A Stochastic Disaster Relief Game Theory Network Model

Anna Nagurney* and Mojtaba Salarpour
Operations and Information Management
Isenberg School of Management
University of Massachusetts
Amherst, Massachusetts 01003
June Dong
Department of Marketing & Management
School of Business
State University of New York
Oswego, New York 13126
and
Ladimer S. Nagurney
Department of Electrical and Computer Engineering
University of Hartford
West Hartford, Connecticut 06117

December 2019; revised February and March 2020

To appear in *SN Operations Research Forum*

Abstract: In this paper, we construct a novel game theory model for multiple humanitarian organizations engaged in disaster relief. Each organization is faced with a two stage stochastic optimization problem associated with the purchase and storage of relief items pre-disaster, subject to a budget constraint, and, if need be, additional purchases and shipments post the disaster. The model integrates logistical and financial components, in that the humanitarian organizations compete for financial donations, as well as freight service provision, and each seeks to maximize its expected utility. The expected utility function of each humanitarian organization depends on its strategies and on those of the other organizations, and their feasible sets do, as well, since the organizations are subject to common lower and upper bound demand constraints. The governing concept is that of a Stochastic Generalized Nash Equilibrium. We provide alternative variational inequality formulations of the model and propose an algorithmic scheme, which at each iteration yields closed form expressions for the product purchase/storage/shipment variables and the associated constraint Lagrange multipliers. Numerical examples illustrate the modeling and computational framework.

Key words: disaster relief, humanitarian operations, stochastic optimization, game theory, variational inequalities

* Corresponding Author's email: nagurney@isenberg.umass.edu

1. Introduction

The International Federation of Red Cross and Red Crescent Societies (2019) defines a disaster as "a sudden, calamitous event that seriously disrupts the functioning of a community or society and causes human, material, and economic or environmental losses that exceed the community's or society's ability to cope using its own resources. Though often caused by nature, disasters can have human origins." The number of disasters and the number of people affected by them have increased in recent years (Nagurney and Qiang (2009)), inflicting great economic as well as personal losses. For example, natural disasters such as Hurricane Katrina in 2005 and the earthquake that struck in Haiti in 2010 had severe impacts on people's lives and assets, which, in many instances, were already limited. Hurricane Katrina, the costliest natural disaster in US history, resulted in over 1,800 people dead, with many others left homeless, and was associated with economic losses of $250 billion (Amadeo (2019)). According to Pierre-Louis (2018), the National Oceanic and Atmospheric Administration reported that extreme weather events resulted in a total of $306 billion in damage in the United States in 2017, making that year the most expensive year on record for natural disasters in the US. The triple of hurricanes: Harvey, Irma, and Maria were the major natural disasters impacting the US that year (NOAA (2017)). The deadliest natural disasters in 2018 affected multiple countries on the continents of Europe (including wildfires in Greece), Asia (earthquakes and a tsunami in Indonesia and flooding in Japan), and Africa (floods in Nigeria), as well as North America, in the form of wildfires in California (see US News & World Report (2018)).

Researchers, along with practitioners, are now stepping up their efforts to identify useful strategies to reduce the damage caused by natural disasters. This is especially critical since, as noted in Flavelle and Plumer (2019), one of the consequences of global warming is that natural disasters, such as fires and floods, are intensified.

Disaster management has four phases consisting of: mitigation, preparedness, response, and recovery. Although all four phases are essential and should be taken into consideration, both the preparedness phase and the response phase are critical ones in reducing injuries, suffering, and loss of lives. According to Salmeron and Apte (2010), humanitarian organizations (HOs) have only 72 hours to provide vital relief items to victims, since their survival depends on having water, food, medical supplies, etc. Hence, the response phase begins immediately after a disaster strikes. However, it is important to note that, due to the nature of disasters, the region's infrastructure may be compromised, or even destroyed, which makes humanitarian operations in disaster response even more challenging and difficult (Nagurney, Yu, and Qiang (2011)). Furthermore, many natural disasters are highly unpredictable in terms of severity, timing, and location. This uncertainty makes the decision-making process associated with disaster management very difficult and challenging. For a collection of studies on the dynamics of disasters, see the edited volumes by Kotsireas, Nagurney, and Pardalos (2016, 2018).

In many cases, governments alone cannot assume full responsibility for humanitarian operations in response due to limited resources and also high costs. Government assets, as was the case post the

Haiti 2010 earthquake, may also be severely compromised. Therefore, humanitarian organizations, both national, and, in many major disasters, international ones, as well, come to assist (Nagurney, Salarpour, and Daniele (2019)). The multiplicity of organizations that may be involved in disaster response may, in turn, lead to materiel convergence (too many of the wrong supplies in the wrong locations resulting in congestion and additional work for relief workers) and a lack of coordination, which may lead to unmet demand and, hence, increases in suffering and loss of life. For example, as noted in Nagurney (2017), within three weeks following the 2010 earthquake in Haiti, 1,000 nongovernmental organizations (NGOs) were operating in Haiti.

Furthermore, what is less recognized, although also very important, is that many of the humanitarian organizations engaged in disaster relief are reliant on donors for their financial sustainability. Hence, they compete with one another for financial funds and may also compete for freight service provision, while seeking to provide the necessary relief items to disaster victims. As noted in Nagurney, Alvarez Flores, and Soylu (2016), donors respond to the visibility of HOs in disaster response in the media and, hence, what may occur, without some intervention by a central authority, is that, while HOs are engaged in the noncooperative competition, victims in locations that are easier to access may receive a surplus of supplies, whereas those in less accessible locations may experience shortages. On the other hand, as also revealed in that paper, if lower and upper bounds on the demands at locations for distribution for victims are imposed, as could occur, for example, in the case of the United Nations Logistics Cluster, or other cognizant governmental authority, then not only are the needs met, but the HOs can also gain financially. That paper presented a case study inspired by Hurricane Katrina and was the first example of a Generalized Nash Equilibrium (GNE) model for disaster relief. In a GNE, in contrast to a classical Nash (1950, 1951) equilibrium, the utility functions of a player/decision-maker depend not only on his own strategies but also on those of the other players, and the feasible sets do as well. In a subsequent paper, Nagurney et al. (2018) generalized the earlier work through the concept of a Variational Equilibrium and also demonstrated similar practical implications on a case study modeled on tornados hitting western Massachusetts, as actually occurred in June 2011.

The above noted game theory models, however, were deterministic. In this paper, we take up the challenge of incorporating uncertainty in a game theory network model for disaster relief that includes both preparedness and response phases and that integrates financial and logistical components. Each HO is faced with a two-stage stochastic optimization problem, with the first stage corresponding to the preparedness phase and the second stage to the response phase. In the second stage there are multiple scenarios and the uncertainty lies in the prices of the relief item kits procured, the costs associated with freight service provision, the demands at the demand points, and the donations that the HOs expect to attain given their response and visibility in the media.

Such extended features are important for several reasons, including:

1. Some of the population may have perished in the disaster;

2. Some of the population may have migrated from a location either before, during, or imme-

diately after the disaster;

3. Prices of relief items post the disaster may increase due to competition and the demand;

4. Since the logistical costs also assume timely deliveries, there may be uncertainty due to the possibly compromised transportation and other infrastructure, along with the costs of freight service provision;

5. Finally, the HOs depend on financial donations for their sustainability and there may also be uncertainty surrounding the willingness of donors to give post a disaster and their level of donations. In addition, we retain a weighted altruistic benefit function in the objective functions of the humanitarian organizations engaged in the disaster relief operation(s) and the associated competition, since they are nonprofit organizations (see, e.g., Nagurney, Alvarez Flores, and Soylu (2016), Nagurney et al. (2018), and the references therein).

2. Literature Review and Our Contributions

We now provide a deeper literature review in order to further emphasize the gaps in the literature that we attempt to address in this paper. This paper focuses on a game theory framework for disaster relief with stochastic elements, so the literature review does as well.

2.1 Game Theory and Humanitarian Operations in Disaster Relief

Since various governmental and non-governmental agencies are involved in the humanitarian supply chain, each with its limitations and goals, game theory is a powerful tool to address the interactions among them. However, Muggy and Heier Stamm (2014) in their survey of game theory and humanitarian operations noted that the associated literature has been limited. Seaberg, Devine, and Zhuang (2017), more recently, reviewed 57 papers from 2006 to 2016 on disaster management and game theory and noted that the response phase of disaster management had been the phase researched the most intensively. Coles, Zhang, and Zhuang (2018) in their recent, novel work, investigate how non-cooperative game theory can be utilized to assist different agencies, such as businesses, government agencies, militaries, and nonprofit organizations, in better decision-making as to partnership selection during the disaster response and recovery phases.

Muggy and Heier Stamm (2014) recognized that relief agencies compete for media exposure as well as for financial funds. Zhuang, Saxton, and Wu (2014) and Toyasaki and Wakolbinger (2014) developed game theory models of humanitarian organizations competing for donations. Nagurney, Alvarez Flores, and Soylu (2016) developed the first Generalized Nash Equilibrium model to address some of the challenges that humanitarian organizations face during disaster relief and integrated the financial side and the logistical side of disaster relief response operations. Each HO sought to maximize his utility function by determining an optimal vector of product shipments to points of demand, while competing for financial donations. As noted earlier, the HOs were subject to common constraints imposed by a higher level authority in the form of demand lower and upper bounds on the number of relief items at points of demand. The authors also identified the Nash Equilibrium (Nash (1950, 1951)) counterpart as a special case.

Nagurney et al. (2018) extended the game theory model of Nagurney, Alvarez Flores, and Soylu (2016) by incorporating more general logistical cost, altruism benefit, and financial donation functions. The previous model enabled an optimization reformulation of the GNE conditions, but the new model, in contrast, could not be reformulated as an optimization problem. Hence, the authors utilized a Variational Equilibrium concept to provide a variational inequality formulation of the governing equilibrium conditions of their model. Nagurney, Salarpour, and Daniele (2019), subsequently, brought additional realism to the model of Nagurney et al. (2018). In that game theory model, the purchasing of products was added to the costs that humanitarian organizations may face in disaster relief with the HOs also having the option of selecting among different purchasing locations, local or nonlocal, with the consideration of different associated prices and shipping costs. On the logistical side, there are multiple freight service providers with different characteristics. Budget constraints of the HOs were also included. The authors also provided a Lagrange analysis of the marginal utilities of the humanitarian organizations in their GNE game theory framework, along with numerical examples that were inspired by Hurricane Harvey hitting Houston, Texas in 2017.

Rodriguez-Espindola, Albores, and Brewster (2018) constructed a dynamic, multi-objective optimization model for humanitarian response operations incorporating multiple organizations but did not utilize game theory. Coles and Zhuang (2011), earlier, discussed the potential of cooperative game theory in disaster recovery operations and provided a critique. Muggy and Heier Stamm (2014) had also emphasized that cooperative models may assist in the identification of methods for partnering agencies to achieve greater impact than what is possible individually and independently. Nagurney and Qiang (2020) identified potential synergies associated with the possible teaming of humanitarian organizations from a supply chain network perspective.

Game theory for freight services associated with disaster relief has been a specific research theme that has been addressed in a number of recent papers. As noted in Nagurney, Salarpour, and Daniele (2019), Nagurney (2018) introduced a multitiered disaster relief network equilibrium model of humanitarian organizations and freight service providers. In that model the supplies were pre-positioned supplies and the demands for relief item deliveries were known and fixed. Gossler et al. (2019) focused on slow-onset disasters in which framework agreements are available for freight service provision. The authors developed a game theory model comprised of two submodels and also utilized a variational equilibrium as in Nagurney et al. (2018). The latter paper extended the game theory model in Nagurney (2016) to multiple humanitarian organizations, while retaining multiple competing freight service providers, and included capacities associated with the freight service providers. It is important to note that the general game theory models highlighted above for disaster relief have been deterministic.

2.2 Optimization Under Uncertainty and Disaster Relief

The need to include uncertainty in disaster management models has intensified the research in this area. However, the optimization approaches have almost exclusively considered a single humanitarian organization and have not utilized game theory, as we do in this paper.

Multicriteria optimization in humanitarian aid, in particular, is very relevant, since a decision-maker may be faced with multiple objectives in decision-making associated in disaster relief, such as optimizing in terms of efficiency, optimizing effectiveness, etc., and these may even be in conflict. For example, Qiang and Nagurney (2012) constructed a supply chain network model for critical needs (food, medicines, etc.) in the case of disruptions. The objective in the model was to minimize the total network costs, which are generalized costs that may include the monetary, risk, time, and social costs. The model assumed that disruptions may have an impact on both the network link capacities as well as on the product demands. Two different cases of disruption scenarios were considered. In the first case, it was assumed that the impacts of the disruptions are minor and that the demands can be met. In the second case, the demands cannot all be satisfied. For these two cases, the authors proposed two individual performance indicators, and constructed a bi-criteria indicator to assess the supply chain network performance for critical need products.

Earlier, Nagurney, Yu, and Qiang (2011) had developed an integrated framework for the design of supply chain networks for critical products such as vaccines, medicines, food, etc., which may be used in preparation (and response) to pandemics, disasters, attacks, etc. The supply chain network model focused on cost minimization as the primary objective and captured rigorously the uncertainty associated with the demand for critical products at the various demand points. It allows investment in enhanced link capacities associated with such supply chain activities as manufacturing, storage, and distribution. Moreover, it handles nonnegative initial capacities on the supply chain activities that the organization controls. The organization contracts the outsource product volumes at a fixed price. Finally, the model assists in the determination of whether the production of the critical product product should be outsourced or produced in-house.

