Fashion Supply Chain Management Through Cost and Time Minimization from a Network Perspective

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August 2010; revised September 2010

In: *Fashion Supply Chain Management: Industry and Business Analysis*,2011,
T.-M. Choi, Editor, IGI Global, Hershey, PA, pp. 1-20.

Abstract: In this paper, we consider fashion supply chain management through cost and time minimization, from a network perspective and in the case of multiple fashion products. We develop a multicriteria decision-making optimization model subject to multimarket demand satisfaction, and provide its equivalent variational inequality formulation. The model allows for the determination of the optimal multiproduct fashion flows associated with the supply chain network activities, in the form of: manufacturing, storage, and distribution, and identifies the minimal total operational cost and total time consumption. The model allows the decision-maker to weigh the total time minimization objective of the supply chain network for the time-sensitive fashion products, as appropriate. Furthermore, we discuss potential applications to fashion supply chain management through a series of numerical examples.

Key words: fashion supply chain, fast fashion, network economics, multiple products, time-sensitive products, multicriteria decision-making, total cost minimization, time performance, optimization

1. Introduction

In recent decades, fashion retailers, such as Benetton, H&M, Topshop, and Zara have revolutionized the fashion industry by following what has become known as the "fast fashion" strategy, in which retailers respond to shifts in the market within just a few weeks, versus an industry average of six months (Sull and Turconi (2008)). Specifically, fast fashion is a concept developed in Europe to serve markets for teenage and young adult women who desire trendy, short-cycle, and relatively inexpensive clothing, and who are willing to buy from small retail shops and boutiques (Doeringer and Crean (2006)). Fast fashion chains have grown quicker than the industry as a whole and have seized market share from traditional rivals (Sull and Turconi (2008)), since they aim to obtain fabrics, to manufacture samples, and to start shipping products with far shorter lead times than those of the traditional production calendar (Doeringer and Crean (2006)).

Nordas, Pinali, and Geloso Grosso (2006) further argued that time is a critical component in the case of labor-intensive products such as clothing as well as consumer electronics, both examples of classes of products that are increasingly time-sensitive. They presented two case studies of the textile and clothing sector in Bulgaria and the Dominican Republic, respectively, and noted that, despite higher production costs than in China, their closeness to major markets gave these two countries the advantage of a shorter lead time that allowed them to specialize in fast fashion products. Interestingly and importantly, the authors also identified that lengthy, time-consuming administrative procedures for exports and imports reduce the probability that firms will even enter export markets for time-sensitive products.

Clearly, superior time performance must be weighed against the associated costs. Indeed, as noted by So (2000), it can be costly to deliver superior time performance, since delivery time performance generally depends on the available capacity and on the operating efficiency of the system. It is increasingly evident that, in the case of time-sensitive products, with fashion being an example par excellence, an appropriate supply chain management framework for such products must capture both the operational (and other) cost dimension as well as the time dimension.

For example, in the literature, the total order cycle time, which refers to the time elapsed in between the receipt of customer order until the delivery of finished goods to the customer, is considered an important measure as well as a major source of competitive advantage (see Bower and Hout (1988) and Christopher (1992)), directly influencing the customer satisfaction level (cf. Gunasekaran, Patel, and Tirtiroglu (2001) and Towill (1997)). Moreover, according to the survey of Gunasekaran, Patel, and McGaughey (2004), performance metrics for time issues associated with planning, purchasing, manufacturing, and delivery are consistently rated as important factors in supply chain management.

Conventionally, there have been several methodological approaches utilized for time-dependent supply chain management, including multiperiod dynamic programming and queuing theory (see, e.g., Guide Jr., Muyldermans, and Van Wassenhove (2005), Lederer and Li (1997), Palaka, Erlebacher, and Kropp (1998), So and Song (1998), So (2000), Ray and Jewkes (2004), and Liu, Parlar, and Zhu (2007)). However, according to the review by Goetschalckx, Vidal, and Dogan (2002), the paper by Arntzen et al. (1995) is the only one that has captured the time issue in the modeling and design of a global logistics system, with the expression of time consumption explicitly in the objective function.

In particular, Arntzen et al. (1995) applied the Global Supply Chain Model (GSCM) to the Digital Equipment Corporation so as to evaluate global supply chain alternatives and to determine the worldwide manufacturing and distribution strategies. In their mixed-integer linear programming model to minimize the weighted combination of total cost and activity days, the authors adopted a weighted activity time to measure activity days throughout the supply chain, which is the sum of processing times for each individual segment multiplied by the number of units processed or shipped through the link. However, we believe that the authors oversimplified the weighted activity time in assuming that the unit processing activity days are fixed, regardless of the facility capacities and the product flows. Also, in some other mathematical models dealing with time-sensitive demand, the lead time is used as the only indicator to differentiate the demand groups (see Cheong, Bhatnagar, and Graves (2004)). We note that Ferdows, Lewis, and Machuca (2004) recognized the nonlinear relationship between capacity and time in the context of the fashion industry and fast response with a focus on Zara and, hence, an appropriate model for fashion supply chain management must be able to handle such nonlinearities.

In this paper, we utilize a network economics approach to develop a mathematical model for fashion supply chain management that allows a firm to determine its cost-minimizing and time-minimizing multiproduct flows, subject to demand satisfaction at the demand markets, with the inclusion of an appropriate weight associated with time minimization. Hence, we utilize a multicriteria decision-making perspective. In addition, we allow the cost on each network link, be it one corresponding to manufacturing (or procurement), to transportation/shipment, and/or to storage, or to any other type of product processing, which may also include administrative processing associated with importing/exporting, to be an increasing function of the flow in order to capture the aspect of capacity and, in effect, congestion, as would result in queuing phenomena. Hence, we take some ideas from the transportation and logistics literature (cf. Nagurney (1999) and the references therein). Similar assumptions we impose on the link time functions since, clearly, the time to process a volume of fashion product should be dependent on the flow. Given the realities of the fashion industry in the US (see, e.g., Sen (2008)), it is imperative to have a methodological framework that can provide decision-makers with both cost and time information associated with the complex network of fashion supply chain activities. As early as Fisher (1997) it has been recognized that different products may require distinct supply chains.

Multicriteria decision-making for supply chain management applications has been applied in both centralized and decentralized decision-making contexts and in the case of general, multitiered networks (see, e.g., Nagurney (2006) and Nagurney and Qiang (2009) and the references therein) with the most popular criteria utilized being cost, quality, and on-time delivery (Ho, Xu and Dey (2010)). Nagurney et al. (2005), in turn, developed a multitiered competitive supply chain network equilibrium model with supply side and demand side risk (see also Dong et al. (2005) and Nagurney and Matsypura (2005)). Nagurney and Woolley (2010) studied the decision-making problem associated with supply chain network integration, in the context of mergers and acquisitions, so as to minimize the cost and the emissions generated. Nagurney and Nagurney (2010) added environmental concerns into a supply chain network design model. In this paper, we capture the explicit time consumption associated with fashion supply chain activities, along with the associated costs, within a network framework. The model in this paper provides decision-makers with insights associated with trade-offs between the operational costs and the time involved in a multiproduct fashion supply chain subject to multimarket demand satisfaction.

This paper is organized as follows. In Section 2, we develop the fashion supply chain management model and reveal the generality of the associated network framework. We provide both the multicriteria decision-making optimization model as well as its equivalent variational inequality formulation. The latter is given, for the sake of generality, since it provides us with the foundation to also develop models for multiproduct competition in the fashion industry, with results on supply chain network design under oligopolistic competition and profit maximization obtained in Nagurney (2010). In addition, the variational inequality form allows for the efficient and effective computation of the multiproduct supply chain network flows. We also provide some qualitative properties.

