The Closed-loop Supply Chain Network with Competition, Distribution Channel Investment, and Uncertainties

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Abstract

In this paper, a closed-loop supply chain network is investigated with decentralized decision-makers consisting of raw material suppliers, retail outlets, and the manufacturers that collect the recycled product directly from the demand market. We derive the optimality conditions of the various decision-makers, and establish that the governing equilibrium conditions can be formulated as a finite-dimensional variational inequality problem. We establish convergence of the proposed algorithm that can allow for the discussion of the effects of competition, distribution channel investment, yield and conversion rates, combined with uncertainties in demand, on equilibrium quantity transactions and prices. Numerical examples are provided for illustration.

Keywords: environmental responsibility; remanufacturing; closed-loop supply chain

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1. Introduction

The path toward sustainability to demonstrate environmental and social responsibility has led to an increasing attention to the lifecycle of a product with a focus on value-added recovery activities [1,2]. For example, in 2007 alone, Kodak collected 120 million single-use cameras [3], and in 2000, Fuji Xerox was the first to achieve zero landfill of used products in Japan [4,5]. The study of closed-loop supply chain (CLSC) networks is used to maximize the value created from product take-back, recovery, and re-distribution which reuses the entire product, and/or some of its modules [6,7,8]. Product take-back activities have received attention, in part, by legislation such as the Paper Recycling Directive, the End-of-Life Vehicle Directive, and the Waste Electrical and Electronic Equipment Directive (WEEE) within the European Union, intended to give manufacturers incentives to reduce the environmental burden of their end-of-life (EOL) products, while also removing the growing waste management cost from municipal governments [9].

Even before the emergence of product take-back laws, some firms were already participating in voluntary product recovery, as reported by Kodak, FujiFilm, Hewlett-Packard, IBM Europe, and Xerox to name a few. An environmentally friendly firm can use a sustainable product program as a strategic tool for not only environmental improvement, but to enhance the environmental image of their brand, generate revenue, serve their customers, and reduce production costs [10]. For example, on a life cycle basis, remanufacturing photocopiers consumes 20-70% less materials, labor, and energy and generates 35-50% less waste than conventional manufacturing using virgin materials [11]. Additionally, production costs are further reduced since the cost for a remanufactured part is generally 30-50% less than a new part would be [9].

There is an abundant amount of research available on the topic of CLSC management. For a comprehensive review of published literatures we refer the reader to the work of [12,13,14,15]. Based on the concept of equilibrium, first explored in a general forward supply chain setting by Nagurney et al. [16], Nagurney and Toyasaki [17] provide a variational inequality CLSC formulation model which involve manufacturers, retailers, and demand markets, with the inclusion of recycling. Subsequently, Yang et al. [18] expanded on the work of Nagurney and Toyasaki [17], and incorporated the work of Hammond and Beullens [19] and Sheu et al. [20], to strategically model the oligopolistic closed-loop supply chain, which include manufacturers who are involved in the production of a homogeneous commodity from raw materials and reusable materials, and recovery centers that can get subsidies from government organizations.

However, there is limited contribution in the literature that addresses the complexity that arises from the large number of actors in a decentralized CLSC system [6], which increases the intensity of competition, combined with significant product EOL issues [21]. This paper provides an innovative framework to study the effects of competition, combined with distribution channel investment, yield and conversion rates, uncertainties in demand, and the resulting implications on equilibrium quantity transactions and prices in the CLSC network. In the subsequent paragraphs, we discuss the motivation for the study of these issues since environmental recovery is an option that is underutilized as firms are unsure how to mitigate the ambiguity surrounding economic performance.

Interestingly, even though the remanufacturing sector is larger than the U.S. domestic steel industry with annual sales over \$53 billion [22], currently, very little, if any, value is recovered by the manufacturer [6] due to various yield factors. For example, Hewlett-Packard estimates that returns cost them as much as 2% of total outbound sales and less than 50% of the value of those product returns are being recovered [23]. The yield rate affects the viability of any recycling option by external factors such as the product's condition as a result of its utilization profile (affected by light vs heavy use and individual care for the products) [24], the ambiguity related to the product material content [25], and the product's structural design and level of modularity, which may make the disassembly process more or less difficult [26].

The economic viability of a recovery program is affected by not only the uncertainty in yield, which takes into account the conversion rates of recycled components to "like-new" products, but also the ambiguity surrounding customer demand [27]. Demand uncertainty is a known problem faced by firms to determine suitable levels of output before demand is known, which is classically known as the "newsboy" problem in operational research literature [28,29,30]. Inderfurth [31] studied the impact of uncertainties on recovery behavior but was restricted by stationary demands and return patterns. They determined, however, that in a non-stationary situation the impact of uncertainties could be even stronger, since excessive returns might happen more often. Shi et al. [32] studied the production planning problem for a multi-product closed loop system, in which the demands and their returns are uncertain and price-sensitive [33], but developed the model to include only the manufacturer's decision-making problem. In particular, we note that Dong et al. [34] studied the demand uncertainties in the decentralized supply chain network; but their model only considers the forward supply chain network.

Finally, we assume the manufacturers' recovery system investment is related to the collection efforts of the EOL products directly from the demand market. A direct collection system is evident in practice in response to increasing customers' demands for the removal of products as a service associated with the sale of new products. EOL products are collected either with monetary compensation or as a free-of-charge disposal where almost no costs are incurred to the end-users. Sprint PCS and Gateway customers may be paid for used, working technology products through trade-in programs; Apple offers free computer take-back and recycling with the purchase of a new Macintosh system; and Hewlett-Packard and Xerox Corporation provide free shipping and cartridge boxes for customer returns [35]. Given this information, it would make sense to include in our CLSC network model the assumption that the return rate and volume of the used product flow depends on the level of manufacturers' investment in the direct collection system [36,37].

In summary, the major innovations and contributions that differentiate this paper from the above mentioned works is: *1.* We consider that the manufacturers can invest in the reverse supply channel to increase the recycled product volume; *2.* The CLSC model captures the uncertainty in demand, which is associated with penalties, namely, inventory and shortage costs; *3.* Uncertain yield rate is modeled. We use the expected value since the probability distribution function of the yield rate is known. This information can be estimated based on historical data [38] and references therein. *4.* We simultaneously consider multiple decision-making entities such as the raw material suppliers, the manufacturers, and the retailers, and study the equilibrium prices and transactions.

The paper is organized as follows: In Section 2, we develop the CLSC network model with decentralized decision-makers consisting of raw material suppliers, retail outlets, and the manufacturers that collect the recycled product directly from the demand market. We derive the optimality conditions of the various decision-makers, and establish that the governing equilibrium conditions can be formulated as a finite-dimensional variational inequality problem.

In Section 3, we study qualitative properties of the equilibrium pattern, and under reasonable conditions, establish existence and uniqueness results. We discuss the characteristics of the functions in the variational inequality that enable us to establish convergence. This can allow for the discussion of the effects of competition, reverse distribution channel investment, yield rates, combined with uncertainties demand on equilibrium quantity transactions and prices. We illustrate the model by applying the modified projection algorithm to CLSC numerical examples in Section 4. The paper concludes with Section 5, in which we provide a summary of the paper and future research directions.

2. The Closed-loop Supply Chain with Competition and Distribution Channel Investment with Uncertainties in Yield and Demand

In this section, we develop the CLSC network model. In particular, we consider W material suppliers involved in the supply of raw material to N manufacturers, who make a homogenous product. Moreover, there are M retailers who deal with the local demand markets and face uncertain demands. Each retailer is assumed to be responsible for dealing with its own demand market. Such an assumption has been used in CLSC literature (See [34] for example). Consumers in these demand markets can return their used products to manufacturers at a price. We assume that the consumer is indifferent in their demand for brand-new products or remanufactured returns into as-new products, which is a common assumption used in the CLSC literature (cf. [39]). Furthermore, manufacturers choose the level of investment in such "reverse distribution channels" for the channel establishment and maintenance. It is reasonable to assume that the higher the investment in these reverse channels, the higher volume of recycled product the manufacturer can collect (we refer the reader to the discussion in the Introduction [24,25]). The structure of the CLSC network is depicted in Figure 1.

We denote a typical raw material supplier by m, a typical manufacturer by i, and a typical retailer/demand market by j. To make the presentation clear, we also list the relevant variables/notations below:

- q^w_{mi} is the amount of raw material supplier m sells to manufacture i (w stands for *raw*). Group these variables into a WN-dimensional column vector Q^w.

