Dynamics of Electric Power Supply Chain Networks under Risk and Uncertainty

Dmytro Matsypura and Anna Nagurney*
Department of Finance and Operations Management
Isenberg School of Management
University of Massachusetts
Amherst, Massachusetts 01003

March 2005

Abstract: Over the past several decades, the electric power industry in the US and in several other countries has been undergoing a fundamental transformation due to deregulation. This transformation has been accompanied by an increase in the number of market participants and changes to the electricity flow patterns with system reliability being increasingly difficult to maintain. The occurrence of several major power outages in 2003 in the US, Canada, England, Switzerland, and Italy suggest that recent changes in the electric power markets require renewed and intense scrutiny. Towards that end, in this paper, we develop a dynamic model for electric power generation, supply, transmission, and consumption under risk and demand uncertainty. We consider the behavior of the various multicriteria decision-makers, who operate in a decentralized manner and include power generators, power suppliers, transmission service providers, as well as the consumers associated with the demand markets. We derive a projected dynamical system that describes the behavior of the entire system; introduce the *speeds of adjustment* for the adjustment processes associated with the electric power transactions and the prices and prove that the equilibrium solution does not change with a change in the adjustment speed(s). We also propose a computational procedure that discretizes and tracks the continuous-time trajectories, and provide several illustrative numerical examples, accompanied by graphs of the iterates over time.

Keywords: Electric power; Supply chains; Networks; Multicriteria optimization; Projected dynamics; Dynamical Systems; Variational inequalities

* Corresponding author. Tel.: +1-413-5455635; fax: +1-413-5453858; e-mail address: nagurney@gbfin.umass.edu

1 Introduction

Systems of electric power generation, supply, and transmission play an extremely important role in modern society. Human civilization has reached the level at which in many aspects of daily life electricity is one of the most widely used sources of energy. Modern society depends on electric power as an essential resource for communication; transportation; for heating, cooling, and lighting; and the powering of computers and electronics. Consumers have grown to expect electricity to be available instantaneously with a flick of a switch.

At the same time, providing reliable electricity is a very complicated and technically challenging task. It involves real-time assessment, coordination, and control of thousands of generating units, the movement of electric power over networks of transmission lines and, finally, the delivery of electric power to the consumers.

In the past couple of decades, several nations have deregulated, or have started the process of deregulating, their electricity markets, with various approaches (see Chao and Hantington (1998)). The deregulation process in the US, which started with the Public Utilities Regulatory Policies Act of 1978 that encouraged the development of nonutility power producers that used renewable energy, has had a dramatic effect on the US power market. By the end of 2000, approximately 20 percent of all electric utility generating capacity had been sold to unregulated companies or had been transferred to unregulated subsidiaries which sell their power in competitive markets. In addition, by February 1, 2005, more than 1200 companies were eligible to sell wholesale power at market-based rates in the US (Source: Federal Energy Regulatory Commission; http://www.ferc.gov/industries/electric/gen-info/pm-over.asp).

With the growth in power marketing companies in the US, the volume of power trades has increased significantly in recent years. For example, in the first quarter of 1995, power marketers traded 1.8 million megawatthours of electricity. By the first quarter of 1999, trade by power marketers had increased to over 400 million megawatthours (source: http://www.eia.doe.gov).

The main motivation behind deregulation is to improve the efficiency and profitability of the electricity market. However, the dramatic increase in the number of market participants trading over the past few years, as well as changes to electricity trading patterns, have made system reliability more difficult to maintain. The North American Electric Reliability Council (NERC) (see North American Electric Reliability Council (1998)) reported that, "[in recent years] the adequacy of the bulk power transmission system has been challenged to support the movement of power in unprecedented amounts and in unexpected directions." Moreover, a US Department of Energy Task Force noted that "there is a critical need to be sure that reliability is not taken for granted as the industry restructures, and thus does not fall through the cracks"(Secretary of Energy Advisory Board's (SEAB) Task Force on Electric System Reliability (1998)).

All these concerns and the level of importance that reliable electric power has for everyday life (as well as for national security) have helped to stimulate and propel significant research activity in the area of electric power supply systems modeling and analysis. Several models have been proposed for simulating the interaction of competing generation companies (see Kahn (1998) and Hobbs et al. (2000)), as well as those that simulate the exercising of market power (Day et al. (2002)). Bolle (1992), Green and Newbery (1992), von der Fehr and Harbord (1993), Green (1996), Newbery and Pollitt (1997), Newbery (1998), and Wolfram (1998, 1999) have all analyzed electricity generation in Great Britain, where two generating companies have dominated the market and, therefore, market power may exist due to a small number of suppliers. A wide range of proposed models is focused on different degrees of decentralization of the electricity market (see, e.g., Schweppe et al. (1988), Hogan (1992), Chao and Peck (1996), Wu et al. (1996), and Willems (2002)). Some researchers, in turn, have proposed model-based comparisons of different market mechanisms (see, for example, Bower and Bunn (2000)).

Although the research and analytical effort has been quite impressive, during August – September 2003 major outages occurred in Northern America and Europe. First, on August 14, 2003, large portions of the Midwest, the Northeastern United States, and Ontario, Canada, experienced an electric power blackout. The outage affected an area with an estimated 50 million people and 61,800 megawatts (MW) of electric load in the states of Ohio, Michigan, Pennsylvania, New York, Vermont, Massachusetts, Connecticut, New Jersey, and the Canadian province of Ontario. Parts of Ontario suffered rolling blackouts for more than a week before full power was restored. Estimates of the total associated costs in the United States ranged between \$4 billion and \$10 billion (U.S. dollars). In Canada, the gross domestic product was down 0.7% in that August; there was a net loss of 18.9 million work hours, and manufacturing shipments in Ontario were down by \$2.3 billion (Canadian dollars) (U.S.-Canada Power System Outage Task Force (2004)). In addition, two other significant outages occurred during the month of September 2003 – one in England and one, initiated in Switzerland, that cascaded over much of Italy. These events clearly indicate that recent changes in the electric power markets require deep and thorough analyses.

In this paper, we build upon the work of Nagurney and Matsypura (2004) which proposed a supply chain network equilibrium model for electric power generation, supply, transmission, and consumption, that allowed for decentralized decision-making. Here we take that theoretical framework further by:

1. introducing dynamics into the electric power supply chain network in the form of adjustment

processes that enable the analysis of the electric power flows (and prices) due to the coupled interactions over time between power generators, power marketers, transmission service providers, and the demand markets;

2. introducing risk minimization as an explicit criterion in the multicriteria decision-making setting;

3. allowing for random demands associated with the demand for electricity at the demand markets, and

4. introducing a speed of adjustment, which may be distinct for each continuous-time adjustment process associated with the electric power supply chain network.

Note that, in wholesale electricity markets, prices typically vary from hour to hour. The vast majority of end-users, however, do not have their consumption metered by the hour. Instead, their consumption is metered several times a year and they are billed on a monthly basis. Since the power marketers must purchase electricity at hourly varying prices and sell it to consumers at flat rates, this system of metering and billing end-users introduces considerable risk to the power marketers. Hence, we believe that it is necessary to consider risk minimization as one of the criteria faced by the decision-makers in the electric power supply network. We model the risk minimization explicitly which makes our model significantly different from recent models (see, e.g., Jing-Yuan and Smeers (1999), Takriti et al. (2000), Boucher and Smeers (2001), and Daxhelet and Smeers (2001)).

Moreover, since we allow for the demand of electricity to be random, our new model is closer to reality for the following reasons. First, the consumption of electric power in residential and commercial sectors is heavily influenced by weather conditions which can be random in nature. Second, the consumption of electric power in industrial sectors is governed by the demands for various goods and services, which may have random demand patterns.

Another issue we deem important in our understanding of the dynamics of the electric power supply chain network is the sensitivity of this system to changes in supply and/or demand, and prices. For example, changes in supply and demand, which lead to price fluctuations, will cause the market to react to these changes. The question of interest to us is: how quickly? We address this question in detail by introducing what we call *speeds of adjustments* and by obtaining associated new theoretical results which are supported by several computational experiments that help to further illuminate the electric power flow and price patterns over time.

The remainder of the paper is organized as follows. In Section 2, we develop the dynamic electric power supply chain network model, describe the various decision-makers and their behavior, and construct the projected dynamical system formulation. The projected dynamical system describes how the electricity transactions evolve over time between the tiers of the supply chain network as well as how the prices associated with the power suppliers and the demand markets evolve. We then prove that the set of stationary points of the projected dynamical system is independent of the speeds of adjustment. In Section 3, we propose an algorithm, which we then apply to several illustrative numerical examples in Section 4. We conclude the paper with Section 5 in which we summarize our results and suggest directions for future research.

2 The Dynamic Electric Power Supply Chain Network Model under Risk and Uncertainty

In this Section, we develop the dynamic electric power supply chain network model in which risk minimization is included as one of the decision-making criteria. In particular, we consider an electric power supply chain network economy in which goods and services are limited to electric energy and transmission services. The economy under consideration is depicted as (cf. Figure 1) a three tiered electric power supply chain network consisting of: *power generators*, *power suppliers*, *demand markets*, and *transmission service providers* with the first three classes of decision-makers associated, respectively, with the top tier, the middle tier, and the bottom tier nodes of the network. Transmission service providers (TSP), in turn, are modeled as different *modes of transaction* (transmission modes) corresponding to distinct links connecting a given supplier node to a given demand market node in Figure 1.

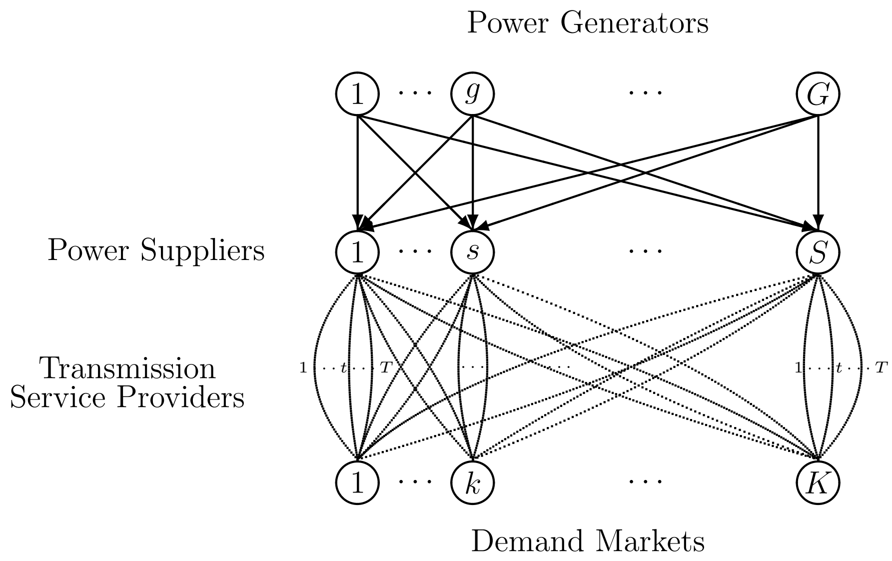


Figure 1: The Electric Power Supply Chain Network

The following description of the decision-making agents is adopted from our previous work (see

Nagurney and Matsypura (2004)) and is included here for completeness.

Power generators are those decision-makers who own and operate electricity generating facilities or power plants. They produce electric power which is then sold to the power suppliers. The prices that generators charge for the electricity that they produce is determined by the competitive wholesale market. There is a total of G power generators, with a typical power generator denoted by g. Power suppliers, in turn, bear a function of an intermediary. They buy electric power from power generators and sell it to the consumers in different demand markets. We denote a typical supplier by s and consider a total of S power suppliers.

Note that every node in the top tier of nodes in Figure 1 is connected to every node in the middle tier. This connectivity represents the assumption that every power supplier (a node in the middle tier) can buy electric energy from any power generator (top tier nodes). In addition, power suppliers do not physically possess electric power at any stage of the supplying process; they merely hold the rights for the electric power. Therefore, the links between the top and the middle tiers of nodes in the network do not represent a physical connectivity but rather a decision-making connectivity between each such pair of nodes. Of course, if there is no possibility of a transaction between a pair of nodes then that link can just be removed from the network.

Electric power, regardless of where and how it is generated, must be delivered to the point of consumption and this service is provided by the transmission service providers. Transmission service providers are those entities that own and operate the electric transmission and distribution systems. These are the companies that transmit electricity from generators via suppliers to demand markets (homes and businesses). We assume that power suppliers cover the direct cost of the physical transaction of electric power from power generators to the demand markets and, therefore, have to make a decision as to from where to acquire the transmission services (and at what level). Hence, transmission service providers do not make decisions as to where the electric power will be acquired and to whom it will be delivered. As a consequence, we do not include them in the model explicitly as nodes. Instead, as was mentioned earlier, their presence in the market is modeled as different *modes of transaction* (transmission modes) corresponding to distinct links connecting a given supplier node to a given demand market node in Figure 1.

We assume that there are T transmission service providers operating in the market, with a typical transmission service provider denoted by t. For the sake of generality, we assume that every power supplier can transact with every demand market using any of the transmission service providers or any combination there of. Therefore, there are T links joining every node in the middle tier of the network with every node at the bottom tier (see Figure 1). Again, depending on the application, certain links can be removed, as necessary.

The fourth type of decision-making agent in the model consists of the demand markets (or consumers). They generate demand for the electric power that drives the generation and transmission in the entire system, and are depicted as the bottom tier of nodes in Figure 1. There is a total of K demand markets, with a typical demand market denoted by k, and distinguished from the others through the use of appropriate characteristics, such as geographic location; the type of consumer; that is, whether a business or household; etc.

We assume a competitive electric power market, meaning that the demand markets can choose between different electric power suppliers (power marketers, brokers, etc.). We also assume that a given power supplier negotiates with the transmission service providers and makes sure that the necessary electric power is delivered. These assumptions fit well into the main idea of the restructuring of the electric power industry that is now being performed in the US, the European Union, and many other countries (see http://www.ferc.gov and http://www.europarl.eu.int).

Clearly, and as already noted, in certain situations, some of the links in the supply chain network for electric power in Figure 1 may not exist (due, for example, to various restrictions, regulations, lack of physical connections, etc.). Such cases can be handled within our framework by eliminating the corresponding link(s) from the supply chain network or (see further discussion below) by assigning an appropriately high transaction cost associated with those link(s).

We now turn to the description of the dynamic adjustment processes associated with the disequilibrium dynamics as the various electric power supply chain decision-makers adjust their electric power transactions between the tiers and the prices associated with the different tiers adjust as well. We begin with a discussion of the price dynamics and then describe the dynamics of the electric power transactions.

2.1 Demand Market Price Dynamics

We begin by describing the dynamics underlying the prices of the electricity associated with the demand markets since the demand for the electric power drives both its generation and the distribution. Let ρ_{3k} denote the price of a unit of electric power associated with demand market k. We assume that the demand for electric power at each demand market k is elastic and depends not only on the price at the corresponding demand market but may, in general, also depend on the entire vector of prices in the electric power supply chain network. This level of generality allows one to facilitate the modeling of competition on the consumption side. For simplicity of notation, we group all such prices into a column vector $\rho_3 = (\rho_{31}, \ldots, \rho_{3k}, \ldots, \rho_{3K})^T$.

Apart from the price, the demand for electric power may depend on additional factors. In residential areas, for example, weather conditions play a significant role because large amounts of electricity are used for heating or cooling. The consumption of electric power in industrial sectors, in turn, is also influenced by the demand for various goods and services. Hence, in order to make our model more realistic we will consider the demand for electric power to be *random* and we will model it in the following way, as suggested by Nagurney et al. (2005) for commodities.

Let $\hat{d}_k(\rho_3)$ denote the demand for electric power (in watts) at the demand market k associated with the vector of prices ρ_3, where $\hat{d}_k(\rho_3)$ is a *random* variable with density function $\Phi_k(x, \rho_3)$, with ρ_3 serving as a parameter. Hence, we assume that the density function may vary with the demand market prices. Let P_k be the probability distribution function of $\hat{d}_k(\rho_3)$, that is, $P_k(x, \rho_3) = P_k(\hat{d}_k \leq x) = \int_0^x \Phi_k(x, \rho_3)dx$.

