Multicommodity International Agricultural Trade Network Equilibrium

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Abstract: International agricultural trade is essential to the well-being of societies and economies and is a pillar of food security. Such networks, consisting of producers at supply markets and consumers at demand markets connected by transportation routes, expand the variety of commodities for consumers through the reallocation of produced commodities from surplus countries to those with deficits. Various crises and disasters, both sudden-onset and slow-onset ones, are exacerbating challenges associated with international agricultural trade. In this chapter, a multicommodity international agricultural trade network equilibrium model is presented with exchange rates as well as capacities across commodities at the country supply markets, the country demand markets, and the transportation routes to capture a spectrum of contemporary real world scenarios. Such an equilibrium approach provides a valuable benchmark for decision-making and policy-making. The methodology used for the formulation, analysis, and solution of the model is the theory of variational inequalities. The model is related to several studies in the literature. Both illustrative examples as well as algorithmically solved ones, with the latter drawn from Russia's ongoing war on Ukraine, demonstrate the applicability of the framework. Finally, suggestions for future research are given.

Key words: agriculture, supply chains, network equilibrium, international trade, spatial price, capacities, food security, variational inequalities

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Glossary

Exchange rate the price of one currency in terms of another currency.

Equilibrium a state of a system in which the properties don't undergo any other changes.

Spatial price equilibrium a steady state in which perfectly competitive spatially separated supply markets and demand markets in the presence of transportation costs equilibrate in terms of flows and prices.

Commodity a raw material or primary agricultural product that can be purchased and sold, such as fresh produce and grain.

International commodity trade the exchange of commodities across international borders or territories because of need or desire for the commodities.

Agriculture another word for farming. It includes growing and harvesting crops. Agriculture provides the food that humans need to survive.

Variational inequality a mathematical formalism and methodology used in the formulation, analysis, and solution of many equilibrium problems, including game theory problems.

1. Introduction

Trade is essential for the exchange of commodities and takes place on networks consisting of producers at supply markets and consumers at demand markets with transportation routes providing the linkages. Agricultural commodities such as wheat, corn, and other cereals and grains, as well as fresh produce consisting of fruits and vegetables plus other perishables of fish, meat, and dairy are staples of international trade. Such agricultural commodities are also critical to food security because of their nutritional value. Agricultural trade expands the choices of consumers through a greater variety of commodities and enhanced quality by reallocating produced commodities from surplus countries to those countries with deficits. Availability is, thus, increased, and prices lowered.

According to the Food and Agricultural Organization of the United Nations (2023), since 1995, food and agricultural trade has increased more than threefold in nominal terms, with low-income countries importing approximately 14% of their food for consumption and high-income ones depending on global trade for about 60% of their food needs. Boysen-Urban, Ehjeij, and Flaig (2023) note that fruits and vegetables are the most traded commodity group by value at 23%, followed by cereals at 14%, fish and meat at 11% each, and oilseeds at 8%. As also reported therein, some regions of the globe are very dependent on cereal imports, including the Democratic Republic of Congo, Libya, Lebanon, and Gabon, as are countries in Central America and the western part of South America, Japan, and South Korea. In contrast, Ukraine, Canada, Argentina, and Australia are net exporters.

Crises and disasters in the form of droughts, floods, wildfires, and extreme weather events, due, in part, to climate change, as well as man-made disasters such as wars, and the associated devastation on agricultural infrastructure, are exacerbating challenges for the international trade of agricultural commodities. In addition, the COVID-19 pandemic (cf. Choi, 2021), a global healthcare disaster, demonstrated the importance of the resilience of agricultural trade networks as well as their vulnerabilities (see Nagurney, 2021).

Given the importance of agricultural commodities to the health and well-being of our societies and to economic growth, a rich literature has arisen to address the modeling and analysis of agricultural trade networks, accompanied by rigorous algorithmic procedures for the solution of such problems. The foundational work dates to the contributions of Samuelson (1952) and Takayama and Judge (1964, 1971) in the construction of spatial price equilibrium models. The governing equilibrium concept in these models, which assume perfect competition, is that there will be a positive volume of flow of a commodity between a pair of supply and demand markets if the supply price at the supply market plus the unit cost of transportation is equal to the demand price at the demand market. If the supply price plus the unit cost exceeds the demand market price (the price the consumers are willing to pay) then there will be no commodity flow between that pair of supply and demand markets.

Spatial price equilibrium models have had wide application in agricultural trade. For example, as emphasized in Nagurney, Li, and Nagurney (2014), spatial price equilibrium models have been applied to agricultural commodities such as eggs (cf. Judge, 1956), potatoes (Howard, 1984), beef (Sohn, 1970), cereal grains (Ruijs et al., 2001), soybeans (Barraza De La Cruz, Pizzolato, and Barraza de La Cruz, 2010), and dairy (Bishop, Pratt, and Novakovic, 1994). Much of the earlier work, however, assumed separable supply price and demand price functions, which were also usually linear and the unit transportation costs were fixed and not dependent on flow. Separability in this context means that the supply price of a commodity at a supply market depends only on the supply (the amount produced) of the commodity at the supply market; similarly, the demand price at a demand market depends only on the demand for the commodity at the demand market. Such an assumption enables an optimization reformulation of the spatial price equilibrium conditions. This is also possible if the functions admit symmetric Jacobians (see also, e.g., Nagurney, 1999). However, such assumptions preclude greater realism needed for multicommodity trade modeling. Florian and Los (1982) brought the methodology of variational inequality theory to this application domain allowing for the further advancement of such models (see, also, e.g., Dafermos, 1980; Dafermos and Nagurney, 1984; Friesz, Harker, and Tobin, 1984; Harker, 1985; Nagurney, Thore, and Pan, 1996; Nagurney, 1999). For example, recently, spatial price equilibrium models have been constructed using variational inequality theory and applied to international agricultural trade of corn and wheat by Nagurney et al. (2024a) in which the impacts of various disruption scenarios following Russia's full-scale invasion of Ukraine on February 24, 2022 were investigated quantitatively. Furthermore, variational inequality theory enables the modeling of a spectrum of policy interventions, which are very important to international agricultural trade and include: ad valorem tariffs (Nagurney, Nicholson, and Bishop, 1996), tariff rate quotas and, of course, unit tariffs and quotas (see Nagurney, Besik, and Dong, 2019) or even subsidies (see Nagurney and Besedina, 2023), along with price floors and price ceilings (cf. Nagurney, 1999).

Although in some of the spatial price equilibrium models in which there are multiple countries it is assumed that a common currency is used and the data is translated accordingly, recent research has included exchange rates explicitly. Devadoss and Sabala (2020) note that their study is the first spatial price equilibrium model to analyze the effects of exchange rate changes. The authors developed a single commodity spatial price equilibrium model for cotton markets and focused on the yuan-dollar exchange rate. The model could be reformulated as an optimization problem. Nagurney et al. (2023) constructed a multicommodity international trade spatial price equilibrium model of relevance to agriculture in which exchange rates are included as well as policy instruments in the form of tariffs, subsidies, and quotas. In addition, there could be multiple routes joining each pair of supply and demand markets, and a route could consist of one or more links (see also Dafermos and Nagurney, 1984). The governing equilibrium conditions were formulated as a variational inequality problem. The works of Nagurney and Besedina (2023) and Nagurney et al. (2024a, b) also include explicit exchange rates in models of international trade based on spatial price equilibrium.

