Competition for Medical Supplies Under Stochastic Demand in the Covid-19 Pandemic: A Generalized Nash Equilibrium Framework

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Abstract: The Covid-19 pandemic has negatively impacted virtually all economic and social activities across the globe. Presently, since there is still no vaccine and no curative treatments for this disease, medical supplies in the form of Personal Protective Equipment and ventilators are sorely needed for healthcare workers and certain patients, respectively. The fact that this healthcare disaster is not limited in time and space has resulted in intense global competition for medical supplies. In this paper, we construct the first Generalized Nash Equilibrium model with stochastic demands to model competition among organizations at demand points for medical supplies. The model includes multiple supply points and multiple demand points, along with prices of the medical items and generalized costs associated with transportation. The theoretical constructs are provided and a Variational Equilibrium utilized to enable alternative variational inequality formulations. A qualitative analysis is presented and an algorithm proposed, along with convergence results. Illustrative examples are detailed as well as numerical examples that are solved with the implemented algorithm. The results reveal the impacts of the addition of supply points as well as of demand points on the medical item product flows. The formalism may be adapted to multiple medical items both in the near term and in the longer term (such as for vaccines).

Keywords: pandemic, Covid-19, medical supplies, game theory, networks, healthcare

1. Introduction

The Covid-19 pandemic, which was declared a pandemic by the World Health Organization on March 11, 2020 (cf. Secon, Woodward, and Mosher (2020)), has disrupted the globe, altering economic and social activities, and negatively impacting education, travel, work, and even leisure. Healthcare systems around the world continue to face great challenges as the battle against the novel coronavirus that causes this disease continues. The great need for medical items from Personal Protective Equipment (PPEs) to ventilators and, now, even convalescent plasma, has led to intense competition for medical supplies among healthcare institutions and even regions, including states, as well as nations. Although a vaccine is not yet available and a cure does not yet exist, scientific advances are adding to knowledge regarding possible treatments. However, even when a vaccine becomes available, one can expect, because of the great demands and potential insufficiency of manufacturing capacity as well as vaccine components for distribution, that competition will be a reality for the foreseeable future for even vaccines. The same holds for medicinal treatments for patients suffering from Covid-19.

Indeed, the competition for PPEs, to start, is reasonable, since it has been scientifically established that one of the most effective ways to mitigate contagion associated with the novel coronavirus (Johns Hopkins Medicine (2020)) is to use Personal Protective Equipment (PPE), for healthcare and other essential workers (see Jacobs, Richtel, and Baker (2020)) as well as those in social proximity (CDC (2020c), Herron et al. (2020)). China has historically produced half of the world's face masks, but with the coronavirus originating in Wuhan, China, the country dedicated the majority of the supply for their own citizens, whereas other countries, such as Germany, even banned the export of PPEs (Lopez (2020)). The intense competition for PPEs led to a dramatic increase in the price, with some prices rising by more than 1,000%, according to the report by The Society for Healthcare Organization Procurement Professionals (2020). For example (cf. Diaz, Sands, and Alesci (2020) and Berklan (2020)), the price of N95 masks grew from \$0.38 to \$5.75 each (a 1,413% increase); isolation protective gowns experienced a price increase from \$0.25 to \$5.00 (a 1900% increase), with the price of reusable face shields going from \$0.50 to \$4.00 (a 700% increase). According to Glenza (2020), demand and prices for PPEs, as of the end of June 2020, are dramatically increasing again across the United States as coronavirus cases continue to rise in more than half of states. Furthermore, shortages of PPEs are again being reported in the United States in July as medical and dental practices reopen and with the reopening of some schools also on the horizon.

In addition, because the coronavirus SARS-CoV-2 that causes Covid-19 may result in severe respiratory problems in certain individuals, various healthcare organizations, including hospitals, were clamoring for ventilators for their patients (Gelles and Petras (2020), Namendys-Silva (2020)). This is an example of, yet, another medical item for which there was and continues to be intense competition globally, and with limited supply availability (see Goudie et al. (2020), Kamdar (2020), Pifer (2020), SCCM (2020), Schlanger (2020)). The supply chain for ventilators is quite complex, with components sourced from different countries.

There is a growing demand for another medical product which has become critical in the health care system due to the pandemic. It is the plasma or liquid part of blood obtained from recovered Covid-19 patients, also known as convalescent plasma. It contains antibodies that can fight the virus SARS-CoV-2 causing the Covid-19 disease (Hererra (2020)). The pandemic has given rise to a rather unique competitive market for convalescent plasma as blood banks and hospitals are seeking this antibody rich serum to directly transfuse and treat critically ill patients, while pharmaceutical companies are collecting it to produce plasma derived medicine such as hyperimmune globulin that can act as a cure for Covid-19 patients (Aleccia (2020), Nagurney and Dutta (2020)). Even though the efficacy and safety of both the treatments are still under investigation worldwide, there exist studies on patients with other infectious diseases, severe acute viral respiratory infections, including those caused by related coronaviruses (SARS-CoV and MERS-CoV) that found therapeutic benefits of convalescent plasma (Mair-Jenkins et al. (2015), Winkler and Koepsell (2015), Van Griensven et al. (2016), Ferguson et al. (2020)). Both the non-profit and profit-making organizations competing in this market for convalescent plasma are taking measures to raise awareness, to generate confidence regarding the safety of the donation process, and to recruit donors (American Red Cross (2020), Grifols (2020)).

In the United States, according to the guidelines issued by the Food and Drug Administration (2020), individuals who have fully recovered from Covid-19 and have shown no symptoms for at least two weeks prior to donation are eligible to donate plasma. In addition to meeting the regular donor criteria, convalescent plasma donors need to provide documentation of prior Covid-19 diagnosis. According to Harvard Health Publishing (2020), one donor can produce sufficient plasma to treat three patients. As the world continues to wait for the availability of vaccines, and more studies show promising results of convalescent plasma therapy (Casadevall and Pirofski (2020), Duan et al. (2020), Johnson (2020)), the demand for this product and competition among hospitals, medical facilities, and pharmaceutical companies for the limited donor pool is going to become more prominent.

2. Literature Review and Our Contributions

Since the pandemic was declared only several months ago, although for many it feels like an eternity, the research is nascent, but ongoing and vigorous. Queiroz et al. (2020) presented a research agenda through a structured literature review of Covid-19 related work and supply chain research on earlier epidemics. Ivanov (2020), in turn, discussed simulation-based research focused on the potential impacts on global supply chains of the Covid-19 pandemic. Nagurney (2020) developed a supply chain network optimization model for perishable food in the Covid-19 pandemic, which included the critical labor resource. The model can be used to investigate the impacts of labor disruptions, due to illnesses, death, etc., on prices and product flows.

In this paper, we construct a competitive game theory network model for medical supplies inspired by the Covid-19 pandemic. It features salient characteristics of the realities of this pandemic in terms of competition among organizations/institutions for supplies under limited capacities globally as well as uncertain demands due to the fact that so much about this novel coronavirus remains unknown and has yet to be discovered. Since the organizations, notably, healthcare ones such as hospitals and nursing homes but also medical practices, etc., compete with one another for the limited supplies, given the prices and their associated logistical costs as well as the expected loss due to possible shortages or surpluses, the model is a Generalized Nash Equilibrium (GNE) model (cf. Debreu (1952); see also Arrow and Debreu (1954)) rather than a Nash equilibrium one (cf. Nash (1950, 1951)). To-date, there have been very few GNE models in the setting of disaster relief. Here we are dealing with a global healthcare disaster on a monumental scale, which, unlike other disasters (cf. Nagurney and Qiang (2009), Kotsireas, Nagurney, and Pardalos (2016, 2018)), is not limited in space and time. Furthermore, our model has stochastic elements.

