transportation Costs at Equilibrium	**Ex. 1**	**Ex. 2**	**Ex. 3**
$c_{11}^1(Q^*)$	46.65	49.20	44.52
$c_{12}^1(Q^*)$	86.68	123.32	109.17
$c_{13}^1(Q^*)$	89.07	116.10	104.04
$c_{21}^1(Q^*)$	80.00	80.00	80.00
$c_{22}^1(Q^*)$	50.91	67.38	58.26
$c_{23}^1(Q^*)$	59.09	75.54	65.43
$c_{31}^1(Q^*)$	80.00	80.00	80.00
$c_{32}^1(Q^*)$	50.00	65.08	55.64
$c_{33}^1(Q^*)$	45.53	63.64	53.30
Wheat Demand Prices at Equilibrium	**Ex. 1**	**Ex. 2**	**Ex. 3**
$\rho_1^1(d^*)$	308.35	305.80	310.48
$\rho_2^1(d^*)$	342.75	318.94	330.90
$\rho_3^1(d^*)$	345.92	324.06	334.26

3,000. Wheat farmers get the highest price per ton of wheat in Example 2 and the lowest in Example 1. This shows that, through the subsidization of consumers, producers can also benefit. The highest commodity shipments are obtained in Example 2 across all the country trade flows. The prices are reasonable - recall that they are for a ton of grain (see, also, Nagurney et al. (2023)).

Sensitivity analysis is now conducted using Example 3. Specifically, with the major invasion of Ukraine by Russia on February 24, 2022, farmers in Ukraine have been faced with many challenges and the challenges are affecting the supply prices of Ukrainian wheat (cf. Nagurney et al. (2023)). In Figure 3, the consumer subsidies are displayed that Ukraine, Egypt, and Lebanon would need to pay to guarantee that the caloric needs of their respective population are met. Recall that the intercept in Ukraine's supply price function in Examples 1 through 3 is 230. This term is increased, which would reflect disruptions to farming, and the results reported in Figure 3.

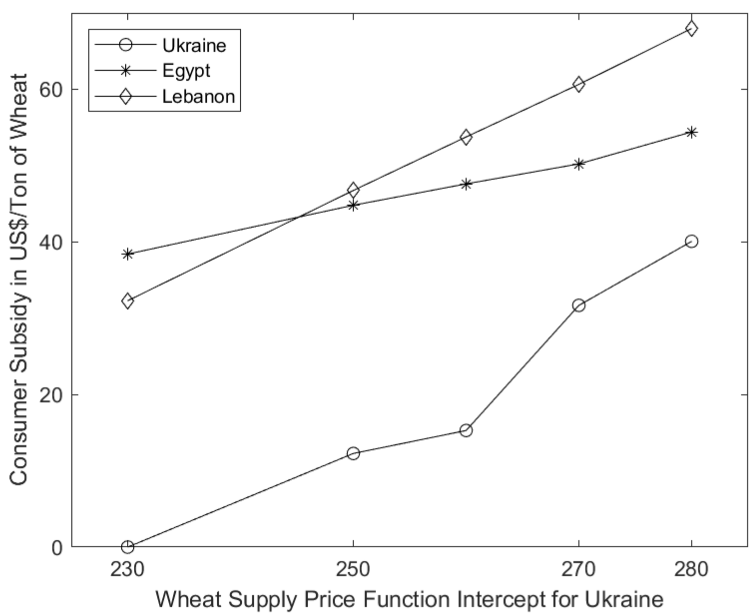

Figure 3: Sensitivity Analysis for the Supply Price Function Intercept for Ukraine in Example 3

As can be seen from Figure 3, when the supply price intercept in Ukraine for wheat increases, all countries need to increase their consumer subsidies to ensure that the caloric needs of their population are met. Disruptions to agriculture in one country, because of international trade and connectivity, can affect food security in other countries. The examples are stylized but demonstrate that a network equilibrium model for international trade with minimum nutritional standards and consumer subsidies can yield useful insights. It is important to address food security issues and challenges through rigorous mathematical models.

6. Summary and Conclusions

Food insecurity and hunger have been growing globally with climate change, the COVID-19 pandemic, as well as major conflicts including Russia's war on Ukraine exacerbating the pain and suffering. International trade has served as a mechanism for the allocation of resources and products including agricultural ones. Nutritious food is essential to a thriving, healthy citizenry and, thus, governments have, historically, made use of various policies for agricultural trade from the imposition of tariffs and quotas to subsidies for farmers. In this paper, we provide an integrated framework for multicommodity international trade with a focus on food security through minimum nutritional standards that are enabled through consumer subsidies. The theoretical framework is that of variational inequalities, which is utilized for the formulation of the governing equilibrium conditions (and alternative such formulations are provided), qualitative analysis, as well as for the construction of an algorithm that has nice features for implementation for our model. In addition, we propose explicit formulae for the consumer subsidies.

