

**Mergers and Acquisitions in Blood Banking Systems:  
A Supply Chain Network Approach**

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**Abstract:** Blood banking systems in the United States over the past decade have been faced with a volatile demand for blood, specifically, a decrease in demand for red blood cells, for a variety of reasons. This change in the blood supply chain landscape, accompanied by an increasing emphasis on cost efficiency, is a driver of Mergers & Acquisitions between blood banks. In this paper, we first present supply chain network optimization pre- and post-merger models. The models handle perishability of the life-saving product of blood, include both operational and discarding costs of waste, capture the uncertainty associated with the demand points, as well as the expected total blood supply shortage cost and the total discarding cost at demand points. They also incorporate capacities on the links. Their solution yields the optimal path and link flows plus the frequencies of activities associated

with blood collection, shipment, testing and processing, storage, and distribution, and incurred total costs. We provide a cost efficiency (synergy) measure associated with a merger or acquisition in the blood banking industry, as well as measures capturing the expected supply shortage and surplus. The methodological framework and its applicability are then illustrated via a large-scale blood supply chain network example inspired by a pending merger in the real-world in both status quo and disaster scenarios.

**Keywords:** blood banking, supply chains, mergers and acquisitions, efficiency measures, perishable product

## 1. Introduction

Blood services in the United States, a vital part of the healthcare system, are at a crossroads with American blood suppliers experiencing a volatile environment over the past decade. Prior to 2008, blood shortages were consistently reported every year by hospitals and other medical centers resulting in the cancellation and postponement of elective surgeries. The scenario has since dramatically changed, placing increased pressure on this unique industry in which the product must be donated and cannot be manufactured.

According to Whitaker (2011), the number of units of whole blood (WB) and red blood cells (RBC) transfused in 2011 decreased by a statistically significant 8.2% from 2008, with the percentage of decline for the units collected over that period being 9.1%. This downward trend has continued with both numbers suffering 4.4% and 9.4% further declines over the 2011 to 2013 period, respectively (cf. Chung et al. (2016)). The demand for blood continues to drop despite population growth and ever-increasing numbers of senior citizens. This declining trend in blood collection and utilization is raising concerns for disaster preparedness and other unexpected utilization needs (Chung et al. (2016)).

Researchers note that there is not a single factor for the declining utilization of most blood products; rather, a variety of economic factors as well as medical/technological advancements have contributed to the trend. Reduced hospital census due to hospital consolidations (America's Blood Centers (2015)) and historically low occupancy rates at hospitals (down to 60% in 2013 from 64% five years earlier and 77% in 1980) have significantly lowered the demand for blood (Evans (2015)). Furthermore, a lower number of elective surgeries allowed by private insurers has resulted in a reduced need for blood transfusions (Wald (2014)). The implementation of more conservative Patient Blood Management (PBM) initiatives as well as a number of improvements in clinical practices are also a major cause of this declining trend (Chung et al. (2016)).

Figure 1 from Chung et. al (2016) depicts the estimated rate per 1000 population of whole

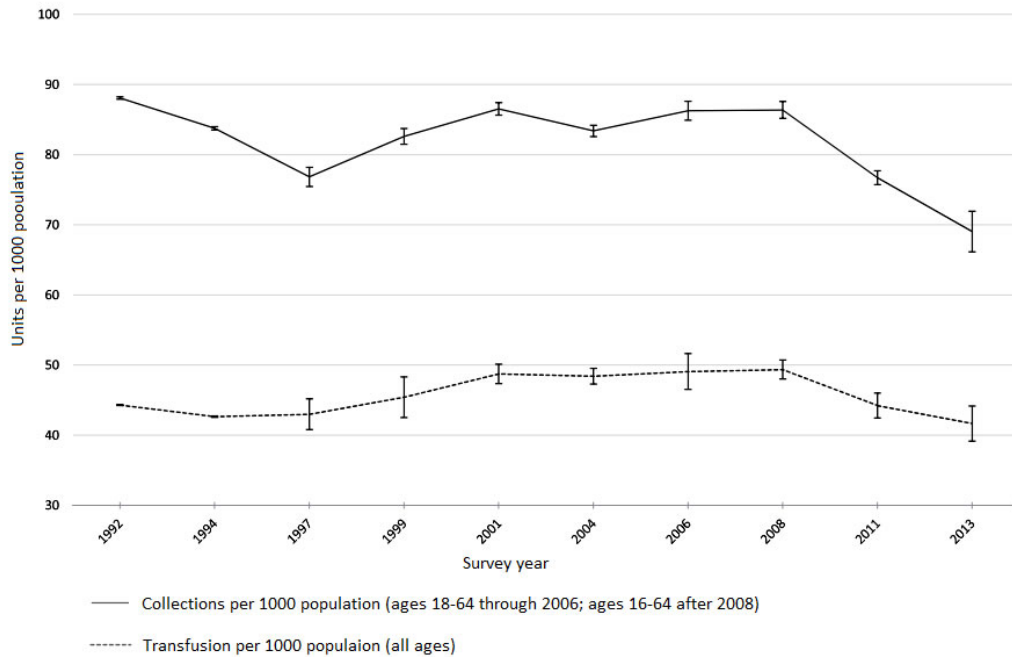


Figure 1: Estimated Rates of Whole Blood and RBC Collections and Transfusions in the US from 1992 to 2013 (Chung et al. (2016))

blood and RBC units collected and transfused in the US over the 1992-2013 period. In the United States, the American Red Cross supplies about 40% of the blood, with America’s Blood Centers, with 600 blood donor centers, providing about 50% (and about one quarter of the blood in Canada) (cf. American Red Cross (2016)). The remainder is collected by hospitals and medical centers themselves or, lately, by profit-maximizing blood suppliers (see, e.g., Carlyle (2012)).

Among the strategic responses of blood banks to the decreasing demand has been to downsize their operations at various stages of their supply chains. As part of this strategy, during a period of only a few years, the Red Cross consolidated their 5 national testing labs into 3, in addition to consolidating their manufacturing zones from 7 to 4. These consolidations were also applied at the regional division level to better coordinate the operations across those zones (Meyer (2015)). The Red Cross is also holding fewer mobile blood drives and, instead, operates larger blood collection sites in an effort to increase the efficiency of their supply chain (Meyer (2015)).

A rapidly accelerating trend in the blood banking industry over the past few years is the formation of partnerships between the suppliers at a variety of depths and different configurations. According to the America's Blood Centers, nearly 20 partnerships and mergers were formed in the 2010-2015 period among their member blood banks (reducing the size of the network from 87 to 68 members). It is interesting to note that only 19 mergers took place during the 1990s (America's Blood Centers (2015)). Even outside of the America's Blood Centers network, the formation of partnerships between blood banks demonstrates that the industry is currently going through a wave of mergers (Carden (2009)). For example, in April 2010, the Blood Center of Iowa and the Sioux Fall Blood Bank merged operations to become the LifeServe Blood Center (LifeServe Blood Center (2016)). In August 2013, The American Red Cross acquired Delta Bank, which was a nonprofit blood bank serving several counties in California (Bonnett (2013)). In September 2013, five nationally recognized blood centers announced the formation of a new alliance known as the HemeXcel Purchasing Alliance LLC., with a goal towards cost savings (BloodWorks Northeast (2013)). In December 2015, San Francisco's blood bank and Sacramento's BloodSource announced their merger plan (Robertson (2015)).

In July 2014, OneBlood, Inc., a blood center serving Florida, Georgia, and Alabama, and The Institute for Transfusion Medicine, Inc. (ITxM), a blood center operating in Pennsylvania, Illinois, Virginia, Ohio, and West Virginia, announced that they had reached an agreement to pursue a merger of the two organizations. According to officials, the two blood banks were seeking enhanced safety, availability, and affordability of blood for their hospital partners and the patients through the merger process. It is interesting that both organizations (or their parent organizations) had undergone a number of mergers and acquisitions of their own in the past (Lifesource (2014)). Later that year, the two parties suspended the merger talks after "several months of thorough and careful due diligence" on the feasibility of the merger. In August 2015, OneBlood announced another merger, this time with The Blood Alliance, Inc., a smaller blood center located in Jacksonville. This merger expanded

OneBlood’s reach in North Florida, deeper into Georgia, Alabama as well as South Carolina (Anguiano (2015)). Given the changes in the blood services landscape in the United States and the growing number of mergers and acquisitions between blood service organizations, this paper focuses on identifying potential operational efficiency synergies associated with such M&As. The specific pending M&A that that we utilize to illustrate the applicability of our methodological framework is that of OneBlood, Inc., and ITxM.

## **2. Literature Review**

In this section we review the relevant articles in the area of blood supply chains in addition to the literature corresponding to network optimization and blood supply chains as well as mergers and acquisitions.

### **2.1 Blood Supply Chains**

Perishable products, in general, and human blood, in particular, from a supply chain perspective, have long been studied by numerous researchers. Among the remarkable early works in the area of perishable product supply chains is the paper of Nahmias and Pierskalla (1973), which developed a multiperiod ordering model for a perishable product with a fixed lifetime. Federgruen, Prastacos, and Zipkin (1986), in turn, presented a combined problem for the allocation and distribution of a perishable product from a regional distribution center to a given set of locations with random demands.