In Section 3 we illustrate the model and its potential applications to fashion supply chain management through a series of numerical examples. In Section 4, we summarize the results in this paper and provide suggestions for future research.

2. The Fashion Supply Chain Management Model

We assume that the fashion firm is involved in the production, storage, and distribution of multiple fashion products and is seeking to determine its optimal multiproduct flows to its demand points (markets) under total cost minimization and total time minimization, with the latter objective function weighted by the fashion firm.

We consider the fashion supply chain network topology depicted in Figure 1 but emphasize that the modeling framework developed here is not limited to such a network. This network is only representative, for definiteness. The origin node in the network in Figure 1 consists of node 1, which represents the beginning of the product processing, and the destination nodes, $R_1, \ldots, R_{n_R}$, are the demand points (markets) located at the bottom tier of the network. The paths joining the origin node to the destination nodes represent sequences of supply chain network activities corresponding to directed links that ensure that the fashion products are produced and, ultimately, delivered to the demand points. Hence, different supply chain network topologies to that depicted in Figure 1 correspond to distinct fashion supply chain network problems. For example, if the fashion product(s) can be delivered directly to the demand points from a manufacturing plant, then there would be, as depicted, links joining the corresponding nodes.

We assume that the fashion producing firm is involved in the production, storage, and transportation / distribution of J products, with a typical product denoted by j. In particular, as depicted in Figure 1, we assume that the firm has, at its disposal, n_M manufacturing facilities/plants; n_D distribution centers, and must serve the n_R demand points. The links from the top-tiered node are connected to the manufacturing facility nodes of the firm, which are denoted, respectively, by: $M_1, \ldots, M_{n_M}$. The links from the manufacturing facility nodes, in turn, are connected to the distribution/storage center nodes of the firm, which are denoted by $D_{1,1}, \ldots, D_{n_D,1}$. Here we allow for the possibility of multiple links joining each such pair of nodes to reflect possible alternative modes of transportation/shipment between the manufacturing facilities and the distribution centers, an issue highly relevant to the fashion industry.

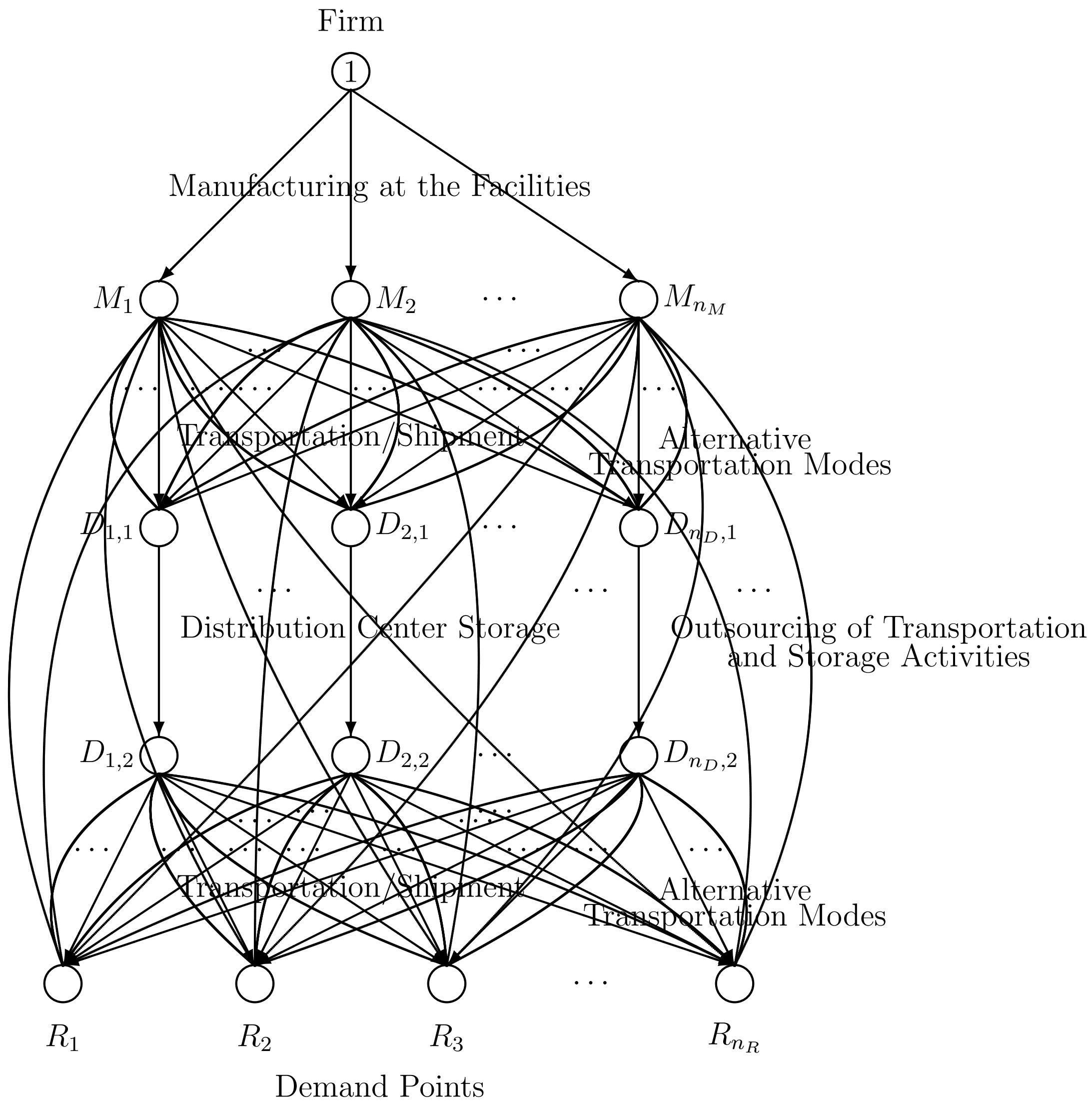

Figure 1: The Fashion Supply Chain Network Topology

The links joining nodes $D_{1,1}, \ldots, D_{n_D,1}$ with nodes $D_{1,2}, \ldots, D_{n_D,2}$ correspond to the possible storage links for the products. Finally, there are multiple transportation/shipment links joining the nodes $D_{1,2}, \ldots, D_{n_D,2}$ with the demand nodes: $R_1, \ldots, R_{n_R}$. Distinct such links also correspond to different modes of transportation/shipment.

The outermost links in Figure 1 can also depict the option of possible outsourcing of the transportation and storage activities, with appropriate assigned costs and time values, as will be discussed below. Indeed, our supply chain network framework is sufficiently general and flexible to also capture alternatives (such as outsourcing of some of the supply chain network activities) that may be available to the fashion firm.

We assume that in the supply chain network topology there exists one path (or more) joining node 1 with each destination node. This assumption for the fashion supply chain network model guarantees that the demand at each demand point will be satisfied. We denote the supply chain network consisting of the graph $G = [N, L]$, where N denotes the set of nodes and L the set of directed links.

The demands for the fashion products are assumed as given and are associated with each product and each demand point. Let d_k^j denote the demand for the product j; $j = 1, \ldots, J$, at demand point R_k. A path consists of a sequence of links originating at the top node and denotes supply chain activities comprising manufacturing, storage, and transportation/shipment of the products to the demand nodes. Note that, if need be, one can also add other tiers of nodes and associated links to correspond to import/export administrative activities. Let x_p^j denote the nonnegative flow of product j on path p. Let P_k denote the set of all paths joining the origin node 1 with destination (demand) node R_k. The paths are assumed to be acyclic.