- q^f_{ij} is the amount of product manufacture i sells to retailer j. Group these variables into a NM-dimensional column vector Q^f.
- q^b_{ji} is the amount of used product sold by the consumers on demand market j to manufacture i (b stands for *backward*) which depends on the manufacturer i's distribution channel investment I_{ji}.
- I_{ji} is the investment of manufacturer i on the reverse distribution channel with the demand market j regarding recycled products. Group these variables into a MN-dimensional column vector I.
- β_{mi} is the conversion rate from the raw material of supplier m to products produced by manufacturer i;
- β_i is the probability that a recycled product can be converted to a new product. We assume that its probability distribution is known, denoted by $\mathcal{F}(\beta_i)$. $\bar{\beta}_i = \int \beta_i d\mathcal{F}(\beta_i)$ is the expected conversion rate from the used product;
- θ^b_i unit landfill cost for manufacture i;
- B_i budget of manufacturer i to invest in the reverse distribution channel.

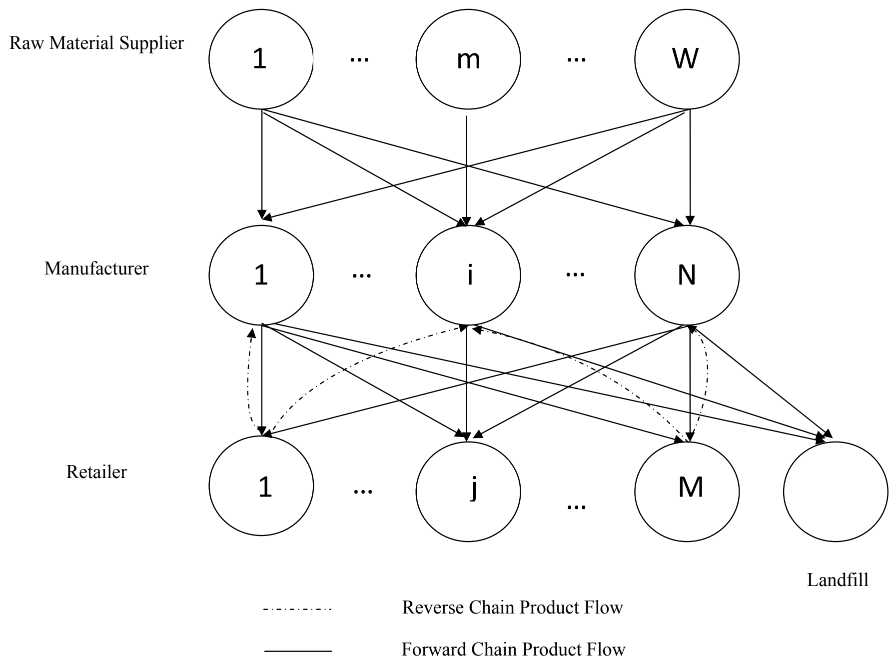

Figure 1: Closed-loop Supply Chain

2.1 The Problem Faced by The Raw Material Suppliers

We assume that raw material supplier m faces a procurement cost function f^w_m, which can generally depend upon the material volume Q^w. c^w_{mi} is the transaction cost associated with raw material supplier m transacting with manufacturer i. We assume that

c^w_{mi} also depends on Q^w. Denote ρ^{w*}_{1m} as the price charged by raw material supplier m to all the manufacturers. Given the above notation, we can express the criterion of profit maximization for raw material supplier m as:

$$\text{Max} \sum_{i=1}^{N} \rho^{w*}_{1m} q^w_{mi} - f^w_m(Q^w) - \sum_{i=1}^{N} c^w_{mi}(Q^w) \tag{1}$$

where $q^w_{mi} \geq 0 \ \forall i$.

We assume that $f_m(Q^w)$ and $c^w_{mi}(Q^w)$ are continuous and convex. Furthermore, we assume that raw material suppliers compete in a noncooperative fashion. Hence, the optimality conditions for all raw material suppliers simultaneously (cf. [34]) can be expressed as the following variational inequality: determine $Q^{w*} \in R^{WN}_+$, satisfying

$$\sum_{m=1}^{W} \sum_{i=1}^{N} (-\rho^{w*}_{1m} + \frac{\partial f^w_m(Q^{w*})}{\partial q^w_{mi}} + \frac{\partial c^w_{mi}(Q^{w*})}{\partial q^w_{mi}}) \times (q^w_{mi} - q^{w*}_{mi}) \geq 0, \ \ \forall Q^w \in R^{WN}_+. \tag{2}$$

2.2 The Problem Faced by The Manufacturers

Manufacturer i, who makes a profit by producing a homogeneous product (from both raw and reusable materials), incurs production costs from both raw materials and recycled materials, which is denoted by f_i. Note that the production cost function, in general, can depend on the entire production outputs of all manufacturers, which is Q^f. Similarly, we denote c^f_{ij} as the transaction cost of manufacturer i dealing with retailer j in the forward supply chain, which also depends on Q^f. Furthermore, c^b_{ji} is the transaction cost of manufacturer i dealing with the consumer at the market j in the reverse distribution channel, which depends on the flow on the reverse distribution channel, Q^b. We further assume that manufacturers can invest in these reverse distribution channels and therefore, q^b_{ji} depends on the investment level I_{ji}.

Let ρ^{f*}_{2i} denote the price charged by manufacture i for the transactions with the retailers. ρ^{b*}_{4j} is the price manufacturers pay the consumers at demand market j to collect the recycled product. Furthermore, $1 - \bar{\beta}_i$ is the expected unusable portion of the recycled product for manufacturer i, who face a unit landfill cost θ^b_i. Therefore, we can express the criterion of the expected profit maximization for manufacturer i as:

$$\text{Max} \sum_{j=1}^{M} \rho^{f*}_{2i} q^f_{ij} - f_i(Q^f) - \sum_{j=1}^{M} c^f_{ij}(Q^f) - \sum_{j=1}^{M} c^b_{ji}(Q^b) - \sum_{m=1}^{W} \rho^{w*}_{1m} q^w_{mi} - \sum_{j=1}^{M} \rho^{b*}_{4j} q^b_{ji}(I_{ji})$$
$$- \sum_{j=1}^{M} \theta^b_i (1 - \bar{\beta}_i) q^b_{ji}(I_{ji}) - \sum_{j=1}^{M} I_{ji} \tag{3}$$

subject to:

$$\sum_{j=1}^{M} I_{ji} \leq B_i \tag{4}$$

$$\sum_{j=1}^{M} q^f_{ij} \leq \sum_{m=1}^{W} \beta_{mi} q^w_{mi} + \sum_{j=1}^{M} \bar{\beta}_i q^b_{ji}(I_{ji}) \tag{5}$$

and $q_{ij}^f, q_{ji}^b, q_{mi}^w, I_{ji} \geq 0 \; \forall \; i, j, m.$

In Equation (3), the first term is the revenue of the manufacturer i from selling product to retailers. The second, third, and fourth terms are his various cost functions. The fifth term is his total payment to the raw material suppliers and the sixth term is his total payment to the consumers to collect recycled products. Furthermore, the seventh term is the landfill cost and finally, the last term is his total investment in the reverse distribution channels. Equation (4) is manufacturer i's investment budget constraint. Equation (5) indicates the total output of manufacturer i cannot exceed the supply from both raw material suppliers and the recycled products from consumers.

Furthermore, we assume that the production and transaction costs for each manufacturers are continuous and convex. We also assume that manufacturers compete in a noncooperative fashion. Hence, the optimality conditions for all manufacturers simultaneously (cf. [41]) can be expressed as the following variational inequality: determine $(Q^{w*}, Q^{f*}, I^*, \gamma^*, \lambda^*) \in R_+^{WN+NM+MN+N+N}$, satisfying

$$\sum_{i=1}^{N}\sum_{j=1}^{M}\left[-\rho_{2i}^{f*} + \frac{\partial f_i(Q^{f*})}{\partial q_{ij}^f} + \frac{\partial c_{ij}^f(Q^{f*})}{\partial q_{ij}^f} + \lambda_i^*\right] \times [q_{ij}^f - q_{ij}^{f*}]$$

$$+ \sum_{m=1}^{W}\sum_{i=1}^{N}[\rho_{1m}^{w*} - \lambda_i^* \beta_{mi}] \times [q_{mi}^w - q_{mi}^{w*}]$$

$$+\sum_{i=1}^{N}\sum_{j=1}^{M}\left[\frac{\partial c_{ji}^b(Q^b)}{\partial q_{ji}^b}\frac{dq_{ji}^b(I_{ji}^*)}{dI_{ji}} + \rho_{4j}^{b*}\frac{dq_{ji}^b(I_{ji}^*)}{dI_{ji}} + \theta_i^b(1-\bar{\beta}_i)\frac{dq_{ji}^b(I_{ji}^*)}{dI_{ji}} - \lambda_i^*\bar{\beta}_i\frac{dq_{ji}^b(I_{ji}^*)}{dI_{ji}} + 1 + \gamma_i^*\right]$$

$$\times[I_{ji} - I_{ji}^*] + \sum_{i=1}^{N}\left[B_i - \sum_{j=1}^{M} I_{ji}^*\right] \times [\gamma_i - \gamma_i^*] + \sum_{i=1}^{N}\left[\sum_{m=1}^{W}\beta_{mi}q_{mi}^{w*} + \sum_{j=1}^{M}\bar{\beta}_i q_{ji}^b(I_{ji}^*) - \sum_{j=1}^{M} q_{ij}^{f*}\right]$$

$$\times[\lambda_i - \lambda_i^*] \geq 0 \;\; \forall (Q^w, Q^f, I, \gamma, \lambda) \in R_+^{WN+NM+MN+N+N}, \tag{6}$$

