Competition for Blood Donations

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Abstract

In this paper, we focus on the blood services industry in the US, which is undergoing major changes, including a rise in competition, and we develop a novel game theory model for blood donations among blood service organizations in which the organizations seek to maximize their transaction utilities and compete on the quality of service that they provide at the blood collection sites in different regions. The governing equilibrium concept is that of Nash Equilibrium. We formulate the equilibrium as a variational inequality problem and prove the existence of the equilibrium quality level service pattern. We also provide conditions for uniqueness and present a Lagrange analysis and interpretation associated with the lower and upper bounds on the quality service levels. An algorithm with nice features for computations is proposed and then applied to numerical examples of increasing complexity. The results demonstrate that enhanced competition can improve the quality service level and that blood service organizations can also benefit from having collection sites in multiple regions.

Keywords: game theory, blood supply chains, competition for donations, variational inequalities, healthcare
1. Introduction

A multi-billion dollar industry has evolved out of the demand for and supply of blood, with the global market for blood products projected to reach $41.9 billion by 2020 (Global Industry Analysts, Inc. (2015)). The United States constitutes the largest market for blood products in the world, with approximately 21 million blood components transfused every year in the nation (cf. American Red Cross (2016)). Blood transfusions are integral parts of any major surgeries. The number of units transfused can vary from one pint to dozens of pints for critical surgeries such as organ transplants. In addition, blood transfusions are used to treat certain diseases, including, for example, malaria-related anemia (see Obonyo et al. (1998)). Moreover, blood transfusions are often needed in the case of accidents as well as natural disasters because of injuries sustained by the victims. Despite advancements in technology enabling minimally invasive surgeries the aging populations of many countries ensure a steady demand for blood. Furthermore, since blood is a perishable product, with platelets lasting 5 days and with red blood cells having a lifetime of 42 days, regular replenishment of the blood supply of regions and nations is necessary.

A unique feature of the blood banking industry is that the supply of the product is solely dependent on donations by individuals to the blood banks and blood service organizations collecting blood, which, for the most part, are nonprofits. Blood can neither be manufactured nor substituted by any other product. On average, 13.6 million whole blood and red blood cell units are collected in the United States per year. The American Red Cross (2015) reports that the number of donors in US in a year is approximately 6.8 million. An estimated 38% of the US population is eligible to donate blood at any given time. However, less than 10% of that eligible population actually donates blood each year. In Britain, according to Gregory (2015), just 4% of the residents regularly donate blood with the National Health Service stating that it can no longer guarantee sufficient supplies. In New Zealand, also about 4% of the population donates blood (see New Zealand Blood (2016)). According to the World Health Organization (2010), blood donation by 1% of the population is generally the minimum needed to meet a nation’s most basic requirements for blood; the requirements are higher in countries with more advanced health care systems. However, the average donation rate is 15 times lower in developing countries than in developed countries. Globally, more than 70 countries had a blood donation rate of less than 1% in 2006.

There have been several studies on the factors that motivate people to become blood donors. While in some cases blood donations are incentivized even with monetary compensations, some economists and industry practitioners believe that it is altruism that mainly drives donors to donate their blood with the idea that it is going to help save the lives
of people, at times, even in their communities, and, sometimes, even themselves (see, e.g., Lacetera, Macis, and Slonim (2012) and the references therein). Much of the theoretical works on blood donor motivation found in existing literature focus on altruism (Andreoni (1990), Mellström and Johanesson (2008), Evans and Ferguson (2013)). Others speculate that blood donors may be motivated by a notion of duty rather than unselfishness (see Wildman and Hollingsworth (2009)). However, there are also several operational aspects of the blood collection centers that can motivate or deter potential donors. It is clearly of value and importance for blood service organizations to understand the reasons behind blood donation decisions in order to increase the efficiency of blood banking systems and to ensure that there is a sufficient supply of blood and blood products. While motivating donors to give blood is one of the most challenging tasks in blood supply chain management, retaining donors has also proved pivotal and extremely difficult in blood supply operations.

In their paper, Gillespie and Hillyer (2002) look at the studies conducted on the topic of blood donation decisions over the preceding three decades, focusing on both first time and return donors. They identify blood donation process measures such as the general donation experience for first time donors in terms of comfort, convenience of the process, and treatment by the staff in charge of technical and administrative activities as factors affecting donation decisions. It is further mentioned that negative donation experiences account for 6-19 percent attrition for all donors and 20 to as high as 41 percent of the drop-out rate for first time donors. In Charbonneau et al. (2016) the authors report deterrents among regular whole blood donors, lapsed whole blood donors and plasma/platelet donors. Based on their survey they found that for a significant percentage of donors in all three categories too much waiting time is a deterrent. These claims are also supported by the empirical evidence provided in Yuan et al. (2011), Finck et al. (2016) and Schreiber et al. (2006). These papers suggest that, while motivators are mainly altruistic, the deterrents are all factors controlled by the blood service organizations. Hence, there is reason to believe that improving such aspects can have a positive impact on blood donations. In this paper we conceptualize these operational factors combined together as the service quality of the blood collection sites. Personal experience and donor satisfaction from the blood donation process, along with the image or awareness of the impact of the organization collecting blood, have been pointed out as significant factors in donor motivation and retention in several studies (Nguyen et al. (2008), Aldamiz-echevarria and Aguirre-Garcia (2014)). In other words, it can be said that the quality of services provided by the blood banks during the collection or donation process plays a significant role in the decision-making process of first time as well as return donors.
There also exists literature on the assessment of blood banks in terms of service quality as perceived by donors, which, again, suggests a direct relationship between donor satisfaction and donation decisions (Al-Zubaidi and Al-Asousi (2012), Jain, Doshit, and Joshi (2015)). For example, Schlumpf et al. (2008), in a survey of over 7,900 blood donors, determined that prior donation frequency, intention to return, donation experience, and having a convenient location appear to significantly predict donor return. Craig et al. (2016), in turn, estimated the effect of wait time on the satisfaction, intention to donate, and actual return behavior of blood donors and found that for a 38% increase in wait time there is a 14% decrease in whole blood donations. Thus, longer wait times entail substantial social costs and also attest to the importance of the quality of service for blood organizations in terms of donations. Convenience is also identified in the literature as a factor that can influence donor behavior (Schreiber et al. (2006)) in terms of clinic accessibility, which also attests to the relevance of quality of service. Cimarolli (2012) in her thesis, which focused on blood donations in Canada, emphasized that it is of utmost importance to improve the experiences of those donating blood, especially first-timers, by optimizing clinic locations and resources, minimizing negative reactions, lowering wait time, and increasing donor comfort. Perera et al. (2015) specifically noted, in their study of blood donor programs in Sri Lanka, that blood donor programs could be improved there by addressing the provision of quality service.

In this paper, we develop a model of competition for blood donations in which the decision-makers are blood service organizations. The organizations compete with one another for blood donations through the levels of quality of service that they provide at their respective blood collection sites in different regions. We focus on competition and provide a game theory Nash Equilibrium network framework since changes in the blood industry have resulted in greater competition among blood collection organizations where the services are not nationalized. Indeed, as noted by Meckler and Neergaard (2002), the blood banking industry has seen a rise in competition among blood banks in terms of recruiting and retaining donors as the number of organizations providing blood services has grown. Donors in parts of the US, for example, may have the option of giving to the American Red Cross, which controls about 44% of the blood nationally, to a local community blood center or hospital, or to America’s Blood Centers’ member organizations which make up North America’s largest network of nonprofit community blood centers, and operate more than 600 blood donation collection sites, or to the United Blood Services, if in proximity. Medical centers and hospitals, in turn, may be juggling shipments of tested blood from multiple suppliers, whose prices may differ. Indeed, it is also important to emphasize that blood organizations incur huge costs for organizing blood donation drives and other operations such as testing, processing of collected blood, transportation, and storage which have to be met through revenue generation. This
is achieved by charging money per pint to hospitals, healthcare clinics, and trauma-centers etc. that require blood (cf. Nagurney, Masoumi, and Yu (2012)). The revenue generated covers costs and can be invested back into the process to improve the services provided by the blood banks and collection agencies. Moreover, organizations involved in blood banking and supply derive a utility in terms of satisfaction from providing quality service. Hence, it is beneficial for them to increase the number of donors and the amount of blood donated.