As emphasized in Nagurney et al. (2023), exchange rates capture the value (price) of one currency relative to another currency. Exchange rates are highly relevant to international trade, impacting the decision-making of governments, businesses, and consumers. A distinct exchange rate exists for each pair of independent currencies, such as the US dollar and the Ukrainian hryvnia, the hryvnia and the euro, the euro and the Japanese yen, etc. Changes in the exchange rate have a direct effect on the prices of goods and services produced in a given country relative to those produced in another country. If the US dollar appreciates, which means that the exchange rate increases, the relative price of domestic goods as well as services increases whereas the relative price of foreign goods and services falls. Such a change in relative prices will decrease exports from the US but will increase its imports. As noted in Nagurney et al. (2023), over the period June 1, 2021, through May 31, 2022, the US dollar increased by 12% against the euro; it increased 9% against the British pound and 16% against the yen. On the other hand, during the first year of the COVID-19 pandemic, the dollar weakened with respect to the euro, the British pound, and the yen.

Given the importance of multicommodity international agricultural trade in a world with many significant challenges including disruptions impacting production as well as transportation of agricultural commodities and, therefore, food security, attention has been directed to the development of mathematical models that are holistic and that include the relevant constraints. Such models, and accompanying algorithmic procedures can provide important managerial insights for decision-making as well as policy-making.

In this chapter, we revisit the multicommodity international trade network equilibrium model of Nagurney et al. (2024a) and provide an extension to allow for a different number of routes between each pair of supply and demand markets as well as bounds on the commodities at the demand markets to reflect possible capacities associated with storage, for example. In addition, the model includes upper bounds associated with the production of agricultural commodities at the supply markets as well as upper bounds on the transportation of commodities on routes between the supply and demand markets. Such a modeling framework allows for the investigation of numerous real world scenarios from the impacts of the drought on the Panama Canal, which has reduced the flow of commodities on this critical transportation link, to the exploration of alternative routes for transportation of grain from Ukraine in wartime, along with which grains and cereals and oilseeds should be planted under restricted land availability due to the mining and destruction of arable lands, and even storage facilities (see, also, e.g., Nagurney et al., 2024b). We note that the variational inequality model in Nagurney et al. (2024b) includes capacities on supply, transportation, and demand, but these are commodity-specific and not market- and route-specific as in the model in this chapter. We also highlight important definitions in Section 2 and provide variational inequality formulations of the governing equilibrium conditions plus we relate the model to others in the literature. In Section 3, a spectrum of illustrative examples is presented to demonstrate the insights that can be gained from the framework. Section 4 recalls an algorithm and provides explicit closed form expressions for the commodity shipment variables and the Lagrange multipliers at a given iteration for the solution of the international agricultural trade network equilibrium model. Section 5, for completeness, provides results for several numerical examples drawn from contemporary real world scenarios. Section 6 presents the conclusions and takeaways.

2. The Multicommodity International Agricultural Trade Network Equilibrium Model with Supply, Transportation, and Demand Capacities

The model is now presented. It is a generalization of the model in Nagurney et al. (2024a) to allow for a different number of routes between each pair of country supply and demand markets as well as capacities at the demand markets. The notation is also modified. The international trade network is depicted in Figure 1 and consists of m countries that are supply markets where the K agricultural commodities are produced and n countries where the commodities are consumed. A typical supply market is denoted by i, a typical demand market by j, and a typical commodity by k. Each pair (i, j) of country supply and demand markets is connected by n_{ij} routes with a typical route denoted by r for a total of P routes in the trade network. We note that a route r associated with distinct market pairs need not be the same one. In addition, we associate with each pair of country supply and demand markets (i, j) an exchange rate e_{ij} for $i = 1, \ldots, m$; $j = 1, \ldots, n$. Of course, if the country of the supply market is the same as the country of the demand market, then the corresponding exchange rate is set equal to 1.

Let s_i^k represent the supply of commodity k produced at country supply market i with all the commodity

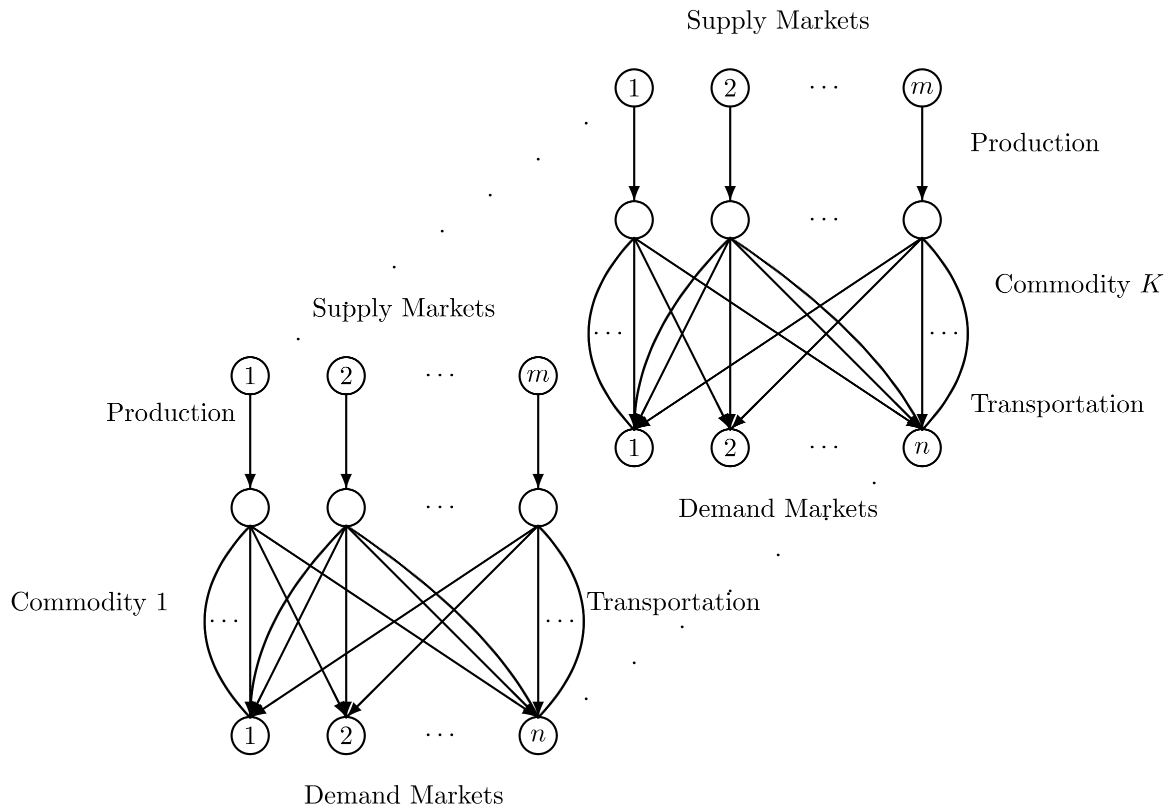

Figure 1: The Multicommodity International Trade Network

supplies grouped into the vector $s \in R_+^{Km}$. The demand for commodity k at country demand market j, in turn, is denoted by d_j^k, with all the commodity demands grouped into the vector $d \in R_+^{Kn}$. Note that the products are homogeneous and that the model assumes perfect competition, which is standard in spatial price equilibrium modeling of agricultural applications. All vectors are assumed to be column vectors.