The following conservation of flow equations must hold for each product j and each demand point R_k:

$$\sum_{p \in P_k} x_p^j = d_k^j, \quad j = 1, \ldots, J; \quad k = 1, \ldots, n_R, \tag{1}$$

that is, the demand for each product must be satisfied at each demand point.

Links are denoted by a, b, etc. Let f_a^j denote the flow of product j on link a. We must have the following conservation of flow equations satisfied:

$$f_a^j = \sum_{p \in P} x_p^j \delta_{ap}, \quad j = 1, \ldots, J; \quad \forall a \in L, \tag{2}$$

where $\delta_{ap} = 1$ if link a is contained in path p and $\delta_{ap} = 0$, otherwise. In other words, the flow of a product on a link is equal to the sum of flows of the product on paths that contain that link. Here P denotes the set of all the paths in Figure 1. The path flows must be nonnegative, that is,

$$x_p^j \geq 0, \quad j = 1, \ldots, J; \quad \forall p \in P. \tag{3}$$

We group the path flows into the vector x and the link flows into the vector f, respectively.

Below we present the optimization problems in path flows and in link flows.

There is a unit operational cost associated with each product and each link (cf. Figure 1) of the network. We denote the unit cost on a link a associated with product j by c_a^j. The unit cost of a link associated with each product, be it a manufacturing link, a transportation/shipment link, or a storage link, etc., is assumed, for the sake of generality, to be a function of the flow of all the products on the link. Hence, we have that

$$c_a^j = c_a^j(f_a^1, \ldots, f_a^J), \quad j = 1, \ldots, J; \quad \forall a \in L. \tag{4}$$

Note that in the case of an outsourcing link for a fashion product the unit cost may be fixed, as per the negotiated contract.

Let C_p^j denote the unit operational cost associated with product j; $j = 1, \ldots, J$, on a path p, where

$$C_p^j = \sum_{a \in L} c_a^j \delta_{ap}, \quad j = 1, \ldots, J; \quad \forall p \in P. \tag{5}$$

Then, the total operational cost for product j; $j = 1, \ldots, J$, on path p; $p \in P$, in view of (2), (4), and (5), can be expressed as:

$$\hat{C}_p^j(x) = C_p^j(x) \times x_p^j, \quad j = 1, \ldots, J; \quad \forall p \in P. \tag{6}$$

The total cost minimization problem, hence, is formulated as:

$$\text{Minimize} \quad \sum_{j=1}^{J} \sum_{p \in P} \hat{C}_p^j(x) \tag{7}$$

subject to constraints (1) and (3).

In addition, the firm also seeks to minimize the time consumption associated with the demand satisfaction for each product at each demand point. Let t_a^j denote the average unit time consumption for product j; $j = 1, \ldots, J$, on link a, $a \in L$. We assume that

$$t_a^j = t_a^j(f_a^1, \ldots, f_a^J), \quad j = 1, \ldots, J, \quad \forall a \in L, \tag{8}$$

that is, the link average unit time consumption is, also, for the sake of generality, a function of the flow of all the products on that link.

Therefore, the average unit time consumption for product j on path p is:

$$T_p^j = \sum_{a \in L} t_a^j \delta_{ap}, \quad j = 1, \ldots, J, \quad \forall p \in P, \tag{9}$$

with the total time consumption for product j on path p, in view of (2), (8), and (9), given by:

$$\hat{T}_p^j(x) = T_p^j(x) \times x_p^j, \quad j = 1, \ldots, J; \quad \forall p \in P. \tag{10}$$

The objective of time minimization problem is to minimize the total time associated with the supply chain network processing of all the products, which yields the following optimization problem:

$$\text{Minimize} \quad \sum_{j=1}^{J} \sum_{p \in P} \hat{T}_p^j(x), \tag{11}$$

subject to constraints (1) and (3).

The optimization problems (7) and (11) can be integrated into a single multicriteria objective function (cf. Dong et al. (2005)) using a weighting factor, ω, representing the preference of the decision-making authority. Please note that ω here can be interpreted as the monetary value of a unit of time. Consequently, the multicriteria decision–making problem, in path flows, can be expressed as:

$$\text{Minimize} \quad \sum_{j=1}^{J} \sum_{p \in P} \hat{C}_p^j(x) + \omega \sum_{j=1}^{J} \sum_{p \in P} \hat{T}_p^j(x), \tag{12}$$

subject to constraints (1) and (3).

The optimization problem (12), with the use of (2), (4), (5), (8), and (9), can be equivalently reformulated in link flows, rather than in path flows, as done above, as:

$$\text{Minimize} \quad \sum_{j=1}^{J} \sum_{a \in L} \hat{c}_a^j + \omega \sum_{j=1}^{J} \sum_{a \in L} \hat{t}_a^j, \tag{13}$$

subject to constraints (1) – (3), where $\hat{c}_a^j \equiv c_a^j(f_a^1, \ldots, f_a^J) \times f_a^j$ and the $\hat{t}_a^j \equiv t_a^j(f_a^1, \ldots, f_a^J) \times f_a^j$. We assume that the total link cost functions $\hat{c}_a^j$ and total time functions $\hat{t}_a^j$ are convex and continuously differentiable, for all products j and all links $a \in L$.

Let K denote the feasible set such that

$$K \equiv \{x | (1) \text{ and } (3) \text{ are satisfied}\}. \tag{14}$$

We now state the following result in which we derive the variational inequality formulations of the problem in both path flows and link flows, respectively. Having alternative formulations allows for the application of distinct algorithms (see, e.g., Nagurney (2006)).

Theorem 1

A path flow vector $x^* \in K$ is an optimal solution to the optimization problem (12), subject to constraints (1) and (3), if and only if it is a solution to the variational inequality problem in path flows: determine the vector of optimal path flows, $x^* \in K$, such that:

$$\sum_{j=1}^{J}\sum_{p\in P}\left[\frac{\partial \hat{C}_p^j(x^*)}{\partial x_p^j}+w\frac{\partial \hat{T}_p^j(x^*)}{\partial x_p^j}\right]\times(x_p^j-x_p^{j*})\geq 0, \quad \forall x\in K, \tag{15}$$

where $\frac{\partial \hat{C}_p^j(x)}{\partial x_p^j}\equiv\sum_{l=1}^{J}\sum_{a\in L}\frac{\partial \hat{c}_a^l(f_a^1,\ldots,f_a^J)}{\partial f_a^j}\delta_{ap}$, *and* $\frac{\partial \hat{T}_p^j(x)}{\partial x_p^j}\equiv\sum_{l=1}^{J}\sum_{a\in L}\frac{\partial \hat{t}_a^l(f_a^1,\ldots,f_a^J)}{\partial f_a^j}\delta_{ap}$.

A link flow vector $f^* \in K^1$ is an optimal solution to the optimization problem (13), subject to constraints (1) – (3), in turn, if and only if it is a solution to the variational inequality problem in link flows: determine the vector of optimal link flows, $f^* \in K^1$, such that:

$$\sum_{j=1}^{J}\sum_{l=1}^{J}\sum_{a\in L}\left[\frac{\partial \hat{c}_a^l(f_a^{1*},\ldots,f_a^{J*})}{\partial f_a^j}+\omega\frac{\partial \hat{t}_a^l(f_a^{1*},\ldots,f_a^{J*})}{\partial f_a^j}\right]\times(f_a^j-f_a^{j*})\geq 0, \quad \forall f\in K^1, \tag{16}$$

where $K^1\equiv\{f|(1)-(3)\, are\, satisfied\}$.

