Quality in Competitive Fresh Produce Supply Chains
with
Application to Farmers' Markets

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Abstract: Fresh produce supply chains have special characteristics, notably, that the quality of the product (fruit or vegetable) deteriorates continuously over time, even under ideal conditions. In this paper, we begin with explicit formulae for fresh produce quality deterioration based on chemistry and temperature and provide a path-based framework. We then focus on farmers' markets, the popularity of which has been growing due to consumers' greater awareness of and interest in product quality and emphasis on health. Farmers' markets, as examples of direct to consumer channels and shorter supply chains, are studied in the framework of game theory in both uncapacitated and capacitated versions. A case study of apples in Massachusetts, under various scenarios, including production disruptions, provides quantitative evidence of the applicability of our supply chain network approach.

Key words: quality, food supply chains, fresh produce, oligopolistic competition, food deterioration, product differentiation

1. Introduction

Food is essential to life and to well-being. Fresh produce in the form of fruits and vegetables, in particular, is noted for its nutrients in terms of vitamins and minerals and is an important component of healthy diets of both children and adults alike (O'Connor (2013)). Eating nutritious foods can reduce the incidence of illnesses and malnutrition and can even prolong life. Children who are well-nourished are better able to concentrate and to learn and grow. Adults who lack food security cannot prosper and neither can their families. According to Neff et al. (2009) food hardship is closely correlated with obesity and related diseases of the heart, diabetes, and cancers.

Knowledgeable modern consumers are increasingly demanding high quality in their food products, including fresh produce, and, yet, they may be unaware of the great distances the food has traveled through intricate supply chains and the length of time from the initial production or "picking" of the fruits and vegetables to the ultimate delivery. Moreover, consumers, faced with information asymmetry, may not know how long the food may have been lying on the grocers' and retailers' shelves, even once delivered and unpacked. The great distances traveled create issues in terms of quality since fresh produce is perishable (see Nahmias (2011) and Nagurney et al. (2013)). As noted in Yu and Nagurney (2013), food supply chains are distinct from other product supply chains in that the quality of food products is decreasing with time, even with the utilization of the most advanced facilities and conditions (Sloof, Tijskens, and Wilkinson (1996) and Zhang, Habenicht, and Spieß (2003)).

The transformation of global food supply chains since the early 1900s has been nothing short of remarkable. As reported in Martinez et al. (2010), in the early 1900s, much of the food bought and consumed in the United States was grown locally and about 40% of Americans resided on farms, whereas in 2000 only 1% did (cf. Pirog (2009)). Consumers over a century ago obtained information as to the quality of the foods through direct contact with farmers. Except for various food preservation activities, few foods were processed or packaged, and fresh produce, fish, and dairy products usually traveled less than 24 hours to market (see Giovannucci, Barham, and Pirog (2010)). Foods were consumed based on their availability and growing seasons.

According to a report by DeWeerdt (2016), in 1993, a Swedish researcher determined that the ingredients of a typical Swedish breakfast consisting of an apple, bread, butter, cheese, coffee, cream, orange juice, and sugar, had traveled a distance equal to the circumference of our planet before reaching the consumer. In 2005, Pirog and Benjamin, through the use of a mathematical formula, computed that the milk, sugar, and strawberries that make

up a container of strawberry yogurt jointly journeyed 2,216 miles only to the processing plant. According to Pirog et al. (2010), calculations made by Hendrickson (1997) examining transportation and fuel requirements estimated that fresh produce in the United States traveled an estimated 1,500 miles with fresh produce arriving in Austin, Texas estimated to have traveled an average of 1,129 miles.

Consumers tend to connect the terms 'fresh,' 'good quality,' and 'tasty' to the products that are being locally produced (Holloway and Kneafsey (2000)). The definition of the term 'local' varies between regions and communities (Martinez (2010)); however, the most common definition is that local food is food grown within a county or within a state (Wilkins, Bokaer-Smith, and Hilchey (1996)). According to a national survey conducted by Bond, Thilmany, and Bond (2009), four out of five people living in the United States buy local food. Thus, the demand for locally grown fresh produce is also increasing and demand and supply management becomes more crucial for farmers. Successful harvest and post-harvest decisions can help farms to meet growing demand in local fresh produce markets.

It has now been established that the quality of food, including that of fresh produce, along with quality preservation and degradation, can be determined using chemical formulae, which include both time and temperature (cf. Labuza (1982), Taoukis and Labuza (1989), Tijskens and Polderdijk (1996), Rong, Akkerman, and Grunow (2011)). Hence, in order to capture quality associated with fresh produce in a supply chain one must be aware of the various supply chain network economic activities such as "production," storage, transportation, etc., as well as the durations and temperatures associated with these activities. For example, in terms of kinetics (cf. Labuza (1982)), the quality degradation of food such as meat and fish follows first order reactions whereas that of many fresh fruits and vegetables follows zero-order reactions with the order of the reaction corresponding to the power of the differential equation for quality.

Consequently, the network topology of a fresh produce supply chain and, in particular, the length of a path in terms of time from an origin node to a destination node can significantly influence the quality of the fresh produce that consumers purchase and consume. This gives impetus to the investigation of local food systems and associated shorter marketing channels. For example, a U.S. Congressionally mandated report by the U.S. Department of Agriculture (cf. Low et al. (2015)) noted that the sale of food through direct-to-consumer channels, such as through farmers' markets, and intermediated marketing channels, that is, sales to institutions, including hospitals and schools, or regional distributors, appears to be increasing. As reported therein, there were 8,268 farmers' markets in the United States in 2014, with the number having increased by 180% since 2006. 163,675 farms, or 7.8% of the

U.S. ones, were marketing foods locally in 2012 and of these farms, 70% used only direct-to-consumer channels. This growth implies increasing consumer interest in locally sourced fresh produce due to quality and health perspectives. Interestingly, and perhaps paradoxically, as noted by Babiak (2013), concurrent with the growth in farmers' markets across the United States, food insecurity has reached epidemic levels with nearly 1 in 5 Americans, or 47.8 million people, relying on food stamp benefits.

Various authors have emphasized the change in quality of food products in the supply chain until the final points of demand (see Sloof, Tijskens, and Wilkinson (1996), Van der Vorst (2000), Lowe and Preckel (2004), Ahumada and Villalobos (2009, 2011), Blackburn and Scudder (2009), Akkerman, Farahani, and Grunow (2010), and Aiello, La Scalia, and Micale (2012), Yu and Nagurney (2013)). Amorim, Costa, and Almada-Lobo (2014) utilize demand functions that depend on product quality and also price and then construct demands for different products based on age. They propose deterministic and stochastic production planning models that capture consumers' desire for fresher products. Liu, Zhang, and Tang (2015) also utilize demand functions that depend on price and quality but they depend continuously on time. The authors determine the dynamic pricing and investment strategies to reduce the deterioration rate of the quality for perishable foods. However, there has been only limited research done in terms of farmers' markets, which are examples of short supply chains (cf. Fabbrizzi, Menghini, and Marinelli (2014) for a review), and provide an excellent setting in which quality can be captured over time since consumers can obtain information directly from the producers, that is, the farmers.

Moreover, optimization has been the primary methodological framework, in the case of food supply chain model settings, including longer ones, whereas it is clear that farmers compete in food supply chains including in direct to consumer chains as in farmers' markets. Our supply chain model utilizes game theory. Farmers' operational costs can depend on their fresh produce flows as well as those of the others since they compete for resources. Furthermore, demand prices are differentiated by brand since consumers reveal their preferences through their demands for the quality of the individual farmer's produce as well as that of the others'. Hence, in order to capture competition, both operational costs as well as demand price functions depend on vectors of variables. The work of Yu and Nagurney (2013) is one of the few that proposes a game theory model for oligopolistic competition in brand differentiated fresh produce supply chains with perishability. This paper differs from that paper is multiple significant ways. It models fresh produce quality through kinetics, and, hence, quality, which is time-dependent, is based on actual physical and chemical parameters. The quality of the fresh produce that arrived at the demand markets in the model

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of Yu and Nagurney (2013) was identical. Also, both uncapacitated and capacitated versions are presented here.

The majority of research on local food markets, especially farmers' markets, pertains mostly to consumer behavior and there are not many mathematical models that consider demand or the supply side in local food supply chains. The literature on consumer behavior in farmers' markets provides valuable insights from the consumers' and farms' perspectives. For instance, Holloway and Kneafsey (2000) argue that the quality criteria of the consumers can be met by assigning food products to new consumption places, specifically, farmers' markets. According to the authors, consumers in the United Kingdom regard farmers' markets as an alternative space for supermarkets and even find them nostalgic. Asebo et al. (2007) have also worked on farmer and consumer attitudes at farmers' markets in Norway. They found that, in Norway, consumers consider the origin of the food and the production processes to be significant. Farmers' markets fit perfectly into these considerations, since consumers can meet the farmers face-to-face and learn about the production processes. Farmers also benefit from farmers' markets by being able to keep more of the retail price to themselves, passing the middleman in the mainstream supply chain and meeting face to face with their customers (Trobe (2001)). Besides, 19,000 farmers use farmers' markets as their only sale channel in the United States. Between 1998 and 2009, the number of farmers' markets increased by 92 percent, which reveals the positive tendency of consumers towards farmers' markets. Therefore, it is important to reinforce the relationship between farms and the consumers in local food markets, by proposing solutions to increase the profitability of farms and consequently providing higher quality fresh food to consumers. However, despite the significant interest in local food, a supply chain network model, which combines the quality criteria of consumers and offers strategies for farms to increase their profits in farmers' markets, does not exist in the literature.

Tong, Ren, and Mack (2012) present several of the few mathematical models for farmers' markets, focusing on site selection using location theory in a spatio-temporal context and provide a case study in Tucson, Arizona to illustrate the benefits of their approach. They capture the distance that consumers may need to travel to the farmers' markets and also include the times of operation. They propose two types of linear mathematical programming models to locate farmers' markets, which are also extensions of the *p-median* problem in the literature. In the first model, they propose an optimal location for farmers' markets, while minimizing the total overall additional travel distance of consumers. The additional travel distance of an individual to a farmers' market, in a time-frame, is represented by a parameter in the objective function. The authors differentiate consumers as worker and non-workers in

the second model and suggest an optimal site for farmers' markets. In addition, Bosona et al. (2011) investigate the local food supply chain in Sweden, and propose alternative routes to optimize transportation between farms and demand points. Furthermore, Aregu (2014), from the Netherlands, creates a software called 'Farmers Decision Enhancement Studio' for farmers to enhance their pricing decisions in farmers' markets in Uganda.