Let Q_{ijr}^k denote the amount of commodity k produced at country supply market i and transported on route r to country demand market j. The commodity shipments are grouped into the vector $Q \in R_+^{KP}$.

The Conservation of Flow Equations

The conservation of flow equations are:

$$s_i^k = \sum_{j=1}^{n} \sum_{r=1}^{n_{ij}} Q_{ijr}^k, \quad k = 1, \dots, K; i = 1, \dots, m, \tag{1}$$

$$d_j^k = \sum_{i=1}^{m} \sum_{r=1}^{n_{ij}} Q_{ijr}^k, \quad k = 1, \dots, K; j = 1, \dots, n. \tag{2}$$

According to (1), the supply of a commodity produced at a country supply market is equal to the shipments of the commodity to all the country demand markets on the transportation routes.

The equations in (2) state that the demand for each commodity at each country demand market must be equal to the commodity shipments from all the commodity supply markets on the transportation routes.

Furthermore, the commodity shipments must be nonnegative; that is:

$$Q_{ijr}^k \geq 0, \quad k = 1, \dots, K; i = 1, \dots, m; j = 1, \dots, n; r = 1, \dots, n_{ij}. \tag{3}$$

The Constraints

Here, $\bar{Q}_{ijr}$ denotes the transportation capacity of route r between country supply market i and country demand market j, for all i, j, r. Usually, in agricultural trade, the units of flow for the commodities are in tons, which we also assume here.

Let $\bar{S}_i$ denote the supply (production) capacity of country supply market i across all the commodities. The below supply capacity constraints must be met:

$$\sum_{k=1}^{K} s_i^k \leq \bar{S}_i, \quad i = 1, \ldots, m. \tag{4a}$$

Due to the conservation of flow equations (1), constraints (4a) can be re-expressed as:

$$\sum_{k=1}^{K} \sum_{j=1}^{n} \sum_{r=1}^{n_{ij}} Q_{ijr}^k \leq \bar{S}_i, \quad i = 1, \ldots, m. \tag{4b}$$

Expressions (4a) or (4b) guarantee that a country supply market i cannot exceed its aggregate supply capacity. These constraints capture competition among commodities that farmers plant due to the capacities on the supplies (production outputs) at the country supply markets.

The transportation capacity constraints are:

$$\sum_{k=1}^{K} Q_{ijr}^k \leq \bar{Q}_{ijr}, \quad i = 1, \ldots, m; j = 1, \ldots, n, r = 1, \ldots, n_{ij}. \tag{5}$$

The constraints in (5) guarantee that the sum of commodity shipments from country supply market i on route r to country demand market j cannot exceed the shipment capacity of that route. Expressions in (5) enable the modeling of competition among the commodities for transportation services along particular routes.

In addition, here we allow for capacities on demand at the demand markets across the commodities of the following form:

$$\sum_{k=1}^{K} \sum_{i=1}^{m} \sum_{r=1}^{n_{ij}} Q_{ijr}^k \leq \bar{D}_j, \quad j = 1, \ldots, n. \tag{6}$$

A country may be restricted in terms of commodity storage facilities, for example.

The Multicommodity Functions

The country supply price functions π_i^k, for all k, i, are:

$$\pi_i^k = \pi_i^k(s), \quad k = 1, \ldots, K; i = 1, \ldots, m. \tag{7a}$$

Observe that, due to the conservation of flow equations (1), one may define country supply price functions $\tilde{\pi}_i^k$, for all k, i, such that:

$$\tilde{\pi}_i^k(Q) \equiv \pi_i^k(s), \quad k = 1, \ldots, K; i = 1, \ldots, m. \tag{7b}$$

According to (7a) or (7b), the supply price of a commodity at a country supply market can, in general, be a function not only of the supply of the commodity in the country (the amount produced) but also of the supplies of other commodities in the country as well as the supplies of the commodities in all other countries.

The demand price of a commodity k in country j, ρ_j^k, in turn, can depend on the entire vector of demands of the commodities:

$$\rho_j^k = \rho_j^k(d), \quad k = 1, \ldots, K; j = 1, \ldots, n. \tag{8a}$$

Similarly, due to (2), one may define new country demand price functions $\tilde{\rho}_j^k$, for all k, j, such that:

$$\tilde{\rho}_j^k(Q) \equiv \rho_j^k(d), \quad k = 1, \ldots, K; j = 1, \ldots, n. \tag{8b}$$

The unit transportation cost associated with transporting commodity k from country supply market i to country demand market j on transportation route r is denoted by c_{ijr}^k, where:

$$c_{ijr}^k = c_{ijr}^k(Q), \quad k = 1, \ldots, K; i = 1, \ldots, m; j = 1, \ldots, n; r = 1, \ldots, n_{ij}. \tag{9}$$

It is important to recognize that, according to (9), the unit transportation cost associated with a commodity can depend on the vector of commodity shipments between all pairs of country supply and demand markets. These functions are quite general and also allow for the modeling of competition for transportation services among commodities.

Here it is assumed that the multicommodity supply price, the demand price, as well as the unit transportation cost functions are all continuous.

We introduce Lagrange multipliers: μ_i; $i = 1, \ldots, m$, λ_{ijr}; $i = 1, \ldots, m$; $j = 1, \ldots, n$; $r = 1, \ldots, n_{ij}$, and γ_j; $j = 1, \ldots, n$, associated with the capacity constraints (4b), (5), and (6), respectively. We then group these Lagrange multipliers into the vectors: $\mu \in R_+^m$, $\lambda \in R_+^P$ and $\gamma \in R_+^n$, respectively. Lagrange multipliers have an interpretation of being "shadow prices" and a problem, once solved, yields valuable information through their numerical values.

The Equilibrium Conditions

The multicommodity international agricultural trade network equilibrium conditions are now stated.

Definition 1: The Multicommodity International Agricultural Trade Network Equilibrium Conditions Under Supply, Transportation, and Demand Capacities