Using the most natural and popular among economists assumption about the forces that influence the behavior of the prices in the market economy (see, for example, Samuelson (1941), Metzler (1945), and Arrow and Hurwicz (1958)), we assume that the rate of change of the price ρ_{3k}, denoted by $\dot{\rho}_{3k}$, is proportional to the difference between the *expected* demand, where the expected demand is $d_k(\rho_{3k}) \equiv E(\hat{d}_k(\rho_{3k}))$, and the total amount transacted with the demand market. This function is assumed to be continuous. Moreover, the rate of change of the price ρ_{3k} has the same sign as the difference between the expected demand and the total amount transacted with the demand market. Let now ϕ_k denote the *speed of adjustment* associated with the price at demand market k. This term or factor is assumed to be positive. Hence, we have that for each demand market k:

$$\dot{\rho}_{3k} = \begin{cases} \phi_k(d_k(\rho_3) - \sum_{s=1}^{S}\sum_{t=1}^{T} q_{sk}^t), & \text{if} \quad \rho_{3k} > 0 \\ \max\{0, \phi_k(d_k(\rho_3) - \sum_{s=1}^{S}\sum_{t=1}^{T} q_{sk}^t)\} & \text{if} \quad \rho_{3k} = 0, \end{cases} \tag{1}$$

where q_{sk}^t denotes the amount of electric power transacted (in watts) between nodes s and k through link t.

Note that ϕ_k may also be interpreted as the sensitivity of the specific demand market to the changes in the supply of and/or demand for the electric power. Each demand market may have its own unique sensitivity, which, as will be shown later in this Section and numerically in Section 4, affects the dynamics of this particular demand market (and, hence, the rest of the dynamical system). However, the value of the speed of adjustment will not affect the equilibrium pattern, as we shall establish theoretically and also illustrate computationally.

Hence, if the expected demand for the electricity in the demand market (at any given instant in time) exceeds the amount available, the price of the electric power in that demand market will increase with the speed ϕ_k; if the amount available exceeds the demand at a particular price, then the price in the demand market will decrease with the same speed ϕ_k. Furthermore, we guarantee that the prices do not become negative.

2.2 The Dynamics of the Prices at the Power Suppliers

Distinct power suppliers may be willing to pay a different price for electric power. The price must reflect the supply and demand conditions, similar to that in the case of the demand markets. We denote this price by γ_s and emphasize that it may be interpreted as a clearing-type price for supplier s. Let $\dot{\gamma}_s$ denote the rate of change of this price. Thus, we assume that the price γ_s, associated with power supplier s, evolves over time according to:

$$\dot{\gamma}_s = \begin{cases} \phi_s(\sum_{k=1}^{K}\sum_{t=1}^{T} q_{sk}^t - \sum_{g=1}^{G} q_{gs}), & \text{if} \quad \gamma_s > 0 \\ \max\{0, \phi_s(\sum_{k=1}^{K}\sum_{t=1}^{T} q_{sk}^t - \sum_{g=1}^{G} q_{gs})\}, & \text{if} \quad \gamma_s = 0, \end{cases} \tag{2}$$

and that such dynamics hold true for all the power suppliers in the system. Here q_{gs} denotes the amount of electric power in watts being transmitted between nodes g and s. The term ϕ_s is the speed of adjustment of the price for the electricity at power supplier s. This term is also assumed to be positive for all s; $s = 1, \dots, S$.

In other words, if the amount of the electric power transacted by the demand markets (at any given instant in time) exceeds that available from a power supplier, then the price associated with the power supplier will increase with the speed ϕ_s; if the amount available is greater than that desired by the demand market, then the price associated with the power supplier will decrease with the speed ϕ_s. As in the case of the demand market prices, we guarantee that this price remains nonnegative.

2.3 Precursors to the Dynamics of the Electric Power Transactions

Before we describe the dynamics of the electric power transactions, it is necessary to describe the behavior of the top two tiers of decision-makers, that is, the power generators and the suppliers, and also to introduce some additional notation.

Let q_g denote the electric power generation output (in watts) of generator g, which is a nonnegative quantity. We group the generation outputs of all generators into the column vector $q \in R_+^G$ and assume that each power generator g has a power generation cost function f_g, which can depend, in general, on the entire vector of production outputs, that is,

$$f_g = f_g(q), \quad \forall g. \tag{3}$$

Hence, we consider the general situation in which the generation cost of a particular power generator can depend not only on his production output but also on the production outputs of the other generators. This allows one to model competition.

All power generating cost functions are assumed to be convex and continuously differentiable.

We let q_{gs} denote the electric power transacted between generator g and supplier s. Then, due to the conservation of electric power flows, we must have that:

$$q_g = \sum_{s=1}^{S} q_{gs}, \quad \forall g. \tag{4}$$

We group all the electric power flows (transactions) q_{gs} between the first and the second tier of nodes into a column vector $Q^1 \in R_+^{GS}$. Without loss of generality, we can now write, in view of the power generation cost functions (3) and the conservation of flow equations (4):

$$f_g = f_g(Q^1), \quad \forall g. \tag{5}$$

As the production output reaches the capacity of a given generator, we can expect the production cost to become very large.

While the electric power is being transmitted from a point of generation (node g), there will be transaction costs associated with the transmission process. Let c_{gs} denote the g-th power generator's transaction cost function, associated with transacting with power supplier s. Since different power generators may utilize the same physical lines for the energy transmission the transaction cost of a particular power generator may be a function of not only the amount of power transacted by this particular power generator, but also the amount of power transacted by all other power generators and supplier combinations, sharing the same physical line(s). Hence, for the sake of generality, we let

$$c_{gs} = c_{gs}(Q^1), \quad \forall g. \tag{6}$$

These transaction cost functions are assumed to be convex and continuously differentiable.

Multicriteria Decision-Making Behavior of the Power Generators

It is assumed that each power generator seeks to maximize his profit which is the difference between his total revenue and the total costs incurred. Let ρ_{1gs} denote the price per unit of electric power transacted between generator g and supplier s. We allow the power generator to set different prices for different power suppliers. Hence, the profit maximization problem of power generator g can be expressed as follows:

$$\text{Maximize} \quad \sum_{s=1}^{S} \rho_{1gs} q_{gs} - f_g(Q^1) - \sum_{s=1}^{S} c_{gs}(Q^1) \tag{7}$$

subject to: $q_{gs} \geq 0, \quad \forall s.$

In addition to the criterion of profit maximization, we also assume that each power generator is concerned with risk minimization. Here, for the sake of generality, we assume, as given, a risk function r_g, for generator g, which is assumed to be convex and continuously differentiable and a function of not only the product transactions associated with the particular generator but also of those of other generators. Hence, we assume that

$$r_g = r_g(Q^1), \quad \forall g. \tag{8}$$

Therefore, the second criterion of power generator g can be expressed as:

$$\text{Minimize} \quad r_g(Q^1), \tag{9}$$

subject to: $q_{gs} \geq 0$, for all s. The risk function may be distinct for each power generator.

Each power generator g considers the risk minimization criterion to be of a certain level of importance, but this level is not necessarily the same for all generators. Consequently, we associate a nonnegative weight α_g with the risk minimization criterion (9), with the weight associated with the profit maximization criterion (7) serving as the numeraire and being set equal to 1. Thus, we can construct a *value function* for each power generator (cf. Fishburn (1970), Chankong and Haimes (1983), Yu (1985), Keeney and Raiffa (1993)) using a constant additive weight value function. Consequently, letting U_g denote the multicriteria objective function faced by generator g, the decision-making problem for generator g, for $g = 1, \ldots, G$, can be transformed into:

$$\text{Maximize} \quad U_g = \sum_{s=1}^{S} \rho_{1gs} q_{gs} - f_g(Q^1) - \sum_{s=1}^{S} c_{gs}(Q^1) - \alpha_g r_g(Q^1) \tag{10}$$

subject to:

$$q_{gs} \geq 0, \qquad \forall s. \tag{11}$$

We assume that the generators compete in a noncooperative manner according to Nash (1950, 1951). In this context, we will have that each generator will determine his optimal generation quantity and transactions, given the optimal ones of his competitors. We ignore, for the time being, the nonnegativity constraints on the transaction variables.

Let the gradient of power generator g's multicriteria objective function U_g with respect to the q_g variables be denoted by: $\nabla_{q_g} U_g = (\frac{\partial U_g}{\partial q_{g1}}, \ldots, \frac{\partial U_g}{\partial q_{gS}})$. This gradient represents power generator g's *idealized direction* with respect to q_g, with the gs-component of $\nabla_{q_g} U_g$ being given by:

$$\left(\rho_{1gs} - \frac{\partial f_g(Q^1)}{\partial q_{gs}} - \frac{\partial c_{gs}(Q^1)}{\partial q_{gs}} - \alpha_g \frac{\partial r_g(Q^1)}{\partial q_{gs}} \right). \tag{12}$$

Subsequently, we will include the nonnegativity assumption on the electric power transactions, when we construct the projected dynamical system that captures the dynamics of all the electric power transactions and prices.

Multicriteria Decision-Making Behavior of the Power Suppliers

We now turn to the description of the behavior of the power suppliers. The term power supplier refers to power marketers, traders, and brokers, who arrange for the sale and purchase of the output of the generators to other suppliers or load-serving entities, or in many cases, serve as load-serving entities themselves. They play a fundamental role in the model since they are responsible for acquiring electricity from power generators and delivering it to the demand markets. Therefore, power suppliers are involved in transactions with both power generators and the demand markets through transmission service providers.

We assume that a typical power supplier s is faced with two kinds of costs. The first type of cost is the *operating cost* associated with maintaining the office, licensing, etc. We denote this cost by c_s. Recall from our previous discussion that q^t_{sk} denotes the amount of electricity being transacted between power supplier s and demand market k via the link corresponding to the transmission service provider t. We group all transactions associated with power supplier s and demand market k into the column vector $q_{sk} \in R^T_+$. We then further group all such vectors associated with all the power suppliers into a column vector $Q^2 \in R^{STK}_+$. For the sake of generality and to enhance the modeling of competition, we assume that

$$c_s = c_s(Q^1, Q^2), \quad \forall s. \tag{13}$$

Let $\hat{c}_{gs}$, in turn, denote the transaction cost associated with power supplier s acquiring electric power from power generator g, where we assume that:

$$\hat{c}_{gs} = \hat{c}_{gs}(Q^1), \quad \forall g, \forall s. \tag{14}$$

Let c^t_{sk} denote the transaction cost associated with power supplier s transmitting electric power to demand market k via transmission service provider t, where:

$$c^t_{sk} = c^t_{sk}(Q^2), \quad \forall t, \forall s, \forall k. \tag{15}$$

We assume that all the above transaction cost functions are convex and continuously differentiable.

Let ρ^t_{2sk} denote the price associated with the transaction between power supplier s and demand market k via transmission service provider t. The total revenue the power supplier obtains from his transactions is equal to the sum over all the modes of transmission and all the demand markets of the price times the amount of electric power transacted with the demand market using the particular transmission mode. Indeed, the total revenue of power supplier s can mathematically be expressed as follows:

$$\sum_{k=1}^{K}\sum_{t=1}^{T}\rho_{2sk}^{t}q_{sk}^{t}. \tag{16}$$

It is assumed, as in the case of power generators, that each power supplier seeks to maximize his profit which is the difference between his total revenue and the total costs incurred. Hence, the optimization problem of the power supplier s can be expressed as follows:

$$\text{Maximize} \quad \sum_{k=1}^{K}\sum_{t=1}^{T}\rho_{2sk}^{t}q_{sk}^{t} - c_s(Q^1,Q^2) - \sum_{g=1}^{G}\rho_{1gs}q_{gs} - \sum_{g=1}^{G}\hat{c}_{gs}(Q^1) - \sum_{k=1}^{K}\sum_{t=1}^{T}c_{sk}^{t}(Q^2) \tag{17}$$

subject to:

$$\sum_{k=1}^{K}\sum_{t=1}^{T}q_{sk}^{t} \leq \sum_{g=1}^{G}q_{gs} \tag{18}$$

$$q_{gs} \geq 0, \quad \forall g \tag{19}$$

$$q_{sk}^{t} \geq 0, \quad \forall k, \forall t. \tag{20}$$

Constraint (18) is a conservation of flow constraint and states that a power supplier cannot transact more electric power than he has obtained from the various power generators. Constraints (19) and (20), in turn, are nonnegativity assumptions.

We assume that power supplier s is faced with his own perception of risk associated with obtaining and supplying the electric power, which we assume is incorporated into a continuous risk function of a general form given by:

$$r_s = r_s(Q^1,Q^2), \quad \forall s. \tag{21}$$

Note that we allow the risk function of a particular supplier to depend on the transactions to and from all the power suppliers in the electric power supply chain network. Therefore, the second criterion of power supplier s can be expressed as:

$$\text{Minimize} \quad r_s(Q^1,Q^2), \tag{22}$$

subject to: $q_{gs} \geq 0, \quad \forall g$ and $q_{sk}^{t} \geq 0$, $\forall k, \forall t$.

We assume also that each power supplier s associates a weight of 1 with the profit maximization criterion (17) and a weight of β_s with his risk level. Hence, the multicriteria decision-making problem for supplier s; $s = 1, \ldots, S$, faced now with a multicriteria objective function denoted by U_s can be transformed directly into the optimization problem:

$$\text{Maximize} \quad U_s = \sum_{k=1}^{K}\sum_{t=1}^{T} \rho_{2sk}^t q_{sk}^t - c_s(Q^1, Q^2) - \sum_{g=1}^{G} \rho_{1gs} q_{gs} - \sum_{g=1}^{G} \hat{c}_{gs}(Q^1) - \sum_{k=1}^{K}\sum_{t=1}^{T} c_{sk}^t(Q^2) - \beta_s r_s(Q^1, Q^2) \tag{23}$$

subject to:

$$\sum_{k=1}^{K}\sum_{t=1}^{T} q_{sk}^t \leq \sum_{g=1}^{G} q_{gs} \tag{24}$$

$$q_{gs} \geq 0, \quad \forall g \tag{25}$$

$$q_{sk}^t \geq 0, \quad \forall k, \forall t. \tag{26}$$

Objective function (23) represents a value function for power supplier s with β_s having the interpretation as a conversion rate in dollar value.

For simplicity of notation, we let $q_s = (q_{1s}, \ldots, q_{gs}, \ldots, q_{Gs})$ and $q^s = (q_{s1}^1, \ldots, q_{sk}^t, \ldots, q_{sK}^T)$.

Ignoring, for the time being, the nonnegativity constraints (as was done above for the generators), and constraint (24), $\nabla_{q_s} U_s = (\frac{\partial U_s}{q_{1s}}, \ldots, \frac{\partial U_s}{q_{Gs}})$ represents supplier s's *idealized direction* in terms of q_s, where component gs is given by:

$$\left(-\frac{\partial c_s(Q^1, Q^2)}{\partial q_{gs}} - \frac{\partial \hat{c}_{gs}(Q^1)}{\partial q_{gs}} - \rho_{1gs} - \beta_s \frac{\partial r_s(Q^1, Q^2)}{\partial q_{gs}} \right), \tag{27}$$

whereas $\nabla_{q^s} U_s = (\frac{\partial U_s}{q_{s1}^1}, \ldots, \frac{\partial U_s}{q_{sK}^T})$ represents supplier s's idealized direction in terms of the q^s variables, with component tsk given by:

$$\left(\rho_{2sk}^t - \frac{\partial c_s(Q^1, Q^2)}{\partial q_{sk}^t} - \frac{\partial c_{sk}^t(Q^2)}{\partial q_{sk}^t} - \beta_s \frac{\partial r_s(Q^1, Q^2)}{\partial q_{sk}^t} \right). \tag{28}$$

However, generator g and supplier s must agree on the volume of q_{gs} in order for the transaction to actually take place. Therefore, direction (12) of generator g must coincide with direction (27) of supplier s, and also respond to the price signal γ_s, yielding, after algebraic simplification, a *combined direction*, which signifies cooperation, given by:

$$\left(\gamma_s - \frac{\partial f_g(Q^1)}{\partial q_{gs}} - \frac{\partial c_{gs}(Q^1)}{\partial q_{gs}} - \alpha_g \frac{\partial r_g(Q^1)}{\partial q_{gs}} - \frac{\partial c_s(Q^1, Q^2)}{\partial q_{gs}} - \frac{\partial \hat{c}_{gs}(Q^1)}{\partial q_{gs}} - \beta_s \frac{\partial r_s(Q^1, Q^2)}{\partial q_{gs}} \right). \tag{29}$$

Moreover, (29) must hold for all g and s for the supply chain to allow for links between the top two tiers of nodes.