Adding to the competition in the US are new startups, such as General Blood, that are profit-maximizers and provide blood services (cf. Carlyle (2012)). Pierskalla (2005), on page 141, emphasized in his overview of the supply chain management of blood banks that “To some extent there is a war out there. Many of the suppliers are in heavy, mostly negative competition among themselves and with many of the HBBs (hospital blood banks)” (see also Cohen, Pierskalla, and Sassetti (1987)). According to the European Blood Alliance (EBA) (2012), in the European Union, emerging competition is coming almost entirely from for-profit companies (making decisions on who and what products to supply on a commercial basis), and paying donors to obtain their blood. Hence, competition can and does arise between blood services, as also emphasized in Lowalekar and Ravichandran (2014), notably in the case of hospital blood banks. The competition has also been recognized in the popular press. As early as 1989, Gorman, reporting on competition among blood banks in California, remarked that the Palomar-Pomerado Hospital District established its own laboratory, staff, and recruiting drive to collect blood for use primarily at its two hospitals striking out against the decades old San Diego Blood Bank. Barton (2002) reported that in Florida the competition between Community Blood Centers and South Florida Blood Banks had become “ferocious.” More recently, in Tennessee (cf. Potts (2015)), a new blood center was opened by Blood Assurance in the town of Athens, where another blood center, Medic, currently uses six mobile blood centers with Medic officials noting their concern about the competition and whether they could honor their existing contracts with hospitals. Medic needs about 36 donors per day in the county to keep two medical hospitals stocked; whereas Blood Assurance requires at least 540 donors each day for the 75 regional hospitals.

There are several recent studies on blood supply chain optimization while few focus specifically on the collection operation of the blood centers or the supply side issues. In their most recent paper, Osorio, Brailsford, and Smith (2018) distinguish between blood collection methods and present a multiobjective optimization problem that deals with the issue of assigning donors to a certain collection method while considering stochastic demand. There are several other studies that capture the uncertainty in the demand for blood (Dillon, Oliveira, and Abbasi (2017), Zahiri and Pishvaee (2016), Nagurney, Masoumi and Yu
Beliën and Force (2012) provide an excellent review of the supply chain management of blood products to that date, while the work of Osorio, Brailsford, and Smith (2015) classifies the literature based on quantitative models for various stages of the blood supply chain. In their paper, Ramezanian and Behboodi (2017) address and solve a deterministic location/allocation problem for blood collection facilities while taking into consideration the utility of blood donors in order to motivate them. El-Amine, Bish, and Bish (2017) handle the issue of blood screening and consider that the budget-constrained blood center’s goal is to construct a “robust” post donation blood screening scheme that minimizes the risk of an infectious donation being released into the blood supply. Hosseinifard and Abbasi (2016) consider the effect of centralized inventory in a two echelon supply chain model and show how centralization of the hospital level inventory increases the sustainability and resilience of the blood supply chain. In Ayer et al. (2017) the authors focus on when and from which mobile collection sites to collect blood for cryo production, so that the weekly collection target is met and the collection cost is minimized. Further, in Ayer et al. (2018), the authors develop a dynamic programming approach to the problem and create a decision support tool to help the American Red Cross select cryo collection sites. Some recent works by Salehi, Mahootchi, and Husseini (2017) and Fahimnia et al. (2017) have also modeled robust blood supply chain networks to deal with crisis and disasters. Masoumi, Yu, and Nagurney (2017) provide a cost efficiency (synergy) measure associated with a merger or acquisition in the blood banking industry, as well as measures capturing the expected supply shortage and surplus, and illustrate their supply chain network approach in a case study focusing on the status quo and a disaster. Abbasi, Vakili, and Chesneau (2017), in turn, consider the body of literature, both supporting and opposing, the concept that older transfused blood can have a negative impact on patient outcomes and provide an illuminating case study on the blood supply chain in New South Wales, Australia, demonstrating the impacts of reducing the shelf life of Red Blood Cells (RBCs) and associated ramifications.

While there exists a body of literature on the optimization of blood supply chains (cf. Osorio et al. (2017) and the references therein) to ensure that demand is met as closely as possible and that shortages are minimized, with a view towards the perishability of this life-saving product (see, e.g., Nagurney et al. (2013), Duan and Liao (2014), Masoumi, Yu, and Nagurney (2012)), a thorough search of the published journal literature fails to return any significant work on the modeling of competition for donors among blood service organizations. Blood donations comprise the very basis of blood supply chains and play a crucial role in the stability of the entire blood supply system. Hence, it is of academic as well as practical significance to develop a model capturing the competition among organizations collecting blood. Game theory is a powerful modeling and analytical framework with

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which to formulate and solve a spectrum of problems in which there is competition. It has been used to model competition not only among profit-making organizations but also in the nonprofit sector (see Ortmann (1996), Castaneda, Garen, and Thornton (2008), Nagurney, Alvarez Flores and Soylu (2016)). However, it has been primarily applied in the case of for-profit entities in the operations research community in the case of supply chains, where there has arisen a rich literature. In this paper, in contrast, we develop a game theoretical network framework to model the competition among blood service organizations for blood donations, based on the quality of service that they provide. Stewart (1992) and Janssen and Mendys-Kamphorst (2004) presented theoretical models for blood donor decisions. A review of service quality models is provided by Seth and Deshmukh (2005), but not for blood service organizations. In our model the quality of service variables are continuous and lie between lower and upper bounds; the upper bounds may, of course, be equal to 100, which could represent perfect quality.

In this paper, we develop a game theory model, whose solution yields the desired service quality levels at the blood collection sites and the corresponding quantities of blood received. We can also observe the financial implications in terms of cost and revenue for blood service organizations facing competition for donors. The results obtained from such a model can provide managerial insights to the blood service organizations and help them make decisions about improving the service quality levels at their various blood collection sites to increase the amount of blood donations that they receive. This paper is organized as follows. In Section 2, we present the competitive network model for blood donations, identify the Nash Equilibrium conditions, and provide the variational inequality formulation. We also establish that the equilibrium quality level service pattern of the blood service organizations is guaranteed to exist and provide conditions for uniqueness of the solution. We note that here, unlike the work of Li, Nagurney, and Yu (2017), our focus is on quality of services rather than on product quality. Moreover, the competition here is among nonprofits.

In Section 3, we derive an equivalent formulation to the variational inequality problem for competition among the blood service organizations which utilizes Lagrange multipliers associated with the lower and upper bounds of the quality levels that the blood service organizations can provide in the different regions in which they have or may desire to have blood collection services. We then give a richer interpretation of the underlying economic behavior of the blood service organizations. An illustrative example is provided and subsequently expanded into a series of numerical examples in Section 4. In Section 4, we also outline an algorithm, which yields closed form expressions at each iteration for the quality service levels, until convergence is achieved and then demonstrate the generality of the modeling and
algorithmic framework through a series of numerical examples with accompanying insights. We summarize our results and present our conclusions in Section 5.

2. The Competitive Network Model for Blood Donations

In this section, we present the game theory model in which the blood service organizations compete for blood donations. There are $m$ blood service organizations responsible for collecting the blood donations, which are then tested, processed, and distributed to hospitals and other medical facilities. A typical blood service organization is denoted by $i$. There are $n$ regions in which the blood collections can take place. The collection sites may be fixed or mobile. A typical region is denoted by $j$. The time horizon for our model is flexible but we have in mind a week or a month. The illustrative example in Section 3 provides an actual context for an application of the model, which is then further expanded in our numerical examples in Section 4.

The network structure of the problem is depicted in Figure 1.