Nagurney and Nagurney (2016) extended the model of Nagurney, Masoumi, and Yu (2015) by including stochastic link costs, and with the objective function including the minimization of the expected costs as well as the variance with an associated weight for the latter to denote the humanitarian organization's value of risk reduction. Their disaster relief supply chain network topology allows for the procurement and pre-positioning of supplies locally.

He and Zhuang (2016) constructed a two-stage, dynamic model to assess the trade-off between pre-disaster preparedness and post-disaster relief with the goal of minimizing the total expected damage and the costs of preparedness and relief. The authors modeled the disaster magnitude as a random parameter following either a discrete or continuous probability distribution, while the damage was a function of pre-disaster preparedness, disaster magnitude, and post-disaster relief. The authors used backward induction to obtain optimal preparedness and relief both analytically and numerically, with relevant insights for decision and policy makers.

Nagurney and Qiang (2020) also considered multiple humanitarian organizations engaged in disaster relief and constructed multiproduct supply chain network models that include uncertainty associated with costs of supply chain activities, including procurement, storage, and distribution, under multiple disaster scenarios, along with uncertainty associated with the demand for the disaster relief products at the demand points. Therein, the humanitarian organizations seek to identify the disaster relief multiproduct flows that minimize their expected total cost and risk subject to expected demand satisfaction. Again, a mean-variance approach was used to model the risk associated with cost uncertainty. The authors also constructed a synergy measure for the assessment of the potential strategic advantages of cooperation for HOs engaged in disaster management.

Gutjahr and Nolz (2016) presented an excellent, very thorough survey on multicriteria optimization in humanitarian aid and included references to both deterministic and stochastic models. The authors, in their future research directions section, noted, on page 364, that: "In general, there is a need for papers that explicitly consider the diverging interests of multiple and sometimes competing stake-holders, such as various humanitarian organizations with different missions or religious mandates, different political interests, private company interests, donor interests, and the interests and needs of beneficiaries." For a survey of uncertainty in humanitarian logistics in disaster management, we refer the interested reader to Liberatore et al. (2013). And, for an even more recent survey, see Hoyos, Morales, and Akhavan-Tabatabaei (2015). The latter survey reviews the literature on Operations Research (OR) models with some stochastic component applied to Disaster Operations Management (DOM), along with an analysis of these stochastic components and the techniques used by different authors.

Our game theory model for disaster relief under uncertainty considers both the pre-disaster preparedness phase and the post-disaster response phase. Hence, the model is a two-stage model for each humanitarian organization. We recognize that there have been multiple two-stage stochastic optimization models proposed for various phases of disaster management (see, e.g., Rawls and Turnquist (2010), Mete and Zabinsky (2010), Falasca and Zobel (2011), Grass and Fischer (2016)). In a disaster management two-stage stochastic programming framework, pre-disaster decisions such as pre-positioning of supplies are referred to as first-stage decisions and decisions regarding shipment of disaster relief items to the demand nodes, such as flows on network links, are second-stage decisions. First-stage decisions should be made prior to realizing the uncertain factors, taking into account all possible scenarios of the incident. When the event happens, and unknown factors are revealed, second-stage decisions are modified and carried out (Grass and Fischer (2016)).

Indeed, since 2010 and, specifically, following the work by Rawls and Turnquist (2010), the number of publications on two-stage stochastic programming in disaster management has increased significantly (see Grass and Fischer (2016)). Rawls and Turnquist (2010) presented a model that assists decision-makers in determining the relief item quantities and the location of facilities for pre-positioning in terms of preparedness for a possible hurricane. The objective in their model is to minimize the total costs. Mete and Zabinsky (2010) developed a stochastic programming model that, in the first stage, determines the location of warehouses and the inventory level for medical supplies. In the second stage, based on the disaster scenario, the model selects the quantities of the medical supplies to be transported to the hospitals. Falasca and Zobel (2011) provided a two-stage stochastic model for procurement. In the first stage, immediately after the disaster, humanitarian organizations find out how much relief is needed. In the second stage, when the actual demand has been revealed, they can purchase additional items. The amount of donations is another parameter that is uncertain in this model; the authors consider three possible levels of demand and donations for their analysis.

For background on the two-stage scenario-based stochastic programming problem, we refer the interested reader to Dupacova (1996) and Barbarosoglu and Arda (2004). The books by Birge and Louveaux (1997) and Shapiro, Dentcheva, and Ruszczynski (2009) have excellent coverage of the theory and applications of stochastic programming, with the book by Derman et al. (1973) serving as a fundamental resource for probability theory and applications.

Liu and Nagurney (2013) focused on competition in commercial supply chains and proposed a two-stage, game theory framework for supply chain networks with global outsourcing and quick-response production under demand and cost uncertainty. They utilized the methodology of variational inequalities for formulation, analysis, and computations. That is the methodology that we also utilize herein but emphasize that this is the first time that this is done for a disaster relief game theory model with stochastic features that integrates logistical and financial elements. Furthermore, in order to meet the demands of the victims, but not add to materiel convergence, common demand constraints are imposed by a higher level authority, under uncertainty, leading to a Generalized Nash Equilibrium model under uncertainty.

2.3 Our Contributions

As mentioned above, the state of the art in terms of game theory approaches to disaster relief and management, despite the numerous ways in which humanitarian organizations compete in practice (from financial funds to obtaining freight service for distribution), has a much more recent literature than that associated with optimization and disaster relief. Hence, it is not surprising that the topic of game theory for disaster relief under uncertainty remains essentially unexplored. Our paper hopes to fill this gap in the literature, while also advancing the development of game theory models that integrate the financial and logistical sides associated with disaster response of humanitarian organizations. Hence, this paper builds on the work of Nagurney, Alvarez Flores, and Soylu (2016), Nagurney et al. (2018), and Nagurney, Salarpour, and Daniele (2019) but with the following significant extensions/modifications.

1. Each decision-maker is faced with a two-stage stochastic optimization problem.

2. There is demand uncertainty associated with the relief items, which can differ at the various demand locations, post the disaster, with the demand depending also on the disaster level at the location.

3. There is price uncertainty, post the disaster, regarding the relief items, at the puchase locations (PLs) and the humanitarian organizations can purchase supplies from multiple locations prior to and post the disaster.

4. The logistical costs associated with having the relief items delivered to the different points of demand are uncertain and can be distinct for the different freight service providers (FSPs), which is reasonable, since some may have suffered greater (or lesser) disruptions during the disaster.

5. The financial donations that the HOs are expecting to obtain can differ for the HOs and also are associated with the distinct demand points. Moreover, the donation functions depend on the severity of the disaster.

6. We introduce stochastic elements into the Generalized Nash Equilibrium model, due to the common constraints corresponding to the lower bounds and the upper bounds on the relief item volumes at demand points, which are affected by the scale of the disaster.

This paper integrates two vital issues in humanitarian logistics that have been addressed separately in research works in recent years in order to take an essential step towards bringing disaster management models closer to reality. Uncertainty is one of the most important characteristics of natural disasters, and understanding and capturing the interactions among different organizations is a crucial factor for the success or failure of relief operations. In our new model, using two-stage stochastic modeling, we show how organizations by taking into account uncertainty, while having access to several options for supply and distribution of items, allocate their initial budgets to achieve the best result. In addition, through the utilization of game theory, our model illustrates how organizations can optimize their decision-making, both in advance and, after a disaster, given the competitive environment.

This paper is organized as follows: In Section 3, we construct the stochastic disaster relief game theory network model. We identify the behavior of each humanitarian organization in terms of the two-stage stochastic optimization problem that each solves and describe the competitive behavior among the humanitarian organizations. We present the governing conditions for both the Stochastic General Nash Equilibrium and the Stochastic Nash Equilibrium and construct their variational inequality formulations. We also discuss existence of a solution and provide illustrative numerical examples. In Section 4, we present an alternative variational ienquality formulation and outline an algorithm for computations. It resolves the variational inequality problem into a series of problems, in the product volumes and the Lagrange multipliers, each of which can be solved exactly and in closed form. The algorithm is then applied in Section 5 to numerical examples. We summarize our results and present our conclusions in Section 6.

3. The Stochastic Disaster Relief Game Theory Network Model

The primary goals of humanitarian organizations in disaster response are to assist victims of a disaster and to save lives. The relief operation, along with delivering vital relief items to the victims, should be carried out as quickly as feasible (Salmeron and Apte (2010), Nagurney, Masoumi, and Yu (2015)). This characteristic of relief operations, coupled with the unpredictability of natural disasters, demonstrates the importance of careful planning and full preparedness at the time that the disaster strikes. The uncertainty exhibits itself in many ways: the type of disaster and the time of the incident, its location, its severity, and its effect on the region are among the most unpredictable parameters.

In two-stage stochastic programming, a common way to deal with uncertainty is to use the scenario approach; a scenario includes information about the location, time, and extent of the damage caused by the disaster (Grass and Fischer (2016)). Snyder (2006) states that using the scenario approach makes the model more tractable and allows the parameters to be statistically dependent; dependency plays an important role in bringing the model closer to reality. There is a great number of research works that have used this method to model pre- and post-disaster management. Rawls and Turnquist (2010) utilize disaster scenarios to capture the uncertainties such as demand and damage levels in their work on the prepositioning of emergency supplies for disaster response. Chang, Tseng, and Chen (2007) use a scenario planning approach to solve the location-allocation model for the flood emergency logistics preparation. Barbarosoglu and Arda (2004) develop a scenario-based two-stage stochastic programming model to plan the transportation of vital first-aid items in earthquake emergency response.

The humanitarian organizations try to be prepared to respond to the disasters when they occur, given the likelihood of each scenario. This preparation involves planning and executing operations in two stages. The first stage is before the disaster, when humanitarian organizations are required to take some actions to increase their preparedness for possible disaster scenarios, such as locating and allocating warehouses and shelters, and keeping them ready for the time of the disaster. The second stage is related to the aftermath of the incident when the uncertain parameters such as time, location, severity, and demand have been revealed and humanitarian organizations know which disaster scenario has occurred. In this stage, the humanitarian organizations take advantage of their previous actions in Stage 1 as well as taking immediate actions in Stage 2 to assist victims and to provide the critical disaster relief items. For background on the construction of scenarios, with a focus on practitioners, see the book by Alexander (2002).

In our new stochastic game theory network model, we incorporate some of the mentioned unpredictable disaster parameters, along with the options available for humanitarian organizations, to illustrate how they operate and compete in the two stages. These two stages are also referred to by practitioners as two of the four *disaster phases* of disaster management – preparedness and response, with the other two corresponding to mitigation and recovery (see Altay and Green (2006)).

The fundamental notation utilized our model is given in Table 1.

Table 1: Notation for the Stochastic Game Theory Network Model

Notation	**Definition**
$\omega \in \Omega$	the disaster scenarios.
$q^{i1}_{hj,l}$	the amount of the disaster relief items purchased by HO i and carried by FSP l from PL h to Hub j. We group all the h, j, l elements into the vector q^{i1} and then group such vectors for all i into the vector q^1.
$q^{i2,\omega}_{hk,l}$	the amount of the disaster relief items purchased by HO i and carried by FSP l directly from PL h to demand point k when the disaster scenario ω happens. We group all the h, k, l, ω elements into the vector q^{i2}, and then group such vectors for all i into the vector q^2.
$q^{i3,\omega}_{jk,l}$	the amount of the disaster relief items purchased by HO i and carried by FSP l from Hub j to demand point k when the disaster scenario ω occurs. We group all the j, k, l, ω elements into the vector q^{i3}, and then group these vectors for all i into the vector q^3. We group the q^1, q^2, q^3 vectors into the vector $q \in R_+^{IHJL+\|\Omega\|(IHKL+IJKL)}$.
q^ω	the vector of relief item shipments for scenario ω: $\{q^{2,\omega}, q^{3,\omega}\}$; $\forall \omega \in \Omega$, where $q^{2,\omega}$=$\{q^{i2,\omega}_{hk,l}\}$, $\forall i, h, k, l$ and $q^{3,\omega}$=$\{q^{i3,\omega}_{jk,l}\}$, $\forall i, j, k, l$.
$U_i(q)$	HO i's utility; $i = 1, \ldots, I$.
ρ_h	the price of the disaster relief item at purchasing location h before the disaster; $h = 1, \ldots, H$.
π_j	the price of storage per unit at storage/hub location j; $j = 1, \ldots, J$.
β_i	the weight imposed by HO i on the altruism component of his utility function in stage 2; $i = 1, \ldots, I$.
$\rho_{h,\omega}$	the price of disaster relief item at purchasing location h when the disaster scenario ω occurs; $h = 1, \ldots, H; \omega \in \Omega$.
$c^{i1}_{hj,l}(q^1)$	the transportation cost encumbered by HO i to have its relief items delivered to hub j by freight service provider l from purchasing location h before the disaster.
$c^{i2,\omega}_{hk,l}(q^{2,\omega})$	the transportation cost that HO i pays to have its relief items delivered to demand point k by freight service provider l from purchasing location h when the disaster scenario ω occurs.
$c^{i3,\omega}_{jk,l}(q^{3,\omega})$	the transportation cost encumbered by HO i to have his relief items delivered to demand point k by freight service provider l from hub j when the disaster scenario ω occurs.
B_i	HO i's budget in stage 1; $i = 1, \ldots, I$.
$P^\omega_{ik}(q^\omega)$	the donations received by HO i; $i = 1, \ldots, I$ due to visibility at location k; $k = 1, \ldots, K$, when disaster scenario $\omega \in \Omega$ strikes, with $E(P_i(q)) = \sum_{\omega \in \Omega} p_\omega \sum_{k=1}^{K} P^\omega_{ik}(q^\omega)$.
$\underline{d}_k$	base level of the lower bound on demand at demand point k; $k = 1, \ldots, K$.
$\bar{d}_k$	base level of the upper bound on demand at demand point k; $k = 1, \ldots, K$.
$\underline{\gamma}_\omega$	coefficient reflecting the effect of disaster scenario ω on the lower bound demands; $\forall \omega \in \Omega$.
$\bar{\gamma}_\omega$	coefficient reflecting the effect of disaster scenario ω on the upper bound demands; $\forall \omega \in \Omega$.
p_ω	the probability of scenario ω; $\forall \omega \in \Omega$.