Nahmias (2011), Karaesmen, Scheller-Wolf, and Deniz (2011), Belien and Force (2012), as well as Osorio, Brailsford, and Smith (2015) have done comprehensive reviews of the perishable products supply chain management literature including the study of various inventory policies in the blood banking systems.

Jacobs, Silan, and Clemson (1996), Pierskalla (2004), and Ghandforoush and Sen (2010) are among the papers that discuss the application of integer programming optimization models for the operations/design/redesign of blood supply chains. These models are generally

used to address the facility location problem, routing problems, set covering, and the allocation of whole blood and other blood products. Furthermore, Amorim, Gunther, and Almada-Lobo (2012) presented an integrated approach for the production and distribution of highly perishable products using a multi-objective mixed-integer programming framework.

As noted by Belien and Force (2012), simulation techniques have gained popularity among researchers of blood banking systems due to the inherent complexity of inventory management in such systems. Ryttila and Spens (2006) and Mustafee, Katsaliaki, and Brailsford (2009) are among the notable simulation works in the area of blood supply chains. Furthermore, Duan and Liao (2014) developed a simulation optimization framework for the inventory management of blood while taking into account the ABO blood group compatibility.

Van Dijk et al. (2009) used a Markov dynamic programming and simulation approach for the production of platelets with data from Dutch blood banks. Haijema (2014) presented a stochastic dynamic programming model for perishable product inventory by introducing an optimal stockage dependent ordering and disposal policy. Pauls-Worm et al. (2016) modeled a single-item, single-echelon production planning problem of a perishable product with a fixed life-time using a stochastic programming technique with a chance constraint. More recently, Dillon, Oliveira, and Abbasi (2017) proposed a two-stage stochastic programming model with recourse for supporting hospital's inventory management decisions in the blood supply chain. The model was reformulated and solved as a mixed integer linear programming problem. Hosseinifard and Abbasi (2016), in turn, consider uncertainty on the supply side and on the demand side and demonstrate that the number of hospitals serviced by the blood bank has a significant impact on the performance of the supply chain. Also, the authors' results reveal that centralizing the hospitals may improve the outdate at hospitals and the shortage at hospitals and the blood bank.

## **2.2 Network Optimization and Blood Supply Chains**

Network optimization methods have been widely used of late in modeling the design and operations of blood banking systems. This is mainly due to the versatility of such models in capturing various complexities of large blood supply chain networks in addition to the applicability of solution algorithms from the domain of network theory. Nagurney et al. (2013) presented an analytical framework for the supply chain of several perishable and time-sensitive products including blood banking systems. Jabbarzadeh, Fahimnia, and Seuring (2014) proposed a network optimization approach for the location-allocation problem of blood banks after natural disasters such as earthquakes. Nagurney, Masoumi, and Yu (2012) constructed a multicriteria supply chain network model for the operations management of a regional blood bank subject to uncertain demand while minimizing the overall cost as well as the supply-side risk. A generalized network approach with arc multipliers was utilized in that model to capture the perishability/loss/waste during various activities of the blood supply chain. Nagurney and Masoumi (2012) developed a supply chain optimization model for the design/redesign problem of a sustainable blood bank under various scenarios of uncertain demand.

## **2.3 Mergers and Acquisition Policies**

Concurrent with the wave of mergers and acquisitions in various industries, several researchers have studied the process of a merger – however, mostly in the context of commercial supply chains. Kumar (2009) compared numerous cases of mergers and acquisitions in developed countries to those in emerging economies from a strategic perspective. Cho (2014) analyzed the effects of a merger in a decentralized supply chain concluding that attention should be paid to where a merger takes place in a supply chain, and to whether a supply chain is led by upstream or downstream firms. Comez-Dolgan and Tanyeri (2015) in their study of 270 cases of mergers of American companies over a period of three decades found that the inventory turnover of the merging firms improves once a successful merger is complete



as a result of greater economies of scale. In the context of healthcare systems, Zuckerman (2011) identified the key forces reshaping the industry including the insurance industry consolidation and cost inflation in a period of integration for healthcare organizations.

In their book chapter, Havila and Salmi (2002) investigated international mergers and acquisitions from a network perspective, and suggested that failures in such integration efforts might be a result of the influences of the merging companies' network contexts. Mattsson (2003) stated that a network view of the supply chain would help in analyzing the potential reasons for the success and failure of integration in globalized markets. Nagurney (2009) presented a system-optimization framework for the quantification of strategic advantages associated with horizontal mergers. Nagurney and Qiang (2009) discussed a multicommodity supply chain network model, which could be utilized to assess potential synergies associated with the teaming of humanitarian organizations for disaster relief. Nagurney (2010), subsequently, considered competitive firms in an oligopolistic supply chain network setting and quantified synergies associated with mergers with the goal of explaining the merger paradox. Nagurney and Woolley (2010) developed a multicriteria supply chain network framework to assess the environmental effects of a merger or acquisition as well as the resulting synergy from possible strategic gains. Liu and Nagurney (2011) constructed a supply chain network framework to quantify the potential gains through integration in terms of risk reduction and cost synergy using a mean-variance approach. Oberg, Henneberg, and Mouzas (2007) and Zhang et al. (2010) are among the other notable studies that investigate pre-merger and post-merger issues from a network perspective.

This paper is the first to present a mathematical framework for the modeling of a merger or acquisition between two or more blood banks as well as the acquisition of a blood bank by another organization. The developed framework enables the decision-makers of blood banking systems to fully investigate the pros and cons of a potential merger or acquisition with respect to the goals of *operational efficiency* at a time of reduced demand, as is being faced by the industry, as well as at a time of a natural disaster, terrorist attack, or pandemic,

when there may be a surge in demand. The number of disasters is growing as well as the number of people impacted (cf. Nagurney and Qiang (2009)) and, hence, preparing for them and having a sustainable blood supply is of critical importance (see Mulcahy et al. (2016)).

The proposed models in this paper are built on a network optimization platform with arc multipliers representing the throughput of product flow subject to the perishability/waste/loss of blood across the supply chain. In both the pre-merger and post-merger problems, the organizations aim to minimize the total operational and discarding costs subject to the uncertain demand being satisfied as closely as possible. Moreover, we present specific efficiency measures to evaluate the extent of success of a potential merger or acquisition taking into account the total cost as well as the shortage/surplus of blood units in the pre- and post-merger models, while capturing perishability. Another contribution of this paper to the blood supply chain literature is the inclusion of the frequency associated with each supply chain activity as a decision variable in the model. We note that the idea of activity frequency has been utilized in other applications of network theory (see, e.g., Nagurney, Yu, and Floden (2013) and Nagurney (2015)), but not in the case of mergers and acquisitions or in the case of product perishability. Given the existing capacities, the frequency associated with a supply chain activity, be it collection, testing and processing, storage, or distribution, captures the repetition rate of that activity within a specified time interval.

The rest of this paper is organized as follows. In Section 3, we discuss the structure of the blood banking system which includes the various associated activities of these critical supply chains. Next, network topologies are provided to represent the cases of pre- and post-merger between two or more blood banks. We construct the optimization formulations of both cases, and then establish that the optimization problems are equivalent to variational inequality problems. Such an association provides us with an effective computational framework. The efficiency measures are also discussed in this section. In Section 4, we present a large-scale numerical example for the case of a merger between two blood banks. The algorithm which yields the optimal level of blood product flows in the supply chain network(s) as well the

frequencies of the activities in the supply chain, before and after the merger, is given, for completeness, in the Appendix. We then apply the proposed algorithm to compute the solution to both problems in order to illustrate the modeling and analyze the efficiency and responsiveness of the merger/acquisition from different perspectives, including the occurrence of a disaster. We also discuss the significance of the results. In Section 5, we present our conclusions.

### **3. The Supply Chain Network Model of a Blood Bank Merger or Acquisition**

We first present the supply chain network model for a blood bank and then its merger/acquisition counterpart. Without loss of generality, we can assume that our model represents the blood banking system of a regionalized blood bank (in case of a large organization such as the American Red Cross Blood Services with multiple regions across the country), or an autonomous blood center (such as one chapter of the America's Blood Centers serving one or a few neighboring states). Even though our network framework is sufficiently general to capture any vertically integrated blood supply chain network, it should be noted that these modules might be called differently from one organization to another. Furthermore, the proposed supply chain network model for blood banking systems with arc multipliers and frequencies of supply chain activities can also be adapted to other perishable products (see, e.g., the case of fresh produce in Yu and Nagurney (2013) and that for medical nuclear products in Nagurney and Nagurney (2012)).