A multicommodity shipment and Lagrange multiplier pattern $(Q^*, \mu^*, \lambda^*, \gamma^*) \in \mathcal{K}^1$, *where*

$$\mathcal{K}^1 \equiv \{(Q, \mu, \lambda, \gamma) | (Q, \mu, \lambda, \gamma) \in R_+^{KP+m+P+n}\}$$

is a multicommodity international agricultural trade network equilibrium with exchange rates, under supply, transportation, and demand capacities, if the following conditions hold: For all commodities k; $k = 1, \ldots, K$; *for all country supply and demand market pairs:* (i, j); $i = 1, \ldots, m$; $j = 1, \ldots, n$, *and for all routes* r; $r = 1, \ldots, n_{ij}$:

$$(\tilde{\pi}_i^k(Q^*) + c_{ijr}^k(Q^*))e_{ij} + \mu_i^* + \lambda_{ijr}^* + \gamma_j^* \begin{cases} = \tilde{\rho}_j^k(Q^*), & \textit{if } Q_{ijr}^{k*} > 0, \\ \geq \tilde{\rho}_j^k(Q^*), & \textit{if } Q_{ijr}^{k*} = 0; \end{cases} \tag{10}$$

also, for all country supply markets i; $i = 1, \ldots, m$:

$$\mu_i^* \begin{cases} \geq 0, & \textit{if } \sum_{k=1}^K \sum_{j=1}^n \sum_{r=1}^{n_{ij}} Q_{ijr}^{k*} = \bar{S}_i, \\ = 0, & \textit{if } \sum_{k=1}^K \sum_{j=1}^n \sum_{r=1}^{n_{ij}} Q_{ijr}^{k*} < \bar{S}_i, \end{cases} \tag{11}$$

and for all country market pairs (i,j)*;* $i=1,\ldots,m$*;* $j=1,\ldots,n$*, and all routes* r*;* $r=1,\ldots,n_{ij}$*:*

$$\lambda^*_{ijr} \begin{cases} \geq 0, & \textit{if } \sum_{k=1}^{K} Q^{k*}_{ijr} = \bar{Q}_{ijr}, \\ = 0, & \textit{if } \sum_{k=1}^{K} Q^{k*}_{ijr} < \bar{Q}_{ijr}, \end{cases} \tag{12}$$

plus for all country demand markets j*;* $j=1,\ldots,n$*:*

$$\gamma^*_j \begin{cases} \geq 0, & \textit{if } \sum_{k=1}^{K}\sum_{i=1}^{m}\sum_{r=1}^{n_{ij}} Q^{k*}_{ijr} = \bar{D}_j, \\ = 0, & \textit{if } \sum_{k=1}^{K}\sum_{i=1}^{m}\sum_{r=1}^{n_{ij}} Q^{k*}_{ijr} < \bar{D}_j. \end{cases} \tag{13}$$

The equilibrium conditions (10), along with the Lagrange multipliers corresponding to capacity constraints in (11), (12), and (13), expand the classical spatial price equilibrium conditions of Samuelson (1952) and Takayama and Judge (1971) to include exchange rates, and supply, transportation, and demand capacities. Furthermore, the underlying multicommodity supply price, demand price, and unit transportation cost functions in this international trade model need not be separable (nor symmetric), and the unit transportation cost functions are flow-dependent. If the demand capacity constraints are removed and the number of paths is equal to P for each pair of supply and demand markets, then the above model and accompanying equilibrium conditions collapse to the model constructed in Nagurney et al. (2024a). In addition, the above model is related to the one introduced by Nagurney and Besedina (2023) that focused on policy measures in the form of non-tariff ones in that there are multiple commodities and exchange rates. However, in the earlier paper, although there are exchange rates, there is only a single route between each pair of supply and demand markets. Plus, there are no supply constraints and no demand constraints but there are constraints to correspond to quotas on individual commodities on individual routes as well as quality variables and opportunity costs.

We now provide an interpretation of the equilibrium conditions (10) through (13). The multicommodity international agricultural trade network equilibrium conditions (10) through (13) state that, if there is a positive flow of a commodity on a route between a pair of country supply and demand markets, and the route is not at its capacity, and the supply at the country supply market is not at its capacity, and neither is the country demand market, then the supply price of the commodity at the country supply market plus the unit transportation cost associated with transporting the commodity on the route, multiplied by the exchange rate between the two countries is equal to the demand price of the commodity at the country demand market. On the other hand, if the route is at its capacity, or the supply is at its capacity at the country supply market, or the demand at the demand market is at its capacity, and the flow of the commodity on a route is positive, then the demand price of the commodity at the country demand market is greater than or equal to the sum of commodity supply price and its unit transportation cost multiplied by the applicable exchange rate, with the sum of the corresponding Lagrange multipliers equal to the nonnegative difference. If there is no flow of a commodity on a route, then the country demand market price of the commodity is less than or equal to the country supply market price plus the unit transportation cost multiplied by the appropriate exchange rate plus the Lagrange multipliers.

We remark that, to the above equilibrium conditions, it is easy to add unit tariffs, ad valorem tariffs, and/or subsidies as done in Nagurney and Besedina (2023).

The concept of equilibrium is a very powerful one and serves as a baseline from which to ascertain the performance of a system. Equilibrium also generalizes optimization since the former necessarily involves not just a single decision-maker. Equilibrium problems arise in numerous disciplines, including the sciences, social sciences, as well as the management sciences and operations research, and, hence, methodologies such as that of variational inequality theory, applied widely for the formulation, analysis, and computation of solutions to equilibrium problems, are very valuable and relevant.

We now present a variational inequality formulation of the above equilibrium conditions in a theorem, the proof of which follows using similar arguments as in Theorem 1 in Nagurney et al. (2024a).

Theorem 1: Variational Inequality Formulation of the Multicommodity International Agricultural Trade Network Equilibrium Conditions Under Supply, Transportation, and Demand Capacities

A multicommodity shipment and Lagrange multiplier pattern $(Q^*, \mu^*, \lambda^*, \gamma^*) \in \mathcal{K}^1$ is a multicommodity international agricultural trade network equilibrium with exchange rates, under supply, transportation, and demand capacities, according to Definition 1, if and only if it satisfies the variational inequality:

$$\sum_{k=1}^{K}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{n_{ij}}\Big[(\tilde{\pi}_i^k(Q^*)+c_{ijr}^k(Q^*))e_{ij}+\mu_i^*+\lambda_{ijr}^*+\gamma_j^*-\tilde{\rho}_j^k(Q^*)\Big]\times(Q_{ijr}^k-Q_{ijr}^{k*})$$
$$+\sum_{i=1}^{m}\Big[\bar{S}_i-\sum_{k=1}^{K}\sum_{j=1}^{n}\sum_{r=1}^{n_{ij}}Q_{ijr}^{k*}\Big]\times(\mu_i-\mu_i^*)+\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{n_{ij}}\Big[\bar{Q}_{ijr}-\sum_{k=1}^{K}Q_{ijr}^{k*}\Big]\times(\lambda_{ijr}-\lambda_{ijr}^*)$$
$$+\sum_{j=1}^{n}\Big[\bar{D}_j-\sum_{k=1}^{K}\sum_{i=1}^{m}\sum_{r=1}^{n_{ij}}Q_{ijr}^{k*}\Big]\times(\gamma_j-\gamma_j^*)\geq 0,\quad \forall (Q,\mu,\lambda,\gamma)\in\mathcal{K}^1. \tag{14}$$

We now provide an alternative variational inequality to the one in (14). We define the feasible set: $\mathcal{K}^2 \equiv \{Q|Q \in R_+^{KP} \text{ and } (4b), (5), \text{ and } (6) \text{ hold}\}$. Then, using similar arguments as in the proof of Theorem 2 in Nagurney et al. (2024a), the following theorem holds true.