The stochastic disaster relief game theory network model that we propose considers a two-stage network, with each stage corresponding to a time stage. The first stage is the pre-disaster preparedness stage and the second stage is the post-disaster response stage. In the first stage, each HO i; $i = 1, \ldots, I$, seeks to determine how many of the relief items to purchase at each of the H purchasing locations and how many of these are to be delivered by each of the L freight service providers for storage at each of the J storage/distribution hubs (cf. Figure 1). In other words, each HO i; $i = 1, \ldots, I$, determines its vector q^{i1}, and does so before the information on the disaster scenario ω is revealed. Each HO is subject to a budget constraint in the first stage as to its purchasing, freight service provision, and storage decisions and the associated prices and costs.

In the second stage, the disaster scenario is revealed and the relief items that were prepositioned in the first stage are made available for further shipment to the disaster victims at points of demand. The HOs must also decide how many additional items they must purchase from the purchase locations to be shipped directly to the points of demand via an appropriate freight service provider or providers from the purchasing locations. There are multiple disaster scenarios ω that the HOs must consider. Hence, each HO i; $i = 1, \ldots, I$ must decide his q^{i2} and his q^{i3}.

In the second stage, the HOs also receive financial donations based on their visibility in the media as to the response, etc. They are not subject to a budget constraint since they must respond to the disaster and the victims' needs. Each HO is faced with a weighted altruism function since the humanitarian organizations are nonprofits. The HOs face common constraints as to the lower and upper bounds on the effective demands at the demand points, which are also uncertain and depend on the disaster scenario.

In Figure 1, stages are displayed separately to better distinguish the first and second stage activities and links. We can see that the two stages are connected through the HOs and the Hubs. The Hubs $1, \ldots, J$ in Stage 2 are the same Hubs fulfilled by the HOs in Stage 1 and the same HOs operate in both stages as do the FSPs. As can be seen in Figure 1, at the first stage, links enter the Hubs in order to store the relief items purchased by the HOs $1, \ldots, I$ at the PLs $1, \ldots, H$, which were delivered by the FSPs. At the second stage, links only exit the Hubs (in order to capture the delivery of the relief items to the demand points. We also see that organizations in the second stage still have the opportunity to provide relief items purchased at the PLs, post the disaster, directly to the demand points via the FSPs.

3.1 The HOs' Behavior and Optimality Conditions

The humanitarian organizations determine their relief item product purchases and shipments q^1, via one or more freight service providers, to the storage hubs, in Stage 1. They decide their post-disaster purchases and direct shipments to points of demand, as well as the shipments from the hubs (and the freight service providers utilized), that is, their q^2 and q^3, in Stage 2. Each humanitarian organization maximizes its expected utility over all the scenarios. The complete optimal set of decisions for humanitarian organization i, hence, includes his optimal purchasing/shipment/storage

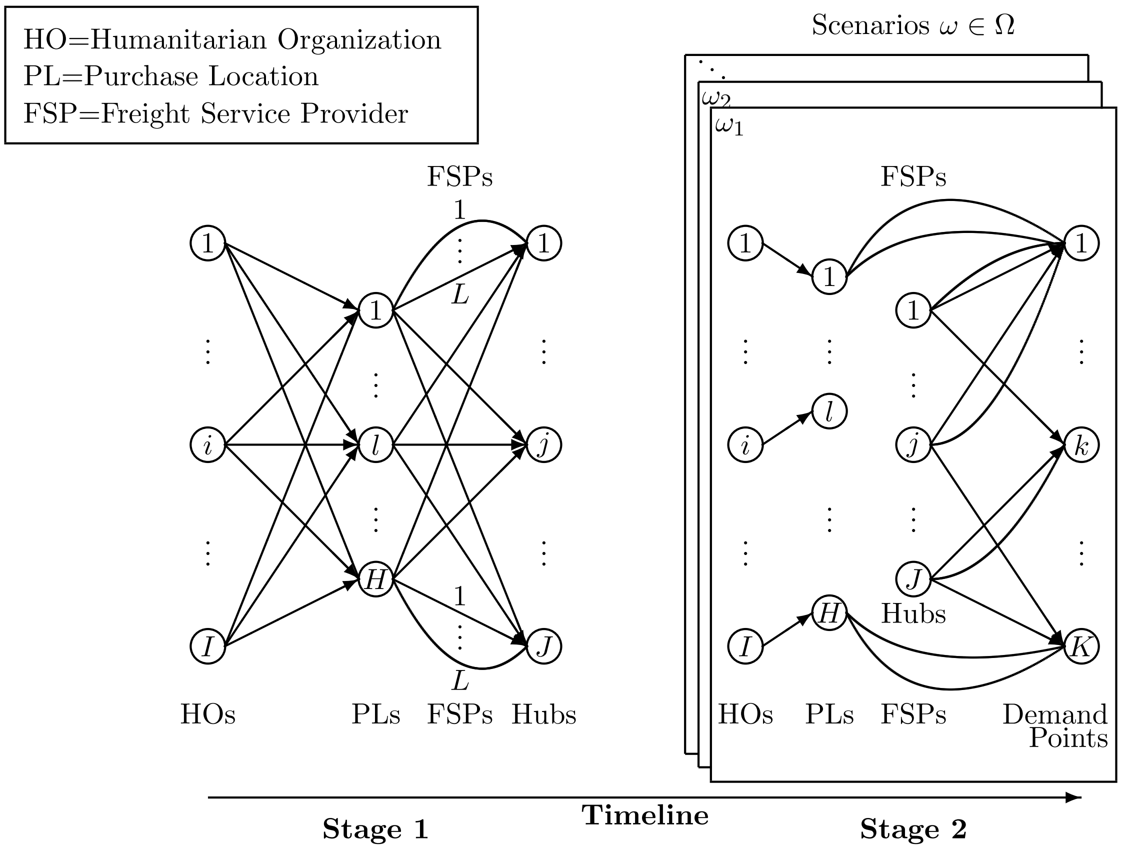

Figure 1: The Timeline of the Humanitarian Organizations

quantities as well as his optimal disaster response plan under scenario ω in Stage 2. Each humanitarian organization maximizes his expected utility across all the disaster scenarios, and, when the optimality conditions of all HOs hold simultaneously, an equilibrium state is reached in which no humanitarian organization is better off by changing his decision strategies.

We first describe each HO's decision-making problem as a two-stage stochastic programming problem and then define the equilibrium state of all the humanitarian organizations, under the Stochastic Generalized Nash Equilibrium. All vectors throughout this paper are assumed to be column vectors unless otherwise noted. The equilibrium solution is denoted by "*".

3.1.1 The HOs' Optimization Problems

In the first stage, the humanitarian organizations, acting independently, seek to determine their relief item volumes q^1. If a humanitarian organization purchases too many relief items, he may end up with too many relief items in the second stage, while, if an HO purchases too few relief item kits, and the demand for them, because of the disaster that strikes, is high in the second stage, he may have to purchase at a higher cost more of the items and have them shipped directly from the purchase locations, which also may be at a higher freight service provision cost. In the second stage, after the disaster scenario is revealed, each HO i determines the second stage purchase and direct shipment levels, the elements of its vector of strategies q^{i2}, and the shipments from the hubs to the demand points, the q^{i3}s. Hence, the humanitarian organizations need to determine their $q^{i1}_{hj,l}$s, $q^{i2,\omega}_{hk,l}$s, and $q^{i3,\omega}_{jk,l}$s in order to maximize the expected utilities, in equilibrium, across all scenarios. In particular, the optimization problem faced by humanitarian organization i: $i = 1, \ldots, I$, can be expressed as a two-stage stochastic programming problem as follows, corresponding to expected utility maximization:

$$\text{Maximize } \; E(U^i(q)) = -\sum_{h=1}^{H} \rho_h \sum_{j=1}^{J} \sum_{l=1}^{L} q^{i1}_{hj,l} - \sum_{h=1}^{H} \sum_{j=1}^{J} \sum_{l=1}^{L} c^{i1}_{hj,l}(q^1) - \sum_{j=1}^{J} \pi_j \sum_{h=1}^{H} \sum_{l=1}^{L} q^{i1}_{hj,l}$$
$$+E_{\Omega}\left[Q_i(q^1, q^2, q^3, \omega)\right] \tag{1}$$

subject to:

$$\sum_{h=1}^{H} \rho_h \sum_{j=1}^{J} \sum_{l=1}^{L} q^{i1}_{hj,l} + \sum_{h=1}^{H} \sum_{j=1}^{J} \sum_{l=1}^{L} c^{i1}_{hj,l}(q^1) + \sum_{j=1}^{J} \pi_j \sum_{h=1}^{H} \sum_{l=1}^{L} q^{i1}_{hj,l} \leq B_i, \tag{2}$$

$$q^{i1}_{hj,l} \geq 0, \quad h = 1, \ldots, H; j = 1, \ldots, J; l = 1, \ldots, L. \tag{3}$$

The first term in the objective function (1) is minus the total payout of HO i for purchases of relief items, pre-disaster. The second term is minus the total payout to the FSPs for shipment to the hubs for storage and the third term is minus the total cost associated with storage. The fourth term in the objective function is the expected value of HO i's utility in Stage 2, $Q_i(q^1, q^2, q^3, \omega)$, over all scenarios. Specifically, $Q_i(q^1, q^2, q^3, \omega)$ is the optimal value of the following problem:

$$\text{Maximize } \; -\sum_{h=1}^{H} \rho_{h,\omega} \sum_{k=1}^{K} \sum_{l=1}^{L} q^{i2,\omega}_{hk,l} - \sum_{h=1}^{H} \sum_{k=1}^{K} \sum_{l=1}^{L} c^{i2,\omega}_{hk,l}(q^{2,\omega}) - \sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{l=1}^{L} c^{i3,\omega}_{jk,l}(q^{3,\omega})$$
$$+\beta_i(\sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{l=1}^{L} q^{i3,\omega}_{jk,l} + \sum_{h=1}^{H} \sum_{k=1}^{K} \sum_{l=1}^{L} q^{i2,\omega}_{hk,l}) + \sum_{k=1}^{K} P^{\omega}_{ik}(q^{\omega}) \tag{4}$$

subject to:

$$\sum_{k=1}^{K} \sum_{l=1}^{L} q^{i3,\omega}_{jk,l} \leq \sum_{h=1}^{H} \sum_{l=1}^{L} q^{i1}_{hj,l}, \quad j = 1, \ldots, J; h = 1, \ldots, H; l = 1, \ldots, L, \tag{5}$$

$$q^{i3,\omega}_{jk,l} \geq 0, \quad j = 1, \ldots, J; k = 1, \ldots, K; l = 1, \ldots, L, \tag{6}$$

$$q^{i2,\omega}_{hk,l} \geq 0, \quad h = 1, \ldots, H; k = 1, \ldots, K; l = 1, \ldots, L, \tag{7}$$

and the shared constraints imposed by a higher level authority in order, as discussed earlier, to ensure that the demands for relief items by the victims are ensure, while at the same time, not contributing to materiel convergence:

$$\underline{\gamma_{\omega} d}_k \leq \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{l=1}^{L} q^{i3,\omega}_{jk,l} + \sum_{i=1}^{I} \sum_{h=1}^{H} \sum_{l=1}^{L} q^{i2,\omega}_{hk,l}, \quad k = 1, \ldots, K; \forall \omega \in \Omega, \tag{8}$$

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L} q_{jk,l}^{i3,\omega} + \sum_{i=1}^{I}\sum_{h=1}^{H}\sum_{l=1}^{L} q_{hk,l}^{i2,\omega} \leq \overline{\gamma}_{\omega}\overline{d}_k, \quad k=1,\ldots,K; \forall \omega \in \Omega. \tag{9}$$

Objective function (4) represents the utility of humanitarian organization i in the second stage when disaster scenario ω occurs. The first term in this function is minus the cost that the organization pays to purchase the relief items after the disaster, the second term is minus the cost to transfer the purchased items in the second stage to the demand points, the third term is minus the cost that must be paid by the HO to ship the relief items that have been stored in the hubs in the first stage to the demand points. The fourth term in (4) is the benefit/altruism that the HO receives from providing help to the disaster victims and the last term is the amount of donations that the HO receives through the visibility in the media.

Observe that the purchased quantities, the $q_{hj,l}^{i1}$s, have been determined in the first stage and cannot be altered in the second stage. These variables must satisfy the budget constraint (2) and the nonnegativity constraints (3). Note that in the budget constraint the first term corresponds to the total purchase payments for the relief items in the first stage; the second term corresponds to the total payments for freight service provision in the first stage, and the third term to the storage of the relief items at the hubs. Constraint (5) reflects that the total amount of the relief items shipped out by HO i from the storage hubs to the points of demand cannot exceed the volume that the HO has stored in the hubs. Constraints (6) and (7) are the nonnegativity constraints on the relief item shipments from the hubs and those purchased post-disaster and shipped directly from the purchase locations, respectively. Constraints (8) and (9) guarantee, respectively, that the victims receive at least the expected needed minimum volumes of relief items, with expected upper bounds not exceeded (to reduce materiel convergence).