#### **3.1 Overview of the Supply Chain of a Blood Bank**

In order to be consistent with the majority of real-world cases, we consider a blood banking system that conducts and distributes blood in a regional manner (cf. Nagurney, Masoumi, and Yu (2012)). In our model, there exists a *Blood Bank* serving a specific geographic area which is also in charge of administration of its lower-level units. The blood bank could also be called a *Blood Chapter*, or a *Blood Organization* depending on the case. We refer to it as

an “organization” henceforth in this paper. Figure 2 depicts a supply chain network topology corresponding to the pre-merger problem consisting of  $I$  organizations. In this topology, the top level node (origin) of each organization represents the *blood chapter management* or the *regional division management*. Every other node within the network of each organization denotes a facility in the system. For every organization  $i \in \{1, \dots, I\}$  there exist one or more paths, each of which connects the corresponding origin node  $i$  to one of its destination nodes, that is, the demand nodes representing the hospitals or medical centers. This assumption guarantees that the uncertain demand for blood at each such facility will be met as closely as possible. Every path consists of a sequence of directed links representing the supply chain network activities starting with the collection of blood all the way to the distribution. According to Figure 2, each organization services, pre-merger, a different set of demand points.

Next, we present a brief description of the activities taking place between every two tiers of the facilities belonging to Organization  $i$ ;  $i \in \{1, \dots, I\}$ , as depicted in Figure 2. The links/arcs connecting these nodes (facilities) represent the five major activities in the blood supply chain as described below:

- *Blood Collection*: occurs in either mobile or permanent blood “collection sites” or “donor centers” and is represented by the set of links connecting each top node to the collection sites. We assume Organization  $i$  has  $n_c^i$  of such facilities denoted by  $C_1^i, C_2^i, \dots, C_{n_c^i}^i$ .
- *Shipment*: is displayed by the second set of links connecting the second and the third tier of nodes, and captures the process of shipping the collected units of whole blood (WB) to  $n_B^i$  “blood centers” of Organization  $i$  denoted by  $B_1^i, B_2^i, \dots, B_{n_B^i}^i$ .
- *Testing and Processing*: are typically done concurrently yet at different locations sometimes hundreds of miles away. Processing refers to the separation of the collected blood

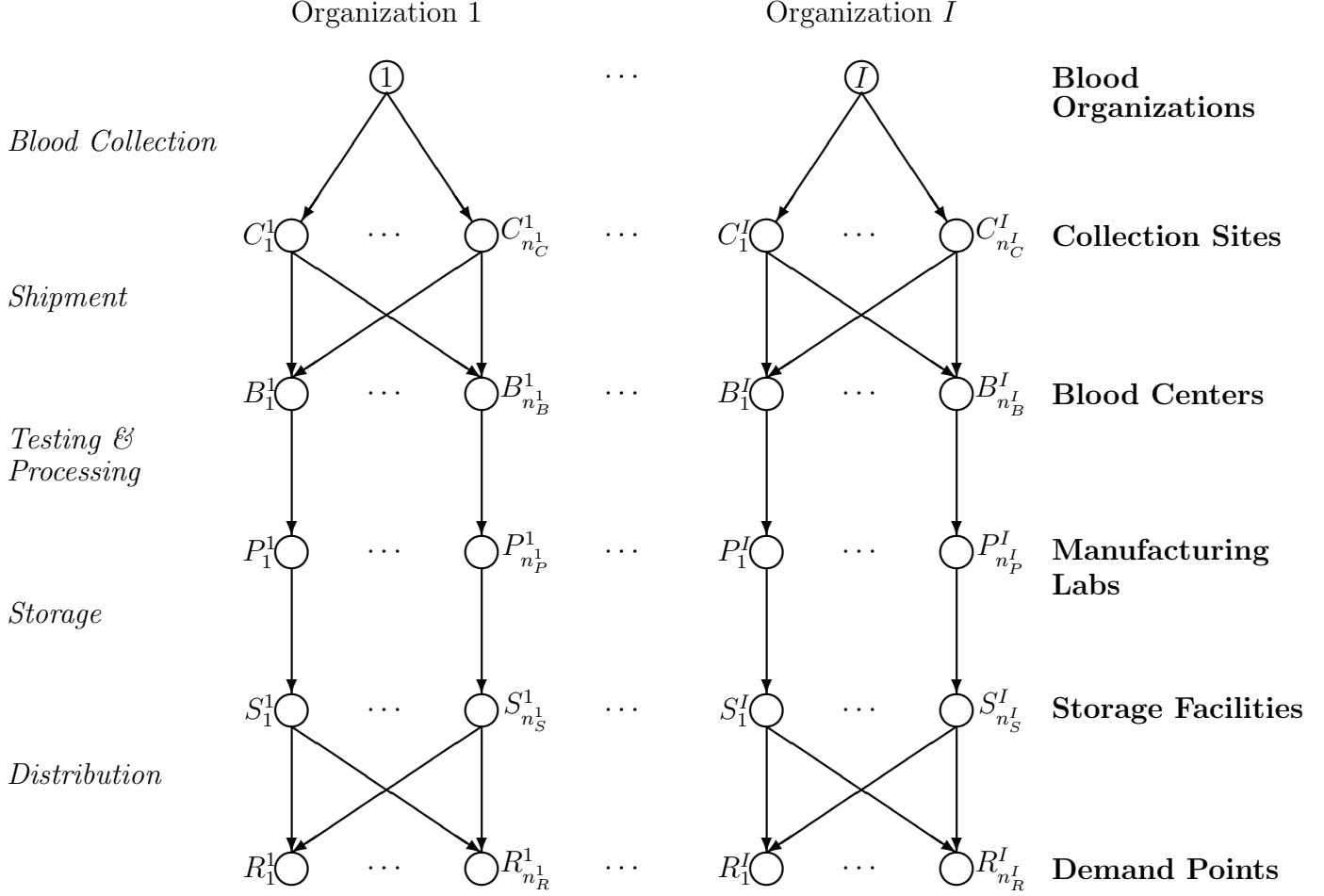


Figure 2: Supply Chain Network Topology for  $I$  Blood Organizations in the Pre-Merger Problem

into parts (red blood cells, white blood cells, plasma and platelets) at  $n_P^i$  “manufacturing labs” of Organization  $i$  denoted by  $P_1^i, P_2^i, \dots, P_{n_P^i}^i$ . Also known as “component labs”, manufacturing labs are very often located within the blood center locations. Testing, unlike processing, is conducted at the “testing labs” to which a small sample of every unit of collected blood is sent overnight. These samples are discarded regardless of the test result once the tests are over, and, in our model, the respective amounts of waste are addressed using the arc multipliers. The cost on these sets of links represents the operational cost of testing and processing combined.

- *Storage*: refers to the short-term storing of processed units of red blood cells (RBC) in one of the  $n_S^i$  “storage facilities” of Organization  $i$  before being distributed. In the

proposed network topology for a blood bank, storage facilities are located on the fifth tier of the nodes, and are denoted by  $S_1^i, S_2^i, \dots, S_{n_S^i}^i$ . Note that, in practice, storage facilities are usually located in the same place as the manufacturing labs, serving as “distribution centers” in the blood supply chain.

- *Distribution*: corresponds to the last set of links connecting the bottom two tiers of nodes in the supply chain network topology, and captures the delivery of the red blood cells to  $n_R^i$  “demand points” served by Organization  $i$  in the pre-merger problem. Hospitals and other medical surgical centers denoted by  $R_1^i, R_2^i, \dots, R_{n_R^i}^i$  constitute our demand points.

It is necessary to emphasize that the proposed supply chain network topology is process-based rather than location-based. As mentioned earlier, specific modules of the blood banking system belonging to one organization may physically coincide with some others of the same organization. The ultimate solution of the complete model yields the optimal flows of blood on various links of the network as well as the optimal frequency of such activities at minimum total cost. Next, we present the mathematical model that captures the pre-merger problem, beginning with the notation utilized.

### 3.2 Mathematical Model for the Pre-Merger Blood Banking Problem

In the proposed model for the pre-merger problem, there exists a total of  $I$  blood banking organizations. Detailed definitions of the notation are given in Table 1.