We emphasize that in the case of Generalized Nash Equilibrium models not only do the objective functions of the players in the game depend on the strategies of the other players but the feasible sets do as well (see, e.g., Fischer, Herrich, and Schonefeld (2014)). Nagurney, Alvarez-Flores, and Soylu (2016) constructed the first disaster relief GNE model integrating financial and logistical aspects of humanitarian organizations activities and demonstrated that, because of the underlying functions, an optimization reformulation was possible. Subsequently, Nagurney et al. (2018) generalized the results to a broader class of functions and used the concept of a Variational Equilibrium (cf. Kulkarni and Shabhang (2012)), which enabled a finite-dimensional variational inequality formulation and solution procedures. However, these models were deterministic. The first stochastic GNE model for disaster relief was constructed by Nagurney et al. (2020) with each humanitarian organization facing a two-stage stochastic optimization problem and with the common, that is, the shared constraints being on the demand side and associated with relief items to be delivered to the victims at the various demand points. There were no bounds on the availability of supplies.

In this paper, in contrast, and as is vividly occurring in the Covid-19 pandemic, the supplies of the items, which in our model are medical items, are constrained. Also, the demand for the medical items is uncertain with associated penalties for shortages or surpluses, with the former expected to be much higher due to potential loss of life, increased pain and suffering, etc. The constructs that we utilize for handling the uncertain demands for medical items are based on results of Dong, Zhang, and Nagurney (2004), who introduced a supply chain equilibrium model with random demands, and on the results of Nagurney, Yu, and Qiang (2011) and Nagurney, Masoumi, and Yu (2012, 2015), who focused on optimization models in disaster relief and healthcare. Nagurney and Nagurney (2016) developed a supply chain network model for disaster relief under cost and demand uncertainty, but again, therein, there was a single-decision-maker and, hence, game theory was not needed. Mete and Zabinsky (2010) introduced a two-stage stochastic optimization model for storage and distribution of medical supplies but also considered a single decision-maker. Adida, DeLaurentis, and Lawley (2011) consider hospital stockpiling of medical supplies with a focus on shortages in the system and a common population. The authors because of their assumptions could derive closed form expressions for solutions. In our model there are multiple independent demand points and they compete for the medical item supplies with one another. Our model also include general transportation costs and each demand point is subject to uncertain demand for the medical supplies. Moreover, our model is a Generalized Nash Equilibrium model and not a Nash equilibrium model.

Muggy and Heier Stamm (2014) provide a review of game theory in humanitarian operations to that date and note that there remain many unexplored modeling research opportunities. The excellent survey article of Gutjahr and Nolz (2016) on multicriteria optimization in humanitarian aid includes references to both deterministic and stochastic models. The authors, in their future research directions section, emphasize the need for papers that consider the diverging interests of multiple and sometimes competing stake-holders. Such a research gap is addressed in this paper.

This remainder of the paper is organized as follows. In Section 3, we present the Generalized Nash Equilibrium network model for medical supplies and provide alternative variational inequality formulations of the governing equilibrium conditions. In Section 4, we discuss some qualitative properties of the model as well as the function that enters the variational inequality that we utilize to solve the numerical examples in Section 5. Section 6 summarizes our results, presents our conclusions, and also gives suggestions for future research.

3. The Generalized Nash Equilibrium Network Model for Medical Supplies Under Stochastic Demand

We consider m locations that are supply locations for the medical supplies, with a typical supply point denoted by i, and n locations that are demand points, with a typical demand point denoted by j. Note that supply points can be locations in different regions, states, or even countries. Demand points are locations where the medical supplies are needed such as hospitals, nursing homes, medical clinics, prisons, etc. The bipartite structure of the game theory problem is depicted in Figure 1. The notation for the model is given in Table 1. All vectors are column vectors.

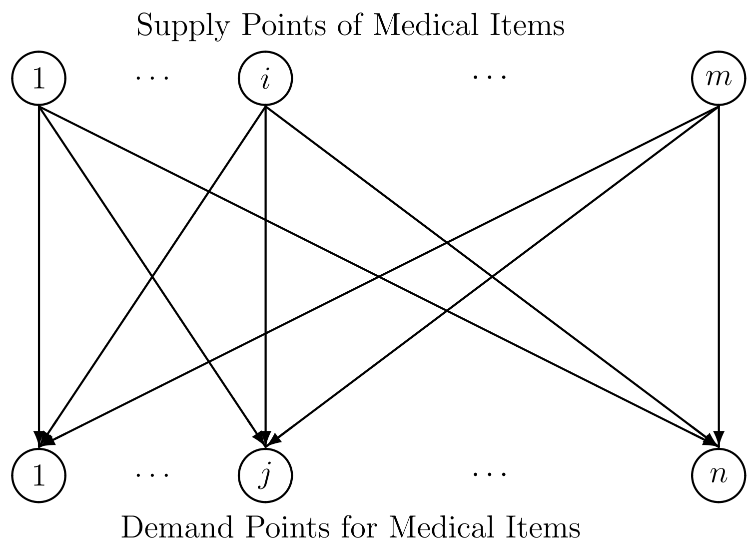

Figure 1: The Network Structure of the Competitive Game Theory Model for Medical Supplies

The demand for the medical item at the demand points is uncertain due to the unpredictability of the actual demand at the demand points. The literature contains examples of supply chain network models with uncertain demand and associated shortage and surplus penalties (see, e.g., Dong, Zhang, and Nagurney (2004), Nagurney, Yu, and Qiang (2011), Nagurney and Masoumi (2012), Nagurney, Masoumi, and Yu (2015)). Nagurney and Nagurney (2016) develop a model for disaster relief under cost and demand uncertainty. The probability distribution of demand for PPEs can be obtained using census data and/or information gathered during the pandemic disaster preparedness phase.

Before constructing the objective function, we present some needed preliminaries.

Table 1: Notation for the Medical Supply Generalized Nash Equilibrium Network Model

Notation	Definition
q_{ij}	the amount of the medical item purchased from supply location i by j. We first group all the i elements $\{q_{ij}\}$ into the vector q_j and then we group such vectors for all j into the vector $q \in R^{mn}_{+}$.
v_j	the projected demand at demand point j; $j = 1, \ldots, n$.
d_j	the actual (uncertain) demand for the medical item at demand location j; $j = 1, \ldots, n$.
Δ_j^-	the amount of shortage of the medical item at demand point j; $j = 1, \ldots, n$.
Δ_j^+	the amount of surplus of the medical item at demand point j; $j = 1, \ldots, n$.
λ_j^-	the unit penalty associated with a shortage of the the medical item at demand point j; $j = 1, \ldots, n$.
λ_j^+	the unit penalty associated with a surplus of the medical item at demand point j; $j = 1, \ldots, n$.
ρ_i	the price of the medical item at supply location i; $i = 1, \ldots, m$.
$c_{ij}(q)$	the generalized cost of transportation associated with transporting the the medical item from supply location i to demand location j, which includes the financial cost, any tariffs/taxes, time, and risk. We group all the generalized costs into the vector $c(q) \in R^{mn}$.
S_i	the nonnegative amount of the medical item available for purchase at supply location i; $i = 1, \ldots, m$.
μ_i	the nonnegative Lagrange multiplier associated with the supply constraint at supply location i. We group the Lagrange multipliers into the vector $\mu \in R^m_+$.

