An Integrated Disaster Relief Supply Chain Network Model with Time Targets and Demand Uncertainty

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Abstract: As the number of natural disasters and their impacts increase across the globe, the need for effective preparedness against such events becomes more vital. In this paper, we construct a supply chain network optimization model for a disaster relief organization in charge of obtaining, storing, transporting, and distributing relief goods to certain disaster-prone regions. Our system-optimization approach minimizes the total operational costs on the links of the supply chain network subject to the uncertain demand for aid at the demand points being satisfied as closely as possible. A goal programming approach is utilized to enforce the timely delivery of relief items with respect to the pre-specified time targets at the demand points. A solution algorithm for the model is also provided. A spectrum of numerical examples illustrates the modeling and computational framework, which integrates the two policies of pre-positioning relief supplies as well as their procurement once the disaster has occurred.

Keywords: supply chains, disaster relief, humanitarian logistics, network optimization, cost minimization, goal programming, time constraints, variational inequalities.

1. Introduction

A natural disaster, by definition, is *some rapid, instantaneous or profound impact of the natural environment upon the socio-economic system* (Alexander (1993)). Cyclones, tornadoes, hurricanes, landslides, earthquakes, tsunamis, volcanic eruptions, and floods are among the natural phenomena ravaging our planet each year, causing immense loss of life as well as suffering. The number of natural disasters and the sizes of the populations affected by such events have been growing (Schultz, Koenig, and Noji (1996) and Nagurney and Qiang (2009)). Scientists are warning that we can expect more frequent extreme weather events in the future. For instance, tropical cyclones – which include hurricanes in the US – are expected to be stronger as a result of global warming (Sheppard (2011) and Borenstein (2012)).

The amount of damage and loss following a disaster depends on the vulnerability of the affected region, and on its ability to respond (and recover) in a timely manner, also referred to as *resilience*. Disasters are believed to occur when hazards meet vulnerability (Blaikie et al. (1994)). Hence, being prepared against potential disasters leads to reduced vulnerability and a lower number of fatalities. As the institutional experts say: "during a natural disaster, one has only two options: to become a victim, or to become a responder" (Alvendia-Quero (2012)). Thus, viable resiliency against natural disasters has to be achieved and sustained not only by the cognizant organizations, from governmental to humanitarian ones, and private enterprises, but also by individuals.

Disasters necessarily affect regions and pose challenges in all phases of disaster management. Vivid examples of disasters such as Hurricane Katrina in August 2005, the Haiti earthquake in January 2010, Fukushima in March 2011, Superstorm Sandy in October 2012, and tropical cyclone Haiyan in November 2013 have challenged researchers, practitioners, as well as policy-makers and other decision-makers and have yielded multidisciplinary approaches to models, methods, and techniques with major contributors being from regional science (cf. West and Lenze (1994), Israelevich et al. (1997), Rose et al. (1997), Okuyama, Hewings, and Sonis (1999), Cho et al. (2001), Okuyama (2004), Rose and Liao (2005), Ham, Kim, and Boyce (2005), Greenberg, Lahr, and Mantell (2007), Grubesic et al. (2008), Reggiani and Nijkamp (2009), Nagurney and Qiang (2009), and Rose (2009), among others). Background on the quantification of the economic impacts of disasters under risk and uncertainty can be found in the book by Dacy and Kunreuther (1969) (see also Kunreuther (1967)) with an updated treatment in Kunreuther and Michel-Kerjan (2012) and with a critique of the former book by Okuyama (2003).

The complexity of disaster relief supply chains, in turn, originates from several inherent factors. The associated large demands for relief products pose challenges to the logistics planning authorities (Lin (2010)) with the level of uncertainty adding to the complexity. According to Beamon and Kotleba (2006), there may exist irregularities in the size, the timing, and the location of relief product demand patterns. In addition, disaster-driven supply chains are, typically, formed as incident-responsive ones with temporary configurations of disparate resources. Commercial supply chains, on the other hand, often involve supplier-buyer relationships that have evolved over years, along with the refinement of policies. In contrast, disaster relief systems develop new networks of relationships within days or even hours, and have very short life-cycles (Oloruntoba and Gray (2006) and Denning (2006)). Thus, *time* plays a substantial role in the construction and operation of such networks. As noted by Tzeng, Cheng, and Huang (2007), once a disaster such as an earthquake strikes, effective disaster efforts can mitigate the damage, reduce the number of fatalities, and bring relief to the survivors.

The fact that time is a critical element in disaster relief is also noted in the key benchmarks defined for the US Federal Emergency Management Agency (FEMA)'s response and recovery. The key benchmarks are: to meet the survivors' initial demands within 72 hours, to restore basic community functionality within 60 days, and to return to as normal of a situation within 5 years (Fugate (2012)). According to FEMA's first national preparedness report, states have developed fatality management plans, yet not all of them are adequate and practical. Specifically, it is challenging to measure the progress of preparedness activities, according to a US Department of Homeland Security (2012) report. Walton, Mays, and Haselkorn (2011) further emphasize that the importance of speed is noted in leading emergency response guidelines with disaster relief operations intrinsically requiring the need for speed (see also USAID (2005) and UNHCR (2007)).

The timely and efficient delivery of relief goods to the affected population not only decreases the fatality rate but may also prevent chaotic situations. In the case of cyclone Haiyan, for example, the strongest typhoon ever recorded in terms of wind speed, which devastated areas of Southeast Asia, particularly the Philippines, where 11 million people were affected, slow relief delivery efforts forced people to seek any possible means to survive. A number of relief trucks were attacked and had food stolen, and some areas were reported to be on the brink of anarchy (Chicago Tribune (2013) and CBS News (2013)).

In this paper, we propose an integrated supply chain network model for disaster relief. Our mathematical framework is of system-optimization type where the organization aims to satisfy the uncertain demands subject to the minimization of total operational costs while the sequences of activities leading to the ultimate delivery of the relief good are targeted to be completed within a certain time. The first criterion in this model captures the total costs of all the activities in the supply chain as well as the expected shortage and surplus penalties at the demand points. The second criterion consists of the penalties associated with the time deviations on paths of the relief items to the demand points with respect to the pre-specified target times. Our model allows this time requirement to vary from one demand point to another. In addition, our model integrates the possibility of the pre-positioning of relief items before the occurrence of the disaster as well as the case in which the organization procures after the disaster hits the region. The solution to our model yields the optimal levels of activities associated with procurement, storage, and transportation of the relief items, whether purchased/procured in advance or after the disaster strikes – if need be. The solution also provides the optimal values of the over-the-target time deviations. This feature enables the organization to estimate the anticipated delays in the delivery of the relief items to the demand points.

We now review the related literature in the area of disaster relief supply chains. About half of the research has applied quantitative techniques (Charles and Lauras (2011)). The other half utilizes case studies and empirical research to investigate past disasters and to present best practices. Time is frequently a criterion/objective aimed to be maintained along with the total cost being minimized (see, e.g., Zhenling (2009) and Tzeng, Cheng, and Huang (2007)). Such a formulation, however, may result in excessive delivery times to the demand points if the distribution to certain locations is too costly. In our model, in contrast, rather than evenly minimizing all travel/delivery times, we seek to minimize the time deviations of activities on paths with respect to the pre-determined target times. Thus, one can prioritize the demand points based on their locations, demand patterns, number of elderly and children, etc.

Sheu (2010) constructed a dynamic fuzzy model of disaster relief response in large-scale problems. Nagurney, Yu, and Qiang (2011) presented a supply chain network design model for critical needs with the possibility of outsourcing. Nagurney, Yu, and Qiang (2012) developed a multiproduct supply chain for the production and distribution of disaster relief items. Nagurney et al. (2013) presented supply chain network frameworks for various time-sensitive, perishable healthcare products such as human blood, pharmaceuticals, medical nuclear products, etc. Nagurney and Yu (2014), in turn, constructed a game theory supply chain network model for the case of time–based oligopolistic competition.

Hale and Moberg (2005) proposed a set covering location model to identify secure sites for the storage of emergency supplies. Barbarosoglu and Arda (2004) and Falasca and Zobel

(2011) developed two-stage stochastic models for the procurement and transportation of the vital disaster relief items. Also, Mete and Zabinsky (2010) proposed a two-stage stochastic model for the storage and distribution of medical supplies to be used in case of emergencies. Balcik and Beamon (2005) studied facility location in humanitarian relief. Huang, Smilowitz, and Balcik (2012) presented performance measures for the efficiency, efficacy, and equity of relief distribution.

Liu and Nagurney (2011), in turn, constructed a supply chain network model with quick-response production and outsourcing under uncertain demand and cost. Nagurney and Qiang (2012) developed network robustness and performance measures in addition to synergy measurement of network integration in the case of humanitarian partnerships. Qiang and Nagurney (2012) proposed a bi-criteria indicator to evaluate the performance of supply chains of critical needs under capacity and demand disruptions. MacKenzie and Barker (2011) integrated a risk-management approach with a Multiregional Input-Output model using ideas from Isard et al. (1998) to quantify the regional economic impacts of a supply shortage. Simpson and Hancock (2009) applied simulation to the case of resource allocation in an emergency response system. Rottkemper, Fischer, and Blecken (2012) presented a bi-criteria mixed-integer programming model for the inventory relocation of relief items. Furthermore, Ortuño, Tirado, and Vitoriano (2011) and Vitoriano et al. (2011) developed goal programming frameworks for the distribution of relief goods while considering targets for attributes such as the cost and travel time. A recent edited volume on disaster management and emergencies is by Vitoriano, Montero, and Ruan (2013) with a survey on decision aid models for humanitarian logistics therein by Ortuño et al. (2013).