In this paper, we develop a modeling and algorithmic framework for competitive farmers' markets. The model is network-based and the farmers engage in Cournot competition over space and time. The governing Nash equilibrium conditions are formulated as a variational inequality problem. The novelty of the framework lies in that the quality of the fresh produce product is captured as the produce propagates in the supply chain over space and time with the consumers at the markets responding both to the price and the quality of the fresh produce. Both uncapacitated and capacitated versions of the model are presented. The latter can capture limitations in supply due to harvest problems or damage during the growing season, limitations in transport and storage capabilities, and/or labor for harvesting and processing. The game theory model can address questions of farmers as to which farmers' markets they should serve; what the impact of a new competitor may be at one or more markets in terms of profits, as well as the effects of capacity disruptions (or enhancements) in their individual and others' short supply chains. In addition, the model can ascertain the impacts of changes in link parameters that capture quality. Policy makers, in turn, can also obtain useful information as to the impacts of a greater number or fewer farms represented at various markets and how reducing quality decay can affect farmers' profits.

The paper is organized as follows. In Section 2, we provide some preliminaries, focusing on the quality differential equations, which we then generalize to a path concept since the fresh produce will proceed on multiple supply chain links from the harvesting point to ultimate purchase at the farmers' markets. Each link has associated with it both a time element as well as a factor such as temperature, which also affects the fresh produce quality.

In Section 3, we first present the uncapacitated competitive supply chain network model for farmers' markets, give the governing Nash equilibrium conditions, and derive the variational inequality formulations in path flows and links flows. In addition, we construct the capacitated path analogue and provide its variational inequality formulation under Nash equilibrium. In Section 4, we present the algorithm, which resolves the variational inequality problems into subproblems in the path flows (and in the case of the capacitated model, also in terms of Lagrange multipliers) that can be solved explicitly using formulae that we present. In Section 5 we present a case study on apples in farmers' markets. We summarize and present our conclusions in Section 6.

2. Preliminaries on Food Quality Deterioration

Before we present the fresh food supply chain network model for farmers' markets, it is important to provide some preliminaries on the definition of quality, and the kinetics behind the quality decay, since quality has major relevance for the fresh food industry. Afterwards, we provide the generalization of the quality differential equations for fresh produce supply chain networks by capturing them through the path concept.

Quality can be defined in multiple dimensions such as physical, emotional, and even philosophical. Therefore, it is not easy to find a proper global definition of quality which is valid for every type of industry and consumer. Reeves and Beednar (1994) present a good discussion on the evolution of quality definitions. Nagurney and Li (2014) investigate minimum quality standards in a spatial price equilibrium model and also consider information asymmetry. Nagurney and Li (2016) present a plethora of competitive supply chain network models with a focus on quality, where quality is defined as conformance to specifications as in manufactured products. Nagurney et al. (2013), in turn, focus on perishable product supply chains. Murdoch, Marsden, and Banks (2000) also present a valuable discussion on the connection between quality and nature, especially in the food sector, from the perspective of social sciences. However, there are more specific definitions of quality in the literature that are construed specifically for the food sector. According to Kader (1997), the quality of fresh fruits can be defined over the combination of their attributes, properties, or characteristics. Furthermore, these attributes can be categorized by: color and appearance, flavor (taste and aroma), texture, and nutritional value (Barrett, Beaulieu, and Shewfelt (2010)). Consumers make the decision of purchase based on their sensory evaluations which consist of sight, smell, taste, touch, and hearing (Abbott (1994)). Moreover, reports suggest that consumers consider the quality attributes of appearance, firmness, freshness, and ripeness during the initial phase of purchase (see Kader (1997) and Zind (1990)). Therefore, retaining the quality of these fresh produce attributes is very important for both consumers and farmers.

Since fresh foods are biological products, they deteriorate and, consequently, lose quality over time (Schouten et al. (2004), Singh and Anderson (2004)). Thus, an understanding of the biochemical/physicochemical reactions, which cause the deterioration, is necessary in order to be able to present an explanation of the quality loss (Singh and Cadwallader (2004)). According to Taoukis and Labuza (1989), the rate of quality deterioration can be given as a function of its microenvironment, gas composition, relative humidity, and temperature.

In this regard, Labuza (1984) captures the quality decay of a food attribute, Q, over time

t, through the differential equation:

$$\frac{\partial Q}{\partial t} = kQ^n = Ae^{(-E/RT)}Q^n. \tag{1}$$

Here, k is the reaction rate and is defined by the Arrhenius formula, $Ae^{(-E/RT)}$, where A is a pre-exponential constant, T is the temperature, E is the activation energy, and R is the universal gas constant (Arrhenius (1889)). Moreover, n is the reaction order, which is a non-negative integer and belongs to the set $Z^* = \{0\} \cup Z^+$. In general, the quality decay function of the food attribute can be shown in terms of its reaction order. When the reaction order n is zero, that is, $\frac{\partial Q}{\partial t} = k$, the quality decay rate of the food attribute Q can be expressed as the function:

$$f_0(Q) = -kt. \tag{2}$$

Furthermore, assuming that the initial quality is known and given as Q_0, we can define the remaining quality Q_t at time t, by using the zero order quality decay function as in Tijskens and Polderdijk (1996):

$$Q_t = Q_0 + f_0(Q) = Q_0 - kt. \tag{3}$$

Recall that the reaction constant k can be written by the Arrhenius formula to show the relationship between temperature and quality decay explicitly. Furthermore, having the reaction order be 1 – also referred to as a first order reaction – leads to an exponential function, which is observed commonly in food quality decay (Tijskens and Polderdijk (1996)). This type of quality decay is given by the expression:

$$f_1(Q) = e^{-kt}. \tag{4}$$

Notice that, since this quality decay function has an exponential component, the quality Q_t at time t should be written as a multiplication of the initial quality Q_0 and the quality decay function, as shown in the following expression:

$$Q_t = Q_0 f_1(Q) = Q_0 e^{-kt}. \tag{5}$$

Table 1 shows the decay kinetics, with the related quality attributes, of some fruits and vegetables. For a detailed description of the quality decay kinetics for vegetables, see Aamir et al. (2013). Notice that the quality attributes, in Table 1, are related to the appearance and texture which are the two most important fresh produce characteristics for consumers.

We can now generalize each type of quality deterioration function in terms of the path concept in a given fresh produce food supply chain network. We define a path p joining an

Attribute	Fresh Produce	Reaction Order	Reference
Color Change	Peaches	First	Toralles et al. (2005)
Color Change	Raspberries	First	Ochoa et al. (2001)
Color Change	Blueberries	First	Zhang, Guo, and Ma (2012)
Nutritional (Vitamin C)	Strawberries	First	Castro et al. (2004)
Color Change	Watermelons	Zero	Dermesonlouoglou, Giannakourou, and Taoukis (2007)
Moisture Content	Tomatoes	First	Krokida et al. (2003)
Color Change	Cherries	First	Ochoa et al. (2001)
Texture Softening	Apples	First	Tijskens (1979)
Nutritional (Vitamin C)	Pears	First	Mrad et al. (2012)
Texture Softening	Avocados	First	Maftoonazad and Ramaswamy (2008)
Nutritional (Vitamin C)	Pineapples	First	Karim and Adebowale (2009)
Color Change	Spinach	Zero	Aamir et al. (2013)
Color Change	Asparagus	First	Aamir et al. (2013)
Color Change	Peas	First	Aamir et al. (2013)
Texture Softening	Beans	First	Aamir et al. (2013)
Texture Softening	Brussel Sprouts	First	Aamir et al. (2013)
Texture Softening	Carrots	First	Aamir et al. (2013)
Texture Softening	Peas	First	Aamir et al. (2013)
Color Change	Coriander Leaves	First	Aamir et al. (2013)

Table 1: Fresh Produce Attributes and Decay Kinetics

origin node i with a destination node j through directed links that it is comprised of in the link set L. Furthermore, let β_a denote the quality decay incurred on link a, which depends on the reaction order n, reaction rate k_a, and time t_a on link a, according to:

$$
\beta_a \equiv \begin{cases} -k_a t_a, & \text{if } n = 0, \forall a \in L, \\ e^{-k_a t_a}, & \text{if } n \neq 0, \forall a \in L. \end{cases}
$$

(6)

Here, k_a is the reaction constant related to the link a. Since each link on a path can have different temperature conditions, the differentiation over the temperature of the links is necessary. Thus, the reaction rate is described in the following equation for each link a by the Arrhenius formula with the same parameters as in (1), except that the temperature is now denoted for each link a as T_a, where:

$$
k_a = A e^{(-E_A/RT_a)}.
$$

(7)

In addition, since (7) contains the exponential decay as a function and the other terms are parameters, β_a is defined generally for the reaction orders greater than zero. Before we show

the quality decay over a given path p, it is worthwhile to introduce the quality parameter q_{0i}, which represents the initial quality of the fresh produce, produced at the origin node i. Also, P_j^i represents the set of all paths that have origin i and destination j. We now can define the quality q_p, over a path p, joining the origin farm node, i, with a destination node farmer market, j, while incorporating the quality deterioration of the fresh produce as:

$$q_p \equiv \begin{cases} q_{0i} + \sum_{a \in p} \beta_a, & \text{if } n = 0, \forall a \in L, \ p \in P_j^i, \forall i, j, \\ \\ q_{0i} \prod_{a \in p} \beta_a, & \text{if } n = 1, \forall a \in L, \ p \in P_j^i, \forall i, j. \end{cases} \tag{8}$$

By the above generalization, we differentiate our model from the existing works in the literature. Although we utilize Tijskens and Polderdijk (1996)'s quality decay formulations, the authors have not provided a quality deterioration formulation for a path in a food supply chain network. Rong, Akkerman, and Grunow (2011) build the food supply chain through a mixed-integer programming model and incorporate the food quality decay into this model. However, in this paper, we present a competitive supply chain game theory model where the demand markets and the quality decay are product specific. Furthermore, Ketzenberg, Bloemhof, and Gaukler (2015) present a model to assess the value of time and temperature information of a perishable product. The authors provide a simulation study to evaluate the remaining shelf life of fresh fish with randomly realized temperature and time information. In their work, they provide a different food quality metric than ours which is not based on quality kinetics and is not applied on a supply chain network.

We are now ready to present the competitive fresh produce supply chain network model for farmers' markets.

3. The Fresh Produce Farmers' Market Supply Chain Network Models

In this section, we capture the economic behavior of farmers selling at farmers' markets and competing on quality and quantity of their fresh produce a la Cournot (1838) through both an uncapacitated and a capacitated oligopoly model and we derive the variational inequality formulations of the governing Nash (1950, 1951) equilibrium conditions. The models consist of a finite number of I farms, that are run by farmers, typically, denoted by i, and a finite number of demand points, M, which correspond to farmers' markets, and with a typical one denoted by j. Each farmers' market takes place in a region and on a given day of the week. We assume that the demand points correspond to farmers' markets on different days of the week since farmers may not have sufficient staff to be at multiple markets on the same day. Farmers' markets, on any given day, usually last no more than 6 hours and are repeated in the same location and day over a season, which, in the Northeast of the United States, for example, can last from May through October.