Since d_j denotes the actual (uncertain) demand at destination point j, we have:

$$P_j(D_j) = P_j(d_j \leq D_j) = \int_0^{D_j} \mathcal{F}_j(t)dt, \qquad j = 1, \ldots, n, \tag{1}$$

where P_j and $\mathcal{F}_j$ denote the probability distribution function, and the probability density function of demand at point j, respectively.

Recall from Table 1 that v_j is the "projected demand" for the medical item at demand point $j; j = 1, \ldots, n$. The amounts of shortage and surplus at demand point j are calculated, respectively, according to:

$$\Delta_j^- \equiv \max\{0, d_j - v_j\}, \qquad j = 1, \ldots, n, \tag{2a}$$

$$\Delta_j^+ \equiv \max\{0, v_j - d_j\}, \qquad j = 1, \ldots, n. \tag{2b}$$

The expected values of shortage and surplus at each demand point are, hence:

$$E(\Delta_j^-) = \int_{v_j}^{\infty} (t - v_j)\mathcal{F}_j(t)dt, \qquad j = 1, \ldots, n, \tag{3a}$$

$$E(\Delta_j^+) = \int_0^{v_j} (v_j - t)\mathcal{F}_j(t)dt, \qquad j = 1, \ldots, n. \tag{3b}$$

The expected penalty incurred by demand point j due to the shortage and surplus of the medical item is equal to:

$$E(\lambda_j^- \Delta_j^- \; + \; \lambda_j^+ \Delta_j^+) = \lambda_j^- E(\Delta_j^-) + \lambda_j^+ E(\Delta_j^+), \qquad j = 1, \ldots, n. \tag{4}$$

We assume that $\lambda_j^+ + \lambda_j^-$ is greater than zero, for each demand point j.

The projected demand at demand point j, v_j, is equal to the sum of flows of the medical item to j, that is:

$$v_j \equiv \sum_{i=1}^{m} q_{ij}, \qquad j = 1, \ldots, n. \tag{5}$$

Each demand location j seeks to minimize the total costs associated with the purchasing of the medical item plus the total cost of transportation plus the expected cost due to a shortage or surplus at j.

The objective function of each demand point j is, hence, given by:

$$\text{Minimize} \quad \sum_{i=1}^{m} \rho_i q_{ij} + \sum_{i=1}^{m} c_{ij}(q) + \lambda_j^- E(\Delta_j^-) + \lambda_j^+ E(\Delta_j^+) \tag{6}$$

subject to the following constraints:

$$\sum_{j=1}^{n} q_{ij} \leq S_i, \quad i = 1, \ldots, m, \tag{7}$$

$$q_{ij} \geq 0, \quad i = 1, \ldots, m. \tag{8}$$

The first term in the objective function (6) represents the purchasing costs, whereas the second term represents the generalized total transportation costs. The third term in (6) captures the expected cost due to shortage or surplus of the medical items at the demand point of the organization. We expect that the weight λ_j^- would be significantly higher than the weight λ_j^+ for each j since a shortage of the medical items can result in greater suffering and loss of life.

The constraints (7) represent common, that is, a shared constraints in that the demand locations compete for the medical items that are available for purchase at the supply locations at a maximum available supply. The constraints in (8) are the nonnegativity assumption on the medical item purchase volumes.

We assume that the total generalized transportation cost functions are continuously differentiable and convex. Note that, in our model, the transportation costs can, in general, depend upon the vector of medical item flows since there is competition for freight service provision in the pandemic.

We now present some preliminaries that allow us to express the partial derivatives of the expected total shortage and discarding costs of the medical items at the demand points only in terms of the medical item flow variables. We then prove that the third term in the Objective Function (6) is also convex.

Note that that, for each demand point j:

$$\frac{\partial E(\Delta_j^-)}{\partial q_{ij}} = \frac{\partial E(\Delta_j^-)}{\partial v_j} \times \frac{\partial v_j}{\partial q_{ij}}, \qquad \forall i. \tag{9}$$

By Leibniz's integral rule, we have:

$$\frac{\partial E(\Delta_j^-)}{\partial v_j} = \frac{\partial}{\partial v_j}\left(\int_{v_j}^{\infty}(t-v_j)\mathcal{F}_j(t)d(t)\right) = \int_{v_j}^{\infty}\frac{\partial}{\partial v_j}(t-v_j)\mathcal{F}_j(t)d(t)$$

$$= P_j(v_j) - 1, \qquad j = 1, \ldots, n. \tag{10a}$$

Therefore,

$$\frac{\partial E(\Delta_j^-)}{\partial v_j} = P_j\left(\sum_{i=1}^{m} q_{ij}\right) - 1, \qquad j = 1, \ldots, n. \tag{10b}$$

On the other hand, we have:

$$\frac{\partial v_j}{\partial q_{ij}} = \frac{\partial}{\partial q_{ij}}\sum_{l=1}^{m} q_{lj} = 1, \qquad \forall i; j = 1, \ldots, n. \tag{11}$$

Therefore, from (10b) and (11), we conclude that

$$\frac{\partial E(\Delta_j^-)}{\partial q_{ij}} = \left[P_j\left(\sum_{i=1}^{m} q_{ij}\right) - 1\right], \qquad \forall i; j = 1, \ldots, n. \tag{12}$$

Analogously, for the surplus, we have:

$$\frac{\partial E(\Delta_j^+)}{\partial q_{ij}} = \frac{\partial E(\Delta_j^+)}{\partial v_j} \times \frac{\partial v_j}{\partial q_{ij}}, \qquad \forall i; j = 1, \ldots, n, \tag{13}$$

$$\frac{\partial E(\Delta_j^+)}{\partial v_j} = \frac{\partial}{\partial v_j}\left(\int_0^{v_j}(v_j - t)\mathcal{F}_j(t)d(t)\right) = \int_0^{v_j}\frac{\partial}{\partial v_j}(v_j - t)\mathcal{F}_j(t)d(t)$$
$$= P_j(v_j), \qquad j = 1, \ldots, n. \tag{14a}$$

Thus,

$$\frac{\partial E(\Delta_j^+)}{\partial v_j} = P_j\left(\sum_{i=1}^m q_{ij}\right), \qquad j = 1, \ldots, n. \tag{14b}$$

From (14b) and (11) we have:

$$\frac{\partial E(\Delta_j^+)}{\partial q_{ij}} = P_j\left(\sum_{i=1}^m q_{ij}\right), \qquad \forall i; j = 1, \ldots, n. \tag{15}$$

Lemma 1

The expected shortage and surplus cost function $\lambda_j^- E(\Delta_j^-) + \lambda_j^+ E(\Delta_j^+)$ is convex.