Our model, in contrast to much of the above cited literature, does not consider targets for cost; instead, it minimizes the total operational costs of the activities in the supply chain network. This, we believe, makes more sense in that the relief operations budget largely depends on the intensity of damage and the fatality toll which may be difficult be determine a priori. In addition, our model – unlike the aforementioned ones – allows for the pre-disaster and the post-disaster procurement of relief items, and involves the time and the cost associated with each strategy or a combination of both. Moreover, we handle nonlinear costs, which capture congestion effects, a big issue in disaster relief, and an aspect that has been missing from much of the literature on the topic (cf. Haghani and Oh (1996)).

The organization of this paper is as follows. In Section 2, we present the supply chain network topology of the disaster relief organization, construct the optimization problem, and establish its variational inequality formulation which provides us with nice computational features. We also present several illustrative numerical examples and conduct sensitivity analysis. In Section 3, we propose an algorithm that, when applied, computes the incurred optimal levels of disaster relief product flows in the supply chain network as well as the optimal path time deviations. The algorithm is then used to solve a large-scale numerical example in Section 4, along with a variant. In Section 5, we summarize the results and present our conclusions.

2. The Integrated Disaster Relief Supply Chain Network Model

In this Section, we develop the integrated disaster relief supply chain network model. We mention that, depending on the structure of the specific disaster relief (humanitarian) organization, the scale/intensity of the disaster, and the region or the country that the disaster takes place in, the supply chain network may vary in specific details. Nevertheless, the general framework of the majority of non-profit disaster aid organizations follows a pattern similar to the one that we describe below.

2.1 Components of the Disaster Relief Supply Chain Network

The disaster relief supply chain network model, as we shall demonstrate, captures both the *preparedness* phase and the *response* phase of the disaster management cycle (with the other two phases being *mitigation* and *recovery*) (Tomasini and Van Wassenhove (2009)). We take into account the pre-disaster preparations including the procurement, the pre-positioning, and the storage of disaster relief items given the estimated demand in disaster-prone areas. We also take into consideration the relevant issues surrounding the transportation and the ultimate distribution of the relief goods to the demand points once a potential disaster takes place. Furthermore, the case where an organization procures the humanitarian aid items after the occurrence of a disaster is also integrated into our model.

Regardless of the strategy that the relief/humanitarian organization will eventually adopt – the procurement of relief goods in advance or subsequent to the disaster or a combination of both – the ultimate delivery of the items to each demand point is aimed to be completed within a specific time target associated with that point as closely as possible. Such goal constraints enhance the reliability of a disaster relief system. All disaster relief organizations, whether owned and run by the public or the private sector, are non-profit. Thus, a centralized system-optimization approach is adopted as recommended by Nagurney, Yu, and Qiang (2011).

Figure 1 depicts the network topology of a disaster relief supply chain associated with a humanitarian organization responsible for the procurement and the delivery of a disaster relief product. The progression of the required activities in such systems over time is graphically depicted. Depending on the location and the type of the relief product, be it water, food, medicines or vaccines, for example, the disaster relief organization can represent the US National Guard, the American Red Cross, the International Federation of Red Cross and Red Crescent Societies (IFRC), Doctors without Borders (MSF), etc. The organization, in Figure 1, is denoted by the origin node. Every other node in the supply chain network represents a facility in the supply chain. A path connecting the origin node to a demand node (destination) consists of a sequence of directed links with each one corresponding to an activity in the supply chain. This sequence of activities ensures that the disaster relief item is procured, positioned, stored, and distributed to the people in need. We assume that there exists at least one path in the supply chain network connecting the origin (node 1) with each demand point located at the rightmost (final) tier of the supply chain (with additional assumptions stated later).

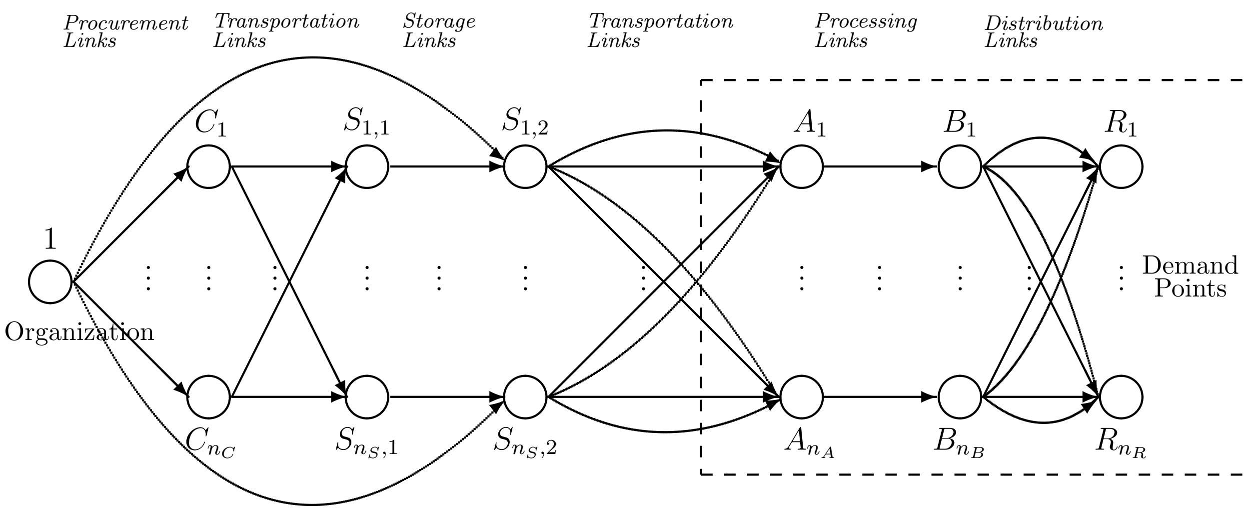

Figure 1: Network Topology of the Integrated Disaster Relief Supply Chain

In the topology in Figure 1, we assume that the organization is considering n_C procurement facilities, denoted by $C_1, C_2, \ldots, C_{n_C}$. These are the locations where the purchased aid item is received from, or the item is collected from donors, such as people, companies, etc. Associated with each type of procurement is an individual cost function. Collection nodes constitute the second tier of the supply chain, and the links connecting the origin to these nodes correspond to the activities of "procurement."

The next two sets of nodes, i.e., those associated with tiers 3 and 4, represent the storage facilities. These are the facilities to which the collected goods are shipped, and then stored until needed. There are n_S of such nodes in the network, and the links connecting tiers 2 with 3, and 3 with 4 are referred to as "transportation" and "storage" links, respectively.

The reason that the nodes in tiers 3 and 4 are sharing similar notation (e.g., $S_{1,1}$ and $S_{1,2}$) is that they correspond to the same geographical locations but reflect different points in time.

As mentioned earlier, our model not only allows for the pre-planned procurement and storage of relief goods, but also integrates the direct procurement of aid items. The case of immediate procurement is reflected in the network in Figure 1 with the curved links connecting the origin node to the storage facilities. This is the case where the organization initiates procurement after the occurrence of the disaster. The organization may benefit from avoiding storage costs. Even if the organization decides to "store beforehand" as its dominant strategy rather than to "procure afterward," the latter can be expected to be available as a backup plan especially under severe circumstances.

The next set of links, connecting tiers 4 and 5 of nodes, are of "transportation" type, and correspond to the shipment of the product – either stored or freshly procured – to the affected region. These links are used once the disaster strikes a region. As seen in Figure 1, there may exist multiple links connecting every pair of nodes in tiers 4 and 5. This is to allow for different modes of transportation: air, ground, and/or sea, as feasible, for the shipment. Thus, nodes $A_1, A_2, \ldots, A_{n_A}$ represent the arrival portals of disaster aid items into the affected region – which is represented by the dashed box in the network topology.

The next set of links represent the "processing" of the received items at the portals of the affected region. Typically, a variety of goods are sent to the areas hit by disasters – some of which may even be useless or inappropriate. As a result, the unloading, unpacking, sorting, and the loading of the goods have to be done before the ultimate distribution takes place. Processing is often done at the same location as the arrival portals (e.g., airports, ports, etc.), and the corresponding facilities are denoted by $B_1, B_2, \ldots, B_{n_B}$. Processing at international locations may include, in addition, customs inspections and payments, etc.

Finally, the last set of nodes reflects the actual demand points in the affected region, denoted by $R_1, R_2, \ldots, R_{n_R}$. These nodes can be the hospitals, community centers, or points of distribution of the relief product to the disaster victims. The links connecting the last two sets of nodes are "distribution" links, and are represented by multiple links to allow for the possibility of various modes of transportation within the affected region. This may be crucial when parts of the infrastructure have been severely damaged by the disaster.

The supply chain network topology is denoted by $G = [N, L]$, where N and L, respectively, denote the sets of nodes and links (arcs).

2.2 Formulation of the Disaster Relief Supply Chain Network Model

Let f_a denote the flow of the disaster relief product on link a, be it a procurement, storage, transportation, processing, or distribution link. Let $c_a(f_a)$ and $\hat{c}_a(f_a)$ denote the unit operational cost function and the total operational cost function on link a, respectively. The link total cost functions are assumed to be convex and continuously differentiable. We have:

$$\hat{c}_a(f_a) = f_a \times c_a(f_a), \qquad \forall a \in L. \tag{1}$$

$\mathcal{P}_k$ denotes the set of paths connecting the origin (node 1) to demand point k with $\mathcal{P}$ denoting the set of all paths joining the origin node to the destination nodes. The total number of paths in the supply chain, i.e., the number of elements in $\mathcal{P}$ is given by n_p.

In the model, we assume that the demand is uncertain due to the unpredictability of the actual demand at the demand points. Similar examples of system-optimized models with uncertain demand and associated shortage and surplus penalties can be found in the literature (see, e.g., Dong, Zhang, and Nagurney (2004), Nagurney, Yu, and Qiang (2011), Nagurney, Masoumi, and Yu (2012), and Nagurney and Masoumi (2012)).

