

**Criminal Networks and the Authorities:
An Integrated Equilibrium Framework for Disrupting Illicit Product Trade**

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Abstract: We develop an integrated equilibrium framework for the disruption of illicit product trade. A criminal network and the authorities compete in a noncooperative game that leads to a Nash Equilibrium, rather than in the more common Stackelberg structure in which one player leads and the other follows. The criminal network selects illicit product shipments over multiple supply markets, multiple demand markets, and multiple transportation routes that join them. What emerges is a spatial price equilibrium in which the supply and demand prices are determined within the model and clear the markets. The authorities allocate a finite budget across the routes in order to disrupt the trade. They evaluate that allocation through a multicriteria objective, one that weighs the disruption achieved against the monetized societal benefit of enforcement, with a weight that governs the balance between the two. We formulate the equilibrium conditions as a variational inequality problem and then derive alternative formulations of it. We establish monotonicity, along with conditions under which a modified projection method converges, and that method yields closed form expressions for the product shipments, the budget allocations, and the Lagrange multiplier at each iteration. The framework applies to illicit trade in drugs, minerals, counterfeit personal protective equipment, cybercrime in financial services, and stolen grain. We then present a realistic case study on stolen grain from the occupied territories of Ukraine, calibrated entirely from open sources. In the computed equilibria, enforcement erodes criminal revenue and raises the prices that consumers pay, the societal benefit weight shapes the budget allocation, and disrupting transportation corridors suppresses the trade while pressure on a single destination market only displaces it.

Key words: criminal networks, illicit supply networks, Nash Equilibrium, spatial price equilibrium, variational inequalities, multicriteria optimization

Practitioner Summary: Illicit product trade is a global problem that costs the world economy trillions of dollars each year. It spans illegal drugs, smuggled gold and other minerals, counterfeit personal protective equipment, cybercrime in financial services, and grain stolen from the occupied territories of Ukraine. The criminal networks behind this trade are highly adaptive. When the authorities put pressure on one transportation route or market, the networks reroute their shipments to others. In this paper, we develop an integrated equilibrium framework in which a criminal network and the authorities compete in a noncooperative game.

The criminal network in our framework selects shipments of an illicit product over multiple

supply markets, multiple demand markets, and multiple transportation routes, with the supply and demand prices determined within the model. The authorities, in turn, allocate a limited budget across the routes in order to disrupt the trade. Their objective weighs the disruption achieved against the monetized societal benefit of enforcement. We also provide an algorithm with computationally convenient features, which yields closed form iterative updates for all the variables.

We illustrate the framework with a case study on stolen grain from the occupied territories of Ukraine, calibrated entirely from open sources. The results offer important policy and managerial insights. Enforcement reduces the revenue of the criminal network and raises the prices that consumers of the illicit product pay. The authorities place more of their budget on the busier and cheaper transportation corridors.

For the authorities, the results highlight that pressure on a single destination market mainly displaces the trade to other markets. A budget spread across the transportation corridors, in contrast, suppresses the trade. The framework can support enforcement agencies and policymakers in the design and evaluation of interdiction strategies in many illicit trade sectors.

1. Introduction

Trade of illicit products is an immense global issue with major threats to economies, the security of regions and nations, public health, and even the environment (cf. Transnational Alliance to Combat Illicit Trade (2023)). Sectors that have been especially amenable to illicit trade of tangible products include, among others: agrifood, pharmaceuticals, electronics, precious metals, including gold, and energy such as petroleum, firearms, tobacco, as well as wildlife. According to Center Forward (2025), illicit trade is the illegal production, transport, and sale of goods and services, with operations frequently run by criminal networks. Center Forward (2025) estimates that the global economic impact of illicit trade amounts to \$2.2 trillion a year – a sum greater than the economies of such countries as: Brazil, Italy, and Canada, and of the size of Mexico and Indonesia combined. Furthermore, and as noted by Shelley (2018), illicit trade also includes intangible products that are bought and sold in cyberspace, costing consumers and organizations billions globally in terms of compromised identities, financial funds stolen from bank accounts, etc.

Given the scope of illicit product trade and the associated damages, international organizations are increasingly partnering to disrupt such trade. For example, the World Customs Organization (2023) reported that in a major operation, organized by it, the UN Office on Drugs and Crime (UNODC), and the Australian Border Force, along with members of the World Shipping Council, in late Fall 2022, 58 countries worked together to carry out dozens of arrests and drug seizures, resulting in the seizure of 98,734 kilograms of cocaine and 314 kilograms of cannabis herbals. This operation vividly illustrated how authorities can disrupt criminal networks that are taking advantage of different transportation routes for the movement of illicit products. According to Interpol (2019), Operation Pangea, which it coordinates, is an ongoing international effort to disrupt the e-commerce of counterfeit and illicit health products. Operation Pangea, since its founding in 2008, until 2019, resulted in the removal of more than 105 million units (pills, ampoules, sachets, bottles and so on) from circulation and made more than 3,000 arrests. And, in its most recent rendition, in mid-March 2026 (see Interpol (2026)) Interpol coordinated the operation spanning 90 countries and territories, resulting in the seizure of 6.42 million doses of unapproved and counterfeit pharmaceuticals valued at \$15.5 million. In terms of illicit intangible products, Interpol, very recently (see SC Media (2026)), orchestrated a major investigation, Operation Ramz, involving 13 countries in the Middle East and North Africa to break apart cybercrime operations, identifying both suspects and victims.

As Stavridis (2013) emphasizes, criminal networks engaged in illicit product trade affect everyone in our modern, globalized world. Although trade in illegal narcotics has received the most attention, awareness is rapidly growing in terms of the illicit trafficking of counterfeit items, weapons, natural resources, and even of humans, as well as of money, by well-resourced, villainous adversaries. The Council on Foreign Relations (2026), in their report, detail the various drug cartels in Mexico that are major suppliers of illicit narcotics (cocaine, heroin, methamphetamine, etc.) to the United States. Felbab-Brown (2025), in testimony before the U.S. Senate Caucus on International Narcotics Control, describes the role of Chinese networks not only in drug trafficking in Latin America,

but also in illegal fishing, wildlife trafficking, illegal logging and mining, plus human smuggling. A Europol (2024) report notes that European Union (EU) Member States and third countries identified 821 of the most threatening criminal networks, with over 25,000 individuals involved, active in the European Union, and impacting the region’s internal security. For the first time, as noted in Europol (2024), an ‘ABCD’ classification was introduced for what were deemed to be the most threatening criminal networks – with A, B, C, and D corresponding, respectively, to: Agile, Borderless, Controlling, and Destructive. For example, the ‘A’ for Agile in the Europol (2024) classification points to a feature of contemporary criminal networks that has become increasingly apparent: their use of multiple, often substitutable transportation routes for moving illicit products. As enforcement pressure tightens on one corridor, criminal networks redirect shipments onto alternative ones, sustaining illicit flows in the face of interdiction. The European Union Drugs Agency and the World Customs Organization (2025) document this dynamic in the maritime cocaine trade, where intensified inspections at Antwerp and Rotterdam have pushed shipments toward smaller secondary ports. Europol (2026) reports a parallel diversification of trafficking *modi operandi*, and SwissAid (2025) describes analogous routing flexibility in the illicit gold trade through transit countries such as Uganda, Rwanda, Chad, and Libya. This multi-route adaptability is central to the resilience of criminal networks, and motivates the explicit modeling of multiple transportation routes between each pair of supply and demand markets in our framework.

Clearly, authorities recognize that criminal networks can be transnational organizations engaged in illicit trade where there is demand and profit to be made.

Given the importance of understanding and disrupting criminal networks engaged in illicit product trade, researchers have been engaged in the modeling and analysis of illicit supply networks and their interdiction using various methodologies. Game theory, in particular, has emerged as a relevant, powerful methodological tool for the development of mathematical models for the interaction between criminal stakeholders and authorities. Much of the related literature has assumed a Stackelberg perspective. In this paper, in contrast, we introduce an integrated equilibrium framework in which the criminal network competes directly with the authorities at the same level in a noncooperative game leading to a Nash (1950, 1951) Equilibrium. The formalism that we utilize for the formulation, analysis, and algorithmic solution of our game theory model is the theory of variational inequalities (cf. Dafermos (1980), Gabay and Moulin (1980), Nagurney (1999, 2006)). The objective function faced by the criminal network is that of maximizing the net payoff with supply price functions associated with different production locations included as well as the demand price functions associated with demand markets for the illicit product. The pairs of supply and demand markets are joined by multiple transportation routes, with associated costs, which are flow-dependent, and with each route having the option of including different modes of transportation. The net payoff of the criminal network also includes the budget allocation on each route selected by the authorities for disruption, which becomes an added total cost with the trade flow on the route included. The authorities, in turn, face a budget constraint, and they judge their budget allocation by two things. One is the disruption it imposes on the criminal network, given the illicit product shipments between the supply and demand markets on the routes. The other

is the monetized societal benefit that enforcement generates, weighted by the importance that the authorities place on it. The structure of the criminal network is, therefore, that of a network with the solution of the game theory problem yielding a spatial price equilibrium (cf. Samuelson (1952), Takayama and Judge (1964, 1971), Florian and Los (1982), Dafermos and Nagurney (1984)) in illicit product flows as well as the optimal budget allocations.

The central contribution of this paper is the integration of a spatial price equilibrium model of a criminal network with a strategic, budget constrained enforcement authority in a noncooperative game leading to a Nash Equilibrium. To our knowledge, this is the first framework for illicit trade in which endogenous market-clearing supply and demand prices, multiple substitutable transportation routes, and a strategic, multicriteria enforcement authority interacting with the criminal network on the same decision-making level are captured simultaneously. In support of this central contribution, the paper also provides:

1. A spatial price equilibrium model for illicit product trade of relevance to multiple sectors, including: agrifood trade (with stolen grain from Ukraine), illegal drug trade, mineral trade, cybercrime in financial services, and counterfeit Personal Protective Equipment (PPEs).
2. A multicriteria optimization problem for the authorities that weighs the disruption achieved against the monetized societal benefit of enforcement.
3. Alternative variational inequality formulations for the analysis of the equilibrium state, with one especially amenable to algorithmic solution.
4. An algorithm with explicit closed form expressions at each iteration for all the variables, accompanied by conditions for convergence that are reasonable for the underlying economic functions.
5. A case study, with full input and output data, on the disruption of the trade of stolen grain from occupied Ukrainian territories.

2. Related Literature Review and Organization of the Paper

The modeling and disruption of illicit supply networks has been studied through several distinct methodological traditions, drawing principally on optimization and network interdiction, on game-theoretic analytical models, on the economics of illicit markets and prices, and on spatial economic equilibrium theory. We organize this review by tradition. We begin with optimization and interdiction, both in their general form and as applied specifically to illicit trade. We then turn to the analytical game-theoretic literature on illicit supply networks, identify what distinguishes the present paper from that body of work, and then locate our contribution within the economics of prices in illicit markets and the spatial price equilibrium tradition that anchors our modeling framework.

2.1 Optimization and Network Interdiction

A substantial body of work approaches the problem through optimization, in particular through network interdiction, in which an interdictor seeks to maximize disruption on a flow network subject

to a resource constraint. The recent survey by Smith and Song (2020) covers the network interdiction literature in operations research, with foundational contributions including those of Wood (1993) on deterministic max-flow interdiction and of Israeli and Wood (2002) on shortest-path interdiction. Most of this work is formulated as Stackelberg leader-follower problems or as bilevel optimization problems; Colson, Marcotte, and Savard (2007) provide the standard survey of bilevel optimization, with Bracken and McGill (1973) being the originating reference. Adjacent traditions of defender-attacker-defender modeling (Brown et al. (2006)) and facility interdiction with fortification (Scaparra and Church (2008)) have also been used to analyze the protection and disruption of critical infrastructure networks. Across this body of work, the interdictor is typically modeled as a single strategic actor with a fixed budget, the adversary is modeled as a simple flow-maximizer or shortest-path follower, and the criminal economic environment in which the flow takes place is exogenous to the model.