Proof: We have:

$$\frac{\partial^2}{\partial {q_{ij}}^2}\left[\lambda_j^- E(\Delta_j^-) + \lambda_j^+ E(\Delta_j^+)\right] = \lambda_j^- \, \frac{\partial^2 E(\Delta_j^-)}{\partial {q_{ij}}^2} + \lambda_j^+ \, \frac{\partial^2 E(\Delta_j^+)}{\partial {q_{ij}}^2}, \qquad \forall i; j = 1, \ldots, n. \tag{16a}$$

Substituting the first order derivatives from (12) and (15) into (16a) yields:

$$\frac{\partial^2}{\partial {q_{ij}}^2}\left[\lambda_j^- E(\Delta_j^-) + \lambda_j^+ E(\Delta_j^+)\right] = \lambda_j^- \, \frac{\partial}{\partial q_{ij}}\left[P_j\left(\sum_{i=1}^m q_{ij}\right) - 1\right] \; + \; \lambda_j^+ \, \frac{\partial}{\partial q_{ij}} \, P_j\left(\sum_{i=1}^m q_{ij}\right)$$
$$= (\lambda_j^- + \lambda_j^+)\mathcal{F}_j\left(\sum_{i=1}^m q_{ij}\right) \geq 0, \qquad \forall i; j = 1, \ldots, n. \tag{16b}$$

The above inequality holds provided that $(\lambda_j^- + \lambda_j^+)$, i.e., the sum of shortage and surplus penalties, is positive. Hence, $\lambda_j^- E(\Delta_j^-) + \lambda_j^+ E(\Delta_j^+)$, and, as a consequence, the objective function in (6) is also convex. □

We refer to the objective function (6) for j as the disutility of j and denote it by $DU_j(q)$; $j = 1, \ldots, n$.

We define the feasible sets $K_j \equiv \{q_j \geq 0\}$; $j = 1, \ldots, n$. We define $K \equiv \prod_{i=1}^I K_i$. We also define the feasible set $\mathcal{S} \equiv \{q | q \, \text{satisfying} \, (7))\}$, which consists of the shared constraints.

Definition 1: Generalized Nash Equilibrium for Medical Items

A vector of medical items $q^* \in K \cap \mathcal{S}$ is a Generalized Nash Equilibrium if for each demand point j; $j = 1, \ldots, n$:

$$DU_j(q_j^*, \hat{q}_j^*) \leq DU_j(q_j, \hat{q}_j^*), \quad \forall q_j \in K_j \cap \mathcal{S}, \tag{17}$$

where $\hat{q}_j^* \equiv (q_1^*, \ldots, q_{j-1}^*, q_{j+1}^*, \ldots, q_n^*)$.

According to (17), an equilibrium is established if no demand point has any incentive to unilaterally change its vector of medical item purchases/shipments. Observe that in our model not only does the objective function of a demand point depend not only on the vector of strategies of its own strategies and on those of the other demand points, but the feasible set does as well. Hence, this model is not a Nash (1950, 1951) model, but, rather, it is a Generalized Nash Equilibrium model. Our model captures the reality of the intense competitive landscape in the Covid-19 pandemic.

Here, we utilize the concept of a *Variational Equilibrium*, which allows us to formulate the above GNE conditions as the solution to a finite-dimensional variational inequality problem. Hence, rather than making use of quasi-variational inequalities, for which the algorithms are not as advanced, we can apply variational inequality algorithms to solve numerically the model. Indeed, as emphasized in Nagurney, Yu, and Besik (2017), in Nagurney, Salarpour, and Daniele (2019), and in Nagurney et al. (2020), we can define a Variational Equilibrium which is a refinement and a specific type of GNE (cf. Kulkarni and Shabhang (2012)) that enables a variational inequality formulation.

We define the feasible set $\mathcal{K} \equiv K \cap \mathcal{S}$.

Definition 2: Variational Equilibrium

A vector of medical items $q^* \in \mathcal{K}$ is a Variational Equilibrium of the above Generalized Nash Equilibrium problem if it is a solution to the following variational inequality:

$$\sum_{j=1}^{n} \sum_{i=1}^{m} \frac{\partial DU_j(q^*)}{q_{ij}} \times (q_{ij} - q_{ij}^*) \geq 0, \quad \forall q \in \mathcal{K}, \tag{18}$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in mn-dimensional Euclidean space.

In expanded form, the variational inequality in (18) is: determine $q^* \in \mathcal{K}$ such that

$$\sum_{j=1}^{n} \sum_{i=1}^{m} \left[\rho_i + \sum_{l=1}^{m} \frac{\partial c_{lj}(q^*)}{\partial q_{ij}} + \lambda_j^+ P_j(\sum_{l=1}^{m} q_{lj}^*) - \lambda_j^- (1 - P_j(\sum_{l=1}^{m} q_{lj}^*)) \right] \times \left[q_{ij} - q_{ij}^* \right] \geq 0, \quad \forall q \in \mathcal{K}. \tag{19}$$

Note that the variational equilibrium guarantees that the Lagrange multipliers associated with the common constraints are the same for all the demand points. This feature yields and elegant fairness and equity interpretation, which is very relevant during this pandemic.

We now put variational inequality (19) into standard form. Recall (cf. Nagurney (1999)) that the finite-dimensional variational inequality problem, VI($F, \mathcal{K}$), is to determine a vector $X^* \in \mathcal{K} \subset R^N$, such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{20}$$

where F is a given continuous function from $\mathcal{K}$ to R^N, and $\mathcal{K}$ is a given closed, convex set.

We let $X \equiv q$ and $F(X)$ be the vector with elements: $\{\frac{\partial DU_j(q^*)}{q_{ij}}\}$, $\forall j, i$ with $\mathcal{K}$ as originally defined and $N = mn$. Then, clearly, variational inequality (19) can be put into standard form (20), under our assumptions.

Also it is worth noting that existence of a solution q^* to variational inequality (19) is guaranteed under the classical theory (see Kinderlehrer and Stampacchia (1980)) since the function that enters the variational inequality is continuous and the feasible set $\mathcal{K}$ is not only convex but also compact because the supplies of the medical items are bounded. Hence, the following theorem is immediate.

Theorem 1: Existence

A solution to variational inequality (19) exists.

We now provide an alternative variational inequality to (18) (and (19)). We associate a nonnegative Lagrange multiplier μ_i with constraint (7), for each supply location $i = 1, \ldots, m$. We group all the Lagrange multipliers into the vector $\mu \in R^m_+$. We define the feasible set $\mathcal{K}^2 \equiv \{(q, \mu) | q \geq 0, \mu \geq 0\}$.

Then, using arguments as in Nagurney, Salarpour, and Daniele (2019), an alternative variational inequality for (19) is: determine $(q^*, \mu^*) \in \mathcal{K}^2$ such that

$$\sum_{j=1}^{n}\sum_{i=1}^{m}\left[\rho_i + \sum_{l=1}^{m}\frac{\partial c_{lj}(q^*)}{\partial q_{ij}} + \lambda_j^+ P_j(\sum_{l=1}^{m} q^*_{lj}) - \lambda_j^-(1 - P_j(\sum_{l=1}^{m} q^*_{lj}) + \mu^*_i\right] \times [q_{ij} - q^*_{ij}]$$

$$+ \sum_{i=1}^{m}\left[S_i - \sum_{j=1}^{n} q^*_{ij}\right] \times [\mu_i - \mu^*_i] \geq 0, \quad \forall (q, \mu) \in \mathcal{K}^2. \tag{21}$$

Variational inequality (21) can also be put into standard form (20) if we define $X \equiv (q, \mu)$ and $F(X) \equiv (F^1(X), F^2(X))$ where $F^1(X)$ has as its (i,j)-th component: $\rho_i + \sum_{l=1}^{m} \frac{\partial c_{lj}(q)}{\partial q_{ij}} + \lambda_j^+ P_j(\sum_{l=1}^{m} q_{lj}) - \lambda_j^-(1 - P_j(\sum_{l=1}^{m} q_{lj}) + \mu_i$; $i = 1, \ldots, m$; $j = 1, \ldots, n$, and the i-th component of $F^2(X)$ is $S_i - \sum_{j=1}^{n} q_{ij}$, for $i = 1, \ldots, m$. Furthermore, $\mathcal{K} \equiv \mathcal{K}^2$ and $N = mn + m$.