Proof: The result follows from the standard theory of variational inequalities (see the book by Nagurney (1999) and the references therein) since the functions comprising the objective functions are convex and continuously differentiable under the imposed assumptions and the respective feasible sets consisting of the constraints are nonempty, closed, and convex. □

In addition, the following theoretical results in terms of the existence of solutions as well as the uniqueness of a link flow solution are immediate from the theory of variational inequalities. Indeed, the existence of solutions to (15) and (16) is guaranteed since the underlying feasible sets, K and K^1, are compact and the corresponding functions of marginal total costs and marginal total time are continuous, under the above assumptions. If the total link cost functions and the total time functions are strictly convex, then the solution to (16) is guaranteed to be unique.

It is worth noting that the above model contains, as a special case, the multiclass system-optimization transportation network model of Dafermos (1972) if we set $\omega = 0$. The fashion supply chain management network model developed here is novel since it captures both the reality of multiple products in this application domain as well as the significant relevant criteria of cost minimization as well as time minimization in the production and delivery of the fashion products to the demand markets.

Variational inequality (15) can be put into standard form (see Nagurney (1999)): determine $X^* \in \mathcal{K}$ such that:

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{17}$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in n-dimensional Euclidean space. Indeed, if we define the column vectors: $X \equiv x$ and

$$F(X) \equiv \left[\frac{\partial \hat{C}_p^j(x)}{\partial x_p^j} + \omega \frac{\partial \hat{T}_p^j(x)}{\partial x_p^j}; \; j = 1, \ldots, J; \; p \in P \right], \tag{18}$$

and $\mathcal{K} \equiv K$ then (15) can be re-expressed as (17).

Similarly, if we define the column vectors: $X \equiv f$ and

$$F(X) \equiv \left[\frac{\partial \hat{c}_a^l(f_a^1, \ldots, f_a^J)}{\partial f_a^j} + \omega \frac{\partial \hat{t}_a^l(f_a^1, \ldots, f_a^J)}{\partial f_a^j}; \; j = 1, \ldots, J; \; l = 1, \ldots, J; \; a \in L \right], \tag{19}$$

and $\mathcal{K} \equiv K^1$ then (16) can be re-expressed as (17).

Note that the above model may be transformed into a single product network model by making as many copies of the network in Figure 1 as there are products and by constructing appropriate link total cost and time functions, which would be nonseparable, and by redefining the associated link flows, path flows, and demands accordingly. For details, see Nagurney and Qiang (2009) and the references therein.

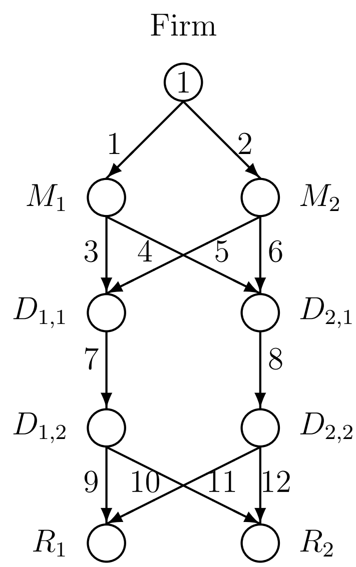

Figure 2: The Supply Chain Network Topology for the Numerical Examples

3. Numerical Examples

We now, for illustration purposes, present fashion supply chain numerical examples, both single product and multiproduct ones.

3.1 Single Product Fashion Supply Chain Examples

We assume that the fashion firm is involved in the production of a single fashion product and has, at its disposal, two manufacturing plants and two distribution centers. It must supply two different demand points. Hence, the topology is as depicted in Figure 2.

The manufacturing plant M_1 is located in the U.S., while the manufacturing plant M_2 is located off-shore and has lower operating cost. The average manufacturing time consumption of one unit of product is identical at these two plants, while the related costs vary mainly because of the different labor costs. The total cost functions and the total time functions for all the links are given in Table 1.

The demands for this fashion product at the demand points are:

$$d_1 = 100, \quad d_2 = 200,$$

that is, the market at demand point R_1 is half that at demand market R_2.

We used the the general equilibration algorithm of Dafermos and Sparrow (1969) (see also, e.g., Nagurney (1999)) for the solution of the numerical examples.

Table 1: Total Link Operational Cost and Total Time Functions

Link a	$\hat{c}_a(f_a)$	$\hat{t}_a(f_a)$
1	$10f_1^2 + 10f_1$	$f_1^2 + 10f_1$
2	$f_2^2 + 5f_2$	$f_2^2 + 10f_2$
3	$f_3^2 + 3f_3$	$.5f_3^2 + 5f_3$
4	$f_4^2 + 4f_4$	$.5f_4^2 + 7f_4$
5	$2f_5^2 + 30f_5$	$.5f_5^2 + 25f_5$
6	$2f_6^2 + 20f_6$	$.5f_6^2 + 15f_6$
7	$.5f_7^2 + 3f_7$	$f_7^2 + 5f_7$
8	$f_8^2 + 3f_8$	$f_8^2 + 2f_8$
9	$f_9^2 + 2f_9$	$f_9^2 + 5f_9$
10	$2f_{10}^2 + f_{10}$	$f_{10}^2 + 3f_{10}$
11	$f_{11}^2 + 5f_{11}$	$f_{11}^2 + 2f_{11}$
12	$f_{12}^2 + 4f_{12}$	$f_{12}^2 + 4f_{12}$

We conducted sensitivity analysis by varying the value of time, ω, for $\omega = 0, 1, 2, 3, 4, 5$. The computed optimal link flows are reported in Table 2.

We now display the optimal link flows as ω varies for the manufacturing links in Figure 3; for the first set of transportation links in Figure 4; for the set of storage links in Figure 5, and for the bottom tier of transportation links in Figure 6.

It is interesting to note from Figure 3 that, with the increase of the value of time, part of the fashion production is shifted from offshore manufacturing plant M_2 to onshore facility M_1, due to the onshore facility's advantage of shorter transportation time to distribution centers (or demand markets). Consequently, there is an increase in transportation flow from the onshore facility M_1 to the distribution centers, as depicted in Figure 4. Figure 5, in turn, illustrates that distribution center D_2 is getting to be an appealing choice as the time performance concern increases, although the storage cost there is slightly higher than at D_1. Also, as the value of time increases, a volume of the fashion product flow switches from transportation link 9 (or link 12) to transportation link 11 (or link 10), to reduce the total time consumption of the distribution activities (as shown in Figure 6).