![Figure 1: The Network Structure of the Game Theory Model for Blood Donations](image)

The blood service organizations have, as their strategic variables, the quality of service that they provide donors at their collection sites in the regions. It is well-recognized that and, as highlighted in the Introduction, the quality of service at the blood collection sites plays a big role in repeat donations. Moreover, donors receive no financial remuneration in our model and this is quite reasonable since, in many countries, payments for blood donations are not permitted. Let $Q_{ij}$ denote the quality of service that $i$ provides in region $j$. The formal definition of the word “quality” implies the standard of something as measured against other things of similar kind or, in other words, the degree of excellence of something. Service quality pertaining to the blood collection process at different facilities operated by blood service organizations would include operational characteristics such as cleanliness, wait time, hours of donation (convenience), location of the facilities, and treatment by staff.
that affect donation decisions as supported by empirical findings in the existing literature (Gillespie and Hillyer (2002), Nguyen et al. (2008), Aldamiz-echevarria and Aguirre-Garcia (2014), Al-Zubaidi and Al-Asousi (2012), Yuan et al. (2011), Finck et al. (2016), Schreiber et al. (2006)). Different facilities, even operated by the same blood service organization, may have different associated service quality, depending on the size, the resources available there, and location of the service. For example, a permanent unit may be more comfortable to some donors, and be viewed as providing a higher level of quality service than a mobile unit. Moreover, the service quality may vary from one organization to another for similar reasons and even the experience of personnel can be a factor. Hence, it is reasonable to have the flexibility of allowing for upper and lower bounds of quality. Lower bounds could be employed as a target for a blood services organization or correspond to a minimum set by a regulatory body. An upper bound would represent the maximum achievable quality service level by an organization in a region.

We group the strategic variables for each blood service organization into the vector \( Q_i \in \mathbb{R}^n_{+} \), and then we group the quality of service levels for all blood service organizations into the vector \( Q \in \mathbb{R}^{mn}_{+} \). So \( Q \) is essentially a \( m \times n \) matrix. There is a nonnegative lower bound and a positive upper bound imposed on each strategic variable, such that

\[
Q_{ij} \leq Q_{ij} \leq \bar{Q}_{ij}, \quad j = 1, \ldots, n. \tag{1}
\]

We define the feasible set \( K_i \) for blood service organization \( i; \ i = 1, \ldots, m \), as \( K_i \equiv \{ Q_i | (1) \text{ holds} \} \). The feasible set underlying all players in the game, that is, the blood service organizations, is denoted by \( K \) where \( K \equiv \prod_{i=1}^{m} K_i \).

We now describe the components of the transaction utility faced by each blood service organization that capture the total cost associated with the blood collection in the different regions, the utility corresponding to providing the quality levels, and the revenue from the blood donations. Each blood service organization seeks to maximize its transaction utility, which depends on the quality levels not only that it controls but also on those determined by the competing blood service organizations in the various regions.

Each blood service organization \( i \) encumbers a total cost \( \hat{c}_{ij} \) associated with collecting blood in region \( j \), where

\[
\hat{c}_{ij} = \hat{c}_{ij}(Q), \quad j = 1, \ldots, n, \tag{2}
\]

where \( \hat{c}_{ij} \) is assumed to be convex and continuously differentiable for all \( i, j \). Note that the total cost depends, in general, on the quality of service that the blood service organization \( i \) provides at its facility in each region \( j \), as well as on the quality levels of the other blood
service organizations. Some facilities may be more spacious, have more staff, provide greater comfort, have shorter waiting times, and also be cleaner, all factors that enter into the quality of the service and experience of the blood donors. Also, the blood service organizations may be competing for blood service professionals as well as other resources so that the generality of the functions in (2) allows for greater modeling flexibility. The total cost functions in (2) also include the cost of supplies for collecting the blood.

Each blood service organization $i$, since it values the service that it provides to donors, enjoys a utility associated with the service given by: $\omega_i \sum_{j=1}^{n} \gamma_{ij} Q_{ij}$, where the $\omega_i$ and the $\gamma_{ij}s$; $j = 1, \ldots, n$, take on positive values (cf. Nagurney, Alvarez Flores, and Soylu (2016) and the references therein). This component of the transaction utility represents a monetized utility reflecting the value that the blood service organization places on providing collection services at quality levels in the regions under study. If the blood service organization does not wish to consider this component, then $\omega_i$ can be set equal to zero. However, for organizations such as the American Red Cross, these operations might give more visibility and create a goodwill among donors that can eventually aid in fundraising for their other humanitarian operations.

In addition, each blood service organization $i$ receives a volume of blood donations in region $j$, denoted by $P_{ij}$; $j = 1, \ldots, n$, where

$$P_{ij} = P_{ij}(Q), \quad (3)$$

where each $P_{ij}$ is assumed to be concave and continuously differentiable. These blood donation functions capture competition for blood donations among the blood service organizations based on the levels of quality of service that they provide. If need be, these functions can include parameters associated with the level of organizational effectiveness as well as the impact preference of donors. Donors can be expected to be more willing to give to reputable blood collection service organizations. Also, donors often prefer that the blood that they donate be used in a region in proximity to them. For example, Mews and Boenigk (2013), in an online experiment with 144 potential donors, found that organizational reputation is easily damaged by negative news in the press and that this leads to a significantly lower willingness to donate blood for such an organization among potential donors. They also note, that in highly competitive markets, as is the case in Germany (and other countries), intangible assets such as organizational reputation and nonprofit brands have been proven to be of critical importance.

Since blood service organizations charge for the blood that they provide and different organizations can and do price differently, we associate an average price $\pi_i$ for blood (typically,
measured in pints) for blood service organization $i; i = 1, \ldots, m$. These prices correspond to the price associated with the blood collection activity and, hence, would be fraction of the price charged for a pint of blood to hospitals and other medical facilities. For example, the price for a pint of blood can range in the US from about $150 to as much as $300 and this price would also cover testing and delivery. The revenue that blood service organization $i$ achieves that is associated with its blood collection activities over the time horizon is, hence, given by $\pi_i \sum_{j=1}^{n} P_{ij}(Q)$. 

We are now ready to construct the optimization problem faced by blood service organization $i; i = 1, \ldots, m$. Each blood service organization $i$ seeks to maximize its transaction utility, $U_i$, with the transaction utility capturing income due to contractual payments for the blood that it will distribute as well as the monetized utility that the organization gains from providing the collection services to various regions and the costs associated with collection. In particular, the optimization problem is as follows:

$$\text{Maximize } U_i = \pi_i \sum_{j=1}^{n} P_{ij}(Q) + \omega_i \sum_{j=1}^{n} \gamma_{ij}Q_{ij} - \sum_{j=1}^{n} \hat{c}_{ij}(Q) \quad (4)$$

subject to (1).

For additional background on utility functions for nonprofit and charitable organizations, see Rose-Ackerman (1982) and Malani, Philipson, and David (2003).

We now state the Nash Equilibrium conditions for the noncooperative game (cf. Nash (1950, 1951)).

**Definition 1: Nash Equilibrium for Blood Donations**

A quality service level pattern $Q^* \in K$ is said to constitute a Nash Equilibrium in blood donations if for each blood service organization $i; i = 1, \ldots, m$,

$$U_i(Q^*_i, \hat{Q}^*_i) \geq U_i(Q_i, \hat{Q}^*_i), \quad \forall Q_i \in K^i, \quad (5)$$

where

$$\hat{Q}^*_i \equiv (Q^*_1, \ldots, Q^*_{i-1}, Q^*_{i+1}, \ldots, Q^*_m). \quad (6)$$

According to (5), a Nash Equilibrium is established if no blood service organization can improve upon its transaction utility by altering its quality service levels, given that the other organizations have decided on their quality service levels.

We then have the following Theorem.
Theorem 1: Variational Inequality Formulation of the Nash Equilibrium for Blood Donations

A quality service level pattern \( Q^* \in K \) is a Nash Equilibrium according to Definition 1 if and only if it satisfies the variational inequality problem:

\[
- \sum_{i=1}^{m} \sum_{j=1}^{n} \frac{\partial U_i(Q^*)}{\partial Q_{ij}} \times (Q_{ij} - Q^*_{ij}) \geq 0, \quad \forall Q \in K,
\]

(7)

or, equivalently, the variational inequality:

\[
\sum_{i=1}^{m} \sum_{j=1}^{n} \left[ \sum_{k=1}^{n} \frac{\partial \hat{c}_{ik}(Q^*)}{\partial Q_{ij}} - \omega_i \gamma_{ij} - \pi_i \sum_{k=1}^{n} \frac{\partial P_{ik}(Q^*)}{\partial Q_{ij}} \right] \times [Q_{ij} - Q^*_{ij}] \geq 0, \quad \forall Q \in K.
\]

(8)

Proof: We know that each feasible set \( K^i; i = 1, \ldots, m \), is convex since it consists of simple box constraints. Hence, it follows that the Cartesian product \( K \) of these sets is also convex. Under the imposed conditions on the blood donation functions \( P_{ij}(Q) \), and the total cost functions \( \hat{c}_{ij}(Q) \), for all \( i, j \), we also know that the utility functions \( U_i; i = 1, \ldots, m \), are concave and continuously differentiable, since the utility functions consist of such functions, and a linear expression. Therefore, according to Proposition 2.2 in Gabay and Moulin (1980), which established the equivalence between the solution to a Nash equilibrium problem and the solution to the corresponding variational inequality problem, we know that each blood service organization \( i; i = 1, \ldots, m \), maximizes its utility according to Definition 1 if and only if:

\[
- \sum_{i=1}^{m} \sum_{j=1}^{n} \frac{\partial U_i(Q^*)}{\partial Q_{ij}} \times (Q_{ij} - Q^*_{ij}) \geq 0, \quad \forall Q \in K,
\]

which is precisely variational inequality (7).