Table 1: Notation for the Pre-Merger Blood Banking Problem

Notation	Definition
Sets:	
$G_i = [N_i, L_i]$	The graph consisting of nodes $N_i$ and directed links $L_i$ representing the activities associated with each Organization $i$ ; $i = 1, \dots, I$ .
$G^0 = [N^0, L^0]$	The graph consisting of the set of nodes and the set of links in the topology for the pre-merger problem; that is, $G^0 = [N^0, L^0] = \cup_{i=1, \dots, I} [N_i, L_i]$ . $n_{L^0}$ denotes the number of all the links.
$\mathcal{P}_{R_k^i}^0$	The set of paths joining the top node of each Organization $i$ to a specific demand node; i.e., the origin/destination pair $(i, R_k^i)$ ; $i = 1, \dots, I$ and $k = 1, \dots, n_R^i$ .
$\mathcal{P}^0$	The set of all paths in the pre-merger problem (Figure 2). $n_{\mathcal{P}^0}$ denotes the number of all paths.
Parameters:	
$d_{ik}$	The stochastic parameter representing the actual demand for red blood cells (RBC) at demand market $R_k^i$ ; $i = 1, \dots, I$ and $k = 1, \dots, n_R^i$ during the time period of one week.
$\bar{u}_a$	The existing capacity of link $a$ ; $a \in L^0$ .
$\delta_{ap}$	$\delta_{ap} = 1$ , if link $a$ is contained in path $p$ and $\delta_{ap} = 0$ , otherwise; $a \in L^0$ and $p \in \mathcal{P}^0$ .
$\alpha_a$	The arc multiplier associated with link $a$ , which represents the percentage of throughput on link $a$ . $\alpha_a \in (0, 1]$ ; $a \in L^0$ .
$\alpha_{ap}$	The arc-path multiplier, which is the product of the multipliers of the links on path $p$ that precede link $a$ ; $a \in L^0$ and $p \in \mathcal{P}^0$ ; that is, $\alpha_{ap} \equiv \begin{cases} \delta_{ap} \prod_{b \in \{a' < a\}_p} \alpha_b, & \text{if } \{a' < a\}_p \neq \emptyset, \\ \delta_{ap}, & \text{if } \{a' < a\}_p = \emptyset, \end{cases}$ where $\{a' < a\}_p$ denotes the set of the links preceding link $a$ in path $p$ .
$\mu_p$	The multiplier corresponding to the percentage of throughput on path $p$ ; that is, $\mu_p \equiv \prod_{a \in p} \alpha_a$ ; $p \in \mathcal{P}^0$ .
$\lambda_{ik}^-$	The penalty associated with the shortage of a unit of blood at demand point $R_k^i$ ; $i = 1, \dots, I$ and $k = 1, \dots, n_R^i$ .

Notation	Definition
$\lambda_{ik}^+$	The penalty associated with the surplus of a unit of blood at demand point $R_k^i$ ; $i = 1, \dots, I$ and $k = 1, \dots, n_R^i$ .
Decision Variables:	
$v_{ik}$	The projected demand for RBC at the demand point $R_k^i$ ; $i = 1, \dots, I$ and $k = 1, \dots, n_R^i$ during the time period of one week. We group the projected demands into the $\sum_{i=1}^I n_R^i$ -dimensional vector $v$ .
$x_p$	The nonnegative flow of RBC on path $p$ ; $p \in \mathcal{P}^0$ . We group the path flows into the $n_{\mathcal{P}^0}$ -dimensional vector $x$ .
$f_a$	The initial flow of RBC on link $a$ ; $a \in L^0$ . We group the link flows into the $n_{L^0}$ -dimensional vector $f$ .
$\gamma_a$	The activity frequency of link $a$ ; $a \in L^0$ in the time period. We group the activity frequencies into the $n_{L^0}$ -dimensional vector $\gamma$ .
Functions:	
$\hat{c}_a = \hat{c}_a(f_a, \gamma_a)$	The total operational cost on link $a$ which is assumed to be a function of the product flow on that link as well as the activity frequency of that link; $a \in L^0$ .
$\mathcal{F}_{ik}(t)$	The probability density function of the actual demand at demand market $R_k^i$ ; $i = 1, \dots, I$ and $k = 1, \dots, n_R^i$ .
$P_{ik}(v_{ik})$	The cumulative distribution function of $d_{ik}$ ; that is, $P_{ik}(v_{ik}) = P_{ik}(d_{ik} \leq v_{ik}) = \int_0^{v_{ik}} \mathcal{F}_{ik}(t) d(t)$ .
$\hat{z}_a = \hat{z}_a(f_a)$	The total discarding cost function associated with the product flow on link $a$ ; $a \in L^0$ .
$\Delta_{ik}^- = \Delta_{ik}^-(v_{ik})$	The shortage of blood at demand point $R_k^i$ ; that is, $\Delta_{ik}^- \equiv \max\{0, d_{ik} - v_{ik}\}$ ; $i = 1, \dots, I$ and $k = 1, \dots, n_R^i$ . The expected shortage is given by $E(\Delta_{ik}^-) = \int_{v_{ik}}^{\infty} (t - v_{ik}) \mathcal{F}_{ik}(t) d(t)$ .
$\Delta_{ik}^+ = \Delta_{ik}^+(v_{ik})$	The surplus of blood at demand point $R_k^i$ ; that is, $\Delta_{ik}^+ \equiv \max\{0, v_{ik} - d_{ik}\}$ ; $i = 1, \dots, I$ and $k = 1, \dots, n_R^i$ . The expected surplus is given by $E(\Delta_{ik}^+) = \int_0^{v_{ik}} (v_{ik} - t) \mathcal{F}_{ik}(t) d(t)$ .

The total operational cost functions and the total discarding cost functions of all links are assumed to be convex and continuously differentiable.

In the model,  $\alpha_a$  represents the throughput of product flow on link  $a$  subject to the percentage of perishability/waste/loss on that link (activity). For example, if the expected percentage of waste during the testing and processing phase of a blood center is 5%, the



corresponding throughput factor,  $\alpha_a$ , will be 0.95. Given the network topology, the following conservation of flow constraints must hold across the supply chain of all blood organizations:

$$f_a = \sum_{p \in \mathcal{P}^0} x_p \alpha_{ap}, \quad \forall a \in L^0. \quad (1)$$

Equation (1) states that the initial product flow (excluding waste/perishability/loss) on link  $a$  is the sum of the product flows along paths which contain that link, with the consideration of possible losses in the preceding activities.

Furthermore, each organization determines the shipments of blood product based on the projected demand at its own demand points subject to the throughput factor on its paths, which must satisfy the following conservation of flow equation:

$$v_{ik} = \sum_{p \in \mathcal{P}_{R_k^i}^0} x_p \mu_p, \quad i = 1, \dots, I; k = 1, \dots, n_R. \quad (2)$$

Note that in our model, the flow on each link  $a$ ; i.e.,  $f_a$ , represents the number of units of red blood cells (RBC) on that link (rather than the number of units of whole blood (WB)) subject to the waste/loss on the preceding links. As a matter of fact, over the first three sets of links (blood collection, shipment of collected blood, as well as testing and processing) the unprocessed whole blood constitutes the product whereas in the bottom two tiers of links (storage and distribution), we have the processed blood cells. This assumption is in line with the real-world problem as one unit (one pint) of collected whole blood can be separated into one unit of red blood cells in addition to plasma, platelets, and other by-products of the human blood. Similarly, the value of path flow,  $x_p$ , as well as the actual and projected demand at demand market  $R_i^k$ ; i.e.,  $d_{ik}$  and  $v_{ik}$ , respectively, represent the number of units of red blood cells corresponding to that path or that demand point.

It should also be noted here that the focus of this paper is on the supply chain of red

blood cells rather than the other derivatives of blood. This is mainly due to the fact that various derivatives of blood have very different shelf-lives (5 days for platelets, 5-6 weeks for whole blood and red blood cells, and up to a year for plasma when kept frozen). With red blood cells constituting the majority of transfusions among all products of blood (cf. Whitaker (2011)), we exclude the other derivatives from our model. In addition, platelets and plasma can be obtained during a different process called apheresis in which blood is passed through an apparatus separating out one particular constituent - plasma or platelets – and returning the remainder to the donor.

In addition, the path flows must be nonnegative; that is,

$$x_p \geq 0, \quad \forall p \in \mathcal{P}^0. \quad (3)$$

The model assumes a capacity for every link of the blood supply chain networks. These capacities reflect the upper bound on the volume of various activities – collection, shipment, and so forth during a period (of one week). Given the existing link capacities, our model guarantees that the flow on each link does not exceed the effective capacity (subject to the frequency of that activity) in that time period. Thus, the following constraints must hold:

$$f_a \leq \bar{u}_a \gamma_a, \quad \forall a \in L^0. \quad (4)$$

For each organization, the total cost minimization objective faced by the organization includes the total cost of operating the various links, the total discarding cost of waste over the links, and the expected total blood supply shortage cost as well as the total discarding cost of outdated blood at the demand points. Since the optimization problems for the organizations are independent, the solution of the following optimization problem, also yields

the optimal solution for each Blood Organization  $i$ ;  $i = 1, \dots, I$ ,

$$\text{Minimize } \sum_{i=1}^I \sum_{a \in L_i} \hat{c}_a(f_a, \gamma_a) + \sum_{i=1}^I \sum_{a \in L_i} \hat{z}_a(f_a) + \sum_{i=1}^I \sum_{k=1}^{n_R^i} (\lambda_{ik}^- E(\Delta_{ik}^-) + \lambda_{ik}^+ E(\Delta_{ik}^+)), \quad (5)$$

subject to: constraints (1)–(4).

We define  $\eta_a$ ;  $a \in L^0$  as the Lagrange multipliers associated with constraint (4); and we group the Lagrange multipliers into the  $n_{L^0}$ -dimensional vector  $\eta$ . We know that the objective function (5) for each  $i = 1, \dots, I$ , is convex under our imposed assumptions on the total operational cost and the total discarding cost functions since  $\sum_{k=1}^{n_R^i} (\lambda_{ik}^- E(\Delta_{ik}^-) + \lambda_{ik}^+ E(\Delta_{ik}^+))$  is also convex, as established in Nagurney, Masoumi, and Yu (2012), and the sum of convex functions is also a convex function.