Table 2: Computed Optimal Link Flows f_a^* as ω Increases

Link a	$\omega = 0$	$\omega = 1$	$\omega = 2$	$\omega = 3$	$\omega = 4$	$\omega = 5$
1	48.66	66.02	78.41	87.71	94.94	100.73
2	251.34	233.98	221.59	212.29	205.06	199.27
3	31.06	36.68	42.13	46.49	49.97	52.80
4	17.60	29.34	36.29	41.22	44.97	47.93
5	127.66	116.89	109.82	104.67	100.72	97.58
6	123.68	117.09	111.76	107.62	104.34	101.69
7	158.72	153.57	151.95	151.16	150.69	150.38
8	141.28	146.43	148.05	148.84	149.31	149.62
9	75.23	62.87	58.43	56.14	54.74	53.80
10	83.49	90.70	93.51	95.02	95.95	96.58
11	24.77	37.13	41.57	43.86	45.26	46.20
12	116.51	109.30	106.49	104.98	104.05	103.42

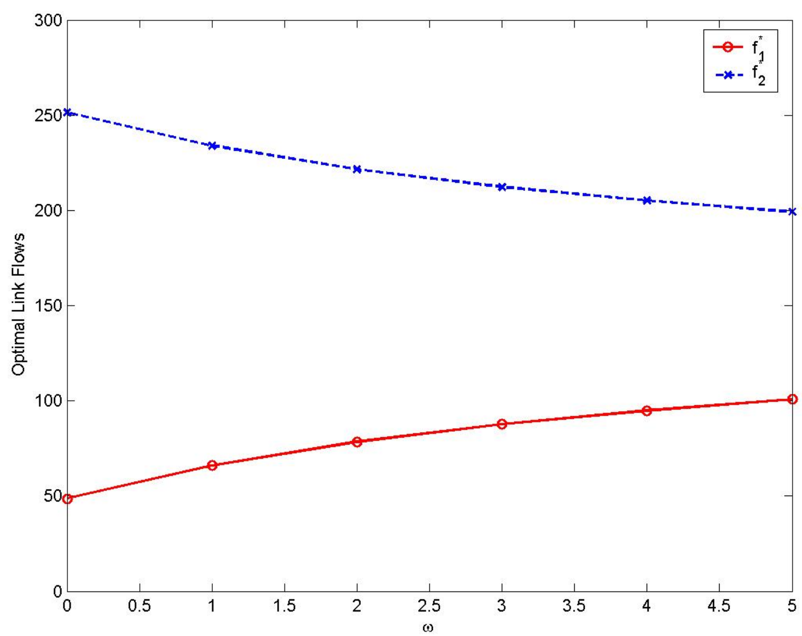

Figure 3: Optimal Link Flows on Manufacturing Links 1 and 2 as ω Increases

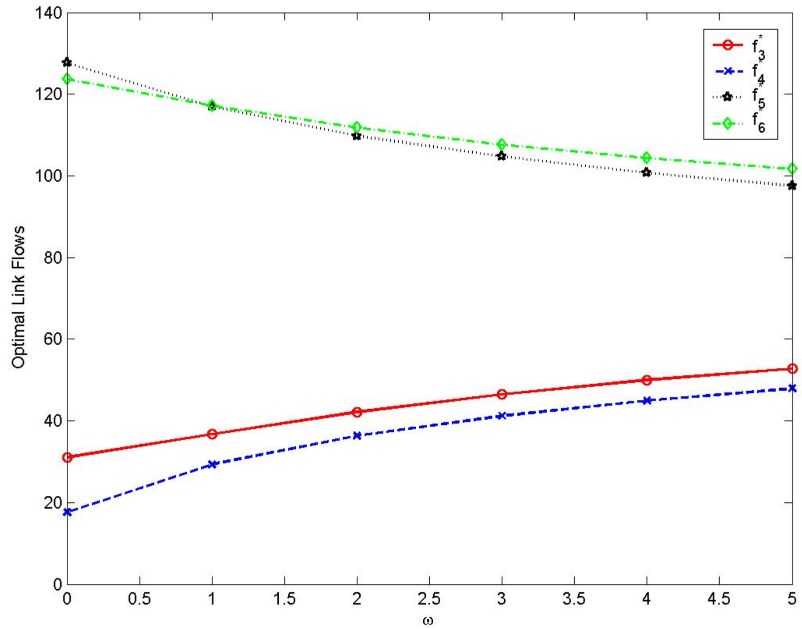

Figure 4: Optimal Link Flows on Transportation Links 3, 4, 5, and 6 as ω Increases

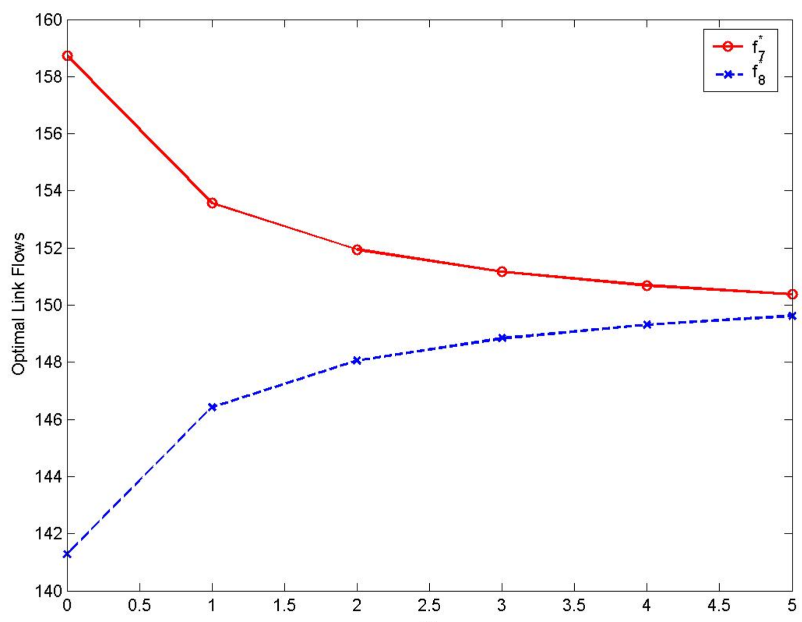

Figure 5: Optimal Link Flows on Storage Links 7 and 8 as ω Increases

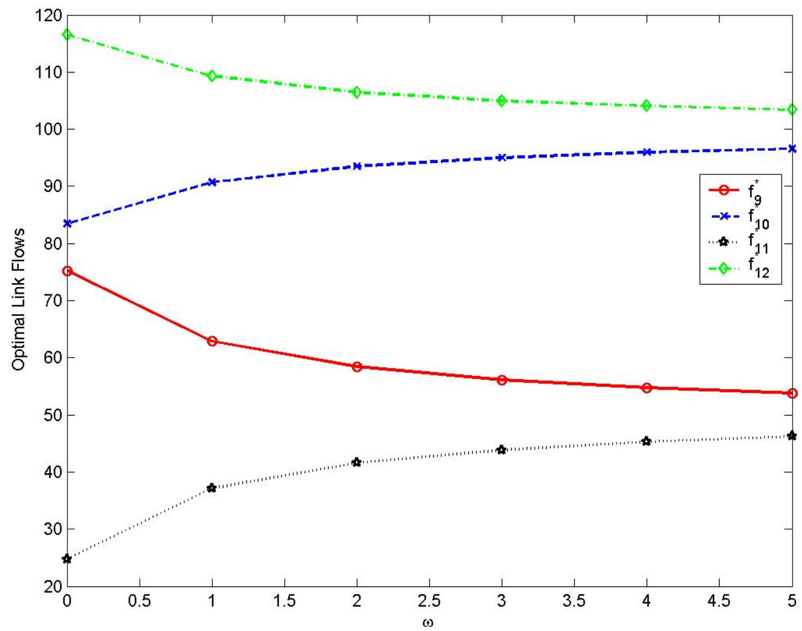

Figure 6: Optimal Link Flows on Transportation Links 9, 10, 11, and 12 as ω Increases

Table 3: Total Costs and Total Times as ω Increases

	$\omega = 0$	$\omega = 1$	$\omega = 2$	$\omega = 3$	$\omega = 4$	$\omega = 5$
Total cost	227,590.89	231,893.93	239,656.04	247,949.30	255,864.62	263,121.79
Total time	164,488.11	154,652.53	149,329.07	145,965.20	143,684.11	142,061.79

In Table 3, we provide the values of the total costs and the total time at the optimal solutions for the examples as ω increases.