The probability distribution of demand is assumed to be available. It may be derived using census data and/or information gleaned and obtained over the course of the preparedness phase. If d_k denotes the actual (uncertain) demand at destination point k, we have:

$$P_k(D_k) = P_k(d_k \leq D_k) = \int_0^{D_k} \mathcal{F}_k(t)dt, \qquad k = 1, \ldots, n_R, \tag{2}$$

where P_k and $\mathcal{F}_k$ denote the probability distribution function, and the probability density function of demand at point k, respectively.

Let v_k be the "projected demand" for the disaster relief item at point $k; k = 1, \ldots, n_R$. The amounts of shortage and surplus of the aid item at destination node k are denoted by Δ_k^- and Δ_k^+, respectively, and are calculated as follows:

$$\Delta_k^- \equiv \max\{0, d_k - v_k\}, \qquad k = 1, \ldots, n_R, \tag{3a}$$

$$\Delta_k^+ \equiv \max\{0, v_k - d_k\}, \qquad k = 1, \ldots, n_R. \tag{3b}$$

Hence, based on the probability distribution of the demand, the expected values of shortage and surplus at each demand point are:

$$E(\Delta_k^-) = \int_{v_k}^{\infty} (t - v_k)\mathcal{F}_k(t)dt, \qquad k = 1, \ldots, n_R, \tag{4a}$$

$$E(\Delta_k^+) = \int_0^{v_k} (v_k - t)\mathcal{F}_k(t)dt, \qquad k = 1, \ldots, n_R. \tag{4b}$$

Therefore, the expected penalty assigned to the humanitarian organization due to the shortage and surplus of the relief item at each demand point is equal to:

$$E(\lambda_k^- \Delta_k^- \ + \ \lambda_k^+ \Delta_k^+) = \lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+), \qquad k = 1, \ldots, n_R, \tag{5}$$

where λ_k^- is the unit penalty corresponding to the shortage of the relief item at demand point k which can represent the social cost of a death due to the inefficiency of the relief distribution system. In order to avoid such shortages, a large penalty should be assigned by the authorities, which can vary across the demand points. Furthermore, λ_k^+ denotes the unit penalty associated with the surplus of the relief item. This penalty is taken into account so as to minimize the over-shipping of goods, which results in congestion and additional efforts. Depending on the criticality of the situation, the unit surplus penalty can be expected to be lower than that of shortage, for a given demand point, and can be equal to zero. Similar ideas have been applied in the case of critical needs (Nagurney, Yu, and Qiang (2011)) and human blood (Nagurney, Masoumi, and Yu (2012) and Nagurney and Masoumi (2012)).

x_p represents the flow of the disaster relief goods on path p joining node 1 with a demand node which must be nonnegative, since the goods are procured, stored, and shipped in nonnegative quantities, that is,

$$x_p \geq 0, \qquad \forall p \in \mathcal{P}. \tag{6}$$

The projected demand at destination node k, v_k, is equal to the sum of flows on all paths belonging to $\mathcal{P}_k$, that is:

$$v_k \equiv \sum_{p \in \mathcal{P}_k} x_p, \qquad k = 1, \ldots, n_R. \tag{7}$$

The relationship between the flow on link a, f_a, and the path flows is as follows:

$$f_a = \sum_{p \in \mathcal{P}} x_p \ \delta_{ap}, \qquad \forall a \in L. \tag{8}$$

Here, δ_{ap} is an indicator of link a's relation with path p, and is equal to 1 if link a is contained in path p and is 0, otherwise.

Next, we present the expressions that capture the *time* aspect of our integrated disaster relief supply chain model.

Let τ_a denote the completion time of the activity on link a, which is assumed to be a linear function of the flow of the product on that link. We have:

$$\tau_a(f_a) = g_a f_a + h_a, \qquad \forall a \in L, \tag{9}$$

where $h_a \geq 0$, and $g_a \geq 0$. We allow (some of) these terms to take on zero values for modeling flexibility purposes, as we shall show in a forthcoming numerical example.

Considering the above link time functions, we note that, in the context of transportation science, g_a can be referred to as the *congestion factor* on link a while h_a is the *uncongested term*, also known as the *free-flow travel time* on link a (see Dafermos and Sparrow (1969) and Beckman, McGuire, and Winsten (1956)). Here, the congestion factor addresses the impact of the volume of the product on the time associated with the link activity, whether it is a procurement link, a storage link, a processing link, or a transport/shipment/distribution link. The uncongested cost term on a link represents the best case scenario time associated with completing the activity on the link.

The completion time of the sequence of activities on path p is denoted by τ_p, and is derived as follows:

$$\tau_p = \sum_{a \in L} \tau_a(f_a)\delta_{ap} = \sum_{a \in L}(g_a f_a + h_a)\delta_{ap}, \qquad \forall p \in \mathcal{P}. \tag{10}$$

The sum of the uncongested terms h_as on path p is constant and is denoted by h_p:

$$h_p = \sum_{a \in L} h_a \delta_{ap}, \qquad \forall p \in \mathcal{P}. \tag{11}$$

Hence,

$$\tau_p = h_p + \sum_{a \in L} g_a f_a \delta_{ap}, \qquad \forall p \in \mathcal{P}. \tag{12}$$

Note that, at this point in the model development, the completion time of each path in accordance with the demand points can be expressed as:

$$\tau_p \leq T_k, \qquad \forall p \in \mathcal{P}_k; \quad k = 1, \ldots, n_R, \tag{13}$$

where T_k is the target for the completion time of the activities on paths corresponding to demand point k determined by the organization's decision-maker. In other words, all activities on paths connecting the origin node to a specific demand point are aimed to be completed by a certain time. The time targets at the demand points, T_ks, may vary from one demand node to another since the decision-maker may need to prioritize demand points based on their strategic locations, potential injury tolls, classes of populations (elderly, children, etc.). Hence, the choice of these time goals by the authorities has to be done carefully. Using (12), inequality (13) can be re-written as:

$$\sum_{a \in L} g_a f_a \delta_{ap} \leq T_k - h_p, \qquad \forall p \in \mathcal{P}_k; \quad k = 1, \ldots, n_R. \tag{14}$$

We also define the target time T_{kp} for demand point k with respect to path p. This new target time is defined as the difference between target time T_k and h_p:

$$T_{kp} = T_k - h_p, \qquad \forall p \in \mathcal{P}_k; \quad k = 1, \ldots, n_R. \tag{15}$$

We assume that the values of the T_{kp}'s are all nonnegative; otherwise, we remove path p from the network due to infeasibility. We further assume that there is at least one path $p \in \mathcal{P}_k$ for each k such that $T_{kp} > 0$; otherwise, the organization will have to relax the corresponding time target T_k (make it less restrictive).

Thus, inequality (14) can be re-written as:

$$\sum_{a \in L} g_a f_a \delta_{ap} \leq T_{kp}, \qquad \forall p \in \mathcal{P}_k; \quad k = 1, \ldots, n_R. \tag{16}$$

Note that the goal constraints introduced here are not hard constraints meaning that, under certain circumstances, the organization might be forced to deviate from the goal. However, this deviation will be minimized, as we shall see, along with the minimization of total cost throughout the network subject to the uncertain demand. For example, depending on the actual completion time, the sequence of activities on path p leading to the delivery of the relief products to demand point k will be completed either at, before, or after the determined time goal target. Let z_p denote the amount of deviation with respect to target time T_{kp} corresponding to the "late" delivery of product to point k on path p, which was assumed to be nonnegative. Using (16), we now construct the following constraints:

$$\sum_{a \in L} g_a f_a \delta_{ap} - z_p \leq T_{kp}, \qquad \forall p \in \mathcal{P}_k; \quad k = 1, \ldots, n_R. \tag{17}$$

The path time deviations must be nonnegative for all paths in the network; that is,

$$z_p \geq 0, \qquad \forall p \in \mathcal{P}_k; \quad k = 1, \ldots, n_R. \tag{18}$$

Using (8), we can replace the link flows with the path flows in (17), so that

$$\sum_{q \in \mathcal{P}} \sum_{a \in L} g_a x_q \delta_{aq} \delta_{ap} - z_p \leq T_{kp}, \qquad \forall p \in \mathcal{P}_k; \quad k = 1, \ldots, n_R. \tag{19}$$

Let $\gamma_k(z)$ denote the tardiness penalty function corresponding to demand point k which is a function of the time deviations on paths leading to that point. These functions are assumed to be convex and continuously differentiable.

Interestingly, Nagurney, Thore, and Pan (1996) utilized goal targets in the case of spatial economic markets, whereas Nagurney and Ramanujam (1996) considered penalties associated with transportation targets with associated penalty functions that could be nonlinear (as in the case above).

$\hat{C}_p(x)$ denotes the total operational cost function on path p and is constructed as:

$$\hat{C}_p(x) = x_p \times C_p(x) = x_p \times \sum_{a \in L} c_a(f_a)\delta_{ap}, \qquad \forall p \in \mathcal{P}, \tag{20}$$

with notice to (8), where C_p denotes the unit operational cost on path p.

The disaster relief supply chain network optimization problem can be expressed as follows. The organization seeks to determine the optimal levels of the disaster relief item processed on each supply chain link as well as the optimal amounts of the time deviations on paths, subject to the minimization of the total operational cost while satisfying the uncertain demand as closely as possible. Therefore, the optimization problem is constructed as:

$$\text{Minimize} \sum_{p \in \mathcal{P}} \hat{C}_p(x) \;+\; \sum_{k=1}^{n_R} (\lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+)) + \sum_{k=1}^{n_R} \gamma_k(z), \tag{21}$$

subject to: constraints (6), (18), and (19).