A smaller but rapidly growing literature applies these methodological tools specifically to the modeling and disruption of illicit supply networks. Anzoom, Nagi, and Vogiatzis (2022) provide a comprehensive review of research on illicit supply chain networks and the analytical, decision-support, and network-science approaches that have been brought to bear on them. Jabarzare, Zolfagharinia, and Najafi (2020) develop a dynamic interdiction model with applications to illicit supply chains, formulating the problem as a sequential game over time. Kosmas et al. (2023) develop a max-flow interdiction model in which the criminal network can restructure by adding arcs after the interdictor acts, and apply it to city-scale drug trafficking; their finding that conventional interdiction can simply displace activity to other arcs resonates with the route-substitution behavior our equilibrium framework produces endogenously. Magliocca et al. (2019) model cocaine trafficking and counterdrug interdiction as a complex adaptive system, combining agent-based simulation with spatial optimization. Anzoom, Nagi, and Vogiatzis (2024) take a graph-theoretic approach to uncovering illicit supply networks and their interfaces to licit counterparts under limited information, as part of a special issue of *IISE Transactions* dedicated to analytical and engineering approaches for detecting, disrupting, and dismantling illicit trafficking operations and markets. Wildlife trafficking has also attracted dedicated treatment, including, for example, the review by Keskin et al. (2023) of quantitative investigations of wildlife trafficking supply chains.

2.2 Game-Theoretic Analytical Models of Illicit Supply Networks

A parallel body of work applies game-theoretic analytical models to the strategic interaction between criminal actors and either authorities or licit firms. The counterfeit-products literature is particularly developed. Cho, Fang, and Tayur (2015) study combating strategic counterfeiters in licit and illicit supply chains, providing a foundational model of the trade-offs facing a brand-name firm that must decide between pursuing legal action, lowering its own price, or improving its quality in response to entry by counterfeit competitors. Yi, Yu, and Cheung (2022) extend this analysis to the global supply chain context, examining how counterfeits influence supply chain design choices and what anti-counterfeit actions are warranted. Ding, Guan, and Ke (2024) consider counterfeit competition under strategic consumers in a two-period durable-good setting, while

Bian et al. (2025) examine the strategic interaction between an authentic brand firm and a deceptive counterfeiter, showing that the authentic firm can paradoxically be worse off under heavier counterfeit penalties. On the smuggling side, Bakker et al. (2020) develop a multi-game model of counter-smuggling that captures the impacts of changes in port screening policies, detection-device capabilities, shipping subsidies, and worldwide corruption. In the conflict minerals setting, Zhang, Aydin, and Heese (2023) study equilibrium sourcing decisions in a supply network of manufacturers and smelters, with each manufacturer deciding whether to commit to compliance with certification requirements. A related Stackelberg security games tradition, surveyed for the wildlife protection and illegal fishing settings in Fang, Stone, and Tambe (2015), has produced deployed systems for adversarial patrol planning under limited resources. An earlier and frequently-cited foundation for equilibrium models of counterfeit trade is Grossman and Shapiro (1988), who develop an equilibrium model of counterfeit-product trade and provide a welfare analysis of border inspection.

Several features distinguish this analytical game-theoretic work from the framework developed in the present paper. First, the existing models are typically stylized in the number of strategic actors, often involving a single brand firm against a single counterfeiter, a customs authority against a single smuggling antagonist, or a patrol planner against a generic adversary. Second, the criminal network is rarely modeled as embedded in a spatial transportation network with multiple routes and substitution between them; the question of which corridor a flow takes, and how authority intervention reallocates flows across corridors, is largely absent. Third, the economic behavior of the criminal network in the form of endogenous market-clearing supply and demand prices is rarely captured; prices, when modeled, are typically exogenous or determined by simple penalty and detection probabilities. Fourth, mitigation strategies on the enforcement side are commonly evaluated by simulation or by a try-and-evaluate approach over a small set of candidate policies, rather than derived from a strategic optimization with an explicit budget constraint and an endogenous equilibrium response from the criminal network. The present paper is motivated by the gap that these features collectively define.

2.3 Economics of Prices in Illicit Markets

A separate empirical and theoretical literature, located primarily in economics, takes prices in illicit markets seriously. The economic theory of crime, beginning with Becker (1968), treats criminal participation as a rational economic response to expected returns and penalties, providing the conceptual basis for treating the criminal network as an optimizing actor responsive to costs imposed by enforcement. The drug-economics tradition, beginning with Reuter and Kleiman (1986) on the role of enforcement risk in determining drug prices and reviewed comprehensively in Caulkins and Reuter (2010), documents that enforcement risk is a major component of the price of illicit products, that prices vary substantially across space and time, and that adjustments to enforcement propagate through the supply chain in ways that static interdiction models do not capture. Galenianos, Pacula, and Persico (2012) construct a formal search-theoretic equilibrium model of the retail drug market with matching frictions and price formation. The classical theoretical literature on smuggling, beginning with Bhagwati and Hansen (1973), similarly emphasizes

equilibrium prices, though it treats smuggling as competitive tariff evasion rather than as organized criminal activity on a spatial transportation network. Other illicit markets have received analogous treatment; for example, Wheaton, Schauer, and Galli (2010) frame human trafficking as a monopolistically competitive industry with explicit supply and demand sides. Despite the depth of these traditions taken individually, models that combine endogenous market-clearing prices on both supply and demand sides with a spatial transportation network and a strategic budget-constrained enforcement authority remain rare. We note the recent work of Gabriel, de la Barra, and Salo (2026) in developing a mathematical programming with equilibrium constraints (MPEC) framework for the mitigation of trade volumes in illicit supply networks in which the behavior of those that are engaged in illicit supply networks is captured via a supply chain network equilibrium model of Nagurney, Dong, and Zhang (2002), for the first time applied to illicit supply networks, along with the optimizing behavior of a decision-maker associated with resource allocation of mitigation strategies. That model is a bilevel one, in contrast to the one in this paper, and does not include multiple routes between illicit actors.

2.4 Spatial Price Equilibrium and the Present Paper

Capturing the equilibrium economic behavior of criminal networks together with endogenous prices motivates the use of a spatial price equilibrium framework where the importance of transportation and different routes continues to garner attention. The spatial price equilibrium tradition originated in the work of Samuelson (1952) and Takayama and Judge (1971), with subsequent variational inequality and network formulations developed by Florian and Los (1982), Dafermos and Nagurney (1984), Friesz, Harker, and Tobin (1984), Harker (1985), and Nagurney (1999). Within the licit-trade domain, the framework has been applied widely, including to minerals (Labys and Yang, 1991; Labys, 1999), agricultural products (Nagurney et al., 2023, 2024a, 2024b; Hassani et al., 2025), and essential goods in pandemics (Nagurney, Salarpour, and Dong, 2022). It has also been extended in several substantive directions, including dynamic adjustment (Nagurney and Aronson, 1989), information asymmetry and minimum quality standards (Nagurney, Li, and Nagurney, 2014), exchange rates and non-tariff measures (Nagurney and Besedina, 2023), and multicommodity trade with quality deterioration and transportation capacities (Besik and Nagurney, 2025). Within the illicit domain, Nagurney (2015) constructed a multiproduct spatial price equilibrium model of cybercrime in financial services, in which the authorities could impose additional transaction costs in order to disrupt trade. To our knowledge, this is the only prior spatial price equilibrium treatment of an illicit trade, and the present paper extends it by introducing the authorities as a strategic actor with an explicit budget allocation, by formulating the interaction as a Nash Equilibrium rather than as an externally chosen cost increase, and by enriching the network structure to allow multiple transportation routes between each pair of supply and demand markets.

Our treatment of the authorities' objective also draws on a long tradition of multicriteria and goal programming formulations in network equilibrium, where a decision maker weighs several objectives within one problem (cf. Nagurney, Thore, and Pan (1996)). We take that view here. The authorities weigh the disruption they achieve against the monetized societal benefit of enforcement,

and the weight α sets the balance between the two. To the best of our knowledge, such a weighted objective has not been combined before with a spatial price equilibrium criminal network and a budget constrained enforcement authority.

The present paper draws on each of these threads. We model the criminal network as a spatial-price-equilibrium actor with elastic supply and demand prices, multiple supply markets, multiple demand markets, and multiple transportation routes between each pair of markets. The authorities are modeled as a strategic actor allocating a finite budget across these routes in order to disrupt trade. The interaction is governed by a Nash Equilibrium, rather than the more common Stackelberg leader-follower structure, on the grounds that both players adapt continuously and simultaneously to each other in practice. The resulting equilibrium conditions are formulated as a variational inequality problem and alternative formulations are given. The combination of spatial price equilibrium with multi-route transportation, endogenous market-clearing prices on both supply and demand sides, and Nash interaction with a budget-constrained enforcement authority is, to our knowledge, new in the literature on illicit trade. This work also adds to the literature on operations research, networks, and policies (cf. Nagurney (2026) and references therein).

A few additional words on the choice of a Nash Equilibrium, as opposed to a Stackelberg, structure are warranted. A Stackelberg formulation is appropriate when the authorities can credibly commit to an enforcement posture before the criminal network responds, and when the criminal network can observe that posture, as in the design of fixed infrastructure or in one-shot interdiction planning. The Nash Equilibrium formulation, in contrast, is appropriate when both players adjust their strategies repeatedly, on comparable time scales, with neither able to commit to a strategy over the relevant horizon. The empirical record on illicit trade, including the rerouting of cocaine shipments away from Antwerp and Rotterdam and the redirection of stolen grain cargoes across corridors and destination markets documented in Section 5 and in Appendix B, is consistent with such continuous mutual adaptation, and, in such settings, a leader-follower construct would attribute to the authorities a first-mover advantage that they do not, in practice, possess. The Nash Equilibrium computed here can also be interpreted as the rest point of the tatonnement process discussed in Section 4, in which both players iteratively adjust their strategies in response to one another. When commitment power is present, however, a bilevel, Stackelberg formulation, as in the network interdiction literature reviewed in Subsection 2.1, remains the preferable construct. The two approaches are, hence, complementary, with the appropriate one determined by the institutional setting.

2.5 Organization of the Paper

The paper is organized as follows. In Section 3, the competitive behavior of both the criminal network and that of the authorities is presented, along with their optimization problems. The Nash Equilibrium is then shown to satisfy, under suitable assumptions, a variational inequality problem. Alternative variational inequalities are then derived, with one utilized for algorithmic solution in the case study, and with the proof of the associated theorem, along with the conditions for convergence of the algorithm, given in Appendix A. In Subsection 3.1, specific applications of our framework are described. In Section 4, the algorithmic scheme is discussed, and its resolution in terms of the game theory model given, with explicit formulae for the illicit product shipments, the budget allocations on the routes, and the Lagrange multiplier associated with the budget constraint, at each of the two steps of the algorithm. An interpretation of the algorithm as a tatonnement or dynamic adjustment process is also presented. In Section 5, a realistic case study is presented on stolen grain from occupied territories of Ukraine, with the mapping to the empirical record and the construction of the calibrated dataset from open sources detailed in Appendix B. Section 6 discusses the implications of this work for the theory and practice of OR/MS, and Section 7 summarizes the results, presents our conclusions, and suggests directions for future research.

3. The Integrated Equilibrium Framework

In this Section, we introduce the integrated equilibrium framework that includes the criminal network overseeing the production and trade of an illicit product, subject to the prices the consumers are willing to pay, and the authorities that seek to determine how much of their budget to optimally allocate to disrupt the criminal network's transportation routes. The criminal network's optimization problem yields the production, trade, and demand pattern satisfying a spatial price equilibrium in the presence of the imposed increased transportation costs by the authorities, which are determined through a Nash Equilibrium game between the criminal network and the authorities.