The values of the minimal total costs and the minimal total time for varying ω are displayed graphically in Figure 7. As can be seen from Figure 7, as the weight ω increases the minimal total time decreases, as expected, since a higher value of ω represents an increase in the decision-maker's valuation of time as a criterion.

3.2 Multiproduct Fashion Supply Chain Examples

We then considered multiproduct fashion supply chain problems. We assumed that the fashion firm provides two different fashion products with the same supply chain network topology as depicted in Figure 2. The total cost functions and the total time functions for

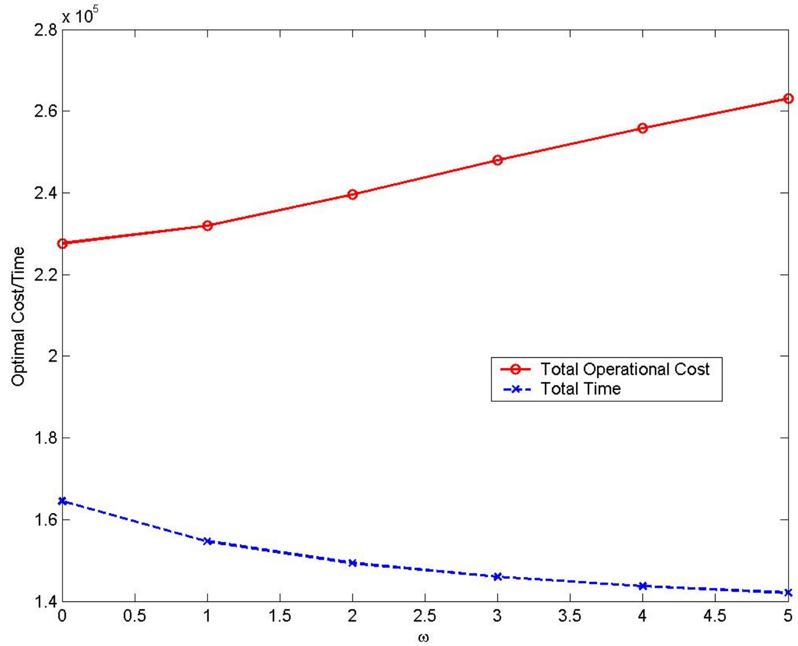

Figure 7: Minimal Total Costs and Minimal Total Times as ω Increases

all the links associated with product 1 and product 2 are given in Table 4 and 5, respectively.

The demands for the two fashion products at the demand points are:

$$d_1^1 = 100, \quad d_2^1 = 200, \quad d_1^2 = 300, \quad d_2^2 = 400.$$

To solves these problems, we used the modified projection method of Korpelevich (1977), embedded with the general equilibration algorithm of Dafermos and Sparrow (1969) (see also, e.g., Nagurney (1999)).

We also conducted sensitivity analysis, as in Section 3.1, by varying the value of time, ω, for $\omega = 0, 1, 2, 3, 4, 5$. The computed optimal link flows associated with products 1 and 2 are, respectively, reported in Tables 6 and 7.

We display the optimal link flows of products 1 and 2 as ω varies for the manufacturing links in Figure 8; for the first set of transportation links in Figure 9; for the set of storage links in Figure 10, and for the bottom tier of transportation links in Figure 11.

With the increase of the value of time, parts of the production of fashion products 1 and 2 are shifted from offshore manufacturing plant M_2 to onshore facility M_1 (as depicted in Figure 8), resulting in an increase in transportation flow from M_1 to the distribution centers for both

Table 4: Total Link Operational Cost and Total Time Functions for Product 1

Link a	$\hat{c}_a^1(f_a^1, f_a^2)$	$\hat{t}_a^1(f_a^1, f_a^2)$
1	$10(f_1^1)^2 + 1f_1^1f_1^2 + 10f_1^1$	$1(f_1^1)^2 + .3f_1^1f_1^2 + 10f_1^1$
2	$1(f_2^1)^2 + .4f_2^1f_2^2 + 5f_2^1$	$1(f_2^1)^2 + .3f_2^1f_2^2 + 10f_2^1$
3	$1(f_3^1)^2 + .3f_3^1f_3^2 + 3f_3^1$	$.5(f_3^1)^2 + .2f_3^1f_3^2 + 5f_3^1$
4	$1(f_4^1)^2 + .2f_4^1f_4^2 + 4f_4^1$	$.5(f_4^1)^2 + .2f_4^1f_4^2 + 7f_4^1$
5	$2(f_5^1)^2 + .25f_5^1f_5^2 + 30f_5^1$	$.5(f_5^1)^2 + .1f_5^1f_5^2 + 25f_5^1$
6	$2(f_6^1)^2 + .3f_6^1f_6^2 + 20f_6^1$	$.5(f_6^1)^2 + .1f_6^1f_6^2 + 15f_6^1$
7	$.5(f_7^1)^2 + .1f_7^1f_7^2 + 3f_7^1$	$1(f_7^1)^2 + .5f_7^1f_7^2 + 5f_7^1$
8	$1(f_8^1)^2 + .1f_8^1f_8^2 + 3f_8^1$	$1(f_8^1)^2 + .5f_8^1f_8^2 + 2f_8^1$
9	$1(f_9^1)^2 + .5f_9^1f_9^2 + 2f_9^1$	$1(f_9^1)^2 + .2f_9^1f_9^2 + 5f_9^1$
10	$2(f_{10}^1)^2 + .3f_{10}^1f_{10}^2 + 1f_{10}^1$	$1(f_{10}^1)^2 + .4f_{10}^1f_{10}^2 + 3f_{10}^1$
11	$1(f_{11}^1)^2 + .6f_{11}^1f_{11}^2 + 5f_{11}^1$	$1(f_{11}^1)^2 + .25f_{11}^1f_{11}^2 + 2f_{11}^1$
12	$1(f_{12}^1)^2 + .7f_{12}^1f_{12}^2 + 4f_{12}^1$	$1(f_{12}^1)^2 + .25f_{12}^1f_{12}^2 + 4f_{12}^1$

Table 5: Total Link Operational Cost and Total Time Functions for Product 2

Link a	$\hat{c}_a^2(f_a^1, f_a^2)$	$\hat{t}_a^2(f_a^1, f_a^2)$
1	$8(f_1^2)^2 + 1f_1^1f_1^2 + 10f_1^2$	$1(f_1^2)^2 + .5f_1^1f_1^2 + 8f_1^2$
2	$1(f_2^2)^2 + .5f_2^1f_2^2 + 4f_2^2$	$1(f_2^2)^2 + .5f_2^1f_2^2 + 8f_2^2$
3	$1.5(f_3^2)^2 + .2f_3^1f_3^2 + 3f_3^2$	$1(f_3^2)^2 + .1f_3^1f_3^2 + 3f_3^2$
4	$1(f_4^2)^2 + .3f_4^1f_4^2 + 4f_4^2$	$1(f_4^2)^2 + .2f_4^1f_4^2 + 3f_4^2$
5	$2(f_5^2)^2 + .3f_5^1f_5^2 + 25f_5^2$	$.8(f_5^2)^2 + .1f_5^1f_5^2 + 20f_5^2$
6	$3(f_6^2)^2 + .4f_6^1f_6^2 + 20f_6^2$	$.8(f_6^2)^2 + .2f_6^1f_6^2 + 12f_6^2$
7	$1(f_7^2)^2 + .1f_7^1f_7^2 + 3f_7^2$	$1(f_7^2)^2 + .4f_7^1f_7^2 + 4f_7^2$
8	$.5(f_8^2)^2 + .2f_8^1f_8^2 + 3f_8^2$	$1(f_8^2)^2 + .6f_8^1f_8^2 + 4f_8^2$
9	$2(f_9^2)^2 + .3f_9^1f_9^2 + 2f_9^2$	$1(f_9^2)^2 + .1f_9^1f_9^2 + 7f_9^2$
10	$1(f_{10}^2)^2 + .5f_{10}^1f_{10}^2 + 1f_{10}^2$	$1(f_{10}^2)^2 + .3f_{10}^1f_{10}^2 + 6f_{10}^2$
11	$2(f_{11}^2)^2 + .5f_{11}^1f_{11}^2 + 8f_{11}^2$	$1(f_{11}^2)^2 + .3f_{11}^1f_{11}^2 + 3f_{11}^2$
12	$1(f_{12}^2)^2 + .4f_{12}^1f_{12}^2 + 7f_{12}^2$	$1(f_{12}^2)^2 + .5f_{12}^1f_{12}^2 + 4f_{12}^2$