Next, we present the partial derivatives of the shortages and the surpluses solely in terms of path flows, which will be used later in developing the variational inequality formulation of the problem. The respective partial derivatives of the expected values of shortage and surplus of the disaster relief item at each demand point with respect to the path flows, derived in Dong, Zhang, and Nagurney (2004), Nagurney, Yu, and Qiang (2011), and Nagurney, Masoumi, and Yu (2012), are given by:

$$\frac{\partial E(\Delta_k^-)}{\partial x_p} = P_k \left(\sum_{q \in \mathcal{P}_k} x_q \right) - 1, \quad \forall p \in \mathcal{P}_k; \quad k = 1, \ldots, n_R, \tag{22a}$$

and,

$$\frac{\partial E(\Delta_k^+)}{\partial x_p} = P_k \left(\sum_{q \in \mathcal{P}_k} x_q \right), \quad \forall p \in \mathcal{P}_k; \quad k = 1, \ldots, n_R. \tag{22b}$$

Let K denote the feasible set such that:

$$K = \{(x, z, \omega) | x \in R_+^{n_p}, z \in R_+^{n_p}, \text{ and } \omega \in R_+^{n_p}\}, \tag{23}$$

where x is the vector of path flows of the relief item, z is the vector of time deviations on paths, and ω is the vector of Lagrange multipliers corresponding to the constraints in (19).

We now derive the variational inequality of the integrated disaster relief supply chain network problem.

Theorem 1

The optimization problem (21), subject to its constraints (6), (18), and (19), is equivalent to the variational inequality problem: determine the vector of optimal path flows, the vector of optimal path time deviations, and the vector of optimal Lagrange multipliers $(x^*, z^*, \omega^*) \in K$*, such that:*

$$\sum_{k=1}^{n_R} \sum_{p\in\mathcal{P}_k} \left[\frac{\partial \hat{C}_p(x^*)}{\partial x_p} + \lambda_k^+ P_k(\sum_{q\in\mathcal{P}_k} x_q^*) - \lambda_k^-(1 - P_k(\sum_{q\in\mathcal{P}_k} x_q^*)) + \sum_{q\in\mathcal{P}} \sum_{a\in L} \omega_q^* g_a \delta_{aq}\delta_{ap} \right] \times [x_p - x_p^*]$$

$$+ \sum_{k=1}^{n_R} \sum_{p\in\mathcal{P}_k} \left[\frac{\partial \gamma_k(z^*)}{\partial z_p} - \omega_p^* \right] \times [z_p - z_p^*]$$

$$+ \sum_{k=1}^{n_R} \sum_{p\in\mathcal{P}_k} \left[T_{kp} + z_p^* - \sum_{q\in\mathcal{P}} \sum_{a\in L} g_a x_q^* \delta_{aq}\delta_{ap} \right] \times [\omega_p - \omega_p^*] \geq 0, \qquad \forall (x, z, \omega) \in K, \qquad (24)$$

where

$$\frac{\partial \hat{C}_p(x)}{\partial x_p} \equiv \sum_{a\in L} \frac{\partial \hat{c}_a(f_a)}{\partial f_a} \delta_{ap}, \quad \forall p \in \mathcal{P}_k; \; k = 1, \ldots, n_R. \qquad (25)$$

Proof: Consider the optimization formulation in (21). The convexity of the objective function follows, under the imposed assumptions, from Nagurney, Masoumi, and Yu (2012), and since the $\gamma_k(z)$ functions are assumed to be convex.

According to Bertsekas and Tsitsiklis (1989) (page 287), the optimization problem (21), along with its inequality constraints (19), is equivalent to the below inequality which is resulted from the Karush-Kuhn-Tucker (KKT) (cf. Karush (1939) and Kuhn and Tucker (1951)) conditions:

$$\sum_{k=1}^{n_R} \sum_{p\in\mathcal{P}_k} \left[\frac{\partial}{\partial x_p} \left(\hat{C}_p(x^*) + \lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+) + \gamma_k(z^*) \right) + \sum_{q\in\mathcal{P}} \sum_{a\in L} \omega_q^* g_a \delta_{aq}\delta_{ap} \right] \times [x_p - x_p^*]$$

$$+ \sum_{k=1}^{n_R} \sum_{p\in\mathcal{P}_k} \left[\frac{\partial}{\partial z_p} \left(\hat{C}_p(x^*) + \lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+) + \gamma_k(z^*) \right) - \omega_p^* \right] \times [z_p - z_p^*]$$

$$+ \sum_{k=1}^{n_R} \sum_{p\in\mathcal{P}_k} \left[T_{kp} + z_p^* - \sum_{q\in\mathcal{P}} \sum_{a\in L} g_a x_q^* \delta_{aq}\delta_{ap} \right] \times [\omega_p - \omega_p^*] \geq 0, \qquad \forall (x, z, \omega) \in K. \qquad (26)$$

Substituting the partial derivatives in (26), and using (25), (22a), and (22b), one obtains the variational inequality (24). □

Variational inequality (24) can be put into standard form (Nagurney (1999)) as follows: determine $X^* \in \mathcal{K}$ such that:

$$\left\langle F(X^*), X - X^* \right\rangle \geq 0, \qquad \forall X \in \mathcal{K}, \tag{27}$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in n-dimensional Euclidean space. If the feasible set is defined as $\mathcal{K} \equiv K$, and the column vectors $X \equiv (x, z, \omega)$ and $F(X) \equiv (F_1(X), F_2(X), F_3(X))$, where:

$$F_1(X) = \left[\frac{\partial \hat{C}_p(x)}{\partial x_p} \; + \; \lambda_k^+ P_k(\sum_{q \in \mathcal{P}_k} x_q) \; - \; \lambda_k^-(1 - P_k(\sum_{q \in \mathcal{P}_k} x_q)) + \; \sum_{q \in \mathcal{P}} \sum_{a \in L} \omega_q g_a \delta_{aq} \delta_{ap}, \right.$$

$$\left. p \in \mathcal{P}_k; \quad k = 1, \ldots, n_R \right],$$

$$F_2(X) = \left[\frac{\partial \gamma_k(z)}{\partial z_p} - \omega_p, \qquad p \in \mathcal{P}_k; \quad k = 1, \ldots, n_R \right],$$

and

$$F_3(x) = \left[T_{kp} + z_p - \sum_{q \in \mathcal{P}} \sum_{a \in L} g_a x_q \delta_{aq} \delta_{ap}, \qquad p \in \mathcal{P}_k; \quad k = 1, \ldots, n_R, \right], \tag{28}$$

then variational inequality (24) can be re-expressed as standard form (27).

We utilize variational inequality (24) for our computations to obtain the optimal path flows and the optimal path time deviations. Then, we use (8) to calculate the optimal link flows of disaster relief items in the supply chain network.

2.3 An Illustrative Example and Two Variants with Sensitivity Analysis

We now present an illustrative numerical example as well as two variants along with their solutions, accompanied by some sensitivity analysis, before proceeding to the solution algorithm, which can be applied to solve large-scale disaster relief supply chain networks in practice.

2.3.1 Illustrative Example

Consider the simple disaster relief supply chain network topology in Figure 2. The organization is assumed to possess a single procurement facility and a single storage facility, and aims to deliver the relief goods to one demand point through one arrival portal and one processing facility in the affected region. We allow two modes of transportation from the storage facility to the portal of the affected region. The links are labeled as in Figure

2, i.e., a, b, c, d, e, f, and g, where links d and e represent ground and air transportation, respectively.

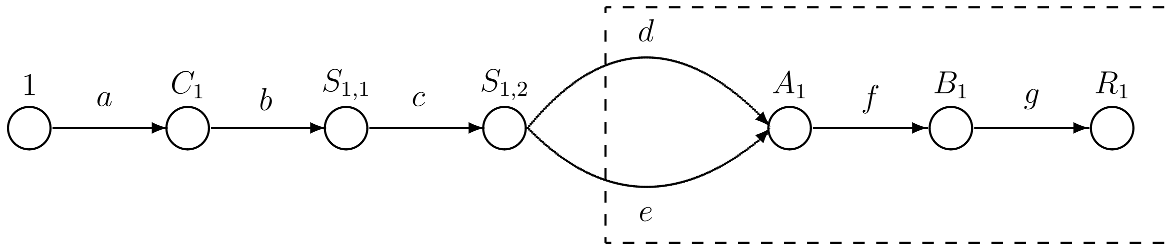

Figure 2: Supply Chain Network Topology for the Illustrative Numerical Example

The total operational cost functions on the links are:

$$\hat{c}_a(f_a) = 3f_a^2 + 2f_a, \quad \hat{c}_b(f_b) = f_b^2 + 3f_b, \quad \hat{c}_c(f_c) = 2f_c^2 + f_c,$$

$$\hat{c}_d(f_d) = 4f_d^2 + 3f_d, \quad \hat{c}_e(f_e) = 7f_e^2 + 5f_e,$$

$$\hat{c}_f(f_f) = f_f^2 + 4f_f, \quad \hat{c}_g(f_g) = 3f_g^2 + 2f_g.$$

There are two paths in this network defined as: $p_1 \equiv (a, b, c, d, f, g)$ and $p_2 \equiv (a, b, c, e, f, g)$. The set of paths, $\mathcal{P}$, is identical to the set of paths connecting the origin 1 to the destination, R_1, i.e., $\mathcal{P}_{R_1}$, where

$$\mathcal{P} = \mathcal{P}_{R_1} = \{p_1, p_2\}.$$

The demand for the relief item at the demand point followed a uniform distribution on the interval [5,10]; therefore, the probability distribution function of demand at the demand point is:

$$P_{R_1}(v_{R_1}) = \frac{v_{R_1} - 5}{10 - 5} = \frac{x_{p_1} + x_{p_2} - 5}{5}.$$

The unit shortage and surplus penalties were: $\lambda^-_{R_1} = 5000$ and $\lambda^+_{R_1} = 100$.