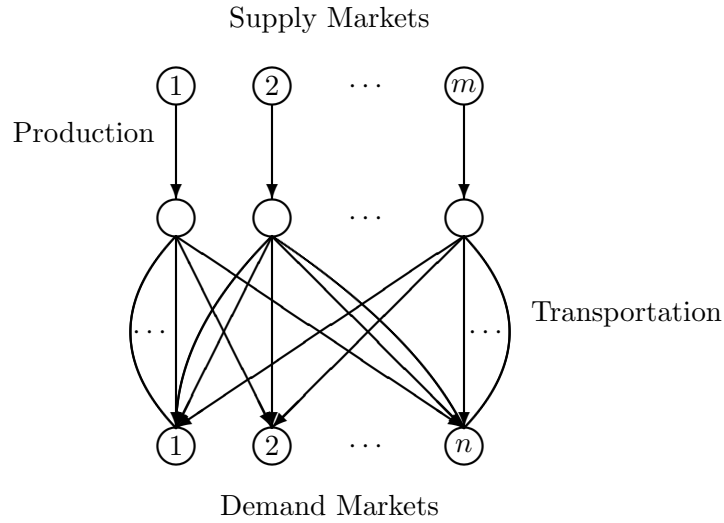


Figure 1: The Criminal Network for Illicit Product Trade

We begin with the criminal network model and then turn to the optimization problem of the authorities. In terms of noncooperative game theory, the strategies of the former are the illicit product shipments (from which one can calculate both the production outputs at the supply markets and the consumption amounts at the demand markets). The strategies of the latter, in turn, are the budget allocations for disruption on the transportation routes. The objective function of the criminal network depends on both its strategies and those of the authorities whereas the objective function of the authorities depends on their strategies and those of the criminal network. Each of these two players (the criminal network and the authorities) in the noncooperative game has their own feasible set; that is, the constraints that must be satisfied. The criminal network engaged in illicit product trade is depicted in Figure 1 and consists of m supply markets where the illicit product is produced and n demand markets where the products can be transported to and consumed based on the demand. A typical supply market is denoted by i , and a typical demand market is denoted by j . The supply and demand markets can be in the same or in different countries. This is important to emphasize since criminal networks engaged in illicit product supply and trade can be transnational organized crime networks.

Furthermore, since illicit product trade can involve multiple transportation routes, we allow for multiple routes connecting each supply market with each demand market. Each pair (i, j) of supply and demand markets is connected by n_{ij} routes with a typical route denoted by r for a total of n_P routes in the criminal network. We emphasize that a route r associated with distinct market pairs need not be the same one. A route may involve multiple modes of transportation, such as ships, trucks, air, rail, etc.

Henceforth, when we refer to a “product” we consider it to be an illicit one. The products are assumed to be homogeneous. In Subsection 3.1, we identify specific relevant applications of our modeling framework.

Let s_i represent the supply of the product produced at supply market i with all the supplies grouped into the vector $s \in R_+^m$. The demand for the product at demand market j is denoted by d_j , with all the product demands grouped into the vector $d \in R_+^n$. All vectors are assumed to be column vectors.

Let Q_{ijr} denote the amount of the product produced at supply market i and transported on route r to demand market j . The product shipments; equivalently, the product trade flows, are grouped into the vector $Q \in R_+^{n_P}$.

The Conservation of Flow Equations of the Criminal Network

The conservation of flow equations of the criminal network are:

$$s_i = \sum_{j=1}^n \sum_{r=1}^{n_{ij}} Q_{ijr}, \quad i = 1, \dots, m, \quad (1)$$

$$d_j = \sum_{i=1}^m \sum_{r=1}^{n_{ij}} Q_{ijr}, \quad j = 1, \dots, n. \quad (2)$$

The conservation of flow equations in (1) state that the supply of the product produced at a supply market is equal to the sum of the product shipments to all the demand markets on the transportation routes.

The equations in (2), in turn, state that the demand for the product at each demand market must be equal to the sum of the product shipments from all the supply markets on the transportation routes to the demand market.

The product shipments must be nonnegative; that is:

$$Q_{ijr} \geq 0, \quad i = 1, \dots, m; j = 1, \dots, n; r = 1, \dots, n_{ij}. \quad (3)$$

The Criminal Network Functions

We assume that the decision-maker associated with the criminal network is a “rational” economic decision-maker and, therefore, responsive to supply and demand and to the associated prices as well as to the transportation costs.

The supply price functions π_i , for all i , are expressed as:

$$\pi_i = \pi_i(s_i), \quad i = 1, \dots, m. \quad (4a)$$

Observe that, due to the conservation of flow equations (1), one may define supply price functions $\tilde{\pi}_i$, for all i , such that:

$$\tilde{\pi}_i(Q) \equiv \pi_i(s_i), \quad i = 1, \dots, m. \quad (4b)$$

The demand price of the product at demand market j , ρ_j , for all j , is as follows:

$$\rho_j = \rho_j(d_j), \quad j = 1, \dots, n. \quad (5a)$$

Similarly, due to (2), one may define new demand price functions $\tilde{\rho}_j$, for all j , such that:

$$\tilde{\rho}_j(Q) \equiv \rho_j(d_j), \quad j = 1, \dots, n. \quad (5b)$$

The unit transportation cost associated with transporting the illicit product from supply market i to demand market j on transportation route r is denoted by c_{ijr} , where:

$$c_{ijr} = c_{ijr}(Q_{ijr}), \quad i = 1, \dots, m; j = 1, \dots, n; r = 1, \dots, n_{ij}. \quad (6)$$

We assume that the supply price, the demand price, as well as the unit transportation cost functions are all continuous. Also, we assume that the supply price functions and the unit transportation cost functions are monotonically increasing and that the demand price functions are monotonically decreasing. Increasing transportation costs, as a function of product shipments, are reasonable due to possible congestion effects and also the increased possibility of detection.

The Criminal Network's Optimization Problem

The criminal network seeks to maximize its utility, u^1 , which is represented by its net payoff. The concept of net payoff in spatial price equilibrium problems is due to Samuelson (1952); see also Takayama and Judge (1964, 1971), but there the context was not that of criminal networks but of legal commodity trade.

Introducing now the budgetary allocations of the authorities to disrupt trade via the transportation routes with b_{ijr} denoting the budgetary allocation on route r joining markets (i, j) , and grouping all these values into the vector $b \in R_+^{nP}$, where each b_{ijr} has the interpretation of the additional cost per unit of flow that the authorities' enforcement activity imposes on the criminal network on that route, through, for example, inspections, seizures, sanction exposure, insurance frictions, and the costs of evasive rerouting, so that $b_{ijr}Q_{ijr}$ is the total additional cost inflicted on the criminal network's shipments there, the utility function of the criminal network, $u^1(Q, b)$, is given by:

$$\begin{aligned}
& u^1(Q, b) \\
&= - \sum_{i=1}^m \int_0^{\sum_{j=1}^n \sum_{r=1}^{n_{ij}} Q_{ijr}} \tilde{\pi}_i(x) dx - \sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^{n_{ij}} \int_0^{Q_{ijr}} c_{ijr}(y) dy - \sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^{n_{ij}} b_{ijr} Q_{ijr} \\
&\quad + \sum_{j=1}^n \int_0^{\sum_{i=1}^m \sum_{r=1}^{n_{ij}} Q_{ijr}} \tilde{\rho}_j(z) dz.
\end{aligned} \tag{7}$$

The criminal network seeks to maximize its utility, $u^1(Q, b)$, via its selection of its product shipment values on the routes and the imposition of the budget allocations of the authorities for transportation disruption. Hence, its optimization problem is given by:

$$\text{Maximize } u^1(Q, b) \tag{8}$$

subject to:

$$Q_{ijr} \geq 0, \quad \forall i, j, r. \tag{9}$$

The Authorities' Optimization Problem

The authorities, in turn, judge their budget allocations by two criteria. The first is the disruption imposed on the criminal network. On a given route, that disruption grows with the budget placed on the route and with the volume of illicit product moving along it. The second criterion is the monetized societal benefit that enforcement generates on a route. It can include the value recovered by legitimate producers and markets, the revenue denied to the criminal network, and, depending on the product, reductions in illnesses, deaths, and other social harms. We capture this second criterion through functions $h_{ijr}(b_{ijr})$, $\forall i, j, r$, which we take to be increasing, concave, and continuously differentiable, with $h_{ijr}(0) = 0$. Concavity reflects the diminishing societal returns to further enforcement on a route. The utility function of the authorities, $u^2(Q, b)$, is then the

weighted sum of the two criteria:

$$u^2(Q, b) = \sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^{n_{ij}} b_{ijr} Q_{ijr} + \alpha \sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^{n_{ij}} h_{ijr}(b_{ijr}), \quad (10)$$

where the weight $\alpha \geq 0$ reflects how much importance the authorities place on the societal benefit criterion. Setting $\alpha = 0$ recovers the pure disruption objective as a special case. Weighing several criteria within one objective in this way has a long history in network equilibrium and goal programming models (cf. Nagurney, Thore, and Pan (1996)). In practice, the functions h_{ijr} would be estimated by monetizing the dominant harm averted in the specific application: reduced overdoses, illnesses, and social costs of addiction in the case of illegal drugs (cf. Caulkins and Reuter (2010)); the value recovered by legitimate producers and markets, and food security restored, in the case of stolen grain; averted fraud losses in cybercrime in financial services; the protection of public health in the case of counterfeit PPEs, and reduced environmental damage in minerals crime, with agencies' existing harm indices and damage assessments providing natural starting points. The weight α , in turn, is a policy parameter of the authorities: $\alpha = 0$ recovers a pure disruption mandate, whereas a positive α expresses the importance that the authority places on broader societal returns relative to disruption. Since a precise value of α may be difficult to select, we investigate the sensitivity of the equilibrium to it in the case study; see Table 3, where the aggregate disruption is essentially unaffected by moving from $\alpha = 0$ to a substantial positive weight, whereas the budget allocation becomes smoother and differentiated across the corridors. The authorities then determine the best allocation of their budget, B , by solving the following optimization problem:

$$\text{Maximize} \quad u^2(Q, b) \quad (11)$$

subject to:

$$\sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^{n_{ij}} b_{ijr} \leq B. \quad (12)$$

$$b_{ijr} \geq 0, \quad \forall i, j, r. \quad (13)$$

Note that, in our framework, the authorities can correspond to one or more anticriminal organizations or even governments. Because each h_{ijr} is concave and the first term in (10) is linear in b , the utility $u^2(Q, b)$ is concave in the authorities' strategy vector b , a property used in the equilibrium analysis that follows.

The Nash Equilibrium and Variational Inequality Formulations

In order to conform to noncooperative game theory notation (see, e.g., Nagurney (1999)), we make the following definitions. We define the vectors: $x^1 \equiv Q$, $x^2 = b$, and $x \equiv (x^1, x^2)$ plus $\hat{u}^1(x) \equiv u^1(Q, b)$ and $\hat{u}^2(x) \equiv u^2(Q, b)$.

We also define the feasible sets:

$$K^1 \equiv \{x^1 | x^1 \geq 0\}, \quad (14a)$$

$$K^2 \equiv \{x^2 | x^2 \geq 0 \text{ and satisfies (12) with } x^2 \text{ replacing } b\}, \quad (14b)$$

$$K \equiv \prod_{i=1}^2 K^i. \quad (14c)$$

We now state the governing Nash Equilibrium conditions.