In addition from the consumers' perspective, we have that *idealized direction* is one where the amount of flow of the electric power between a supplier/demand market pair (s,k) transacted via transmitter t is given by:

$$\rho_{3k} - \rho^t_{2sk} - \hat{c}^t_{sk}(Q^2), \quad \forall t, \tag{30}$$

where the constraints are ignored for the time being.

Furthermore, since the consumers at demand market k must agree with the supplier s as to the amount of flow q^t_{sk}, adding (28) plus the term (30), and noting that they, in turn, must also be responsive to the price signals at the suppliers, yields a *combined direction* of:

$$\left(\rho_{3k} - \hat{c}^t_{sk}(Q^2) - \frac{\partial c_s(Q^1,Q^2)}{\partial q^t_{sk}} - \frac{\partial c^t_{sk}(Q^2)}{\partial q^t_{sk}} - \beta_s \frac{\partial r_s(Q^1,Q^2)}{\partial q^t_{sk}} - \gamma_s\right). \tag{31}$$

We are now ready to express the dynamics of the electric power transactions between the tiers of decision-makers in the electric power supply chain network. Moreover, we now explicitly include the nonnegativity constraints associated with the electricity transactions and note that (24) has actually been subsumed within the power supplier price dynamics given by (2). We then provide a unified expression in the form of a projected dynamical system for the dynamic evolution of all the electric power transactions and the electricity prices simultaneously.

The adjustment assumptions to be introduced are in concert with what suggested by a number of prominent economists (see, for example, Fisher (1961)) and are based on the following reasoning. The rate of change in the amount of good (here, electric power) produced/transacted by a particular decision-maker is proportional to his marginal profit and has the same sign. In other words, a typical decision-maker will try to increase his output for as long as the marginal profit is nonnegative, and will try to decrease his output for as long as the marginal profit is negative until the output is zero (that is, hits the boundary for the variable). As was stated earlier, for any transaction to take place, the decision-makers involved have to agree on the amount to be transacted. Thus, the marginal profit of the transaction is represented by the combined direction. Therefore, we can expect that the rate of change in the amount of product transacted between two particular decision-makers is proportional to their combined optimal direction.

2.4 The Dynamics of the Electricity Transactions between Power Generators and Suppliers

Let $\phi_{gs} > 0$ denote the speed of adjustment associated with the amount of electric power q_{gs} transacted between power generator g and power supplier s. In order to guarantee that the electricity transactions do not assume negative quantities, we propose the following dynamics for the transactions between each power generator g and power supplier s with $\dot{q}_{gs}$ denoting the rate of change of q_{gs}:

$$\dot{q}_{gs} = \begin{cases} \phi_{gs}(\gamma_s - \frac{\partial f_g(Q^1)}{\partial q_{gs}} - \frac{\partial c_{gs}(Q^1)}{\partial q_{gs}} - \alpha_g \frac{\partial r_g(Q^1)}{\partial q_{gs}} \\ -\frac{\partial c_s(Q^1,Q^2)}{\partial q_{gs}} - \frac{\partial \hat{c}_{gs}(Q^1)}{\partial q_{gs}} - \beta_s \frac{\partial r_s(Q^1,Q^2)}{\partial q_{gs}}), & \text{if} \quad q_{gs} > 0 \\ \max\{0, \phi_{gs}(\gamma_s - \frac{\partial f_g(Q^1)}{\partial q_{gs}} - \frac{\partial c_{gs}(Q^1)}{\partial q_{gs}} - \alpha_g \frac{\partial r_g(Q^1)}{\partial q_{gs}} \\ -\frac{\partial c_s(Q^1,Q^2)}{\partial q_{gs}} - \frac{\partial \hat{c}_{gs}(Q^1)}{\partial q_{gs}} - \beta_s \frac{\partial r_s(Q^1,Q^2)}{\partial q_{gs}})\}, & \text{if} \quad q_{gs} = 0. \end{cases} \tag{32}$$

Another way of interpreting the above expression is the following. Whenever there is a positive difference between the shadow price of the power supplier and the aggregated marginal costs and risks associated with a transaction, the amount of flow on that particular link will increase. Otherwise, the amount of flow will either decrease or remain the same; see also, Nagurney and Dong (2002).

2.5 The Dynamics of the Electricity Transactions between Power Suppliers and Demand Markets

The combined direction associated with a transaction between a power supplier and demand market pair via a particular transmission service provider, in turn, is given by expression (31) but now we guarantee that the evolution of the electric power transactions cannot yield negative values. We also introduce a speed adjustment parameter $\phi^t_{sk} > 0 \; \forall s, k, t$ similar to the above. Hence, letting $\dot{q}^t_{sk}$ denote the rate of change of the electric power transaction between supplier s and demand market k, through a transmission service provider t we obtain the following dynamics, which we assume hold true for all power suppliers, all demand markets, and all transmission service providers:

$$\dot{q}^t_{sk} = \begin{cases} \phi^t_{sk}(\rho_{3k} - \hat{c}^t_{sk}(Q^2) - \frac{\partial c_s(Q^1,Q^2)}{\partial q^t_{sk}} - \frac{\partial c^t_{sk}(Q^2)}{\partial q^t_{sk}} - \beta_s \frac{\partial r_s(Q^1,Q^2)}{\partial q^t_{sk}} - \gamma_s), & \text{if} \quad q^t_{sk} > 0 \\ \max\{0, \phi^t_{sk}(\rho_{3k} - \hat{c}^t_{sk}(Q^2) - \frac{\partial c_s(Q^1,Q^2)}{\partial q^t_{sk}} - \frac{\partial c^t_{sk}(Q^2)}{\partial q^t_{sk}} - \beta_s \frac{\partial r_s(Q^1,Q^2)}{\partial q^t_{sk}} - \gamma_s)\}, & \text{if} \quad q^t_{sk} = 0. \end{cases} \tag{33}$$

2.6 The Projected Dynamical System

Consider now a dynamical system in which all the demand market prices evolve according to (1); all the prices at the power suppliers evolve according to (2); the amounts of electric power transacted between power generators and suppliers evolve according to (32), and those between suppliers and demand markets via transmission service providers all evolve according to (33).

Let now X denote the aggregate column vector $(Q^1, Q^2, \gamma, \rho_3)$ in the feasible set $\mathcal{K} \equiv R_+^{GS+SKT+S+K}$. Define the column vector

$$F(X) \equiv (F_{gs}, F_{sk}^t, F_s, F_k)_{g=1,\ldots,G; s=1,\ldots,S; t=1,\ldots,T; k=1,\ldots,K},$$

with components: $-F_{gs}$ given by (29), $\forall g, s$; $-F_{sk}^t$ given by (31), $\forall s, k, t$; $-F_s \equiv \phi_s(\sum_{k=1}^K \sum_{t=1}^T q_{sk}^t - \sum_{g=1}^G \sum_{s=1}^S q_{gs})$, $\forall s$; $-F_k \equiv \phi_k(d_k(\rho_3) - \sum_{s=1}^S \sum_{t=1}^T q_{sk}^t)$, $\forall k$.

Then the dynamic model described by (1), (2), (32), and (33) for all g, s, k, t, can be rewritten as a *projected dynamical system* (PDS) (see Nagurney and Zhang (1996)) defined by the following initial value problem:

$$\dot{X} = \Pi_{\mathcal{K}}(X, -F(X)), \quad X(0) = X_0, \tag{34}$$

where $\Pi_{\mathcal{K}}$ is the projection operator of $-F(X)$ onto $\mathcal{K}$ at X and $X_0 = (Q^{10}, Q^{20}, \gamma^0, \rho_3^0)$ is the initial point corresponding to the electric power flow and price pattern. Note that since the feasible set $\mathcal{K}$ is simply the nonnegative orthant the projection operation takes on a very simple form as revealed through (1), (2), (32), and (33).

A Stationary/Equilibrium Point

Recall that a stationary point of a dynamical system is that point where $\dot{X} = 0$ and, hence, in the context of our model, when there is no change in the electric power transactions in the electric power supply chain network and no change in the prices.

The following result is immediate from the results in Dupuis and Nagurney (1993).

Theorem 1: Set of Stationary Points Coincides with the Set of Solutions of a Variational Inequality Problem

Since the feasible set $\mathcal{K}$ is a convex polyhedron, the set of stationary points of the projected dynamical system given by (34), that is, X^* such that $0 = \Pi_{\mathcal{K}}(X^*, -F(X^*))$, coincides with the set of solutions to the variational inequality problem given by: determine $X^* \in \mathcal{K}$, such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{35}$$

where $F(X)$ and X are as defined above and $\langle \cdot, \cdot \rangle$ denotes the inner product in N-dimensional Euclidian space where here $N = GS + SKT + S + K$, or, explicitly, we have that an electric power transaction and price pattern $X^* \equiv (Q^{1*}, Q^{2*}, \gamma^*, \rho_3^*) \in \mathcal{K}$ is a stationary point of the projected *dynamical system (34) if and only if it satisfies the variational inequality problem:*

$$\sum_{g=1}^{G}\sum_{s=1}^{S}\phi_{gs}\Bigg[\frac{\partial f_g(Q^{1*})}{\partial q_{gs}}+\frac{\partial c_{gs}(Q^{1*})}{\partial q_{gs}}+\alpha_g\frac{\partial r_g(Q^1)}{\partial q_{gs}}+\frac{\partial c_s(Q^{1*},Q^{2*})}{\partial q_{gs}}+\frac{\partial \hat{c}_{gs}(Q^{1*})}{\partial q_{gs}}+$$

$$\beta_s\frac{\partial r_s(Q^1,Q^2)}{\partial q_{gs}}-\gamma_s^*\Bigg]\times[q_{gs}-q_{gs}^*]$$

$$+\sum_{s=1}^{S}\sum_{k=1}^{K}\sum_{t=1}^{T}\phi_{sk}^t\left[\frac{\partial c_s(Q^{1*},Q^{2*})}{\partial q_{sk}^t}+\frac{\partial c_{sk}^t(Q^{2*})}{\partial q_{sk}^t}+\hat{c}_{sk}^t(Q^{2*})+\beta_s\frac{\partial r_s(Q^1,Q^2)}{\partial q_{sk}^t}+\gamma_s^*-\rho_{3k}^*\right]\times[q_{sk}^t-q_{sk}^{t*}]$$

$$+\sum_{s=1}^{S}\phi_s\left[\sum_{g=1}^{G}q_{gs}^*-\sum_{k=1}^{K}\sum_{t=1}^{T}q_{sk}^{t*}\right]\times[\gamma_s-\gamma_s^*]+\sum_{k=1}^{K}\phi_k\left[\sum_{s=1}^{S}\sum_{t=1}^{T}q_{sk}^{t*}-d_k(\rho_3^*)\right]\times[\rho_{3k}-\rho_{3k}^*]\geq 0,$$

$$\forall(Q^1,Q^2,\gamma,\rho_3)\in\mathcal{K}. \tag{36}$$

We now present several properties of the PDS defined by the initial value problem (34) that follow from the theory of projected dynamical systems.

Theorem 2: Existence and Uniqueness

Assume that function F that enters the initial value problem (34) is Lipschitz continuous. Then, for any $X_0 \in \mathcal{K}$, there exists a unique solution $X_0(t)$ to the initial value problem (34).

Proof: Follows from Theorem 2.5 in Nagurney and Zhang (1996).

Definition 1: Stability of the System

The system defined by (38) is stable if, for every X_0 and every equilibrium point X^*, the Euclidean distance $\|X^* - X_0(t)\|$ is a monotone nonincreasing function of time t.

Using Definition 1, a global stability result for the dynamical system (34) can be stated as follows:

Theorem 3: Stability of the Electric Power Supply Chain Network

Assume that function F that enters the initial value problem (34) is monotone. Then the dynamical system (34) underlying the electric power supply chain network with risk and uncertainty is stable.

Proof: $F(X)$ is monotone and, hence, the conclusion follows directly from Theorem 4.1 of Zhang and Nagurney (1995).

2.7 More Theoretical Results

We now investigate the following question: what can be said regarding the set of stationary points of the projected dynamical system (34), which coincides with the set of solutions to variational inequality (35), and the set of stationary points associated with the projected dynamical system in which all of the speeds of adjustment are identical, with the special case being that all speeds are equal to one (and the equivalent equilibria)? In other words, do the speeds of adjustment affect the solutions?

We provide the answer in the form of the following theorem, but, first, we need to introduce some notation. We express the components of $F(X)$ above, for simplicity, as $F_1,\ldots,F_N$ and recall that the feasible set is the nonnegative orthant. We denote variational inequality (35) by $\mathrm{VI}(F,\mathcal{K})$.

Theorem 4

Assume that $\mathcal{K}$ is the convex polyhedron given by R^N_+ and that $\phi \equiv (\phi_1,\ldots,\phi_N)$ is a vector of positive terms. Then, the set of stationary points of the projected dynamical system given in (34), which coincides with the set of solutions X^* to variational inequality (35), is equivalent to the set of solutions to the variational inequality $\mathrm{VI}(F',\mathcal{K})$ *where:*

$$F' \equiv (F'_1,\ldots,F'_N)$$

and

$$F \equiv (\phi_1 F'_1,\ldots,\phi_n F'_N). \tag{37}$$

Proof: From Theorem 1 we know that the set of stationary points of (34) coincides with the set of solutions to $\mathrm{VI}(F,\mathcal{K})$. We now prove that the solutions to $\mathrm{VI}(F,\mathcal{K})$ coincide with the solutions to $\mathrm{VI}(F',\mathcal{K})$, that is, when the speeds of adjustment are all identically equal to one.

We first prove that if X^* solves $\mathrm{VI}(F,\mathcal{K})$ then it also solves $\mathrm{VI}(F',\mathcal{K})$. Since $X^* \equiv (X^*_1, X^*_2,\ldots,X^*_N)$ is a solution to $\mathrm{VI}(F,\mathcal{K})$, the following must hold, by definition:

$$\sum_{i=1}^{N}[F_i(X^*)]\times[X_i - X^*_i] = F_1(X^*)\times[X_1 - X^*_1]+\ldots+F_N(X^*)\times[X_N - X^*_N]$$

$$= \phi_1 F'_1(X^*)\times[X_1-X^*_1]+\phi_1 F'_2(X^*)\times[X_2-X^*_2]+\ldots+\phi_1 F'_N(X^*)\times[X_N-X^*_N]\geq 0, \quad \forall X\in\mathcal{K}. \tag{38}$$

Setting now $X_2 = X^*_2,\ldots,X_N = X^*_N$ and substituting the resultants into (38), yields:

$$\phi_1 F'_1(X^*)\times[X_1 - X^*_1]\geq 0 \Leftrightarrow F'_1(X^*)\times[X_1 - X^*_1]\geq 0, \quad \forall X_1\in R_+, \tag{39}$$

and for any scalar $\phi_1 > 0$.

In a similar manner, we may set $X_1 = X_1^*, X_3 = X_3^*, \ldots, X_N = X_N^*$ with substitution into (38) yielding:

$$\phi_2 F_2'(X^*) \times [X_2 - X_2^*] \geq 0 \Leftrightarrow F_2'(X^*) \times [X_2 - X_2^*] \geq 0, \quad \forall X_2 \in R_+, \tag{40}$$

for any scalar $\phi_2 > 0$.

Proceeding in the same fashion with all the remaining X_i variables, we obtain:

$$F_1'(X^*) \times [X_1 - X_1^*] \geq 0, \quad \forall X_1 \in R_+, \quad F_2'(X^*) \times [X_2 - X_2^*] \geq 0, \quad \forall X_2 \in R_+,$$
$$\ldots \quad F_N'(X^*) \times [X_N - X_N^*] \geq 0, \quad \forall X_N \in R_+. \tag{41}$$

Summation of all the inequalities in (41) yields:

$$\sum_{i=1}^{N} F_i'(X^*) \times [X_i - X_i^*] = \langle F'(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K} \equiv R_+^N. \tag{42}$$

Hence, we have shown that X^* is also a solution to VI$(F', \mathcal{K})$.