We now present the variational inequality formulation of the above optimization problem, once, in terms of path flows and, then, link flows. The proof is in the Appendix. We provide the variational inequality (VI) formulation since it then allows us to exploit the special structure for computational purposes.

### Theorem 1: Variational Inequality Formulation

*The optimization problem (5), subject to (1) through (4), is equivalent to the variational inequality problem: determine the vector of optimal path flows, the vector of optimal activity frequencies, and the vector of Lagrange multipliers,  $(x^*, \gamma^*, \eta^*) \in K$ , such that:*

$$\begin{aligned} & \sum_{i=1}^I \sum_{k=1}^{n_R^i} \sum_{p \in \mathcal{P}_{R_k^i}^0} \left[ \frac{\partial \hat{C}_p(x^*, \gamma^*)}{\partial x_p} + \frac{\partial \hat{Z}_p(x^*)}{\partial x_p} + \lambda_{ik}^+ \mu_p P_{ik} \left( \sum_{q \in \mathcal{P}_{R_k^i}^0} x_q^* \mu_q \right) \right. \\ & \left. - \lambda_{ik}^- \mu_p \left( 1 - P_{ik} \left( \sum_{q \in \mathcal{P}_{R_k^i}^0} x_q^* \mu_q \right) \right) + \sum_{a \in L_i} \eta_a^* \alpha_{ap} \right] \times [x_p - x_p^*] \\ & + \sum_{i=1}^I \sum_{a \in L_i} \left[ \frac{\partial \hat{C}_p(x^*, \gamma^*)}{\partial \gamma_a} - \bar{u}_a \eta_a^* \right] \times [\gamma_a - \gamma_a^*] \end{aligned}$$

$$+ \sum_{i=1}^I \sum_{a \in L_i} \left[ \bar{u}_a \gamma_a^* - \sum_{q \in \mathcal{P}^0} x_q^* \alpha_{aq} \right] \times [\eta_a - \eta_a^*] \geq 0, \quad \forall (x, \gamma, \eta) \in K, \quad (6)$$

where  $K \equiv \{(x, \gamma, \eta) | x \in R_+^{n_{\mathcal{P}^0}}, \gamma \in R_+^{n_{L^0}}, \eta \in R_+^{n_{L^0}}\}$  and for each path  $p$ ;  $p \in \mathcal{P}_{R_k^i}^0$ ;  $i = 1, \dots, I$ ;  $k = 1, \dots, n_R^i$ ,

$$\frac{\partial \hat{C}_p(x, \gamma)}{\partial x_p} \equiv \sum_{a \in L^i} \frac{\partial \hat{c}_a(f_a, \gamma_a)}{\partial f_a} \alpha_{ap}, \quad (7a)$$

$$\frac{\partial \hat{Z}_p(x)}{\partial x_p} \equiv \sum_{a \in L^i} \frac{\partial \hat{z}_a(f_a)}{\partial f_a} \alpha_{ap}. \quad (7b)$$

For each link  $a$ ;  $a \in L_i$ ;  $i = 1, \dots, I$ ,

$$\frac{\partial \hat{C}_p(x, \gamma)}{\partial \gamma_a} \equiv \frac{\partial \hat{c}_a(f_a, \gamma_a)}{\partial \gamma_a}. \quad (7c)$$

The variational inequality (6), in turn, can be rewritten in terms of link flows as: determine the vector of optimal link flows, the vector of optimal projected demands, the vector of optimal activity frequencies, and the vector of Lagrange multipliers,  $(f^*, v^*, \gamma^*, \eta^*) \in K^1$ , such that:

$$\begin{aligned} & \sum_{i=1}^I \sum_{a \in L_i} \left[ \frac{\partial \hat{c}_a(f_a^*, \gamma_a^*)}{\partial f_a} + \frac{\partial \hat{z}_a(f_a^*)}{\partial f_a} + \eta_a^* \right] \times [f_a - f_a^*] \\ & + \sum_{i=1}^I \sum_{k=1}^{n_R^i} [\lambda_{ik}^+ P_{ik}(v_{ik}^*) - \lambda_{ik}^- (1 - P_{ik}(v_{ik}^*))] \times [v_{ik} - v_{ik}^*] \\ & + \sum_{i=1}^I \sum_{a \in L_i} \left[ \frac{\partial \hat{c}_a(f_a^*, \gamma_a^*)}{\partial \gamma_a} - \bar{u}_a \eta_a^* \right] \times [\gamma_a - \gamma_a^*] \\ & + \sum_{i=1}^I \sum_{a \in L_i} [\bar{u}_a \gamma_a^* - f_a^*] \times [\eta_a - \eta_a^*] \geq 0, \quad \forall (f, v, \gamma, \eta) \in K^1, \end{aligned} \quad (8)$$

where  $K^1 \equiv \{(f, v, \gamma, \eta) | \exists x \geq 0, (1) - (3) \text{ hold, and } \gamma \geq 0, \eta \geq 0\}$ .

### 3.3 Mathematical Model for the Post-Merger Blood Banking Problem

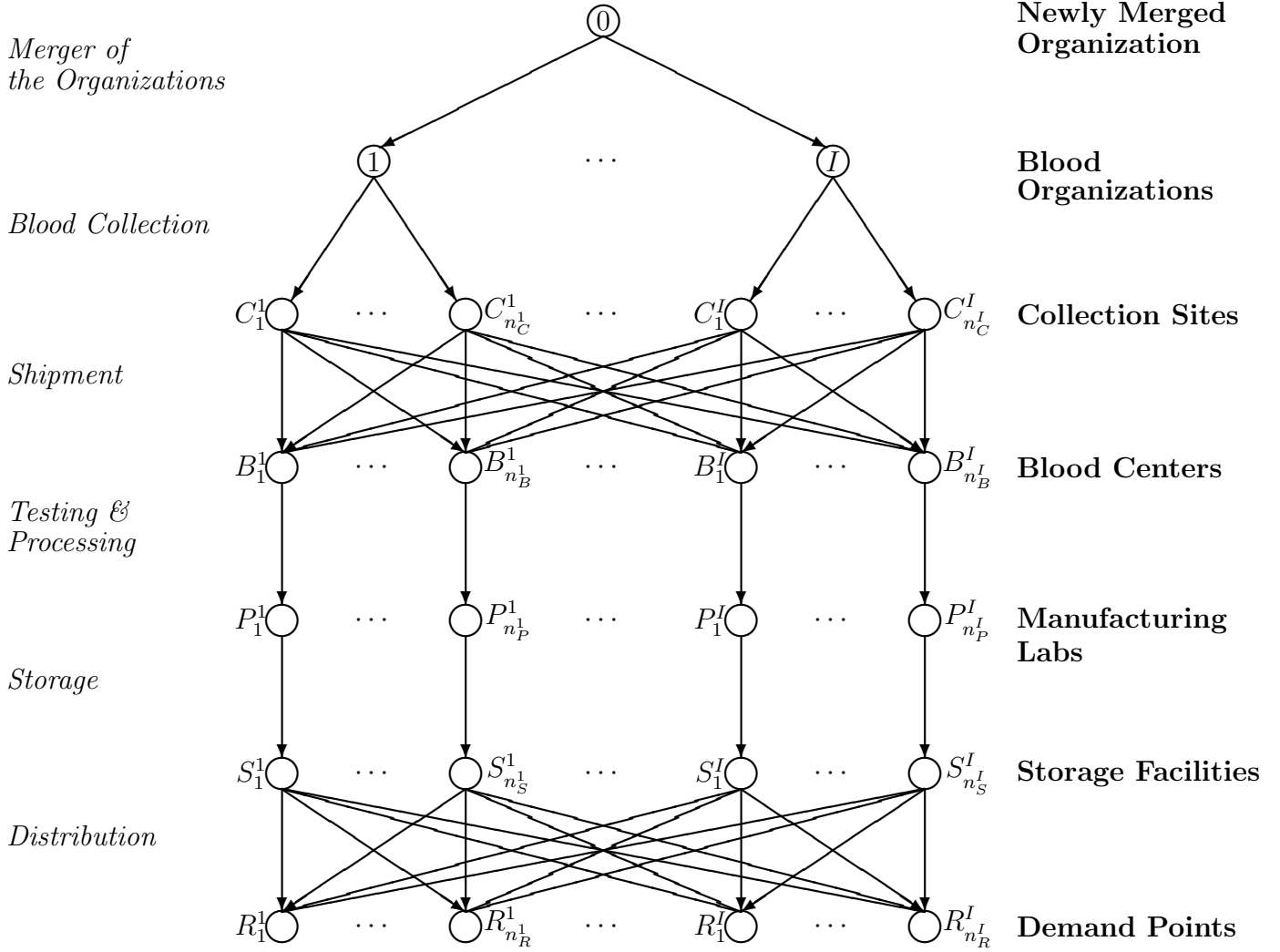


Figure 3: Supply Chain Network Topology for the Post-Merger Problem

We now consider the situation where the  $I$  individual blood organizations have merged to form a new organization as shown in Figure 3. We denote the network topology for the post-merger problem by  $G^1 = [N^1, L^1]$  where  $N^1 = N^0 \cup \text{node } 0$ ,  $L^1 = L^0 \cup \text{the additional links}$ , and  $n_{L^1}$  is the number of the links in this new topology. Supernode 0 represents the newly merged blood banking organization. The links connecting node 0 and node  $i$ ;  $i = 1, \dots, I$  reflect the merger of those blood organizations. Since the focus of this model is on the operational aspects associated with the merger, we assume that the total costs associated

with merging are negligible. If the merger/acquisition costs are to be incorporated then they would be assigned to the uppermost links in the network in Figure 3.