The organization is interested in the pre-positioning strategy; i.e., it wishes to determine the amount of the relief item that should be stored beforehand. Thus, the organization will only ship the pre-positioned supplies of relief goods and will not procure post the disaster. Consequently, the completion time on links a, b, and c (procurement, transportation, and storage) is set to zero:

$$\tau_a(f_a) = \tau_b(f_b) = \tau_c(f_c) = 0.$$

The completion time functions on the rest of the links were:

$$\tau_d(f_d) = 9f_d + 6, \quad \tau_e(f_e) = 2f_e + 2,$$

$$\tau_f(f_f) = 1.5f_f + 2, \quad \tau_g(f_g) = 5f_g + 4.$$

The target time at demand point R_1 was 72 hours:

$$T_{R_1} = 72, \quad \forall p \in \mathcal{P}_{R_1}.$$

The decision-maker assigned a higher tardiness penalty to p_2 in that the expectation of on-time delivery from the path with the air transportation link was higher, so the tardiness penalty function at the demand point was:

$$\gamma_{R_1}(z) = 3.5z_{p_1}^2 + 8z_{p_2}^2.$$

Solution: For the variational inequality (24) to hold for all $(x, z, \omega) \in K$, at the optimal solution (x^*, z^*, ω^*), we must have that the term in each of the three left-hand-side brackets is equal to zero, assuming that the optimal value of each of the variables is positive. Thus, we have the following six equations with $x_{p_1}^*, x_{p_2}^*, z_{p_1}^*, z_{p_2}^*, \omega_{p_1}^*$, and $\omega_{p_2}^*$ as unknowns:

$$\frac{\partial \hat{C}_{p_1}(x^*)}{\partial x_{p_1}} + \lambda_{R_1}^+ P_{R_1}(x_{p_1}^* + x_{p_2}^*) - \lambda_{R_1}^-(1 - P_{R_1}(x_{p_1}^* + x_{p_2}^*)) + \omega_{p_1}^*(g_d + g_f + g_g) + \omega_{p_2}^*(g_f + g_g) = 0,$$

$$\frac{\partial \hat{C}_{p_2}(x^*)}{\partial x_{p_2}} + \lambda_{R_1}^+ P_{R_1}(x_{p_1}^* + x_{p_2}^*) - \lambda_{R_1}^-(1 - P_{R_1}(x_{p_1}^* + x_{p_2}^*)) + \omega_{p_1}^*(g_f + g_g) + \omega_{p_2}^*(g_e + g_f + g_g) = 0,$$

$$\frac{\partial \gamma_{R_1}(z^*)}{\partial z_{p_1}} - \omega_{p_1}^* = 0,$$

$$\frac{\partial \gamma_{R_1}(z^*)}{\partial z_{p_2}} - \omega_{p_2}^* = 0,$$

$$T_{R_1 p_1} + z_{p_1}^* - (x_{p_1}^*(g_d + g_f + g_g) + x_{p_2}^*(g_f + g_g)) = 0, \text{ and}$$

$$T_{R_1 p_2} + z_{p_2}^* - (x_{p_1}^*(g_f + g_g) + x_{p_2}^*(g_e + g_f + g_g)) = 0.$$

Note that, in the first two and the last two equations above, $g_a = g_b = g_c = 0$, and several δ_{ap}'s and δ_{aq}'s are zero. Also, note that:

$$T_{R_1 p_1} = T_{R_1} - h_{p_1} = T_{R_1} - (h_d + h_f + h_g) = 60$$

and

$$T_{R_1 p_2} = T_{R_1} - h_{p_2} = T_{R_1} - (h_e + h_f + h_g) = 64.$$

Next, using (25) to calculate the partial derivatives of the total path costs, and substituting the given parameters into the above equations, we obtain the following system of equations:

$$\begin{cases} 1048x^*_{p_1} & +1040x^*_{p_2} & & & +15.5\omega^*_{p_1} & +6.5\omega^*_{p_2} & = 10085 \\ 1040x^*_{p_1} & +1054x^*_{p_2} & & & +6.5\omega^*_{p_1} & +8.5\omega^*_{p_2} & = 10083 \\ & & 7z^*_{p_1} & & -\omega^*_{p_1} & & = 0 \\ & & & 16z^*_{p_2} & & -\omega^*_{p_2} & = 0 \\ -15.5x^*_{p_1} & -6.5x^*_{p_2} & +z^*_{p_1} & & & & = -60 \\ -6.5x^*_{p_1} & -8.5x^*_{p_2} & & +z^*_{p_2} & & & = -64. \end{cases}$$

Solution of the above system yields:

$$x^*_{p_1} = 1.04 \text{ and } x^*_{p_2} = 7.50.$$

Hence, the optimal values of link flows are:

$$f^*_a = f^*_b = f^*_c = f^*_f = f^*_g = x^*_{p_1} + x^*_{p_2} = 8.54, \qquad f^*_d = x^*_{p_1} = 1.04, \text{ and } f^*_e = x^*_{p_2} = 7.50.$$

As seen above, the optimal flow of the disaster relief item on link e (air transportation to the affected region) was considerably higher than that on link d (ground transportation). This is because the humanitarian organization, in this example, chooses the quicker mode of transportation, but at a higher cost. The optimal time deviations on paths p_1 and p_2 with respect to the target of 72 hours are:

$$z^*_{p_1} = 4.85 \text{ and } z^*_{p_2} = 6.47.$$

Neither of the two transportation modes to the affected area would be able to satisfy the target time requirement. Interestingly, the time deviation is higher on the path that contains the air route, which is due to the majority of the load being allocated to this mode.

The value of the projected demand at point R_1 was:

$$v^*_{R_1} = x^*_{p_1} + x^*_{p_2} = 8.54,$$

which is the amount that needs to be pre-positioned at the storage facility. The projected demand was closer to the upper bound of the uniform distribution range of the demand for the relief item at point R_1. If the organization seeks to reduce the consequences of the shortage of relief items in the affected region, the decision-maker should assign a higher unit shortage penalty so as to be able to better meet the uncertain demand within the given time limit. This obviously will result in a higher operational cost and, yet, in a lower social cost.

The optimal values of the Lagrange multipliers corresponding to the time goal constraints were:

$$\omega^*_{p_1} = 33.97 \text{ and } \omega^*_{p_2} = 103.55.$$

2.3.2 Illustrative Example – Variant I

We then considered the following variant of the above illustrative example. We assumed a worst case scenario in the form of unavailability of ground transport. Hence, link d would no longer be available in the supply chain (See Figure 3). The data were as above with the expressions (and constraints) associated with path p_1 and link d removed.

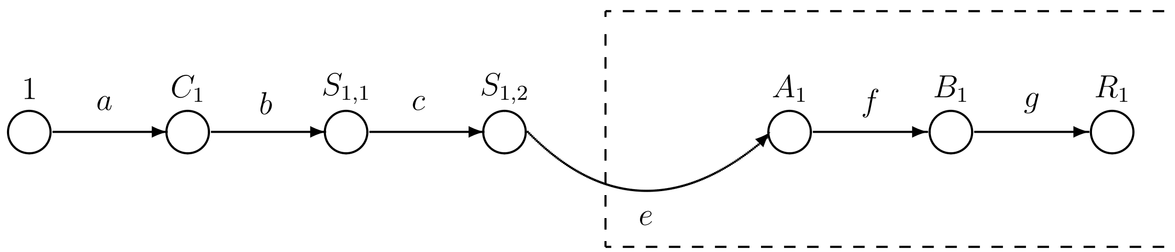

Figure 3: Supply Chain Network Topology for the Illustrative Numerical Example – Variant I

The new solution had to satisfy the following equations, under the assumption that $x^*_{p_2} > 0$, $z^*_{p_2} > 0$, and $\omega^*_{p_2} > 0$:

$$\frac{\partial \hat{C}_{p_2}(x^*)}{\partial x_{p_2}} + \lambda^+_{R_1} P_{R_1}(x^*_{p_2}) - \lambda^-_{R_1}(1 - P_{R_1}(x^*_{p_2})) + \omega^*_{p_2}(g_e + g_f + g_g) = 0,$$

$$\frac{\partial \gamma_{R_1}(z^*)}{\partial z_{p_2}} - \omega^*_{p_2} = 0, \text{ and}$$

$$T_{R_1 p_2} + z^*_{p_2} - (x^*_{p_2}(g_e + g_f + g_g)) = 0.$$

Similar to the original example, substitution of the partial derivatives and the numerical values into the above equations leads to the following system of equations:

$$\begin{cases} 1054x^*_{p_2} & & +8.5\omega^*_{p_2} & = 10083 \\ & 16z^*_{p_2} & -\omega^*_{p_2} & = 0 \\ -8.5x^*_{p_2} & +z^*_{p_2} & & = -64. \end{cases}$$

Solution of the above system of equations yields:

$$x^*_{p_2} = 8.50.$$

Hence,

$$f_a^* = f_b^* = f_c^* = f_e^* = f_f^* = f_g^* = 8.50.$$

Also, the optimal value of the projected demand at demand point R_1, $v_{R_1}^* = 8.50$. Therefore, removal of the ground transportation link slightly decreased the projected demand.

The new optimal time deviation on path p_2 was:

$$z_{p_2}^* = 8.26,$$

which is higher than the value in the preceding example. The value of the tardiness penalty function, $\gamma_{R_1}(z^*)$, has increased from 438.39 in the illustrative example to 579.61 in its variant. Also, removal of the ground transportation link results in a higher total operational cost in that the sole mode of transportation to the affected region would be the costlier air transport mode.

The optimal value of Lagrange multiplier on path p_2 was:

$$\omega_{p_2}^* = 132.12,$$

which shows a 28% increase.

2.3.3 Illustrative Example – Variant II

The second variant of the original example addresses the post-disaster procurement strategy – as opposed to pre-positioning of the supplies. We assumed that the organization did not store disaster items beforehand. In other words, the organization would procure relief items only once a disaster struck. Thus, the network did not include links a, b, and c – corresponding to pre-disaster procurement, shipment, and storage of relief goods – and instead incorporated link h representing post-disaster procurement, as depicted in Figure 4.