The uncapacitated model is presented in Section 3.1 and its capacitated analogue then outlined in Section 3.2.

Before introducing the variables and the notation for the models, we provide a summary of the assumptions that we have made:

1. The supply chain network topology illustrates a fixed time horizon in a given season of the fresh fruit or vegetable, typically, a week.

2. The demand points represent the direct-to-consumer local farmers' markets, in different regions of a county or a state.

3. Farmers pick the harvest right before the beginning of the time horizon; therefore, a farmers' market that is available at the beginning of this time horizon sells the fresh produce without storage. Farmers' markets taking place on subsequent days store the fresh produce before transporting it to the markets.

4. Consumers can buy products that are substitutes of one other within or across the demand points. That means that the farmers' markets provide multiple options of fresh produce to consumers over space and time.

Since the local supply chains are short, the intermediate activities can be defined as: harvesting, processing and packaging, storage, and transportation. Farmers compete at the local farmers' markets noncooperatively in an oligopolistic manner. The products are

differentiated at the markets according to their quality, which are presented by the quality deterioration functions described in Section 2. In this regard, the corresponding fresh produce food supply chain network topology is as in Figure 1. Based on the case study of concern here, that is, apples, the time horizon of a week and the picking of apples once a week is not unreasonable, according to experts that we have interviewed (Clements (2016a) and Drew (2016)). If more choices were to be added in terms of multiple times to pick, for example, then the network would be adapted/extended accordingly. Here our goal is to lay the foundations for the modeling of both short and long food supply chain networks under competition and with quality deterioration based on kinetics on paths consisting of distinct economic activities such as transport, storage, etc.
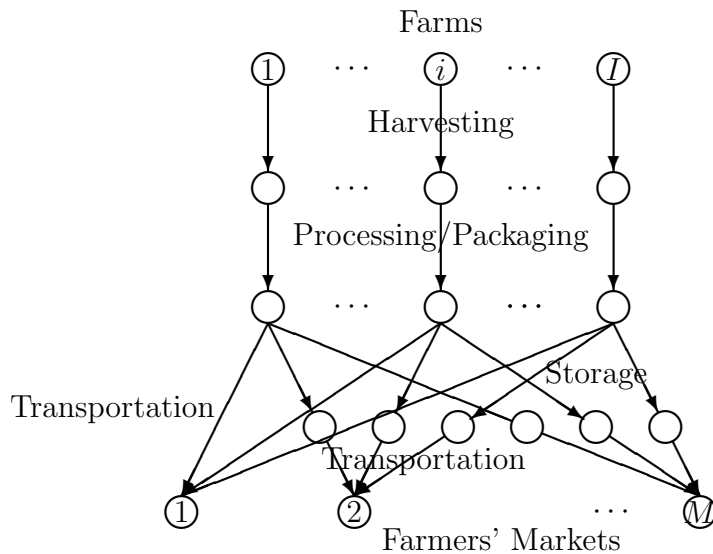

Figure 1: The Supply Chain Network Topology of the Farmer's Market Competitive Fresh Produce Problem

Let $G = [N, L]$ denote the graph with set of nodes N and links L in Figure 1. Each farm i; $i = 1, \ldots, I$, delivers fresh produce to the markets through its harvesting, processing, storage, and transportation facilities, and seeks to determine its optimal strategies in terms of how much of the produce to bring to each market in order to maximize profit. The link set L consists of the sets of links $L_1 \cup L_2 \ldots \cup L_I$, where L_i is the sequence of directed links of farm i. Moreover, the links on the network correspond to the economic activities associated with the farms.

The first set of directed links illustrates the harvesting/picking activity of each farmer i; $i = 1, \ldots, I$. The second set of directed links illustrates the processing activities such as cleaning, sorting and labeling, and also packaging/packing of the fresh produce. The last set of directed links corresponds to the storage and transport of the fresh produce. If the

demand point is a farmers' market that is open on the first day of the week, the links only capture the transportation cost. Otherwise, there is a storage link followed by a transport link each subsequent demand point, that is, farmers' market, as illustrated in Figure 1.

Recall that P_j^i denotes the set of paths joining farm node i with farmer market node j. In addition, we let P^i denote the set of paths emanating from farm node i to all farmer market nodes j; $j = 1, \ldots, M$. P then denotes the set of all paths in the supply chain network in Figure 1. There are n_P paths in the network and n_L links.

3.1 The Uncapacitated Model

We now present the uncapacitated model.

The flow on each path, joining the farmer node i to the farmers' market node j, is denoted by x_p, and all path flows must be nonnegative, that is,

$$x_p \geq 0, \quad \forall p \in P_j^i; \quad i = 1, \ldots, I; j = 1, \ldots, M. \tag{9}$$

Furthermore, the flow on a link a is equal to the sum of the path flows x_p, on paths that include the link a. This conservation of flow equation is expressed as:

$$f_a = \sum_{p \in P} x_p \delta_{ap}, \quad \forall a \in L, \tag{10}$$

where δ_{ap} is equal to 1 if the link a is included in the path p, and 0, otherwise. In the local food supply chain network, there is usually a single path between farm i and the market j; however, the path set P_j^i is introduced to present a more general definition.

Furthermore, the demand at the farmers' market j for the fresh produce product of farmer i is given by:

$$\sum_{p \in P_j^i} x_p = d_{ij}, \quad p \in P_j^i; i = 1, \ldots, I; j = 1, \ldots, M. \tag{11}$$

We group the demands between all farms and farmer market pairs (i, j) into the vector d and also the quality of their fresh produce on paths $p \in P_j^i$, $\forall i, j$, $\{q_p\}$, into the vector q. We assume that all vectors are column vectors.

The price of the product of farm i, in turn, at farmers' market j, is denoted by ρ_{ij}, and depends not only on the demand for the farm's fresh produce, but also on the quality of that product, which we capture by the quality decay kinetics in (11). Also, the price depends, in general, on the quantities of the competitors' fresh produce at the markets as well as the quality of their products.

Hence, the demand price function ρ_{ij} for farm i's product at the farmers' market j, is:

$$\rho_{ij} = \rho_{ij}(d, q), \quad i = 1, \ldots, I; \; j = 1, \ldots, M. \tag{12}$$

In view of (11) and (12), the demand price functions can be rewritten as:

$$\hat{\rho}_{ij} = \hat{\rho}_{ij}(x, q) \equiv \rho_{ij}(d, q), \quad i = 1, \ldots, I; \; j = 1, \ldots, M. \tag{13}$$

The demand price functions are assumed to be continuous, continuously differentiable, and monotone decreasing. Recall that the quality parameter vector q is defined with respect to the decay function of the fresh produce which farms sell at the farmers' markets and is time-dependent. Thus, the demand price for a farm's product is higher when the quality of its fresh produce product is high and, if the demand is high, the price is lower. Note that each farm is aware of the temperature conditions and the time associated with the links in its supply chain network.

The competition among the farms for resources is reflected in the total operational costs incurred in harvesting, processing, storage, and transportation. The quality-keeping cost is related to the food deterioration and is also considered through the transportation and the storage link costs. In general, the total operational cost of each link a, denoted by $\hat{c}_a$, depends on the flows on all the links in the fresh produce supply chain network, that is,

$$\hat{c}_a = \hat{c}_a(f), \quad \forall a \in L, \tag{14}$$

where f is the vector of all the link flows. The total cost on each link is assumed to be convex and continuously differentiable.

Let X_i denote the vector of path flows associated with farm i; $i = 1, \ldots, I$, where $X_i \equiv \{\{x_p\} | p \in P^i\} \in R_+^{n_{P^i}}$, $P^i \equiv \cup_{j=1,\ldots,M} P_j^i$, and n_{P^i} denotes the number of paths from farm i to the farmers' markets. Thus, X is the vector of all the farmers' strategies, that is, $X \equiv \{\{X_i\} | i = 1, \ldots, I\}$.

The profit function of farm i is defined as the difference between its revenue and its total costs, where the total costs are the total operational costs over L_i. The profit function, which we present is novel in terms of capturing the quality of the fresh produce in the demand price functions, which is time-dependent.

The profit/utility function of farm i, denoted by U_i, is given by:

$$U_i = \sum_{j=1}^{M} \rho_{ij}(d, q) d_{ij} - \sum_{a \in L^i} \hat{c}_a(f). \tag{15}$$

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In lieu of the conservation of flow expressions (10) and (11), and the functional expressions (13) and (14), we can define $\hat{U}_i(X) \equiv U_i$ for each farm i; $i = 1, \ldots, I$, with the I-dimensional vector $\hat{U}$ being the vector of the profits of all the farms with respect to their earnings at the farmers' markets over a typical week:

$$\hat{U} = \hat{U}(X). \tag{16}$$

We now state the fresh produce supply chain network Cournot-Nash Equilibrium conditions for farmers' markets in a region.

Definition 1: Fresh Produce Supply Chain Network Cournot-Nash Equilibrium for Farmers' Markets in the Uncapacitated Case

A path flow pattern $X^* \in K = \prod_{i=1}^{I} K_i$ constitutes a fresh produce supply chain network Cournot-Nash equilibrium if for each farm i; $i = 1, \ldots, I$:

$$\hat{U}_i(X_i^*, \hat{X}_i^*) \geq \hat{U}_i(X_i, \hat{X}_i^*), \quad \forall X_i \in K_i, \tag{17}$$

where $\hat{X}_i^* \equiv (X_1^*, \ldots, X_{i-1}^*, X_{i+1}^*, \ldots, X_I^*)$ and $K_i \equiv \{X_i | X_i \in R_+^{n_{P^i}}\}$.

A Cournot-Nash Equilibrium is established if no farm can unilaterally improve its profit by changing its product flows throughout its supply chain network, given the product flow decisions of the other farms.

Next, we derive the variational inequality formulations of the Cournot-Nash equilibrium for the fresh produce supply chain network under oligopolistic competition satisfying Definition 1, in terms of both path flows and link flows (see, e.g., Cournot (1838), Nash (1950, 1951), Gabay and Moulin (1980), Nagurney (2006), and Nagurney et al. (2013)).