Based on standard stochastic programming theory, we can reformulate HO i's two-stage optimization problem as the following maximization problem:

$$\text{Maximize } E(U^i(q)) = -\sum_{h=1}^{H}\rho_h\sum_{j=1}^{J}\sum_{l=1}^{L} q_{hj,l}^{i1} - \sum_{h=1}^{H}\sum_{j=1}^{J}\sum_{l=1}^{L} c_{hj,l}^{i1}(q^1) - \sum_{j=1}^{J}\pi_j\sum_{h=1}^{H}\sum_{l=1}^{L} q_{hj,l}^{i1}$$

$$+\sum_{\omega\in\Omega} p_{\omega}\Bigg[-\sum_{h=1}^{H}\rho_{h,\omega}\sum_{k=1}^{K}\sum_{l=1}^{L} q_{hk,l}^{i2,\omega} - \sum_{h=1}^{H}\sum_{k=1}^{K}\sum_{l=1}^{L} c_{hk,l}^{i2,\omega}(q^{2,\omega}) - \sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{l=1}^{L} c_{jk,l}^{i3,\omega}(q^{3,\omega})$$

$$+\beta_i\Big(\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{l=1}^{L} q_{jk,l}^{i3,\omega} + \sum_{h=1}^{H}\sum_{k=1}^{K}\sum_{l=1}^{L} q_{hk,l}^{i2,\omega}\Big) + \sum_{k=1}^{K} P_{ik}^{\omega}(q^{\omega})\Bigg] \tag{10}$$

subject to:

$$\sum_{k=1}^{K}\sum_{l=1}^{L} q_{jk,l}^{i3,\omega} \leq \sum_{h=1}^{H}\sum_{l=1}^{L} q_{hj,l}^{i1}, \quad j=1,\ldots,J; h=1,\ldots,H; l=1,\ldots,L; \forall\omega\in\Omega, \tag{11}$$

$$q_{jk,l}^{i3,\omega} \geq 0, \quad j=1,\ldots,J; k=1,\ldots,K; l=1,\ldots,L; \forall\omega\in\Omega, \tag{12}$$

$$q^{i2,\omega}_{hk,l} \geq 0, \quad h = 1, \ldots, H; k = 1, \ldots, K; l = 1, \ldots, L; \forall \omega \in \Omega, \tag{13}$$

$$\sum_{h=1}^{H} \rho_h \sum_{j=1}^{J} \sum_{l=1}^{L} q^{i1}_{hj,l} + \sum_{h=1}^{H} \sum_{j=1}^{J} \sum_{l=1}^{L} c^{i1}_{hj,l}(q^1) + \sum_{j=1}^{J} \pi_j \sum_{h=1}^{H} \sum_{l=1}^{L} q^{i1}_{hj,l} \leq B_i, \tag{14}$$

$$q^{i1}_{hj,l} \geq 0, \quad h = 1, \ldots, H; j = 1, \ldots, J; l = 1, \ldots, L, \tag{15}$$

$$\underline{\gamma}_\omega \underline{d}_k \leq \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{l=1}^{L} q^{i3,\omega}_{jk,l} + \sum_{i=1}^{I} \sum_{h=1}^{H} \sum_{l=1}^{L} q^{i2,\omega}_{hk,l}, \quad k = 1, \ldots, K; \forall \omega \in \Omega, \tag{16}$$

$$\sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{l=1}^{L} q^{i3,\omega}_{jk,l} + \sum_{i=1}^{I} \sum_{h=1}^{H} \sum_{l=1}^{L} q^{i2,\omega}_{hk,l} \leq \overline{\gamma}_\omega \overline{d}_k, \quad k = 1, \ldots, K; \forall \omega \in \Omega. \tag{17}$$

Recall that p_ω is the probability of disaster scenario ω, such that $\sum_{\omega \in \Omega} p_\omega = 1$.

3.1.2 The Optimality Conditions of All the Humanitarian Organizations and Variational Inequality Formulation

We assume that the cost functions for freight service provision are continuously differentiable and convex (hence, they could be linear) and that the financial donation functions are continuously differentiable and concave. Also, we assume that the humanitarian organizations compete in a noncooperative manner in the sense of Nash (1950, 1951). Since we are interested in both the Nash Equilibrium and the Generalized Nash Equilibrium we now define the necessary feasible sets. We define

$$\mathcal{K}_i \equiv \{q^i \,\text{such that}\, (11) - (15) \,\text{hold}\}$$

and we let $\mathcal{K}^1 \equiv \prod_{i=1}^{I} \mathcal{K}_i$. Note that each $\mathcal{K}_i$ depends only on the strategy vector of HO i. Also, we define the feasible set of common constraints $\mathcal{S}$ as

$$\mathcal{S} \equiv \{q | (16) \,\text{and}\, (17) \,\text{hold}\}.$$

The feasible set

$$\mathcal{K}^2 \equiv \mathcal{K}^1 \cap \mathcal{S}.$$

Adapting Nagurney, Salarpour, and Daniele (2019), we state the following definition:

Definition 1: Stochastic Generalized Nash Equilibrium for the Humanitarian Organizations

A relief item flow vector $q^* \in \mathcal{K}^2$ is a Stochastic Generalized Nash Equilibrium if for each HO i; $i = 1, \ldots, I$:

$$E(U^i(q^{i*}, \hat{q^{i*}})) \geq E(U^i(q^i, \hat{q^{i*}})), \quad \forall q^i \in K_i \cap \mathcal{S}, \tag{18}$$

where $\hat{q^{i*}} \equiv (q^{1*}, \ldots, q^{i-1*}, q^{i+1*}, \ldots, q^{I*})$.

The above definition states that not one of the HOs is willing to deviate from his current relief item flow pattern, given the relief flow item patterns of the other HOs. Observe that each HO's utility depends not only on his own strategies but also on those of the others' strategies, and so do their feasible sets, since their feasible sets are linked because of the shared constraints. The latter condition makes the problem a Generalized Nash Equilibrium (GNE) model (Debreu (1952)). We know that the feasible sets $\mathcal{K}_i$ are convex for each i, as is the set $\mathcal{S}$, under the imposed assumptions. Although Generalized Nash Equilibrium problems can be formulated as quasivariational inequality problems (cf. Fischer, Herrich, and Schonefeld (2014)), algorithms for such problems are not as advanced as those for variational inequalities.

As noted in Nagurney, Yu, and Besik (2017) and in Nagurney, Salarpour, and Daniele (2019), one may take advantage of a refinement of the GNE known as a Variational Equilibrium, which is a specific type of GNE (cf. Kulkarni and Shabhang (2012)), and which enables a variational inequality formulation. It is used for the first time here in a stochastic setting for disaster relief.

Definition 2: Variational Equilibrium

A relief item flow vector q^* is a Variational Equilibrium of the above Stochastic Generalized Nash Equilibrium problem if $q^* \in \mathcal{K}^2$ is a solution to the following variational inequality:

$$-\sum_{i=1}^{I} \langle \nabla_{q^i} E(U^i(q^*)), q^i - q^{i*} \rangle \geq 0, \quad \forall q \in \mathcal{K}^2, \tag{19}$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in $\mathcal{N}$-dimensional Euclidean space, where $\mathcal{N}$ here is equal to $IHJL + |\Omega|(IHKL + IJKL)$ and ∇ is the gradient.

Notably, the variational equilibrium corresponds to Lagrange multipliers associated with the shared constraints being the same for all the humanitarian organizations. This characteristic feature allows for an appropriate fairness and equity interpretation and is quite reasonable for humanitarian organizations involved in disaster relief. Also, in our stochastic, two-stage model, the lower and upper bounds on the relief item demands depend on the scenarios. In the case that the shared constraints are removed, it follows then from classical results (cf. Gabay and Moulin (1980), Bazaraa et al. (1993), Nagurney (1999)) that the resulting stochastic Nash Equilibrium for the humanitarian organizations satisfies the following variational inequality.

Theorem 1: Variational Inequality Formulation of Stochastic Nash Equilibrium

A relief item flow vector q^* is a Variational Equilibrium of the associated Stochastic Nash Equilibrium if $q^* \in \mathcal{K}^1$ is a solution to the following variational inequality:

$$-\sum_{i=1}^{I} \langle \nabla_{q^i} E(U^i(q^*)), q^i - q^{i*} \rangle \geq 0, \quad \forall q \in \mathcal{K}^1. \tag{20}$$

Proof: When the shared constraints (16) and (17) are removed, we can simply assume $\mathcal{S} \equiv R^{\mathcal{N}}$. Then $\mathcal{K}^2 \equiv \mathcal{K}^1 \cap \mathcal{S} \equiv \mathcal{K}^1$. The Theorem is proved.

We now expand variational inequality (19) and then put it into standard form (cf. Nagurney (1999)). In particular, the VI in (19) is equivalent to the following VI: determine $q^* \in \mathcal{K}^2$ such that

$$\sum_{i=1}^{I}\sum_{h=1}^{H}\sum_{j=1}^{J}\sum_{l=1}^{L}\left[\rho_h+\sum_{r=1}^{H}\sum_{s=1}^{J}\sum_{t=1}^{L}\frac{\partial c^{i1}_{rs,t}(q^{1*})}{\partial q^{i1}_{hj,l}}+\pi_j\right]\times\left[q^{i1}_{hj,l}-q^{i1*}_{hj,l}\right]$$

$$+\sum_{\omega\in\Omega}p_\omega\sum_{i=1}^{I}\sum_{h=1}^{H}\sum_{k=1}^{K}\sum_{l=1}^{L}\left[\rho_{h,\omega}+\sum_{r=1}^{H}\sum_{u=1}^{K}\sum_{t=1}^{L}\frac{\partial c^{i2,\omega}_{ru,t}(q^{2,\omega*})}{\partial q^{i2,\omega}_{hk,l}}-\beta_i-\sum_{o=1}^{K}\frac{\partial P^{\omega}_{io}(q^{\omega*})}{\partial q^{i2,\omega}_{hk,l}}\right]\times\left[q^{i2,\omega}_{hk,l}-q^{i2,\omega*}_{hk,l}\right]$$

$$+\sum_{\omega\in\Omega}p_\omega\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{l=1}^{L}\left[\sum_{s=1}^{J}\sum_{u=1}^{K}\sum_{t=1}^{L}\frac{\partial c^{i3,\omega}_{su,t}(q^{3,\omega*})}{\partial q^{i3,\omega}_{jk,l}}-\beta_i-\sum_{o=1}^{K}\frac{\partial P^{\omega}_{io}(q^{\omega*})}{\partial q^{i3,\omega}_{jk,l}}\right]\times\left[q^{i3,\omega}_{jk,l}-q^{i3,\omega*}_{jk,l}\right]\geq 0,$$

$$\forall q \in \mathcal{K}^2. \qquad (21)$$

Moreover, VI (20) has the same structural form as (21) except that it is defined over the simpler feasible set $\mathcal{K}^1$.

Recall that the finite-dimensional variational inequality problem, VI$(F,\mathcal{K})$, is to determine a vector $X^* \in \mathcal{K} \subset R^{\mathcal{N}}$, such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \qquad (22)$$

where F is a given continuous function from $\mathcal{K}$ to $R^{\mathcal{N}}$, $\mathcal{K}$ is a given closed, convex set, and $\langle \cdot,\cdot \rangle$ denotes the inner product in $\mathcal{N}$-dimensional Euclidean space. Clearly, both VIs (19) and (20) can be put into the above standard form. Specifically, we can define $X \equiv q$, $\mathcal{K} \equiv \mathcal{K}^2$ for VI (19) and $\mathcal{K} \equiv \mathcal{K}^1$ for VI (20). Furthermore, we can define $F(X) \equiv (F^1(X), F^2(X), F^3(X))$, where the components of $F^1(X)$ correspond to the $IHJL$ elements with a typical $ihjl$ element as preceding the first multiplication sign in (21); the components of $F^2(X)$ correspond to the $|\Omega|\ IHKL$ elements with a typical such element as immediately preceding the second multiplication sign, and so on.

We emphasize that the variational inequality framework has been used to formulate, analyze, and solve a plethora of complex network equilibrium problems in a wide range of applications, such as: supply chains, transportation, finance, economics, and electric power (see, for example, Dong et al. (2005), Cruz and Wakolbinger (2008), Nagurney (1999, 2006), Nagurney and Ke (2006), Liu and Nagurney (2009), Barbagallo, et al. (2014), Saberi et al. (2018), Yu, Cruz, and Li (2019), Liu and Wang (2019)). In this paper, we utilize this methodology for the first time for disaster relief associated with humanitarian organizations, in which each such decision-maker is faced with a two-stage stochastic optimization problem.

Existence of a solution to variational inequality (19) is guaranteed from the classical theory of variational inequalities (cf. Kinderlehrer and Stampacchia (1980)) since the feasible set $\mathcal{K}^2$ is compact and the function that enters the VI, $F(X)$, is continuous, under our imposed assumptions. Compactness follows because of the budget constraints and the lower and upper bounds on the demands at the demand points under all the scenarios.

3.2 Illustrative Examples

In this Section, we present three small numerical examples for illustrative purposes. The first such example is depicted in Figure 2, the second one in Figure 3, and the third one in Figure 4.

Illustrative Example 1

In the first example, there is a single HO, a single purchasing location, a single hub, and a single freight service provider, along with one demand point. There is one scenario $\omega = \omega_1 = 1$ with probability $p_{\omega_1} = 1$.