Once the merger (or acquisition) is complete, collection sites of Organization  $i$  (which no longer exists as an individual entity) will have the option to transport the collected units of whole blood to the blood centers of the other organization(s) for subsequent testing, processing, etc. The solution of the model will determine whether or not this leads to cost-saving for the merged organization. In Figure 3, the newly added cross-organizational links connecting the third and the fourth tiers of facilities enable such added capabilities which are an immediate outcome of a completed merger or acquisition. Furthermore, the newly added links between the last two tiers of facilities in the network reveal that Organization  $i$  will now be able to serve the hospitals and medical centers under contract with the other blood provider(s), assuming that the optimal solution recommends this.

The solution to the model with the two new sets of links in the post-merger problem allows for an a priori evaluation of whether one can expect an improved operational efficiency for the blood organizations as well as an enhanced stability of the blood supply. These outcomes, as expressed earlier in the introduction, are among the major motives for the blood banks to consider mergers and acquisitions.

Let  $\mathcal{P}_{R_k^i}^1$  denote the set of the paths joining supernode 0 with demand point  $R_k^i$ ;  $i = 1, \dots, I$  and  $k = 1, \dots, n_R^i$ .  $\mathcal{P}^1$  and  $n_{\mathcal{P}^1}$  denote the set of all the paths and the number of all paths in the post-merger problem, respectively.

Consequently, the constraints (1)–(4) are updated for the new problem as follows:

$$f_a = \sum_{p \in \mathcal{P}^1} x_p \alpha_{ap}, \quad \forall a \in L^1, \quad (9)$$

$$v_{ik} = \sum_{p \in \mathcal{P}_{R_k^i}^1} x_p \mu_p, \quad i = 1, \dots, I; k = 1, \dots, n_R^i, \quad (10)$$

$$x_p \geq 0, \quad \forall p \in \mathcal{P}^1, \quad (11)$$

$$f_a \leq \bar{u}_a \gamma_a, \quad \forall a \in L^1. \quad (12)$$

Therefore, for the merged blood organization, the total cost minimization problem can be expressed as:

$$\text{Minimize} \quad \sum_{a \in L^1} \hat{c}_a(f_a, \gamma_a) + \sum_{a \in L^1} \hat{z}_a(f_a) + \sum_{i=1}^I \sum_{k=1}^{n_R^i} (\lambda_{ik}^- E(\Delta_{ik}^-) + \lambda_{ik}^+ E(\Delta_{ik}^+)), \quad (13)$$

subject to: constraints (9)–(12).

Similar to the pre-merger model, the solution to the post-merger total cost minimization problem can be obtained as a solution to a variational inequality problem akin to (6) (or (8)) where now  $a \in L^1$ ,  $p \in \mathcal{P}_{n_k}^1$ , for  $i = 1, \dots, I$ , and  $k = 1, \dots, n_R^i$ , and the vectors  $f$ ,  $x$ ,  $v$ ,  $\gamma$ ,  $\eta$  have identical definitions as before, but are re-dimensioned accordingly. Set  $K$  is then replaced by  $K^2 \equiv \{(x, \gamma, \eta) | x \in R_+^{n_{\mathcal{P}^1}}, \gamma \in R_+^{n_{L^1}}, \eta \in R_+^{n_{L^1}}\}$ ; and set  $K^1$  is replaced by  $K^3 \equiv \{(f, v, \gamma, \eta) | \exists x \geq 0, (9) - (12) \text{ hold, and } \gamma \geq 0, \eta \geq 0\}$ .

### 3.4 Efficiency Measures Associated with the Merger or Acquisition of the Blood Organizations

In this section, we provide measures for the evaluation of the efficiency associated with the merger or acquisition of the blood organizations. The measure that we utilize to capture the gains, if any, is denoted by  $\mathcal{E}^{TC}$ , and is referred to as the *total cost efficiency*, defined as follows:

$$\mathcal{E}^{TC} \equiv \left[ \frac{TC^0 - TC^1}{TC^0} \right] \times 100\%, \quad (14)$$

where  $TC^0$  and  $TC^1$  represent the optimal objective values of the pre-merger model and the post-merger model, respectively. Note that  $\mathcal{E}^{TC}$  can be interpreted as *synergy* in terms of cost savings (cf. Nagurney (2009), Nagurney and Woolley (2010), and Liu and Nagurney

(2011)). The higher the value of  $\mathcal{E}^{TC}$ , the more beneficial the merger/acquisition.

In addition, we propose the following two measures, which capture the changes in the supply shortage and surplus associated with the merger or acquisition:

$$\mathcal{S}^- = \left[ \frac{S^{0-} - S^{1-}}{S^{0-}} \right] \times 100\%, \quad (15)$$

$$\mathcal{S}^+ = \left[ \frac{S^{0+} - S^{1+}}{S^{0+}} \right] \times 100\%, \quad (16)$$

where  $S^{0-}$  and  $S^{1-}$  represent the expected total supply shortage costs (penalties) associated with the pre-merger model and the post-merger model, respectively, i.e.,  $\sum_{i=1}^I \sum_{k=1}^{n_R^i} \lambda_{ik}^- E(\Delta_{ik}^-)$ , with the  $E(\Delta_{ik}^-)$  values evaluated at the optimal solution for the pre-merger problem for  $S^{0-}$  and at the optimal solution for the post-merger problem for  $S^{1-}$ .  $S^{0+}$  and  $S^{1+}$ , in turn, represent the expected total supply surplus costs associated with the pre-merger model and the post-merger model, respectively, that is,  $\sum_{i=1}^I \sum_{k=1}^{n_R^i} \lambda_{ik}^+ E(\Delta_{ik}^+)$  with the  $E(\Delta_{ik}^+)$  values evaluated at the optimal solution for the pre-merger problem for  $S^{0+}$  and for the post-merger problem for  $S^{1+}$ .

#### 4. Case Study: A Merger Between Two Blood Banking Systems

In order to illustrate the proposed framework for the merger/acquisition between two or more blood banking systems, we are inspired by the recent pending case of the merger between OneBlood, Inc., and The Institute for Transfusion Medicine, Inc. (ITxM). As mentioned earlier in the Introduction, the two blood banks announced their agreement to pursue a merger in July 2014 which would create the largest independent not-for-profit blood center in the United States distributing nearly 2 million units of blood annually, with combined revenues of \$480 million and employing more than 3,500 people. The merger process is said to have become suspended in 2015 - as discussed earlier in the text. Through the following case study, we analyze the potential benefits as well as the risks the two blood organizations would take as a result of the merger under a status quo scenario and also in



the case of a disaster.

Based in Orlando, Florida, OneBlood is the larger of the two organizations, with about 2,000 employees and \$295 million in revenues. Distributing nearly one million units of blood annually, OneBlood serves about 200 hospital partners, covering 80 percent of Florida and parts of Alabama and Georgia. Samples of all blood units – collected in their 80 donor centers or their 200 specialized big red buses – are delivered via a OneBlood courier to the Creative Testing Solutions (CTS), the state-of-the-art testing facility located in St. Petersburg, Florida. As mandated by the Federal laws, more than a dozen of tests are conducted on every sample in a period of 12–14 hours, including screening for Zika virus. Test results are sent back to the blood centers in less than 24 hours. OneBlood owns 9 biologics manufacturing labs (component labs) spread across the region. On average, OneBlood sends around 1,000 deliveries daily to their partner hospitals 24 hours a day, 7 days a week, 365 days a year. It typically takes an additional 2-3 days for the delivered blood to be transfused to a patient in need (Oneblood.org (2016)).

The other organization involved in the merger, The Institute for Transfusion Medicine, Inc. (ITxM), is one of the nation’s foremost nonprofit organizations specializing in transfusion medicine and related services. It operates in Pittsburgh, Chicago, Virginia, and parts of Ohio, and West Virginia serving over one million units of blood components to the hospitals in the region. ITxM has multiple blood centers including: (1) LifeSource located in Chicago providing over a half million blood products to approximately 60 hospitals across Illinois, and (2) Central Blood Bank located in Pittsburgh distributing more than 160,000 units of blood annually to more than 40 hospitals from the blood collected in their 25 community donor centers (The Institute for Transfusion Medicine (2016)).

For the numerical example corresponding to the pre-merger problem, without loss of generality, we focus on a fraction of the facilities belonging to the actual supply chain network of the two aforementioned organizations. It should be emphasized that the presented mod-

eling framework as well as the developed mathematical formulation in Section 3 can capture the case of merger/acquisition between any number of blood organizations consisting of any given number of facilities.