Table 6: Computed Optimal Link Flows f_a^{1*} as ω Increases for Product 1

Link a	$\omega = 0$	$\omega = 1$	$\omega = 2$	$\omega = 3$	$\omega = 4$	$\omega = 5$
1	53.81	71.31	83.38	92.24	99.02	104.39
2	246.19	228.69	216.62	207.76	200.98	195.61
3	41.58	43.44	47.03	50.23	52.90	55.10
4	12.23	27.87	36.35	42.01	46.13	49.29
5	125.73	114.20	106.97	101.86	98.02	95.01
6	120.45	114.49	109.64	105.90	102.96	100.60
7	167.32	157.64	154.00	152.09	150.91	150.11
8	132.68	142.36	146.00	147.91	149.09	149.89
9	73.27	63.60	60.07	58.25	57.14	56.39
10	94.05	94.04	93.93	93.84	93.78	93.73
11	26.73	36.40	39.93	41.75	42.86	43.61
12	105.95	105.96	106.07	106.16	106.22	106.27

Table 7: Computed Optimal Link Flows f_a^{2*} as ω Increases for Product 2

Link a	$\omega = 0$	$\omega = 1$	$\omega = 2$	$\omega = 3$	$\omega = 4$	$\omega = 5$
1	150.04	192.14	219.13	237.85	251.58	262.07
2	549.96	507.86	480.87	462.15	448.42	437.93
3	17.38	63.50	87.28	102.35	112.87	120.65
4	132.66	128.64	131.85	135.50	138.71	141.42
5	309.57	278.09	259.11	246.45	237.40	230.61
6	240.39	229.77	221.75	215.70	211.02	207.32
7	326.95	341.59	346.39	348.80	350.27	351.26
8	373.05	358.41	353.61	351.20	349.73	348.74
9	141.99	145.71	147.19	148.00	148.52	148.88
10	184.96	195.88	199.20	200.80	201.75	202.38
11	158.01	154.29	152.81	152.00	151.48	151.12
12	215.04	204.12	200.80	199.20	198.25	197.62

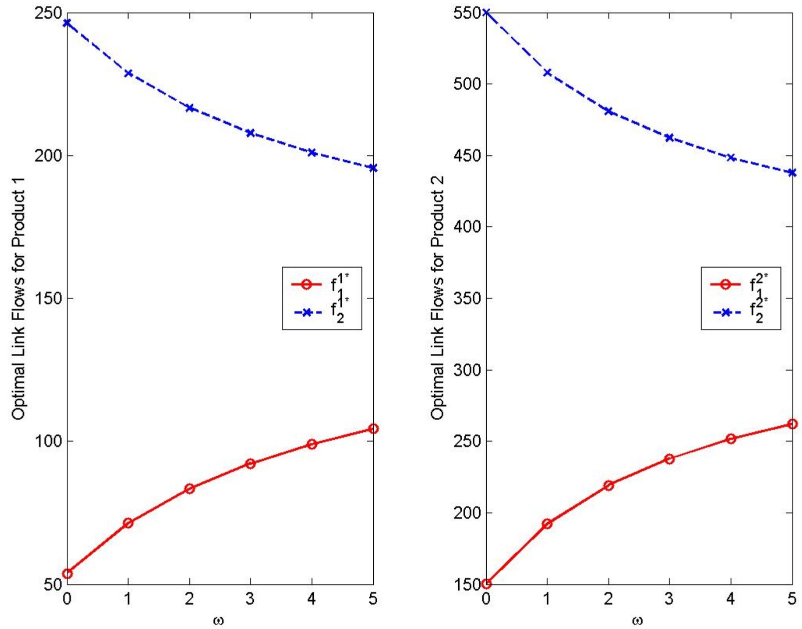

Figure 8: Optimal Link Flows on Manufacturing Links 1 and 2 as ω Increases

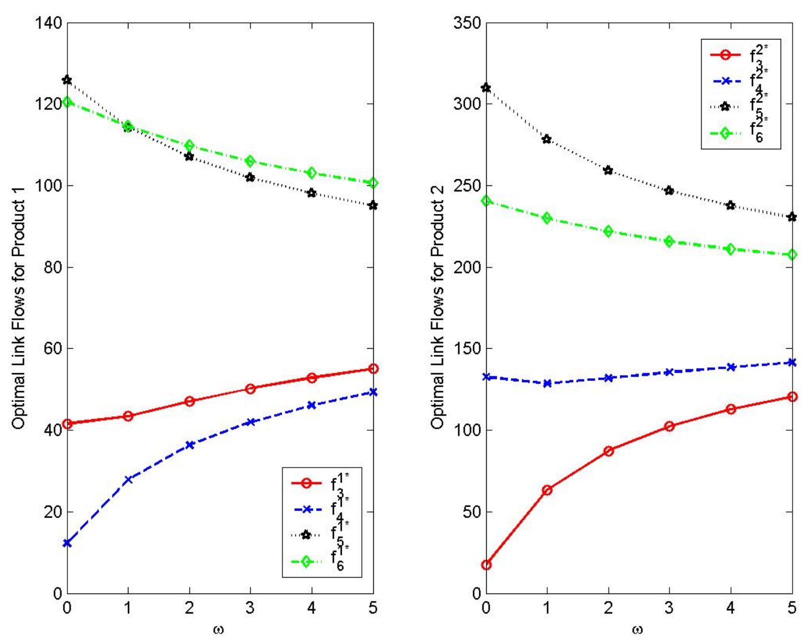

Figure 9: Optimal Link Flows on Transportation Links 3, 4, 5, and 6 as ω Increases