In the above equation, λ_i is the Lagrangian multiplier for the constraint (5) for manufacturer i. γ_i is the Lagrangian multiplier for manufacturer i's budget constraint.

2.3 The Problem Faced by The Retailers

The retailers, in turn, must decide how much to order from the manufacturers in order to cope with the random demand while still seeking to maximize their profits. A retailer j is also faced with a handling cost, which may include, for example, the display and storage cost associated with the product. This cost is denoted by C^r, which, in turn, depends on Q^f.

Let ρ_{3j} denote the demand price of the product associated with retailer j. We assume that $\hat{d}_j(\rho_{3j})$ is the random demand for the product at the demand price ρ_{3j} at retailer outlet j. The probability density function of the random demand at the demand market j is $f_j^d(x, \rho_{3j})$, with ρ_{3j} serving as a parameter. Hence, we assume the density function may vary with the demand price. Let P_j be the cumulative density function of $\hat{d}_j(\rho_{3j})$, that is, $P_j(x, \rho_{3j}) = P_j(\hat{d}_j \leq x) = \int f_j^d(x, \rho_{3j})dx$. Let $s_j = \sum_{i=1}^N q_{ij}^f$. Note that the expected values of excess supply and excess demand of retailer j are scalar functions of s_j. In particular, let e_j^+ and e_j^- denote, respectively, the expected values: $E\left[\triangle_j^+\right]$ and $E\left[\triangle_j^-\right]$, that is,

$$e_j^+(s_j, \rho_{3j}) \equiv E\left[\triangle_j^+\right] = \int_0^{s_j} (s_j - x) f_j^d(x, \rho_{3j})dx, \tag{7}$$

$$e_j^-(s_j, \rho_{3j}) \equiv E\left[\triangle_j^-\right] = \int_{s_j}^{\infty} (x - s_j) f_j^d(x, \rho_{3j})dx. \tag{8}$$

Given the above notation, we can express the criterion of profit maximization for retailer j as:

$$\text{Max } E\left[\rho_{3j}^* \min\{s_j, \hat{d}_j\}\right] - E\left[\pi_j^+\triangle_j^+ + \pi_j^-\triangle_j^-\right] - C_j^r(Q^f) - \sum_{j=1}^n \rho_{2i}^{f*} q_{ij}^f \tag{9}$$

Where π_j^+ refers to the unit inventory cost, π_j^- refers to the unit shortage cost. Objective function (9) thus expresses that the difference between the expected revenue minus the expected inventory and shortage cost, the handling cost, and the payout to the manufacturers is to be maximized.

Applying now the definitions of $\triangle_j^+$ and $\triangle_j^-$, we know that $\min\{s_j, \hat{d}_j(\rho_{3j})\} = \hat{d}_j - \triangle_j^-$. Therefore, the objective function (9) can be expressed as

$$\text{Max } \rho_{3j}^* d_j(\rho_{3j}^*) - (\rho_{3j}^* + \pi_j^-)e_j^-(s_j, \rho_{3j}^*) - \pi_j^+ e_j^+(s_j, \rho_{3j}^*) - C_j^r(Q^f) - \sum_{j=1}^n \rho_{2i}^{f*} q_{ij}^f \tag{10}$$

where $d_j(\rho_{3j}) \equiv E[\hat{d}_j(\rho_{3j})]$ is a scalar function of ρ_{3j}.

We now consider the optimality conditions of the retailers assuming that each retailer is faced with the optimization problem (10). Here, we also assume that the retailers compete in a noncooperative manner so that each maximizes his profit, given the actions of the other retailers. Note that, at this point, we consider that retailers seek to determine the amount that they wish to obtain from the manufacturers. First, however, we make the following derivation and introduce the necessary notation:

$$\frac{\partial e_j^+(s_j, \rho_{3j}^*)}{\partial q_{ij}^f} = P_j(s_j, \rho_{3j}^*) \tag{11}$$

$$\frac{\partial e_j^-(s_j, \rho_{3j}^*)}{\partial q_{ij}^f} = P_j(s_j, \rho_{3j}^*) - 1. \tag{12}$$

Assuming that the handling cost for each retailer is continuous and convex, then the optimal conditions for all retailers satisfy the variational inequality: determine $Q^{f*} \in$

R_+^{NM}, satisfying:

$$\sum_{i=1}^{N}\sum_{j=1}^{M}\left[(\rho_{3j}^*+\pi_j^-)(P_j(s_j^*,\rho_{3j}^*)-1)+\pi_j^+P_j(s_j^*,\rho_{3j}^*)+\frac{\partial C_j^r(Q^{f*})}{\partial q_{ij}^f}+\rho_{2i}^{f*}\right]\times[q_{ij}^f-q_{ij}^{f*}]\geq 0,$$

$$\forall Q^f \in R_+^{NM} \tag{13}$$

2.4 The Demand Market Equilibrium Conditions

The equilibrium conditions associated with the transactions that take place between the retailers and the consumers are the *stochastic economic equilibrium* conditions, which, mathematically, take on the following form: for any retailer j; $j=1,\ldots,M$:

$$\hat{d}_j(\rho_{3j}^*)\begin{cases}\leq \sum_{i=1}^N q_{ij}^{f*}, & \text{a.e., if} \quad \rho_{3j}^*=0,\\ = \sum_{i=1}^N q_{ij}^{f*}, & \text{a.e., if} \quad \rho_{3j}^*>0\end{cases} \tag{14}$$

where a.e. means that the corresponding equality or inequality holds almost everywhere.

Condition (14) states that, if the demand price at the outlet j that consumers are willing to pay is positive, then the quantities purchased by the retailer from the manufacturers in the aggregate is equal to the demand, with exceptions of zero probability. These conditions correspond to the well-known equilibrium conditions. Related equilibrium conditions were proposed in [40].

Equilibrium condition (14) is equivalent to the following variational inequality problem, after taking the expected value and summing over all retailers j: determine $\rho_{3j}^* \in R_+^M$ satisfying:

$$\sum_{j=1}^{M}(\sum_{i=1}^{N} q_{ij}^{f*}-d_j(\rho_{3j}^*))\times[\rho_{3j}-\rho_{3j}^*]\geq 0,\ \forall \rho_3\in R_+^M, \tag{15}$$

where ρ_3 is the M-dimensional vector with components: $\rho_{31},\ldots,\rho_{3M}$.

2.5 The Equilibrium Conditions for the Reverse Supply Chain

In the reverse chain, the consumer markets behavior can be characterized by equation (16) subject to the constraint (17).

$$\alpha_j(Q^b(I^*))\begin{cases}\geq \rho_{4j}^{b*}, & \text{if} \quad I_{ji}^*=0,\forall i\\ = \rho_{4j}^{b*}, & \text{if} \quad I_{ji}^*\geq 0,\forall i\end{cases} \tag{16}$$

subject to

$$\sum_{i=1}^{N} q_{ji}^b(I_{ji}^*)\leq \zeta_j(\sum_{i=1}^{N} q_{ij}^{f*}) \tag{17}$$

Where α_j represents the disutility that consumers face at demand market j for returning the used product. We assume it is an increasing function of the recycle volume

$Q^b(I^*)$, which means that the more a consumer returns the recycled products, the more inconvenience he/she will encounter. ρ^b_{4j} refers to the price of the recyclable products that the manufacturers pay to collect from demand market j. Thus equation (16) states that if a manufacturer opt out in investing in the establishment and maintenance of the reverse distribution channel, the consumer's compensation for returning the recycled product is smaller than his/her disutility. On the other hand, if a manufacturer invests in the reverse distribution channel, the consumers' disutility will be compensated. Constraint (17) states that the amount of the recycled products at the demand market j must not exceed the amount purchased from the retailers multiply the return ratio ζ_j.