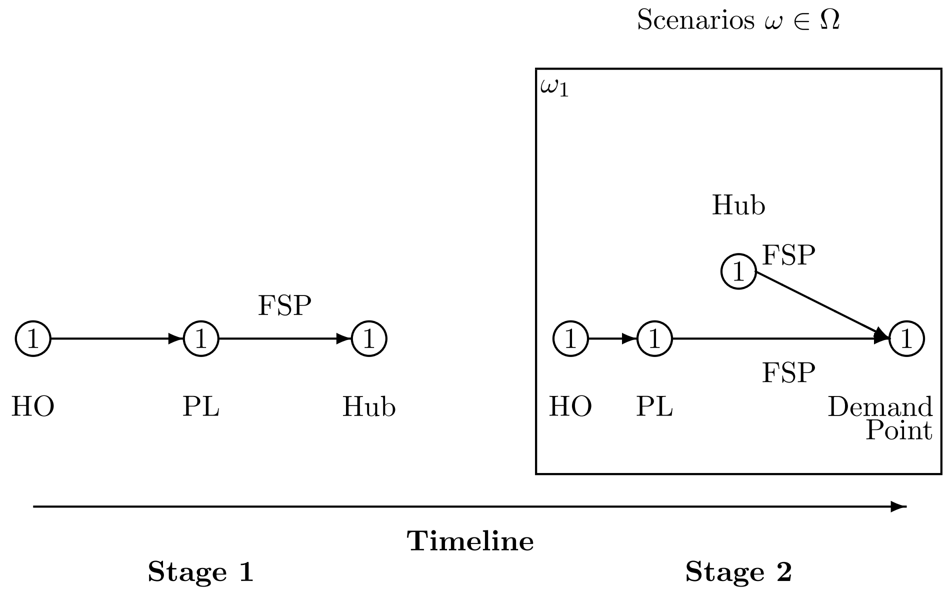

Figure 2: The Timeline of Illustrative Example 1

The data are as follows: the purchasing prices of the relief items and unit storage costs are, respectively: $\rho_1 = 47$, $\rho_{1,1} = 100$, and $\pi_1 = 2$. The transportation costs, in turn, are:

$$c^{11}_{11,1}(q^1) = q^{11}_{11,1}, \quad c^{12,1}_{11,1}(q^{2,1}) = 10q^{12,1}_{11,1}, \quad c^{13,1}_{11,1}(q^{3,1}) = 5q^{13,1}_{11,1}.$$

The budget for HO 1 is: $B_1 = 10,000$, and the financial donation function is: $P^1_{11}(q^1) = 100\sqrt{2(q^{12,1}_{11,1} + q^{13,1}_{11,1})}$ with the altruism weight being: $\beta_1 = 50$.

Also, cf. (8), in this example $\underline{\gamma}_\omega \underline{d}_1 = 100$ and $\overline{\gamma}_\omega \overline{d}_1 = 300$.

Because of the simplicity of this numerical example it is easy to see that the solution of the associated variational inequality (19) is:

$$q^{11*}_{11,1} = 200.00, \quad q^{12,1*}_{11,1} = 0.00, \quad q^{13,1*}_{11,1} = 200.00.$$

Indeed, we now show that VI (21) (equivalently, VI (19)) holds. We first note that the solution lies in the feasible set $\mathcal{K}^2$; that is, the constraints are satisfied. Moreover, constructing the associated functions in (21) using the above data for the example, we obtain:

$$[47+1+2] \times \left[q_{11,1}^{11} - 200\right] + [100+10-50-5] \times \left[q_{11,1}^{12,1} - 0\right]$$

$$+ [5-50-5] \times \left[q_{11,1}^{13,1} - 200\right]$$

$$= [50] \times \left[q_{11,1}^{11} - 200\right] + [55] \times \left[q_{11,1}^{12,1} - 0\right] + [-50] \times \left[q_{11,1}^{13,1} - 200\right]$$

$$= 50 \times q_{11,1}^{11} - 50 \times q_{11,1}^{13,1} + 55 q_{11,1}^{12,1} \geq 0,$$

since, from the definition of the feasible set we know that $q_{11,1}^{11} \geq q_{11,1}^{13,1}$ and all the variables are nonnegative. Hence, in this example, the HO should preposition the supplies and have them delivered from the hub once the disaster strikes. The computed $P_{11}^1(q^{1*}) = 2000.00$ with expected donations $E(P_1(q^*))$ also equal to $2,000$. The humanitarian organization experiences an expected utility under this solution of: $E(U^1(q^*)) = 1,000.00$.

Illustrative Example 2

In the second illustrative example there is one scenario and two HOs, with the figure graphic for the timeline as in Figure 3. This example was constructed from Example 1 and the data are as follows. The data for HO 1 remain as in Example 1. HO 2, for simplicity, has the same associated cost functions as those of HO 1, the same budget, the same weight associated with altruism, etc., but, of course, they are a function of HO 2's strategies: $q_{11,1}^{21}$, $q_{11,1}^{22,1}$, and $q_{11,1}^{23,1}$. In order to capture competition for donations, HO 1's financial donation function is now the following: $P_{11}^1(q^1) = 50\sqrt{2(q_{11,1}^{12,1} + q_{11,1}^{13,1}) - (q_{11,1}^{22,1} + q_{11,1}^{23,1})}$ since the donations that it stands to receive are affected (negatively) by the relief item deliveries of HO 2 to the demand point. HO 2's donation function is: $P_{21}^1(q^1) = 50\sqrt{2(q_{11,1}^{22,1} + q_{11,1}^{23,1}) - (q_{11,1}^{12,1} + q_{11,1}^{13,1})}$.

It is easy to see that the following is the solution to VI (19) for this problem:

$$q_{11,1}^{21*} = q_{11,1}^{11*} = 100.00, \quad q_{11,1}^{22,1*} = q_{11,1}^{12,1*} = 0.00, \quad q_{11,1}^{23,1*} = q_{11,1}^{13,1*} = 100.00,$$

with the expected utilities: $E(U^2(q^*)) = E(U^1(q^*)) = 0.00$. Because of competition, HO 1 and HO 2 each encounter an expected utility of 0.00, but the victims of the disaster, nevertheless, still have 200 relief item kits delivered (as in Example 1). The HOs' computed financial donations are, respectively, $P_{11}^1(q^{1*}) = 500.00$ and $P_{21}^1(q^{1*}) = 500.00$, with these values also corresponding to their expected donations. With double the number of humanitarian organizations now involved in disaster relief, as compared to that in Example 1, the expected donations of the original HO 1 now drop and, in fact, the total volume of donations is now lower (\$1,000) than in Example 1 (\$2,000). Observe that these declines are also, due, in part, to the fact that the coefficient in the donation functions in Example 2 is 50 for each humanitarian organization, whereas it was 100 for the single one (HO 1) in Example 1.

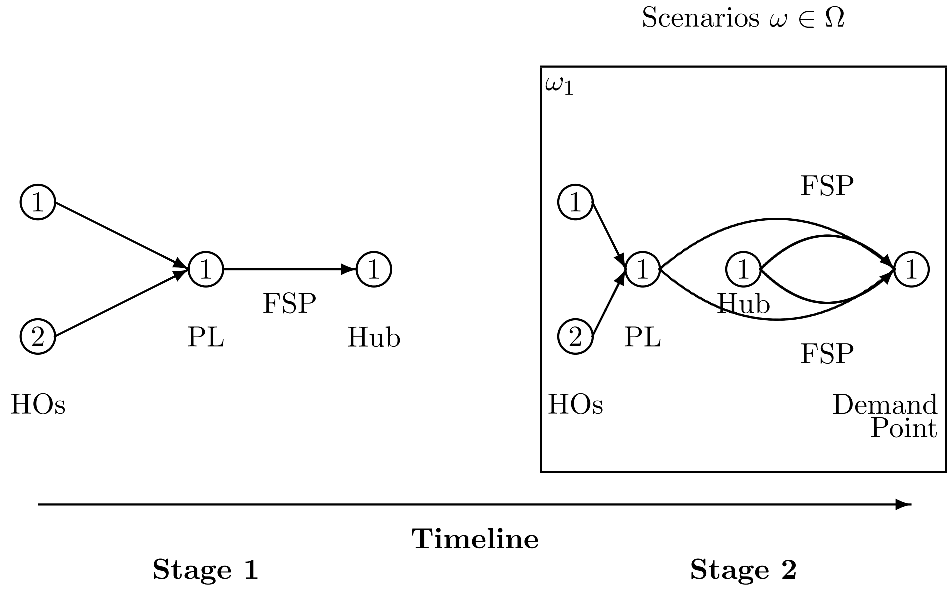

Figure 3: The Timeline of Illustrative Example 2

Illustrative Example 3

In the third illustrative example there are two scenarios $\omega = \omega_1 = 1$ and $\omega = \omega_2 = 2$ and two HOs. This example was constructed from Example 2 and the complete data are now reported.

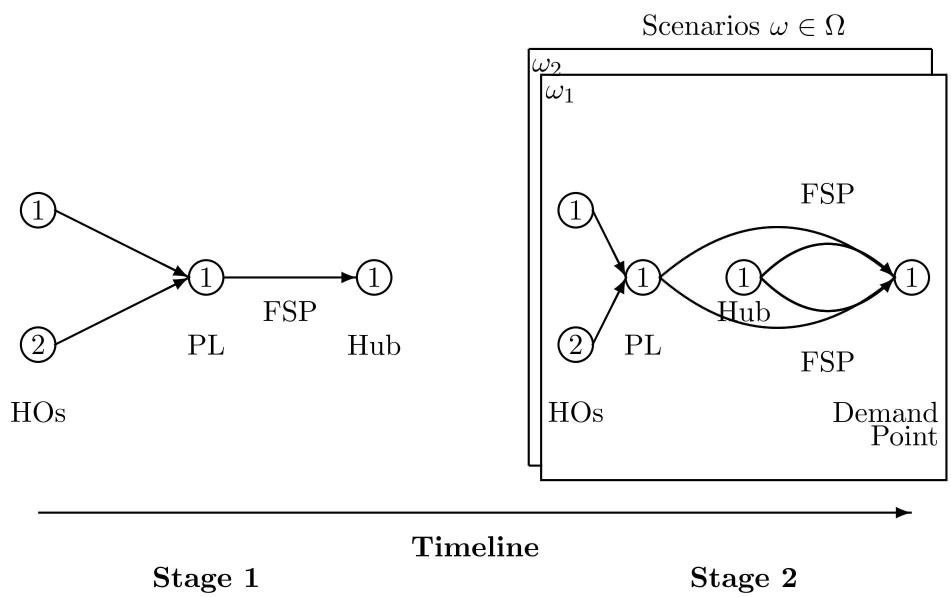

Figure 4: The Timeline of Illustrative Example 3

The various cost functions are:

$$c^{i1}_{11,1}(q^1) = q^{i1}_{11,1}, \quad i = 1, 2,$$

$$c^{i2,1}_{11,1}(q^{2,1}) = 10q^{i2,1}_{11,1}, \quad i = 1, 2,$$
$$c^{i3,1}_{11,1}(q^{3,1}) = 5q^{i3,1}_{11,1}, \quad i = 1, 2,$$
$$c^{i2,2}_{11,1}(q^{2,2}) = 12q^{i2,2}_{11,1}, \quad i = 1, 2,$$
$$c^{i3,2}_{11,1}(q^{3,2}) = 7q^{i3,2}_{11,1}, \quad i = 1, 2.$$

The budgets are:

$$B_i = 10,000.00, \quad i = 1, 2.$$

The price of the relief items is 47 and the prices in Stage 2 are:

$$\rho_{1,1} = 100, \quad \rho_{1,2} = 110.$$

The probabilities of two events, with the second event having a higher impact are:

$$p_1 = .4, \quad p_2 = .6.$$

The financial donation functions are:

$$P^1_{11}(q^1) = 50\sqrt{2(q^{13,1}_{11,1} + q^{12,1}_{11,1}) - (q^{23,1}_{11,1} + q^{22,1}_{11,1})},$$
$$P^1_{21}(q^1) = 50\sqrt{2(q^{23,1}_{11,1} + q^{22,1}_{11,1}) - (q^{13,1}_{11,1} + q^{12,1}_{11,1})},$$

and

$$P^2_{11}(q^2) = 60\sqrt{2(q^{13,2}_{11,1} + q^{12,2}_{11,1}) - (q^{23,2}_{11,1} + q^{22,2}_{11,1})},$$
$$P^2_{21}(q^2) = 60\sqrt{2(q^{23,2}_{11,1} + q^{22,2}_{11,1}) - (q^{13,2}_{11,1} + q^{12,2}_{11,1})}.$$

Note that the coefficient in the donation functions under scenario ω_2 is 60, whereas under scenario ω_1, the coefficient if 50, reflecting higher expected donations in the case of the more severe disaster.

In this example $\underline{\gamma}_\omega \underline{d}_1 = 100$ and $\overline{\gamma}_\omega \overline{d}_1 = 300$ for $\omega = 1$ and $\underline{\gamma}_\omega \underline{d}_1 = 200$ and $\overline{\gamma}_\omega \overline{d}_1 = 500$ for $\omega = 2$.

The Stochastic Generalized Nash Equilibrium solution is:

$$q^{11*}_{11,1} = q^{21*}_{11,1} = 150.00,$$

$$q^{12,\omega*}_{11,1} = q^{22,\omega*}_{11,1} = 0.00, \quad \omega = 1, 2,$$
$$q^{13,\omega*}_{11,1} = q^{23,\omega*}_{11,1} = 150.00, \quad \omega = 1, 2.$$

In Example 3, each humanitarian organization prepositions 150 disaster relief items in the Hub in Stage 1 and, in both disaster scenarios, the HOs supply the victims with all the stored relief items from the hub, once the disaster strikes. They do not purchase any additional items once the disaster strikes. Qualitatively, these strategies are as in Examples 1 and 2. Quantitatively, however, they differ. In Example 3, with two possible scenarios, and the second one corresponding to a more severe disaster, each HO prepositions a larger volume of relief items in the hub (150) than in Example 2 (100), which is very reasonable. Furthermore, in terms of the financial donations to the two humanitarian organizations, we now have that:

$$P^1_{11}(q^{1*}) = 612.37, \quad P^1_{21}(q^{1*}) = 612.37,$$

$$P^2_{11}(q^{2*}) = 734.85, \quad P^2_{21}(q^{2*}) = 734.85,$$

$$E(P_1(q^*)) = 685.86, \quad E(P_2(q^*)) = 685.86.$$

In the case of two scenarios, with one being severe, the HOs in Example 3 stand to gain higher expected financial donations that in Example 2. However, their expected utilities are now negative: $E(U^1(q^*)) = E(U^2(q^*)) = -244.14$ since the humanitarian organizations are required to meet at least the lower bounds for the demand for relief items and now there is also the possibility of a more severe disaster scenario.