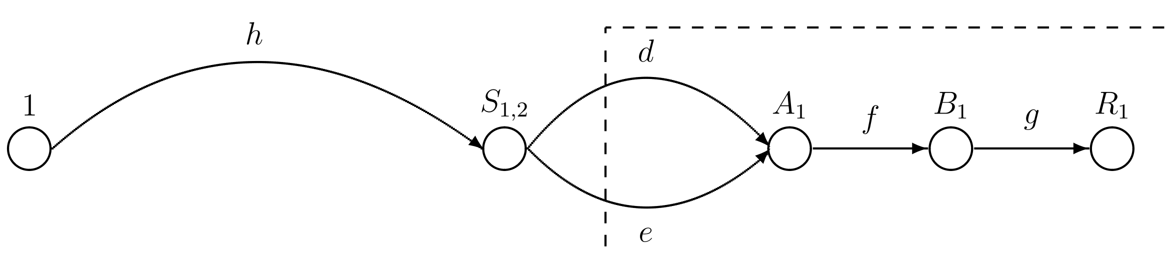

Figure 4: Supply Chain Network Topology for the Illustrative Numerical Example – Variant II

The total cost functions and the time completion functions on links d, e, f, and g remained the same as in the illustrative example. As for link h, we had:

$$\hat{c}_h(f_h) = 5f_h^2 + 3f_h \quad \text{and} \quad \tau_h(f_h) = 3f_h + 3.$$

In this example, the total operational cost on the post-disaster procurement link, h, was higher than that on the pre-disaster procurement link a in the example in Section 2.3.1. That is because the organization is assumed to be able to procure the relief items at a lower price under the pre-positioning strategy given the ample time that it has for preparedness. In contrast, procuring the relief goods after a disaster hits may leave the organization with fewer purchasing options since large quantities of goods have to be purchased/procured quickly (and, depending on the sources, there may even be competition for purchasing). In addition, in the network topology of this example, the post-disaster procurement activity may also include some transportation/shipment, further raising the total operational cost on link h as compared to that on link a in the previous examples.

The set of paths in this problem was $\mathcal{P} = \{p_3, p_4\}$ where $p_3 \equiv (h, d, f, g)$ and $p_4 \equiv (h, e, f, g)$. The demand distribution, the shortage and surplus penalties, as well as the target time were identical to the previous problems. The tardiness penalty function at the demand point was the same, except that now it was a function of the time deviations on new paths:

$$\gamma_{R_1}(z) = 3.5z_{p_3}^2 + 8z_{p_4}^2.$$

Under the assumption of positive optimal solutions, the following system of equations had to be satisfied:

$$\frac{\partial \hat{C}_{p_3}(x^*)}{\partial x_{p_3}} + \lambda_{R_1}^+ P_{R_1}(x_{p_3}^* + x_{p_4}^*) - \lambda_{R_1}^- (1 - P_{R_1}(x_{p_3}^* + x_{p_4}^*)) + \omega_{p_3}^*(g_h + g_d + g_f + g_g) + \omega_{p_4}^*(g_h + g_f + g_g) = 0,$$

$$\frac{\partial \hat{C}_{p_4}(x^*)}{\partial x_{p_4}} + \lambda_{R_1}^+ P_{R_1}(x_{p_3}^* + x_{p_4}^*) - \lambda_{R_1}^- (1 - P_{R_1}(x_{p_3}^* + x_{p_4}^*)) + \omega_{p_3}^*(g_h + g_f + g_g) + \omega_{p_4}^*(g_h + g_e + g_f + g_g) = 0,$$

$$\frac{\partial \gamma_{R_1}(z^*)}{\partial z_{p_3}} - \omega_{p_3}^* = 0,$$

$$\frac{\partial \gamma_{R_1}(z^*)}{\partial z_{p_4}} - \omega_{p_4}^* = 0,$$

$$T_{R_1 p_3} + z_{p_3}^* - (x_{p_3}^*(g_h + g_d + g_f + g_g) + x_{p_4}^*(g_h + g_f + g_g)) = 0, \text{ and}$$

$$T_{R_1 p_4} + z_{p_4}^* - (x_{p_3}^*(g_h + g_f + g_g) + x_{p_4}^*(g_h + g_e + g_f + g_g)) = 0,$$

where

$$T_{R_1 p_3} = T_{R_1} - h_{p_3} = T_{R_1} - (h_h + h_d + h_f + h_g) = 57, \text{ and}$$

$$T_{R_1p_4} = T_{R_1} - h_{p_4} = T_{R_1} - (h_h + h_e + h_f + h_g) = 61.$$

After substitution of the partial derivatives and the other numerical values into the above system, we have the following six equations:

$$\begin{cases} 1046x^*_{p_3} & +1038x^*_{p_4} & & & +18.5\omega^*_{p_3} & +9.5\omega^*_{p_4} & = 10088 \\ 1038x^*_{p_3} & +1052x^*_{p_4} & & & +9.5\omega^*_{p_3} & +11.5\omega^*_{p_4} & = 10086 \\ & & 7z^*_{p_3} & & -\omega^*_{p_3} & & = 0 \\ & & & 16z^*_{p_4} & & -\omega^*_{p_4} & = 0 \\ -18.5x^*_{p_3} & -9.5x^*_{p_4} & +z^*_{p_3} & & & & = -57 \\ -9.5x^*_{p_3} & -11.5x^*_{p_4} & & +z^*_{p_4} & & & = -61. \end{cases}$$

Solution of the above yields:

$$x^*_{p_3} = 0.33 \quad \text{and} \quad x^*_{p_4} = 6.26,$$

which yields the following optimal link flows:

$$f^*_h = f^*_f = f^*_g = x^*_{p_3} + x^*_{p_4} = 6.59, \;\; f^*_d = x^*_{p_3} = 0.33, \quad \text{and} \quad f^*_e = x^*_{p_4} = 6.26.$$

Note that, under a post-disaster procurement strategy, the optimal flow on the ground transportation link is significantly lower than that in the pre-disaster procurement strategy. This is due to the fact that the duration of the procurement activity is now being incorporated, and the organization wants to avoid excessive delays caused by assigning large quantities to the ground transportation path. In addition, the optimal flow on the air transportation link has experienced a 17% decrease which is a consequent of increased operational costs.

The value of the projected demand is:

$$v^*_{R_1} = x^*_{p_3} + x^*_{p_4} = 6.59,$$

which is 23% lower than that for the example in Section 2.3.1. Furthermore, the projected demand is now closer to the lower bound of the demand uniform distribution. Therefore, in the case that pre-positioning of supplies is not available, the decision-maker must be careful in assessing the shortage penalties. A sensitivity analysis on the shortage penalty value and the impact on the solution is presented in Section 2.3.4.

The optimal time deviations on paths p_3 and p_4 are:

$$z^*_{p_3} = 8.54 \quad \text{and} \quad z^*_{p_4} = 14.09,$$

which are higher than their respective values in the first example. The incurred tardiness penalty value, $\gamma_{R_1}(z^*)$, is equal to $1,844.16$, i.e., a 321% increase from that in the first example.

Finally, the optimal Lagrange multipliers on paths p_3 and p_4 are:

$$\omega^*_{p_3} = 59.77 \text{ and } \omega^*_{p_4} = 225.49.$$

2.3.4 Sensitivity Analysis for Variant II

In the illustrative example in Section 2.3.1 and its variants in Sections 2.3.2 and 2.3.3, we investigated the impact of the shortage penalty on the optimal solution.

Table 1 displays the optimal values of the path flows, $x^*_{p_3}$ and $x^*_{p_4}$, the path time deviations, $z^*_{p_3}$ and $z^*_{p_4}$, and the Lagrange multipliers, $\omega^*_{p_3}$ and $\omega^*_{p_4}$, as the unit shortage penalty, $\lambda^-_{R_1}$, is increased from $2,500$ to $12,500$ in Variant II.

Table 1: Sensitivity Analysis of the Optimal Solution to the Unit Shortage Penalty at the Demand Point for Variant II

$\lambda^-_{R_1}$	$x^*_{p_3}$	$x^*_{p_4}$	$z^*_{p_3}$	$z^*_{p_4}$	$\omega^*_{p_3}$	$\omega^*_{p_4}$	Value of Objective Function (21)
$2,500$	0.50	5.56	5.09	7.66	35.66	122.58	$5,081.96$
$5,000$	0.33	6.26	8.54	14.09	59.77	225.49	$8,440.02$
$7,500$	0.20	6.79	11.18	19.02	78.25	304.39	$11,021.81$
$10,000$	0.09	7.22	13.26	22.91	92.80	366.49	$13,035.31$
$12,500$	0.01	7.56	14.94	26.05	104.57	416.72	$14,655.25$

As seen in Table 1, as the shortage penalty increases, the organization will be fulfilling a higher projected demand, $x^*_{p_3} + x^*_{p_4}$, by assigning higher quantities to the path that uses air transportation to the affected region. At $\lambda^-_{R_1} = 12,500$, the optimal path flow on the ground transportation path is almost zero, which means that the organization relies on the air transport mode. Handling larger volumes of goods increases the congestion on paths which, in turn, worsens the lateness of deliveries to the region.

3. The Algorithm

In this Section, we recall the Euler method, which is induced by the general iterative scheme of Dupuis and Nagurney (1993). Its realization for the solution of disaster relief supply chain network problems governed by variational inequality (24) (and (27)) yields subproblems that can be solved explicitly and in closed form. Specifically, recall that at an iteration τ of the Euler method (see also Nagurney and Zhang (1996)) one computes:

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - a_{\tau}F(X^{\tau})), \tag{29}$$

where $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$ and F is the function that enters the variational inequality problem: determine $X^* \in \mathcal{K}$ such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{30}$$

where $\langle \cdot, \cdot \rangle$ is the inner product in n-dimensional Euclidean space, $X \in R^n$, and $F(X)$ is an n-dimensional function from $\mathcal{K}$ to R^n, with $F(X)$ being continuous (see also (28)).