The above equilibrium conditions correspond to the following variational inequality: determine $(I^*, Q^{f*}, \xi^*) \in R_+^{MN+NM+M}$ satisfying

$$\sum_{i=1}^{N}\sum_{j=1}^{M}\left[\alpha_j(Q^b)\frac{dq^b_{ji}(I^*_{ji})}{dI_{ji}} - \rho^{b*}_{4j}\frac{dq^b_{ji}(I^*_{ji})}{dI_{ji}} + \xi^*_j\frac{dq^b_{ji}(I^*_{ji})}{dI_{ji}}\right]\times(I_{ji}-I^*_{ji}) - \sum_{i=1}^{N}\sum_{j=1}^{M}(\zeta_j\xi^*_j)\times(q^f_{ij}-q^{f*}_{ij})$$

$$+\sum_{j=1}^{M}(\zeta_j(\sum_{i=1}^{N} q^{f*}_{ij}) - \sum_{i=1}^{N} q^b_{ji}(I^*_{ji}))\times(\xi_j-\xi^*_j) \geq 0, \ \forall I_{ji}\in R^{MN}_+, \ \forall q^f_{ij}\in R^{MN}_+, \ \forall \xi_j \in R^M_+. \tag{18}$$

By combining the consumers market behavior in both the forward and reverse chains, the equilibrium conditions of the consumer markets can be formulated as the following variational inequality: determine $(Q^{f*}, I^*, \rho_3^*, \xi^*) \in R_+^{WN+NM+M+M}$ satisfying:

$$\sum_{i=1}^{N}\sum_{j=1}^{M}(-\zeta_j\xi^*_j)\times[q^f_{ij}-q^{f*}_{ij}]$$

$$+\sum_{j=1}^{M}(\sum_{i=1}^{N} q^{f*}_{ij} - d_j(\rho^*_{3j}))\times[\rho_{3j}-\rho^*_{3j}] + \sum_{i=1}^{N}\sum_{j=1}^{M}\left[\alpha_j(Q^b)\frac{dq^b_{ji}(I^*_{ji})}{dI_{ji}} - \rho^{b*}_{4j}\frac{dq^b_{ji}(I^*_{ji})}{dI_{ji}} + \xi^*_j\frac{dq^b_{ji}(I^*_{ji})}{dI_{ji}}\right]$$

$$\times(I_{ji}-I^*_{ji}) + \sum_{j=1}^{M}(\zeta_j(\sum_{i=1}^{N} q^{f*}_{ij}) - \sum_{i=1}^{N} q^b_{ji}(I^*_{ji}))\times(\xi_j-\xi^*_j) \geq 0, \ \forall(Q^f, I, \rho_3, \xi)\in R_+^{WN+NM+M+M} \tag{19}$$

2.6 The Equilibrium Conditions of the Closed-loop Supply Chain Network

In equilibrium, we must have that the sum of the optimality conditions for all raw material suppliers, as expressed by inequality (2), the optimality conditions of the manufactures, as expressed by condition (6), the optimality conditions for all retailers, as expressed by inequality (13), and the market equilibrium conditions for both forward chain and reverse chain, as expressed by inequality (19) must be satisfied. Hence, the shipments that the suppliers ship to the manufactures, after being converted into the product, must be equal to the shipments that the manufacturers accept from the suppliers. In addition, the shipments of the recycled products from the consumers to the manufacturers, must be equal to those accepted by the manufacturers. We state this explicitly in the following definition:

Definition 1: The Closed-loop Supply Chain Network Equilibrium *The equilibrium state of the CLSC with uncertain yield and demand is one where the product flow between the tiers of decision makers coincide the product shipment and prices satisfy the sum of the optimality conditions (2), (6), (13), and conditions (19).*

The summation of the inequalities (2), (6), (13), and (19), after algebraic simplification, results in the following:

Theorem 1: The Variational Inequality Formulation
The equilibrium conditions governing the CLSC network model with uncertain yield and random demands are equivalent to the solution of the variational inequality problem given by: determine $(Q^{w*}, Q^{f*}, I^*, \gamma^*, \lambda^*, \rho_3^*, \xi^*)$
$\in R_+^{WN+NM+MN+N+N+M+M}$

$$\sum_{m=1}^{W}\sum_{i=1}^{N}(\frac{\partial f_m^w(Q^{w*})}{\partial q_{mi}^w} + \frac{\partial c_{mi}^w(Q^{w*})}{\partial q_{mi}^w} - \lambda_i^*\beta_{mi}) \times (q_{mi}^w - q_{mi}^{w*})$$

$$+\sum_{i=1}^{N}\sum_{j=1}^{M}[(\rho_{3j}^* + \lambda_j^-)(P_j(s_j^*, \rho_{3j}^*) - 1) + \lambda_j^+ P_j(s_j^*, \rho_{3j}^*) + \frac{\partial C_j^r(Q^{f*})}{\partial q_{ij}^f} + \frac{\partial f_i(Q^{f*})}{\partial q_{ij}^f} + \frac{\partial c_{ij}^f(Q^{f*})}{\partial q_{ij}^f}$$

$$+\lambda_i^* - \zeta_j\xi_j^*] \times [q_{ij}^f - q_{ij}^{f*}]$$

$$+\sum_{i=1}^{N}\sum_{j=1}^{M}[\frac{\partial c_{ji}^b(Q^b)}{\partial q_{ji}^b}\frac{dq_{ji}^b(I_{ji}^*)}{dI_{ji}} + \theta_i^b(1-\bar{\beta}_i)\frac{dq_{ji}^b(I_{ji}^*)}{dI_{ji}} - \lambda_i^*\bar{\beta}_i\frac{dq_{ji}^b(I_{ji}^*)}{dI_{ji}} + 1 + \gamma_i^*$$

$$+\alpha_j(Q^b)\frac{dq_{ji}^b(I_{ji}^*)}{dI_{ji}} + \xi_j^*\frac{dq_{ji}^b(I_{ji}^*)}{dI_{ji}}] \times (I_{ji} - I_{ji}^*)$$

$$+\sum_{i=1}^{N}\left[B_i - \sum_{j=1}^{M} I_{ji}^*\right] \times [\gamma_i - \gamma_i^*]$$

$$+\sum_{i=1}^{N}\left[\sum_{m=1}^{W}\beta_{mi}q_{mi}^{w*} + \sum_{j=1}^{M}\bar{\beta}_i q_{ji}^b(I_{ji}^*) - \sum_{j=1}^{M} q_{ij}^{f*}\right] \times [\lambda_i - \lambda_i^*]$$

$$+\sum_{j=1}^{M}(\sum_{i=1}^{N} q_{ij}^{f*} - d_j(\rho_{3j}^*)) \times [\rho_{3j} - \rho_{3j}^*]$$

$$+\sum_{j=1}^{M}(\zeta_j(\sum_{i=1}^{N} q_{ij}^{f*}) - \sum_{i=1}^{N} q_{ji}^b(I_{ji}^*)) \times (\xi_j - \xi_j^*) \geq 0$$

$$\forall (Q^w, Q^f, I, \gamma, \lambda, \rho_3, \xi) \in R_+^{WN+NM+MN+N+N+M+M}, \tag{20}$$

Proof: The formulation is developed using the standard variational inequality theory (cf. Nagurney [41]).

For easy reference in the subsequent sections, variational inequality problem (20) can be rewritten in standard variational inequality form (cf. [41]) as follows:

$$F(X)^T \cdot (X - X^*) \geq 0, \quad \forall X \in \mathcal{K}. \tag{21}$$

where $\mathcal{K} \equiv \{(Q^w, Q^f, I, \gamma, \lambda, \rho_3, \xi) | q^w_{mi} \geq 0,\ q^f_{ij} \geq 0,\ I_i \geq 0,\ \gamma_i \geq 0,\ \lambda_i \geq 0, \rho_{3j} \geq 0,$ and $\xi_j \geq 0,\ \forall m, i, j\}$. Note that ρ_3 is the M-dimensional vector with components: $\rho_{31}, \ldots, \rho_{3M}$.

$$X \equiv \{(Q^w, Q^f, I, \gamma, \lambda, \rho_3, \xi)\}$$

and $F(X) \equiv \{F^1_{mi}, F^2_{ij}, F^3_{ji}, F^4_i, F^5_i, F^6_j, F^7_j\}_{m=1,\ldots,W, i=1,\ldots,N\, j=1,\ldots,M}$ where the terms of F correspond to the terms preceding the multiplication signs in inequality (20).