3.1 Illustrative Examples

In this Subsection, we present three small numerical examples for illustrative purposes. These examples are inspired by the Covid-19 pandemic and associated challenges in procuring N95 face masks, which are among the most needed medical products in dealing with this healthcare disaster. We emphasize that the equilibrium Lagrange multipliers provide valuable information since they represent the shadow prices of the supply constraints. In particular, if an equilibrium Lagrange multiplier is positive then this is the amount of the cost (or the loss) that could be saved with an extra unit of the supply of the medical item.

Illustrative Example 1: One Supply Point and One Demand Point

In this example there is a single supply point and a single demand point, as depicted in Figure 2.

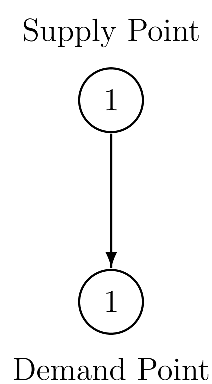

Figure 2: Network Topology for Illustrative Example 1

The supply point sells 20-pack N95 masks in the form of large bulks of 1000 packs each; therefore, one unit of item flow from the supply point to a demand point, q_{ij}, represents 1000 of 20-pack N95 masks. The demand at the demand point is uniformly distributed between 100 and 1,000 of large bulks. To determine the price of a unit item flow, ρ_i at supply point i, we assume that the price of each 20-pack N95 mask during the pandemic is \$25, so that the purchase price of each large bulk is $\rho_1 = 25,000$. Although a face mask is not, under normal conditions, an expensive product, it has been proved to essential in reducing the spread of the virus. Based on this, we assume that, for every 2,000 people who do not use the face mask, one person would die due to the disease. Although it is not easy to value people's lives, we assume a $200,000 equivalent for each loss. As a result, the penalty, λ_1^-, on the shortage of one item flow, which is equivalent to 20,000 N95 masks, is set at $2,000,000. Also, since the supply chain has been severely disrupted at the time of the declaration of the pandemic, overloading can cause many problems in transportation and processing at entry points for countries. To prevent this, we also consider a penalty of $\lambda_1^+ = 100,000$ on surplus item flows. The data for this example is as follows:

$$\rho_1 = 25,000, \quad S_1 = 1,000, \quad c_{11}(q) = q_{11}^2 + 3q_{11}, \quad \lambda_1^- = 2,000,000, \quad \lambda_1^+ = 100,000.$$

We can rewrite variational inequality (21) for this example as: determine $(q^*, \mu^*) \in \mathcal{K}^2$ such that:

$$\left[25000 + 2q_{11}^* + 3 + 100000(\frac{q_{11}^* - 100}{900}) - 2000000(\frac{1000 - q_{11}^*}{900}) + \mu_1^*\right] \times [q_{11} - q_{11}^*]$$

$$+ [1000 - q_{11}^*] \times [\mu_1 - \mu_1^*] \geq 0, \quad \forall (q, \mu) \in \mathcal{K}^2$$

The solution to the above variational inequality, which we obtained analytically, is:

$$q_{11}^* = 945.62, \quad \mu_1^* = 0.00.$$

Observe that the organization at the demand point procures a huge number of masks because of the great importance of PPEs in preventing the further spread of the virus and the potential damage that could be caused by an insufficient number of N95 face masks. The projected demand value $v_1 = 945.62$ lies between the lower and the upper bounds of the uniform distribution range. Note that the projected demand is very close to the upper bound. The decision-makers at the organization at the demand point are aware of the importance of the masks and have assigned a much larger penalty on a shortage as compared to the surplus penalty. The disutility of the organization in this logistical operation is equal to 67,543,534.04.

Illustrative Example 2: Two Supply Points and One Demand Point

In the second illustrative example, a new supply point has been added to the supply chain network, as depicted in Figure 3.

Hence, now, the decision-makers at the demand point have two options for procuring the face masks. The new supply point offers masks for less than half the price of the other

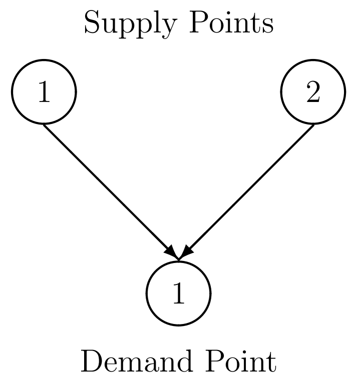

Figure 3: Network Topology for Illustrative Example 2

supply point, but its supply capacity is half that of the previous one. Also, the generalized transportation cost rate from the origin of the N95 masks of the new supply point to the demand point is higher than the rate of the other supply point. The data on the new supply point are as follows.

$$\rho_2 = 10,000, \quad S_2 = 500, \quad c_{21}(q) = 2q_{21}^2 + 4q_{21}.$$

Variational inequality (21) can be rewritten as follows for this example: determine $(q^*, \mu^*) \in \mathcal{K}^2$ such that

$$\left[25000 + 2q_{11}^* + 3 + 100000(\frac{q_{11}^* + q_{21}^* - 100}{900}) - 2000000(\frac{1000 - q_{11}^* - q_{21}^*}{900}) + \mu_1^*\right] \times [q_{11} - q_{11}^*]$$

$$+ \left[10000 + 4q_{21}^* + 4 + 100000(\frac{q_{11}^* + q_{21}^* - 100}{900}) - 2000000(\frac{1000 - q_{11}^* - q_{21}^*}{900}) + \mu_2^*\right] \times [q_{21} - q_{21}^*]$$

$$+ [1000 - q_{11}^*] \times [\mu_1 - \mu_1^*] + [500 - q_{21}^*] \times [\mu_2 - \mu_2^*] \geq 0, \quad \forall (q, \mu) \in \mathcal{K}^2.$$

The solution to the above variational inequality, obtained analytically, is:

$$q_{11}^* = 446.05, \quad q_{21}^* = 500.00, \quad \mu_1^* = 0.00, \quad \mu_2^* = 13,891.80.$$

Observe that, with the addition of a new supply point, the decision-makers' strategy has changed. Since the price of the product offered by the new supply point is much lower than that at the first supply point, the decision-makers purchase more items from supply point 2, despite the fact that the generalized transportation cost to the demand point from supply point 2 is higher than that from supply point 1. However, the supply capacity of the new supply point is half that of the first supply point, and we see that all its capacity has been used. Therefore, the associated equilibrium Lagrange multiplier is positive. Again, the projected demand falls between the lower and the upper bounds of the uniform distribution and is closer to the upper bound for the same reason as in the previous example. But, now, with greater flexibility in the supply chain due to the addition of a new supply point, the disutility of the organization at the demand point has declined, dropping to 59,860,548.75.