Theorem 2: Alternative Variational Inequality Formulation of the Multicommodity International Agricultural Trade Network Equilibrium Conditions Under Supply, Transportation, and Demand Capacities

A multicommodity shipment pattern $Q^{**} \in \mathcal{K}^2$ is a multicommodity international agricultural trade network equilibrium with exchange rates, under supply, transportation, and demand capacities, according to Definition 1, if and only if it satisfies the variational inequality:

$$\sum_{k=1}^{K}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{n_{ij}}\Big[(\tilde{\pi}_i^k(Q^{**})+c_{ijr}^k(Q^{**}))e_{ij}-\tilde{\rho}_j^k(Q^{**})\Big]\times(Q_{ijr}^k-Q_{ijr}^{k**})\geq 0,\quad \forall Q\in\mathcal{K}^2, \tag{15}$$

where μ_i^{**} is the Lagrange multiplier associated with constraint (4b), for all i; λ_{ijr}^{**} is the Lagrange multiplier associated with constraint (5), for all i, j, r, and γ_j^{**} is the Lagrange multiplier associated with constraint (6), for all j.

In addition, we provide a variational inequality formulation of the international trade network equilibrium conditions in the absence of capacity constraints (4b), (5), and (6), in which case only the nonnegativity constraints (3) are imposed. We define the feasible set $\mathcal{K}^3 \equiv \{Q| \in R_+^{KP}\}$ and have the following theorem.

Theorem 3: Variational Inequality Formulation of the Multicommodity International Agricultural Trade Network Equilibrium Conditions Under Only Commodity Nonnegativity Constraints

A multicommodity shipment pattern $Q^{***} \in \mathcal{K}^3$ is a multicommodity international agricultural trade network equilibrium with exchange rates, under only commodity nonnegativity constraints, if and only if it satisfies the variational inequality:

$$\sum_{k=1}^{K}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{n_{ij}}\Big[(\tilde{\pi}_i^k(Q^{**})+c_{ijr}^k(Q^{**}))e_{ij}-\tilde{\rho}_j^k(Q^{**})\Big]\times(Q_{ijr}^k-Q_{ijr}^{k**})\geq 0,\quad \forall Q\in\mathcal{K}^3. \tag{16}$$

Each of the above variational inequalities (14), (15), and (16) can be put into standard form (cf. Nagurney, 1999), VI$(F,\mathcal{K})$, where one seeks to determine a vector $X^*\in\mathcal{K}\subset R^{\mathcal{N}}$, such that

$$\langle F(X^*), X-X^*\rangle \geq 0,\quad \forall X\in\mathcal{K}, \tag{17}$$

with F being a given continuous function from $\mathcal{K}$ to $R^{\mathcal{N}}$, where $\mathcal{K}$ is a given closed, convex set, and $\langle\cdot,\cdot\rangle$ denotes the inner product in $\mathcal{N}$-dimensional Euclidean space.

For example, we now put variational inequality (14) into standard form (17). We define $X\equiv(Q,\mu,\lambda,\gamma)$, $\mathcal{K}\equiv\mathcal{K}^1$, and $\mathcal{N}\equiv KP+m+P+n$ plus $F(X)\equiv(F_1(X),F_2(X),F_3(X),F_4(X))$ where $F_1(X)$ consists of the elements: $\big[(\tilde{\pi}_i^k(Q)+c_{ijr}^k(Q))e_{ij}+\mu_i+\lambda_{ijr}+\gamma_j-\tilde{\rho}_j^k(Q)\big],\forall k,i,j,r$. The elements of $F_2(X)$ are: $\big[\bar{S}_i-\sum_{k=1}^K\sum_{j=1}^n\sum_{r=1}^{n_{ij}}Q_{ijr}^k\big],\forall i$. The elements of $F_3(X)$ are: $\big[\bar{Q}_{ijr}-\sum_{k=1}^K Q_{ijr}^k\big],\forall i,j,r$ and those of $F_4(X)$ are: $\big[\bar{D}_j-\sum_{k=1}^K\sum_{i=1}^m\sum_{r=1}^{n_{ij}}Q_{ijr}^k\big],\forall j$.

It is important to mention that qualitative properties of existence and uniqueness of a solution to a variational inequality problem can be obtained under appropriate monotonicity conditions on the function $F(X)$ that enters the variational inequality (17). However, if $\mathcal{K}$ is compact and $F(X)$ is continuous then existence of a solution X^* is guaranteed. Uniqueness of X^* then follows solely from $F(X)$ being strictly monotone. If $\mathcal{K}$ is not compact, but $F(X)$ is strongly monotone, then a solution X^* exists and is unique. For additional background on qualitative properties of variational inequalities, see Kinderlehrer and Stampacchia (1980) and Nagurney (1999).

The theory of variational inequalities has been applied to a wide range of supply chain problems, traffic network problems, oligopolistic market equilibrium problems operating under imperfect competition, and even knowledge networks and problems of human migration. We refer the interested reader to the books by Nagurney (1999, 2006, 2023), Nagurney et al. (2013), Nagurney and Li (2016), and the references therein.

3. Illustrative Examples

In order to fix ideas, a series of illustrative examples is now presented. The examples consist of a single country supply market and a single country demand market and two commodities. There is a single route joining the supply market with the demand market. Example 1 serves as a baseline. Example 2 has the same data as Example 1 but with a lower exchange rate whereas Example 3 also has the same data as Example 1 but with a higher exchange rate. Please refer to Figure 2.

The multicommodity country supply price functions are:

$$\pi_1^1(s)=s_1^1+5,\quad \pi_1^2(s)=5s_1^2+5.$$

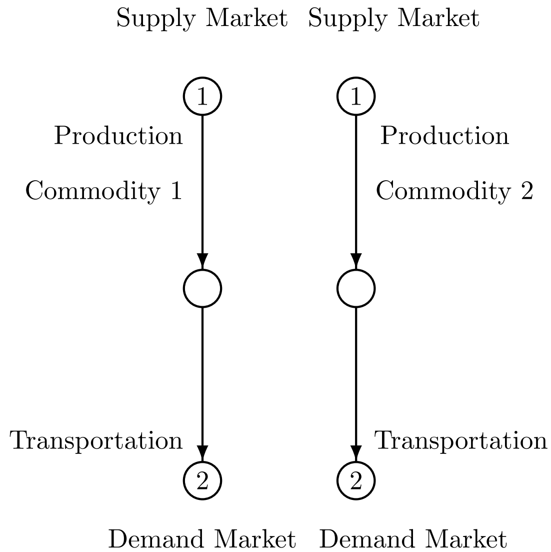

Figure 2: The International Trade Network Topology for the Illustrative Examples

The multicommodity unit transportation costs are:

$$c^1_{121}(Q) = Q^1_{121} + 2, \quad c^2_{121}(Q) = Q^2_{121} + 1$$

and the multicommodity country demand price functions are:

$$\rho^1_2(d) = -d^1_2 + 37, \quad \rho^2_2(d) = -d^2_2 + 20.$$

The capacities are: $\bar{S}_1 = \bar{Q}_{111} = \bar{D}_1 = 15.00$.

Example 1

In Example 1 we assume that the exchange rate $e_{12} = 1.00$.

By applying the equilibrium conditions (10), we obtain two individual equations, whose solution yields:

$$Q^{1*}_{121} = 10.00, \quad Q^{2*}_{121} = 2.00,$$

and, of course, we have that: $s^{1*}_1 = 10.00$ and $d^{1*}_2 = 10.00$ plus $s^{2*}_1 = 2.00$ and $d^{2*}_2 = 2.00$, where $\pi^1_1 = 15.00$, $c^1_{121} = 12.00$, and $\rho^1_2 = 27.00$ and $\pi^2_1 = 15.00$, $c^2_{121} = 3.00$, and $\rho^2_2 = 18.00$. All the Lagrange multipliers are equal to 0.00.