Note that the variables in the model (and which can be determined from the solution of either variational inequality (20) or (21)) are: the equilibrium raw material shipments given by Q^w, the equilibrium product sales from the manufacturers to the retailers denoted by Q^f, the investment level I, as well as the equilibrium demand prices ρ_3. We now discuss how to recover the prices ρ_1 that suppliers charge manufacturers, ρ_2 associated with the middle tier of the supply chain and ρ_4 associated with the reversed supply chain.

First note that from (2), we have that if $q^{w*}_{mi} > 0$, then the price $\rho^{w*}_{1m} = \frac{\partial f^w_m(Q^{w*})}{\partial q^w_{mi}} + \frac{\partial c^w_{mi}(Q^{w*})}{\partial q^w_{mi}}$. Also, from (6) it follows that if $q^{f*}_{ij} > 0$, then the price $\rho^{f*}_{2i} = \frac{\partial f_i(Q^{f*})}{\partial q^f_{ij}} + \frac{\partial c^f_{ij}(Q^{f*})}{\partial q^f_{ij}} + \lambda^*_i$. Furthermore, from (18), if the optimum investment quantity $I^*_{ji} > 0$, then the buy-back price $\rho^{b*}_{4j} = \alpha_j(Q^b) + \xi^*_j$.

3. Qualitative Properties

In this section, we provide some qualitative properties of the solution to variational inequality (20). In particular, we establish the existence of a unique solution satisfying variational inequality (20).

Since the feasible set is not compact, we cannot derive existence simply from the assumption of the continuity of the functions. Nevertheless, we can impose a rather weak condition to guarantee the existence of a solution.

Let $\mathcal{K}_b \equiv \{(Q^f, Q^w, I, \gamma, \lambda, \rho_3, \xi) | 0 \leq (Q^f, Q^w, I, \gamma, \lambda, \rho_3, \xi) \leq b\}$ where $b = (b_1, b_2, B, b_3, b_4, b_5, b_6) \geq 0$ and $Q^f \leq b_1,\ Q^w \leq b_2,\ I \leq B,\ \gamma \leq b_3,\ \lambda \leq b_4, \rho_3 \leq b_5,$ and $\xi \leq b_6$. Indeed $\mathcal{K}_b$ is a bounded closed convex subset of $R^{WN+NM+MN+N+N+M+M}_+$. Thus, the variational inequality (20) admits at least one solution $X_b \in \mathcal{K}_b$, from the standard theory of variational inequalities, since $\mathcal{K}_b$ is compact and the functions are continuous. Following Kinderlehrer and Stampacchia [42] (see also Theorem 1.5 in Nagurney [41]), we then have:

Theorem 2

Variational inequality (20) admits a solution if and only if there exists $b > 0$, such that variational inequality (20) admits a solution in $\mathcal{K}_b$ with $Q^f < b_1$, $Q^w < b_2$, $Q^b < b_3$, $\gamma < b_4$, $\lambda < b_5, \rho_3 < b_6, \xi < b_7$, and $I \leq B$.

Theorem 3: ***Existence***
Suppose that there exist positive constants R, S, T, with $T > 0$, such that:

$$\frac{\partial f^w_m(Q^{w*})}{\partial q^w_{mi}} + \frac{\partial c^w_{mi}(Q^{w*})}{\partial q^w_{mi}} - \lambda^*_i \beta_{mi} \geq R, \ \forall Q^w \text{ with } q^w_{mi} \geq S, \forall m, i \tag{22}$$

$$(\rho^*_{3j} + \lambda^-_j)(P_j(s^*_j, \rho^*_{3j}) - 1) + \lambda^+_j P_j(s^*_j, \rho^*_{3j}) + \frac{\partial C^r_j(Q^{f*})}{\partial q^f_{ij}} + \frac{\partial f_i(Q^{f*})}{\partial q^f_{ij}} + \frac{\partial c^f_{ij}(Q^{f*})}{\partial q^f_{ij}} \geq R,$$

$$\forall Q^f \text{ with } q^f_{ij} \geq S, \forall i, j \tag{23}$$

$$\frac{\partial c^b_{ji}(Q^b)}{\partial q^b_{ji}} \frac{\partial q^b_{ji}(I^*_{ji})}{\partial I_{ji}} + \theta^b_i(1 - \bar{\beta}_i)\frac{\partial q^b_{ji}(I^*_{ji})}{\partial I_{ji}} - \lambda^*_i \bar{\beta}_i \frac{\partial q^b_{ji}(I^*_{ji})}{\partial I_{ji}} + 1 + \gamma^*_i + \alpha_j(Q^b)\frac{\partial q^b_{ji}(I^*_{ji})}{\partial I_{ji}} +$$

$$\xi^*_j \frac{dq^b_{ji}(I^*_{ji})}{dI_{ji}} \geq R, \ \forall I \text{ with } I_{ji} \geq S, \forall i, j \tag{24}$$

$$d_j(\rho^*_3) \leq S, \forall \rho_3 \text{ with } \rho_{3j} > T, \forall j \tag{25}$$

Then, variational inequality (20) admits at least one solution.

Assumptions (22)-(25) are reasonable from an economics perspective. In particular, according to (22), when the raw material shipment from a supplier to a manufacturer is large, we can expect that the corresponding sum of the marginal costs associated with the procurement, the production, the shipment and the holding of the raw material will exceed a positive lower bound. Similar rationale exists for assumption (23). If the amount of transaction between a manufacture and retailer is large, the sum of expected inventory cost, opportunity cost, marginal holding cost and related production and transaction cost will exceed a lower bound. On the other hand, (24) shows a fact from consumer's view point. If the manufacturer shows enough environmental consciousness and invest in recycling, the consumer would be able to sell recyclable products at a fairly good price and therefore, the sum of cost associated with recycling will exceed a lower bound too. Moreover, according to assumption (25), if the price of the product at the retailer is high, we can expect that the demand for the product will be bounded from above at that market.

Theorem 4: ***Monotonicity***
Suppose that the production cost functions f_i; $i = 1, \ldots, N$ are additive and can be written as $f^1_i(q^1_i) + f^2_i(\bar{q_i}^1)$, where $f^1_i(q^1_i)$ is the internal production cost that depends solely on the firm i's own output level q^1_i, and $f^2_i(\bar{q_i}^1)$ is the interdependent part of the production cost that is a function of all the other firms' output levels $\bar{q_i}^1 = (q_1, ..., q_{i-1}, q_{i+1}, ..., q_m)$, and furthermore, assume that f^1_i; $i = 1, \ldots, N$, are

convex functions. Similarly, the procurement cost functions f_m^w; $m = 1, \ldots, W$ are additive as well. If the c_{mi}^w, c_{ij}^f, C_j^r and c_{ji}^b functions are convex; the α_j functions are monotone increasing; and d_j functions are monotone decreasing functions for all m, i, j. We further assume that $q_{ji}^b(I_{ji})$ is a linear function of I_{ji} with non-negative slope. Then the vector function F defined in (20) is monotone, that is,

$$(F(X') - F(X''))^T \cdot (X' - X'') \geq 0, \quad \forall X', X'' \in \mathcal{K}. \tag{26}$$