The model is illustrated via simple examples as well as a series of examples drawn from the real world - that of Russia's was on Ukraine and the impacts on food insecurity. Specifically, we present and solve numerical examples for the countries of Ukraine, Egypt, and Lebanon and the commodity of wheat. Sensitivity analysis results are also displayed illustrating the changes in consumer subsidies needed as the supply price function for Ukraine, often referred to as the breadbasket of Europe, is modified to reflect increases in cost due to war, for example. The results demonstrate that a local disruption can have impacts on food security in multiple countries. This work adds to the literature on international trade and food security with an emphasis on nutrition and government interventions.

Future research is possible in many different directions, including: incorporating different classes of consumers in countries and their nutritional minimal requirements; including budget constraints of governments for nutritional subsidies; comparing subsidies for farmers (on the production side) with subsidies for consumers (on the consumption side), and factoring in accessibility of consumers to the agricultural products for their nutritional needs, among other topics.

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References

Albornoz, V.M., Cancela, H., Mac Cawley, A., Maturana, S., Weintraub, A., 2023. Special issue: OR and big data in agriculture. International Transactions in Operational Research, 30(4), 1579-2131.

An, R., 2013. Effectiveness of subsidies in promoting healthy food purchases and consumption: A review of field experiments. Public Health Nutrition, 16(7), 1215-1228.

Barnes, J., 2023. The Ukraine war, grain trade and bread in Egypt. Middle East Research and Information Project. February 22. Available at:
https://merip.org/2023/02/the-ukraine-war-grain-trade-and-bread-in-egypt/

Barraza De La Cruz, B.C., Pizzolato, N.D., Barraza De La Cruz, A., 2010. An application of the spatial equilibrium model to soybean production in tocantins and neighboring states in Brazil. Pesquisa Operacional, 30(2), 443-464.

Besik, D., Nagurney, A., 2017. Quality in competitive fresh produce supply chains with application to farmers' markets. Socio-Economic Planning Sciences, 60, 62-76.

Birge, J., Chan, T.C.Y., Pavlin, J.M., Zhu, I.Y., 2022. Spatial price integration in commodity markets with capacitated transportation networks. Operations Research, 70(3), 1739-1761.

Bishop, P., Pratt, J., Novakovic, A., 1994. Using a joint-input, multi-product formulation to improve spatial price equilibrium models. Cornell University, Department of Agricultural Economics Staff Paper no. 94-06, Ithaca, NewYork.

Casey, C.A., Cimino-Isaacs, C.D., 2021. Export restrictions in response to the COVID-19 pandemic. Congressional Research Service, April 21. Available at:
https://crsreports.congress.gov/product/pdf/IF/IF11551

Dafermos, S., Nagurney, A., 1984. Sensitivity analysis for the general spatial economic equilibrium problem. Operations Research, 32, 1069-1086.

Dupuis, P., Nagurney, A., 1993. Dynamical systems and variational inequalities. Annals of Operations Research, 44, 7-42.

Euroactive, 2022. How the Ukraine war is reshaping the CAP. April 4. Available at:
https://www.euractiv.com/section/agriculture-food/special_report/dg-agri-sr-draft-page/

Florian, M., Los, M. 1982. A new look at static spatial price equilibrium models. Regional Science and Urban Economics, 12, 579-597.

Food and Agricultural Organization, 2016. Save and grow in practice: Maize, rice, wheat. A guide to sustainable cereal production. Available at:
http://www.fao.org/ag/save-and-grow/MRW/index_en.html

Food and Agricultural Organization, 2021. The state of food security and nutrition in the world 2021. Available at:
https://www.fao.org/state-of-food-security-nutrition/2021/en/

Food and Agricultural Organization, 2023. Hunger and food insecurity. Available at:
https://www.fao.org/hunger/en/

Howard, E.A., 1984. An analysis of interregional competition in the U.S. summer potato market, Master of Science dissertation, Department of Agricultural Economics, Texas A&M University, College Station, Texas.

Hua, J., Yao, K., 2021. China to give farmers $3.1 bln in subsidies amid rising costs, Reuters, June 18. Available at:
https://www.nasdaq.com/articles/china-to-give-farmers-$3.1-bln-in-subsidies-amid-rising-costs-2021-06-18

Judge, G.G., 1956. A spatial equilibrium model for eggs. Connecticut Agricultural Experiment Station, Storrs, Connecticut.

Kinderlehrer, D., Stampacchia, G., 1980. Variational Inequalities and Their Applications. Academic Press, New York.

Korpelevich G.M., 1977. The extragradient method for finding saddle points and other problems. Matekon, 13, 35-49.

Nagurney, A., 1999. Network Economics: A Variational Inequality Approach, second and revised edition. Kluwer Academic Publishers, Boston, Massachusetts.

Nagurney, A., Aronson, J., 1989. A general dynamic spatial price network equilibrium model with gains and losses. Networks, 19(7), 751-769.

Nagurney, A., Besik, D., 2022. Spatial price equilibrium networks with flow-dependent arc multipliers. Optimization Letters, 26, 2483-2500.

Nagurney, A., Besik, D., Dong, J., 2019. Tariffs and quotas in world trade: A unified variational inequality framework. European Journal of Operational Research, 275(1), 347-360.