As seen in Figure 4, Organization 1, representing OneBlood, Inc., in our pre-merger problem, is composed of three blood collection sites, one of which ( $C_1^1$ ) is a permanent donor center, and the other two are assumed to be big red buses ( $C_2^1$  and  $C_3^1$ ). In addition, there exist two blood centers, two manufacturing labs, two storage facilities, and two hospitals in the supply chain network of OneBlood, Inc. We assume Blood Center  $B_1^1$ , Manufacturing Lab  $P_1^1$ , and Storage Facility  $S_1^1$  are all located in a consolidated facility in Fort Lauderdale area whereas  $B_2^1$ ,  $P_2^1$ , and  $S_2^1$  are located in Tampa.  $R_1^1$  and  $R_2^1$  are the two partner hospitals also located in Florida, large and medium-size, respectively. We consider a time horizon of a week since we are interested in a potential M&A and such a time period is more reflective of supply chain activities than a single day. The weekly demand for red blood cells is assumed to follow a continuous uniform distribution on the intervals [200,400] and [150,250], respectively. Thus, the cumulative distribution functions corresponding to these demand points are as follows:

$$P_{11}\left(\sum_{p \in \mathcal{P}_{R_1^1}^0} \mu_p x_p\right) = \frac{\sum_{p \in \mathcal{P}_{R_1^1}^0} \mu_p x_p - 200}{400 - 200}, \quad P_{12}\left(\sum_{p \in \mathcal{P}_{R_2^1}^0} \mu_p x_p\right) = \frac{\sum_{p \in \mathcal{P}_{R_2^1}^0} \mu_p x_p - 150}{250 - 150},$$

where  $\mathcal{P}_{R_1^1}^0$  and  $\mathcal{P}_{R_2^1}^0$  represent the sets of paths connecting the top node of Organization 1, One Blood, Inc., with the two demand points in the pre-merger problem. Each of these sets contains 6 paths demonstrating 6 possible sequences of links corresponding to each demand point. Furthermore,  $G_1 = [N_1, L_1]$ , the graph corresponding to Organization 1 in the pre-merger problem, consists of 12 nodes in  $N_1$  and 17 directed links in  $L_1$  as seen in Figure 4.

Similarly, Organization 2, representing ITxM, is composed of two collection sites, two

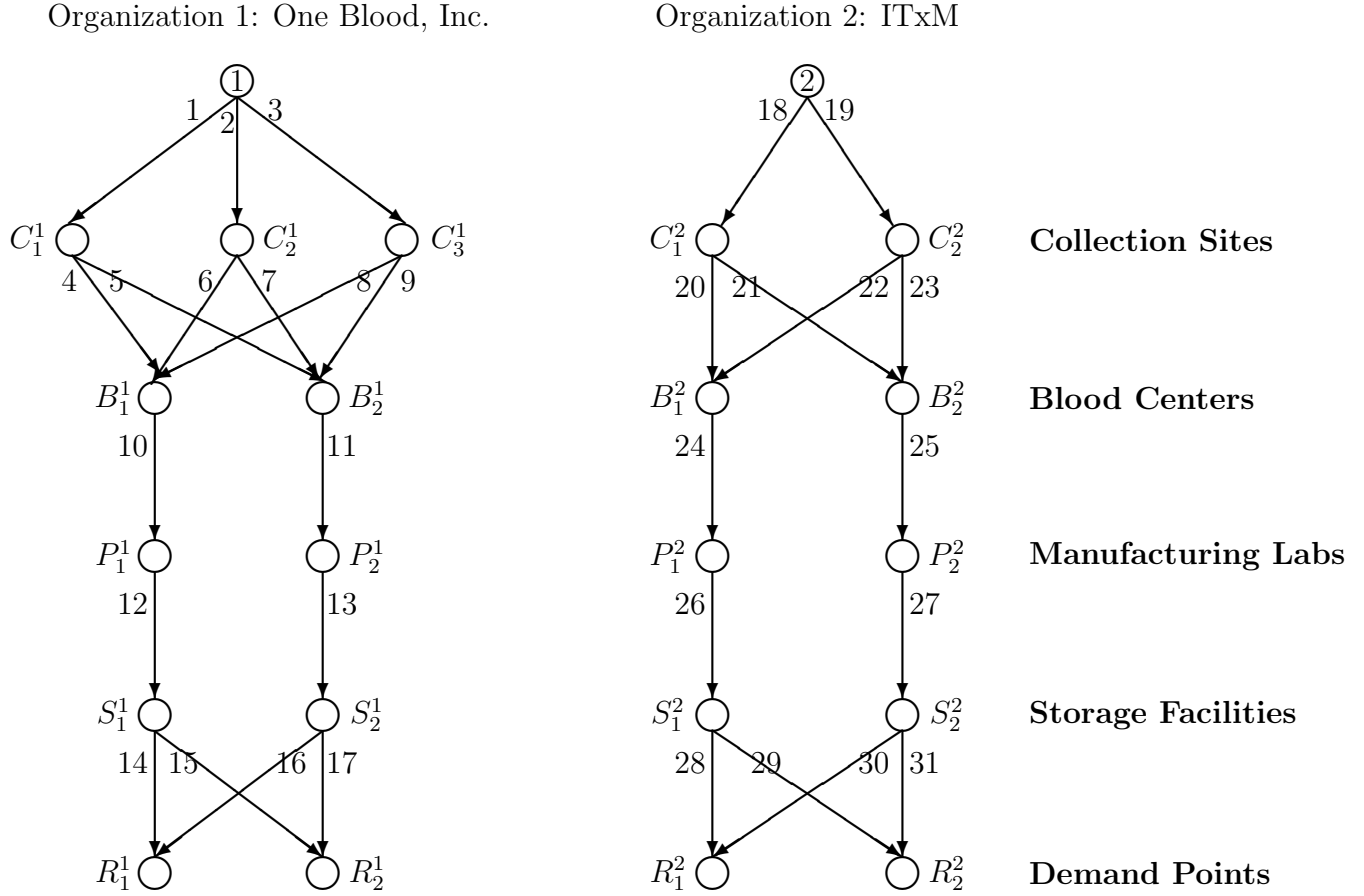


Figure 4: Supply Chain Network Topology for the Pre-Merger Problem in the Case Study

blood centers, two manufacturing labs, two storage facilities, and two hospitals. Similar to the other organization, each blood center is assumed to be co-located with one manufacturing lab and one storage facility. Thus, the first consolidated facility (housing  $B_1^2$ ,  $P_1^2$ , and  $S_1^2$ ) is located in Chicago, and the other one (consisting of  $B_2^2$ ,  $P_2^2$ , and  $S_2^2$ ) is in Pittsburgh. Weekly demand at their two partner hospitals,  $R_1^2$  and  $R_2^2$  located in Chicago and Pittsburgh, respectively, are assumed to follow a continuous uniform distribution on the intervals  $[220,370]$  and  $[80,110]$  representing a large hospital and a small surgical center, respectively.

The cumulative distribution functions corresponding to these demand points are:

$$P_{21}\left(\sum_{p \in \mathcal{P}_{R_1^0}^0} \mu_p x_p\right) = \frac{\sum_{p \in \mathcal{P}_{R_1^0}^0} \mu_p x_p - 220}{370 - 220}, \quad P_{22}\left(\sum_{p \in \mathcal{P}_{R_2^0}^0} \mu_p x_p\right) = \frac{\sum_{p \in \mathcal{P}_{R_2^0}^0} \mu_p x_p - 80}{110 - 80},$$

where  $\mathcal{P}_{R_1^0}^0$  and  $\mathcal{P}_{R_2^0}^0$  represent the sets of paths connecting the top node of Organization 2; i.e., ITxM, with its demand points in the pre-merger problem. Each of these sets contains 4 paths corresponding to each demand point. Similarly,  $G_2 = [N_2, L_2]$ , the graph corresponding to Organization 2 in the pre-merger problem, consists of 11 nodes in  $N_2$  and 14 directed links in  $L_2$ .

In total,  $n_{\mathcal{P}^0}$ , the number of paths in the pre-merger problem corresponding to both organizations combined, is equal to  $2 \times 6 + 2 \times 4 = 20$ . Also,  $G^0 = [N^0, L^0]$ , the graph consisting of the set of all nodes and links in the topology for the pre-merger problem is composed of  $17 + 14 = 31$  links where links  $\{1 - 17\}$  and  $\{18 - 31\}$  belong to  $L_1$  and  $L_2$ , respectively.