As shown in Dupuis and Nagurney (1993); see also Nagurney and Zhang (1996), for convergence of the general iterative scheme, which induces the Euler method, among other methods, the sequence $\{a_{\tau}\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_{\tau} = \infty$, $a_{\tau} > 0$, $a_{\tau} \to 0$, as $\tau \to \infty$. Specific conditions for convergence of this scheme can be found for a variety of network-based problems, similar to those constructed here, in Nagurney and Zhang (1996) and the references therein.

Explicit Formulae for the Euler Method Applied to the Disaster Relief Supply Chain Network Variational Inequality (24)

The elegance of this procedure for the computation of solutions to the disaster relief supply chain network problem modeled in Section 2 can be seen in the following explicit formulae. Indeed, (29) for the supply chain network problem governed by variational inequality problem (24) yields the following closed form expressions for the product path flows, the time deviations, and the Lagrange multipliers, respectively:

$$x_p^{\tau+1} = \max\{0, x_p^{\tau} + a_{\tau}(\lambda_k^-(1 - P_k(\sum_{q \in \mathcal{P}_k} x_q^{\tau})) - \lambda_k^+ P_k(\sum_{q \in \mathcal{P}_k} x_q^{\tau}) - \frac{\partial \hat{C}_p(x^{\tau})}{\partial x_p} - \sum_{q \in \mathcal{P}} \sum_{a \in L} \omega_q^{\tau} g_a \delta_{aq} \delta_{ap})\},$$

$$\forall p \in \mathcal{P}_k; \quad k = 1, \ldots, n_R, \tag{31}$$

$$z_p^{\tau+1} = \max\{0, z_p^{\tau} + a_{\tau}(\omega_p^{\tau} - \frac{\partial \gamma_k(z^{\tau})}{\partial z_p})\}, \qquad \forall p \in \mathcal{P}_k; \quad k = 1, \ldots, n_R, \text{ and} \tag{32}$$

$$\omega_p^{\tau+1} = \max\{0, \omega_p^{\tau} + a_{\tau}(\sum_{q \in \mathcal{P}} \sum_{a \in L} g_a x_q^{\tau} \delta_{aq} \delta_{ap} - T_{kp} - z_p^{\tau}\}, \qquad \forall p \in \mathcal{P}_k; \qquad k = 1, \ldots, n_R. \tag{33}$$

In the next Section, we solve additional disaster relief supply chain network problems using the above algorithmic scheme.

4. A Larger Numerical Example and Variant

The scenario for the first numerical example is built on the possibility of another earthquake striking Haiti. We then construct a variant.

Figure 5 displays the disaster relief supply chain network topology corresponding to the case of a Haiti earthquake. Node 1 is assumed to represent the American Red Cross. We assumed that the Red Cross could utilize two of its disaster aid zones in the US, one in Maryland – representing the Northeast and the East – and the other one in Florida – the closest state to the Caribbean. Each of the two zones is assumed to possess a single procurement facility, a single storage facility, and a single departure portal.

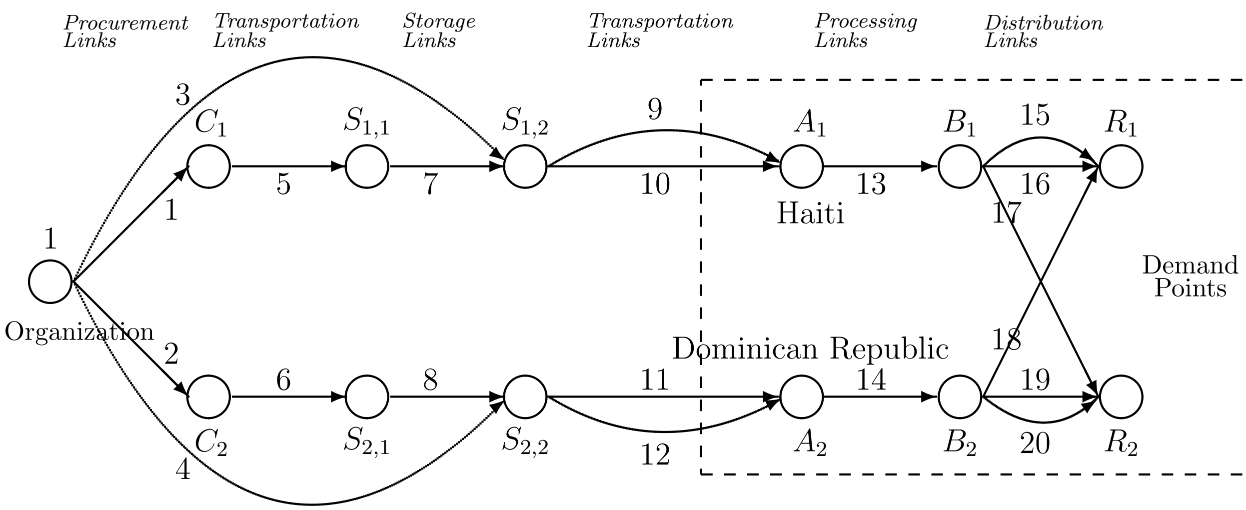

Figure 5: Network Topology of the Larger Disaster Relief Supply Chain Numerical Example

Both locations have the ability to start procuring the relief goods after an earthquake strikes when and if the need arises (links 3 and 4). These procured/purchased goods at the zones are directly sent to their respective departure portals bypassing the storage phase. Next, from these departure facilities, the collected items – whether pre-positioned beforehand or just procured – are sent via air or sea to the affected region. We assumed that the facility $S_{1,2}$ – hypothetically, Maryland – would cover the arrival facility A_1 in Haiti, and the facility $S_{2,2}$ would serve the arrival port A_2 in the Dominican Republic. Links 9 and 12 represent air transportation whereas links 10 and 11 correspond to marine transportation.

After the arrived cargo is sorted and processed, relief items are distributed to the points of demand, R_1 and R_2, both located in Haiti. Links 15 and 20 correspond to the distribution of goods by helicopter, whereas links 16 through 19 represent ground distribution. Note that we allow for each of the two processing facilities located in Haiti and the Dominican Republic, i.e., B_1 and B_2, to ship to both of the two demand points in Haiti.

Now we present the available information as well as the solution to the problem. For purposes of transparency and reproducibility, we provide both the input and the output data.

Table 2 presents the total operational cost functions and completion time functions corresponding to the 20 links of the network. Similar to the illustrative example and its variants, the completion time functions on links $1, 2, 5, 6, 7, 8$ – pre-disaster procurement, transportation and storage – are set equal to zero since these activities are assumed to have been completed before the occurrence of the disaster.

Table 2: Total Operational Costs, Completion Time Functions, and the Optimal Flows on Links in the Numerical Example

Link	$\hat{c}_a(f_a)$	$\tau_a(f_a)$	f_a^*
1	$3f_1^2 + 2f_1$	0	19.22
2	$2f_2^2 + 2.5f_2$	0	20.02
3	$5f_3^2 + 4f_3$	$3f_3 + 3$	0.00
4	$4.5f_4^2 + 3f_4$	$4f_4 + 2$	0.00
5	$f_5^2 + 2f_5$	0	19.22
6	$f_6^2 + .5f_6$	0	20.02
7	$2.5f_7^2 + 3f_7$	0	19.22
8	$3.5f_8^2 + 2f_8$	0	20.02
9	$7f_9^2 + 5f_9$	$2f_9 + 2$	19.22
10	$4f_{10}^2 + 6f_{10}$	$10f_{10} + 6$	0.00
11	$2.5f_{11}^2 + 4f_{11}$	$7.5f_{11} + 5$	0.23
12	$4.5f_{12}^2 + 5f_{12}$	$1.5f_{12} + 1.5$	19.79
13	$2f_{13}^2 + 4f_{13}$	$2f_{13} + 2$	19.22
14	$f_{14}^2 + 3f_{14}$	$1.5f_{14} + 1$	20.02
15	$4f_{15}^2 + 5f_{15}$	$3f_{15} + 3$	13.95
16	$2.5f_{16}^2 + 2f_{16}$	$5f_{16} + 4$	5.28
17	$3f_{17}^2 + 4f_{17}$	$6.5f_{17} + 3$	0.00
18	$4f_{18}^2 + 4f_{18}$	$7f_{18} + 5$	6.85
19	$3f_{19}^2 + 3f_{19}$	$4f_{19} + 5$	5.68
20	$3.5f_{20}^2 + 5f_{20}$	$3.5f_{20} + 4$	7.49

There are 24 paths in the supply chain network connecting node 1 to the two demand points. The set of paths connecting the origin to demand points R_1 and R_2, respectively, are denoted by $\mathcal{P}_{R_1}$ and $\mathcal{P}_{R_2}$, and each contains 12 paths as shown in Table 3.

R_1 is assumed to have a higher demand for relief goods due to a larger population and its potential higher vulnerability to the disasters as compared to R_2. The demand for the relief item at R_1 and at R_2 is assumed to follow a uniform distribution on the intervals [25,45] and

[10,20], respectively.

The unit shortage and surplus penalties at the demand points are:

$$\lambda^-_{R_1} = 10,000, \ \lambda^+_{R_1} = 100,$$

$$\lambda^-_{R_2} = 7,500, \ \lambda^+_{R_2} = 150.$$

The target times of delivery at demand points R_1 and R_2 are:

$$T_{R_1} = 72, \ T_{R_2} = 70.$$

Using (15), the T_{kp}'s are also shown in Table 3.