Nagurney, A., Besik, D., Nagurney, L.S., 2019. Global supply chain networks and tariff rate quotas: Equilibrium analysis with application to agricultural products. Journal of Global Optimization, 75(2), 439-460.

Nagurney, A., Hassani, D., Nivievskyi, O., Martyshev, P. 2023. Exchange rates and multicommodity international trade: Insights from spatial price equilibrium modeling with policy instruments via variational inequalities. Journal of Global Optimization, 87, 1-30.

Nagurney, A., Li, D., Nagurney, L.S., 2014. Spatial price equilibrium with information asymmetry in quality and minimum quality standards. International Journal of Production Economics, 158, 300-313.

Nagurney, A., Nicholson, C.F., Bishop, P.M., 1996. Massively parallel computation of large-scale spatial price equilibrium models with discriminatory ad valorem tariffs. Annals of Operations Research, 68, 281-300.

Nagurney, A., Salarpour, M., Dong, J., 2022. Modeling of Covid-19 trade measures on essential products: A multiproduct, multicountry spatial price equilibrium framework. International Transactions in Operational Research, 29(1), 226-258.

Nagurney, A., Thore, S., Pan, J., 1996. Spatial market policy modeling with goal targets. Operations Research, 44(2), 393-406.

Nicholas, K., Brady, M., 2020. Revealed: How billions in EU farming subsidies are being misspent. Lund University, Sweden, August 24. Available at:
https://www.lunduniversity.lu.se/article/revealed-how-billions-eu-farming-subsidies-are-being-misspent

O'Neill Hayes, T., Kerska, K., 2023. Primer: Agricultural subsidies and their influence on the composition of U.S. food supply and consumption. American Action Forum, Novembe 3. Available at:
https://www.americanactionforum.org/research/primer-agriculture-subsidies-and-their-influence-on-the-composition-of-u-s-food-supply-and-consumption/

Pang, J.S., 1985. Asymmetric variational inequality problems over product sets: Applications and iterative methods. Mathematical Programming, 31, 206-219.

Pla, L.M., Sanders, D.L., Higgins, A.J., 2014. A perspective on operational research prospects for agriculture. The Journal of the Operational Research Society, 65(7), 1078-1089.

Reuters, 2020. Trade restrictions on food exports due to the coronavirus pandemic. April 3.

Available at:
https://www.reuters.com/article/us-health-coronavirus-trade-food-factbox/trade-restrictions-on-food-exports-due-to-the-coronavirus-pandemic-idUSKBN21L332

Ruijs, A., Schweigman, C., Lutz, C., Sirpe, G., 2001. Cereal trade in developing countries: Stochastic spatial equilibrium models. Technical report, University of Groningen, The Netherlands.

Samuelson, P.A., 1952. Spatial price equilibrium and linear programming. American Economic Review, 42, 283-303.

Schweigman, C., 2008. Food security problems in sub-Saharan Africa: Operations Research as a tool of analysis. International Transactions in Operational Research, 15(2), 173 - 193.

Sohn, H.K., 1970. A spatial equilibrium model of the beef industry in the United States, PhD Dissertation, Department of Agricultural Economics, University of Hawaii.

Takayama, T., Judge, G.G., 1964. An intertemporal price equilibrium model. Journal of Farm Economics, 46, 477-484.

Takayama, T., Judge, G.G., 1971. Spatial and Temporal Price and Allocation Models. Amsterdam, The Netherlands: North-Holland.

The Economist, 2023. What will be the impact of India's rice-export ban? July 26. Available at:
https://www.economist.com/the-economist-explains/2023/07/26/what-will-be-the-impact-of-indias-rice-export-ban

Thompson, R.L., 1989. Spatial and temporal price equilibrium agricultural models. In: Quantitative Methods for Market Oriented Economic Analysis Over Space and Time. W.C. Labys, T. Takayama, and N. Uri, Editors, Gower Publishing Co. Ltd., Brookfield, Vermont, pp 49-65.

United Nations, 2023. Sustainable Development Goal 2. Department of Economic and Social Affairs. Available at:
https://sdgs.un.org/goals/goal2

van Berkum, 2021. How trade can drive inclusive and sustainable food system outcomes in food deficit low-income countries. Food Security, 13, 1541-1554.

van Dooren, C., 2018. A review of the use of linear programming to optimize diets, nutritiously, economically and environmentally. Frontiers in Nutrition, 5, June 21.

Weintraub, A., Romero, C., 2006. Operations research models and the management of agricultural and forestry resources: A review and comparison. INFORMS Journal on Applied Analytics, 36(5), 446-457.

World Food Programme, 2023. A global food crisis. Rome, Italy. Available at: https://www.wfp.org/global-hunger-crisis

World Food Summit, 1996. Rome declaration on world food security.

Zimmermann, A., Rapsomanikis, G., 2023. Trade and sustainable food systems. In: von Braun, J., Afsana, K., Fresco, L.O., Hassan, M.H.A., Editors, Science and Innovations for Food Systems Transformation. Springer, Cham. https://doi.org/10.1007/978-3-031-15703-5_36