Theorem 1: Variational Inequality Formulations of the Uncapacitated Model

Assume that for each fresh produce farm i; $i = 1, \ldots, I$, the profit function $\hat{U}_i(X)$ is concave with respect to the variables in X_i and is continuously differentiable. Then $X^* \in K$ is a fresh produce supply chain network Cournot-Nash equilibrium for farmers' markets according to Definition 1 if and only if it satisfies the variational inequality:

$$-\sum_{i=1}^{I} \langle \nabla_{X_i} \hat{U}_i(X^*), X_i - X_i^* \rangle \geq 0, \quad \forall X \in K, \tag{18}$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in the corresponding Euclidean space and $\nabla_{X_i} \hat{U}_i(X)$ denotes the gradient of $\hat{U}_i(X)$ with respect to X_i. Variational inequality (18), in turn, for

our uncapacitated model, is equivalent to the variational inequality that determines the vector of equilibrium path flows $x^* \in K^1$ such that:

$$\sum_{i=1}^{I}\sum_{j=1}^{M}\sum_{p\in P_j^i}\left[\frac{\partial \hat{C}_p(x^*)}{\partial x_p} - \hat{\rho}_{ij}(x^*, q) - \sum_{l=1}^{M}\frac{\partial \hat{\rho}_{il}(x^*, q)}{\partial x_p}\sum_{r\in P_l^i}x_r^*\right]\times [x_p - x_p^*] \geq 0, \quad \forall x \in K^1, \quad (19)$$

where $K^1 \equiv \{x | x \in R_+^{n_P}\}$, and for each path p; $p \in P_j^i$; $i = 1, \ldots, I$; $j = 1, \ldots, M$,

$$\frac{\partial \hat{C}_p(x)}{\partial x_p} \equiv \sum_{a\in L^i}\sum_{b\in L^i}\frac{\partial \hat{c}_b(f)}{\partial f_a}\delta_{ap} \text{ and } \frac{\partial \hat{\rho}_{il}(x, q)}{\partial x_p} \equiv \frac{\partial \rho_{il}(d, q)}{\partial d_{ij}}. \quad (20)$$

Variational inequality (19) can also be rewritten in terms of link flows as: determine the vector of equilibrium link flows and the vector of equilibrium demands $(f^*, d^*) \in K^2$, such that:

$$\sum_{i=1}^{I}\sum_{a\in L^i}\left[\sum_{b\in L^i}\frac{\partial \hat{c}_b(f^*)}{\partial f_a}\right]\times [f_a - f_a^*]$$

$$+\sum_{i=1}^{I}\sum_{j=1}^{M}\left[-\rho_{ij}(d^*, q) - \sum_{l=1}^{M}\frac{\partial \rho_{il}(d^*, q)}{\partial d_{ik}}d_{il}^*\right]\times [d_{ij} - d_{ij}^*] \geq 0, \quad \forall(f, d) \in K^2, \quad (21)$$

where $K^2 \equiv \{(f, d) | x \geq 0, \text{ and } (10) \text{ and } (11) \text{ hold}\}$.

Proof: (18) follows from Gabay and Moulin (1980); see, also, Masoumi, Yu, and Nagurney (2012). (19) and (21) then follow using algebraic substitutions. $\square$

Variational inequalities (19) and (21) can be put into standard form (see Nagurney (1999)): determine $X^* \in \mathcal{K}$ such that:

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \quad (22)$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in N-dimensional Euclidean space. Let $X \equiv x$ and

$$F(X) \equiv \left[\frac{\partial \hat{C}_p(x)}{\partial x_p} - \hat{\rho}_{ij}(x, q) - \sum_{l=1}^{M}\frac{\partial \hat{\rho}_{il}(x, q)}{\partial x_p}\sum_{r\in P_l^i}x_r;\right.$$

$$\left. p \in P_j^i; i = 1, \ldots, I; j = 1, \ldots, M\right], \quad (23)$$

$\mathcal{K} \equiv K^1$, and $N = n_P$, then (19) can be re-expressed as (22). Similarly, for the variational inequality in terms of link flows, if we define the column vectors: $X \equiv (f, d)$ and $F(X) \equiv (F_1(X), F_2(X))$, where

$$F_1(X) = \left[\sum_{b\in L^i}\frac{\partial \hat{c}_b(f)}{\partial f_a}; a \in L^i; i = 1, \ldots, I\right],$$

16

$$F_2(X) = \left[-\hat{\rho}_{ij}(x,q) - \sum_{l=1}^{M} \frac{\partial \hat{\rho}_{il}(x,q)}{\partial x_p} \sum_{r \in P_l^i} x_r; \ p \in P_j^i; \ i = 1, \ldots, I; \ j = 1, \ldots, M \right], \quad (24)$$

$\mathcal{K} \equiv K^2$, and $N = n_L + M$, then (21) can be re-written as (22).

Since the feasible set K^1 is not compact, and the same holds for K^2, we cannot obtain the existence of a solution simply based on the assumption of the continuity of F. However, the demand d_{ij} for each farm i's product; $i = 1, \ldots, I$, at every farmers' market j; $j = 1, \ldots, M$, may be assumed to be bounded, since the population requiring these products is finite (although it may be large). Consequently, in light of (12), we have that:

$$\mathcal{K}_c \equiv \{x \mid 0 \leq x \leq c, \}, \quad (25)$$

where $c > 0$ and $x \leq c$ means that $x_p \leq c$ for all $p \in P_j^i$; $i = 1, \ldots, I$, and $j = 1, \ldots, M$. Then $\mathcal{K}_c$ is a bounded, closed, and convex subset of K^1. Thus, the following variational inequality

$$\langle F(X^c), X - X^c \rangle \geq 0, \quad \forall X \in \mathcal{K}_c, \quad (26)$$

admits at least one solution $X^c \in \mathcal{K}_c$, since $\mathcal{K}_c$ is compact and F is continuous. Therefore, following Kinderlehrer and Stampacchia (1980) (see also Theorem 1.5 in Nagurney (1999)), we have the following theorem:

Theorem 2: Existence

There exists at least one solution to variational inequality (19) (equivalently, to (21)), since there exists a $c > 0$, such that variational inequality (29) admits a solution in $\mathcal{K}_c$ with

$$x^c \leq c. \quad (27)$$

In addition, we now provide a uniqueness result.

Theorem 3: Uniqueness

With Theorem 2, variational inequality (26) and, hence, variational inequalities (19) and (21) admit at least one solution. Moreover, if the function $F(X)$ of variational inequality (21), as defined in (22), is strictly monotone on $\mathcal{K} \equiv K^2$, that is,

$$\langle (F(X^1) - F(X^2)), X^1 - X^2 \rangle > 0, \quad \forall X^1, X^2 \in \mathcal{K}, \ X^1 \neq X^2, \quad (28)$$

then the solution to variational inequality (21) is unique, that is, the equilibrium link flow pattern and the equilibrium demand pattern are unique.

3.2 The Capacitated Model

As noted in the Introduction, challenges that farmers may be faced with include: labor shortages, lower yield harvests due to weather conditions, as well as lower capacities in both storage facilities and transport, which may arise due to disruptions. Hence, in this subsection, we provide the capacitated version of the model in Section 3.1.

Specifically, we retain the objective function (15), for each farm i; $i = 1, \ldots, I$, the nonnegativity constraints (9), with conservation of flow equations (11), as well as the previous notation, but now we have that

$$f_a \leq u_a, \quad \forall a \in L, \tag{29a}$$

or, in view of (10):

$$\sum_{p \in P} x_p \delta_{ap} \leq u_a, \quad \forall a \in L, \tag{29b}$$

where u_a is the positive upper bound on the flow on link a.

We define the feasible set K_i^3 faced by farm i in the capacitated case as: $K_i^3 \equiv \{X_i | X_i \in R_+^{n_{Pi}} \text{ and } (29b) \text{ holds for } a \in L^i\}$. Also, we define $K^3 \equiv \prod_{i=1}^{I} K_i^3$.

We, hence, have the following definition.

Definition 2: Fresh Produce Supply Chain Network Cournot-Nash Equilibrium for Farmers' Markets in the Capacitated Case

A path flow pattern $X^* \in K^3$ constitutes a fresh produce supply chain network Cournot-Nash equilibrium in the capacitated case if for each farm i; $i = 1, \ldots, I$:

$$\hat{U}_i(X_i^*, \hat{X}_i^*) \geq \hat{U}_i(X_i, \hat{X}_i^*), \quad \forall X_i \in K_i^3, \tag{30}$$

where, as before, $\hat{X}_i^* \equiv (X_1^*, \ldots, X_{i-1}^*, X_{i+1}^*, \ldots, X_I^*)$

Theorem 4: Variational Inequality Formulations of the Capacitated Model

Assume that for each fresh produce farm i; $i = 1, \ldots, I$, the profit function $\hat{U}_i(X)$ is concave with respect to the variables in X_i, and is continuously differentiable. Then $X^* \in K^3$ is a fresh produce supply chain network Cournot-Nash equilibrium for farmers' markets in the capacitated case according to Definition 2 if and only if it satisfies the variational inequality:

$$-\sum_{i=1}^{I} \langle \nabla_{X_i} \hat{U}_i(X^*), X_i - X_i^* \rangle \geq 0, \quad \forall X \in K^3. \tag{31}$$

Variational inequality (31), in turn, for our capacitated model, is equivalent to the variational inequality that determines the vector of equilibrium path flows $x^* \in K^3$ such that:

$$\sum_{i=1}^{I}\sum_{j=1}^{M}\sum_{p\in P_j^i} \left[\frac{\partial \hat{C}_p(x^*)}{\partial x_p} - \hat{\rho}_{ij}(x^*,q) - \sum_{l=1}^{M} \frac{\partial \hat{\rho}_{il}(x^*,q)}{\partial x_p} \sum_{r\in P_l^i} x_r^* \right] \times [x_p - x_p^*] \geq 0, \quad \forall x \in K^3. \quad (32)$$

Moreover, variational inequality (32) is equivalent to the variational inequality problem: determine $(x^*, \lambda^*) \in K^4$, where $K^4 \equiv \{x \in R_+^{n_P}, \lambda \in R_+^{n_L}\}$, such that:

$$\sum_{i=1}^{I}\sum_{j=1}^{M}\sum_{p\in P_j^i} \left[\frac{\partial \hat{C}_p(x^*)}{\partial x_p} - \hat{\rho}_{ij}(x^*,q) - \sum_{l=1}^{M} \frac{\partial \hat{\rho}_{il}(x^*,q)}{\partial x_p} \sum_{r\in P_l^i} x_r^* + \sum_{a\in L} \lambda_a^* \delta_{ap} \right] \times [x_p - x_p^*]$$

$$+ \sum_{a\in L} \left[u_a - \sum_{p\in P} x_p^* \delta_{ap} \right] \times [\lambda_a - \lambda_a^*] \geq 0, \quad \forall (x,\lambda) \in K^4, \quad (33)$$

where $\frac{\partial \hat{C}_p(x)}{\partial x_p}$ and $\frac{\partial \hat{\rho}_{il}(x,q)}{\partial x_p}$ are as defined in (20).

Proof: Variational inequality (31) again follows directly from Gabay and Moulin (1980). Variational inequality (33) follows, in turn, from Bertsekas and Tsitsiklis (1989) (see also Nagurney (2010)) with notice that λ^* corresponds to the vector of optimal Lagrange multipliers associated with constraints (29b) and λ is the vector of Lagrange multipliers associated with the upper bound constraints on all links $a \in L$.