4. The Algorithm and Alternative Variational Inequality Formulation

In this section, we first present the algorithmic scheme and then provide an alternative variational inequality formulation to that of (19), governing the Stochastic Generalized Nash Equilibrium, to which the algorithm, the modified projection method (cf. Korpelevich (1997) and Nagurney (1999)), is applied to compute solutions to numerical examples in the next section. A straightforward adaptation allows also for computation of solutions to the Stochastic Nash Equilibrium VI (20). The modified projection method is guaranteed to converge if the function $F(X)$ that enters the variational inequality is monotone and Lipschitz continuous, and that a solution exists.

The statement of the modified projection method is as follows, where τ denotes an iteration counter:

The Modified Projection Method

Step 0: Initialization

Start with $X^0 \in \mathcal{K}$ (cf. (22)). Set $\tau := 1$ and select a, such that $0 < a \leq \frac{1}{L}$, where L is the Lipschitz continuity constant for $F(X)$.

Step 1: Construction and Computation

Compute $\overline{X}^{\tau-1}$ by solving the variational inequality subproblem:

$$\langle \overline{X}^{\tau-1} + (aF(X^{\tau-1}) - X^{\tau-1}), X - \overline{X}^{\tau-1} \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{23}$$

Step 2: Adaptation

Compute X^τ by solving the variational inequality subproblem:

$$\langle X^\tau + (aF(\overline{X}^{\tau-1}) - X^{\tau-1}), X - X^\tau \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{24}$$

Step 3: Convergence Verification

If $| X_l^\tau - X_l^{\tau-1} | \leq \epsilon$, for all l, with $\epsilon > 0$, a prespecified tolerance, then stop; else set $\tau := \tau + 1$, and go to step 1.

In order to construct the alternative VI, which will allow for closed form expressions of (23) and (24) for the relief item flows, and the Lagrange multipliers associated with the budget constraints, the hub flow inequality constraints, and the lower and upper bound demand constraints at the demand points, we define the necessary Lagrange multipliers. We let η_i; $i = 1, \ldots, I$, be the Lagrange multiplier associated with the budget constraint (14) for each HO i, respectively, and we group all these into the vector $\eta \in R_+^I$. We let $\alpha_j^{i\omega}$; $i = 1, \ldots, I$; $j = 1, \ldots, J$; $\omega \in \Omega$, be the Lagrange multiplier associated with HO i's Hub j inequality constraint (11) under scenario ω, respectively, and we group all these Lagrange multipliers into the vector $\alpha^{|\Omega|IJ}$. Finally, we associate the Lagrange multiplier λ_k^ω with the lower bound constraint (16) at demand point k and ω for $k = 1, \ldots, K$ and $\omega \in \Omega$, and the Lagrange multiplier μ_k^ω with the upper bound constraint (17) at demand point k and scenario ω for $k = 1, \ldots, K$ and $\omega \in \Omega$. We group the Lagrange multipliers $\{\lambda_k^\omega\}$ into the vector $\lambda \in R_+^{|\Omega|K}$, and we group all the $\{\mu_k^\omega\}$ into the vector $\mu \in R_+^{|\Omega|K}$. We define the feasible set $\mathcal{K}^3$ as follows:

$$\mathcal{K}^3 \equiv \{(q, \eta, \alpha, \lambda, \mu) | q \in R_+^{IHJL+|\Omega|(IHKL+IJKL)}, \eta \in R_+^I, \alpha \in R_+^{|\Omega|IJ}, \text{and}\, \lambda \in R_+^{|\Omega|K}, \mu \in R_+^{|\Omega|K}\}.$$

Following arguments as in Nagurney, Salarpour, and Daniele (2019), an alternative variational inequality to VI (19); equivalently, VI (21) is: determine $(q^*, \eta^*, \alpha^*, \lambda^*, \mu^*) \in \mathcal{K}^3$ such that

$$\sum_{i=1}^{I}\sum_{h=1}^{H}\sum_{j=1}^{J}\sum_{l=1}^{L}\left[(\rho_h + \sum_{r=1}^{H}\sum_{s=1}^{J}\sum_{t=1}^{L}\frac{\partial c_{rs,t}^{i1}(q^{1*})}{\partial q_{hj,l}^{i1}} + \pi_j)(1+\eta_i^*) - \sum_{\omega\in\Omega}\alpha_j^{i,\omega *}\right] \times [q_{hj,l}^{i1} - q_{hj,l}^{i1*}]$$

$$+\sum_{\omega\in\Omega}p_\omega\sum_{i=1}^{I}\sum_{h=1}^{H}\sum_{k=1}^{K}\sum_{l=1}^{L}\left[\rho_{h,\omega} + \sum_{r=1}^{H}\sum_{u=1}^{K}\sum_{t=1}^{L}\frac{\partial c_{ru,t}^{i2,\omega}(q^{2,\omega *})}{\partial q_{hk,l}^{i2,\omega}} - \beta_i - \sum_{o=1}^{K}\frac{\partial P_{io}^\omega(q^{\omega *})}{\partial q_{hk,l}^{i2,\omega}} - \lambda_k^{\omega *} + \mu_k^{\omega *}\right] \times \left[q_{hk,l}^{i2,\omega} - q_{hk,l}^{i2,\omega *}\right]$$

$$+\sum_{\omega\in\Omega}p_\omega\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{l=1}^{L}\left[\sum_{s=1}^{J}\sum_{u=1}^{K}\sum_{t=1}^{L}\frac{\partial c_{su,t}^{i3,\omega}(q^{3,\omega *})}{\partial q_{jk,l}^{i3,\omega}} - \beta_i - \sum_{o=1}^{K}\frac{\partial P_{io}^\omega(q^{\omega *})}{\partial q_{jk,l}^{i3,\omega}} + \alpha_j^{i,\omega *} - \lambda_k^{\omega *} + \mu_k^{\omega *}\right] \times \left[q_{jk,l}^{i3,\omega} - q_{jk,l}^{i3,\omega *}\right]$$

$$+\sum_{i=1}^{I}\left[B_i - \sum_{h=1}^{H}\rho_h\sum_{j=1}^{J}\sum_{l=1}^{L}q_{hj,l}^{i1*} - \sum_{h=1}^{H}\sum_{j=1}^{J}\sum_{l=1}^{L}c_{hj,l}^{i1}(q^{1*}) - \sum_{j=1}^{J}\pi_j\sum_{h=1}^{H}\sum_{l=1}^{L}q_{hj,l}^{i1*}\right] \times [\eta_i - \eta_i^*]$$

$$+\sum_{\omega\in\Omega}\sum_{i=1}^{I}\sum_{j=1}^{J}\left[\sum_{h=1}^{H}\sum_{l=1}^{L}q_{hj,l}^{i1*}-\sum_{k=1}^{K}\sum_{l=1}^{L}q_{jk,l}^{i3,\omega*}\right]\times\left[\alpha_j^{i,\omega}-\alpha_j^{i,\omega*}\right]$$

$$+\sum_{\omega\in\Omega}\sum_{k=1}^{K}\left[\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L}q_{jk,l}^{i3,\omega*}+\sum_{i=1}^{I}\sum_{h=1}^{H}\sum_{l=1}^{L}q_{hk,l}^{i2,\omega*}-\underline{\gamma}_\omega\underline{d}_k\right]\times\left[\lambda_k^\omega-\lambda_k^{\omega*}\right]$$

$$+\sum_{\omega\in\Omega}\sum_{k=1}^{K}\left[\overline{\gamma}_\omega\overline{d}_k-\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L}q_{jk,l}^{i3,\omega*}-\sum_{i=1}^{I}\sum_{h=1}^{H}\sum_{l=1}^{L}q_{hk,l}^{i2,\omega*}\right]\times\left[\mu_k^\omega-\mu_k^{\omega*}\right]\geq 0,$$

$$\forall(q,\eta,\alpha,\lambda,\mu)\in\mathcal{K}^3. \qquad (25)$$

We now provide the explicit formulae for the modified projection method at a given iteration for Step 1 above (cf. (23)). Analogous constructs are easily obtained for Step 2. Specifically, we have that:

Explicit Formulae for the Product Purchase/Storage Quantities in Stage 1

For each i,h,j,l, compute

$$\bar{q}_{hj,l}^{(i1)\tau-1}=\max\{0,q_{hj,l}^{(i1)\tau-1}-a((\rho_h+\sum_{r=1}^{H}\sum_{s=1}^{J}\sum_{t=1}^{L}\frac{\partial c_{rs,t}^{i1}(q^{1\tau-1})}{\partial q_{hj,l}^{i1}}+\pi_j)(1+\eta_i^{\tau-1})-\sum_{\omega\in\Omega}\alpha_j^{i,\omega\tau-1})\}; \quad (26)$$

Explicit Formulae for the Product Purchase and Direct Shipment Quantities in Stage 2

For each ω,i,h,k,l, compute

$$\bar{q}_{hk,l}^{(i2,\omega)\tau-1}=\max\{0,q_{hk,l}^{(i2,\omega)\tau-1}-ap_\omega(\rho_{h,\omega}+\sum_{r=1}^{H}\sum_{u=1}^{K}\sum_{t=1}^{L}\frac{\partial c_{ru,t}^{i2,\omega}(q^{2,\omega\tau-1})}{\partial q_{hk,l}^{i2,\omega}}-\beta_i-\sum_{o=1}^{K}\frac{\partial P_{io}^{\omega}(q^{\omega\tau-1})}{\partial q_{hk,l}^{i2,\omega}}-\lambda_k^{\omega\tau-1}+\mu_k^{\omega\tau-1})\}; \quad (27)$$

Explicit Formulae for the Product Shipment Quantities from the Hubs in Stage 2

For each ω,i,j,k,l, compute

$$\bar{q}_{jk,l}^{(i3,\omega)\tau-1}=\max\{0,q_{jk,l}^{(i3,\omega)\tau-1}-ap_\omega(\sum_{s=1}^{J}\sum_{u=1}^{K}\sum_{t=1}^{L}\frac{\partial c_{su,t}^{i3,\omega}(q^{3,\omega\tau-1})}{\partial q_{jk,l}^{i3,\omega}}-\beta_i-\sum_{o=1}^{K}\frac{\partial P_{io}^{\omega}(q^{\omega\tau-1})}{\partial q_{jk,l}^{i3,\omega}}+\alpha_j^{i,\omega\tau-1}-\lambda_k^{\omega\tau-1}+\mu_k^{\omega\tau-1})\}; \quad (28)$$

Explicit Formulae for the Lagrange Multipliers Associated with the Budget Constraints

For each i, compute

$$\bar{\eta}_i^{\tau-1}=\max\{0,\eta_i^{\tau-1}-a(B_i-\sum_{h=1}^{H}\rho_h\sum_{j=1}^{J}\sum_{l=1}^{L}q_{hj,l}^{i1\tau-1}-\sum_{h=1}^{H}\sum_{j=1}^{J}\sum_{l=1}^{L}c_{hj,l}^{i1}(q^{1\tau-1})-\sum_{j=1}^{J}\pi_j\sum_{h=1}^{H}\sum_{l=1}^{L}q_{hj,l}^{i1\tau-1})\}; \quad (29)$$

Explicit Formulae for the Lagrange Multipliers Associated with the Hub Shipment Inequality Constraints

For each ω, i, j, compute

$$\bar{\alpha}_j^{(i,\omega)\tau-1} = \max\{0, \alpha_j^{(i,\omega)\tau-1} - a(\sum_{h=1}^{H}\sum_{l=1}^{L} q_{hj,l}^{i1\tau-1} - \sum_{k=1}^{K}\sum_{l=1}^{L} q_{jk,l}^{i3,\omega\tau-1})\}; \tag{30}$$

Explicit Formulae for the Lagrange Multipliers Associated with the Demand Lower Bound Constraints

For each ω, k, compute

$$\bar{\lambda}_k^{\omega\tau-1} = \max\{0, \lambda_k^{\omega\tau-1} - a(\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L} q_{jk,l}^{i3,\omega\tau-1} + \sum_{i=1}^{I}\sum_{h=1}^{H}\sum_{l=1}^{L} q_{hk,l}^{i2,\omega\tau-1} - \underline{\gamma}_\omega \underline{d}_k)\}; \tag{31}$$

Explicit Formulae for the Lagrange Multipliers Associated with the Demand Upper Bound Constraints

For each ω, k, compute

$$\bar{\mu}_k^{\omega\tau-1} = \max\{0, \mu_k^{\omega\tau-1} - a(\overline{\gamma}_\omega \overline{d}_k - \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L} q_{jk,l}^{i3,\omega\tau-1} - \sum_{i=1}^{I}\sum_{h=1}^{H}\sum_{l=1}^{L} q_{hk,l}^{i2,\omega\tau-1})\}. \tag{32}$$

We utilize the above modified projection method in the next section for computational purposes.

5. Additional Numerical Examples

We implemented the modified projection method as described in Section 4 in FORTRAN and utilized a Linux system at the University of Massachusetts Amherst for the computations. The convergence tolerance was $\epsilon = 10^{-5}$, meaning that the absolute value of the difference of each variable computed at two successive iterations was less than or equal to this ϵ. We set a in the algorithm (see (23) and (24)) equal to .1.