Definition 1: Nash Equilibrium Between the Criminal Network and the Authorities

The vector $x^* \in K$ is a Nash Equilibrium between the criminal network and the authorities if the following equilibrium condition holds:

$$\hat{u}^i(x^{i*}, x^{-i*}) \geq \hat{u}^i(x^i, x^{-i*}), \quad \forall x^i \in K^i, \quad i = 1, 2, \quad (15)$$

where

$$x^{-1*} \equiv (x^{2*}) \quad (16a)$$

$$x^{-2*} \equiv (x^{1*}). \quad (16b)$$

According to Definition 1, a Nash Equilibrium is established if neither the criminal network nor the authorities can improve upon their utility. We note that, under our assumptions, the utility functions of both the criminal network, $\hat{u}^1(x)$, and that of the authorities, $\hat{u}^2(x)$, are continuous, continuously differentiable, and concave with respect to their respective strategy vectors: x^1 and x^2 . Hence, we consider the game to consist of two major players and they are on the same “level” and can interact simultaneously, in contrast to a leader-follower type of game.

The variational inequality (VI) formulation of the above Nash Equilibrium conditions is now given.

Theorem 1: Variational Inequality Formulation of the Nash Equilibrium Between the Criminal Network and the Authorities

The vector $x^* \in K$ consisting of the product shipments and budget allocations on routes to disrupt transportation is a Nash Equilibrium according to Definition 1 if and only if it satisfies the variational inequality:

$$-\sum_{i=1}^2 \nabla_{x^i} \hat{u}^i(x^*) \times (x^i - x^{i*}) \geq 0, \quad \forall x \in K, \quad (17)$$

where $\nabla_{x^i} \hat{u}^i(x^*)$ denotes the gradient of the utility function $\hat{u}^i(x)$ with respect to the strategy vector x^i .

Proof: Under the imposed assumptions on the utility functions of the criminal network and of the authorities, which are concave with respect to their vectors of strategic variables and continuously

differentiable, we know that, according to Gabay and Moulin (1980); see also Nagurney (1999) and Nash (1950, 1951), the associated Nash Equilibrium pattern can be formulated as the solution to the variational inequality: Determine $x^* \in K$, satisfying (17). $\square$

We now, for completeness, expand variational inequality (17) as follows: Determine $(Q^*, b^*) \in R_+^{2n_P}$ and (12) holds, satisfying:

$$\begin{aligned} & \sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^{n_{ij}} (\tilde{\pi}_i(Q^*) + c_{ijr}(Q_{ijr}^*) + b_{ijr}^* - \tilde{\rho}_j(Q^*)) \times (Q_{ijr} - Q_{ijr}^*) \\ & + \sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^{n_{ij}} (-Q_{ijr}^* - \alpha \frac{\partial h_{ijr}(b_{ijr}^*)}{\partial b_{ijr}}) \times (b_{ijr} - b_{ijr}^*) \geq 0, \quad \forall (Q, b) \in R_+^{2n_P} \text{ and (12) holds.} \end{aligned} \quad (18)$$

Immediately one can see from VI (18) that the term: $\sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^{n_{ij}} (\tilde{\pi}_i(Q^*) + c_{ijr}(Q_{ijr}^*) + b_{ijr}^* - \tilde{\rho}_j(Q^*)) \times (Q_{ijr} - Q_{ijr}^*)$, when greater than equal to zero, corresponds to a variational inequality for a spatial price equilibrium problem in product shipment variables (see, e.g., Florian and Los (1982), Dafermos and Nagurney (1984), Nagurney (1999), Nagurney, Besik, and Dong (2019), Nagurney (2022), Nagurney and Besedina (2023), and Besik and Nagurney (2025)) but with the added costs of b_{ijr}^* , $\forall i, j, r$.

We now construct alternative variational inequalities to the one in (18) that includes Lagrange multipliers. One of these is especially amenable to effective algorithmic solution, as we will demonstrate. We will also, subsequently, further amplify the relevance of the alternative variational inequalities to spatial price equilibrium problems and the trade of illicit products.

Define $V(Q, b)$ as

$$\begin{aligned} V(Q, b) \equiv & \sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^{n_{ij}} (\tilde{\pi}_i(Q^*) + c_{ijr}(Q_{ijr}^*) + b_{ijr}^* - \tilde{\rho}_j(Q^*)) \times (Q_{ijr} - Q_{ijr}^*) \\ & + \sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^{n_{ij}} (-Q_{ijr}^* - \alpha \frac{\partial h_{ijr}(b_{ijr}^*)}{\partial b_{ijr}}) \times (b_{ijr} - b_{ijr}^*) \end{aligned} \quad (19)$$

and note that the VI (18) can be rewritten as

$$\text{Minimize}_K V(Q, b) = V(Q^*, b^*) = 0. \quad (20)$$

In order to construct the Lagrange function, we reformulate the constraints with the associated Lagrange multiplier next to the corresponding constraint as follows:

$$e_{ijr} = -Q_{ijr} \leq 0, \quad \epsilon_{ijr}, \quad \forall i, \forall j, \forall r, \quad (21)$$

$$f_{ijr} = -b_{ijr} \leq 0, \quad \mu_{ijr}, \quad \forall i, \forall j, \forall r, \quad (22)$$

$$g = -B + \sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^{n_{ij}} b_{ijr} \leq 0, \quad \lambda, \quad (23)$$

and

$$\Gamma(Q, b) = (e_{ijr}, f_{ijr}, g), \quad \forall i, \forall j, \forall r. \quad (24)$$

We group all the e_{ijr} s into the vector e and the f_{ijr} s into the vector f . The Lagrange function, denoted by $L(Q, b, \epsilon, \mu, \lambda)$, is then given by:

$$\begin{aligned} L(Q, b, \epsilon, \mu, \lambda) \equiv & \sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^{n_{ij}} (\tilde{\pi}_i(Q^*) + c_{ijr}(Q_{ijr}^*) + b_{ijr}^* - \tilde{\rho}_j(Q^*)) \times (Q_{ijr} - Q_{ijr}^*) \\ & + \sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^{n_{ij}} (-Q_{ijr}^* - \alpha \frac{\partial h_{ijr}(b_{ijr}^*)}{\partial b_{ijr}}) \times (b_{ijr} - b_{ijr}^*) \\ & + \sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^{n_{ij}} e_{ijr} \epsilon_{ijr} + \sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^{n_{ij}} f_{ijr} \mu_{ijr} + g \lambda, \\ & \forall Q \in R_+^{n_P}, \forall b \in R_+^{n_P}, \forall \epsilon \in R_+^{n_P}, \forall \mu \in R_+^{n_P}, \forall \lambda \in R_+^1. \end{aligned} \quad (25)$$

The feasible set K is convex and the Slater condition is satisfied. Indeed, we know that $\Gamma(Q, b)$ is convex and $\exists(\bar{Q}, \bar{b}) \in R_+^{2n_P}$ such that $\Gamma(\bar{Q}, \bar{b}) < 0$. Therefore, if (Q^*, b^*) is a minimal solution to (20), there exist Lagrange multipliers $\epsilon^* \in R_+^{n_P}$, $\mu^* \in R_+^{n_P}$, and $\lambda^* \in R_+^1$ such that the vector $(Q^*, b^*, \epsilon^*, \mu^*, \lambda^*)$ is a saddle point of the Lagrange function (25):

$$L(Q^*, b^*, \epsilon, \mu, \lambda) \leq L(Q^*, b^*, \epsilon^*, \mu^*, \lambda^*) \leq L(Q, b, \epsilon^*, \mu^*, \lambda^*) \quad (26)$$

and

$$e_{ijr}^* \epsilon_{ijr}^* = 0, \quad \forall i, \forall j, \forall r, \quad (27)$$

$$f_{ijr}^* \mu_{ijr}^* = 0, \quad \forall i, \forall j, \forall r, \quad (28)$$

$$g^* \lambda^* = 0. \quad (29)$$

From the right-hand side of (26), it follows that $(Q^*, b^*) \in R_+^{2n_P}$ is a minimal point of the function $L(Q, b, \epsilon^*, \mu^*, \lambda^*)$ in the whole space $R_+^{2n_P}$ and, therefore:

$$\frac{\partial L(Q^*, b^*, \epsilon^*, \mu^*, \lambda^*)}{\partial Q_{ijr}} = (\tilde{\pi}_i(Q^*) + c_{ijr}(Q_{ijr}^*) + b_{ijr}^* - \tilde{\rho}_j(Q^*) - \epsilon_{ijr}^*) = 0, \quad \forall i, \forall j, \forall r, \quad (30a)$$

$$\frac{\partial L(Q^*, b^*, \epsilon^*, \mu^*, \lambda^*)}{\partial b_{ijr}} = (-Q_{ijr}^* - \alpha \frac{\partial h_{ijr}(b_{ijr}^*)}{\partial b_{ijr}} - \mu_{ijr}^* + \lambda^*) = 0, \quad \forall i, \forall j, \forall r, \quad (30b)$$

together with conditions (27) - (29).

We now state the following theorem.

Theorem 2: Alternative Variational Inequality Formulations

Conditions (27) through (30b) correspond to a variational inequality equivalent to VI (18) given by: Determine $(Q^*, b^*, \epsilon^*, \mu^*, \lambda^*) \in R_+^{4n_P+1}$ such that:

$$\begin{aligned}
& \sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^{n_{ij}} (\tilde{\pi}_i(Q^*) + c_{ijr}(Q_{ijr}^*) + b_{ijr}^* - \tilde{\rho}_j(Q^*) - \epsilon_{ijr}^*) \times (Q_{ijr} - Q_{ijr}^*) \\
& + \sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^{n_{ij}} (-Q_{ijr}^* - \alpha \frac{\partial h_{ijr}(b_{ijr}^*)}{\partial b_{ijr}} - \mu_{ijr}^* + \lambda^*) \times (b_{ijr} - b_{ijr}^*) \\
& + (B - \sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^{n_{ij}} b_{ijr}^*) \times (\lambda - \lambda^*) \\
& + \sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^{n_{ij}} Q_{ijr}^* \times (\epsilon_{ijr} - \epsilon_{ijr}^*) + \sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^{n_{ij}} b_{ijr}^* \times (\mu_{ijr} - \mu_{ijr}^*) \geq 0, \quad \forall (Q, b, \epsilon, \mu, \lambda) \in R_+^{4n_P+1} \quad (31)
\end{aligned}$$

or simplified as: Determine $(Q^*, b^*, \lambda^*) \in R_+^{2n_P+1}$ such that:

$$\begin{aligned}
& \sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^{n_{ij}} (\tilde{\pi}_i(Q^*) + c_{ijr}(Q_{ijr}^*) + b_{ijr}^* - \tilde{\rho}_j(Q^*)) \times (Q_{ijr} - Q_{ijr}^*) \\
& + \sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^{n_{ij}} (-Q_{ijr}^* - \alpha \frac{\partial h_{ijr}(b_{ijr}^*)}{\partial b_{ijr}} + \lambda^*) \times (b_{ijr} - b_{ijr}^*) \\
& + (B - \sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^{n_{ij}} b_{ijr}^*) \times (\lambda - \lambda^*), \quad \forall (Q, b, \lambda) \in R_+^{2n_P+1}. \quad (32)
\end{aligned}$$

Proof: See Appendix A. $\square$

We now put variational inequality (32) into standard form (cf. Nagurney (1999)): Determine $X^* \in \mathcal{K} \subset R^N$, such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \quad (33)$$

where X and $F(X)$ are N -dimensional vectors, $\mathcal{K}$ is a closed, convex set, and F is a given continuous function from $\mathcal{K}$ to R^N .