We now establish the converse, that is, if X' solves VI$(F', \mathcal{K})$, it also solves VI$(F, \mathcal{K})$. Let X' be a solution to VI$(F', \mathcal{K})$. Then the following must hold:

$$\sum_{i=1}^{N} [F_i'(X')] \times [X_i - X_i'] = F_1'(X') \times [X_1 - X_1'] + \ldots + F_N'(X') \times [X_N - X_N'] \geq 0, \quad \forall X \in \mathcal{K}. \tag{43}$$

Setting now: $X_2 = X_2', \ldots, X_N = X_N'$ (all the variables except for the first) and substituting the resultants into (43), we obtain:

$$F_1'(X') \times [X_1 - X_1'] \geq 0 \Leftrightarrow \phi_1 F_1'(X') \times [X_1 - X_1'] = F_1(X') \times [X_1 - X_1'] \geq 0, \quad \forall X_1 \in R_+, \tag{44}$$

for any scalar $\phi_1 > 0$.

In a similar manner, we set: $X_1 = X_1'$; $X_3 = X_3'$,...; $X_N = X_N'$ with substitution into (43) yielding:

$$F_2'(X') \times [X_2 - X_2'] \geq 0 \Leftrightarrow \phi_2 F_2'(X') \times [X_2 - X_2'] = F_2(X') \times [X_2 - X_2'] \geq 0, \quad \forall X_2 \in R_+, \tag{45}$$

for any scalar $\phi_2 > 0$.

Proceeding in the same manner with all the remaining variables, we conclude that

$$F_1(X') \times [X_1 - X_1'] \geq 0, \quad \forall X_1 \in R_+, \quad F_2(X') \times [X_2 - X_2'] \geq 0, \quad \forall X_2 \in R_+,$$

$$\ldots \quad F_N(X') \times [X_N - X'_N] \geq 0, \quad \forall X_N \in R_+, \tag{46}$$

which leads us to conclude that

$$\sum_{i=1}^{N} F_i(X') \times [X_i - X'_i] = \langle F(X'), X - X' \rangle \geq 0, \quad \forall X \in \mathcal{K} \equiv R^N_+. \tag{47}$$

Therefore, X' solves VI$(F, \mathcal{K})$.

Hence, we have established that the set of stationary points of:

$$\dot{X} = \Pi_{\mathcal{K}}(X, -F(X)), \quad X(0) = X_0 \in \mathcal{K},$$

that is, X^* satisfying:

$$0 = \Pi_{\mathcal{K}}(X^*, -F(X^*)), \tag{48}$$

coincides with the set of solutions of VI$(F', \mathcal{K})$.

In other words we have proven that the set of equilibria of the electric power supply chain network are *independent* of the speeds of adjustment. We explore this issue computationally in Section 4.

3 The Euler Method

In this Section, we consider the computation of the stationary points of (34). Recall that the projected dynamical system (34) is a continuous-time adjustment process. However, in order to provide a means of "tracking" the trajectory, a discrete-time adjustment process is proposed, which is a special case of a general iterative scheme of Dupuis and Nagurney (1993) and is, in fact, an Euler method. It has been applied to-date to solve a plethora of dynamic network models (see, e.g., Nagurney and Zhang (1996), Nagurney and Dong (2002), Nagurney and Matsypura (2003)). Specifically, the complete statement of this method in context of our model takes the following form.

The Euler Method

Step 0: Initialization

Set $X^0 = (Q^{10}, Q^{20}, \gamma^0, \rho_3^0) \in \mathcal{K}$. Let $\mathcal{T}$ denote an iteration counter and set $\mathcal{T} = 1$. Set the sequence $\{\alpha_{\mathcal{T}}\}$ so that $\sum_{\mathcal{T}=1}^{\infty} \alpha_{\mathcal{T}} = \infty$, $\alpha_{\mathcal{T}} > 0$, and $\alpha_{\mathcal{T}} \to 0$, as $\mathcal{T} \to \infty$ (which is a requirement for convergence).

Step 1: Computation

Compute $X^{\mathcal{T}} = (Q^{1\mathcal{T}}, Q^{2\mathcal{T}}, \gamma^{\mathcal{T}}, \rho_3^{\mathcal{T}}) \in \mathcal{K}$ by solving the variational inequality subproblem:

$$\langle X^{\mathcal{T}} + \alpha_{\mathcal{T}} F(X^{\mathcal{T}-1}) - X^{\mathcal{T}-1}, X - X^{\mathcal{T}} \rangle \geq 0, \qquad \forall X \in \mathcal{K}. \tag{49}$$

Step 2: Convergence Verification

If $|X^{\mathcal{T}} - X^{\mathcal{T}-1}| \leq \epsilon$, with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $\mathcal{T} := \mathcal{T} + 1$, and go to Step 1.

Convergence results for the Euler method can be found in Dupuis and Nagurney (1993). See the book by Nagurney and Dong (2002) for applications of this algorithm to other network problems in the context of dynamic supply chains and financial networks with intermediation.

Due to the fact that the feasible set underlying the dynamic electric power supply chain network model is that of the nonnegative orthant, the solution of (49) takes on a very simple form for computational purposes, which we state explicitly below.

Computation of Electric Power Transactions between Tiers Using Explicit Formulae

In particular, we have that the electric power transactions between tiers of decision-makers in the electric power supply chain network can be computed explicitly and in closed form as follows for a given iteration $\mathcal{T}$:

$$q_{gs}^{\mathcal{T}} = \max\{0, q_{gs}^{\mathcal{T}-1} - \alpha^{\mathcal{T}} \phi_{gs} \Big(\frac{\partial f_g(Q^{1(\mathcal{T}-1)})}{\partial q_{gs}} + \frac{\partial c_{gs}(Q^{1(\mathcal{T}-1)})}{\partial q_{gs}} + \alpha_g \frac{\partial r_g(Q^{1(\mathcal{T}-1)})}{\partial q_{gs}} + \frac{\partial c_s(Q^{1(\mathcal{T}-1)}, Q^{2(\mathcal{T}-1)})}{\partial q_{gs}} +$$

$$\frac{\partial \hat{c}_{gs}(Q^{1(\mathcal{T}-1)})}{\partial q_{gs}} + \beta_s \frac{\partial r_s(Q^{1(\mathcal{T}-1)}, Q^{2(\mathcal{T}-1)})}{\partial q_{gs}} - \gamma_s^{\mathcal{T}-1} \Big) \}, \quad \forall g, s; \tag{50}$$

and

$$q_{sk}^{t\mathcal{T}} = \max\{0, q_{sk}^{t(\mathcal{T}-1)} - \alpha^{\mathcal{T}} \phi_{sk}^t \Big(\gamma_s^{\mathcal{T}-1} + \hat{c}_{sk}^t(Q^{2(\mathcal{T}-1)}) + \frac{\partial c_s(Q^{1(\mathcal{T}-1)}, Q^{2(\mathcal{T}-1)})}{\partial q_{sk}^t} + \frac{\partial c_{sk}^t(Q^{2(\mathcal{T}-1)})}{\partial q_{sk}^t} +$$

$$\beta_s \frac{\partial r_s(Q^{1(\mathcal{T}-1)}, Q^{2(\mathcal{T}-1)})}{\partial q_{sk}^t} - \rho_{3k}^{\mathcal{T}-1} \Big) \}, \quad \forall s, k, t. \tag{51}$$

Computation of Electricity Prices Using Explicit Formulae

Similarly, the prices associated with the middle tier and the bottom tier of nodes of the dynamic electric power supply chain network can also be computed explicitly and in closed form, at a given iteration $\mathcal{T}$, according to:

$$\gamma_s^{\mathcal{T}} = \max\{0, \gamma_s^{\mathcal{T}-1} - \alpha^{\mathcal{T}} \phi_s \left(\sum_{g=1}^{G} q_{gs}^{\mathcal{T}-1} - \sum_{k=1}^{K} \sum_{t=1}^{T} q_{sk}^{t(\mathcal{T}-1)} \right) \}, \quad \forall s, \tag{52}$$

and

$$\rho_{3k}^{\mathcal{T}} = \max\{0, \rho_{3k}^{\mathcal{T}-1} - \alpha^{\mathcal{T}} \phi_k \left(\sum_{s=1}^{S} \sum_{t=1}^{T} q_{sk}^{t(\mathcal{T}-1)} - d_k(\rho_3^{\mathcal{T}-1}) \right) \}, \quad \forall k. \tag{53}$$

Moreover, the above computations can be done simultaneously and independently. The Euler method, hence, provides us with a discrete-time adjustment process by which the network decision-makers adjust their electric power transactions and the prices associated with the middle and bottom tiers of the network also adjust.

4 Numerical Examples

In this Section, we apply the Euler method to several numerical examples. The Euler method was coded in Java and Matlab, and the computer used was a Dell Latitude X200 system. The convergence criterion utilized was that the absolute value of the electric power transactions and prices between two successive iterations differed by no more than 10^{-4}. The parameter $\alpha_{\mathcal{T}}$ in the Euler method was set to $\{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, ...\}$ for all the examples.

The structure of the electric power supply chain network for the numerical examples is given in Figure 2. Specifically, we assumed that there were three power generators, two power suppliers, one transmission service provider, and three demand markets. Hence, we had that $G = 3$, $S = 2$, $T = 1$, and $K = 3$.

In all the examples, we assumed that the demands associated with the demand markets followed a uniform distribution. In particular, we assumed that the random demand $\hat{d}_k(\rho_{3k})$ associated with demand market k is uniformly distributed in $\left[0, \frac{b_k}{\rho_{3k}+1}\right]$, with $b_k > 0$; $k = 1, 2, 3$. Therefore, we have that

$$P_k(x, \rho_{3k}) = \frac{x(\rho_{3k} + 1)}{b_k}; \quad k = 1, 2, 3, \tag{54}$$

$$F_k(x, \rho_{3k}) = \frac{\rho_{3k} + 1}{b_k}; \quad k = 1, 2, 3, \tag{55}$$

$$d_k(\rho_{3k}) = E(\hat{d}_k) = \frac{1}{2} \frac{b_k}{(\rho_{3k} + 1)}; \quad k = 1, 2, 3. \tag{56}$$

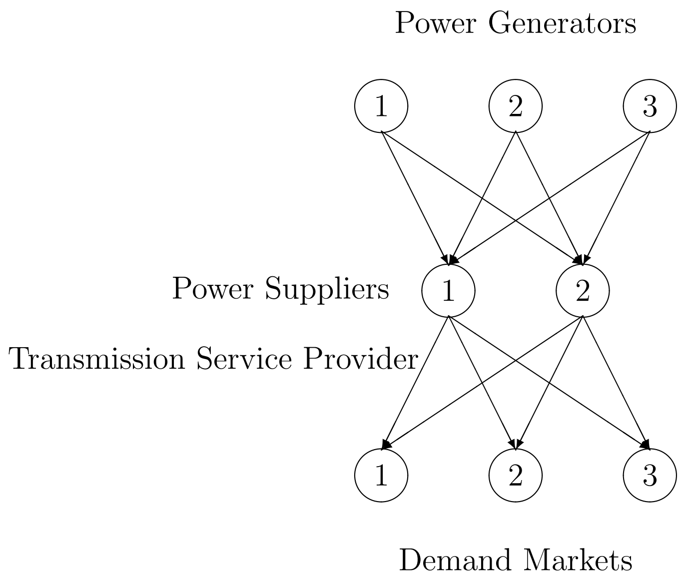


Figure 2: Electric Power Supply Chain Network for the Numerical Examples

It is straightforward to verify that the expected demand function $d_k(\rho_{3k})$ associated with demand market k is a decreasing function of the price at this demand market.

There are four experiments associated with each example. The experiments differ from one another in the values of the vector of speed of adjustment parameters ϕ, where

$$\phi \equiv \{\phi_{11}, \ldots, \phi_{GS}, \phi^1_{11}, \ldots, \phi^T_{SK}, \phi_1, \ldots, \phi_S, \phi_1, \ldots, \phi_K\}.$$

Table 1 presents the values of the ϕ's that were used in each of the experiments for numerical examples 1 through 4.

The Euler method was initialized as follows: all variables were set equal to zero, except for the initial demand market prices ρ^0_{3k}, which were set to 1 for all k. For completeness and illustrative purposes, the graphs of the dynamic trajectories of the variables: q_{11}. q_{12}, q^1_{11}, γ_1, and ρ^1_{31} are given in the Appendix for Examples 1 through 4. The behavior of the remaining variables is similar but space limitations prevent us from including the trajectories of all the variables over time.

Example 1

The power generating cost functions for the power generators were given by:

$$f_1(q) = 2.5q_1^2 + q_1q_2 + 2q_1, \quad f_2(q) = 2.5q_2^2 + q_1q_2 + 2q_2, \quad f_3(q) = .5q_3^2 + .5q_1q_3 + 2q_3.$$

The transaction cost functions faced by the power generators and associated with transacting with the power suppliers were given by:

$c_{11}(Q^1) = .5q_{11}^2 + 3.5q_{11}$, $c_{12}(Q^1) = .5q_{12}^2 + 3.5q_{12}$, $c_{21}(Q^1) = .5q_{21}^2 + 3.5q_{21}$, $c_{22}(Q^1) = .5q_{22}^2 + 3.5q_{22}$,

Associated Variable	Parameter	Experiment 1	Experiment 2	Experiment 3	Experiment 4
q_{11}	ϕ_{11}	1	1	0.5	0.5
q_{12}	ϕ_{12}	1	1	1	0.7
q_{21}	ϕ_{21}	1	1	1	1
q_{22}	ϕ_{22}	1	1	1	1
q_{31}	ϕ_{31}	1	1	1	1
q_{32}	ϕ_{32}	1	1	1	1
q^1_{11}	ϕ^1_{11}	1	1	1	1
q^1_{12}	ϕ^1_{12}	1	1	1	1
q^1_{13}	ϕ^1_{13}	1	1	1	1
q^1_{21}	ϕ^1_{21}	1	1	1	1
q^1_{22}	ϕ^1_{22}	1	1	1	1
q^1_{23}	ϕ^1_{23}	1	1	1	1
γ_1	ϕ_1	1	1	1	0.5
γ_2	ϕ_2	1	1	1	0.3
ρ_{31}	ϕ_1	1	0.5	0.5	0.5
ρ_{32}	ϕ_2	1	0.5	0.5	0.5
ρ_{33}	ϕ_3	1	0.5	0.5	0.5

Table 1: Values of ϕ

$$c_{31}(Q^1) = .5q_{31}^2 + 2q_{31}, \quad c_{32}(Q^1) = .5q_{32}^2 + 2q_{32}.$$

The operating costs of the power suppliers, in turn, were given by:

$$c_1(Q^1, Q^2) = .5(\sum_{i=1}^{3} q_{i1})^2, \quad c_2(Q^1, Q^2) = .5(\sum_{i=1}^{3} q_{i2})^2.$$

The b_ks (cf. (54) – (56)) were set to 1000 for all k.

In Example 1, we set all the weights associated with risk minimization to zero, that is, we had that $\alpha_g = 0$ for $g = 1, 2, 3$ and $\beta_s = 0$ for $s = 1, 2$. This means that in the first example all the generators and all the suppliers were concerned with profit maximization exclusively.

It is evident from Table 2 that in each of the four experiments the electric power supply chain network system achieves the equilibrium state, as substantiated by Theorem 4. In other words, the equilibrium electric power flow and price patterns are essentially the same, regardless of the adjustment speeds.

Another interesting (but expected) feature (cf. Figures 3 through 7 in the Appendix) is that the trajectories of the variables change depending on the values of the ϕ parameters. This behavior is reasonable since, indeed, one would expect that larger, that is, faster speeds of adjustment (i.e.,

Computed Equilibrium Values

	Experiment 1	Experiment 2	Experiment 3	Experiment 4
Number of Iterations	1101	892	794	2702
q_{11}^*	2.12	2.12	2.12	2.12
q_{12}^*	2.12	2.12	2.12	2.13
q_{21}^*	2.12	2.12	2.12	2.12
q_{22}^*	2.12	2.12	2.12	2.13
q_{31}^*	8.99	8.98	8.98	8.99
q_{32}^*	8.99	8.98	8.98	9.00
q_{11}^{*1}	4.41	4.41	4.41	4.41
q_{12}^{*1}	4.41	4.41	4.41	4.41
q_{13}^{*1}	4.41	4.41	4.41	4.41
q_{21}^{*1}	4.41	4.41	4.41	4.41
q_{22}^{*1}	4.41	4.41	4.41	4.41
q_{23}^{*1}	4.41	4.41	4.41	4.41
γ_1^*	46.32	46.27	46.27	46.37
γ_2^*	46.32	46.27	46.27	46.37
ρ_{31}^*	55.73	55.68	55.68	55.78
ρ_{32}^*	55.73	55.68	55.68	55.78
ρ_{33}^*	55.73	55.68	55.68	55.78

Table 2: Solutions to Example 1

those with higher values of ϕ) would imply that the associated equilibrium value of the variable involved would be reached "more quickly."