We will utilize variational inequality (33) for solution of the capacitated model and note that variational inequality (33) can be put into the standard form (22) (see Nagurney (1999)), if the vectors: $X \equiv (x, \lambda)$ and $F(X) \equiv (F_3(X), F_4(X))$, where

$$F_3(X) = \left[\frac{\partial \hat{C}_p(x)}{\partial x_p} - \hat{\rho}_{ij}(x,q) - \sum_{l=1}^{M} \frac{\partial \hat{\rho}_{il}(x,q)}{\partial x_p} \sum_{r\in P_l^i} x_r + \sum_{a\in L} \lambda_a \delta_{ap}, \ p \in P_j^i; \ i = 1, \ldots, I; \ j = 1, \ldots, M \right],$$
$$(34)$$

$$F_4(X) = \left[u_a - \sum_{p\in P} x_p \delta_{ap}, \ a \in L \right]. \quad (35)$$

With $\mathcal{K} \equiv K^4$, (33) can be rewritten as (22).

The existence result below follows from the classical theory of variational inequalities (cf. Kinderlehrer and Stampacchia (1980) and Nagurney (1999)).

Theorem 5: Existence of a Solution to the Capacitated Model

Existence of a solution X^* to variational inequality (31) is guaranteed since the feasible set K^3 is compact.

4. The Algorithm

We now recall the Euler method, which is induced by the general iterative scheme of Dupuis and Nagurney (1993). Specifically, at an iteration τ of the Euler method (see also Nagurney and Zhang (1996)) one computes:

$$X^{\tau+1} = P_{\mathcal{K}}(X^\tau - a_\tau F(X^\tau)), \tag{36}$$

where $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$ and F is the function that enters the variational inequality problem (22).

As shown in Dupuis and Nagurney (1993) and Nagurney and Zhang (1996), for convergence of the general iterative scheme, which induces the Euler method, the sequence $\{a_\tau\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \to 0$, as $\tau \to \infty$. Specific conditions for convergence of this scheme as well as various applications to the solutions of network oligopolies can be found in Nagurney and Zhang (1996), Nagurney, Dupuis, and Zhang (1994), Nagurney (2010), and Masoumi, Yu, and Nagurney (2012).

4.1 Explicit Formulae for the Euler Method Applied to the Uncapacitated Model

The elegance of this procedure for the computation of solutions to the uncapacitated fresh produce farmers' market supply chain network problem in Section 3.1 can be seen in the following explicit formulae. In particular, we have the following closed form expressions for the fresh produce path flows, for each path $p \in P_j^i$, $\forall i, j$:

$$x_p^{\tau+1} = \max\{0, x_p^\tau + a_\tau(\hat{\rho}_{ij}(x^\tau, q) + \sum_{l=1}^M \frac{\partial \hat{\rho}_{il}(x^\tau, q)}{\partial x_p} \sum_{r \in P_l^i} x_r^\tau - \frac{\partial \hat{C}_p(x^\tau)}{\partial x_p})\},$$

$$\forall p \in P_j^i; \ i = 1, \ldots, I; \ j = 1, \ldots, M. \tag{37}$$

Next, the closed form expression to compute the solution of the capacitated problem in Section 3.2 is given.

4.2 Explicit Formulae for the Euler Method Applied to the Capacitated Model

The explicit formulae is shown to compute the solutions to the capacitated fresh produce farmers' market supply chain problem in Section 3.2. The closed form expressions for the fresh produce path flows at iteration $\tau + 1$ are as follows. For each path $p \in P_j^i$, $\forall i, j$, compute:

$$x_p^{\tau+1} = \max\{0, x_p^\tau + a_\tau(\hat{\rho}_{ij}(x^\tau, q) + \sum_{l=1}^{M} \frac{\partial \hat{\rho}_{il}(x^\tau, q)}{\partial x_p} \sum_{r \in P_l^i} x_r^\tau - \frac{\partial \hat{C}_p(x^\tau)}{\partial x_p} - \sum_{a \in L} \lambda_a^\tau \delta_{ap})\},$$

$$\forall p \in P_j^i;\ i = 1, \ldots, I;\ j = 1, \ldots, M. \tag{38}$$

The Lagrange multipliers for each link $a \in L^i; i = 1, \ldots, I$, can be computed as:

$$\lambda_a^{\tau+1} = \max\{0, \lambda_a^\tau + a_\tau(\sum_{p \in P} x_p^\tau \delta_{ap} - u_a)\},\ \forall a \in L. \tag{39}$$

The number of strategic variables x_p, as well as the number of the paths, in the supply chain network, for both of the uncapacitated and capacitated supply chain networks, grow linearly in terms of the number of nodes in the supply chain network, be it a farm, or a farmers' market, etc. Therefore, even a fresh produce supply chain network with hundreds of demand markets is still tractable within our proposed modeling and computational framework.

5. Case Study

We focus on apples for our case study. The United States holds second place in the world's apple production with 4 million metric tons of apples produced, with the leader being China, which produced 33 million metric tons of apples in 2010 (USDA (2012)). Specifically, in our case study we consider Golden Delicious apples. This type of apple is ranked in third place among the other apple varieties in the United States with 25,000 metric tons of production value. Our case study is based on farmers' markets in western Massachusetts in the United States. According to the USDA (2016) and the website of Community Involved in Sustaining Agriculture (CISA), there are 8,558 farmers markets in the United States, 312 in Massachusetts, and 40 in western Massachusetts (CISA (2016)). Also, in Massachusetts, 369 in-state apple orchards produced 34 million pounds of apples in 2010. There are approximately 27 apple orchards in western Massachusetts. For this case study, we selected three of the apple orchards located in western Massachusetts: Apex Orchards, the Park Hill Orchard, and Sentinel Farm. The locations of these orchards/farms in Massachusetts are, respectively, Shelburne, Easthampton, and Belchertown. These orchards/farms have the possibility of selling their apples at the Amherst, Northampton, South Hadley, and/or Belchertown farmers' markets, which are open on different days in a given week during the

season, as shown in Figure 2. The Northampton Farmers' Market is open on Tuesdays, the South Hadley Farmers' Market operates on Thursdays. On Saturdays the Amherst Farmers' Market is open, and the Belchertown Farmers' Market is open on Sundays. Each orchard/farm has its own harvesting, processing, storage, and transportation units. Notice that the demand points in Figure 2 are sequenced depending on the day of the week. For example, since the Northampton Farmers' Market is open on Tuesdays, which is the first farmers' market of the week in this case study, it is numbered as the first demand point. The South Hadley Farmers' Market corresponds to demand point 2, and so on. Additionally, apples are picked on Tuesdays at all the orchards/farms, so that there is no storage for the Golden Delicious apples sold at the Northampton Farmers' Market. However, each farm should store its Golden Delicious apples for 2, 4, and 5 days for the South Hadley, Amherst, and the Belchertown Farmers' Markets, respectively.
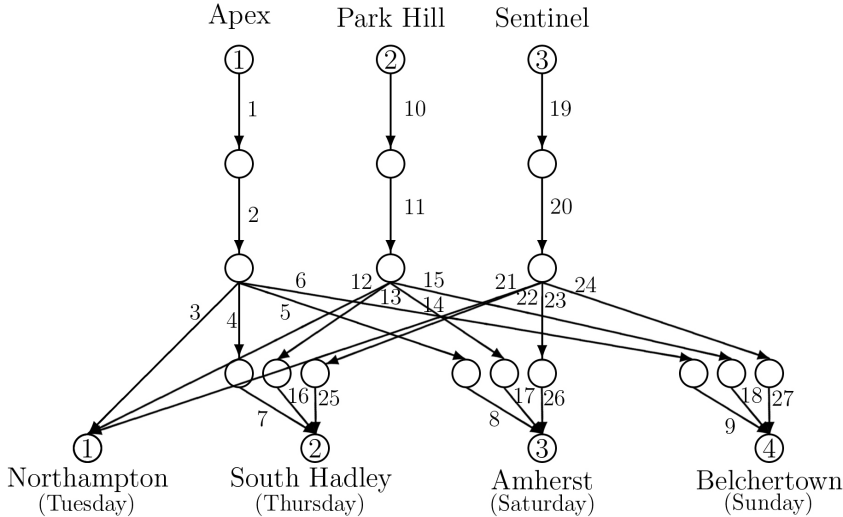

Figure 2: The Supply Chain Network Topology for the Apple Farmers' Markets Case Study – Scenario 1

The Euler method for the uncapacitated (cf. (37)) and the capacitated problems (cf. (38) and (39)) is implemented in MATLAB and the sequence $a_\tau = \{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \ldots\}$, with the convergence tolerance being 10^{-6}, that is, the Euler method is deemed to have converged if the absolute value of the difference of each successive variable iterate differs by no more than this value. The code in Matlab is executed on a Macbook Pro laptop with a 2.8 GHz Intel Core i5 processor and 8GB 1600 MHz DDR3 memory.

Scenario 1. In Scenario 1, it is assumed that the weather conditions in Massachusetts are steady, which means that the temperature in the growing season of Golden Delicious

apples is at its seasonal normal and the harvesting season is for 2015. For this scenario, we assume that orchard/farm i; $i = 1, 2, 3$, in the supply chain network has initial quality, respectively, of: $q_{01} = 1$, $q_{02} = 0.8$, and $q_{03} = 0.7$. Apex Orchards are well-known for the quality of their apples in western Massachusetts and, hence, the value for their apple initial quality is $q_{01} = 1$. In addition, we assume that there is no capacity limit on the links in the supply chain network. According to Table 1, we know that, for Golden Delicious apples, the quality attribute, that of the texture softening, follows first order quality decay. The link quality decay β_a for every link a in the supply chain network and the parameters used in the calculations are provided in Table 2. Also, we include the quality decay at the farmers' markets by considering the selling/purchasing point as being the middle of the operation period, since quality decay is a continuous process.

The orchards/farms in this case study are of different sizes and are located at different altitudes. According to CISA, Apex, the Park Hill Orchards, and the Sentinel Farm have 170, 127, and 8 acres of land, respectively. Also, Apex Orchards are located at a higher altitude than Park Hill and Sentinel Farm, but the altitude of Park Hill and the Sentinel are similar. The harvesting season of apples is between September and October when temperatures are between 19-22 °C. In this case study, the harvesting operation is assumed to be realized at the average temperature of the season depending on the location of the orchards. Notice that Apex Orchards have the lowest harvesting temperature, since they are located at a higher altitude where the temperature is lower. Furthermore, the duration of the link operations are constructed according to the land size and the number of employees. For example, Apex has a larger land size; therefore, the harvesting operation at this orchard takes a longer time. Moreover, we assume that the larger the size of the orchard, the more employees it has. As a result, the duration of harvesting and processing operations are the longest for Sentinel Farm due to it having the lowest number of employees. The duration of transportation is calculated as the summation of the actual transportation time between the orchard and the farmers' market and half of the farmers' market's hours of operation, which the transportation link is connected to in the supply chain network. The hours of operation of the Northampton, South Hadley, Amherst, and Belchertown Farmers' Markets are, respectively: 5, 6, 5.5, and 4 hours. The hours of storage are calculated according to the days between the harvesting and when the farmers' markets open in the week. For instance, Apple Orchards need to store their apples for 2 days to sell at the South Hadley Farmers' Market, which is open on Thursdays. It is assumed that Apex Orchards have the controlled atmospheric storage system to keep the temperature at 0°C which is the optimal storage temperature for apples (Iowa State University Extension (2008)). However, it is assumed that Park Hill and Sentinel Farm have regular storage which can lower the temperatures to 9°C and 12°C, respectively.