In order to obtain variational inequality (8) from variational inequality (7), we note that:

\[
- \frac{\partial U_i(Q^*)}{\partial Q_{ij}} = \sum_{k=1}^{n} \frac{\partial \hat{c}_{ik}(Q^*)}{\partial Q_{ij}} - \omega_i \gamma_{ij} - \pi_i \sum_{k=1}^{n} \frac{\partial P_{ik}(Q^*)}{\partial Q_{ij}}, \quad \forall i, j,
\]

and, therefore, variational inequality (8) also holds. \( \square \)

We now put the above variational inequality formulations of the Nash Equilibrium problem into standard variational inequality form (see Nagurney (1999)), that is: determine \( X^* \in \mathcal{K} \subset \mathbb{R}^N \), such that

\[
\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K},
\]

(9)
where $F$ is a given continuous function from $\mathcal{K}$ to $R^N$ and $\mathcal{K}$ is a closed and convex set.

We define the $mn$-dimensional column vector $X \equiv Q$ and the $mn$-dimensional column vector $F(X)$ with the $(i, j)$-th component, $F_{ij}$, given by

$$F_{ij}(X) \equiv -\frac{\partial U_i(Q)\partial Q_{ij}}{\partial Q_{ij}} = \left[\sum_{k=1}^{n} \frac{\partial \hat{c}_{ik}(Q)}{\partial Q_{ij}} - \omega_i \gamma_{ij} - \pi_i \sum_{k=1}^{n} \frac{\partial P_{ik}(Q)}{\partial Q_{ij}}\right],$$

(10)

and with the feasible set $\mathcal{K} \equiv K$. Then, clearly, variational inequality (7) and (8) can be put into standard form (9).

Existence of solution $Q^*$ to variational inequality (7) and also (8) is guaranteed from the standard theory of variational inequalities (cf. Nagurney (1999)) since the function $F(X)$ that enters the variational inequality is continuous and the feasible set $\mathcal{K}$ is compact.

Moreover, if $F(X)$ is strictly monotone, that is:

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle > 0, \quad \forall X^1, X^2 \in \mathcal{K}, X^1 \neq X^2,$$

(11)

then the equilibrium solution $X^*$ and, hence, $Q^*$ is unique.

3. An Equivalent Formulation of Variational Inequality (7)

We now describe and analyze an equivalent formulation of variational inequality (7) which allows us to provide a deeper analysis of the Lagrange multipliers that are associated with the constraints (1), in the form of lower and upper bounds, on the levels of quality service.

We observe that the feasible set $K$ can be rewritten as

$$K = \{Q \in R^{mn} : Q_{ij} - Q_{ij} \leq 0, Q_{ij} - Q_{ij} \leq 0, i = 1, \ldots, m; j = 1, \ldots, n\}.$$  

(12)

Also, variational inequality (7) can be rewritten as a minimization problem. For example, by setting

$$V(Q) = -\sum_{i=1}^{m} \sum_{j=1}^{n} \frac{\partial U_i(Q^*)}{\partial Q_{ij}} \times (Q_{ij} - Q_{ij}^*),$$

(13)

we have that

$$V(Q) \geq 0 \text{ in } K \text{ and } \min_K V(Q) = V(Q^*) = 0.$$  

(14)

We consider now the Lagrange function:

$$\mathcal{L}(Q, \lambda^1, \lambda^2) = -\sum_{i=1}^{m} \sum_{j=1}^{n} \frac{U_i(Q^*)}{\partial Q_{ij}} \times (Q_{ij} - Q_{ij}^*)$$

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\[ + \sum_{i=1}^{m} \sum_{j=1}^{n} \lambda_{ij}^1 (Q_{ij} - Q_{ij}) + \sum_{i=1}^{m} \sum_{j=1}^{n} \lambda_{ij}^2 (Q_{ij} - \bar{Q}_{ij}), \]  

(15)

where \( Q \in \mathbb{R}^{mn}, \lambda^1, \lambda^2 \in \mathbb{R}^{mn}_+ \), \( \lambda^1 = \{\lambda_{11}, \ldots, \lambda_{mn}\} \), and \( \lambda^2 = \{\lambda_{11}^2, \ldots, \lambda_{mn}^2\} \).

Since for the convex set \( K \) the Slater condition is verified and \( Q^* \) is a minimal solution to problem (14), according to well-known theorems (see Jahn (1994)), there exist Lagrange multiplier vectors \( \bar{\lambda}^1, \bar{\lambda}^2 \in \mathbb{R}^{mn}_+ \) such that the vector \((Q^*, \bar{\lambda}^1, \bar{\lambda}^2)\) is a saddle point of the Lagrange function (15), that is,

\[ \mathcal{L}(Q^*, \bar{\lambda}^1, \bar{\lambda}^2) \leq \mathcal{L}(Q^*, \bar{\lambda}^1, \bar{\lambda}^2) \leq \mathcal{L}(Q, \bar{\lambda}^1, \bar{\lambda}^2), \quad \forall Q \in K, \forall \lambda^1, \lambda^2 \in \mathbb{R}^{mn}_+, \]

(16)

and

\[ \bar{\lambda}_{ij}^1 (Q_{ij} - Q_{ij}^*) = 0, \quad \bar{\lambda}_{ij}^2 (Q_{ij}^* - Q_{ij}) = 0, \quad i = 1, \ldots, m; j = 1, \ldots, n. \]

(17)

From the right-hand side of (16) it follows that \( Q^* \in \mathbb{R}^{mn}_+ \) is a minimal point of \( \mathcal{L}(Q, \bar{\lambda}^1, \bar{\lambda}^2) \) in the whole space \( \mathbb{R}^{mn} \) and, hence, for all \( i \) and \( j \) we have that:

\[ \frac{\partial \mathcal{L}(Q^*, \bar{\lambda}^1, \bar{\lambda}^2)}{\partial Q_{ij}} = - \frac{\partial U_i(Q^*)}{\partial Q_{ij}} - \bar{\lambda}_{ij}^1 + \bar{\lambda}_{ij}^2 = 0 \]

(18)

together with conditions (17).

Conditions (17) and (18), which hold for all \( i \) and \( j \), represent an equivalent formulation of variational inequality (7).

Indeed, it is easy to see that from (17) and (18), variational inequality (7) follows. For example, multiplication of (18) by \((Q_{ij} - Q_{ij}^*)\) yields

\[ - \frac{\partial U_i(Q^*)}{\partial Q_{ij}} \times (Q_{ij} - Q_{ij}^*) - \bar{\lambda}_{ij}^1 (Q_{ij} - Q_{ij}^*) + \bar{\lambda}_{ij}^2 (Q_{ij} - Q_{ij}^*) = 0 \]

and, by taking into account (17), we have

\[ - \frac{\partial U_i(Q^*)}{\partial Q_{ij}} \times (Q_{ij} - Q_{ij}^*) = \bar{\lambda}_{ij}^1 (Q_{ij} - Q_{ij}^*) - \bar{\lambda}_{ij}^2 (Q_{ij} - Q_{ij}^*) = \bar{\lambda}_{ij}^1 (Q_{ij} - Q_{ij}) + \bar{\lambda}_{ij}^2 (\bar{Q}_{ij} - Q_{ij}) \geq 0. \]

(19)

Summation over all \( i \) and \( j \) of (19) yields variational inequality (7).
We now proceed to analyze the marginal transaction utilities of the blood service organizations.