Example 4

This example had the same data as in Illustrative Example 3 (see also Figure 4) except that we modified all of the cost functions, increasing them as follows:

$$c_{11,1}^{i1}(q^1) = \frac{1}{2}(q_{11,1}^{i1})^2 + q_{11,1}^{i1}, \quad i = 1, 2,$$

$$c_{11,1}^{i2,1}(q^{2,1}) = \frac{1}{2}(q_{11,1}^{i2,1})^2 + 10q_{11,1}^{i2,1}, \quad i = 1, 2,$$

$$c_{11,1}^{i3,1}(q^{3,1}) = \frac{1}{2}(q_{11,1}^{i3,1})^2 + 5q_{11,1}^{i3,1}, \quad i = 1, 2,$$

$$c^{i2,2}_{11,1}(q^{2,2}) = \frac{1}{2}(q^{i2,2}_{11,1})^2 + 12q^{i2,2}_{11,1}, \quad i = 1, 2,$$

$$c^{i3,2}_{11,1}(q^{3,2}) = \frac{1}{2}(q^{i3,2}_{11,1})^2 + 7q^{i3,2}_{11,1}, \quad i = 1, 2.$$

Note that all of the above cost functions are transportation cost functions and now they are nonlinear (rather than linear as in Example 3). This enables the better modeling of costs and time delays associated with congestion that can occur prior and post the disaster, but for different reasons.

The modified projection method yielded the following Stochastic Generalized Nash Equilibrium solution. The computed product/shipment quantities are now:

$$q^{11*}_{11,1} = q^{21*}_{11,1} = 55.00,$$

$$q^{12,\omega*}_{11,1} = q^{22,\omega*}_{11,1} = 0.00, \quad \omega = 1,$$

$$q^{12,\omega*}_{11,1} = q^{22,\omega*}_{11,1} = 45.00, \quad \omega = 2,$$

$$q^{13,\omega*}_{11,1} = q^{23,\omega*}_{11,1} = 52.00, \quad \omega = 1,$$

$$q^{13,\omega*}_{11,1} = q^{23,\omega*}_{11,1} = 55.00, \quad \omega = 2.$$

The Lagrange multipliers are:

$$\eta^*_1 = \eta^*_2 = 0.00,$$

$$\lambda^{1*}_1 = 0.00, \quad \lambda^{2*}_1 = 111.00,$$

$$\mu^{1*}_1 = 0.00, \quad \mu^{2*}_1 = 0.00,$$

$$\alpha^{(1,1)*}_1 = \alpha^{(2,1)*}_1 = 0.00, \quad \alpha^{(1,2)*}_1 = \alpha^{(2,2)*}_1 = 105.00.$$

In this example, with the increase in transportation costs, we see a significant change in the humanitarian organizations' strategies as compared to Example 3. In Example 4, the humanitarian organizations change their strategies both quantitatively and qualitatively. Each HO now prepositions only 55 relief items in the Hub in Stage 1. In Stage 2, in response to the first scenario, no additional relief items are purchased and 52 items are shipped by each HO to the victims. In the case of the second scenario, which is associated with more severe damage, in Stage 2, each humanitarian organization purchases 45 additional relief items and these are transported to the victims along with each HO's 55 items that have been stored.

Also, in Example 4, in the case of scenario ω_2, the lower bound on the demand of 200 is precisely met, and, hence, the associated Lagrange multiplier λ^{2*}_1 is positive. The equilibrium conditions hold with excellent accuracy.

In terms of financial donations, we now have the following results:

$$P^1_{11}(q^{1*}) = 360.56, \quad P^1_{21}(q^{1*}) = 360.56,$$

$$P^2_{11}(q^{2*}) = 600.00, \quad P^2_{21}(q^{2*}) = 600.00,$$

with the expected donations for HO 1 and HO 2:

$$E(P_1(q^*)) = 504.22, \quad E(P_2(q^*)) = 504.22.$$

Faced with higher logistical costs, each HO, under each scenario, delivers fewer relief items to the victims at the demand point that in Example 3. With fewer items delivered, donors respond accordingly, and the expected donations are significantly lower than in Example 3.

Example 5

Example 5 was constructed from Example 4 but now we added a new point of demand for the relief items. There were, hence, two HOs, one FSP, one PL, one Hub, two Demand Points, and two Scenarios. The data associated with the first demand point remained as in Example 4. Also, we increased the budget of each HO from 10,000 to 20,000. Therefore, in this example: $B_1 = B_2 = 20,000$. The timeline for this example is as depicted in Figure 5.

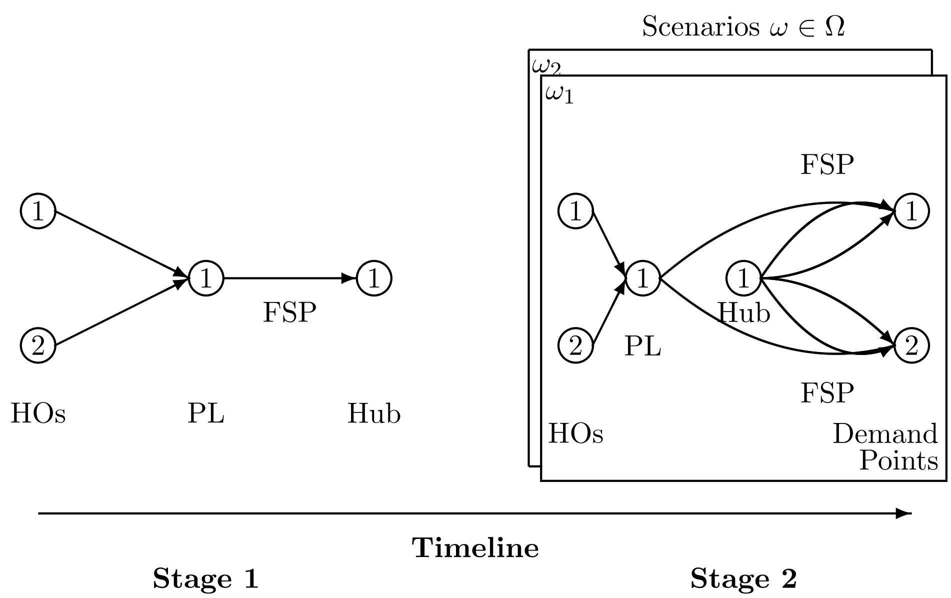

Figure 5: The Timeline of Example 5

The additional data are reported below.

The financial donation functions associated with the second demand point are:

$$P^1_{12}(q^1) = 50\sqrt{2(q^{13,1}_{12,1} + q^{12,1}_{12,1}) - (q^{23,1}_{12,1} + q^{22,1}_{12,1})},$$

$$P^1_{22}(q^1) = 50\sqrt{2(q^{23,1}_{12,1} + q^{22,1}_{12,1}) - (q^{13,1}_{12,1} + q^{12,1}_{12,1})},$$

and

$$P^2_{12}(q^2) = 60\sqrt{2(q^{13,2}_{12,1} + q^{12,2}_{12,1}) - (q^{23,2}_{12,1} + q^{22,2}_{12,1})},$$

$$P_{22}^2(q^2) = 60\sqrt{2(q_{12,1}^{23,2} + q_{12,1}^{22,2}) - (q_{12,1}^{13,2} + q_{12,1}^{12,2})}.$$

The additional cost functions associated with Demand Point 2 are:

$$c_{12,1}^{i2,1}(q^{2,1}) = 8q_{11,1}^{i2,1}, \quad i = 1, 2,$$

$$c_{12,1}^{i3,1}(q^{3,1}) = 4q_{11,1}^{i3,1}, \quad i = 1, 2,$$

$$c_{12,1}^{i2,2}(q^{2,2}) = 9q_{11,1}^{i2,2}, \quad i = 1, 2,$$

$$c_{12,1}^{i3,2}(q^{3,2}) = 5q_{11,1}^{i3,2}, \quad i = 1, 2.$$

The lower and upper bounds at Demand Point 2 under each scenario are the same as at Demand Point 1 in Example 4. The remainder of the data are as in Example 4. The modified projection method converged to the equilibrium solution given below.

The computed product/shipment quantities are:

$$q_{11,1}^{11*} = q_{11,1}^{21*} = 100.00,$$

$$q_{1k,1}^{12,\omega*} = q_{1k,1}^{22,\omega*} = 0.00, \quad \omega = 1; k = 1, 2,$$

$$q_{11,1}^{12,\omega*} = q_{11,1}^{22,\omega*} = 49.75, \quad \omega = 2,$$

$$q_{12,1}^{12,\omega*} = q_{12,1}^{22,\omega*} = 50.25, \quad \omega = 2,$$

$$q_{1k,1}^{13,\omega*} = q_{1k,1}^{23,\omega*} = 50.00, \quad \omega = 1; k = 1, 2,$$

$$q_{11,1}^{13,\omega*} = q_{11,1}^{23,\omega*} = 50.25, \quad \omega = 2,$$

$$q_{12,1}^{13,\omega*} = q_{12,1}^{23,\omega*} = 49.75. \quad \omega = 2.$$

The Lagrange multipliers are:

$$\eta_1^* = \eta_2^* = 0.00,$$

$$\lambda_1^{1*} = 33.43, \quad \lambda_1^{2*} = 115.75, \quad \lambda_2^{1*} = 32.43, \quad \lambda_2^{2*} = 113.25,$$

$$\mu_1^{1*} = \mu_1^{2*} = \mu_2^{1*} = \mu_2^{2*} = 0.00,$$

$$\alpha_1^{(1,1)*} = \alpha_1^{(2,1)*} = 35.50, \quad \alpha_1^{(1,2)*} = \alpha_1^{(2,2)*} = 114.50.$$

In Example 5, in contrast to Example 4, the humanitarian organizations need to plan and be prepared to respond post the disaster to two demand points. In Stage 1, each of the humanitarian organizations stores 100 disaster relief items at the Hub to be able to respond to the needs of the disaster victims in Stage 2. Therefore, in scenario ω_1, the lower bound on the demand, 100, for both demand points is met by using only the items in the hub and there is no need to purchase extra items post the disaster. Under scenario ω_2, each humanitarian organization purchases 100 additional relief items post the disaster and has them transported directly to the demand points along with 100 items from the hub to satisfy the lower bound on the demand of 200, at both demand points.

In Example 5, all the lower bounds hold tightly under both scenarios, and, therefore, the associated Lagrange multipliers are positive. In terms of financial donations, the results for Example 5 are as follows:

$$P^1_{11}(q^{1*}) = 353.55, \quad P^1_{21}(q^{1*}) = 353.55,$$
$$P^2_{11}(q^{2*}) = 600.00, \quad P^2_{21}(q^{2*}) = 600.00,$$
$$P^1_{12}(q^{1*}) = 353.55, \quad P^1_{22}(q^{1*}) = 353.55,$$
$$P^2_{12}(q^{2*}) = 600.00, \quad P^2_{22}(q^{2*}) = 600.00.$$

Hence, HO 1, ω_1:

$$P^1_{11}(q^{1*}) + P^1_{12}(q^{1*}) = 707.10,$$

HO 2, ω_1:

$$P^1_{21}(q^{1*}) + P^1_{22}(q^{1*}) = 707.10,$$

HO 1, ω_2:

$$P^2_{11}(q^{2*}) + P^2_{12}(q^{2*}) = 1200.00,$$

HO 2, ω_2:

$$P^2_{21}(q^{2*}) + P^2_{22}(q^{2*}) = 1200.00,$$

with the expected values of the donations for each humanitarian organization being:

$$E(P_1(q^*)) = 1002.84, \quad E(P_2(q^*)) = 1002.84.$$

With two demand points, each HO can expect to receive financial donations almost double of the amount in Example 4.

5. Summary and Conclusions

In this paper, we constructed, for the first time, a Stochastic Generalized Nash Equilibrium model for disaster relief consisting of multiple humanitarian organizations, multiple purchase locations for the disaster relief items, multiple hubs for storage, and multiple freight service provision options, ultimately, to multiple points of demand. Each humanitarian organization solves a two stage stochastic optimization problem, where, in the first stage, he seeks to determine the optimal purchase quantities for storage at multiple hubs, subject to a budget constraint, and, in the second stage, which handles multiple disaster scenarios with associated probabilities of occurrence, each HO must determine how much to deliver from the hubs to multiple points of demand, and how many additional relief items to purchase, if need be, for delivery. The humanitarian organizations compete for financial donations and consider multiple costs in their objective functions, along with a weighted altruism component, since they are nonprofits.

In addition, in the second stage, the humanitarian organizations are subject to both lower and upper bounds on the demand for the relief items at the demand points. The former constraints guarantee that the victims' needs are met, whereas the latter constraints reduce materiel convergence (and also waste). Each humanitarian organization seeks to maximize its expected utility.

The model is formulated as a finite-dimensional variational inequality problem, utilizing the concept of a Variational Equilibrium and existence established. In addition, an alternative variational inequality formulation is given, which includes Lagrange multipliers associated with the various constraints and enables elegant computations in that, at each iteration of the proposed algorithm scheme, closed form expressions for the product purchase/storage/shipment variables, and the Lagrange multipliers are obtained and presented. Both illustrative examples are presented as well as examples that are computed using the proposed algorithm.