Operations	Link a	Hours	Temp (°C)	β_a
harvesting	1	4.00	19	0.992
processing	2	3.00	19	0.994
transportation	3	0.50	19	0.999
storage (2 days)	4	48.00	0	0.994
storage (4 days)	5	96.00	0	0.988
storage (5 days)	6	120.00	0	0.985
transportation	7	4.00	19	0.993
transportation	8	3.25	19	0.994
transportation	9	4.00	19	0.993
harvesting	10	3.00	22	0.992
processing	11	3.00	22	0.992
transportation	12	0.5	19	0.999
storage (2 days)	13	48.00	9	0.978
storage (4 days)	14	96.00	9	0.957
storage (5 days)	15	120.00	9	0.947
transportation	16	3.75	19	0.993
transportation	17	5.16	19	0.990
transportation	18	3.00	19	0.992
harvesting	19	5.00	22	0.986
processing	20	5.00	22	0.986
transportation	21	0.50	22	0.998
storage (2 days)	22	48.00	12	0.967
storage (4 days)	23	96.00	12	0.936
storage (5 days)	24	120.00	12	0.921
transportation	25	3.75	22	0.990
transportation	26	5.16	22	0.986
transportation	27	3.00	22	0.992

Table 2: Parameters for the Calculation of Quality Decay for the Case Study Scenario 1

The link quality decay β_a is calculated by taking the universal gas constant and the activation energy as 8.314 $Jmol^{-1}K^{-1}$ and 88 $kJmol^{-1}$. The temperature and the time are converted, respectively, to Kelvin and seconds for the quality decay calculations.

The total number of paths in the supply chain network is twelve and they are as follows: path $p_1 = (1, 2, 3)$, $p_2 = (1, 2, 4, 7)$, $p_3 = (1, 2, 5, 8)$, $p_4 = (1, 2, 6, 9)$, $p_5 = (10, 11, 12)$, $p_6 = (10, 11, 13, 16)$, $p_7 = (10, 11, 14, 17)$, $p_8 = (10, 11, 15, 18)$, $p_9 = (19, 20, 21)$, $p_{10} = (19, 20, 22, 25)$, $p_{11} = (19, 20, 23, 26)$, and $p_{11} = (19, 20, 24, 27)$. Also each path has its own quality decay rate, which is calculated according to (11). Furthermore, the demand price functions of the orchard/farms at the farmers' markets are:

Apex Orchards:

$$\rho_{11}(d, q) = -0.04d_{11} - 0.01d_{21} - 0.01d_{31} + 8q_{p_1} - 4q_{p_5} - 3q_{p_9} + 30,$$

$$\rho_{12}(d, q) = -0.02d_{12} - 0.01d_{22} - 0.01d_{32} + 3q_{p_2} - 2q_{p_6} - 2q_{p_{10}} + 25,$$

$$\rho_{13}(d, q) = -0.04d_{13} - 0.02d_{23} - 0.01d_{33} + 8q_{p_3} - 4q_{p_7} - 3q_{p_{11}} + 30,$$

$$\rho_{14}(d, q) = -0.04d_{14} - 0.02d_{24} - 0.02d_{34} + 3q_{p_4} - q_{p_8} - 2q_{p_{12}} + 25,$$

Park Hill Orchard:

$$\rho_{21}(d, q) = -0.04d_{21} - 0.02d_{11} - 0.02d_{31} + 3q_{p_5} - 2q_{p_1} - q_{p_9} + 27,$$

$$\rho_{22}(d, q) = -0.04d_{22} - 0.01d_{12} - 0.02d_{32} + 3q_{p_6} - 2q_{p_2} - q_{p_{10}} + 28,$$

$$\rho_{23}(d, q) = -0.04d_{23} - 0.02d_{13} - 0.02d_{33} + 4q_{p_7} - 2q_{p_3} - q_{p_{11}} + 27,$$

$$\rho_{24}(d, q) = -0.02d_{24} - 0.01d_{14} - 0.01d_{34} + 2q_{p_8} - q_{p_4} - q_{p_{12}} + 28,$$

Sentinel Farm:

$$\rho_{31}(d, q) = -0.04d_{31} - 0.02d_{11} - 0.02d_{21} + 4q_{p_9} - q_{p_1} - 2q_{p_5} + 25,$$

$$\rho_{32}(d, q) = -0.04d_{32} - 0.01d_{12} - 0.02d_{22} + 4q_{p_{10}} - 3q_{p_2} - q_{p_6} + 28,$$

$$\rho_{33}(d, q) = -0.02d_{23} - 0.01d_{13} - 0.01d_{33} + 4q_{p_{11}} - 2q_{p_3} - q_{p_7} + 25,$$

$$\rho_{34}(d, q) = -0.04d_{34} - 0.02d_{14} - 0.02d_{24} + 3q_{p_{12}} - 2q_{p_4} - 2q_{p_8} + 28.$$

The demand price functions are constructed according to the customer and orchard characteristics. According to a former orchard owner, Colnes (2016), customers going to the Amherst Farmers' Market are more affluent and give importance to quality of the apples. The Northampton Farmers' Market is also similar to the Amherst Farmers' Market in terms of the consumers' willingness to pay for higher quality. The demand price functions at the South Hadley and Belchertown Farmers' Markets are similar to one another, with the lowest consumer willingness to pay in the supply chain network. Furthermore, some orchards are assumed to have a more positive reputation at some farmers' markets, which means that the price of the Golden Delicious apples from an orchard may be less affected by the demand for the apples of the other orchards at the same farmers' market. For example, Apex is assumed to have loyal customers at the Amherst Farmers' Market whereas the Sentinel does not; therefore, the price of Apex's Golden Delicious apples is less affected by the demand for the Sentinel's apples.

The total link cost functions and the computed equilibrium link flows are shown in Table 3. The cost functions are constructed based on the price data in Berkett (1994), King and Gomez (2015), and the U.S. Energy Information Administration (2016) for the fuel price. The flow unit is pecks and the total cost functions are constructed based on the dollar price per peck.

Operations	**Link** a	$\hat{c}_a(f)$	f_a^*
harvesting	1	$0.02f_1^2 + 3f_1$	165.8395
processing	2	$0.015f_2^2 + 3f_2$	165.8395
transportation	3	$0.01f_3^2 + 3f_3$	111.9827
storage (2 days)	4	$0.01f_4^2 + 3f_4$	0.0000
storage (4 days)	5	$0.015f_5^2 + 4f_5$	53.8568
storage (5 days)	6	$0.03f_6^2 + 5f_6$	0.0000
transportation	7	$0.02f_7^2 + 6f_7$	0.0000
transportation	8	$0.0125f_8^2 + 4f_8$	53.8568
transportation	9	$0.02f_9^2 + 6.6f_9$	0.0000
harvesting	10	$0.0125f_{10}^2 + 6f_{10}$	94.7414
processing	11	$0.0125f_{11}^2 + 6f_{11}$	94.7414
transportation	12	$0.0045f_{12}^2 + f_{12}$	71.7812
storage (2 days)	13	$0.01f_{13}^2 + 1.67f_{13}$	22.9601
storage (4 days)	14	$0.015f_{14}^2 + 6f_{14}$	0.0000
storage (5 days)	15	$0.015f_{15}^2 + 6.6f_{15}$	0.0000
transportation	16	$0.0075f_{16}^2 + 6f_{16}$	22.9601
transportation	17	$0.01f_{17}^2 + 6f_{17}$	0.0000
transportation	18	$0.02f_{18}^2 + 4f_{18}$	0.0000
harvesting	19	$0.0125f_{19}^2 + 6f_{19}$	98.5294
processing	20	$0.015f_{20}^2 + 4f_{20}$	98.5294
transportation	21	$0.02f_{21}^2 + 4f_{21}$	17.2084
storage (2 days)	22	$0.007f_{22}^2 + 1.67f_{22}$	32.4314
storage (4 days)	23	$0.009f_{23}^2 + 6f_{23}$	0.0000
storage (5 days)	24	$0.01f_{24}^2 + 6f_{24}$	48.8896
transportation	25	$0.005f_{25}^2 + 6f_{25}$	32.4314
transportation	26	$0.005f_{26}^2 + 6f_{26}$	0.0000
transportation	27	$0.0005f_{27}^2 + 0.1f_{27}$	48.8896

Table 3: The Total Link Cost Functions and the Computed Equilibrium Link Flows for the Case Study Scenario 1

The computed equilibrium path flows and the quality decay of the paths are given in Table 4. As mentioned earlier, the unit of the flows is pecks of apples. Notice that, in equilibrium, there are paths and links with zero flows, which indicates the nonprofitable farmers' markets for specific orchards. For example, Apex serves the Northampton and Amherst Farmers' Markets; Park Hill serves the Northampton and South Hadley Farmers' Markets, and Sentinel serves the Northampton, South Hadley, and Belchertown Farmers' Markets. The game theory aspect of our model reveals the profitable markets and the variational inequality solution returns positive path flows for these markets and zero for the nonprofitable ones.

Path p	q_p	x_p^*
p_1	0.9851	111.9827
p_2	0.9733	0.0000
p_3	0.9684	53.8568
p_4	0.9645	0.0000
p_5	0.7864	71.7812
p_6	0.7645	22.9602
p_7	0.7458	0.0000
p_8	0.7395	0.0000
p_9	0.6791	17.2084
p_{10}	0.6514	32.4314
p_{11}	0.6280	0.0000
p_{12}	0.6217	48.8896

Table 4: The Path Quality Decay Rates and the Computed Equilibrium Path Flows for the Case Study Scenario 1

By substituting the equilibrium path flow and quality decay values, the demand prices of the orchard/farms at the profitable demand markets are found, in terms of dollars per peck, as:

Apex Orchards:

$$\rho_{11} = 27.33, \quad \rho_{12} = 24.53, \quad \rho_{13} = 30.72, \quad \rho_{14} = 25.42,$$

Park Hill Orchard:

$$\rho_{21} = 21.25, \quad \rho_{22} = 26.13, \quad \rho_{23} = 26.34, \quad \rho_{24} = 27.40,$$

Sentinel Farm:

$$\rho_{31} = 20.79, \quad \rho_{32} = 25.16, \quad \rho_{33} = 24.29, \quad \rho_{34} = 24.50.$$

According to Clements (2016), who is an educator at the University of Massachusetts Amherst Extension Fruit Program, the retail price of Golden Delicious apples is usually $2 per pound. A peck of apples is equal to 10-12 pounds which means the price of a peck can be between $20-$24. Our results for the demand prices are close to this range and, hence, are consistent with reality. Furthermore, the profits of the orchard/farms, in dollars, at the equilibrium solution, are:

$$U_1(X^*) = 1785.40, \quad U_2(X^*) = 484.03, \quad U_3(X^*) = 460.15.$$

Apex Orchards have the largest profit in Scenario 1, followed by Park Hill Orchard and then the Sentinel Farm.