From (18) we have that

\[-\frac {\partial U_i(Q^*)}{\partial Q_{ij}} - \bar{\lambda}_{ij}^1 + \bar{\lambda}_{ij}^2 = 0, \quad i = 1, \ldots, m; j = 1, \ldots, n.\]

Therefore, if \(Q_{ij} < Q^*_{ij} < \bar{Q}_{ij}\), then we get, using also (8):

\[-\frac {\partial U_i(Q^*)}{\partial Q_{ij}} = \left[\sum_{k=1}^{n} \frac{\partial \hat{c}_{ik}(Q^*)}{\partial Q_{ij}} - \omega_i \gamma_{ij} - \pi_i \sum_{k=1}^{n} \frac{\partial P_{ik}(Q^*)}{\partial Q_{ij}}\right] = 0, \quad i = 1, \ldots, m; j = 1, \ldots, n.\]  

On the other hand, if \(\bar{\lambda}_{ij}^1 > 0\) and, hence, \(Q^*_{ij} = Q_{ij}\) and \(\bar{\lambda}_{ij}^2 = 0\), then we get that

\[-\frac {\partial U_i(Q^*)}{\partial Q_{ij}} = \left[\sum_{k=1}^{n} \frac{\partial \hat{c}_{ik}(Q^*)}{\partial Q_{ij}} - \omega_i \gamma_{ij} - \pi_i \sum_{k=1}^{n} \frac{\partial P_{ik}(Q^*)}{\partial Q_{ij}}\right] = \bar{\lambda}_{ij}^1, \quad i = 1, \ldots, m; j = 1, \ldots, n.\]  

(21)

and if \(\bar{\lambda}_{ij}^2 > 0\) and, hence, \(Q^*_{ij} = \bar{Q}_{ij}\) and \(\bar{\lambda}_{ij}^1 = 0\), we have that

\[-\frac {\partial U_i(Q^*)}{\partial Q_{ij}} = \left[\sum_{k=1}^{n} \frac{\partial \hat{c}_{ik}(Q^*)}{\partial Q_{ij}} - \omega_i \gamma_{ij} - \pi_i \sum_{k=1}^{n} \frac{\partial P_{ik}(Q^*)}{\partial Q_{ij}}\right] = -\bar{\lambda}_{ij}^2, \quad i = 1, \ldots, m; j = 1, \ldots, n.\]  

(22)

We now proceed to analyze (20) through (22). From equality (20), which holds when \(Q_{ij} < Q^*_{ij} < \bar{Q}_{ij}\), we observe that for blood service organization \(i\), which provides a quality level of service of \(Q^*_{ij}\) to blood donors in region \(j\), the marginal transaction utility is zero; that is, the marginal total cost: \(\sum_{k=1}^{n} \frac{\partial \hat{c}_{ik}(Q^*)}{\partial Q_{ij}}\) is equal to the marginal utility associated with providing the service and the marginal revenue associated with the acquired blood donations in region \(j\): \(\omega_i \gamma_{ij} + \pi_i \sum_{k=1}^{n} \frac{\partial P_{ik}(Q^*)}{\partial Q_{ij}}\).

According to equality (21), if \(Q^*_{ij} = Q_{ij}\), then the minus marginal transaction utility is equal to \(\bar{\lambda}_{ij}^1\). In other words, the marginal total cost exceeds the marginal utility associated with providing the service and the marginal revenue. The blood service organization \(i\) then suffers a marginal loss given by \(\bar{\lambda}_{ij}^1\).

Finally, according to (22), in which case \(Q^*_{ij} = \bar{Q}_{ij}\) and \(\bar{\lambda}_{ij}^2 > 0\), minus the marginal transaction utility is equal to \(-\bar{\lambda}_{ij}^2\). In this case the marginal utility plus the marginal revenue exceeds the marginal total cost. Blood service organization \(i\) experiences a marginal gain given by \(\bar{\lambda}_{ij}^2\).
From the above analysis, we see that the equilibrium/optimal Lagrange multiplier vectors of variables $\bar{\lambda}_1$ and $\bar{\lambda}_2$ provide us with a rigorous interpretation of the behavior of the competition with respect to the provision of blood services, under the quality service level bounds. We remark that Langrangean analysis has also yielded useful insights into equilibrium problems in cybersecurity (cf. Daniele, Maugeri, and Nagurney (2017)) and in finance (see Daniele, Giuffre, and Lorino (2016)).

3.1 An Illustrative Example - Example 1

We now present an example to illustrate some of the above concepts. We will expand it in Section 4 to construct a series of numerical examples.

The examples are inspired, in part, by the American Red Cross (cf. Arizona Blood Services Region (2016)) issuing a recent call for donations since its supplies of blood are low due to seasonal colds and flu and the devastating impact of Hurricane Matthew, which made landfall in the US on October 8, 2016, affected such states as Florida, Georgia, and the Carolinas, and disrupted blood donations in many locations in the Southeast of the US. Specifically, we focus on Tucson, Arizona, where the American Red Cross has held recent blood drives at multiple locations and where there are also competitors for blood, including United Blood Services.

Example 1

Example 1 serves as the baseline. It consists of two blood service organizations, the American Red Cross and the United Blood Services, corresponding to uppermost nodes 1 and 2 in Figure 2. There are also two blood collection regions in Tucson, denoted by the lowermost nodes in Figure 2. For example, in its recent call for blood donations (cf. Arizona Blood Services Region (2016)), the America Red Cross had two locations in Tucson for collection. United Blood Services is a nonprofit organization that was founded in 1943 in Arizona and provides blood and services to more than 500 hospitals in 18 states (cf. United Blood Services (2016)). United Blood Services also collects blood in Tucson. The collection site nodes 1 and 2 are fixed, rather than mobile, for both organizations.

The components of the transaction utility functions (4) of the blood service organizations are as follows.

We consider a month of collection of whole blood cells. According to Meyer (2017), Vice President of the American Red Cross (in a private communication), productive Red Cross sites collect, on the average, 700-840 whole blood units a month.
Blood Service Organizations

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Blood Collection Regions

Figure 2: Example 1

The blood donation functions for the American Red Cross are:

\[ P_{11}(Q) = 10Q_{11} - Q_{21} - Q_{22} + 130, \quad P_{12}(Q) = 12Q_{12} - Q_{21} - 2Q_{22} + 135. \]

Note that the fixed terms of 130 and 135 reflect the baseline of repeat donors that the American Red Cross expects over the month in the two regions, respectively.

The blood donation functions for the United Blood Services are:

\[ P_{21}(Q) = 11Q_{21} - Q_{11} - Q_{12} + 123, \quad P_{22}(Q) = 12Q_{22} - Q_{11} - Q_{12} + 135. \]

The United Blood Services has a lower baseline population of donors in these blood collection regions than the America Red Cross. Its attention to quality is as good or higher than that of the American Red Cross, according to the respective functions.

The utility function components of the transaction utilities of these blood service organizations are:

\[ \omega_1 = 9, \quad \gamma_{11} = 8, \quad \gamma_{12} = 9, \]
\[ \omega_2 = 10, \quad \gamma_{21} = 9, \quad \gamma_{22} = 10. \]

The above parameters reflect that blood service organization 2 derives greater utility from providing quality blood collection services than blood service organization 2.

The total costs of operating the blood collection sites over the time horizon, which must cover costs of employees, supplies, and energy, and providing the level of quality service, are:

\[ \hat{c}_{11}(Q) = 5Q_{11}^2 + 10,000, \quad \hat{c}_{12}(Q) = 18Q_{12}^2 + 12,000. \]
\[ \hat{c}_{21}(Q) = 4.5Q_{21}^2 + 12,000, \quad \hat{c}_{22}(Q) = 5Q_{22}^2 + 14,000. \]

The bounds on the quality levels are:

\[ Q_{11} = 50, \quad Q_{11} = 80, \quad Q_{12} = 40, \quad Q_{12} = 70, \]
\[ Q_{21} = 60, \bar{Q}_{21} = 90, \quad Q_{22} = 70, \bar{Q}_{22} = 90. \]

The prices, which correspond to the collection component of the blood supply chain, are: \( \pi_1 = 70 \) and \( \pi_2 = 60. \)

First, observe that the objective function (4) of each blood service organization is, given the above functional forms, concave and continuously differentiable. In order to solve Example 1, we utilize variational inequality (8), and, because of the simplicity of the functions above (which we generalize for our numerical examples), we can easily obtain the equilibrium quality levels.