Proof: The expression $(F(X') - F(X''))^T \cdot (X' - X'')$ is equal to the expression (after some algebraic simplification):

$$\sum_{m=1}^{W}\sum_{i=1}^{N}\left[(\frac{\partial f_m^w(Q^{w'})}{\partial q_{mi}^w} + \frac{\partial c_{mi}^w(Q^{w'})}{\partial q_{mi}^w}) - (\frac{\partial f_m^w(Q^{w''})}{\partial q_{mi}^w} + \frac{\partial c_{mi}^w(Q^{w''})}{\partial q_{mi}^w})\right] \times (q_{mi}^{w'} - q_{mi}^{w''})$$

$$+\sum_{i=1}^{N}\sum_{j=1}^{M}([\rho_{3j}^{'} P_j(s_j^{'}, \rho_{3j}^{'}) + \lambda_j^-(P_j(s_j^{'}, \rho_{3j}^{'}) - 1) + \lambda_j^+ P_j(s_j^{'}, \rho_{3j}^{'}) + \frac{\partial C_j^r(Q^{f'})}{\partial q_{ij}^f} + \frac{\partial f_i(Q^{f'})}{\partial q_{ij}^f}$$

$$+\frac{\partial c_{ij}^f(Q^{f'})}{\partial q_{ij}^f}] - [\rho_{3j}^{''} P_j(s_j^{''}, \rho_{3j}^{''}) + \lambda_j^-(P_j(s_j^{''}, \rho_{3j}^{''}) - 1) + \lambda_j^+ P_j(s_j^{''}, \rho_{3j}^{''}) + \frac{\partial C_j^r(Q^{f''})}{\partial q_{ij}^f} + \frac{\partial f_i(Q^{f''})}{\partial q_{ij}^f}$$

$$+\frac{\partial c_{ij}^f(Q^{f''})}{\partial q_{ij}^f}]) \times [q_{ij}^{f'} - q_{ij}^{f''}]$$

$$+\sum_{i=1}^{N}\sum_{j=1}^{M}((\frac{\partial c_{ji}^b(Q^{b'})}{\partial q_{ji}^b}\frac{dq_{ij}^b(I_{ji}^{'})}{dI_{ji}} + \theta_i^b(1-\bar{\beta}_i)\frac{dq_{ij}^b(I_{ji}^{'})}{dI_{ji}} + \alpha_j(Q^{b'})\frac{dq_{ij}^b(I_{ji}^{'})}{dI_{ji}})$$

$$-(\frac{\partial c_{ji}^b(Q^{b''})}{\partial q_{ji}^b}\frac{dq_{ij}^b(I_{ji}^{''})}{dI_{ji}} + \theta_i^b(1-\bar{\beta}_i)\frac{dq_{ij}^b(I_{ji}^{''})}{dI_{ji}} + \alpha_j(Q^{b''})\frac{dq_{ij}^b(I_{ji}^{''})}{dI_{ji}})) \times [I_{ji}^{'} - I_{ji}^{''}]$$

$$+\sum_{j=1}^{M}(d_j(\rho_{3j}^{''}) - d_j(\rho_{3j}^{'})) \times [\rho_{3j}^{'} - \rho_{3j}^{''}]$$

$$\equiv \sum_{m=1}^{W}\sum_{i=1}^{N}\left[(\frac{\partial f_m^w(Q^{w'})}{\partial q_{mi}^w} - \frac{\partial f_m^w(Q^{w''})}{\partial q_{mi}^w}) + (\frac{\partial c_{mi}^w(Q^{w'})}{\partial q_{mi}^w} - \frac{\partial c_{mi}^w(Q^{w''})}{\partial q_{mi}^w})\right] \times (q_{mi}^{w'} - q_{mi}^{w''})$$

$$+\sum_{i=1}^{N}\sum_{j=1}^{M}((\rho_{3j}^{'} P_j(s_j^{'}, \rho_{3j}^{'}) + \lambda_j^-(P_j(s_j^{'}, \rho_{3j}^{'}) - 1) + \lambda_j^+ P_j(s_j^{'}, \rho_{3j}^{'}))$$

$$-(\rho_{3j}^{''} P_j(s_j^{''}, \rho_{3j}^{''}) + \lambda_j^-(P_j(s_j^{''}, \rho_{3j}^{''}) - 1) + \lambda_j^+ P_j(s_j^{''}, \rho_{3j}^{''}))) \times [q_{ij}^{f'} - q_{ij}^{f''}]$$

$$+\sum_{i=1}^{N}\sum_{j=1}^{M}\left[(\frac{\partial C_j^r(Q^{f'})}{\partial q_{ij}^f} - \frac{\partial C_j^r(Q^{f''})}{\partial q_{ij}^f}) + (\frac{\partial f_i(Q^{f'})}{\partial q_{ij}^f} - \frac{\partial f_i(Q^{f''})}{\partial q_{ij}^f}) + (\frac{\partial c_{ij}^f(Q^{f'})}{\partial q_{ij}^f} - \frac{\partial c_{ij}^f(Q^{f''})}{\partial q_{ij}^f})\right]$$

$$\times[q_{ij}^{f'} - q_{ij}^{f''}]$$

$$+\sum_{i=1}^{N}\sum_{j=1}^{M}((\frac{\partial c^b_{ji}(Q^{b'})}{\partial q^b_{ji}}\frac{dq^b_{ij}(I'_{ji})}{dI_{ji}}-\frac{\partial c^b_{ji}(Q^{b''})}{\partial q^b_{ji}}\frac{dq^b_{ij}(I''_{ji})}{dI_{ji}})+(\alpha_j(Q^{b'})\frac{dq^b_{ij}(I'_{ji})}{dI_{ji}}-\alpha_j(Q^{b''})\frac{dq^b_{ij}(I''_{ji})}{dI_{ji}})$$

$$+(\theta^b_i(1-\bar{\beta}_i)\frac{dq^b_{ij}(I'_{ji})}{dI_{ji}}-\theta^b_i(1-\bar{\beta}_i)\frac{dq^b_{ij}(I''_{ji})}{dI_{ji}}))\times[I'_{ji}-I''_{ji}]$$

$$+\sum_{j=1}^{M}(d_j(\rho''_{3j})-d_j(\rho'_{3j}))\times[\rho'_{3j}-\rho''_{3j}]$$

$$=(I)+(II)+(III)+(IV)+(V) \tag{27}$$

Based on the above assumptions, one can have $(I)\geq 0$,$(III)\geq 0$, $(IV)\geq 0$, $(V)\geq 0$. The proof of $(II)\geq 0$ could be derived from lemma 1 in Dong, Zhang, and Nagurney [34]. Therefore (26) must be greater than or equal to zero, under the above assumptions, and, hence, $F(X)$ is monotone. □

Theorem 5: ***Lipschitz Continuity***
The function F is Lipschitz continuous, that is,

$$\|F(X')-F(X'')\|\leq L\|X'-X''\|,\quad \forall X',X''\in\mathcal{K},\ \textit{where}\ L>0, \tag{28}$$

under the following conditions:
(i). f_i and f^w_m are additive and have bounded second-order derivatives, for all m,i;
(ii). c^w_{mi}, c^f_{ij}, c^b_{ji}, and C^r_j have bounded second-order derivatives, for all m,i,j;
(iii). α_j, d_j and $q^b_{ji}(I_{ji})$ have bounded first-order derivatives for all i,js.

Theorem 6: ***Existence and Uniqueness of a Solution to the Variational Inequality Problem***
Assume the conditions of Theorem 5. Then, the function that enters the variational inequality (20) has a unique solution in $\mathcal{K}$.

Proof: Follows from Theorem 2.5 in Nagurney and Zhang [43]. □

4. Numerical Examples

In this section, the modified projection method of Korpelevich [44] is utilized to solve variational inequality (21). The algorithm is guaranteed to converge if the function F that enters the variational inequality is monotone and Lipschitz Continuous (and that a solution exists). The algorithm was implemented in MATLAB on a Dell E6410 series computer. The convergence criterion, that is, the absolute value of the product shipments and prices between two successive iterations, differed by no more than 10^{-5}. The parameter in the modified projection method (see (29) and (30)) was set to .001 and we assumed, for the sake of illustration, that the demands associated with the retail outlets followed a uniform distribution. Hence, we assumed that the random demand, $\hat{d_k}(\rho_{2j})$, faced by retailer j, is uniformly distributed in $[0,\frac{b_j}{\rho_{2j}}]$, $b_j>0$; $j=1,\ldots,M$. Therefore,

$$P_j(x, \rho_{2j}) = \frac{x\rho_{2j}}{b_j}, \tag{29}$$

$$\mathcal{F}_k(x, \rho_{3k}) = \frac{\rho_{2j}}{b_j} \tag{30}$$

$$d_k(\rho_{3k}) = E(\hat{d_k}) = \frac{1}{2}\frac{b_j}{\rho_{2j}}; \;\; j = 1, \ldots, M \tag{31}$$