Illustrative Example 3: Two Supply Points and Two Demand Points

This example is constructed from the previous examples, with the difference that now there are two demand points trying to procure N95 masks and competing over limited supplies; see Figure 4.

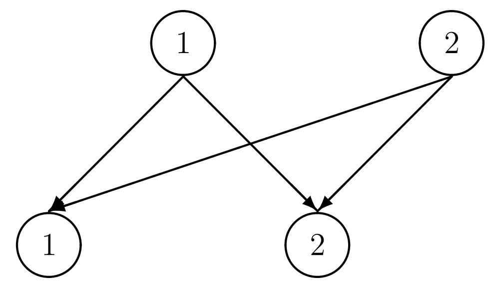

Figure 4: Network Topology for Illustrative Example 3

The demand for the new demand point is uniformly distributed between 100 and 500. The generalized transportation cost functions and the penalty coefficients associated with the second demand point are:

$$c_{12}(q) = 2q_{12}^2 + 3q_{12}, \quad c_{22}(q) = 3q_{22}^2 + 4q_{22}, \quad \lambda_2^- = 2,000,000, \quad \lambda_2^+ = 100,000.$$

Variational inequality (21) for this example is as below: determine $(q^*, \mu^*) \in \mathcal{K}^2$ such that

$$\left[25000 + 2q_{11}^* + 3 + 100000(\frac{q_{11}^* + q_{21}^* - 100}{900}) - 2000000(\frac{1000 - q_{11}^* - q_{21}^*}{900}) + \mu_1^*\right] \times [q_{11} - q_{11}^*]$$

$$+ \left[10000 + 4q_{21}^* + 4 + 100000(\frac{q_{11}^* + q_{21}^* - 100}{900}) - 2000000(\frac{1000 - q_{11}^* - q_{21}^*}{900}) + \mu_2^*\right] \times [q_{21} - q_{21}^*]$$

$$+ \left[25000 + 4q_{12}^* + 3 + 100000(\frac{q_{12}^* + q_{22}^* - 100}{400}) - 2000000(\frac{500 - q_{12}^* - q_{22}^*}{400}) + \mu_1^*\right] \times [q_{12} - q_{12}^*]$$

$$+ \left[10000 + 6q_{22}^* + 4 + 100000(\frac{q_{12}^* + q_{22}^* - 100}{400}) - 2000000(\frac{500 - q_{12}^* - q_{22}^*}{400}) + \mu_2^*\right] \times [q_{22} - q_{22}^*]$$

$$+ [1000 - q_{11}^*] \times [\mu_1 - \mu_1^*] + [500 - q_{21}^*] \times [\mu_2 - \mu_2^*] \geq 0, \quad \forall (q, \mu) \in \mathcal{K}^2.$$

The solution to this variational inequality, again, obtained analytically, is:

$$q_{11}^* = 634.14, \quad q_{21}^* = 311.74, \quad q_{12}^* = 287.71, \quad q_{22}^* = 188.26, \quad \mu_1^* = 0.00, \quad \mu_2^* = 15,020.30.$$

With the addition of another demand point, there is increased competition for the valuable N95 masks. The strategies of the organization at demand point 1 have changed as compared to the previous example. It can be seen that the full capacity of supply point 2 has not been assigned to demand point 1, since the organization at demand point 1 now competed with the organization at demand point 2. As a result, the major part of the demand point 1's procurement of the N95 masks is from supply point 1 that has a larger capacity as compared to supply point 2. And, similar to the previous example, the equilibrium Lagrange multiplier associated with the supply capacity of supply point 2 is positive since it has sold all its available supply of N95 masks, while the other supply point has not exhausted its capacity. Both demand points receive a large amount of face masks and their projected demands lie in their respective uniform probability distribution range. Both projected demands are closer to the upper bound since the penalty on shortage is much higher than the penalty on surplus. The addition of a new demand point to the competition has changed the strategies of the organization at demand point 1, and we can see the impact on its disutility. Its disutility has now increased to 62,580,546.57. The disutility of the second demand point is 28,457,845.74.

4. Qualitative Properties and the Algorithm

We now discuss some properties of the model, specifically, those that guarantee that the conditions for convergence of the modified projection method (cf. Korpelevich (1977) and Nagurney (1999)) that we use to compute solutions to numerical examples in this next section are met. The algorithm is guaranteed to converge to a solution of variational inequality (21) if the function $F(X)$ that enters the variational inequality is monotone and Lipschitz continuous, and that a solution exists. It was recently applied to compute solutions to a stochastic game theory model for disaster relief by Nagurney et al. (2020).

Recall that the function $F(X)$ is said to be monotone, if

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle \geq 0, \quad \forall X^1, X^2 \in \mathcal{K}. \tag{22}$$

Theorem 2: Monotonicity

The function $F(X)$ is monotone, for all $X \in \mathcal{K}$, if all the generalized transportation cost functions c_{ij}, $i = 1, \ldots, m$; $j = 1, \ldots, n$, are convex.

Proof: $\forall X^1, X^2 \in \mathcal{K}$, let $v_j^1 = \sum_{i=1}^m q_{ij}^1$ and $v_j^2 = \sum_{i=1}^m q_{ij}^2$.

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle$$

$$= \sum_{j=1}^n \sum_{i=1}^m \left[\sum_{l=1}^m \frac{\partial c_{lj}(q^1)}{\partial q_{ij}} - \sum_{l=1}^m \frac{\partial c_{lj}(q^2)}{\partial q_{ij}} \right] \times (q_{ij}^1 - q_{ij}^2) \tag{23}$$

$$+ \sum_{j=1}^n (\lambda_j^+ + \lambda_j^-) \times (P_j(v_j^1) - P_j(v_j^2)) \times (v_j^1 - v_j^2). \tag{24}$$

Given the convexity of the generalized transportation cost functions, equation (23) is greater or equal to zero. Since a probability function P_j, $\forall j$, is an increasing function, the expression in equation (24) is greater or equal to zero. Hence, $F(X)$ is monotone. □

If the conditions in Theorem 1 are slightly strengthened so that the the vector function that enters into the variational inequality problem (21) is strictly monotone, then its solution is unique (see, e.g., Nagurney (1999)).

Theorem 3: Uniqueness

The function $F(X)$ is strictly monotone for all $X \in \mathcal{K}$, if all the generalized transportation cost functions c_{ij}; $i = 1, \ldots, m$; $j = 1, \ldots, n$, are strictly convex. Then the variational inequality (21) has a unique solution in $\mathcal{K}$

Theorem 4: Lipschitz Continuity

If the generalized transportation cost functions c_{ij}, for all i and j, have bounded second order partial derivatives, then the function $F(X)$ that enters the variational inequality problem (21) is Lipschitz continuous; that is, there exists a constant $L > 0$, known as the Lipschitz constant, such that

$$\|F(X^1) - F(X^2)\| \leq L\|X^1 - X^2\|, \quad \forall X^1, X^2 \in \mathcal{K}. \tag{25}$$

Proof: Since each probability function P_j; $j = 1, \ldots, n$, is always less than or equal to 1, the result is direct by applying a mid-value theorem from calculus to the vector function $F(X)$

that enters the variational inequality problem (21). See also Nagurney and Zhang (1996) and Nagurney (1999). □

The iterative steps of the modified projection method, with τ denoting an iteration counter, are as follows:

The Modified Projection Method

Step 0: Initialization

Initialize with $X^0 \in \mathcal{K}$. Set the iteration counter $\tau := 1$ and let β be a scalar such that $0 < \beta \leq \frac{1}{L}$, where L is the Lipschitz constant.