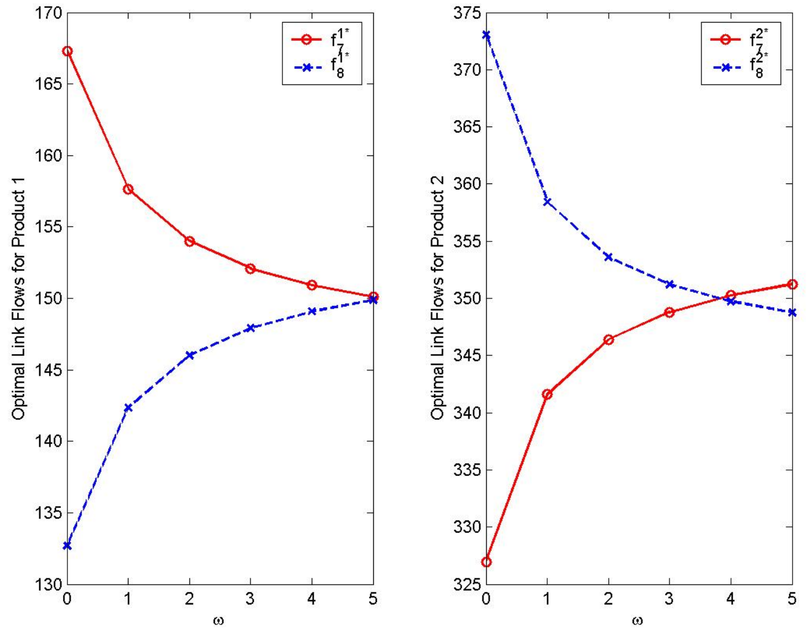

Figure 10: Optimal Link Flows on Storage Links 7 and 8 as ω Increases

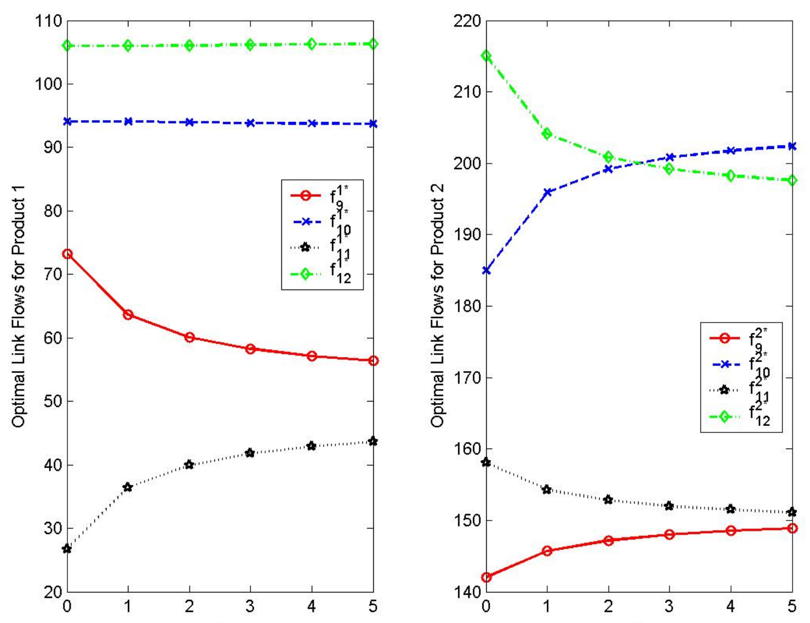

Figure 11: Optimal Link Flows on Transportation Links 9, 10, 11, and 12 as ω Increases

Table 8: Total Costs and Total Times as ω Increases

	$\omega = 0$	$\omega = 1$	$\omega = 2$	$\omega = 3$	$\omega = 4$	$\omega = 5$
Total cost	$1,722,082.05$	$1,745,201.77$	$1,788,457.21$	$1,831,689.80$	$1,870,523.21$	$1,904,398.75$
Total time	$1,291,094.62$	$1,222,959.73$	$1,189,192.37$	$1,169,656.19$	$1,157,297.60$	$1,148,975.34$

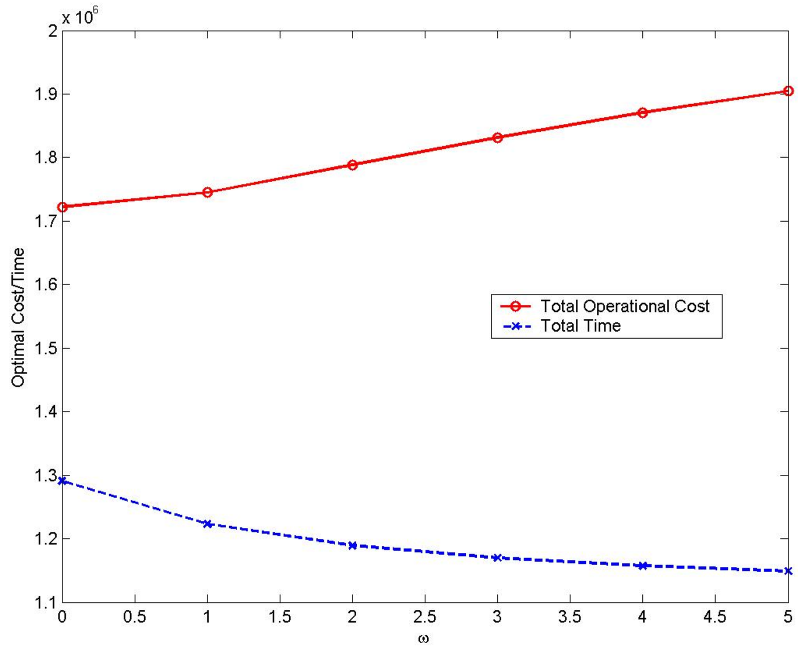

Figure 12: Minimal Total Costs and Minimal Total Times as ω Increases

fashion products (as shown in Figure 9). However, Figure 10 illustrates that the distribution center D_2 is getting to be appealing for product 1 as the value of time increases, while the distribution center D_1 becomes attractive for product 2, since the distribution center D_1 is more time-efficient for product 2. In Figure 11, as the time performance concern increases, a volume of fashion product 1 switches from transportation link 9 to link 11; in contrast, the volume of flow of fashion product 2 on link 9 increases. Also, a volume of fashion product 2 switches from link 12 to link 10, while the flows of fashion product 1 on link 10 and 12 change slightly.

The values of the total costs and the total time at the optimal solutions for the examples as ω increases are provided in Table 8, and displayed graphically in Figure 12. As expected, the minimal total time decreases as ω increases.

4. Summary and Conclusions and Suggestions for Future Research

In this paper, we developed a fashion supply chain management model, using a network economics perspective, that allows for multiple fashion products. The model consists of two objective functions: total cost minimization, associated with supply chain network activities, in the form of: manufacturing, storage, and distribution, and total time consumption minimization. A weighted objective function was then constructed with the weighting factor, representing the monetary value of a unit of time, decided by the firm.

We also provided the optimization model's equivalent variational inequality formulation, with nice features for computational purposes. The solution of the model yields the optimal multiproduct fashion flows of supply chain network activities, with the demands being satisfied at the minimal total cost and the minimal total time consumption. The model is illustrated with a spectrum of numerical examples with potential application to fashion supply chain management.

The fashion supply chain network model allows the cognizant decision-maker to evaluate the effects of changes in the demand for its products on the total operations costs and time. It allows for the evaluation of changes in the cost functions and the time functions on total supply chain network costs and time. In addition, the flexibility of the network framework allows for the evaluation of the addition of various links (or their removal) on the values of the objective function(s). Finally, the model, since it is network-based, is visually graphic.

The research in this paper can be extended in several directions. One can construct a fashion supply chain management model with price-sensitive and time-sensitive demands under oligopolistic competition. One can also incorporate environmental concerns and associated trade-offs. In addition, one can explore computationally as well as empirically large-scale fashion supply chain networks within our modeling framework. We leave such research for the future.

Acknowledgments The authors acknowledge the helpful comments and suggestions of two anonymous reviewers. The authors also thank Professor Jason Choi for the opportunity to contribute to his edited volume.

This research was supported, in part, by the John F. Smith Memorial Fund. This support is gratefully acknowledged.

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