We assume that there are two raw material suppliers, two manufacturers, and two retailers. The conversion rate, β_{mi}, from raw material to final product is equal to 0.8 for $m = 1, 2$ and $i = 1, 2$. The investment budget for both manufactures is equal to 10. The return ratio from the demand market to the manufacturer is set to 0.4 for both demand markets. The unit landfill cost, θ_i^b, is equal to 0.5 for both manufacturers. The overage penalty cost π_j^+ and the shortage cost π_j^- are equal to 0 and 100, respectively, for both retailers. The functions for these examples are constructed as following:

$$f_1^w(Q^w) = 4(\sum_{i=1}^{2} q_{1i})^2 + 2(\sum_{i=1}^{2} q_{1i}) + 1 \;\text{ for } i = 1, 2;$$

$$f_2^w(Q^w) = 2(\sum_{i=1}^{2} q_{2i})^2 + 2(\sum_{i=1}^{2} q_{2i}) + 1 \;\text{ for } i = 1, 2;$$

$$c_{mi}^w(Q^w) = 0.5(\sum_{i=1}^{2} q_{mi}) \text{ for } m = 1, 2; \; i = 1, 2;$$

$$f_i(Q^f) = 0.5(\sum_{j=1}^{2} q_{ij}^f)^2 + (\sum_{j=1}^{2} q_{ij}) \text{ for } i = 1, 2;$$

$$c_{ij}^f(Q^f) = (\sum_{j=1}^{2} q_{ij}^f)^2 \;\text{ for } i = 1, 2;$$

$$c_{ji}^b(Q^b) = (\sum_{i=1}^{2} q_{ji}^b(I_{ji}))^2 \text{ for } j = 1, 2; \;\; q_{ji}^b(I_{ji}) = 5I_{ji} \text{ for } i = 1, 2; \; j = 1, 2;$$

$$C_j^r(Q^f) = (\sum_{i=1}^{2} q_{ij}^f)^2 \text{ for } j = 1, 2;$$

$$\alpha_j(Q^b(I)) = 0.5(\sum_{i=1}^{2} q_{ji}) \text{ for } j = 1, 2.$$

In Figure 2 below, we show the relationship between the expected yield rate $\bar{\beta}_i$ and the total investment level. We see that there is positive correlation between the investment level and the expected yield rate, that is, as one increases so does the other. Manufacturers will be more willing to invest in the recycled product distribution channel if they see a higher return on investment. If firms increase their level of investment, the demand market will increase their return volume, thus benefiting the profits from the reverse supply chain network. Moreover, we also note that the budget for investment is not binding due to the reason that both the yield rate and the demand level prevent the manufacturers from fully investing in the reverse supply chain.

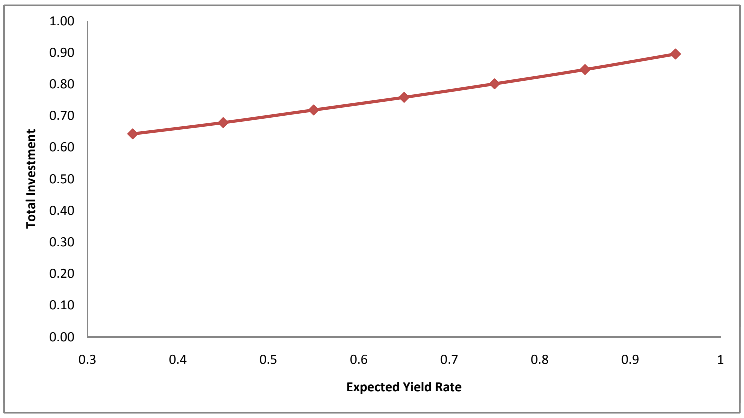

Figure 2: Relationship Between the Total Investment and the Expected Yield Rate

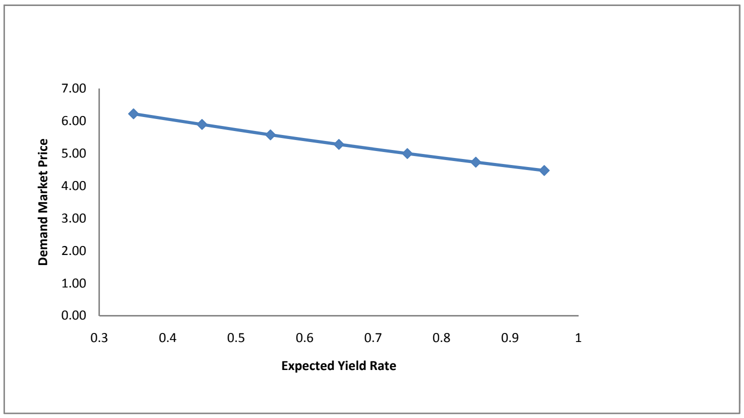

Figure 3: Relationship Between the Demand Market Price and the Expected Yield Rate

In Figure 3, we illustrate the relationship between the expected yield rate with the demand market price. Interestingly, we note that the consumers face a lower market price (and therefore, resulting in higher demand) when the expect yield rate is higher. This is due to the reason that the production of the recycled product is cheaper and a higher expected yield rate enables manufacturers to produce more to satisfy consumers' higher demand.

In Figure 4, we show the relationship between the expected yield rate with the total product flows in the reverse supply chain and that in the forward supply chain. It can be seen that both types of flow increase with the increase of the expected yield rate. However, the product flow volume in the forward chain has a sharper increase than that in the reverse distribution channel. Our interpretation is that the increase in demand due to the yield rate increase (from the investment) will generate more sales volume for manufacturers than for the returned product volume. Therefore, firms have an incentive to invest in their reverse supply which will generate additional sales and benefit to the consumer as well as the firms profit margin.

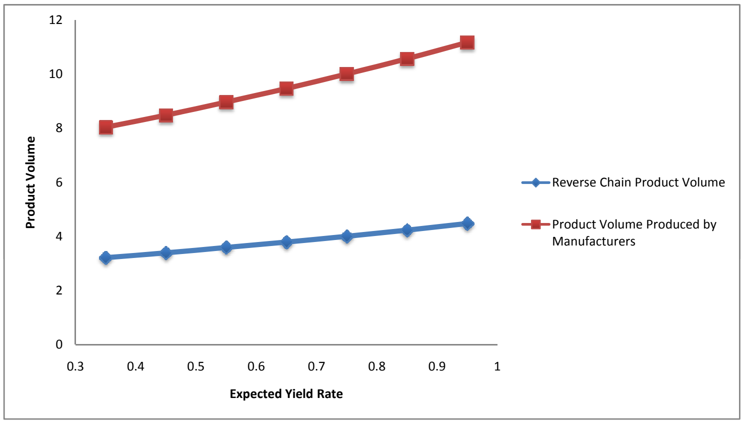

Figure 4: Relationship Between the Product Flows and the Expected Yield Rate

5. Conclusion

In this paper, a CLSC network model was investigated which consisted of raw material suppliers, manufacturers, and retail outlets. The demand of the retailer was satisfied either by newly manufactured products or by remanufactured products which are comparable to the new ones in function and quality, hence the consumer was indifferent in their demand for either products. We assumed the manufacturers collected the recycled product directly from the demand market, similar to the operations of Hewlett-Packard and Xerox Corporation regarding printer cartridges.

We derived the optimality conditions of the various decision-makers, and established that the governing equilibrium conditions could be formulated as a finite-dimensional variational inequality problem. We studied the qualitative properties of the equilibrium pattern, and under reasonable conditions, established existence and uniqueness results. We also discussed the characteristics of the functions in the variational inequality that enabled us to establish convergence of the proposed algorithm that could allow for the discussion of the effects of competition, distribution channel investment, combined with uncertainties in yield and demand on equilibrium quantity transactions and prices. We illustrated the model by presenting numerical examples.

The volume of waste is growing at an alarming rate and environmental recovery is an option that is underutilized since firms are unsure how to mitigate the ambiguity surrounding economic performance. By providing clarification of reverse supply chain issues, and firms re-examine their recovery efforts, the environmental benefits can be pronounced, for example, reducing landfill space, reducing air pollution, and leveraging the earth's natural resources, to name a few. Equipped with our model, one can "fine-tune" the parameters to study the behaviors of different decision makers in the CLSC, which also can generate some implications for the policy maker. Furthermore, although the cost functions in the example are hypothetical, we believe that some interesting managerial implications are reported. Of course, in the future, if the empirical data is available, we can validate our model further.

In our illustrative numerical examples, we note that the consumers face a lower market price (therefore resulting in higher demand) when the expect yield rate is higher since the production of the recycled product is cheaper. If firms increase their level of investment, the demand market will increase their return volume, thus benefiting the profits from the reverse supply chain network. Additionally, we noted that both total product flows in the reverse supply chain and from manufacturers in the forward supply chain decrease as the yield rate decreases. However, manufacturers face a sharper decrease than the reverse channel which signifies that firms have an incentive to invest in their reverse supply which will generate additional sales and benefit to the consumer as well as the firms profit margin.

This topic is of interest and could lead to possible future modifications of the model. Through the development of this model, additional facets of CLSC network models could be considered such as the development of a multiperiod CLSC network model that considers inventory management of remanufacturable products [45], and consider strategic safety stocks in reverse logistics supply chains [46]. Finally, we could take a more comprehensive approach to study CLSC networks with different market structures and provide a comparison of various performance measures.