The results in this paper open up new directions for research in disaster relief, through the synthesis of stochastic elements with game theory, in a unified theoretical and computational framework, along with policy interventions. This paper serves as a "proof of concept" and we can expect additional followup theoretical, computational, as well as empirical work. In particular, this study has taken a major step in investigating the competition among humanitarian organizations in pre- and post-disaster operations by considering uncertainty as a major factor. In this regard, there is much room for future work and development of this model. This problem can be extended to a multi-stage problem where we model the different stages of information emergence and detailed decisions to be made. Similarly, multi-disasters, such as earthquakes and tsunamis, can be examined in different stages, along with appropriate responses. In the model in this paper, we assumed that the supply hubs remain intact after the disaster. In future research one can also consider having warehouses affected by the disaster, which may cause uncertainty in supply. In addition, multiple relief item products may be considered, and the disaster relief items can be divided into perishable and nonperishable categories. In this case, the perishable goods may not be purchased and stored in hubs before the disaster; rather, they must be purchased and shipped to the victims in the second stage, after the disaster. Furthermore, in many relief operations, both before and after the disaster, actors other than humanitarian organizations also play a role. Competition among others, such as freight service providers, can also be considered in future research. This would result in multistage, multitiered stochastic game theory constructs. Another interesting research would be solving large-scale examples using recent disaster data via alternative computational methods as well as identifying possible synergies associated with cooperation among relief organizations under uncertainty.

Acknowledgments The authors are grateful to the two anonymous reviewers and to the Editors for helpful comments and suggestions on earlier versions of this paper.

Statement

No funding was provided for this work.

Conflict of Interest: The authors declare that they have no conflict of interest.

References

Alexander, D., 2002. Principles of Emergency Planning and Management. Terra Publishing, Edinburgh, United Kingdom.

Altay, N., Green III, G.G., 2006. OR/MS Research in disaster operations management. European Journal of Operational Research, 175(1), 475-493.

Amadeo, K., 2019. Hurricane Katrina facts, damage, and costs. thebalance.com, June 25. Available at:
https://www.thebalance.com/hurricane-katrina-facts-damage-and-economiceffects-3306023

Barbagallo, A., Daniele, P., Giuffre, S., Maugeri, A., 2014. Variational approach for a general financial equilibrium problem: The deficit formula, the balance law and the liability formula: A path to the economy recovery. European Journal of Operational Research, 237(1), 231-244.

Barbarosoglu, G., Arda, Y., 2004. A two-stage stochastic programming framework for transportation planning in disaster response. Journal of the Operational Research Society, 55(1), 43-53.

Bazaraa, M.S., Sherali, H.D., Shetty, C.M., 1993. Nonlinear Programming: Theory and Algorithms. John Wiley and Sons. Inc., New York.

Birge, J., Louveaux, F., 1997. Introduction to Stochastic Programming. Springer, New York, NY.

Chang, M.S., Tseng, Y.L., Chen, J.W., 2007. A scenario planning approach for the flood emergency logistics preparation problem under uncertainty. Transportation Research E, 43(6), 737-754.

Coles, J.B., Zhang, J., Zhuang, J., 2018. Partner selection in disaster relief: Partnership formation in the presence of incompatible agencies. International Journal of Disaster Risk Reduction, 27, 94-108.

Coles, J.B., Zhuang, J., 2011. Decisions in disaster recovery operations: A game theoretic perspective on organization cooperation. Journal of Homeland Security and Emergency Management, 8(1), Article 35.

Cruz, J. M., Wakolbinger, T., 2008. Multiperiod effects of corporate social responsibility on supply chain networks, transaction costs, emissions, and risk. International Journal of Production Economics, 116(1), 61-74.

Debreu, G., 1952. A social equilibrium existence theorem. Proceedings of the National Academy of Sciences, 38(10), 886-893.

Derman, C., Gleser, L.J., Olkin, I., 1973. A Guide to Probability Theory and Application. Holt, Rinehart and Winston, New York.

Dong, J., Zhang, D., Yan, H., Nagurney, A., 2005. Multitiered supply chain networks: Multicriteria decision-making under uncertainty. Annals of Operations Research, 135(1), 155-178.

Dupacova, J., 1996. Scenario-based stochastic programs: Resistance with respect to sample. Annals of Operations Research, 64(1), 21-38.

Falasca, M., Zobel, C.W., 2011. A two-stage procurement model for humanitarian relief supply chains. Journal of Humanitarian Logistics and Supply Chain Management, 1(2), 151-169.

Fischer, A., Herrich, M., Schonefeld, K., 2014. Generalized Nash equilibrium problems - Recent advances and challenges. Pesquisa Operacional, 34(3), 521-558.

Flavelle, C., Plumer, B., 2019. California bans insurers from dropping policies made riskier by climate change. The New York Times, December 5.

Gabay, D., Moulin, H., 1980. On the uniqueness and stability of Nash equilibria in noncooperative games. In: Applied Stochastic Control of Econometrics and Management Science, A. Bensoussan, P. Kleindorfer, C.S. Tapiero, Editors, North-Holland, Amsterdam, The Netherlands, pp 271-294.

Gossler, T., Wakolbinger, T., Nagurney, A., Daniele, P., 2019. How to increase the impact of disaster relief: a study of transportation rates, framework agreements and product distribution. European Journal of Operational Research, 274(1), 126-141.

Grass, E., Fischer, K., 2016. Two-stage stochastic programming in disaster management: A literature survey. Surveys in Operations Research and Management Science, 21(2), 85-100.

Gutjahr, W.J., Nolz, P.C., 2016. Multicriteria optimization in humanitarian aid. European Journal of Operational Research, 252, 351-366.

He, F., Zhuang, J., 2016. Balancing pre-disaster preparedness and post-disaster relief. European Journal of Operational Research 252(1), 246-256.

Hoyos, M.C., Morales, R.S., Akhavan-Tabatabaei, R., 2015. OR models with stochastic components in disaster operations management: A literature survey. Computers & Industrial Engineering, 82, 183-197.

International Federation of Red Cross and Red Crescent Societies, 2019. What is a disaster? Available at:
https://www.ifrc.org/en/what-we-do/disaster-management/about-disasters/what-is-a-disaster/

Kinderlehrer, D., Stampacchia, G., 1980. An Introduction to Variational Inequalities and Their Applications. Academic Press, New York.

Korpelevich, G.M., 1977. The extragradient method for finding saddle points and other problems. Matekon, 13, 35-49.

Kotsireas, I.S., Nagurney, A., Pardalos, P.M., Editors, 2016. Dynamics of Disasters: Key Concepts, Models, Algorithms, and Insights. Springer International Publishing Switzerland.

Kotsireas, I.S., Nagurney, A., Pardalos, P.M., Editors, 2018. Dynamics of Disasters: Algorithmic Approaches and Applications. Springer International Publishing Switzerland.

Kulkarni, A. A., Shanbhag, U. V., 2012. On the variational equilibrium as a refinement of the generalized Nash equilibrium. Automatica, 48(1), 45-55.

Liberatore, F., Pizarro, C., Simon de Blas, C., Ortuno, M.Y., Vitoriano, B., 2013. Uncertainty in humanitarian logistics for disaster management: A review. In: Vitoriano, B., Montero, J., Ruan, D., Editors, Decision Aid Models for Disaster Management and Emergencies. Atlantis Computational Intelligence Systems, vol 7. Atlantis Press, Paris, France.

Liu, Z., Nagurney, A., 2009. An integrated electric power supply chain and fuel market network framework: Theoretical modeling with empirical analysis for New England. Naval Research Logistics, 56(7), 600-624.

Liu, Z., Nagurney, A., 2013. Supply chain networks with global outsourcing and quick-response production under demand and cost uncertainty. Annals of Operations Research, 208(1), 251-289.

Liu, Z., Wang, J., 2019. Supply chain network equilibrium with strategic financial hedging using futures. European Journal of Operational Research, 272(3), 962-978.

Mete, H.O., Zabinsky, Z.B., 2010. Stochastic optimization of medical supply location and distribution in disaster management. International Journal of Production Economics, 126(1), 76-84.

Muggy L., Heier Stamm, J.L., 2014. Game theory applications in humanitarian operations: A review. Journal of Humanitarian Logistics and Supply Chain Management, 4(1), 4-23.

Nagurney, A., 1999. Network Economics: A Variational Inequality Approach, second and revised edition. Kluwer Academic Publishers, Dordrecht, The Netherlands.

Nagurney, A., 2006. Supply Chain Network Economics: Dynamics of Prices, Flows and Profits. Edward Elgar Publishing, Cheltenham, England.

Nagurney, A., 2016. Freight service provision for disaster relief: A competitive network model with computations. In: Dynamics of Disasters: Key Concepts, Models, Algorithms, and Insights, I.S. Kotsireas, A. Nagurney, P.M. Pardalos, Editors, Springer International Publishing Switzerland, pp 207-229.

Nagurney, A., 2017. Response to natural disasters like Harvey could be helped with game theory. The Conversation, August 28.

Nagurney, A., 2018. A multitiered supply chain network equilibrium model for disaster relief with capacitated freight service provision. In Dynamics of Disasters: Algorithmic Approaches and Applications, I.S. Kotsireas, A. Nagurney, P.M. Pardalos, Editors, Springer International Publishers

Switzerland, pp 85-108.

Nagurney, A., Alvarez Flores, E., Soylu, C., 2016. A Generalized Nash Equilibrium model for post-disaster humanitarian relief. Transportation Research E, 95, 1-18.

Nagurney, A., Daniele, P., Flores, E.A., Caruso, V., 2018. A variational equilibrium network framework for humanitarian organizations in disaster relief: Effective product delivery under competition for financial funds. In: Dynamics of Disasters: Algorithmic Approaches and Applications, I.S. Kotsireas, A. Nagurney, P.M. Pardalos, Editors, Springer International Publishers Switzerland, pp 109-133.

Nagurney, A., Ke, K., 2006. Financial networks with intermediation: risk management with variable weights. European Journal of Operational Research, 172(1), 40-63.

Nagurney, A., Masoumi, A.H., Yu, M., 2015. An integrated disaster relief supply chain network model with time targets and demand uncertainty. In: Regional Science Matters: Studies Dedicated to Walter Isard, P. Nijkamp, A. Rose, K. Kourtit, Editors, Springer International Publishing Switzerland, pp 287-318.

Nagurney, A., Nagurney, L.S., 2016. A mean-variance disaster relief supply chain network model for risk reduction with stochastic link costs, time targets, and demand uncertainty. In: Dynamics of Disasters: Key Concepts, Models, Algorithms, and Insights, I.S. Kotsireas, A. Nagurney, P.M. Pardalos, Editors, Springer International Publishing Switzerland, pp 231-255.

Nagurney, A., Qiang, Q., 2009. Fragile Networks: Identifying Vulnerabilities and Synergies in an Uncertain World. John Wiley & Sons, Hoboken, New Jersey.

Nagurney, A., Qiang, Q., 2020. Quantifying supply chain network synergy for humanitarian organizations. IBM Journal on Research and Development 64(1/2): (January/March), pp 12:1-12:16.

Nagurney, A., Salarpour, M., Daniele, P., 2019. An integrated financial and logistical game theory model for humanitarian organizations with purchasing costs, multiple freight service providers, and budget, capacity, and demand constraints. International Journal of Production Economics, 212, 212-226.

Nagurney, A., Yu, M., Besik, D., 2017. Supply chain network capacity competition with outsourcing: A variational equilibrium framework. Journal of Global Optimization, 69(1), 231-254.

Nagurney, A., Yu, M., Qiang, Q., 2011. Supply chain network design for critical needs with outsourcing. Papers in Regional Science, 90, 123-142.

Nash, J.F., 1950. Equilibrium points in n-person games. Proceedings of the National Academy of Sciences, USA, 36, 48-49.

Nash, J.F., 1951. Noncooperative games. Annals of Mathematics, 54, 286-298.

NOAA, 2017. Billion-dollar weather and climate disasters: Overview. Available at: https://www.ncdc.noaa.gov/billions/

Pierre-Louis, K., 2018. The billion-dollar natural disasters set a U.S. record in 2017. The New York Times, January 8.

Qiang, P., Nagurney, A., 2012. A bi-criteria indicator to assess supply chain network performance for critical needs under capacity and demand disruptions. Transportation Research A, 46(5), 801-812.

Rawls, C. G., Turnquist, M. A., 2010. Pre-positioning of emergency supplies for disaster response. Transportation Research B, 44(4), 521-534.

Rodriguez-Espindola, O., Albores, P., Brewster, C., 2018. Dynamic formulation for humanitarian response operations incorporating multiple organisations. International Journal of Production Economics, 204, 83-98.

Saberi, S., Cruz, J.M., Sarkis, J., Nagurney, A., 2018. A competitive multiperiod supply chain network model with freight carriers and green technology investment option. European Journal of Operational Research, 266(3), 934-949.

Salmeron, J., Apte, A., 2010. Stochastic optimization for natural disaster asset prepositioning. Production and Operations Management, 19(5), 561-574.

Seaberg, D., Devine, L., Zhuang, J., 2017. A review of game theory applications in natural disaster management research. Natural Hazards, 89, 1461-1483.

Shapiro, A., Dentcheva, D., Ruszczynski, A., 2009. Lectures on Stochastic Programming: Modeling and Theory. Society for Industrial and Applied Mathematics, Philadelphia, Pennsylvania.

Snyder, L.V., 2006. Facility location under uncertainty: A review. IIE Transactions, 38(7), 547-564

Toyasaki, F., Wakolbinger, T., 2014. Impacts of earmarked private donations for disaster fundraising. Annals of Operations Research, 221, 427-447.

US News & World Report, 2018. 10 of the deadliest natural disasters in 2018. Available at: https://wtop.com/world/2018/11/10-of-the-deadliest-natural-disasters-in-2018/

Yu, M., Cruz, J.M., Li, D., 2019. The sustainable supply chain network competition with environmental tax policies. International Journal of Production Economics, 217, 218-231.

Zhuang, J., Saxton, G., Wu, H., 2014. Publicity vs. impact in nonprofit disclosures and donor preferences: A sequential game with one nonprofit organization and N donors. Annals of Operations Research, 221(1), 469-491.