Computed Equilibrium Values

	Experiment 1	Experiment 2	Experiment 3	Experiment 4
Number of Iterations	1117	926	829	2732
q_{11}^*	1.83	1.83	1.83	1.83
q_{12}^*	1.83	1.83	1.83	1.83
q_{21}^*	2.20	2.19	2.19	2.20
q_{22}^*	2.20	2.19	2.19	2.20
q_{31}^*	9.16	9.15	9.15	9.18
q_{32}^*	9.16	9.15	9.15	9.18
q_{11}^{*1}	4.40	4.39	4.39	4.40
q_{12}^{*1}	4.40	4.39	4.39	4.40
q_{13}^{*1}	4.40	4.39	4.39	4.40
q_{21}^{*1}	4.40	4.39	4.39	4.39
q_{22}^{*1}	4.40	4.39	4.39	4.39
q_{23}^{*1}	4.40	4.39	4.39	4.39
γ_1^*	46.51	46.46	46.46	46.57
γ_2^*	46.51	46.46	46.46	46.57
ρ_{31}^*	55.91	55.86	55.86	55.96
ρ_{32}^*	55.91	55.86	55.86	55.96
ρ_{33}^*	55.91	55.86	55.86	55.96

Table 3: Solutions to Example 2

Example 2

Example 2 was constructed from Example 1 as follows. We kept the data as in Example 1 but we assumed now that the first generator was a multicriteria decision-maker and concerned with risk minimization with his risk function being given by:

$$r_1 = (\sum_{s=1}^{2} q_{1s} - 2)^2.$$

The weight associated with his risk measure was $\alpha_1 = 1$.

Addition of a risk minimization criterion to the optimization problem of power generator 1 resulted in an overall decrease of the amount of electric power generated and transacted by this decision-maker (see Table 3). Consequently, the shortages in the supply were partially compensated by power generators 2 and 3. As a result, the prices at demand markets and power suppliers increased. Please refer to Figures 8 through 12 for the dynamic trajectories of selected electric power flow and price variables.

Example 3

Example 3 was constructed from Example 2 as follows. We kept the data as in Example 2 but we

Computed Equilibrium Values

	Experiment 1	Experiment 2	Experiment 3	Experiment 4
Number of Iterations	1120	936	841	2738
q_{11}^*	1.87	1.87	1.87	1.87
q_{12}^*	1.77	1.77	1.77	1.77
q_{21}^*	2.24	2.24	2.24	2.24
q_{22}^*	2.14	2.14	2.14	2.14
q_{31}^*	9.19	9.18	9.18	9.20
q_{32}^*	9.09	9.08	9.08	9.10
q_{11}^{*1}	4.43	4.43	4.43	4.43
q_{12}^{*1}	4.43	4.43	4.43	4.43
q_{13}^{*1}	4.43	4.43	4.43	4.43
q_{21}^{*1}	4.33	4.33	4.33	4.33
q_{22}^{*1}	4.33	4.33	4.33	4.33
q_{23}^{*1}	4.33	4.33	4.33	4.33
γ_1^*	46.61	46.56	46.56	46.66
γ_2^*	46.71	46.66	46.66	46.76
ρ_{31}^*	56.04	55.99	55.99	56.10
ρ_{32}^*	56.04	55.99	55.99	56.10
ρ_{33}^*	56.04	55.99	55.99	56.10

Table 4: Solutions to Example 3

assumed now that the second supplier was a multicriteria decision-maker and concerned with risk minimization with his risk function being given by:

$$r_2 = \sum_{g=1}^{3} q_{g2}.$$

The weight associated with his risk measure was $\beta_2 = 0.5$. Table 4 presents computational results for this Example.

From the results in Table 4 we can observe that an addition of the risk minimization criterion to the optimization problem of power supplier 2 resulted in changes to the overall equilibrium electric power flow pattern. More electric power is now being traded trough power supplier 1 whose overall costs are lower than those of power supplier 2. Because all demand markets can satisfy their demand for electric power through either of the power suppliers, the change in final prices is insignificant. Figures 13 through 17 provide illustrations of the dynamic trajectories for the same variables as the displayed Figures for the preceding Examples.

Example 4

Example 4 was constructed from Example 3 as follows. We kept the data as in Example 3 except

Computed Equilibrium Values

	Experiment 1	Experiment 2	Experiment 3	Experiment 4
Number of Iterations	1186	706	323	2705
q_{11}^*	2.14	2.14	2.14	2.14
q_{12}^*	2.04	2.04	2.04	2.04
q_{21}^*	2.63	2.63	2.63	2.63
q_{22}^*	2.53	2.53	2.53	2.53
q_{31}^*	10.71	10.70	10.70	10.72
q_{32}^*	10.61	10.60	10.60	10.62
q_{11}^{*1}	7.50	7.50	7.50	7.50
q_{12}^{*1}	3.98	3.98	3.98	3.98
q_{13}^{*1}	3.98	3.98	3.98	3.98
q_{21}^{*1}	7.40	7.40	7.40	7.40
q_{22}^{*1}	3.88	3.88	3.88	3.88
q_{23}^{*1}	3.88	3.88	3.88	3.88
γ_1^*	53.59	53.54	53.56	53.64
γ_2^*	53.69	53.64	53.66	53.74
ρ_{31}^*	66.09	66.04	66.06	66.14
ρ_{32}^*	62.57	62.52	62.55	62.63
ρ_{33}^*	62.57	62.52	62.55	62.63

Table 5: Solutions to Example 4

for the parameter b_k (cf. (54) – (56)) for the first demand market which was changed to 2000. The computational results are presented in Table 5.

An interesting fact to note here is that even though the change in demand appears only in the first demand market, it affects the entire electric power supply chain network system and the prices throughout the entire system rise. Figures 18 through 22 provide the dynamic trajectories for this Example of the electric power flow and price variables as described above.

The changes in the equilibrium patterns described above may not be evident from the values presented in Tables 2 through 5 due to the fact that the absolute differences are less than 1. For example, the equilibrium price for demand market 1 in the Experiment 1 is equal to 55.73 in Example 1 and 55.91 in Example 2. It may seem that this difference is insignificant and irrelevant to the changes in the initial data. In order to support our previous conclusions and make the changes in the equilibrium patterns more evident we constructed Examples 5, 6, 7, and 8 where the maximum demand for each demand market was increased a hundredfold. This change led to the overall increase in the magnitude of the computed equilibrium electric power flows and prices and allowed us to observe previously discovered behavior more clearly.

Computed Equilibrium Values

	Example 5	Example 6	Example 7	Example 8
Number of Iterations	2513	2566	2566	2693
q_{11}^*	24.07	18.02	18.07	20.76
q_{12}^*	24.07	18.02	17.97	20.66
q_{21}^*	24.07	25.59	25.64	29.52
q_{22}^*	24.07	25.59	25.54	29.42
q_{31}^*	96.78	100.35	100.37	115.50
q_{32}^*	96.78	100.35	100.27	115.40
q_{11}^{*1}	48.30	47.99	48.02	80.29
q_{12}^{*1}	48.30	47.99	48.02	42.74
q_{13}^{*1}	48.30	47.99	48.02	42.74
q_{21}^{*1}	48.30	47.99	47.92	80.19
q_{22}^{*1}	48.30	47.99	47.92	42.64
q_{23}^{*1}	48.30	47.99	47.92	42.64
γ_1^*	463.31	467.03	467.12	536.89
γ_2^*	463.31	467.03	467.22	536.99
ρ_{31}^*	516.61	520.02	520.15	622.18
ρ_{32}^*	516.61	520.02	520.15	584.63
ρ_{33}^*	516.61	520.02	520.15	584.63

Table 6: Solutions to Examples 5, 6, 7, and 8

Examples 5, 6, 7, and 8

Examples 5, 6, 7, and 8 were constructed from Examples 1 through 4 as follows. In Example 5 we kept the data as in Example 1 except for the parameter b_k (cf. (54) – (56)) for all demand markets, which was increased a hundredfold. Similar changes were done to the data in Examples 2, 3, and 4 to derive the data for Examples 6, 7, and 8, respectively. The values of parameters ϕ were kept equal to 1 for all these Examples. The results are presented in Table 6.

Clearly, the above examples are stylized but they, nevertheless, demonstrate the efficacy of the model and the computational procedure. One may now conduct numerous simulations by modifying the data as well as adding decision-makers with their associated functions and weights and investigating the effects on the equilibrium product transactions and product prices.

5 Summary and Conclusions

In this paper, we have developed a dynamic, multitiered electric power supply chain network model consisting of power generators, suppliers, transmission service providers, and demand markets and analyzed it both from theoretical and computational perspectives. The model handles decision-making under risk and uncertainty. Specifically, the power generators as well as the suppliers are assumed to be multicriteria decision-makers and concerned not only with profit maximization but also with risk minimization. The demands for the electric power, in turn, are random.

The framework permits for the handling of as many power generators, power suppliers, transmission service providers, and demand markets, as required by the specific application. Moreover, the generality of the framework allows for the demand to have almost any distribution as long as it satisfies certain technical conditions.

The dynamic model, which is formulated as a projected dynamical system, provides the evolution of the electric power transactions between tiers of the electric power supply chain network as well as the prices associated with the electric power at different tiers of the network. The model generalizes the recent work of Nagurney and Matsypura (2004) to include supply-side risk and demand uncertainty. Moreover, it provides a natural dynamics as we illustrated through several numerical examples. An important theoretical result, established in Theorem 4, states that the set of equilibria of the electric power supply chain network are *independent* of the speeds of adjustment, a result which was also validated computationally.

As was stated by the US Energy Information Administration: "It is unclear what effect the introduction of competition at the retail level [in electricity markets] will have on future retail prices." (http://www.eia.doe.gov/cneaf/electricity/page/fact_sheets/retailprice.html) With the development of this model we have attempted to make one contribution towards understanding the effects that the process of restructuring has had and will have on the electric power industry.

Acknowledgments

This research was supported, in part, by NSF Grant No.: IIS-0002647. This support is gratefully acknowledged.

References

Arrow, K. J., Hurwicz, L., 1958. On the stability of the competitive equilibrium, I. Econometrica 26, 522-552.

Bazaraa, M. S., Sherali, H. D., Shetty, C. M., 1993. Nonlinear Programming: Theory and

Algorithms. John Wiley & Sons, New York.

Bertsekas, D. P., Tsitsiklis, J. N., 1989. Parallel and Distributed Computation - Numerical Methods. Prentice Hall, Englewood Cliffs, New Jersey.

Bolle, F., 1992. Supply function equilibria and the danger of tacit collusion: The case of spot markets for electricity. Energy Economics April, 94-102.

Borenstein, J., Bushnell, B., Knittel, C. R., 1999. Market power in electricity markets: Beyond concentration measures. The Energy Journal 20, 65-88.

Boucher, J., Smeers, Y., 2001. Alternative models of restructured electricity systems part 1: No market power. Operations Research 49, 821-838.

Bower, J., Bunn, D. W., 2000. Model-based comparisons of pool and bilateral markets for electricity. The Energy Journal 21, 1-29.

Casazza, J., Delea, F., 2003. Understanding Electric Power Systems. John Wiley & Sons, New York.

Chankong, V., Haimes, Y. Y., 1997. Multiobjective Decision Making: Theory and Methodology. North-Holland, New York.

Chao, H-P., Huntington, H. G., 1998. Designing Competitive Electricity Markets. Kluwer Academic Publishers, Boston, Massachusetts.

Chao, H-P., Peck, S., 1996. A market mechanism for electric power transmission. Journal of Regulatory Economics 10, 25-60.

Dafermos, S., Nagurney, A., 1987. Oligopolistic and competitive behavior of spatially separated markets. Regional Science and Urban Economics 17, 245-254.

Daxhelet, O., Smeers, Y., 2001. Variational inequality models of restructured electric systems. In: Applications and Algorithms of Complementarity. M. C. Ferris, O. L. Mangasarian, and J. S. Pang, Editors, Kluwer Academic Publishers, Boston, Massachusetts.

Day, C. J., Hobbs, B. F., Pang, J. S., 2002. Oligopolistic competition in power networks: A conjectured supply function approach. IEEE Transactions on Power Systems 17, 597-607.

Dupuis, P., Nagurney, A., 1993. Dynamical systems and variational inequalities. Annals of Operations Research 44, 9-42.

von der Fehr, N., Harbord, D., 1993. Spot market competition in the UK electricity industry. The Economic Journal 103, 531-546.

Fishburn, P. C., 1970. Utility Theory for Decision Making. John Wiley & Sons, New York.

Fisher, F. M., 1961. The stability of the Cournot oligopoly solution: The effects of speeds of adjustment and increasing marginal costs. The Review of Economic Studies 28, 125-135.

Green, R. J., Newbery, D. M., 1992. Competition in the British electricity spot market. Journal of Political Economy 100, 929-953.

Green, R. J., 1996. Increasing competition in the British electricity spot market. The Journal of Industrial Economics 44, 205-216.

Hobbs, B. F., Metzler, C., Pang, J. S., 2000. Strategic Gaming Analysis for Electric Power Networks: An MPEC Approach. IEEE Transactions on Power Systems 15, 638-645.

Hogan, W., 1992. Contract Networks for Electric Power Transmission. Journal of Regulatory Economics 4, 211-242.

Jing-Yuan, W., Smeers, Y., 1999. Spatial oligopolistic electricity models with Cournot generators and regulated transmission prices. Operations Research 47, 102-112.

Kahn, E. P., 1998. Numerical techniques for analyzing market power in electricity. The Electricity Journal 11, 34-43.

Keeney, R. L., Raiffa, H., 1993. Decisions with Multiple Objectives: Preferences and Value Trade-offs. Cambridge University Press, Cambridge, England.

Metzler, L. A., 1945. Stability of multiple markets: The Hicks conditions. Econometrica 13, 277-292.

Nagurney, A., Cruz, J., Dong, J., Zhang, D., 2005. Supply chain networks, electronic commerce, and supply side and demand side risk. European Journal of Operational Research 164, 120-142.

Nagurney, A., Dong, J., 2002. Supernetworks: Decision-Making for the Information Age. Edward Elgar Publishers, Cheltenham, England.

Nagurney, A., Matsypura, D., 2003. Dynamics of global supply chain supernetworks in a new era of risk and uncertainty. revised and resubmitted to Transportation Research E.

Nagurney, A., Matsypura, D., 2004. A supply chain network perspective for electric power generation, supply, transmission, and consumption. Proceedings of the International Conference on Computing, Communications and Control Technologies, Austin, Texas, Volume VI, pp 127-134; to appear in expanded and revised form in Advances in Computational Economics, Finance and Management Science. Computational Management Science, E. J. Kontoghiorghes and C. Gatu, Editors, Kluwer Academic Publishers, 2005.

Nagurney, A., Zhang, D., 1996. Projected Dynamical Systems and Variational Inequalities with Applications. Kluwer Academic Publishers, Boston, Massachusetts.

Nash, J. F., 1950. Equilibrium points in n-person games. Proceedings of the National Academy of Sciences, USA 36, 48-49.

Nash, J. F., 1951. Noncooperative games. Annals of Mathematics 54, 286-298.

Newbery, D. M., 1998. Competition, contracts, and entry in the electricity spot market. RAND Journal of Economics 29, 726-749.

Newbery, D. M., Pollitt, M. G., 1997. The restructuring and privatisation of Britains CEGB Was it worth it? The Journal of Industrial Economics 45, 269-303.

North American Electric Reliability Council, 1998. Reliability assessment 1998-2007. September.

Public Utilities Regulating Policies Act, 1978. P. L. 95-617 (92 Stat. 3117) 116, U. S. Code Sections 2601.

Samuelson, P. A., 1941. The stability of equilibrium: Comparative statics and dynamics. Econometrica 9, 97-120.