Specifically, using formula (10) for each \( F_{ij} \) we obtain the following equations:

\[
F_{11}(Q^*) = 10Q^*_{11} - 772 = 0, \\
F_{12}(Q^*) = 36Q^*_{12} - 921 = 0, \\
F_{21}(Q^*) = 9Q^*_{21} - 750 = 0, \\
F_{22}(Q^*) = 10Q^*_{22} - 820 = 0.
\]

Note that we also have to be cognizant of the lower and upper bounds on the quality levels. Solving the above equations, we obtain: \( Q^*_{11} = 77.2, \quad Q^*_{12} = 25.5, \quad Q^*_{21} = 83.3, \) and \( Q^*_{22} = 82. \)

Checking whether the values lie within the respective bounds we observe that they all do, except for \( Q^*_{12} \), which we, hence, set to its lower bound so that: \( Q^*_{12} = 40. \)

According to this solution, the Red Cross stands to collect 736.7 units of blood at region 1, since \( P_{11}(Q^*) = 736.7 \) and 367.7 units of whole blood at region 2. United Blood service, on the other hand, stands to collect, since \( P_{21}(Q^*) = 922.1 \), that number of units per month at region 1, and 1001.80 units in region 2 (since \( P_{22}(Q^*) = 1001.8 \)). Hence, United Blood Services collects a larger number of units of blood in the two regions.

We also know, according to the Lagrangean analysis above, that, since only \( Q^*_{12} \) is at its lower bound and no quality service levels are at their upper bounds: \( \bar{\lambda}_{11} = 0, \quad \bar{\lambda}_{21} = 0, \quad \bar{\lambda}_{22} = 0 \), and \( \bar{\lambda}_{12} = 0, \quad \bar{\lambda}_{12}^2 = 0 \). Also, since \( Q^*_{12} = Q^*_{12} \), we compute \( \bar{\lambda}_{12} = \sum_{k=1}^{2} \frac{\partial c_{1k}(Q^*)}{\partial Q_{12}} - \omega_1 \gamma_{12} - \pi_1 \sum_{k=1}^{2} \frac{\partial P_{1k}(Q^*)}{\partial Q_{12}} = 1359. \) The American Red Cross suffers a marginal loss given by \( \bar{\lambda}_{12} \). The transaction utilities at the equilibrium quality levels are: \( U_1(Q^*) = 5,507.20 \) and \( U_2(Q^*) = 40,285.99. \) In this illustrative example, the United Blood Services organization provides a higher level of quality services at each of its locations in Tucson and garners a higher transaction utility than the American Red Cross.
In the numerical examples in Section 4 we consider more general blood donation functions and also add both a competitor and a new region and analyze the resulting blood service organization quality levels and the incurred transaction utilities.

4. The Algorithm and Numerical Examples

Before we present the numerical examples we outline our computational procedure. Specifically, the variational inequalities (7) and (8) are amenable to solution via the Euler method of Dupuis and Nagurney (1993), which, at each iteration, results in closed form expressions for the blood service organization quality levels.

In particular, iteration \( \tau \) of the Euler method where the variational inequality is expressed in standard form (9) is given by:

\[
X_{\tau+1} = P_K(X_{\tau} - a_\tau F(X_{\tau})),
\]

(23)

where \( P_K \) is the projection on the feasible set \( K \) and \( F \) is the function that enters the variational inequality problem (9), where recall that \( X \equiv Q \) and \( F(X) \) consists of the components given in (10).

As established in Dupuis and Nagurney (1993), for convergence of the general iterative scheme, which induces the Euler method, the sequence \( \{a_\tau\} \) must satisfy: \( \sum_{\tau=0}^{\infty} a_\tau = \infty \), \( a_\tau > 0 \), \( a_\tau \to 0 \), as \( \tau \to \infty \). Conditions for convergence for a variety of network-based problems can be found in Nagurney and Zhang (1996) and Nagurney (2006).

Explicit Formulae for the Euler Method Applied to the Blood Donation Service Organization Game Theory Model

The elegance of this algorithm for our variational inequality (8) for the computation of solutions to our model is clear from the following explicit formula. In particular, we have the following closed form expression for the quality service levels \( i = 1, \ldots, m; j = 1, \ldots, n \), at iteration \( \tau + 1 \):

\[
Q_{ij}^{\tau+1} = \max\{Q_{ij}, Q_{ij}^\tau + a_\tau (\pi_i \sum_{k=1}^{n} \frac{\partial P_k(Q^\tau)}{\partial Q_{ij}} + \omega_i \gamma_{ij} - \sum_{k=1}^{n} \frac{\partial \hat{c}_{ik}(Q^\tau)}{\partial Q_{ij}})\}.
\]

(24)

4.1 Numerical Examples

We now present numerical examples focused on an area of Arizona. These build on the illustrative example in Section 3.1. We implemented the Euler method described above in FORTRAN and used a Linux system. The convergence criterion was \( \epsilon = 10^{-5} \), that is, the
Euler method was considered to have converged if, at a given iteration, the absolute value of the difference of each quality service level (see (24)) differed from its respective value at the preceding iteration by no more than the $\epsilon$. We initialized the Euler method by setting all the quality service levels to their lower bounds.

In the subsequent examples, Examples 2 through 6, we add different features. In Example 2, we introduce more general blood donation functions than those used in Example 1 and this changes the solution significantly. In Example 3, we add a collection region to show how that can benefit the competing blood service organizations. Example 4 incorporates an additional blood service organization to increase the competition. With Example 5, we try to show how over time increased competition can affect the outcomes. Lastly, in Example 6, we have three blood service organizations competing for donations in three collection regions. Hence, with each example we add a layer of complexity, which demonstrates the generality of our modeling and computational framework.

Below we provide additional information for each numerical example.

Example 2

Example 2 has the same network topology as Example 1, that is, the one depicted in Figure 2.

The data are identical to those in Example 1 except here, we use significantly more general blood donation functions to observe how that changes the solutions. In particular, the new $P_{ij}$ functions are constructed from the ones in Example 1 thus: $\alpha_{ij} \sqrt{P_{ij}}$ for $i = 1, 2; j = 1, 2$ with $\alpha_{11} = 50$, $\alpha_{12} = 30$, $\alpha_{21} = 40$, and $\alpha_{22} = 20$.

The computed equilibrium quality service levels are:

$$Q_{11}^* = 72.43, \quad Q_{12}^* = 40.00, \quad Q_{21}^* = 64.61, \quad Q_{22}^* = 70.00.$$  

The Euler method requires 34 iterations to converge to this solution. Organization 1 provides a higher level of quality of service in region 1 whereas organization 2 does in region 2. Organization 1 collects $P_{11} = 1341.37$ units of blood in region 1 and $P_{12} = 607.74$ units of blood in region 2, whereas organization 2 collects $P_{21} = 1074.27$ units in region 1 and $P_{22} = 587.39$ units of blood in region 2. The revenue of organization 1 is 136,437.78 and that of organization 2 is: 99,699.67. The monetized service utility component of the transaction utility of organization 1 is 8,455.10 and that of organization 2 is 12,814.97. Organization 1 incurs costs of 77,031.92 and organization 2 incurs costs of 69,285.48. The revenue minus the cost (net revenue) for organization 1 is: 59,405.86, whereas the revenue minus the cost
for organization 2 is: 30,414.19.

The values of the transaction utilities of the blood service organizations at the equilibrium values are, hence: $U_1 = 67,860.96$ and $U_2 = 43,229.16$. Note that both organizations have over the time period revenues that exceed their costs, which is important for the sustainability of their operations. Also, organization 1 garners a larger number of units of blood donated in each region than does organization 2. In region 1 organization 1 has a higher level of quality service than does organization 2 but in region 2 organization 2 provides a higher level of quality service. However, the “brand” of organization 1, which is also reflected in its $P_{12}$ function, dominates, although not significantly in terms of units collected in region 2.