We set $\mathcal{K} \equiv \{X = (Q, b, \lambda) | (Q, b, \lambda) \in R_+^{2n_P+1}\}$, and $N \equiv 2n_P + 1$. Also, we define the vector $F(X) \equiv (F_1(X), F_2(X), F_3(X))$, where

$$F_1(X) = [\tilde{\pi}_i(Q) + c_{ijr}(Q_{ijr}) + b_{ijr} - \tilde{\rho}_j(Q), \quad i = 1, \dots, m; j = 1, \dots, n; r = 1, \dots, n_{ij}], \quad (34)$$

$$F_2(X) = \left[-Q_{ijr} - \alpha \frac{\partial h_{ijr}(b_{ijr})}{\partial b_{ijr}} + \lambda, \quad i = 1, \dots, m; j = 1, \dots, n; r = 1, \dots, n_{ij} \right], \quad (35)$$

$$F_3(X) = \left[B - \sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^{n_{ij}} b_{ijr} \right]. \quad (36)$$

Then, clearly VI (32) can be rewritten as variational inequality (33), with use of the above definitions.

Variational inequalities (18) and (31) can also be put into standard form (33). Note that both of these variational inequalities as well as VI (32) have, prior to their term $(Q_{ijr} - Q_{ijr}^*)$, an expression that includes the supply price at supply market i , $\tilde{\pi}_i(Q^*)$, plus the unit transportation cost, $c_{ijr}(Q^*)$, minus the demand price at demand market j , $\tilde{\rho}_j(Q^*)$, as well as the additional transportation cost imposed by the authorities: b_{ijr}^* . The composition of the first three former noted terms, along with $(Q_{ijr} - Q_{ijr}^*)$, reflects the well-known spatial price equilibrium condition in which, at the equilibrium, if the supply price plus the unit transportation cost is equal to the demand price, then there will be a positive flow of the product between the pair of supply and demand markets. On the other hand, if the supply price plus the unit transportation cost exceeds the demand price, then there will be no trade of the product between the pair of markets at the equilibrium.

3.1 Applications

In this Subsection, we highlight several applications of the above framework. We focus on applications that have been studied as spatial price equilibrium problems or have the potential to be formulated and analyzed thus.

3.1.1 Illegal Drug Trade

Illegal drugs are estimated to account for about 20% of criminal activities (Bussu et al. (2024)), and criminals in this trade avail themselves of different transportation routes, with maritime transportation being quite “popular” (United Nations Office on Drugs and Crime (2013)) – something that our new model captures. As interdiction intensified at the container ports of Antwerp and Rotterdam, criminal networks progressively redirected cocaine shipments toward smaller, less-monitored ports (European Union Drugs Agency and the World Customs Organization (2025)), with Europol (2026) reporting a parallel diversification of trafficking *modi operandi*; see also Magliocca et al. (2019) on cocaine trafficking and its interdiction as an adaptive system. This setting, hence, exhibits the assumptions of our framework: multiple supply and demand markets, substitutable routes, and route-level enforcement. Our model captures such adaptation endogenously through the reallocation of product shipments across routes. Here the societal benefit term $h_{ijr}(b_{ijr})$ monetizes the public health gains from disrupting a route, including fewer overdoses, fewer illnesses, and lower social costs tied to addiction (cf. Caulkins and Reuter (2010)).

3.1.2 Minerals Crime

Spatial price equilibrium models have had wide application in the trade of mineral ores and energy (see Labys and Yang (1991), Hwang et al. (1994), Irwin and Yang (1996), and Labys

(1999)). Growing demand for gold and for critical energy transition minerals is creating economic opportunities for criminal infiltration of mineral production, transportation, and consumption networks, with such activities especially notable in the Andean–Amazon region (United Nations Office on Drugs and Crime (2025), United Nations Interregional Crime and Justice Research Institute (2026)). In the case of gold, SwissAid (2024) estimates that more than 435 tons were smuggled out of Africa in 2022 alone, routed largely through a small set of transit hubs such as Uganda, Rwanda, Chad, and Libya (SwissAid (2025)). The dependence of this trade on a limited set of transportation corridors and transit points, with documented substitution among them, makes it especially suitable for the route-disruption framework developed here, with the societal benefit term capturing the environmental and governance gains from disruption (United Nations Interregional Crime and Justice Research Institute (2026)).

3.1.3 Counterfeit PPEs

The COVID-19 pandemic, during which the trade of PPEs was subject to many trade restrictions (cf. Nagurney, Salarpour, and Dong (2022)), brought an escalation of counterfeiting of PPEs, including N-95 respirators, endangering the health and safety of people (Hashemi (2023)). Because counterfeit PPEs move through the same commercial logistics channels as licit medical goods, with multiple substitutable routings, targeted increases in inspection and handling costs along specific routes offer a natural disruption lever, which is precisely the instrument modeled here through the authorities’ budget allocations. The societal benefit term reflects the protection of public health, since disrupting counterfeit flows keeps substandard equipment, which would otherwise put patients and frontline workers at risk, out of use (Hashemi (2023)).

3.1.4 Cybercrime and Financial Services

Nagurney (2015) constructed a multiproduct spatial price equilibrium model of cybercrime in financial services, with the products being, for example, stolen credit cards, and proposed that authorities could disrupt trade by increasing the transaction costs, which correspond to transportation costs here. The framework in this paper goes further in that it explicitly determines how best to allocate the authorities’ budget across the routes, subject to a budget constraint, with the authorities integrated as a player in a noncooperative game with the criminal network governed by a Nash Equilibrium. In this application the societal benefit term monetizes the fraud losses that are averted and the trust in financial services that is restored (Nagurney (2015)).

3.1.5 Trade in Grain from Occupied Territories in Ukraine

Agriculture is a sector to which spatial price equilibrium models have been widely applied, including to the trade of wheat and corn produced in Ukraine prior to the full-scale invasion by Russia of 2022, under the Black Sea Grain Initiative, and following its collapse (cf. Nagurney et al. (2023, 2024a,b), Hassani et al. (2025)). Melnychuk (2025), Tuliakov (2025), and Shostko et al. (2025) document the illegal market for grain from Ukraine, including that of stolen grain from the occupied territories, a topic also garnering much media attention (Faucon (2024)). Because stolen grain is moved out of the occupied territories through a constrained set of corridors, including overland

routes and a limited number of Black Sea and Crimean ports, the interdiction of transportation routes is a meaningful policy lever, and the societal benefit term monetizes the value recovered by legitimate producers and markets, the revenue denied to the occupying actors, and the restoration of food security when illicit grain flows are disrupted.

Common to all of the above settings, and necessary for the transfer of our framework to any new one, are the following assumptions: multiple supply and demand markets in which prices respond to quantities; multiple, at least partially substitutable, transportation routes joining the market pairs; a criminal actor that responds rationally to prices and costs, and enforcement instruments that can be differentiated by route and that raise the criminal actor's costs per unit of flow. Where routes cannot be substituted, or where enforcement cannot be targeted below the level of a whole market, the single destination market pressure analyzed in Section 5 is the closer construct, and, where the authorities possess commitment power, a Stackelberg formulation may be preferable, as discussed in Section 2.

4. The Algorithm

The algorithm that we use for the solution of the problems in the next section is the modified projection method of Korpelevich (1977). Convergence of this algorithm is guaranteed if the function $F(X)$ that enters the variational inequality problem (33) is monotone and Lipschitz continuous and that a solution exists.

Recall that the function $F(X)$ is monotone if

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle \geq 0, \quad \forall X^1, X^2 \in \mathcal{K}. \quad (37)$$

Also, $F(X)$ is Lipschitz continuous, if there exists an $\eta > 0$, known as the Lipschitz constant, such that

$$\|F(X^1) - F(X^2)\| \leq \eta \|X^1 - X^2\|, \quad \forall X^1, X^2 \in \mathcal{K}. \quad (38)$$

At each iteration, denoted by τ , the modified projection method consists of two steps: a computation step, in which a variational inequality subproblem with a simplified, separable structure is solved at the most recent iterate, followed by an adaptation step, in which a subproblem of the same form is solved at the point produced by the computation step. The iterates are initialized at any point in $\mathcal{K}$, and the parameter β in the subproblems is a scalar satisfying $0 < \beta \leq \frac{1}{\eta}$, where η is the Lipschitz constant in (38). The algorithm is deemed to have converged when the successive iterates differ by no more than a prespecified tolerance, for which we do not introduce a separate symbol in order to avoid any confusion with the Lagrange multipliers ϵ_{ijr} . For the complete formal statement of the algorithm, along with the underlying theory, we refer the reader to Korpelevich (1977) and to Nagurney (1999). In the context of our new game theory model, the two steps of the algorithm yield closed form expressions for the product shipments, the budget allocations, as well as the Lagrange multiplier at each iteration, which we now present. The algorithm in the context of our model is, hence, easy to implement. We conclude this Section with an investigation into

the conditions for convergence of the algorithm for our model, under reasonable conditions on the underlying functions.

The elegance of this algorithm for our variational inequality (33) for the computation of solutions to our model is apparent from the following explicit formulae. In words: at each iteration, the shipment on a route increases if the price that consumers are willing to pay at the demand market exceeds the supply price plus the unit transportation cost and the enforcement cost imposed by the authorities on that route, and decreases otherwise; the budget allocation on a route increases if the flow on the route, together with the marginal societal benefit of enforcement there, exceeds the shadow price of the budget; and the Lagrange multiplier rises when the allocations overspend the budget and falls when they underspend it. The formulae below make these adjustments precise.

Explicit Formulae at Iteration τ for the Product Shipments in the Computation Step

In particular, we have the following closed form expression for each product shipment: $i = 1, \dots, m$; $j = 1, \dots, n$; $r = 1, \dots, n_{ij}$:

$$\bar{Q}_{ijr}^\tau = \max\{0, Q_{ijr}^{\tau-1} + \beta(\tilde{\rho}_j(Q^{\tau-1}) - c_{ijr}(Q_{ijr}^{\tau-1}) - b_{ijr}^{\tau-1} - \tilde{\pi}_i(Q^{\tau-1}))\}. \quad (39a)$$

Explicit Formulae at Iteration τ for the Budget Allocations in the Computation Step

The closed form expression for each budget allocation: $i = 1, \dots, m$; $j = 1, \dots, n$; $r = 1, \dots, n_{ij}$, is:

$$\bar{b}_{ijr}^\tau = \max\{0, b_{ijr}^{\tau-1} + \beta(Q_{ijr}^{\tau-1} + \alpha \frac{\partial h_{ijr}(b_{ijr}^{\tau-1})}{\partial b_{ijr}} - \lambda^{\tau-1})\}. \quad (39b)$$

Explicit Formula at Iteration τ for the Lagrange Multiplier in the Computation Step

For the Lagrange multiplier, we obtain:

$$\bar{\lambda}^\tau = \max\{0, \lambda^{\tau-1} + \beta(-B + \sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^{n_{ij}} b_{ijr}^{\tau-1})\}. \quad (39c)$$

And, for completeness, we also provide the explicit formulae associated with the adaptation step.