Schweppe, F. C., Caramanis, M. C., Tabors, R. E., R. E. Bohn, 1988. Spot Pricing of Electricity, Kluwer Academic Publishers, Boston, Massachusetts.

Secretary of Energy Advisory Board's (SEAB) Task Force on Electric System Reliability, 1998. Maintaining reliability in a competitive U. S. electric industry. September 29.

Takriti, S., Krasenbrink, B., Wu, L. S. -Y., 2000. Incorporating fuel constraints and electricity spot prices into the stochastic unit commitment problem. Operations Research 48, 268-280.

US Energy Information Administration, 2002. The changing structure of the electric power industry 2000: An update.

US-Canada Power System Outage Task Force, 2004. Final report on the August 14th, 2003 blackout in the United States and Canada: Causes and recommendations. April.

Willems, B., 2002. Modeling Cournot competition in an electricity market with transmission constraints. The Energy Journal 23, 95-125.

Wolfram, C. D., 1998. Strategic bidding in a multiunit auction: An empirical analysis of bids to supply electricity in England and Wales.RAND Journal of Economics 29, 703-725.

Wolfram, C. D., 1999. Measuring duopoly power in the British electricity spot market. American Economic Review 89, 805-826.

Wu, F., Varaiya, P., Spiller, P., Oren, S., 1996. Folk theorems on transmission access: Proofs and counterexample. Journal of Regulatory Economics 10, 5-24.

Yu, P. L., 1985. Multiple Criteria Decision Making – Concepts, Techniques, and Extensions. Plenum Press, New York.

Zaccour, G., Editor, 1998. Deregulation of Electric Utilities. Kluwer Academic Publishers, Boston, Massachusetts.

Zhang, D., Nagurney, A., 1995. On the stability of projected dynamical systems. Journal of Optimization Theory and Applications 85 97-124.

Zhang, D., Nagurney, A., 1996. Stability analysis of an adjustment process for oligopolistic market equilibrium modeled as a projected dynamical system. Optimization 36, 263-285.

6 Appendix

In this Appendix we provide graphs of the dynamic trajectories of some of the variables that appear in Examples 1 through 4. In particular we show the dynamics of the flow of electric power between power generator 1 and power supplier 1 (associated variable: q_{11}); between power generator 1 and power supplier 2 (associated variable: q_{12}); between power supplier 1 and demand market 1 (associated variable: q^1_{11}); the price associated with power supplier 1 (associated variable: γ_1); the price associated with demand market 1 (associated variable: ρ_{31}). These variables were selected for illustrative/representative purposes only.

Example 1

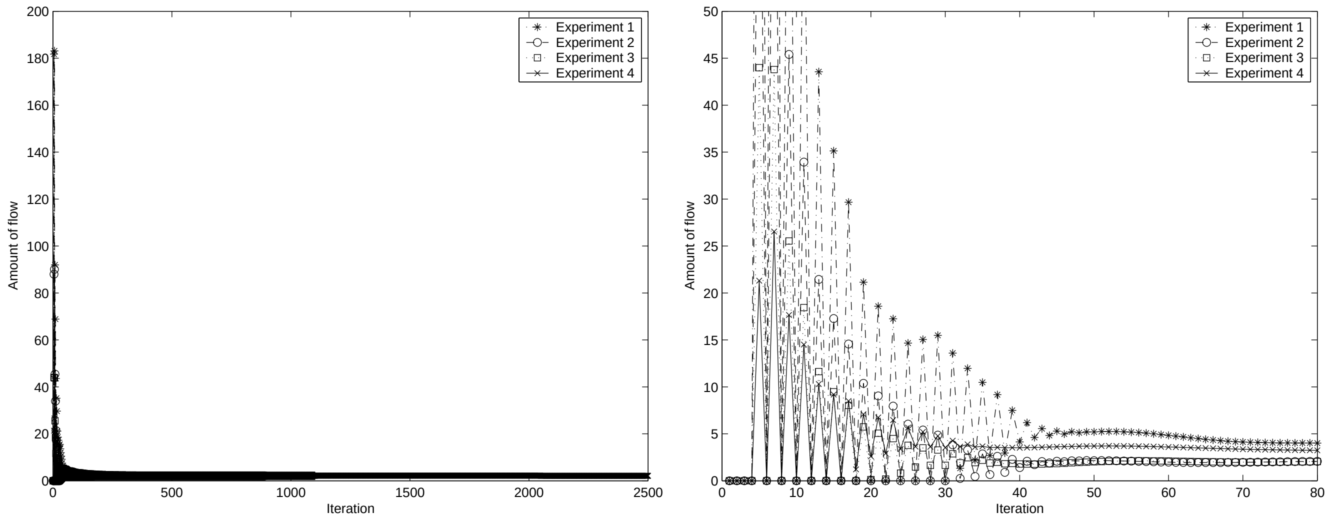


Figure 3: Dynamics of electric power transaction q_{11} complete (left) and first 80 iterations (right).

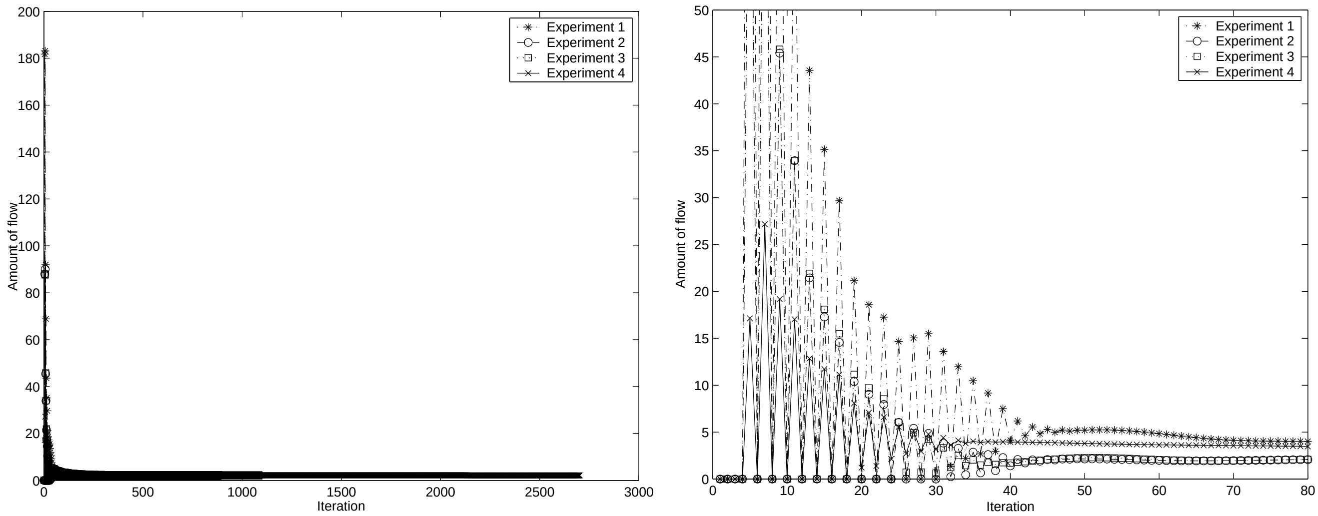


Figure 4: Dynamics of electric power transaction q_{12} complete (left) and first 80 iterations (right).

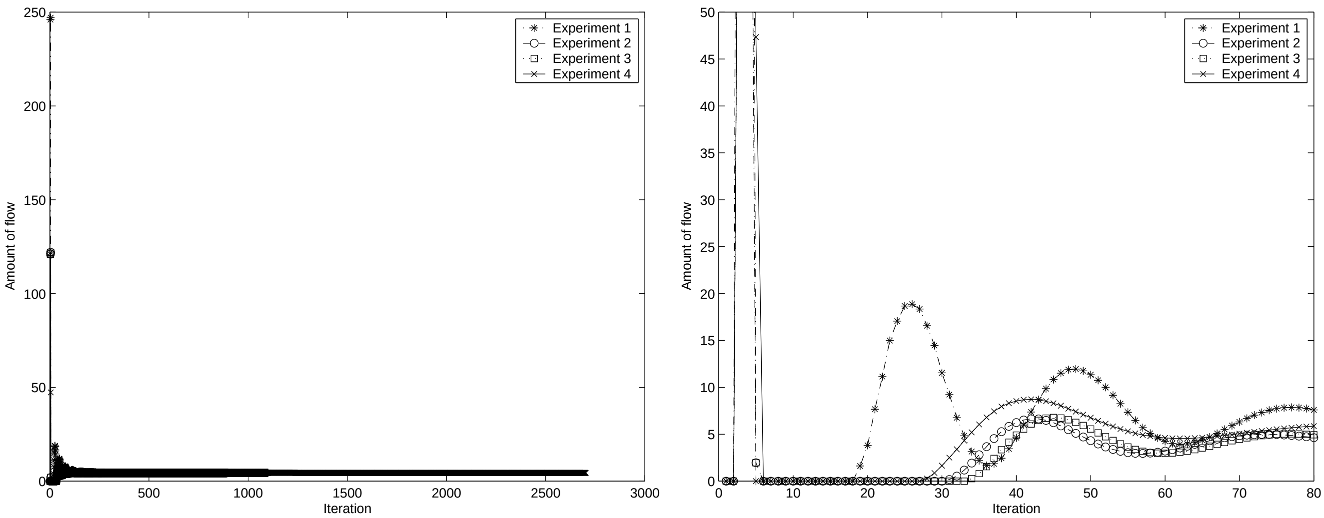


Figure 5: Dynamics of electric power transaction q_{11}^1 complete (left) and first 80 iterations (right).

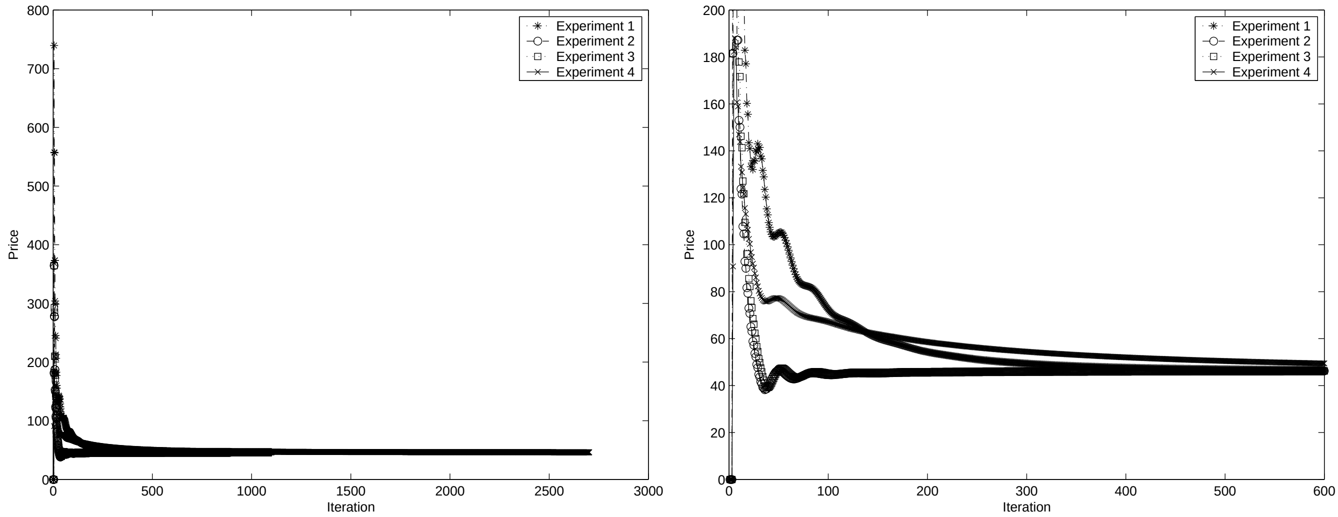


Figure 6: Dynamics of price γ_1 complete (left) and first 600 iterations (right).

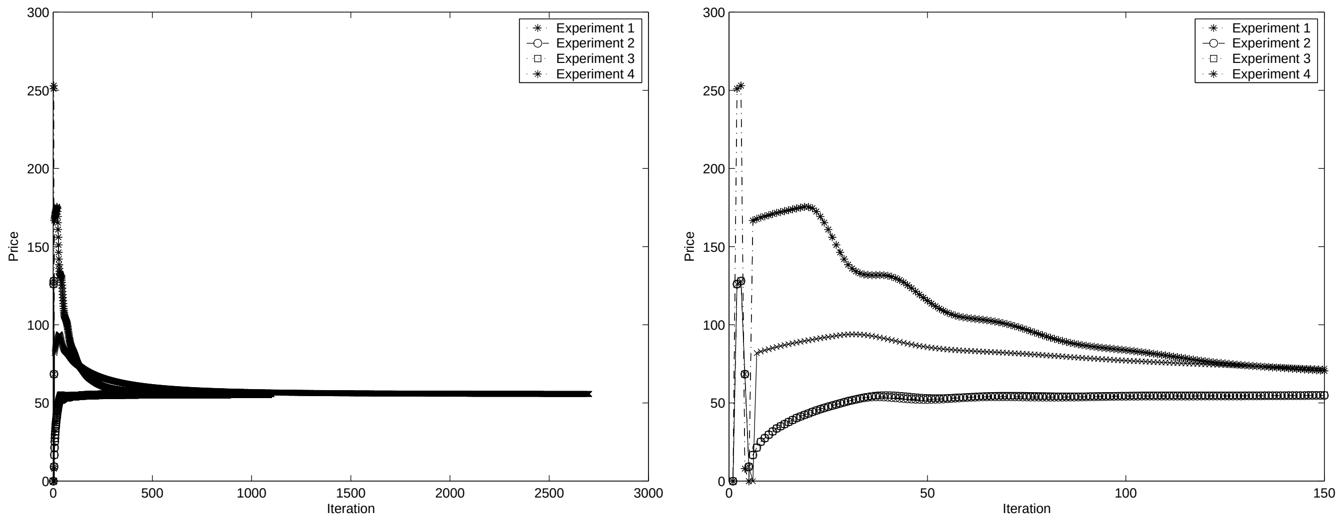


Figure 7: Dynamics of price ρ_{31} complete (left) and first 150 iterations (right).

Example 2

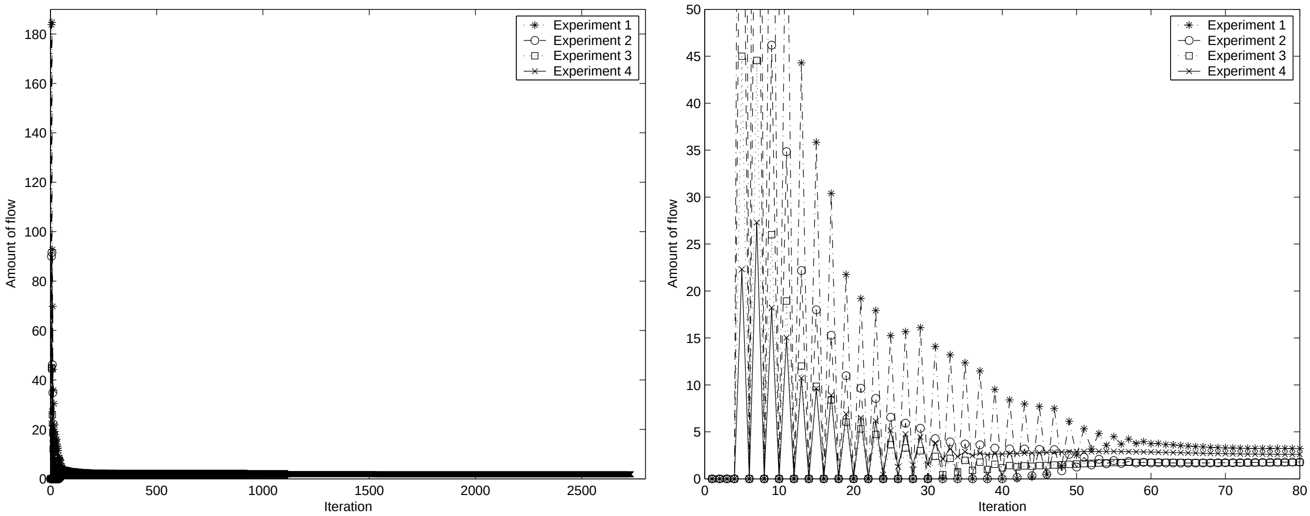


Figure 8: Dynamics of electric power transaction q_{11} complete (left) and first 80 iterations (right).