Observe that both $Q_{12}^*$ and $Q_{22}^*$ are at their lower bounds. Hence, according to the Lagrange analysis theoretical results presented in Section 3 we know that: $\bar{\lambda}_{11}^1 = \bar{\lambda}_{21}^2 = 0$ and $\bar{\lambda}_{21}^1 = \bar{\lambda}_{21}^2 = 0$. Also, we have that $\bar{\lambda}_{12}^1 = 737.03$, $\bar{\lambda}_{12}^2 = 0$, and $\bar{\lambda}_{22}^1 = 354.85$, $\bar{\lambda}_{22}^2 = 0$. Therefore, blood service organization 1 suffers a marginal loss of 737.03 associated with its services in region 2 and blood service organization 2 suffers a marginal loss of 354.85 associated with its services in region 2.

Example 3

Example 3 has the network topology in Figure 3. The data are as in Example 2 but with the addition of a possible new blood collection point in region 3. In this example, the quality service level lower bounds associated with the blood service organizations servicing region 3 in terms of collections are set to 0.

![Network Topology](image)

**Figure 3: Example 3**

The data are as follows: $\alpha_{13} = 40$, $\alpha_{23} = 30$, and

$$P_{13}(Q) = 40\sqrt{10Q_{13} - Q_{23}} + 50, \quad P_{23}(Q) = 30\sqrt{11Q_{23} - Q_{13}} + 50,$$

$$\gamma_{13} = 9, \quad \gamma_{23} = 10,$$

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and
\[ \hat{c}_{13}(Q) = 10Q_{13}^2 + 15,000, \quad \hat{c}_{23}(Q) = 9Q_{23}^2 + 13,000. \]

The lower and upper bounds on the new links, in turn, are:
\[ Q_{13} = 0, \quad Q_{23} = 0, \]
\[ \bar{Q}_{13} = 60, \quad \bar{Q}_{23} = 70. \]

The Euler method converges in 34 iterations to the following equilibrium quality level pattern:
\[ Q_{11}^* = 72.43, \quad Q_{12}^* = 40.00, \quad Q_{13}^* = 38.84, \]
\[ Q_{21}^* = 64.61, \quad Q_{22}^* = 70.00, \quad Q_{23}^* = 33.70. \]

Note that the equilibrium quality levels for \( Q_{11}^* \), \( Q_{12}^* \), and \( Q_{21}^* \), \( Q_{22}^* \) remain as in Example 2 since the underlying functions for these blood service organization and blood region pairs remain as in Example 2. Also, we know from our theoretical analysis in Section 3 that since both \( Q_{13}^* \) and \( Q_{23}^* \) are neither at their lower or at their upper bounds, we have that:
\[ \bar{\lambda}_{13} = \bar{\lambda}_{13} = \bar{\lambda}_{23} = \bar{\lambda}_{23} = 0.00. \]

Organization 1 receives 804.73 units of blood in region 3 whereas organization 2 receives 586.24 units of blood in region 3. The revenue of organization 1 is now 196,739.25 and that of organization 2 is: 134,874.17. The monetized service utility component of the transaction utility is 11,601.49 for organization 1 and 16,185.06 for organization 2. Since now both organizations operate a facility in an additional region the costs for organization 1 are equal to 167,283.03 and for organization 2 the costs are: 127,589.64. The transaction utility for blood service organization 1, \( U_1 = 41,057.70 \), and the transaction utility for blood service organization 2, \( U_2 \), is now 23,469.59.

The revenue for each organization is higher in this example, with a new blood collection facility in a new region, than that obtained in Example 2. However, the cost is also higher. By collecting blood donations from three regions, rather than two, the organizations achieve a higher monetized service utility by serving more regions. The net revenue for organization 1 is now 29,456.22 and it is 7,284.53 for organization 2. For each organization the revenue still exceeds the costs, which means that collecting blood in region 3 does not hurt them financially. However, the net revenue for each organization is lower in this example than in Example 2.
Example 4

Example 4 is constructed from Example 2, and has the same data, except for the new data corresponding to the addition of a new competitor, blood service organization 3, as depicted in Figure 4.

![Blood Service Organizations Diagram](image)

The data for blood service organization 3 are:

\[ P_{31}(Q) = 50\sqrt{11Q_{31} - Q_{21}} + 50, \quad P_{32}(Q) = 40\sqrt{10Q_{32} - Q_{12}} + 2000, \]

\[ \omega_3 = 10, \quad \gamma_{31} = 10, \quad \gamma_{32} = 11, \]

with the total cost functions given by:

\[ \hat{c}_{31}(Q) = 6q_{31}^2 + 10,000 \quad \hat{c}_{32}(Q) = 5Q_{32}^2 + 12,000, \]

and with the lower and upper bounds as follows:

\[ Q_{31} = 50, \quad Q_{31} = 90, \]

\[ Q_{32} = 40, \quad Q_{32} = 80. \]

The price \( \pi_3 = 80 \). Recall that here we just consider the collection component of the blood supply chains.

The Euler method requires 40 iterations for convergence and yields the following equilibrium quality service level pattern:

\[ Q_{11}^* = 72.43, \quad Q_{12}^* = 40, \quad Q_{21}^* = 64.61, \quad Q_{22}^* = 70, \quad Q_{31}^* = 70.73, \quad Q_{32}^* = 66.65. \]

Observe that, since the blood donation functions of the original blood service organizations have not changed, their quality service levels and, hence, their transaction utilities
remain as in Example 2; the same holds for the donations, revenue amounts, costs, as well as the monetized service utility component of the transaction utilities.

Blood service organization 3 has a transaction utility $U_3 = 104,706.44$. The amounts of its blood donations received are: $P_{31} = 1,381.47$ and $P_{32} = 2,049.99$. Its revenue is: 274,516.72 and its monetized service utility component of its transaction utility is: 14,111.81, with its cost equal to 184,922.09. This blood service organization has a net revenue equal to 89,594.63. Blood service organization 3 has the highest net revenue of all the organizations, in this example, since the net revenue for organization 1 in Example 2 was 59,405.86 and that for organization 2: 30,414.19. This is due, in part to organization 3 being able to achieve the highest volume of donations. Its quality levels do not lie at the bounds so that: $\bar{\lambda}_{31}^1 = \bar{\lambda}_{31}^2 = \bar{\lambda}_{32}^1 = \bar{\lambda}_{32}^2 = 0$.

**Example 5**

Example 5 is constructed from Example 4. We assume that some time has transpired and now both blood service organizations 1 and 2 realize that there is more competition from blood service organization 3.

Hence, their blood donation functions are now modified to capture the impact of competition from blood service organization 3 as follows:

For blood service organization 1:

\[
P_{11}(Q) = 50\sqrt{10Q_{1,1} - Q_{2,1} - Q_{2,2} - .5Q_{3,1} + 130},
\]

\[
P_{12}(Q) = 30\sqrt{12Q_{1,2} - Q_{2,1} - 2Q_{2,2} - .3Q_{3,2} + 135},
\]

and for blood service organization 2:

\[
P_{21}(Q) = 40\sqrt{11Q_{2,1} - Q_{1,1} - Q_{1,2} - .2Q_{2,1} + 113},
\]

\[
P_{22}(Q) = 20\sqrt{12Q_{2,2} - Q_{1,1} - Q_{1,2} - .3Q_{3,2} + 135}.
\]

The remainder of the data is identical to that in Example 4.

The Euler method converges in 40 iterations and yields the following quality service level pattern:

\[Q_{11}^* = 73.57, \quad Q_{12}^* = 40, \quad Q_{21}^* = 64.99, \quad Q_{22}^* = 70, \quad Q_{31}^* = 70.73, \quad Q_{32}^* = 66.65.\]

The transaction utilities are now: $U_1 = 64,439.25$, $U_2 = 42,572.30$, and $U_3 = 104,222.39$. The volumes of blood donations are now as follows: for organization 1: $P_{11} = 1,318.43,$
$P_{12} = 592.46$; for organization 2: $P_{21} = 1,059.31, P_{22} = 580.15$, and for organization 3: $P_{31} = 1,381.22$ and $P_{32} = 2,049.99$. Organization 1 has a revenue of 133,762.72, costs equal to 77,860.27, and a monetized service quality component of the transaction utility equal to 8,536.80. Organization 2, in turn, enjoys a revenue of 98,367.77, encumbers costs equal to 68,644.69, and a monetized service quality component of its transaction utility equal to 12,849.21. Organization 3 obtains a revenue of 274,497.03, incurs costs of 185,38.53, and a monetized service quality component of its transaction utility equal to 15,112.89.