Step 1: Computation

Compute $\bar{X}^\tau$ by solving the variational inequality subproblem:

$$\langle \bar{X}^\tau + \beta F(X^{\tau-1}) - X^{\tau-1}, X - \bar{X}^\tau \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{26}$$

Step 2: Adaptation

Compute X^τ by solving the variational inequality subproblem:

$$\langle X^\tau + \beta F(\bar{X}^\tau) - X^{\tau-1}, X - X^\tau \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{27}$$

Step 3: Convergence Verification

If $|X^\tau - X^{\tau-1}| \leq \epsilon$, with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $\tau := \tau + 1$ and go to Step 1.

The modified projection method for the model governed by variational inequality (21) yields closed form expressions for the medical item flows and for the Lagrange multipliers in both Steps (26) and (27). This is a nice features for computer implementation.

Theorem 5: Convergence

Assume that the function that enters the variational inequality (21) (or (19)) has at least one solution and all the generalized transportation cost functions are convex, then the modified projection method described above converges to the solution of the variational inequality (21) (or (19)).

Proof: According to Korpelevich (1977), the modified projection method converges to the solution of the variational inequality problem of the form (20), provided that the function F that enters the variational inequality is monotone and Lipschitz continuous and that a solution exists. Existence of a solution follows from Theorem 1. Monotonicity follows Theorem 2. Lipschitz continuity, in turn, follows from Theorem 4. $\square$

We now provide the explicit formulae for the medical item flows and the Lagrange multipliers at iteration τ for Step 1. The analogues for Step 2 can be easily derived accordingly.

Specifically, we have:

Explicit Formula for the Medical Item Flow for Each i, j at Iteration τ of Step 1

Determine $\bar{q}_{ij}^{\tau}$ for each i, j at Step 1 iteration τ according to:

$$\bar{q}_{ij}^{\tau} = \max\{0, q_{ij}^{\tau-1} + \beta(-\rho_i - \sum_{l=1}^{m} \frac{\partial c_{lj}(q^{\tau-1})}{\partial q_{ij}} - \lambda_j^+ P_j(\sum_{l=1}^{m} q_{lj}^{\tau-1}) + \lambda_j^- (1 - P_j(\sum_{l=1}^{m} q_{lj}^{\tau-1})) - \mu_i^{\tau-1})\}. \tag{28}$$

Explicit Formula for the Lagrange Multiplier for Each i at Iteration τ of Step 1

Determine $\bar{\mu}_i^{\tau}$ for each i at Step 1 iteration τ according to:

$$\bar{\mu}_i^{\tau} = \max\{0, \mu_i^{\tau-1} + \beta(-S_i + \sum_{j=1}^{n} q_{ij}^{\tau-1})\}. \tag{29}$$

5. Numerical Examples

In this Section, we apply the modified projection method to compute solutions to numerical examples. The algorithm was implemented in FORTRAN and the computer system used was a Linux system at the University of Massachusetts Amherst. We initialized the algorithm by setting all the medical item flows and the Lagrange multipliers to 0.00. The convergence condition for all the examples was that the absolute value of two successive variable iterates was less than or equal to 10^{-8}. The β parameter in the modified projection method was set to: .1.

The examples are of increasing complexity. We report all the input and the output data for transparency purposes and reproducibility.

In this Section, we focus on procurement of N95 masks but in the scenario of increasing demand among smaller healthcare organizations in the form of medical practices. With the pandemic in the US continuing in the summer of 2020 and with the opening of schools and universities to a certain degree on the horizon, there are increased pressures on the procurement of PPEs. In particular, we reference the following news article by O'Connell (2020); see also Wan (2020).

Numerical Example 1: One Supply Point and One Demand Point

In the first numerical example, for which we computed the solution using the code that we implemented, there is a single supply point and a single demand point as in the network in Figure 2. The q_{ij}s are in units since these medical practices are small relative to hospitals, etc. We assumed a uniform probability distribution in the range $[100, 1000]$ at the demand point. The additional data for this example are:

$$\rho_1 = 2, \quad S_1 = 1,000, \quad c_{11}(q) = .005q_{11}^2 + .01q_{11}, \quad \lambda_1^- = 1,000, \quad \lambda_1^+ = 10.$$

The computed equilibrium solution is:

$$q_{11}^* = 980.56, \quad \mu_1^* = 0.00.$$

The projected demand of 980.56 is close to the upper bound of the probability distribution at the demand point.

Numerical Example 2: One Supply Point and Two Demand Points

This example has the same data as that in Numerical Example 1 except for added data for the second demand point. The network topology is as in Figure 5.

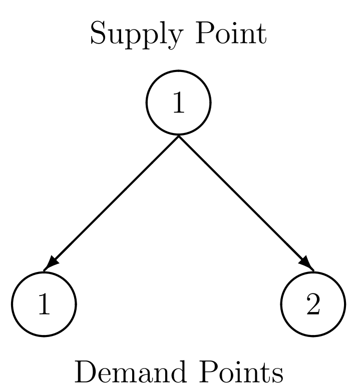

Figure 5: Network Topology for Numerical Example 2

The probability distribution at the second demand point had the same lower and upper bounds as in the first demand point.

This example has the same data as Numerical Example 1 except for the following additional data for the new demand point:

$$c_{12}(q) = .01q_{12}^2 + .02, \quad \lambda_2^- = 1000, \quad \lambda_2^+ = 10.$$

The network topology for this example is as in Figure 5.

The modified projection method converged to the following equilibrium solution:

$$q_{11}^* = 502.20, \quad q_{12}^* = 497.80, \quad \mu_1^* = 541.61.$$

With increased competition for N95 mask supplies from the second demand point, the first demand point has a large reduction in procured supplies, as compared to the volume received in Numerical Example 1. The available supply of 1,000 N95 masks is exhausted between the two demand points, and, hence, the associated Lagrange multiplier μ_1^* is positive. The equilibrium conditions hold with excellent accuracy.

Numerical Example 3: Two Supply Points and Two Demand Points

In Numerical Example 3, we considered the impacts of the addition of a second supply point to Numerical Example 2. The topology was as in Figure 4. Hence, the data are as above with the following additions:

$$S_2 = 500, \quad \rho_2 = 3, \quad c_{21}(q) = .015q_{21}^2 + .03, \quad c_{22}(q) = .02q_{22}^2 + .04q_{22}.$$

The modified projection method yielded the following equilibrium solution:

$$q_{11}^* = 526.31, \quad q_{12}^* = 473.69, \quad q_{21}^* = 225.57, \quad q_{22}^* = 274.43, \quad \mu_1^* = 261.17, \quad \mu_2^* = 258.65.$$

With the addition of a new supply point, both demand points gain significantly in terms of the volume of N95 that each procures and the supplies at each supply point are fully sold out. As a result, both equilibrium Lagrange multipliers are positive.

Numerical Example 4: Two Supply Points and Three Demand Points

Numerical Example 4 was constructed from Numerical Example 3 with demand point 3 added, as in Figure 6.