Explicit Formulae at Iteration τ for the Product Shipments in the Adaptation Step

The closed form expression for each product shipment: $i = 1, \dots, m$; $j = 1, \dots, n$; $r = 1, \dots, n_{ij}$, is:

$$Q_{ijr}^\tau = \max\{0, Q_{ijr}^{\tau-1} + \beta(\tilde{\rho}_j(\bar{Q}^\tau) - c_{ijr}(\bar{Q}_{ijr}^\tau) - \bar{b}_{ijr}^\tau - \tilde{\pi}_i(\bar{Q}^\tau))\}. \quad (40a)$$

Explicit Formulae at Iteration τ for the Budget Allocations in the Adaptation Step

The closed form expression for the budget allocation for each $i = 1, \dots, m$; $j = 1, \dots, n$; $r = 1, \dots, n_{ij}$, is:

$$b_{ijr}^\tau = \max\{0, b_{ijr}^{\tau-1} + \beta(\bar{Q}_{ijr}^\tau + \alpha \frac{\partial h_{ijr}(\bar{b}_{ijr}^\tau)}{\partial b_{ijr}} - \bar{\lambda}^\tau)\}. \quad (40b)$$

Explicit Formula at Iteration τ for the Lagrange Multiplier in the Adaptation Step

For the Lagrange multiplier, we have that:

$$\lambda^\tau = \max\{0, \lambda^{\tau-1} + \beta(-B + \sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^{n_{ij}} \bar{b}_{ijr}^\tau)\}. \quad (40c)$$

Remark

The above expressions are also meaningful from an economic standpoint. According to (39a), if the price the consumers of the illicit product are willing to pay exceeds the supply price plus the unit transportation cost and the added transportation cost imposed by the authorities, then the shipment on that route will increase; otherwise, it will decrease. According to (39b), if the product shipment on a route, together with the marginal societal benefit of enforcement on that route, $\alpha \frac{\partial h_{ijr}(b_{ijr})}{\partial b_{ijr}}$, exceeds the Lagrange multiplier associated with the budget constraint, then the budget allocation on that route should increase – higher shipments, and routes with higher marginal societal benefit from disruption, attract higher allocations. According to (39c), the Lagrange multiplier increases if the sum of the budget allocations exceeds the available budget, and decreases otherwise. The adaptation step expressions (40a) – (40c) have analogous interpretations as adjustments, and the scheme overall can be interpreted as a discrete time adjustment or tatonnement process.

In Appendix A we establish that, under the reasonable assumptions that the supply price, the unit transportation cost, and minus the demand price functions are monotone, and that these functions have bounded first order derivatives with each h_{ijr} having a bounded second order derivative, the function $F(X)$ for our model (cf. (33) – (36)) is monotone (cf. (37)) and Lipschitz continuous (cf. (38)). The conditions for convergence of the modified projection method are, hence, satisfied.

5. Case Study: Disruption of Stolen Grain Trade from Occupied Ukrainian Territories

In this Section we apply the integrated equilibrium framework to the illicit trade in grain looted from the territories of Ukraine occupied by Russia. Of the applications surveyed in Subsection 3.1, this one fits the model especially well, since there is a small set of substitutable transportation corridors, and a budget constrained authority that competes with the criminal network rather than leading it as in a Stackelberg game. Subsection 5.1 describes the empirical setting and the calibrated dataset, with the detailed mapping of each model primitive onto the empirical record, the construction of every parameter from open sources, and a discussion of what the calibration can and cannot support, given in Appendix B. Subsection 5.2 reports the equilibrium solution,

a sensitivity analysis on the enforcement budget, the effect of the societal benefit weight, and a market substitution experiment.

5.1 The Empirical Setting and the Calibrated Dataset

Since the invasion of February 2022, grain has been removed in a systematic way from occupied Ukrainian farmland and exported under a falsified origin by a vertically organized operation that controls elevators, storage, logistics, and ports (Faucon (2024), Melnychuk (2025)) – exactly the criminal network actor of our framework. We model two occupied source regions, $i = 1$ (Zaporizhzhia) and $i = 2$ (Kherson), and two destinations, $j = 1$ (Egypt) and $j = 2$ (Türkiye), joined by two corridors each, for $n_P = 8$ routes in all, as depicted in Figure 2. The first corridor, $r = 1$, is the Crimean port corridor through Sevastopol and Feodosia, served by small shadow fleet vessels and carrying high seizure and sanction exposure. The second, $r = 2$, is the Russian port laundering corridor through Novorossiysk and Temryuk, reached by blending and relabeling the cargo in the Kerch Strait, and enjoying cheaper deep water freight. The Azov ports of Mariupol and Berdiansk serve as feeders into these two corridor types. Product shipments are in kilotonnes per year (kt/yr); prices, costs, and budget allocations are in U.S. dollars per tonne (USD/t); and criminal revenue is in million USD per year. In the tables, C denotes the Crimean port corridor ($r = 1$) and R the Russian port laundering corridor ($r = 2$). Note that the enforcement budget B is expressed in dollars per metric ton, since it is the sum of the per tonne allocations b_{ijr} over the eight routes, and, hence, it is not a total expenditure. The mapping of the model primitives onto the empirical record and the construction of the calibrated functions, reported in Table 1, from open sources, are detailed in Appendix B.

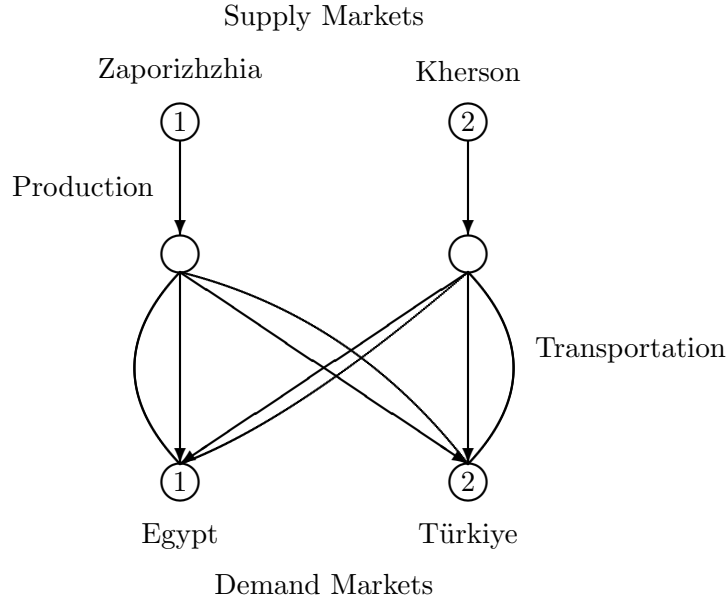


Figure 2: The Criminal Network for the Case Study, with Two Occupied Source Regions, Two Destination Markets, and Two Corridors Joining Each Pair of Markets

Table 1: Calibrated Input Functions (s_i, d_j, Q_{ijr} in kt/yr; Prices and Costs in USD/t)

Object	Functional form
Supply price, $i = 1$ (Zaporizhzhia)	$\tilde{\pi}_1(s_1) = 0.010 s_1 + 40$
Supply price, $i = 2$ (Kherson)	$\tilde{\pi}_2(s_2) = 0.010 s_2 + 50$
Demand price, $j = 1$ (Egypt)	$\tilde{\rho}_1(d_1) = -0.030 d_1 + 255$
Demand price, $j = 2$ (Türkiye)	$\tilde{\rho}_2(d_2) = -0.035 d_2 + 245$
Transport cost (all routes)	$c_{ijr}(Q_{ijr}) = 0.030 Q_{ijr} + t_{ijr}$
intercepts t_{ijr} , $r = 1$ (C) / $r = 2$ (R)	$(1, 1): 75/61, (1, 2): 70/57, (2, 1): 80/66, (2, 2): 75/62$
Societal benefit function	$h_{ijr}(b_{ijr}) = \ln(1 + b_{ijr})$

5.2 Equilibrium, Sensitivity, and a Market Substitution Experiment

Before presenting the numerical results, we emphasize two points. First, as noted in Subsection 5.1, the budget allocation b_{ijr} is the expected disruption cost per tonne that the authorities can impose on a corridor, and it enters the criminal network’s cost on that corridor per unit of flow; the capacity B is, therefore, the sum of the per tonne allocations over the eight routes, expressed in USD/t, and it is not a total enforcement expenditure. The values of B in the tables should be interpreted accordingly. Second, since the calibration is assembled entirely from open sources, as detailed in Appendix B, the numerical values reported below should be read as a defensible, calibrated scenario rather than as precise measurements, and the statements that follow are implications of the computed equilibria of the calibrated model, rather than measured causal effects. The structural insights are, accordingly, more robust than the exact magnitudes.

The modified projection method was implemented in Python 3 with the NumPy library, and the computations were performed on Google Colaboratory using an Intel Xeon processor at 2.20 GHz with 12 GB of memory. With a step size of 0.02, a convergence tolerance of (10^{-7}) , and initialization of the algorithm at the origin, the modified projection method converged for all examples within a few seconds.

Table 2 reports the full equilibrium solutions across enforcement capacities $B = 0, 100, 200, 400$, and 600. For each capacity it gives the product shipments, the budget allocations, the market demands, the budget shadow price, and the criminal revenue. The revenue of the criminal network is defined as the value of its sales at the demand markets at the equilibrium; that is, revenue $\equiv \sum_{j=1}^n \tilde{\rho}_j(Q^*) \times d_j^*$. It is a gross measure of the monies flowing to the criminal network from the consumers of the illicit product; it is not a profit, and it is distinct from the net payoff in (7). In this table we set the societal benefit weight to $\alpha = 0$, which isolates the effect of the enforcement budget. We examine the effect of the term itself subsequently. Here, $B = 0$ is the pure spatial price equilibrium with no intervention. The capacities $B = 50$ and $B = 300$ are left out of the table to save space, and they fall in an orderly way between the reported columns. At those two capacities total throughput is 5.24 Mt (Million tonnes) and 3.97 Mt, and criminal revenue is \$867.5 million and \$736.4 million. In Table 2, kt refers to kilo tonnes.

Three features of the computed equilibria stand out. First, enforcement steadily erodes the

Table 2: Equilibrium Solutions Across Enforcement Capacities (Flows Q_{ijr}^* and Demands in kt/yr; Allocations b_{ijr}^* in USD/t; Criminal Revenue in Million USD/yr)

Variable	$B = 0$	$B = 100$	$B = 200$	$B = 400$	$B = 600$
Q_{111}^*	620.67	650.20	558.75	433.75	308.75
Q_{112}^*	1087.34	650.20	558.75	433.75	308.75
Q_{121}^*	517.12	650.20	558.75	433.75	308.75
Q_{122}^*	950.46	650.20	558.75	433.75	308.75
Q_{211}^*	406.39	650.20	558.75	433.75	308.75
Q_{212}^*	873.06	650.20	558.75	433.75	308.75
Q_{221}^*	302.84	429.64	558.75	433.75	308.75
Q_{222}^*	736.17	650.20	558.75	433.75	308.75
b_{111}^*	—	16.46	33.84	57.59	81.34
b_{112}^*	—	30.46	47.84	71.59	95.34
b_{121}^*	—	6.18	17.66	43.91	70.16
b_{122}^*	—	19.18	30.66	56.91	83.16
b_{211}^*	—	3.67	18.84	42.59	66.34
b_{212}^*	—	17.67	32.84	56.59	80.34
b_{221}^*	—	0.00	2.66	28.91	55.16
b_{222}^*	—	6.38	15.66	41.91	68.16
d_1 (Egypt)	2987.5	2600.8	2235.0	1735.0	1235.0
d_2 (Türkiye)	2506.6	2380.2	2235.0	1735.0	1235.0
λ^*	—	650.20	558.75	433.75	308.75
Revenue (M USD)	888.26	845.14	792.81	671.84	518.36

revenue of the criminal network, from \$888.26 million at $B = 0$ to \$518.36 million at $B = 600$, and it raises the price that consumers pay at both destinations. Across the grid $\tilde{p}_1$ rises from \$165.4/t to \$218.0/t and $\tilde{p}_2$ rises from \$157.3/t to \$201.8/t. Disruption, hence, makes the illicit product more expensive and less abundant. Second, within each source and destination pair the authorities place *more* budget on the cheaper, higher volume laundering corridor $r = 2$ than on the Crimean corridor $r = 1$. At $B = 200$, for instance, $b_{112}^* = 47.84$ exceeds $b_{111}^* = 33.84$, and the busier Zaporizhzhia routes draw more than the Kherson routes. This is the property of objective (10) by which higher flow draws a higher allocation, now operating across a corridor set grounded in the data. Third, at $B = 0$ the laundering corridors carry much larger flows than the closed Crimean corridors. For instance, $Q_{112}^* = 1087.3$ against $Q_{111}^* = 620.7$ kt, which reproduces the documented preference for relabeled routing through Russian ports over the sanctioned Crimean ports.