Example 2

In Example 2, we set $e_{12} = .8$. Solution of the equilibrium conditions yields:

$$s^{1*}_1 = d^{1*}_2 = Q^{1*}_{121} = 12.08, \quad s^{2*}_1 = d^{2*}_2 = Q^{2*}_{121} = 2.62,$$

with all the Lagrange multipliers, again, being equal to 0.00.

As expected, with a lower exchange rate, the supplies and exports of the commodities increase as do the demands.

Example 3

In Example 3, the exchange rate is now raised so that $e_{12} = 1.2$. The equilibrium solution is now:

$$s_1^{1*} = d_2^{1*} = Q_{121}^{1*} = 8.41, \quad s_1^{2*} = d_2^{2*} = Q_{121}^{2*} = 1.56,$$

with the Lagrange multipliers, again, being equal to 0.00. With the higher exchange rate, the multicommodity equilibrium shipments decrease, as do their supplies and demands.

4. The Algorithm

Different variational inequality formulations may suggest different algorithmic schemes. Typically, a variational inequality algorithm resolves the problem into a series of optimization problems. Exploitation of underlying problem network structure, if it exists, may enhance computational efficiency. It is important to apply an appropriate algorithm to a given variational inequality formulation with the understanding of the assumptions needed for convergence. A variety of variational inequality algorithms as well as applications including to supply chain networks as well as to spatial economics and trade can be found in the books by Nagurney (1999, 2006, 2023) and Nagurney and Li (2016).

A class of variational inequality algorithms are projection methods. The modified projection method of Korpelevich (1977), in particular, yields closed form expressions at each iteration, when applied to solve the international agricultural trade network equilibrium problem with variational inequality formation (14). The convergence of this algorithm is guaranteed if the function $F(X)$ that enters the variational inequality problem (14) is monotone and Lipschitz continuous.

The function $F(X)$ is said to be monotone if

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle \geq 0, \quad \forall X^1, X^2 \in \mathcal{K}. \tag{18}$$

$F(X)$ is Lipschitz continuous, if there exists a Lipschitz constant, $\eta > 0$, such that

$$\|F(X^1) - F(X^2)\| \leq \eta \|X^1 - X^2\|, \quad \forall X^1, X^2 \in \mathcal{K}. \tag{19}$$

The steps of the modified projection method are now recalled for ease of reference, with τ denoting an iteration counter. Subsequently, the closed-form expressions of the algorithm, when applied to solve variational inequality (14), for the commodity shipments and the Lagrange multipliers at each iteration are presented.

The Modified Projection Method

Step 0: Initialization

Initialize with $X^0 \in \mathcal{K}$. Set the iteration counter $\tau = 1$ and let β be a scalar such that $0 < \beta \leq \frac{1}{\eta}$, where η is the Lipschitz constant.

Step 1: Computation

Compute $\bar{X}^\tau$ by solving the variational inequality subproblem:

$$\langle \bar{X}^\tau + \beta F(X^{\tau-1}) - X^{\tau-1}, X - \bar{X}^\tau \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{20}$$

Step 2: Adaptation

Compute X^τ by solving the variational inequality subproblem:

$$\langle X^\tau + \beta F(\bar{X}^\tau) - X^{\tau-1}, X - X^\tau \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{21}$$

Step 3: Convergence Verification

If $|X^\tau - X^{\tau-1}| \leq \epsilon$, with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $\tau := \tau + 1$ and go to Step 1.

Due to the structure of the feasible set $\mathcal{K}^1$ underlying the multicommodity international agricultural trade network equilibrium model with supply, transportation, and demand capacities, the solution of each of the subproblems in (20) and (21) can be obtained via closed-form expressions, as in the explicit statements below.

Explicit Formulae at Iteration τ for the Multicommodity Shipments in Step 1

The closed-form expressions for the multicommodity shipments in (20) for the solution of variational inequality (14) are:

$$\bar{Q}^{k\tau}_{ijr} = \max\{0, Q^{k\tau-1}_{ijr} + \beta(\tilde{\rho}^k_j(Q^{\tau-1}) - (\tilde{\pi}^k_i(Q^{\tau-1}) + c^k_{ijr}(Q^{\tau-1}))e_{ij} - \mu^{\tau-1}_i - \lambda^{\tau-1}_{ijr} - \gamma^{\tau-1}_j)\},$$
$$\forall k, i, j, r. \tag{22}$$

Explicit Formulae at Iteration τ for the Supply Capacity Lagrange Multipliers in Step 1

The closed-form expressions for the production capacity Lagrange multipliers in (20) for variational inequality (14) are:

$$\bar{\mu}^\tau_i = \max\{0, \mu^{\tau-1}_i + \beta(\sum_{k=1}^K \sum_{j=1}^n \sum_{r=1}^{n_{ij}} Q^{k\tau-1}_{ijr} - \bar{S}_i)\}, \quad \forall i. \tag{23}$$

Explicit Formulae at Iteration τ for the Transportation Capacity Lagrange Multipliers in Step 1

The closed-form expressions for the transportation capacity Lagrange multipliers for (20) for variational inequality (14) are:

$$\bar{\lambda}^\tau_{ijr} = \max\{0, \lambda^{\tau-1}_{ijr} + \beta(\sum_{k=1}^K Q^{k\tau-1}_{ijr} - \bar{Q}_{ijr})\}, \quad \forall i, j, r. \tag{24}$$

Explicit Formulae at Iteration τ for the Demand Capacity Lagrange Multipliers in Step 1

The closed-form expressions for the demand capacity Lagrange multipliers in (20) for variational inequality (14) are:

$$\bar{\gamma}^\tau_j = \max\{0, \gamma^{\tau-1}_j + \beta(\sum_{k=1}^K \sum_{i=1}^m \sum_{r=1}^{n_{ij}} Q^{k\tau-1}_{ijr} - \bar{D}_j)\}, \quad \forall i. \tag{25}$$

The analogous explicit formulae for the commodity shipment and Lagrange multiplier variables in (21) in Step 2 readily follow.

5. Additional Numerical Examples Based on Real World Scenarios

In this Section, we reprise several examples, previously reported in Nagurney et al. (2024a), that are solved using the algorithm in Section 4. The example data are presented using the notation in this chapter. In the examples there are two commodities: wheat and corn. The country supply market is Ukraine and the country demand markets are Lebanon and Egypt. Lebanon and Egypt are Middle Eastern North African (MENA) countries that rely heavily on cereal grains to feed their populations. The trade network topology for Example 4 is depicted in Figure 3 and that for Example 5 in Figure 4. In Example 4 there are two routes from Ukraine to each of the demand country markets, with the first transportation route representing the export through a Black Sea port in Ukraine, such as that of Odesa, and the second route denoting the transportation of grains via barge, rail, or truck through the western borders of Ukraine to Romania, and then from a Romanian port on the Black Sea, such as the port of Costanza. Further details can be found in Nagurney et al. (2024a). In Example 5, the maritime routes are no longer available.