Numerical Example 4 has the same data as Numerical Example 3 but with the addition of data for demand point 3 as follows:

$$c_{13}(q) = .01q_{13}^2 + .02q_{13}, \quad c_{23}(q) = .015q_{23}^2 + .03q_{23}, \quad \lambda_3^- = 1000, \quad \lambda_3^+ = 10.$$

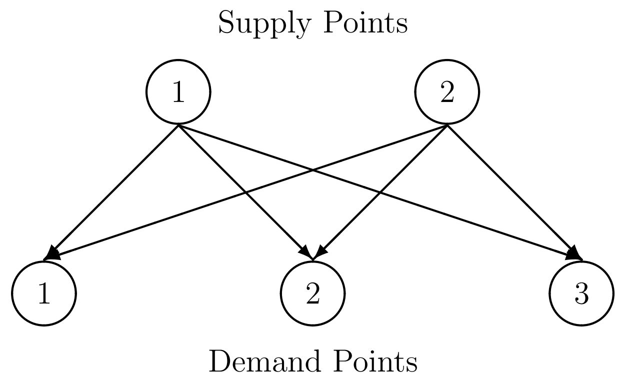

Figure 6: Network Topology for Numerical Example 4

The probability distribution for the N95 masks associated with demand point 3 is uniform with a lower bound of 200 and an upper bound of 1000.

The modified projection method yielded the following equilibrium solution:

$$q_{11}^* = 360.11, \quad q_{12}^* = 318.83, \quad q_{13}^* = 321.06,$$

$$q_{21}^* = 122.29, \quad q_{22}^* = 161.10, \quad q_{23}^* = 216.62, \quad \mu_1^* = 565.25, \quad \mu_2^* = 564.16.$$

Observe that with increasing competition for the N95 masks with another demand point, both demand points 1 and 2 experience decreases in procurement of supplies. The two supply points again fully sell out of their N95 masks and the associated equilibrium Lagrange multipliers are both positive.

Numerical Example 5: Two Supply Points and Four Demand Points

In the final example, Numerical Example 5, we consider yet another demand point addition to the demand points in Numerical Example 4. Please refer to Figure 7. Smaller medical practices are increasingly concerned about being able to secure the much needed PPEs to protect the health of their employees and the viability of their practices.

The data for this example is as the data for Numerical Example 4, and the probability distribution structure for the demand at demand point is the same, with the following additional data for the new demand point 4:

$$c_{14}(q) = .015q_{14}^2 + .03q_{14}, \quad c_{24}(q) = .025q_{24}^2 + .05q_{24}, \quad \lambda_4^- = 1000, \quad \lambda_4^+ = 10.$$

The modified projection method now yielded the following equilibrium solution:

$$q_{11}^* = 260.73, \quad q_{12}^* = 229.36, \quad q_{13}^* = 251.22, \quad q_{14}^* = 258.69,$$

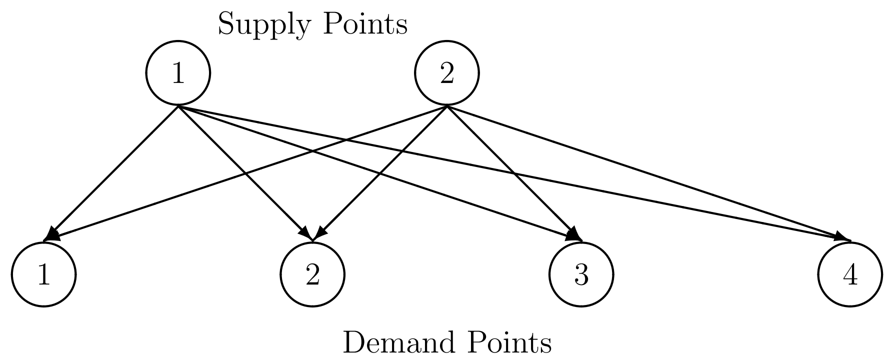

Figure 7: Network Topology for Numerical Example 5

$$q_{21}^* = 79.57, \quad q_{22}^* = 109.17, \quad q_{23}^* = 160.46, \quad q_{24}^* = 150.81, \quad \mu_1^* = 725.71, \quad \mu_2^* = 724.91.$$

Again, the equilibrium conditions hold with excellent accuracy for this example, as was the case for all the other numerical example computed solutions. The suppliers of the N95 sell out their supplies. However, the demand points lose in term of supply procurement for their organizations with the increased demand and competition from and yet another demand point.

We emphasize that although the above numerical examples are stylized, our mathematical, computational framework enables the investigation of numerous scenarios and sensitivity analyses. For example, one can consider the impacts of the removal of supply points and/or demand points; the addition of supply and/or demand points; changes in the prices of the medical item under study, as well as changes to the generalized transportation costs. Furthermore, one can investigate the impacts of alternative probability distribution functions.

The above numerical results are consistent with what one can expect to observe in reality in terms of how organizations would procure critical medical supplies such as N95 masks under demand unpredictability and competition. The findings confirm that more supply points with sufficient supplies are needed to ensure that organizations are not deprived of critical supplies due to competition. As a result of this competition and limited local availability; in particular in the case of supplies such as masks and even coronavirus test kits, we are seeing several countries now setting up local production sites (Bradsher (2020)).

6. Summary and Conclusions and Suggestions for Future Research

Medical supplies are essential in the battle against the coronavirus that causes Covid-19. The demand for medical supplies globally from PPEs to ventilators has created an intense competition. PPEs are essential in protecting healthcare workers and it now has been recognized that masks can reduce the transmission of the novel coronavirus. Ventilators, on the other hand, can be life-saving for patients with severe cases of Covid-19 and convalescent plasma has become a possible interim treatment. With the pandemic, supply chains, including those for medical items, have been disrupted adding to the intense competition for such supplies.

The Covid-19 pandemic is not limited to space or time and, therefore, there have been many shortages of medical items. In order to elucidate the competition for such supplies in this pandemic, we developed a Generalized Nash Equilibrium model that consists of multiple supply points for the medical items and multiple demand points with the demand at the latter being stochastic. Using some recently introduced machinery we were able to provide alternative variational inequality formulations of the equilibrium conditions. We then utilized the variational inequality with not only medical item product flows as variables but also the Lagrange multipliers associated with the supply capacities of the medical items at the supply point. We studied the model both qualitatively and quantitatively - the latter through illustrative examples that we were able to solve analytically as well as via numerical examples for which we utilized an algorithm that we proposed. The algorithm, for which we also provided convergence results, resolved the variational inequality problem into a series of subproblems for which closed form expressions in the variables were identified.

This work adds to the literature on game theory models for disaster relief with the specific features of the Covid-19 pandemic. It can be applied to study the network economics of a spectrum of medical items, both in the near term, and in the longer term, as when vaccines as well as medicines for Covid-19 become available. We also highlight possible extensions of this work. For example, the model is amenable to extension to multiple medical items. It would also be very interesting to have the supplies be elastic, that is, as a function of price. We leave such research endeavors for the future.

Acknowledgments

This paper is dedicated to all essential workers, including: healthcare workers, first responders, freight service providers, grocery store workers, farmers, and educators, who sacrificed so much in the Covid-19 pandemic. Your dedication and courage have graced our planet and we salute you. We also remember all those who perished because of insufficient supplies of PPEs.

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