Recall that in selecting the penalties and the target times, the decision-maker takes into account such data as the population, accessibility, strategic location, etc. In addition, the decision-maker can prioritize certain paths by assigning higher coefficients in the tardiness penalty function. In this example – unlike the illustrative example – all paths are assumed to have equal tardiness penalty weights:

$$\gamma_{R_1}(z) = 3(\sum_{p \in \mathcal{P}_{R_1}} z_p^2), \quad \gamma_{R_2}(z) = 3(\sum_{p \in \mathcal{P}_{R_2}} z_p^2).$$

The Euler method (cf.(31)–(33)) for the solution of variational inequality (24) was implemented in FORTRAN on a PC at the University of Massachusetts Amherst. We set the sequence as $\{a^\tau\} = .1(1, \frac{1}{2}, \frac{1}{2}, \ldots)$, and the convergence tolerance was 10^{-6}. In other words, the absolute values of the differences between each pair of path flows, the path time deviations, and the Lagrange multipliers in two consecutive iterations were less than or equal to this tolerance. We initialized the algorithm by setting all variables equal to zero.

The computed optimal path flows, time deviations, and Lagrange multipliers are reported in Table 3 and the optimal link flows in Table 2.

As seen in Table 2, the optimal flows on links 3 and 4, i.e., the post-disaster procurement links, are zero. Hence, given the demand information and the cost and time functions on the supply chain network links, the organization would be better off by adopting the pre-positioning strategy. In addition, links 10 and 11, corresponding to marine transportation of goods from the US to the affected region have zero or very small flows. Such an outcome is due to the importance of timely deliveries, and, thus, the organization needs to ship via air to minimize the lateness on the demand end. Similarly, among the distribution links, the ones representing shipments by helicopter (links 15 and 20) are assigned relatively higher loads

Table 3: Path Definitions, Target Times, Optimal Path Flows, Optimal Path Time Deviations, and Optimal Lagrange Multipliers for the Numerical Example

	Path Definition	T_{kp}	x_p^*	z_p^*	ω_p^*
$\mathcal{P}_{R_1}$: Set of Paths Corresponding to Demand Point R_1	$p_1 = (1, 5, 7, 9, 13, 15)$	65	13.95	53.66	321.99
	$p_2 = (1, 5, 7, 9, 13, 16)$	64	5.28	39.23	235.39
	$p_3 = (1, 5, 7, 10, 13, 15)$	61	0.00	19.32	115.90
	$p_4 = (1, 5, 7, 10, 13, 16)$	60	0.00	4.83	28.99
	$p_5 = (2, 6, 8, 11, 14, 18)$	61	0.06	18.67	112.03
	$p_6 = (2, 6, 8, 12, 14, 18)$	64.5	6.79	43.12	258.75
	$p_7 = (3, 9, 13, 15)$	62	0.00	56.66	339.99
	$p_8 = (3, 9, 13, 16)$	61	0.00	42.23	253.39
	$p_9 = (3, 10, 13, 15)$	58	0.00	22.34	134.05
	$p_{10} = (3, 10, 13, 16)$	57	0.00	7.84	47.03
	$p_{11} = (4, 11, 14, 18)$	59	0.00	20.71	124.24
	$p_{12} = (4, 12, 14, 18)$	62.5	0.00	45.24	271.46
$\mathcal{P}_{R_2}$: Set of Paths Corresponding to Demand Point R_2	$p_{13} = (1, 5, 7, 9, 13, 17)$	63	0.00	13.87	83.25
	$p_{14} = (1, 5, 7, 10, 13, 17)$	59	0.00	0.00	0.00
	$p_{15} = (2, 6, 8, 11, 14, 19)$	59	0.13	0.00	0.00
	$p_{16} = (2, 6, 8, 11, 14, 20)$	60	0.04	0.00	0.00
	$p_{17} = (2, 6, 8, 12, 14, 19)$	62.5	5.55	19.91	119.44
	$p_{18} = (2, 6, 8, 12, 14, 20)$	63.5	7.45	22.40	134.43
	$p_{19} = (3, 9, 13, 17)$	60	0.00	16.90	101.41
	$p_{20} = (3, 10, 13, 17)$	56	0.00	0.00	0.00
	$p_{21} = (4, 11, 14, 19)$	57	0.00	0.00	0.00
	$p_{22} = (4, 11, 14, 20)$	58	0.00	0.00	0.00
	$p_{23} = (4, 12, 14, 19)$	60.5	0.00	21.96	131.77
	$p_{24} = (4, 12, 14, 20)$	61.5	0.00	24.48	146.85

whereas link 17 corresponding to one of the ground distribution links will not be utilized. Also, note that the optimal flows being almost equal on links 1 and 2 suggests an even split of pre-positioning of the load between the two US aid regions.

Table 3 points out that among the 24 paths in the supply chain network, fewer than one-third have considerable positive flows since the others involve links that are either costlier or more time-consuming. From the optimal values of time deviations on paths, one can observe that significant deviations from the target times have occurred on several paths in the network. This seems to be more of an issue in the paths connecting the origin to demand point R_1, i.e., the hypothetically more vulnerable location. Such an outcome may mandate additional investments on critical transportation/distribution channels to R_1 which can be done in accordance with the optimal values of respective Lagrange multipliers. The higher the value of the Lagrange multiplier on a path, the more improvement in time can be attained by enhancing that path which, in turn, leads to a more efficient disaster response system.

A Variant

We then considered the following variant of the previous example. We assumed that the organization will now procure the items locally and, hence, the time functions associated with the direct procurement links 3 and 4 are now greatly reduced and are given in Table 4 – the remainder of the input data remains as in the previous example. The computed optimal link flow pattern for this variant is also reported in Table 4.

As can be seen from Table 4, now both the storage links for pre-positioning (links 7 and 8) and for post-disaster procurement (links 3 and 4) have positive flows. Hence, with the new data (and decision to procure locally) the organization should engage in both strategies. The optimal solution suggests to the organization how much of the relief good should be stored and in which location and how much should also be procured (and from where) once the disaster strikes.

Table 4: Total Operational Costs, Completion Time Functions, and the Optimal Flows on Links in the Variant

Link	$\hat{c}_a(f_a)$	$\tau_a(f_a)$	f_a^*
1	$3f_1^2 + 2f_1$	0	12.02
2	$2f_2^2 + 2.5f_2$	0	11.21
3	$5f_3^2 + 4f_3$	$.1f_3 + 1$	7.35
4	$4.5f_4^2 + 3f_4$	$.1f_4 + 1$	8.88
5	$f_5^2 + 2f_5$	0	12.02
6	$f_6^2 + .5f_6$	0	11.21
7	$2.5f_7^2 + 3f_7$	0	12.02
8	$3.5f_8^2 + 2f_8$	0	11.21
9	$7f_9^2 + 5f_9$	$2f_9 + 2$	19.37
10	$4f_{10}^2 + 6f_{10}$	$10f_{10} + 6$	0.00
11	$2.5f_{11}^2 + 4f_{11}$	$7.5f_{11} + 5$	0.24
12	$4.5f_{12}^2 + 5f_{12}$	$1.5f_{12} + 1.5$	19.86
13	$2f_{13}^2 + 4f_{13}$	$2f_{13} + 2$	19.37
14	$f_{14}^2 + 3f_{14}$	$1.5f_{14} + 1$	20.10
15	$4f_{15}^2 + 5f_{15}$	$3f_{15} + 3$	14.04
16	$2.5f_{16}^2 + 2f_{16}$	$5f_{16} + 4$	5.33
17	$3f_{17}^2 + 4f_{17}$	$6.5f_{17} + 3$	0.00
18	$4f_{18}^2 + 4f_{18}$	$7f_{18} + 5$	6.84
19	$3f_{19}^2 + 3f_{19}$	$4f_{19} + 5$	5.72
20	$3.5f_{20}^2 + 5f_{20}$	$3.5f_{20} + 4$	7.53

5. Summary and Conclusions

In this paper, we developed a network optimization model for the supply chain management of a disaster relief (humanitarian) organization in charge of procurement and distribution of relief items to a geographic region prone to natural disasters such as earthquakes or hurricanes.

Our proposed framework spans such activities associated with disaster relief supply chains as the pre-disaster procurement, shipment, storage, post-disaster procurement, transportation, and processing, and the ultimate distribution of aid items to the people in need. Our system-optimization approach minimizes the total operational costs of the aforementioned activities subject to the uncertain demand being satisfied as closely as possible. We also utilized goal programming to minimize the time deviations on paths with respect to the pre-determined time targets for deliveries.

The disaster relief supply chain network model has the following features:

- it allows for the integration of two distinct policies by disaster relief organizations, one being pre-positioning the supplies beforehand, as well as the procurement of necessary items once the disaster has occurred;
- it includes penalties associated with shortages/surpluses at the demand points with respect to the uncertain demand, and
- it enables the decision-maker to prioritize the demand points based on the population, geographic location, etc., by assigning different time targets.

We used a variational inequality approach for the model formulation. The developed model was then verified through illustrative numerical examples followed by the sensitivity analysis of the optimal solution to the model parameters. We also used variational inequality theory to present a solution algorithm with nice computational features and solved two larger examples.

Our constructed framework can assist disaster relief organizations to plan their procurement and distribution strategies before the occurrence of unexpected natural catastrophes. The organization's management can also evaluate the timeliness of its service to the survivors of disasters under various scenarios of demand, time targets, and costs. Possible extensions of our framework may include the incorporation of uncertainty associated with link time functions, as well as additional empirical work, which might include the construction of alternative penalty functions and time completion functions. The modeling of cooperation as well as competition among disaster relief agencies could also be addressed. Our variational inequality approach in this paper provides the foundation for such extensions. Indeed, we recognize that having multiple disaster relief agencies – rather than a single organization – may be quite beneficial, provided that the collaboration among the agencies is rigorously planned well ahead of time. Moreover, here the focus was on planning for a single (the next) disaster – it would also be interesting to consider the possibility of multiple disasters in neighboring regions.

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