The solution of the international trade model yields the equilibrium values of the commodity shipments and the Lagrange multipliers at given exchange rates between the country supply and demand markets. The exchange rates, hence, are assumed to be fixed in each of the below numerical examples. The local currency codes are UAH for Ukrainian hryvnia, LBP for the Lebanese pound, EGP for the Egyptian pound, and USD for the United States dollar. Here, the superscript $k = 1$ denotes wheat, and corn is denoted by superscript $k = 2$. The time horizon for each example is a single year and the unit for the commodity shipments is tons with prices and costs also associated with a ton of the specific commodity. The modified projection method was implemented in FORTRAN on a Linux system at the University of Massachusetts Amherst. The modified projection method was deemed to have converged if the absolute value of each computed variable at two successive iterations differed by no more than 10^{-2}.

Example 4 - Scenario Prior to the Full-Scale Invasion of February 24, 2022

Example 4 focuses on the scenario prior to Russia's full-scale invasion of Ukraine on February 24, 2022 when almost all of the grain produced in Ukraine was exported through their Black Sea ports. The exchange rates are from early January 2022, prior to the full-scale invasion. The exchange rates are:

$$e_{12} = 55.0581, \quad e_{13} = .5714,$$

$$USD/UAH = 27.4619, \quad USD/LBP = 1,512.0000, \quad USD/EGP = 15.7300.$$

The supply price functions for wheat and corn per ton in Ukrainian hryvnia are:

$$\pi_1^1(s) = .000136s_1^1 + .000068s_1^2 + 7,001.60, \quad \pi_1^2(s) = .000073s_1^1 + .000142s_1^2 + 6,728.20.$$

The unit transportation cost functions for wheat and corn per ton in Ukrainian hryvnia are:

$$c_{121}^1(Q) = .000556Q_{121}^1 + 2,046.80, \quad c_{122}^1(Q) = .007512Q_{122}^1 + 10,984.60,$$

$$c_{131}^1(Q) = .000185Q_{131}^1 + 2,046.80, \quad c_{132}^1(Q) = .007312Q_{132}^1 + 10,984.60,$$

$$c_{121}^2(Q) = .005566Q_{121}^2 + 2,046.80, \quad c_{122}^2(Q) = .006812Q_{122}^2 + 10,984.60,$$

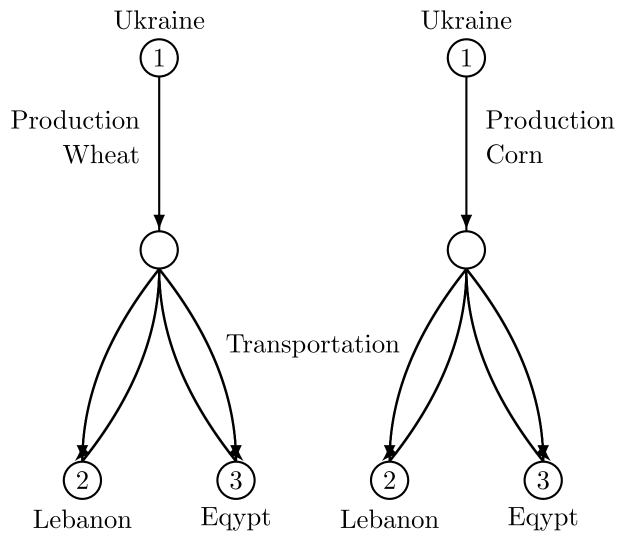

Figure 3: The International Trade Network Topology for Example 4

$$c^2_{131}(Q) = .001259Q^2_{131} + 2,046.80, \quad c^2_{132}(Q) = .007012Q^2_{132} + 10,984.60.$$

The demand price functions for wheat and corn in local currencies are:

$$\rho^1_1(d) = -.15d^1_1 + 602,344.00, \quad \rho^2_1(d) = -.68d^2_1 + 574,560.00,$$

$$\rho^1_2(d) = -.000475d^1_2 + 6,290.00, \quad \rho^2_2(d) = -.000758d^2_2 + 5,980.00.$$

The supply capacity, in tons, in Ukraine is: $\bar{S}_1 = 5,000,000.00$. The transportation capacities, in tons, are:

$$\bar{Q}_{121} = 5,000,000.00, \quad \bar{Q}_{122} = 500,000.00, \quad \bar{Q}_{131} = 5,000,000.00, \quad \bar{Q}_{132} = 500,000.00.$$

Since both the governments in the country demand markets rely on grains to feed their populations, we set the demand capacity to a very high value.

The modified projection method yields the following equilibrium commodity shipment pattern:

$$Q^{1*}_{121} = 477,085.5938, \quad Q^{1*}_{131} = 1,605,672.5000, \quad Q^{1*}_{122} = 0.0000, \quad Q^{1*}_{132} = 0.0000,$$

$$Q^{2*}_{121} = 79,128.0781, \quad Q^{2*}_{131} = 560,130.3750, \quad Q^{2*}_{122} = 0.0000, \quad Q^{2*}_{132} = 0.0000.$$

As noted in Nagurney et al. (2024a), this computed equilibrium commodity shipment pattern is quite close to Ukraine's actual wheat and corn exports to Lebanon and Egypt in 2021 as well as the projected amounts in 2022. Of course, these values are under the assumption that the full-scale invasion would not have occurred. Lebanon, typically, imports more than 70% of its wheat and about 20% of its corn from Ukraine. The analogous percentages for Egypt are 25% and 5%, for wheat and corn, respectively (IndexMundi, 2022a,b; TrendEconomy, 2022a,b). Ukraine's wheat exports to Lebanon were at 520,000 tons in 2021, with an even larger amount of exports expected for 2022 (cf. Hamdan, 2022).

The equilibrium commodity supplies are: $s_1^{1*} = 2,082,758.1250$, $s_1^{2*} = 639,258.4375$ whereas the equilibrium commodity demands are:

$$d_2^{1*} = 477,085.5938, \quad d_2^{2*} = 79,128.0781, \quad d_3^{1*} = 1,605,672.5000, \quad d_3^{2*} = 560,130.3750.$$

The incurred supply prices of the wheat and corn in Ukraine in hryvnia at the equilibrium are:

$$\pi_1^1(s^*) = 7,328.3252 = \$266.8542, \quad \pi_1^2(s^*) = 6,971.0166 = \$253.8432.$$

As recognized in Nagurney et al. (2024a), prior to February 2022, Ukrainian farmers could earn about \$270 per ton for wheat and corn (Associated Press, 2022) and the above computed values are very close; see also Martyshev, Nivievskyi, and Bogonos (2023).

The demand prices at the equilibrium in Lebanon in Lebanese pounds are:

$$\rho_1^1(d^*) = 530,781.1875 = \$351.0457, \quad \rho_1^2(d^*) = 520,752.9063 = \$344.4132,$$

whereas the corresponding demand prices in Egypt in Egyptian pounds are:

$$\rho_2^1(d^*) = 5,527.3057 = \$351.3862, \quad \rho_2^2(d^*) = 5,555.4214 = \$353.1736.$$

These demand prices in Lebanon and Egypt are also very close to the actual prices reported pre-war (cf. Breisinger et al., 2022; El Safty, 2022; Hamdan, 2022).