References

[1] G. Kovacs, Corporate Responsibility in the Supply Chain, Journal of Cleaner Production, 16 (2008) 1571-1578.

[2] P.W. Stonebraker, J. Liao, Supply Chain Integration: Exploring Product and Environmental Contingencies, Supply Chain Management, 11 (2006) 34-43.

[3] The Consumer Electronics Association (CEA) Environmental Sustainability and Innovation in the Consumer Electronics Industry, Arlington, VA, 22202, 2008.

[4] Fuji Xerox Technical Report No.15 2005, Corporate Societal Responsibility: Knowledge Learning through Sustainable Global Supply Chain Management, Company Report, 2004.

[5] S. Benn, D. Dunphy, A Case of Strategic Sustainability: The Fuji Xerox Eco Manufacturing Centre, Innovation: Management, Policy and Practice, 6 (2004) 258-268.

[6] V. Daniel R. Guide Jr., L.N. Van Wassenhove, The Evolution of Closed-Loop Supply Chain Research. Operations Research, 57 (2009) 10-18.

[7] T.J. Barker, Z.B. Zabinsky, A Multicriteria Decision Making Model for Reverse Logistics using Analytical Hierarchy Process, Omega, 39 (2011) 558-573.

[8] W. Wongthatsanekorn, M.J. Realff, J.C. Ammons, Multi-time Scale Markov Decision Process Approach to Strategic Network Growth of Reverse Supply Chains, Omega, 38 (2010) 20-32.

[9] M.W. Toffel, Strategic Management of Product Recovery, California Management Review, 46 (2004) 120-141.

[10] P. Sasikumar, G. Kannan, Issues in Reverse Supply Chains, Part II: Reverse Distribution Issues - An Overview, International Journal of Sustainable Engineering, 1 (2008) 234-249.

[11] M.W. Toffel, Strategic Management of Product Recovery, California Management Review, 46 (2004) 120-141.

[12] A. Atasu, B. Toktay, L.N. Van Wassenhove. The Impact of Collection Cost Structure on Reverse Channel Choice, Under review in Management Science, 2008.

[13] P. Chanintrakul , A.E. Coronado Mondragon, A.E., C. Lalwani , C.Y. Wong, Reverse Logistics Network Design: A State-of-the-Art Literature Review, International Journal of Business Performance and Supply Chain Modelling, 1 (2009) 61-81.

[14] P. Sasikumar, G. Kannan, Issues in Reverse Supply Chains, Part II: Reverse Distribution Issues - An Overview, International Journal of Sustainable Engineering, 1 (2008) 234-249.

[15] P. Sasikumar, G. Kannan, Issues in Reverse Supply Chains, Part III: Classification and Simple Analysis, International Journal of Sustainable Engineering, 2 (2009) 2-27.

[16] A. Nagurney, J. Dong, D. Zhang, A Supply Chain Network Equilibrium Model, Transportation Research Part E: Logistics and Transportation Review, 38 (2002) 281-303.

[17] A. Nagurney, F.Toyasaki, Reverse Supply Chain Management and Electronic Waste Recycling: A Multitiered Network Equilibrium, Transportation Research E: Transportation and Logistics, 41 (2005) 1-28.

[18] G. Yang, Z. Wang, X. Li, The Optimization of the Closed-Loop Supply Chain Network, Transportation Research Part E, 45 (2009) 16-28.

[19] D. Hammond, P. Beullens, Closed-Loop Supply Chain Network Equilibrium Under Legislation. European Journal of Operational Research, 183 (2007) 895-908.

[20] J.B. Sheu, Y.H. Chou, C.C. Hu, An Integrated Logistics Operational Model for Green-Supply Chain Management. Transportation Research, 41 (2005) 287-313.

[21] M. Thierry, M. Salomon, J.V. Nunen, L.V. Wassenhove, Strategic Issues in Product Recovery Management, California Management Review, 37 (1995) 114-135.

[22] H.J. Kim, S. Severengiz, S.J. Skerlos, G. Seliger, Economic and Environmental Assessment of Remanufacturing in the Automotive Industry, 15th CIRP International Conference on Life Cycle Engineering, 2008.

[23] V.D.R. Jr. Guide, G.C. Souza, L.N. Van Wassenhove, J.D. Blackburn, Time Value of Commercial Product Returns, Management Science, 52 (2006) 1200-1214.

[24] M. Klausner, W.M. Grimm, C. Hendrickson, Reuse of Electric Motors in Consumer Products: Design and Analysis of an Electronic Data Log, Journal of Industrial Ecology, 2 (1998) 89-102.

[25] V.D.R Jr Guide, Production Planning and Control for Remanufacturing: Industry Practice and Research Needs, Journal of Operations Management, 18 (2000) 467-83.

[26] H. Krikke, Recovery Strategies and Reverse Logistics Network Design, in J. Sarkis, (Eds), Greener Manufacturing and Operations: From Design to Delivery and Back, Greenleaf Publishing, Sheffield, 256-271, 2001.

[27] M.C. Georgiadis, P. Tsiakis, P. Longinidis, M.K. Sofioglou, Optimal Design of Supply Chain Networks under Uncertain Transient Demand Variations, Omega, 39

(2011) 254-272.

[28] A. Atkinson, Incentives, Uncertainty, and Risk in the Newsboy Problem, Decision Sciences, 10 (1979) 341-357.

[29] D. Huang, H. Zhou, Q.-H. Zhao, A Competitive Multiple-Product Newsboy Problem with Partial Product Substitution, Omega, 39 (2011) 302-312.

[30] L.-H. Chen, Y.-C. Chen, A Multiple-Item Budget-Constraint Newsboy Problem with a Reservation Policy, Omega, 38 (2010) 431-439.

[31] K. Inderfurth, Impact of Uncertainties on Recovery Behavior in a Remanufacturing Environment: A Numerical Analysis, International Journal of Physical Distribution and Logistics Management, 35 (2005) 318-336.

[32] J. Shi, G. Zhang, J. Sha, Optimal Production Planning for a Multi-Product Closed Loop System with Uncertain Demand and Return, Computers and Operations Research, 38 (2011) 641-650.

[33] S. Vadde, A. Zeid, S.V. Kamarthi, Pricing Decisions in a Multi-Criteria Setting for Product Recovery Facilities, Omega, 39 (2011) 186-193.

[34] J. Dong, D. Zhang, A. Nagurney, A Supply Chain Network Equilibrium Model with Random Demands, European Journal of Operational Research, 156 (2004) 194-212.

[35] Businessweek, Remanufacturing: The Original Recycling, www.businessweek.com, 2005.

[36] M. Denizel, M. Ferguson, G. Souza, Multiperiod Remanufacturing Planning With Uncertain Quality of Inputs, IEEE Transactions on Engineering Management, 57 (2010) 394-404.

[37] A. Atasu, B. Toktay, L.N. Van Wassenhove. The Impact of Collection Cost Structure on Reverse Channel Choice, Under review in Management Science, 2008.

[38] Ferrer, G., 2003. Yield information and supplier responsiveness in remanufacturing operations. European Journal of Operational Research 149, 540-556.

[39] Z.P. Bayindir, N. Erkip, R. Gullu, A Model to Evaluate Inventory Costs in a Remanufacturing Environment, International Journal of Production Economics, 81-82 (2003) 597-607.

[40] A. Nagurney, J. Cruz, J. Dong, D. Zhang, Supply Chain Networks, Electronic Commerce, and Supply Side and Demand Side Risk, European Journal of Operational Research, 26 (2005) 120-142.

[41] A. Nagurney Network Economics: A Variational Inequality Approach, Second and Revised Edition, Kluwer Academic Publishers, Boston, Massachusetts, 1999.

[42] D. Kinderlehrer, G. Stampacchia, An Introduction to Variational Inequalities and Applications, Academic Press, New York, 1980.

[43] A. Nagurney, D. Zhang, Stability of Spatial Price Equilibrium Modeled as a Projected Dynamical System. Journal of Economic Dynamics and Control, 20 (1996) 43-63.

[44] G.M. Korpelevich, The Extragradient Method for Finding Saddle Points and Other Problems, Matekon, 13 (1977) 35-49.

[45] L.B. Toktay, L.M. Wein, S.A. Zenios, Inventory Management of Remanufacturable Products. Management Science, 46 (2000) 1412-1426.

[46] S. Minner, Strategic Safety Stocks in Reverse Logistics Supply Chains, International Journal of Production Economics, 71 (2001) 417-428.