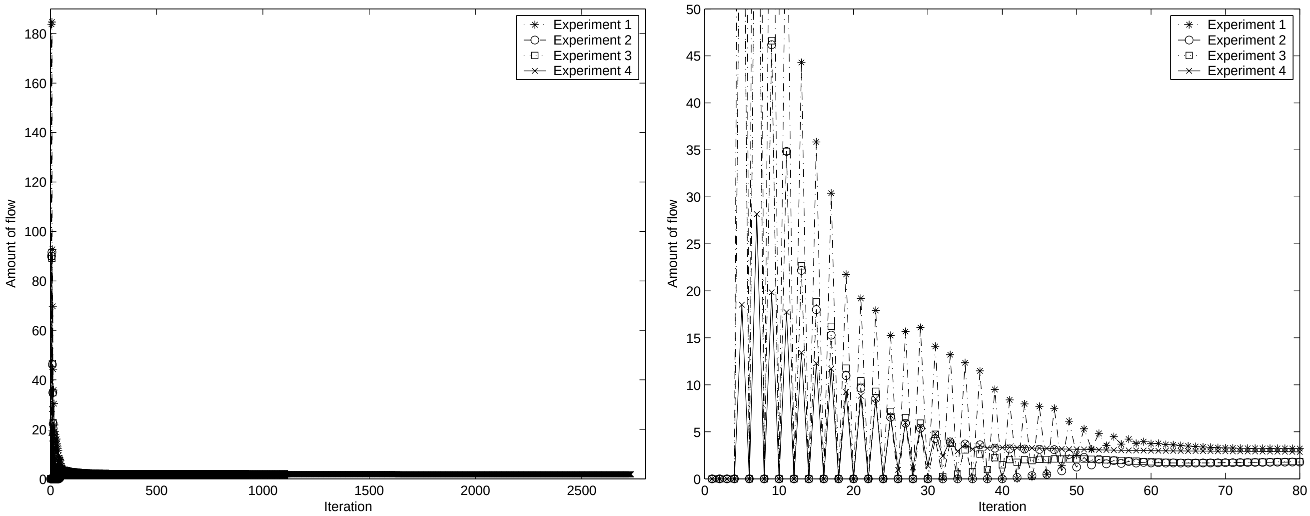


Figure 9: Dynamics of electric power transaction q_{12} complete (left) and first 80 iterations (right).

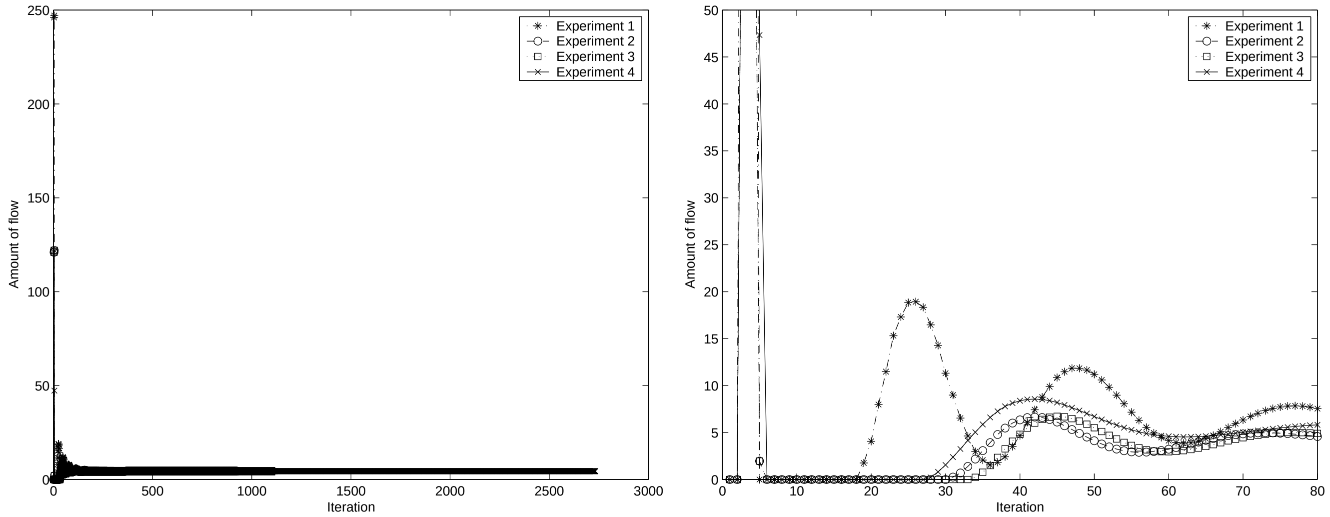


Figure 10: Dynamics of electric power transaction q_{11}^1 complete (left) and first 80 iterations (right).

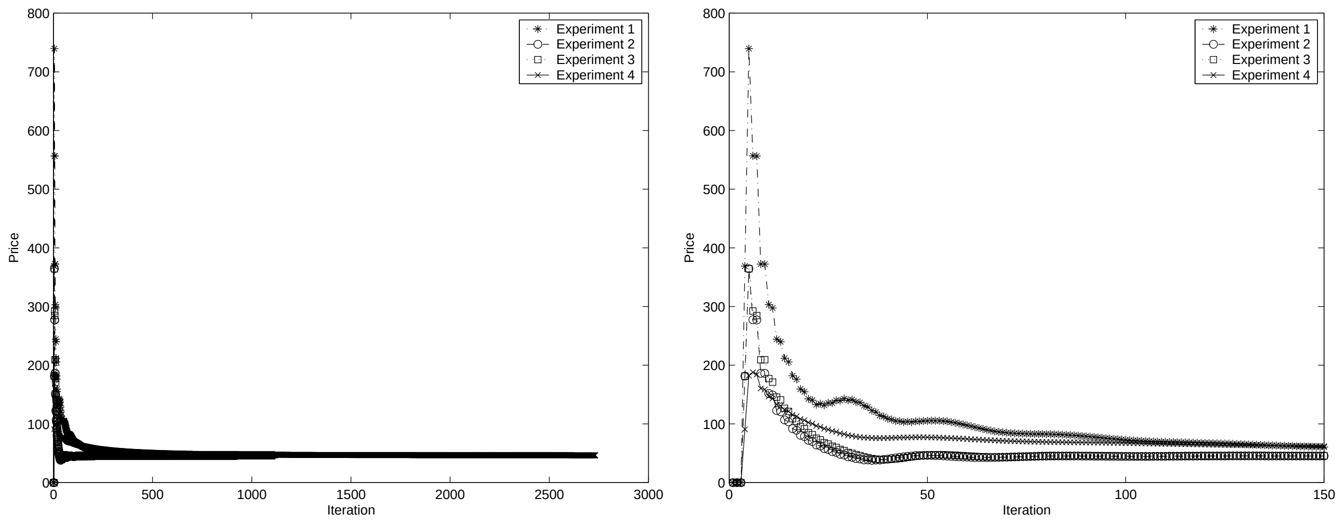


Figure 11: Dynamics of price γ_1 complete (left) and first 150 iterations (right).

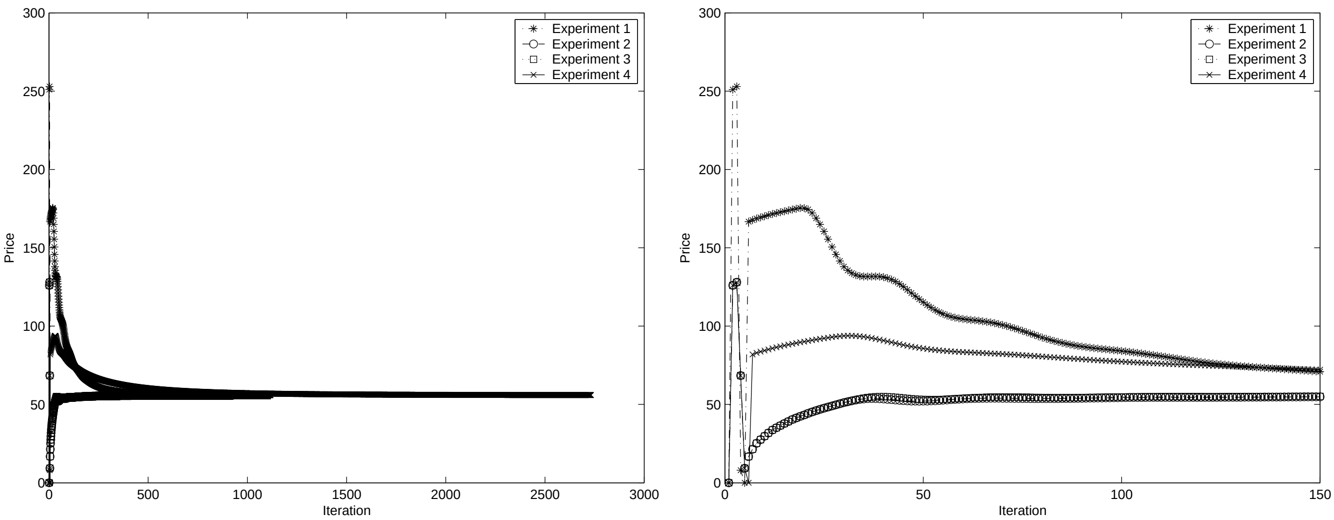


Figure 12: Dynamics of price ρ_{31} complete (left) and first 150 iterations (right).

Example 3

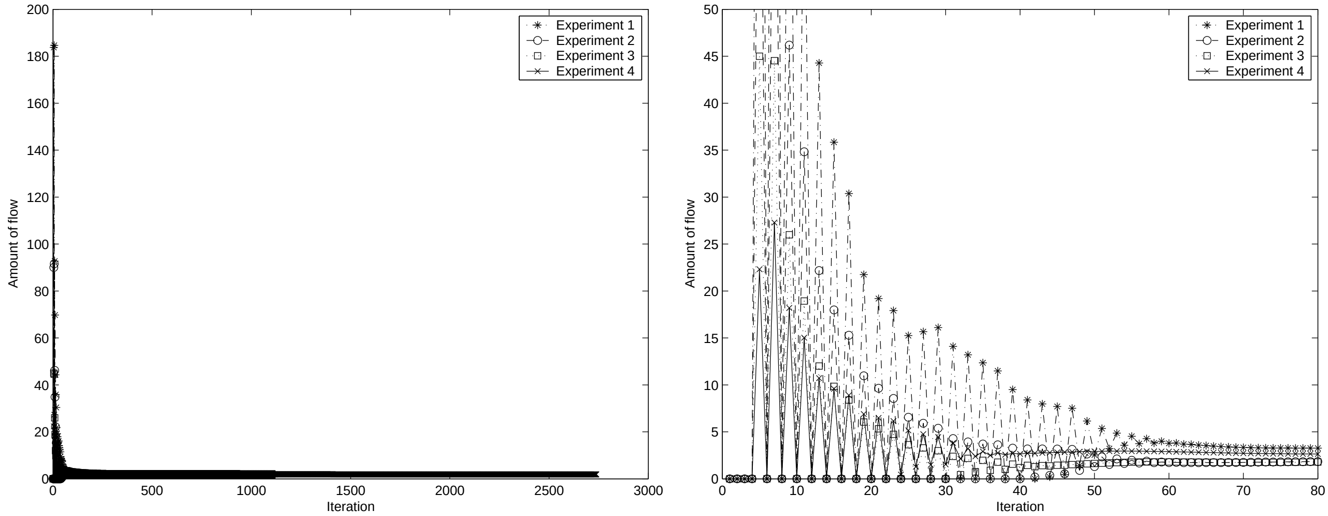


Figure 13: Dynamics of electric power transaction q_{11} complete (left) and first 80 iterations (right).

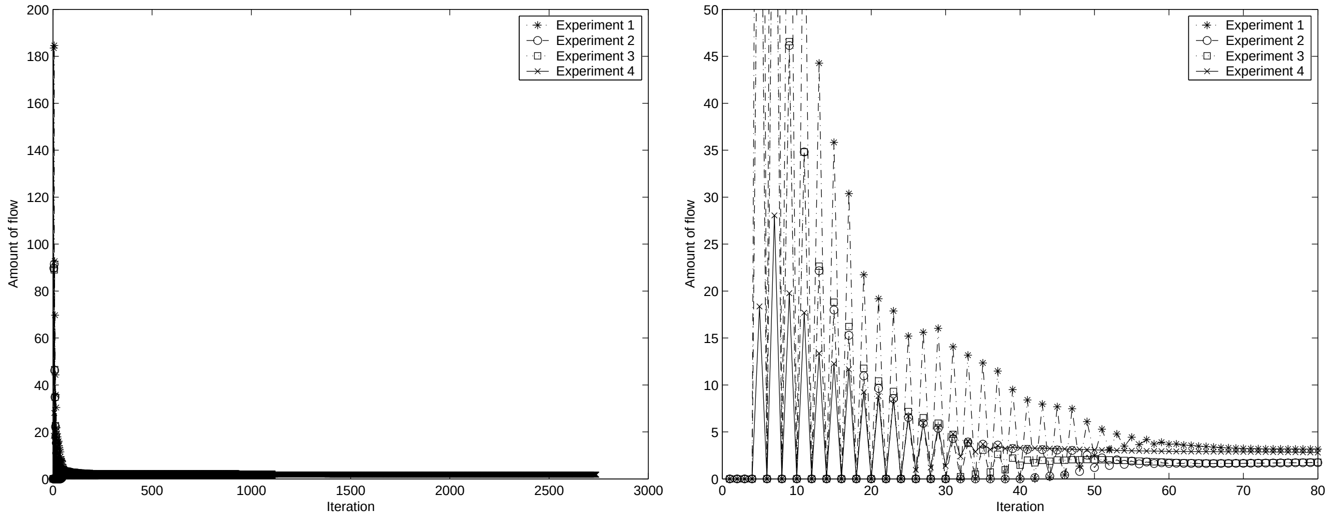


Figure 14: Dynamics of electric power transaction q_{12} complete (left) and first 80 iterations (right).

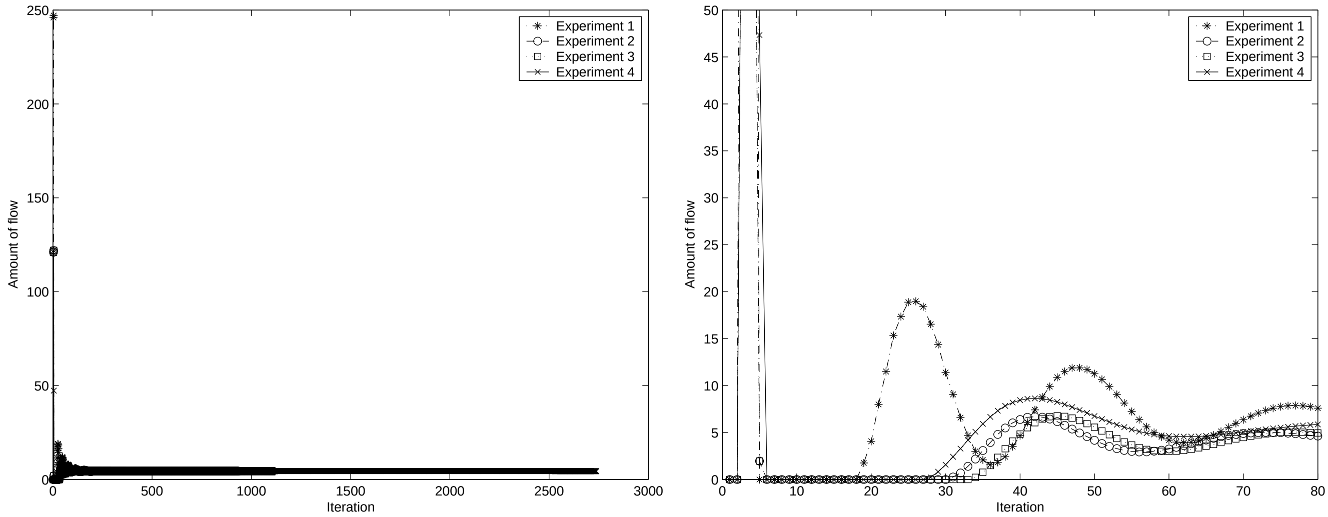


Figure 15: Dynamics of electric power transaction q_{11}^1 complete (left) and first 100 iterations (right).