All the Lagrange multipliers are equal to: 0.0000 at the equilibrium, since the supply, transportation, and demand capacities are greater than the corresponding flows and, hence, the constraints are not tight. Observe, and this is important and very relevant to practice, that only the maritime routes have positive commodity flows.

Example 5: Early Period After the Full-Scale Invasion of February 24, 2022

We now consider the following major disruption, which is actually a man-made disaster. The time period considered is after the full-scale invasion of February 24, 2022, but prior to the Black Sea Grain Initiative of July 2023, which, we might add, that Russia withdrew from the following year. During this extremely challenging period, the Black Sea maritime routes were mined by the Russians and also blockaded, reducing the capacity of these maritime routes to essentially zero because of the dangerous conditions.

Example 5 has the same data as that in Example 4, except that the maritime route links are no longer available. We retain the same superscripts and subscripts as in Example 4 but note that for each pair of supply and demand country market pairs, there is only route 2 available for the transportation of the wheat and corn to Lebanon and Egypt. The network topology for Example 5 with only a single route available for shipment of each commodity from Ukraine to each country demand market is depicted in Figure 4.

The modified projection method computes the following equilibrium commodity shipment pattern:

$$Q_{122}^{1*} = 216,433.1406, \quad Q_{132}^{1*} = 500,000.0000, \quad Q_{122}^{2*} = 0.0000, \quad Q_{132}^{2*} = 0.0000.$$

Since the cheaper maritime routes are no longer operational, the more expensive alternative routes are in use. Nivievskyi (2022) reports that, after the start of the full-scale invasion, the transportation cost of grains

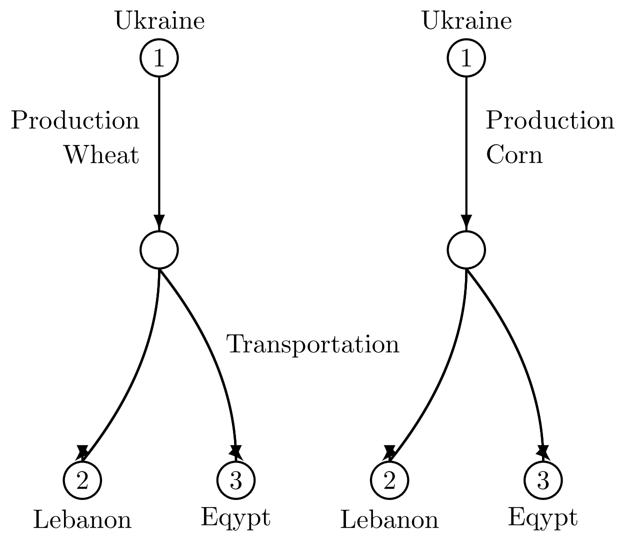

Figure 4: The International Trade Network Topology for Example 5

inside Ukraine reached an unprecedented level of approximately \$200. The alternative routes are used for the export of wheat, but not for corn. Lebanon and Egypt depend significantly on wheat as their main source of nutrition, whereas corn is used primarily as animal feed. Hence, given that wheat is critical for food security in both Lebanon and Egypt, Ukrainian wheat continues to being imported, even with the high transportation costs associated with the alternative routes. On the other hand, these two countries stopped importing corn. It is important to note that Egypt has a population about twenty times that of Lebanon. Hence, the wheat imports are at full capacity of the alternative route. On the other hand, the commodity flow of wheat to Lebanon does not even reach the low capacity of the alternative route, which is due to the high cost of transportation.

The equilibrium commodity supplies are now: $s_1^{1*} = 716,432.1875$, $\quad s_1^{2*} = 0.0000$ whereas the equilibrium commodity demands are:

$$d_1^{1*} = 216,433.1406, \quad d_1^{2*} = 0.0000, \quad d_2^{1*} = 500,000.0000, \quad d_2^{2*} = 0.0000.$$

All Lagrange multipliers are equal to 0.0000 except that $\lambda_{132}^* = 468.4277$.

The supply prices in Ukraine in hryvnia at the equilibrium are now:

$$\pi_1^1(s^*) = 7,099.0347 = \$258.5048, \quad \pi_1^2(s^*) = 6,780.4995 = \$246.9056.$$

Observe that these supply prices are lower than those encountered in Example 4. Ukrainian farmers are selling their wheat at lower prices to make up for the higher cost of transportation after the beginning of the full-scale invasion. In fact, in subsequent months after the start of this war, the supply price for wheat of Ukrainian farmers dropped below \$100 (Arhirova, 2022; Balmforth and Polityuk, 2022; Brower, 2022). This example focuses on the period shortly after the full-scale invasion; hence, the supply prices are just beginning to decrease.

The commodity demand prices at the equilibrium in Lebanon in Lebanese pounds are:

$$\rho_1^1(d^*) = 569,879.0000 = \$376.9041, \quad \rho_1^2(d^*) = 6,052.5005 = \$384.7743,$$

whereas the corresponding demand prices in Egypt in Egyptian pounds are:

$$\rho_2^1(d^*) = 574,560.0000 = \$380, \quad \rho_2^2(d^*) = 5,980.0000 = \$380.1652.$$

Note that the commodity demand prices in both Lebanon and in Egypt are higher in Example 5 than in Example 4. As noted in Nagurney et al. (2024a), in subsequent months, and, close to the establishment of the Black Sea Grain Initiative, commodity demand prices in Lebanon and Egypt reached high levels, of around \$500 (Hernandez, 2022; Rose, 2022). In Example 5, however, the markets are just beginning to respond to the war scenario in terms of higher prices and the accompanying supply and transportation challenges.

Additional examples and insights can be found in Nagurney et al. (2024a).

6. Conclusions and Takeaways

6.1 Summary

In this chapter, we presented an updated multicommodity international agricultural trade network equilibrium model with features of high relevance to practice. The model is built on the variational inequality model of Nagurney et al. (2024a) and includes exchange rates plus capacities not only on supplies across commodities at the supply markets and on shipments on transportation routes across commodities but also on the demands across commodities at the demand markets. The supply and demand markets can be in the same or in different countries. The number of transportation routes between a pair of supply and demand markets can differ from pair to pair. The model is also related to several in the literature.

Illustrative examples are given and full solutions to algorithmically solved multicommodity international agricultural trade network equilibrium problems, drawn from the real world, are presented.

6.2 Takeaways for operations managers / policy makers

The modeling and algorithmic framework, based on the theory of variational inequalities, can be used to assess the impacts of various policies, such as tariffs, quotas, and subsidies, as well as price floors and ceilings (with suitable adaptations) on producers and consumers. With agriculture being essential to the health and well-being of societies as well as economies, having a rigorous, computationally tractable framework to assess the impacts of policies on agricultural trade with relevance also to food security is of high importance.

6.3 Takeaways for operations management researchers

It would be interesting and worthwhile to include product quality and perishability aspects along the lines of the work on fresh produce supply chains in Besik and Nagurney (2017), Nagurney (2022), and Nagurney and Besik (2022). The construction of multiperiod international trade network equilibrium models would also be desirable with the inclusion of commodity losses along the lines of Nagurney and Aronson (1989).

Finally, the development of additional frameworks for nutrition, food security, and international trade would be very worthwhile, along with the exploration of producer subsidies as well as consumer subsidies, with some initial work done in Nagurney (2024).

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