Observe that only $Q^*_{12}$ and $Q^*_{22}$ are, again, at their lower bounds. Hence, according to the Lagrange analysis theoretical results presented in Section 3 we know that: $\lambda^1_{11} = \lambda^2_{11} = 0$, $\lambda^1_{21} = \lambda^2_{21} = 0$, $\lambda^1_{31} = \lambda^2_{31} = 0$ and also $\lambda^1_{32} = \lambda^2_{32} = 0$. Also, we now have that $\lambda^1_{12} = 720.98, \lambda^2_{12} = 0$, and $\lambda^1_{22} = 351.79, \lambda^2_{22} = 0$. Therefore, blood service organization 1 suffers a marginal loss of 720.98 associated with its services in region 2 and blood service organization 2 suffers a marginal loss of 351.79 associated with its services in region 2. These marginal losses are lower than those they suffered in Example 2.

Interestingly, with increased competition, blood donors benefit in that the quality service levels provided are now as high or higher than in Example 4 and both blood service organizations 1 and 2 provide a higher quality service in region 1 than in Example 4. But, of course, this comes at a higher cost so their transaction utilities are lower now than in Example 4. Also, in terms of financial sustainability, we note that for organization 1, its net revenue is now: 55,902.45; for organization 2, this value is: 29,723.08, and for organization 3: 89,109.50. With increased competition, the net revenues decrease for all blood service organizations but these are, nevertheless, still significant and would allow for investment, whether to enhance their operations or, if feasible, to engage in R&D and further innovation for blood services.

In addition, comparing the amounts of blood collected by the two organizations in the two regions in Example 2 with the results obtained here, we see that the blood collections from both regions decrease for organizations 1 and 2. However, due to the presence of a competing organization the overall blood collection increases. This finding is consistent with the empirical findings in Bose (2014).

**Example 6**

Example 6 is constructed from Example 5 but it has a new blood collection region. Hence, as depicted in Figure 5, there are now three regions for blood collection, as well as three blood service organizations.
Blood Service Organizations

Blood Collection Regions

Figure 5: Example 6

The data remain as in Example 5 with the addition of the new data below:

\[ \alpha_{13} = 40, \quad \alpha_{23} = 30, \quad \alpha_{33} = 50, \]

\[ P_{13}(Q) = 40\sqrt{10Q_{13} - Q_{23} - .2Q_{33} + 150}, \]
\[ P_{23}(Q) = 30\sqrt{11Q_{23} - Q_{13} - .2Q_{33} + 150}, \]
\[ P_{33}(Q) = 50\sqrt{10Q_{33} - Q_{23} - .3Q_{13} + 100}, \]
\[ \hat{c}_{13}(Q) = 100Q_{13}^2 + 15,000 \quad \hat{c}_{23}(Q) = 9Q_{23}^2 + 13000, \quad \hat{c}_{33}(Q) = 8Q_{33}^2 + 10000, \]

and with lower and upper bounds on the new links to region 3 given by:

\[ Q_{13} = 0, \quad Q_{23} = 0, \quad Q_{33} = 40, \]
\[ \bar{Q}_{13} = 60, \quad \bar{Q}_{23} = 70, \quad \bar{Q}_{33} = 90. \]

Also, we have that

\[ \gamma_{13} = 9, \quad \gamma_{23} = 10, \gamma_{33} = 10. \]

The Euler method, again, converges in 40 iterations to the following equilibrium pattern:

\[ Q_{11}^* = 73.57, \quad Q_{12}^* = 40, \quad Q_{13}^* = 36.32, \]
\[ Q_{21}^* = 64.99, \quad Q_{22}^* = 70, \quad Q_{23}^* = 31.51, \]
\[ Q_{31}^* = 70.73, \quad Q_{32}^* = 66.65, \quad Q_{33}^* = 56.39. \]

The transaction utilities are now: \( U_1 = 129,918.82, U_2 = 58,877.95, \) and \( U_3 = 168,602.63. \)

All of the Lagrange multipliers are equal to 0 except for the following: \( \bar{\lambda}_{12}^1 = 720.98, \)
\[ \bar{\lambda}_{22}^1 = 351.79. \]
All three blood service organizations enjoy a higher transaction utility by collecting blood in all three regions, rather than just two regions.

The volumes of blood donations are now: for organization 1: $P_{11} = 1,318.43$, $P_{12} = 592.46$, $P_{13} = 867.59$; for organization 2: $P_{21} = 1,059.31$, $P_{22} = 580.15$, $P_{23} = 635.70$, and for organization 3: $P_{31} = 1,381.22$, $P_{32} = 2,049.99$, and $P_{33} = 1,246.49$. Organization 1 has a revenue of 194,493.95, a cost equal to 76,054.13, and a monetized service quality component of the transaction utility equal to 11,478.99. Organization 2, in turn, enjoys a revenue of 136,509.97, a cost 93,632.34, and a monetized service quality component of its transaction utility equal to 16,000.32. Organization 3 obtains a revenue of 374,216.53, incurs a cost 226,36.88, and a monetized service quality component of its transaction utility equal to 20,751.96.

The net revenue of organization 1 is now equal to 118,439.83; that of organization 2 is: 42,877.63, and that of organization 3: 147,850.66. All blood service organizations gain by servicing another region even in the case of competition.

5. Summary and Conclusions

In this paper, we developed a game theory model for blood donations that focuses on blood service organizations, which are increasingly challenged by competition in this unique industry. Indeed, blood is a product that is life-saving, but, at the same time, it cannot be manufactured, but must be donated by individuals. The model is network-based and the governing concept is that of Nash Equilibrium. The blood service organizations compete for blood donations in different regions and donors respond to the quality of service that the blood service organizations provide in blood collection. We formulate the governing equilibrium conditions as a variational inequality problem and prove that the solution is guaranteed to exist. We also provide conditions for uniqueness and establish additional theoretical results based on Lagrange theory associated with the lower and upper bounds on the quality service levels.

The modeling and algorithmic framework is illustrated through a series of examples for which we provide the computed equilibrium quality service levels as well as the Lagrange multipliers, along with analysis. The results obtained for the volume of donations, the revenue, and the costs of each blood service organization, and the monetized component of the transaction utility, which corresponds to the utility associated with providing quality service from the different examples, show how these components vary depending on the intensity of competition. The results also demonstrate how increased competition can yield
benefits for blood donors in terms of quality level of service. In addition, the examples show that the blood service organizations can benefit, from enhanced transaction utility, by providing additional blood collection sites. Importantly, our results also reveal that blood service organizations that do “good,” can also be financially sustainable. This is very important since the blood services industry is undergoing major disruptions brought about by increasing competition and volatile demand and supply.

The results obtained from this model can also provide important managerial insights. For blood collection regions that have lower quality levels, internal assessments can be made by the blood service organizations to figure out the individual factors responsible for such low levels such as longer wait time, unfriendly staff, etc. Accordingly, intervention techniques such as better scheduling, and improved training of staff can be implemented. However, improving the service quality implies increase in cost. The blood service organizations can easily observe the trade-off between the two from our model. It might be of value to blood service organizations to conduct surveys at the blood collection sites and come up with service quality measures that are weighted averages of the scores on the different operational facets mentioned in extant literature that affect donor retention. The blood banking industry is highly regulated by the Food and Drug Administration (FDA). Blood collection procedures have to follow strict protocols stated by the FDA in order to ensure that the collected blood is safe. Our paper suggests that, in addition to emphasizing safety measures for blood collection such as proper donor screening, attention should be given to the service quality aspects of the blood collection sites to maintain a steady supply of blood from motivated donors.

This research adds to the literature on game theory and healthcare and, specifically, to game theory and blood supply chains, which has been very limited, to-date. Future research on this topic entails modeling the effect of cooperation among the blood service organizations and the incorporation of capacity constraints.

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