Scenario 2. In this scenario, it is assumed that a new orchard, which was solely selling to retailers and wholesalers previously, is attracted by the demand for apples at the farmers' markets. This hypothetical new orchard is called New Orchard, and is located in western Massachusetts and enters the local food supply chain as depicted in Figure 3. As in Scenario 1, the uncapacitated variational inequality problem (22) is solved with the Euler method, using the explicit formulae shown in (37).
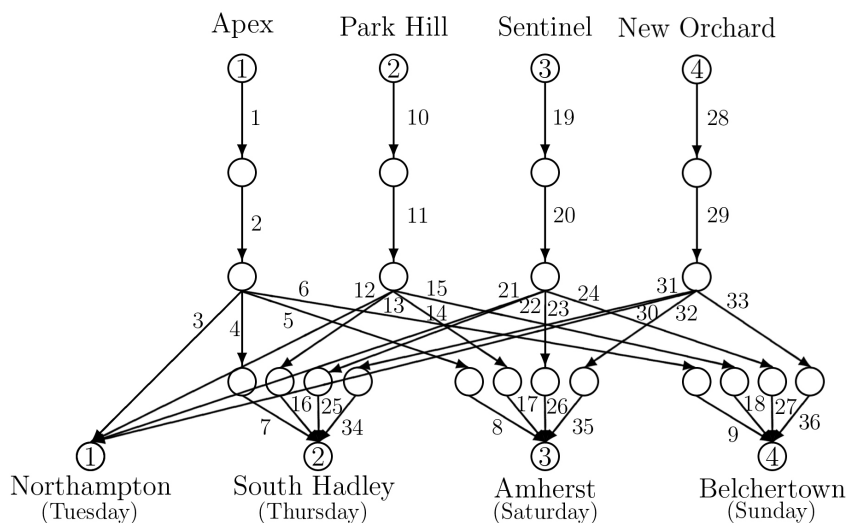

Figure 3: The Supply Chain Network Topology for the Apple Farmers' Markets Case Study - Scenario 2

The quality decay parameters and the link quality decay rates of New Orchard are shown in Table 5 and the total link cost functions are depicted in Table 6. The initial quality for this orchard is, $q_{04} = 1$, and the other orchard/farms have the same initial quality values as

in Scenario 1. New Orchard is very similar to Apex Orchards in terms of its land size, the number of employees, and the storage technology. Therefore, the New Orchards' total link cost functions for harvesting, processing, and storage are very similar to those of Apex. We assume that New Orchard is located near Sentinel Farm in Belchertown at a lower altitude than the other orchards/farms where the harvesting, processing, and transportation take place at higher temperatures. The transportation duration from the New Orchard to the farmers' markets is similar to that of the Sentinel Farm.

Operations	Link a	Hours	Temp (°C)	β_a
harvesting	28	4.00	19	0.988
processing	29	4.00	19	0.988
transportation	30	0.50	19	0.998
storage (2 days)	31	48.00	0	0.968
storage (4 days)	32	96.00	0	0.989
storage (5 days)	33	120.00	0	0.986
transportation	34	3.50	19	0.989
transportation	35	3.00	19	0.991
transportation	36	3.00	19	0.991

Table 5: New Orchard Parameters for the Calculation of Quality Decay for Scenario 2

Since New Orchard has the same initial quality of its Golden Delicious apples as that of the Apex Orchards, it is expected that it may lose some of its loyal customers at the farmers' markets which New Orchard is able to enter.

Operations	Link a	$\hat{c}_a(f)$
harvesting	28	$0.02f_{28}^2 + 3f_{28}$
processing	29	$0.015f_{29}^2 + 3f_{29}$
transportation	30	$0.01f_{30}^2 + 3f_{30}$
storage (2 days)	31	$0.01f_{31}^2 + 6f_{31}$
storage (4 days)	32	$0.015f_{32}^2 + 4f_{32}$
storage (5 days)	33	$0.035f_{33}^2 + 5f_{33}$
transportation	34	$0.025f_{34}^2 + 8f_{34}$
transportation	35	$0.015f_{35}^2 + 4f_{35}$
transportation	36	$0.025f_{36}^2 + 8f_{36}$

Table 6: The Total Link Cost Functions for New Orchard Under Scenario 2

With the entrance of New Orchard into the supply chain, the total number of paths in the supply chain network for Scenario 2 is increased to sixteen with twelve of them being the same as in Scenario 1. The additional paths and links are: path $p_{13} = (28, 29, 30)$, $p_{14} = (28, 29, 31, 34)$, $p_{15} = (28, 29, 32, 35)$, and $p_{16} = (28, 29, 32, 36)$.

Customers at the farmers' markets have enough information about New Orchard's Golden Delicious apples. This means that the demand price functions of the other orchard/farms are affected by the demand for the Golden Delicious apples of New Orchard. The new demand price functions are:

Apex Orchards:

$$\rho_{11}(d, q) = -0.053d_{11} - 0.01d_{21} - 0.01d_{31} - 0.03d_{41} + 8q_{p_1} - 2q_{p_5} - 2q_{p_9} - 4q_{p_{13}} + 30,$$

$$\rho_{12}(d, q) = -0.03d_{12} - 0.01d_{22} - 0.01d_{32} - 0.004d_{42} + 3q_{p_2} - 2q_{p_6} - 2q_{p_{10}} - q_{p_{14}} + 25,$$

$$\rho_{13}(d, q) = -0.053d_{13} - 0.01d_{23} - 0.01d_{33} - 0.03d_{43} + 8q_{p_3} - 2q_{p_7} - 2q_{p_{11}} - 4q_{p_{15}} + 30,$$

$$\rho_{14}(d, q) = -0.03d_{14} - 0.01d_{24} - 0.014d_{34} - 0.004d_{44} + 3q_{p_4} - q_{p_8} - 2q_{p_{12}} - q_{p_{15}} + 25,$$

Park Hill Orchard:

$$\rho_{21}(d, q) = -0.05d_{21} - 0.01d_{11} - 0.01d_{31} - 0.01d_{41} + 3q_{p_5} - q_{p_1} - q_{p_9} - q_{p_{13}} + 27,$$

$$\rho_{22}(d, q) = -0.04d_{22} - 0.01d_{12} - 0.02d_{32} - 0.004d_{42} + 3q_{p_6} - 2q_{p_2} - q_{p_{10}} - q_{p_{14}} + 28,$$

$$\rho_{23}(d, q) = -0.05d_{23} - 0.02d_{13} - 0.01d_{33} - 0.02d_{43} + 4q_{p_7} - 2q_{p_3} - q_{p_{11}} - 2q_{p_{15}} + 27,$$

$$\rho_{24}(d, q) = -0.04d_{24} - 0.01d_{14} - 0.02d_{34} - 0.004d_{44} + 2q_{p_8} - q_{p_4} - q_{p_{12}} - q_{p_{16}} + 28,$$

Sentinel Farm:

$$\rho_{31}(d, q) = -0.05d_{31} - 0.01d_{11} - 0.01d_{21} - 0.01d_{41} + 2q_{p_9} - q_{p_1} - q_{p_5} - q_{p_{13}} + 25,$$

$$\rho_{32}(d, q) = -0.04d_{32} - 0.01d_{12} - 0.02d_{22} - 0.004d_{42} + 4q_{p_{10}} - 3q_{p_2} - q_{p_6} - q_{p_{14}} + 28,$$

$$\rho_{33}(d, q) = -0.05d_{33} - 0.02d_{13} - 0.01d_{23} - 0.02d_{43} + 4q_{p_{11}} - 2q_{p_3} - q_{p_7} - 2q_{p_{15}} + 25,$$

$$\rho_{34}(d, q) = -0.04d_{34} - 0.01d_{14} - 0.02d_{24} - 0.004d_{44} + 3q_{p_{12}} - 2q_{p_4} - 2q_{p_8} - 2q_{p_{16}} + 28.$$

According to the demand price functions, New Orchard is not very strong in the South Hadley Farmers' Market. However, it is very effective in the Northampton, Amherst, and Belchertown Farmers' Markets where Apex and Sentinel have been market leaders. In particular, New Orchard becomes a crucial competitor for Apex Orchards. Its demand price functions are given below.

New Orchard:

$$\rho_{41}(d, q) = -0.053d_{41} - 0.03d_{11} - 0.01d_{21} - 0.01d_{31} + 5q_{p_{13}} - 2q_{p_1} - q_{p_5} - q_{p_9} + 30,$$

$$\rho_{42}(d,q) = -0.03_{42} - 0.006d_{12} - 0.01d_{22} - 0.01d_{32} + 2q_{p_{14}} - q_{p_2} - q_{p_6} - q_{p_{10}} + 25,$$

$$\rho_{43}(d,q) = -0.053d_{43} - 0.03d_{13} - 0.01d_{23} - 0.01d_{33} + 5q_{p_{15}} - 2q_{p_3} - q_{p_7} - q_{p_{11}} + 30,$$

$$\rho_{44}(d,q) = -0.03d_{44} - 0.006d_{14} - 0.01d_{24} - 0.01d_{34} + 2q_{p_{16}} - q_{p_4} - q_{p_8} - q_{p_{12}} + 25.$$

For New Orchard, there is more competition in the Amherst and the Northampton Farmers' Markets where the customers are more health conscious and pay attention to the newcomer. New Orchard's demand price is mostly affected by Apex, which is a very important competitor for New Orchard. For example, the demand price of New Orchard's Golden Delicious apples at the Northampton Farmers' Market is affected mostly by the demand for the Golden Delicious apples from Apex.

The computed equilibrium link flows, path flows, and the quality decay rates of the paths are given in Table 7 and Table 8. New Orchard enters the Northampton and Amherst Farmers' Markets, since they are the only profitable farmers' markets in the supply chain network for it. Notice that some of the equilibrium path flows are lower than in Scenario 1. For instance, the flow on the paths p_1 and p_3 are lower in Table 9 than they are in Table 4. One explanation for this result is that New Orchard is a new market player whose quality of Golden Delicious apples is very similar to that of Apex's; therefore, some of Apex's previous customers now choose to buy apples from New Orchard and cause a drop in Golden Delicious apple sales for Apex.