In every column with a positive budget the active corridors carry a common flow level. The reason is that, with $\alpha = 0$, the first order condition of the authorities for the budget allocation forces $Q_{ijr}^* = \lambda^*$ across the corridors that stay open, and the budget shadow price λ^* falls as B rises. Turning on the societal benefit term breaks this degeneracy. Table 3 contrasts $\alpha = 0$ with $\alpha = 300$ at $B = 200$. The aggregate disruption is essentially unchanged, with criminal revenue of \$793.91 million against \$792.81 million. The budget allocation, however, now becomes *differentiated* across corridors, and the flows spread according to corridor characteristics rather than collapsing to the

single value λ^* . The marginal societal benefit $\alpha \frac{\partial h_{ijr}(b_{ijr})}{\partial b_{ijr}}$ enters the first order condition for the budget (see the Remark in Section 4). The authorities therefore stop equalizing flows and instead trade a little disruption for a smoother and more realistic allocation.

Table 3: Effect of the Societal Benefit Weight at $B = 200$: $\alpha = 0$ Versus $\alpha = 300$, with $h_{ijr}(b_{ijr}) = \ln(1 + b_{ijr})$, $\forall i, j, r$ (cf. (10)); Q^* in kt/yr and b^* in USD/t

Route (i, j, r)	$\alpha = 0$		$\alpha = 300$	
	Q_{ijr}^*	b_{ijr}^*	Q_{ijr}^*	b_{ijr}^*
(1, 1, 1)	558.75	33.84	566.78	32.48
(1, 1, 2)	558.75	47.84	569.41	46.40
(1, 2, 1)	558.75	17.66	560.14	18.23
(1, 2, 2)	558.75	30.66	566.38	31.04
(2, 1, 1)	558.75	18.84	560.12	18.21
(2, 1, 2)	558.75	32.84	566.65	32.01
(2, 2, 1)	558.75	2.66	524.30	4.83
(2, 2, 2)	558.75	15.66	558.88	16.79
λ^*	558.75		575.74	
Revenue (M USD)	792.81		793.91	

Finally, we use the model to contrast two enforcement philosophies. The budget instrument of Table 2 acts on the transportation corridors across the whole network. A different approach, and the one most visible in the news record, puts pressure on a single *destination*, for example through threats of secondary sanctions or buyer refusal aimed at Egypt. We represent such a market tightening as an external access penalty of \$60/t on the two Egypt corridors, with no authority budget ($B = 0$), and we solve the spatial price equilibrium of the criminal network again. Table 4 compares the result with the undisrupted baseline.

Table 4: Market Substitution Experiment at $B = 0$: Undisrupted Baseline Versus a \$60/t Access Penalty on the Egypt Corridors (Flows and Demands in kt/yr; Revenue in Million USD/yr)

Variable	Baseline	Egypt tightening
Q_{111}^* (Zap to Egypt, C)	620.67	263.31
Q_{112}^* (Zap to Egypt, R)	1087.34	729.97
Q_{211}^* (Kher to Egypt, C)	406.39	49.02
Q_{212}^* (Kher to Egypt, R)	873.06	515.69
Q_{121}^* (Zap to Türkiye, C)	517.12	554.74
Q_{122}^* (Zap to Türkiye, R)	950.46	988.07
Q_{221}^* (Kher to Türkiye, C)	302.84	340.45
Q_{222}^* (Kher to Türkiye, R)	736.17	773.79
d_1 (Egypt)	2987.5	1558.0
d_2 (Türkiye)	2506.6	2657.1
Total throughput	5494.0	4215.0
$\tilde{\rho}_1$ (Egypt)	165.38	208.26
Revenue (M USD)	888.26	728.35

Pressuring Egypt alone cuts the flow into Egypt by almost half, from 2987.5 to 1558.0 kt, but it pushes extra volume into Türkiye, which rises from 2506.6 to 2657.1 kt. Total throughput falls by only about 23%, and the network keeps the bulk of its activity. This is exactly the displacement seen in the documented redirection from Egypt to Türkiye (Vlasiuk (2026)). The budget instrument of Table 2 behaves quite differently. By acting across the corridors, it lowers throughput broadly, to 4.47 Mt at $B = 200$ and 2.47 Mt at $B = 600$, because it raises the cost of every escape route at once. The policy reading is direct. Pressure on a single market mostly relocates the trade, whereas a budget spread across the transportation corridors, as in our framework, suppresses it.

6. Implications for the Theory and Practice of OR/MS

This work carries implications for both the theory and the practice of OR/MS, which we now highlight.

In terms of theory, this paper brings the spatial price equilibrium tradition, which originated with Samuelson (1952) and Takayama and Judge (1964, 1971) for licit commodity trade, into an adversarial policy setting, in which a market actor and an enforcement authority interact as players on the same level in a noncooperative game. In contrast to the bilevel, leader-follower constructs that dominate the network interdiction literature, the Nash Equilibrium formulation yields a single variational inequality whose solution delivers, simultaneously, the illicit product flows, the market-clearing supply and demand prices, and the allocation of the enforcement budget. The Lagrange analysis that produces the alternative variational inequality formulations illustrates a general pathway by which policy-relevant constraints, here the enforcement budget, can be internalized so that the resulting formulation admits closed form expressions at each iteration of a provably convergent algorithm. The construction is not specific to illicit trade. It is transferable to other two-player settings in which a resource-constrained authority confronts a network-based economic actor, whether in cybersecurity, in the policing of counterfeit goods, or in the design of sanctions. In addition, the multicriteria objective of the authorities, which weighs the disruption achieved against the monetized societal benefit of enforcement, connects network equilibrium theory with goal programming and multicriteria decision-making, and offers a template for embedding societal considerations into adversarial resource allocation problems.

In terms of practice, the framework delivers quantities that enforcement professionals can act on. The Lagrange multiplier associated with the budget constraint measures the marginal value of an additional unit of enforcement capacity, and, therefore, can support arguments for the sizing of interdiction budgets. The equilibrium budget allocations identify the transportation corridors that merit the greatest pressure, with the busier and cheaper corridors drawing more of the budget, and the societal benefit weight serves as a policy dial that smooths and differentiates the allocation. The case study also demonstrates a policy contrast of direct operational relevance: pressure on a single destination market mainly displaces illicit trade, whereas a budget spread across the transportation corridors suppresses it. Moreover, the construction of the case study dataset entirely from open sources, with every parameter traceable to a cited public anchor, provides a calibration methodology that agencies and researchers can replicate and update as the public record evolves, and the same

solution pipeline accepts primary or restricted data without any change. More broadly, this work adds to the growing literature demonstrating the value of operations research methodologies for policy (cf. Nagurney (2026)), and shows that tools honed on licit supply chains can be turned, with appropriate modeling of the adversary, to the disruption of illicit ones.

7. Summary, Conclusions, and Suggestions for Future Research

In this paper, we developed an integrated equilibrium framework for the disruption of illicit product trade. In it a criminal network and the authorities compete in a noncooperative game governed by a Nash Equilibrium. We modeled the criminal network as a spatial price equilibrium actor that operates over multiple supply markets, multiple demand markets, and multiple substitutable transportation routes, with supply and demand prices that are determined within the model. We modeled the authorities as a strategic actor that allocates a finite budget across the routes. Their objective weighs the disruption achieved against the monetized societal benefit of enforcement, and a weight α governs the balance between the two.

We formulated the governing equilibrium conditions as a variational inequality problem, and we then constructed alternative variational inequality formulations. We established monotonicity together with conditions under which the modified projection method converges, and that method yields closed form expressions for the product shipments, the budget allocations, and the Lagrange multiplier at each iteration. A realistic case study on stolen grain from the occupied territories of Ukraine, calibrated entirely from open sources, demonstrated the framework. It shows that, in the computed equilibria of the calibrated model, enforcement erodes the revenue of the criminal network and raises the prices paid by consumers, that the authorities direct more of their budget to busier and cheaper corridors, that the societal benefit weight yields a smoother and more differentiated allocation, and that disrupting the transportation corridors across the network suppresses the trade while pressure on a single destination market mainly displaces it.

Several directions for future research follow. The framework is static and deterministic, and it could be extended in two ways. One extension is a dynamic setting that captures how an interdiction campaign adjusts over time. Another is a stochastic setting in which detection is represented through an explicit seizure process. The multicriteria objective of the authorities could be broadened to include further criteria, and the societal benefit functions could be estimated from primary data as it becomes available. With suitable data, the framework could also be applied to further illicit sectors and used to inform the design and evaluation of enforcement policy.

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Appendix A. Proof of Theorem 2 and Convergence Conditions

First, note that a vector $(Q^*, b^*, \epsilon^*, \mu^*, \lambda^*)$ satisfying (27) - (30b) also satisfies VI (31). Now we establish that it also satisfies VI (18).

If one multiplies (30a) for a fixed i, j, r , by $(Q_{ijr} - Q_{ijr}^*)$ and sums the resultant over all i, j, r , one gets:

$$\sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^{n_{ij}} (\tilde{\pi}_i(Q^*) + c_{ijr}(Q_{ijr}^*) + b_{ijr}^* - \tilde{\rho}_j(Q^*)) \times (Q_{ijr} - Q_{ijr}^*) = \sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^{n_{ij}} \epsilon_{ijr}^* \times (Q_{ijr} - Q_{ijr}^*). \quad (A1)$$

Expanding the right-hand side of (A1) yields:

$$\sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^{n_{ij}} \epsilon_{ijr}^* \times Q_{ijr} - \sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^{n_{ij}} \epsilon_{ijr}^* Q_{ijr}^*. \quad (\text{A2})$$

The first term in (A2) is greater than equal to zero whereas the second term is equal to zero; therefore:

$$\sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^{n_{ij}} (\tilde{\pi}_i(Q^*) + c_{ijr}(Q_{ijr}^*) + b_{ijr}^* - \tilde{\rho}_j(Q^*)) \times (Q_{ijr} - Q_{ijr}^*) \geq 0, \quad \forall Q \in R_+^{n_P}. \quad (\text{A3})$$

Also, from (30b) it follows that, after multiplying (30b) for a fixed i, j, r , by $(b_{ijr} - b_{ijr}^*)$, summation over all i, j, r , yields:

$$\sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^{n_{ij}} \left(-Q_{ijr}^* - \alpha \frac{\partial h_{ijr}(b_{ijr}^*)}{\partial b_{ijr}} \right) \times (b_{ijr} - b_{ijr}^*) = \sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^{n_{ij}} \mu_{ijr}^* \times (b_{ijr} - b_{ijr}^*) - \sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^{n_{ij}} \lambda^* \times (b_{ijr} - b_{ijr}^*). \quad (\text{A4})$$

Expanding the terms after the equal sign in (A4) gives us:

$$\sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^{n_{ij}} \mu_{ijr}^* \times b_{ijr} - \sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^{n_{ij}} \mu_{ijr}^* b_{ijr}^* - \sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^{n_{ij}} \lambda^* \times b_{ijr} + \sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^{n_{ij}} \lambda^* \times b_{ijr}^*. \quad (\text{A5})$$

The first term in (A5) is greater than equal to zero, whereas the second term is equal to zero.

Furthermore, because of (29) associated with the budget constraint, the third and fourth terms in (A5) are equivalent to:

$$- \sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^{n_{ij}} \lambda^* \times b_{ijr} + \lambda^* B, \quad (\text{A6})$$

which we know is nonnegative. Synthesizing the above, we conclude that:

$$\sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^{n_{ij}} \left(-Q_{ijr}^* - \alpha \frac{\partial h_{ijr}(b_{ijr}^*)}{\partial b_{ijr}} \right) \times (b_{ijr} - b_{ijr}^*) \geq 0, \quad \forall b \in R_+^{n_P} \text{ satisfying (12)}. \quad (\text{A7})$$

Summing up (A3) and (A7), we obtain VI (18). $\square$

Convergence Conditions

We now identify conditions for monotonicity (cf. (37)) of the function $F(X)$ for our model (see (33) – (36)).