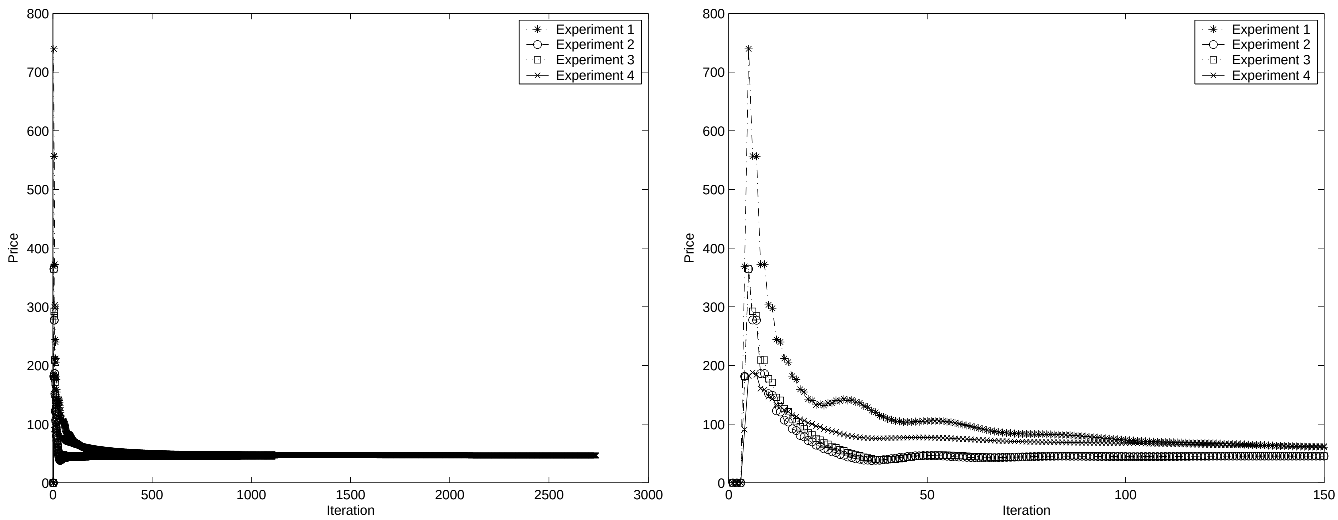
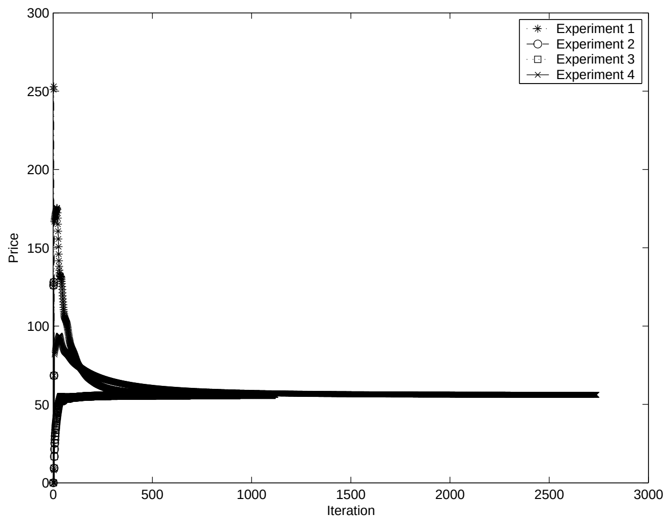


Figure 16: Dynamics of price γ_1 complete (left) and first 150 iterations (right).



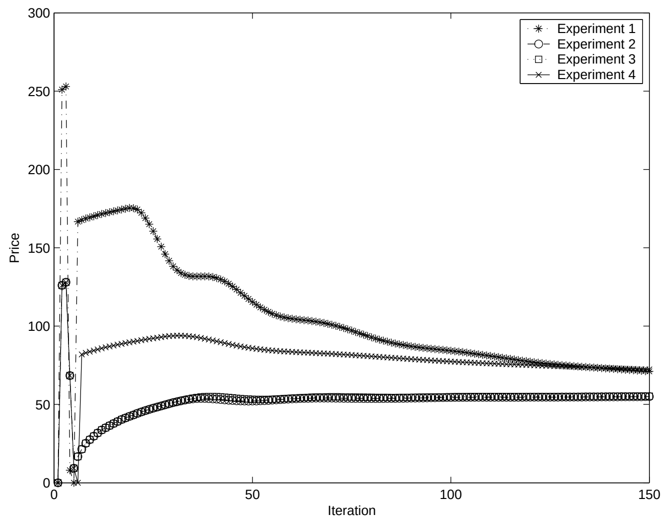


Figure 17: Dynamics of price ρ_{31} complete (left) and first 150 iterations (right).

Example 4

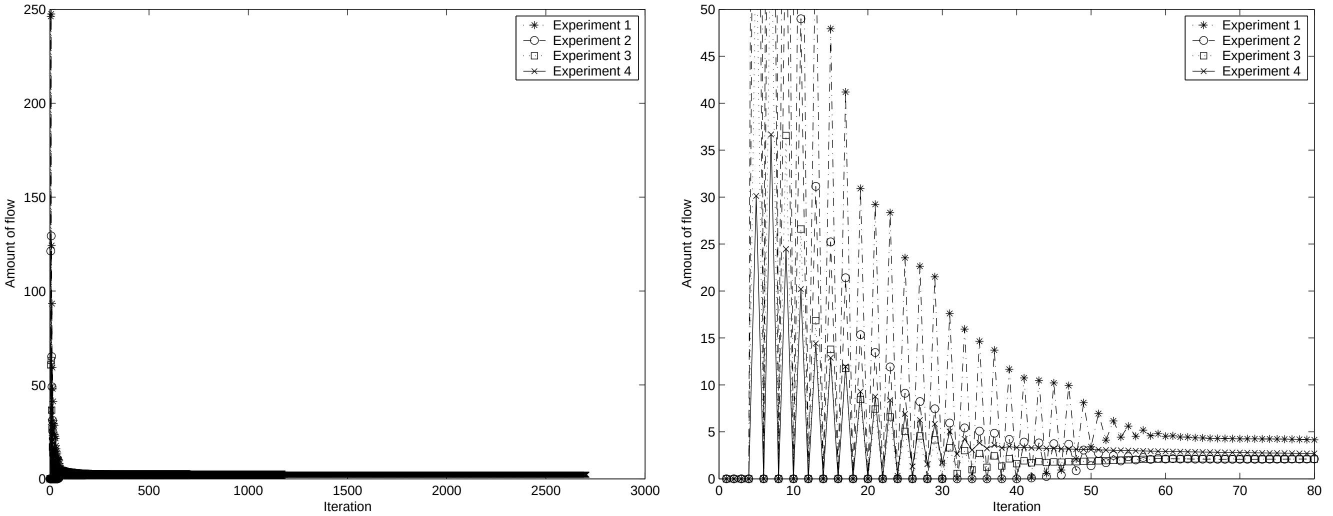


Figure 18: Dynamics of electric power transaction q_{11} complete (left) and first 80 iterations (right).

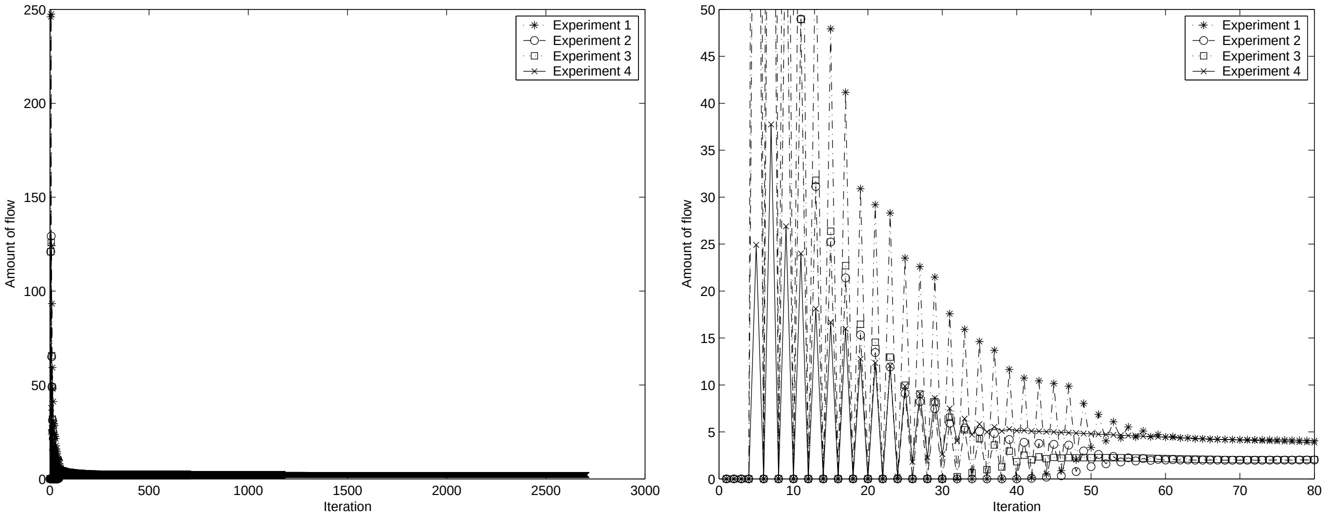


Figure 19: Dynamics of electric power transaction q_{12} complete (left) and first 80 iterations (right).

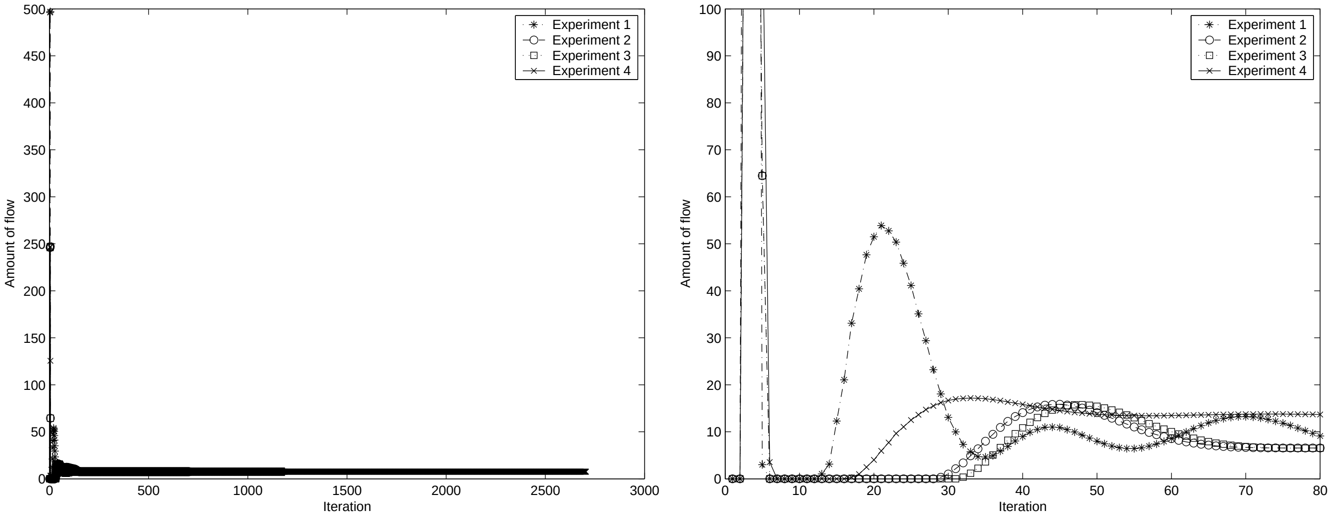


Figure 20: Dynamics of electric power transaction q_{11}^1 complete (left) and first 80 iterations (right).

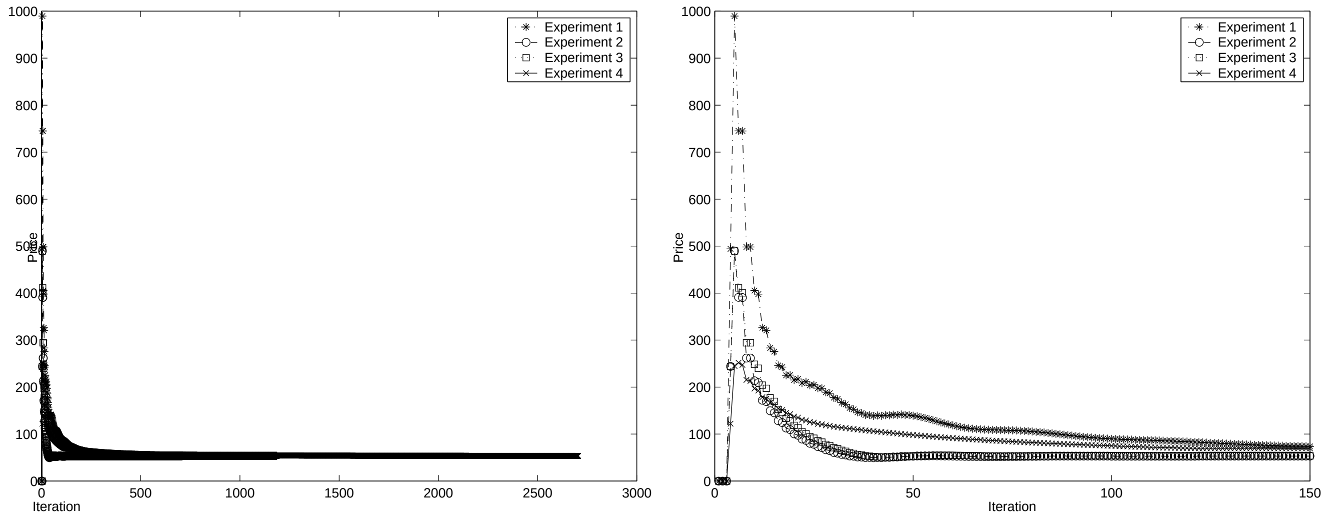


Figure 21: Dynamics of price γ_1 complete (left) and first 150 iterations (right).

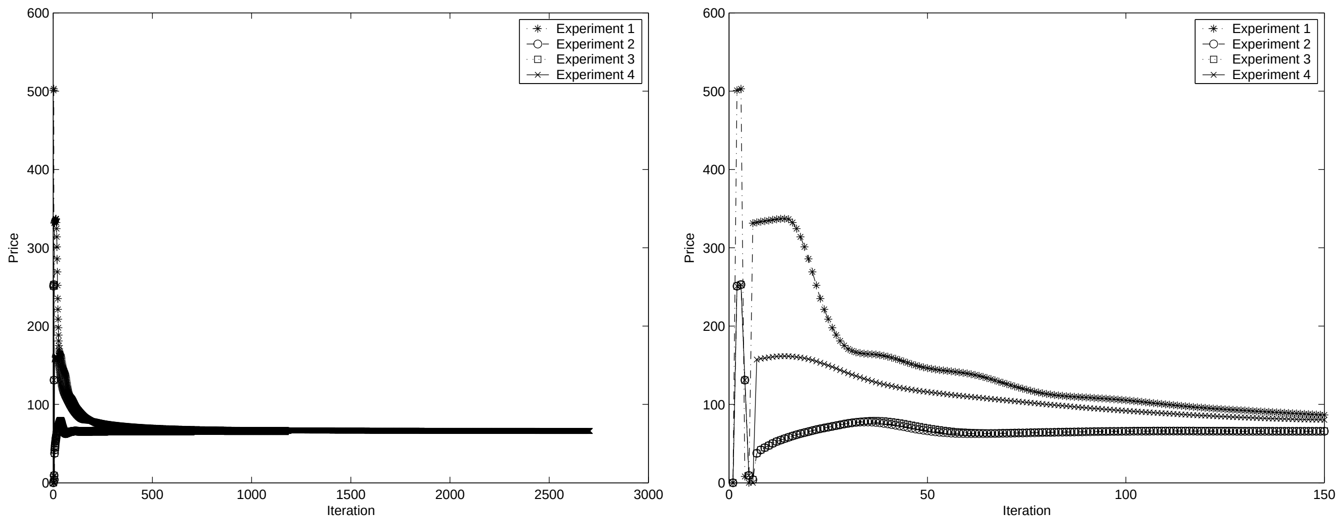


Figure 22: Dynamics of price ρ_{31} complete (left) and first 150 iterations (right).