The demand prices of the orchard/farms, in dollars per peck, for the profitable demand markets are:

Apex Orchards:

$$\rho_{11} = 23.49, \quad \rho_{12} = 23.66, \quad \rho_{13} = 27.49, \quad \rho_{14} = 24.44,$$

Park Hill Orchard:

$$\rho_{21} = 21.46, \quad \rho_{22} = 25.41, \quad \rho_{23} = 25.49, \quad \rho_{24} = 26.20,$$

Sentinel Farm:

$$\rho_{31} = 20.38, \quad \rho_{32} = 24.38, \quad \rho_{33} = 22.91, \quad \rho_{34} = 23.08,$$

New Orchard:

$$\rho_{41} = 23.82, \quad \rho_{42} = 23.99, \quad \rho_{43} = 27.80, \quad \rho_{44} = 24.21.$$

Operations	Link a	f_a^*
harvesting	1	124.0885
processing	2	124.0885
transportation	3	79.5849
storage (2 days)	4	0.0000
storage (4 days)	5	44.5036
storage (5 days)	6	0.0000
transportation	7	0.0000
transportation	8	44.5036
transportation	9	0.0000
harvesting	10	87.4808
processing	11	87.4808
transportation	12	69.2348
storage (2 days)	13	18.2459
storage (4 days)	14	0.0000
storage (5 days)	15	0.0000
transportation	16	18.2459
transportation	17	0.0000
transportation	18	0.0000
harvesting	19	86.0782
processing	20	86.0782
transportation	21	18.3520
storage (2 days)	22	30.9408
storage (4 days)	23	0.0000
storage (5 days)	24	36.7854
transportation	25	30.9408
transportation	26	0.0000
transportation	27	36.7854
harvesting	28	126.1215
processing	29	126.1215
transportation	30	82.0895
storage (2 days)	31	0.0000
storage (4 days)	32	44.0319
storage (5 days)	33	0.0000
transportation	34	0.0000
transportation	35	44.0319
transportation	36	0.0000

Table 7: The Computed Equilibrium Link Flows for the Case Study Scenario 2

New Orchards' entrance causes a price decrease for the other orchard/farms at the farmers' markets due to competition in all prices except for ρ_{21}, which is almost the same as in scenario 1. For example, Apex sells its apples at \$27.33 per peck at the Northampton Farmers' Market in Scenario 1, which decreases to \$23.49 per peck.

Path p	q_p	x_p^*
p_1	0.9851	79.5849
p_2	0.9733	0.0000
p_3	0.9684	44.5036
p_4	0.9645	0.0000
p_5	0.7864	69.2348
p_6	0.7645	18.2460
p_7	0.7458	0.0000
p_8	0.7395	0.0000
p_9	0.6791	18.3520
p_{10}	0.6514	30.9408
p_{11}	0.6280	0.0000
p_{12}	0.6217	36.7854
p_{13}	0.9742	82.0895
p_{14}	0.9345	0.0000
p_{15}	0.9567	44.0319
p_{16}	0.9538	0.0000

Table 8: The Path Quality Decay Rates and the Computed Equilibrium Path Flows for the Case Study Scenario 2

The profits of the orchard/farms are:

$$U_1(X^*) = 1097.39, \quad U_2(X^*) = 471.71, \quad U_3(X^*) = 345.45, \quad U_4(X^*) = 1142.19.$$

The profits of Apex, Park Hill Orchards, and the Sentinel Farm decrease from their values in Scenario 1, as a result of the the entrance of New Orchard, which results in increased competition. The largest profit is gained by New Orchard, followed by Apex, Park Hill, and then Sentinel.

Scenario 3. This scenario is constructed to illustrate the apple shortage experienced in western Massachusetts in 2016. We return to the original farmers as in Scenario 1. The volatile weather conditions of an unexpected cold snap occurred in May. According to various news articles, the cold snap damaged the green apple buds and an apple shortage at the local markets, which includes the farmers' markets, was expected. In this scenario, we capture this apple shortage by imposing link capacities on the harvesting links of the orchard/farms. The Euler method, with the explicit formulae given in (38) and (39) for the capacitated problem, is used to compute the solution of the variational inequality formulation (33). The supply chain network structure for this scenario is the same as that in Figure 2. The total link costs functions, demand price functions of the orchard/farms, and the link quality decay rates are also the same as the corresponding ones in Scenario 1.

The link capacities (in pecks), the computed equilibrium link flows, and the computed equilibrium Lagrange multipliers for this scenario are reported in Table 9.

Operations	Link a	Capacity	f_a^*	λ_a^*
harvesting	1	20	20.0000	16.4077
processing	2	15000	20.0000	0.0000
transportation	3	15000	20.0000	0.0000
storage (2 days)	4	15000	0.0000	0.0000
storage (3 days)	5	15000	0.0000	0.0000
storage (4 days)	6	15000	0.0000	0.0000
transportation	7	15000	0.0000	0.0000
transportation	8	15000	0.0000	0.0000
transportation	9	15000	0.0000	0.0000
harvesting	10	50	50.0000	6.4906
processing	11	15000	50.0000	0.0000
transportation	12	15000	50.0000	0.0000
storage (2 days)	13	15000	0.0000	0.0000
storage (3 days)	14	15000	0.0000	0.0000
storage (4 days)	15	15000	0.0000	0.0000
transportation	16	15000	0.0000	0.0000
transportation	17	15000	0.0000	0.0000
transportation	18	15000	0.0000	0.0000
harvesting	19	60	60.0000	5.6685
processing	20	15000	60.0000	0.0000
transportation	21	15000	13.1918	0.0000
storage (2 days)	22	15000	18.7448	0.0000
storage (3 days)	23	15000	0.0000	0.0000
storage (4 days)	24	15000	28.0624	0.0000
transportation	25	15000	18.7448	0.0000
transportation	26	15000	0.0000	0.0000
transportation	27	15000	28.0624	0.0000

Table 9: Link Capacities and Computed Equilibrium Link Flows for the Case Study in Scenario 3

The capacities in Table 9 reflect the expected level of harvest damage at the orchard/farms. For example, Apex is assumed to experience a larger damage of its Golden Delicious apples, since it is located at a higher altitude, which causes the temperatures to drop lower than the temperatures at the other orchard/farms and damages the green buds more. Therefore, the capacity of Apex's harvesting link is assumed to be the lowest. Park Hill Orchard and Sentinel Farm are located at similar altitudes in western Massachusetts. We assume that they experience similar temperatures which result in similar capacities imposed on the harvesting

links of these orchard/farms. The capacities on the other operational links remain relatively large. Furthermore, the initial quality of Apex (q_{01}), Park Hill Orchard (q_{02}), and Sentinel Farm (q_{03}) apples are assumed to be 0.4, 0.5, and 0.6, respectively. Since the most damage on the apple buds is assumed to happen at the Apex Orchards, the lowest initial quality is assigned to this orchard.

From Table 9 and Table 10 one can see that Apex and Park Hill are now only at the Northampton Farmers' Market, whereas Sentinel serves the Northampton, South Hadley, and Belchertown Farmers' Markets. Moreover, links 1, 10, and 19 are at their capacities and, hence, the associated Lagrange multipliers are positive.

The equilibrium path flows and the path quality decay values are reported in Table 10. Observe that the path flows of all the orchard/farms have decreased substantially with respect to the path flows in Scenario 1. The experienced shortage is especially marked for Apex since the path flows p_1, p_2, p_3, and p_4 have decreased substantially from the results reported in Scenario 1 in Table 4.

Path p	q_p	x_p^*
p_1	0.3940	20.0000
p_2	0.3893	0.0000
p_3	0.3873	0.0000
p_4	0.3858	0.0000
p_5	0.4915	50.0000
p_6	0.4778	0.0000
p_7	0.4662	0.0000
p_8	0.4622	0.0000
p_9	0.5821	13.1918
p_{10}	0.5584	18.7448
p_{11}	0.5383	0.0000
p_{12}	0.5329	28.0624

Table 10: Computed Equilibrium Path Flows for the Case Study Scenario 3

The demand prices of the orchard/farms, in dollars, are:

Apex Orchards:

$$\rho_{11} = 28.01, \quad \rho_{12} = 23.91, \quad \rho_{13} = 29.62, \quad \rho_{14} = 24.35.$$

Park Hill Orchard:

$$\rho_{21} = 24.44, \quad \rho_{22} = 27.72, \quad \rho_{23} = 27.55, \quad \rho_{24} = 27.72,$$

Sentinel Farm:

$$\rho_{31} = 24.02, \quad \rho_{32} = 27.84, \quad \rho_{33} = 25.91, \quad \rho_{34} = 26.78.$$

Since the supply is decreased, the prices of Golden Delicious apples increase in most of the farmers' markets. For example, Apex's Golden Delicious apples are now $28.01 at the Northampton Farmers' Market in this scenario whereas the price was $27.33 in Scenario 1. Additionally, since the quality of Apex's apples is worse than in Scenario 1, this causes the demand price to decrease at the Amherst Farmers' Market. In Scenario 1, the price of its apples at the Amherst Farmers' Market was $30.72; however, now it is $29.62.

The profits of the orchard/farms in this scenario, in dollars, are:

$$U_1(X^*) = 362.15, \quad U_2(X^*) = 498.28, \quad U_3(X^*) = 507.58.$$

The largest profit is achieved by Sentinel, followed by Park Hill, and Apex. In Scenario 1, Apex has the largest profit, which decreases substantially in this scenario due to it having the lowest harvesting capacity and quality.

6. Summary and Conclusions

Fresh produce consists of both fruits and vegetables and such supply chains are especially challenging since the quality of the product deteriorates continuously upon harvesting. At the same time, consumers are demanding fresh products and are increasingly health conscious. Farmers' markets have increased in popularity internationally and, yet, the mathematical modeling associated with such supply chains has been limited.

In this paper, we provide explicit formulae for a variety of fresh produce to capture quality deterioration. We then identify the quality associated with different pathways in supply chain networks. Subsequently, we focus on farmers' markets, which are examples of direct to consumer supply chains and are shorter supply chains since farmers bring the product to markets at which consumers select their purchases. Specifically, we introduce a game theory model for supply chain competition in a network framework for farmers' markets occurring within a period of time, such as a week. The farms are interested in maximizing their profits and the consumers respond to the quality of the product. We provide both qualitative properties of the equilibrium link flow pattern, propose a computational scheme, and also illustrate our framework through numerical examples focused on peaches and then in a case study for Golden Delicious apples and farmers' markets in western Massachusetts. Our framework considers both uncapacitated links in the supply chain network as well as

capacitated ones, which may occur due to crop failures, harvesting problems, labor shortages, etc. This is the first game theory model for farmers' markets and also the first competitive fresh produce supply chain network model in which quality deterioration of fresh produce is explicitly captured.

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