Note that, in forming $\langle F(X^1) - F(X^2), X^1 - X^2 \rangle$, the terms that couple the product shipments with the budget allocations, and the budget allocations with the Lagrange multiplier, cancel. What remains is the product shipment contribution, which, after algebraic simplification and expansion, yields:

$$\sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^{n_{ij}} ((\tilde{\pi}_i(Q^1) + c_{ijr}(Q^1) - \tilde{\rho}_j(Q^1)) - (\tilde{\pi}_i(Q^2) + c_{ijr}(Q^2) - \tilde{\rho}_j(Q^2))) \times (Q_{ijr}^1 - Q_{ijr}^2)$$

$$\begin{aligned}
&= \sum_{i=1}^m (\tilde{\pi}_i(Q^1) - \tilde{\pi}_i(Q^2)) \times \left(\sum_{j=1}^n \sum_{r=1}^{n_{ij}} (Q_{ijr}^1 - Q_{ijr}^2) \right) + \sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^{n_{ij}} (c_{ijr}(Q^1) - c_{ijr}(Q^2)) \times (Q_{ijr}^1 - Q_{ijr}^2) \\
&\quad - \sum_{j=1}^n (\tilde{\rho}_j(Q^1) - \tilde{\rho}_j(Q^2)) \times \left(\sum_{i=1}^m \sum_{r=1}^{n_{ij}} (Q_{ijr}^1 - Q_{ijr}^2) \right). \tag{A8}
\end{aligned}$$

By making use of (1), (2), (4b), and (5b), the right hand side of (A8) becomes:

$$\begin{aligned}
&\sum_{i=1}^m (\pi_i(s^1) - \pi_i(s^2)) \times (s_i^1 - s_i^2) + \sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^{n_{ij}} (c_{ijr}(Q^1) - c_{ijr}(Q^2)) \times (Q_{ijr}^1 - Q_{ijr}^2) \\
&\quad - \sum_{j=1}^n (\rho_j(d^1) - \rho_j(d^2)) \times (d_j^1 - d_j^2). \tag{A9}
\end{aligned}$$

If $\pi(s)$, $c(Q)$, and $-\rho(d)$ are monotone, respectively, in s , Q , and d , then (A9) is greater than equal to zero, so the product shipment contribution in (A8) is greater than equal to zero. The assumption of monotonicity of these functions is quite reasonable from an economics standpoint. The budget allocations contribute, in addition, the term

$$-\alpha \sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^{n_{ij}} \left(\frac{\partial h_{ijr}(b_{ijr}^1)}{\partial b_{ijr}} - \frac{\partial h_{ijr}(b_{ijr}^2)}{\partial b_{ijr}} \right) \times (b_{ijr}^1 - b_{ijr}^2), \tag{A10}$$

which is nonnegative because each h_{ijr} is concave and, hence, $\frac{\partial h_{ijr}}{\partial b_{ijr}}$ is nonincreasing. Combining the two contributions gives $\langle F(X^1) - F(X^2), X^1 - X^2 \rangle \geq 0$. Hence $F(X)$ is monotone.

With respect to Lipschitz continuity of $F(X)$ for our model, we can conclude that $F(X)$ is Lipschitz continuous if the supply price, unit transportation cost, and the demand price functions have bounded first order derivatives and each h_{ijr} has a bounded second order derivative, so that $\frac{\partial h_{ijr}}{\partial b_{ijr}}$ is Lipschitz continuous (see also Nagurney et al. (2002)).

Appendix B. The Empirical Setting, the Mapping to the Model, and the Construction of the Dataset from Open Sources

Since the invasion of February 2022, grain has been removed in a systematic way from occupied Ukrainian farmland. It is harvested and exported under a falsified origin, and it is often blended with grain of Russian origin so that its provenance is laundered (Faucon (2024), Melnychuk (2025), Tuliakov (2025), Shostko et al. (2025)). Independent estimates place the cumulative volume on the order of fifteen million tonnes between 2022 and August 2025, with associated profits of at least one billion U.S. dollars (Faucon (2024), Euromaidan Press (2026)). Roughly 4.8 million tonnes were taken from the occupied territories in 2023 alone (Melnychuk (2025)), and those territories yield an estimated 8 to 10 million tonnes of crops each year (United24 Media (2026)). This is not opportunistic theft. It is a vertically organized operation that controls elevators, storage, logistics, and ports (Melnychuk (2025)), which is exactly the criminal network actor of our framework. In this paper, tonnes refer to metric tonnes, the common unit of measurement in agricultural trade.

The structure of this trade maps cleanly onto the network of Figure 1 and the variables of Section 3.

1. *Supply markets* $i = 1, \dots, m$ are the occupied source regions. Kherson and Zaporizhzhia oblasts are the sources named most often, with Crimea and the occupied parts of Donbas serving as further origins and as areas for storage and consolidation (Kyiv School of Economics (2022), New Eastern Europe (2025), Office of the Director of National Intelligence (2023)). The supply price $\tilde{\pi}_i$ reflects the low cost of acquiring looted grain at the source, together with the cost of consolidating it and staging it for export.

2. *Demand markets* $j = 1, \dots, n$ are the destination markets. This set changes over time, which is itself evidence for the route substitution mechanism. Türkiye absorbed large volumes in 2022 and then drew back. Syria was a major recipient until the change of regime, after which it stopped purchasing. Egypt has since become the main destination and consolidation hub (New Eastern Europe (2025)). Other documented destinations include Syria, Israel, Iran, Lebanon, and Georgia (Office of the Director of National Intelligence (2023)), along with Azerbaijan and Spain (Radio Free Europe/Radio Liberty (2024)). The demand price $\tilde{p}_j$ is anchored to landed Black Sea wheat values, discounted to reflect the laundered character of the cargo.

3. *Transportation routes* $r = 1, \dots, n_{ij}$ between a source region and a destination are the alternative export corridors. Three corridor types recur in the investigative record. The first is the closed Crimean ports of Sevastopol and Feodosia. The second is the occupied Azov ports of Mariupol and Berdiansk. The third is the Russian mainland ports such as Novorossiysk and Temryuk, which are reached after transshipment and blending in the Kerch Strait and are relabeled as being of Russian origin, the so called Temryuk scheme, with roughly 30% of some such cargoes traced back to occupied territory (New Eastern Europe (2025), Euromaidan Press (2026), Vlasiuk (2026)). The unit transportation cost c_{ijr} on a corridor rises with the volume pushed through it, which reflects both congestion at a constrained set of ports and a higher chance of detection.

4. *The authorities* are Ukraine acting together with international partners. Their instruments are concrete and budgeted. Ukraine’s National Security and Defense Council has sanctioned vessels that loaded grain in occupied ports, freezing their assets and, through the Asset Recovery and Management Agency, seizing and auctioning the ships (Office of the President of Ukraine (2025), OCCRP (2025)). The European Union has listed shadow fleet vessels and firms that trade stolen grain in successive sanctions packages (Mayer Brown (2026)). Port state control and naval interdiction by third countries have led to detentions in several jurisdictions. The budget allocation b_{ijr} is the expected disruption cost per tonne that the authorities can impose on a corridor. It bundles sanction exposure, seizure risk, insurance and reflagging frictions, and the risk of secondary sanctions to buyers. The quantity B is the total enforcement capacity of the authorities.

A documented episode makes the equilibrium logic concrete. When Egypt declined a sanctioned vessel, the cargo was simply redirected to Türkiye, even as Egypt continued to accept other shipments, including sanctioned ones (Vlasiuk (2026)). Raising the cost on one corridor or one

market does not extinguish the flow. It moves the flow to other corridors and other markets. Our framework produces this behavior on its own, as the experiment in Subsection 5.2 illustrates.

Primary operational data on this trade are held by national authorities and, under present wartime conditions, are not available to us. We, therefore, assemble the calibration from the open record of published price assessments, freight quotations, investigative reporting, and official sanctions actions, building the price stack for a tonne of grain one component at a time, with every parameter traceable to a cited source and revisable as the public record evolves.

Table B1: Construction of the Price Stack from Open Sources (Values in USD per Metric Ton)

Component	Value used (USD/t)	Basis / open source
Licit Black Sea milling wheat, FOB	≈ 230	H1 2026 FOB Novorossiysk/Ukraine assessments (Fastmarkets (2026), S&P Global Platts (2026))
Licit landed value, CIF Egypt	≈ 252 to 264	Egypt CIF offers, H1 2026 (Fastmarkets (2026), UkrAgroConsult (2026))
Laundered grain discount	destinations priced below licit	“Black” grain reported as very cheap and sold blended or relabeled (Melnychuk (2025), New Eastern Europe (2025))
Demand choke price, Egypt / Türkiye	255 / 245	Licit landed band, net of the laundering discount, with Türkiye below Egypt
At source cost, Zaporizhzhia / Kherson	40 / 50	Looted grain ($\approx$ zero commodity cost) plus harvest, handling, storage, and staging, with Kherson higher because its logistics are more disrupted
Sea freight, Russian port corridor (R)	14 to 18	Panamax Novorossiysk to Egypt 16 to 18, with Türkiye lower (Fastmarkets (2026))
Sea freight plus war and insurance, Crimean corridor (C)	≈ 35 to 45	Small vessel Azov and East Med sailings 23 to 28 plus a shadow fleet risk premium (UkrAgroConsult (2026), Fastmarkets (2026))
Overland, field $\rightarrow$ export port	25 to 40	Long truck and rail hauls from Kherson and Zaporizhzhia to Crimean and Russian ports
Laundering and relabeling, corridor R	≈ 8	Cost of blending and relabeling the origin (New Eastern Europe (2025), Euro-maidan Press (2026))
Enforcement budget B (capacity per tonne, summed over routes)	0 to 600	Swept to bracket weak through strong enforcement (Office of the President of Ukraine (2025), OCCRP (2025), Mayer Brown (2026))

The components of Table B1 combine into the calibrated functions reported in Table 1 in the main text of the paper. Each corridor cost intercept t_{ijr} is the sum of the overland, sea freight,

and risk components above. This makes the Russian port laundering corridor ($r = 2$) consistently cheaper for the criminal network than the closed Crimean corridor ($r = 1$), which matches the documented routing preference. The demand slopes are set so that, with no enforcement ($B = 0$), total throughput is about 5.5 Mt, an undisrupted ceiling. Under moderate enforcement it falls into the range of 4 to 5 Mt per year observed in 2023 (Melnychuk (2025)). This volume check is the one free calibration choice, and even it is disciplined by the published annual estimates rather than simply assumed.

Discussion and Limitations of the Calibration

Two points temper and frame the results. First, the calibration is assembled from secondary and open sources rather than from primary operational data, which are not available under present conditions. The *magnitudes* in Tables 2 to 4 in the main text of the paper, therefore, carry the uncertainty of open source estimation, and they should be read as a defensible base case rather than as precise measurements. The *qualitative* conclusions are more secure. They are structural properties of the model and are robust to the calibration. They include the erosion of revenue and the rise in consumer prices under enforcement, the placing of more budget on busier and cheaper corridors, the preference for the laundering corridor, the smoother and more differentiated allocation that the societal benefit term induces, and the contrast between an instrument that relocates the trade and one that suppresses it. Each parameter in Table B1 is traceable to a cited public anchor, so the base case is reproducible and can be updated as the public record evolves. Should restricted or primary data become available, the same solution pipeline accepts them without any change. Second, the framework is static and deterministic. It characterizes the equilibrium that the criminal network and the authorities settle into, not the moment to moment dynamics of a particular interdiction campaign, and it treats detection through an expected disruption cost per tonne rather than through an explicit stochastic seizure process. These are natural directions for extension, and they do not affect the equilibrium analysis presented here.