The shortage penalties for each of the four demand points reflecting the relative severity of the consequences of shortage of one red blood cell unit at these hospitals and surgical centers are as follows (See Masoumi (2013) for a thorough sensitivity analysis of unit shortage penalties):

$$\lambda_1^{1-} = 7,000, \quad \lambda_2^{1-} = 6,000, \quad \lambda_1^{2-} = 8,000, \quad \text{and} \quad \lambda_2^{2-} = 3,700.$$

Similarly, the outdated penalties representing the extra cost each of the four hospitals incur in case of the surplus of one unit of red blood cells are (cf. Masoumi (2013)):

$$\lambda_1^{1+} = 50, \quad \lambda_2^{1+} = 60, \quad \lambda_1^{2+} = 40, \quad \text{and} \quad \lambda_2^{2+} = 75.$$

The multipliers corresponding to the link throughputs ( $\alpha_a; \forall a \in L^0$ ), the existing link capacities ( $\bar{u}_a; \forall a \in L^0$ ), the total operational cost functions ( $\hat{c}_a(f_a, \gamma_a); \forall a \in L^0$ ), and the total discarding cost functions ( $\hat{z}_a(f_a); \forall a \in L^0$ ) are as reported in Tables 2 and 3. Table 2 demonstrates the data for Organization 1 (OneBlood, Inc.) whereas Table 3 shows the data for Organization 2 (ITxM). The parameters and functions in the pre-merger problem were determined using the available information on the above organizations' websites as well as the average historical data for the American Red Cross Northeast Division Blood Services (cf. Nagurney, Masoumi, and Yu (2012)).

Table 2: Throughput Multipliers, Existing Weekly Capacities, Total Operational Cost and Total Discarding Cost Functions for Organization 1 (OneBlood, Inc.) (Links 1-17) in the Pre-Merger Problem

Link $a$	$\alpha_a$	$\bar{u}_a$	$\hat{c}_a(f_a, \gamma_a)$	$\hat{z}_a(f_a)$
1	.97	200	$.13(f_1)^2 + .2f_1 + (\gamma_1)^2$	$.8f_1$
2	.98	225	$.15(f_2)^2 + .3f_2 + 1.5(\gamma_2)^2$	$.7f_2$
3	.99	225	$.10(f_3)^2 + .25f_3 + (\gamma_3)^2$	$.7f_3$
4	1.00	35	$2f_4 + 2\gamma_4$	0
5	.99	25	$.35(f_5)^2 + .8f_5 + 9(\gamma_5)^2 + 3\gamma_5$	$.8f_5$
6	.99	40	$1.4f_6 + 4\gamma_6$	0
7	1.00	30	$.05(f_7)^2 + .1f_7 + (\gamma_7)^2$	$.7f_7$
8	.99	25	$.18(f_8)^2 + .4f_8 + 3(\gamma_8)^2 + 2\gamma_8$	$.8f_8$
9	1.00	40	$.85f_9 + 2.5\gamma_9$	0
10	.96	1,800	$.55(f_{10})^2 + 2f_{10} + 3(\gamma_{10})^2$	$.8f_{10}$
11	.94	1,600	$.45(f_{11})^2 + 2.5f_{11} + 2(\gamma_{11})^2$	$.7f_{11}$
12	.99	2,000	$.07(f_{12})^2 + .5f_{12} + 2(\gamma_{12})^2$	$.8f_{12}$
13	.99	1,600	$.06(f_{13})^2 + .4f_{13} + 1.5(\gamma_{13})^2$	$.7f_{13}$
14	1.00	36	$.9f_{14} + 3.5\gamma_{14}$	0
15	.99	40	$.25(f_{15})^2 + 2.8f_{15} + 4.5(\gamma_{15})^2 + 4.5\gamma_{15}$	$.8f_{15}$
16	.99	40	$.12(f_{16})^2 + 2.5f_{16} + 4(\gamma_{16})^2 + 5\gamma_{16}$	$.9f_{16}$
17	1.00	35	$1.1f_{17} + 2.5\gamma_{17}$	0

In the Appendix, we outline the algorithm utilized to compute the optimal values of path flows, activity frequencies, and Lagrange multipliers.

Table 3: Throughput Multipliers, Existing Weekly Capacities, Total Operational Cost and Total Discarding Cost Functions for Organization 2 (ITxM) (Links 18-31) in the Pre-Merger Problem

Link $a$	$\alpha_a$	$\bar{u}_a$	$\hat{c}_a(f_a, \gamma_a)$	$\hat{z}_a(f_a)$
18	.98	260	$.11(f_{18})^2 + .3f_{18} + 2(\gamma_{18})^2$	$.6f_{18}$
19	.99	235	$.13(f_{19})^2 + .2f_{19} + (\gamma_{19})^2$	$.7f_{19}$
20	1.00	25	$f_{20} + .5(\gamma_{20})^2 + 4\gamma_{20}$	0
21	.99	35	$.09(f_{21})^2 + .5f_{21} + (\gamma_{21})^2 + 4\gamma_{21}$	$.8f_{21}$
22	.98	35	$.14(f_{22})^2 + .9f_{22} + 1.5(\gamma_{22})^2 + 7.5\gamma_{22}$	$.75f_{22}$
23	1.00	35	$.8f_{23} + 3\gamma_{23}$	0
24	.95	2, 200	$.6(f_{24})^2 + 1.1f_{24} + 2.5(\gamma_{24})^2$	$.6f_{24}$
25	.96	2, 000	$.85(f_{25})^2 + 1.5f_{25} + 3(\gamma_{25})^2$	$.7f_{25}$
26	.99	2, 500	$.05(f_{26})^2 + .4f_{26} + (\gamma_{26})^2$	$.6f_{26}$
27	1.00	2, 150	$.06(f_{27})^2 + .6f_{27} + 2(\gamma_{27})^2$	$.5f_{27}$
28	1.00	30	$.6f_{28} + 4\gamma_{28}$	0
29	.99	40	$.1(f_{29})^2 + .5f_{29} + 2(\gamma_{29})^2 + 5\gamma_{29}$	$.9f_{29}$
30	.99	35	$.4(f_{30})^2 + 1.3f_{30} + 2(\gamma_{30})^2 + 6\gamma_{30}$	$.7f_{30}$
31	1.00	40	$.5f_{31} + 2.5\gamma_{31}$	0

#### 4.1 Solution to the Pre-Merger Problem in the Case Study

We implemented the algorithm, the Euler method provided in the Appendix, using Matlab to the described pre-merger problem to calculate the optimal values of link flows, link frequencies, Lagrange multipliers, among other quantities of interest. The convergence tolerance was  $\epsilon = 10^{-5}$ , and the sequence  $a_\tau = .1(1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \dots)$ . When the absolute value of the difference between successive path flows, link frequencies, and Lagrange multipliers was no more than the above  $\epsilon$ , the algorithm was supposed to have converged. We initialized the algorithm by setting each link frequency and Lagrange multiplier equal to zero. As for the path flows, arbitrary positive values were assigned.

The optimal values for the above quantities for Organizations 1 and 2 in the pre-merger problem can be found in Table 4.

As can be seen from the optimal link flows in Table 4, out of the 31 links belonging to the two blood organizations, 5 have zero or very small flows meaning that such links (including 3

Table 4: Optimal Values of the Weekly Link Flows, Weekly Frequencies of Activities, Lagrange Multipliers as well as the Incurred Values of the Total Operational and Discarding Cost Functions for Organization 1 (OneBlood, Inc.) (Links 1-17) and Organization 2 (ITxM) (Links 18-31) in the Pre-Merger Problem

Organization 1 (OneBlood, Inc.)					Organization 2 (ITxM)				
Link $a$	$f_a^*$	$\gamma_a^*$	$\eta_a^*$	$\hat{c}_a(f_a^*, \gamma_a^*) + \hat{z}_a(f_a^*)$	Link $a$	$f_a^*$	$\gamma_a^*$	$\eta_a^*$	$\hat{c}_a(f_a^*, \gamma_a^*) + \hat{z}_a(f_a^*)$
1	204.95	1.02	.01	\$5,666.81	18	286.37	1.10	.02	\$9,280.89
2	179.42	.80	.01	\$5,009.04	19	214.48	.91	.01	\$6,174.23
3	297.12	1.32	.01	\$9,111.96	20	280.64	11.23	.61	\$388.55
4	193.88	5.59	.08	\$398.94	21	.00	.00	.11	\$0.00
5	4.93	.20	.26	\$17.31	22	22.60	.65	.27	\$114.30
6	125.42	3.06	.17	\$187.84	23	189.73	5.50	.12	\$168.29
7	50.41	1.68	.11	\$170.19	24	302.79	.14	.00	\$55,524.76
8	.00	.00	.08	\$0.00	25	189.73	.09	.00	\$31,016.56
9	294.15	7.30	.04	\$268.26	26	287.65	.11	.00	\$4,424.87
10	318.05	.18	.00	\$56,526.45	27	182.14	.08	.00	\$2,190.97
11	349.43	.22	.00	\$56,063.79	28	284.78	9.40	.04	\$208.45
12	305.33	.15	.00	\$6,922.75	29	.00	.00	.05	\$0.00
13	328.46	.21	.00	\$6,834.69	30	76.10	2.17	.42	\$2,491.03
14	302.28	8.33	.03	\$301.20	31	106.05	2.71	.06	\$59.81
15	.00	.00	.11	\$0.00					
16	83.59	2.09	.54	\$1,150.69					
17	241.59	6.87	.01	\$282.93					

shipment and 2 distribution links) will rarely - if at all - be used in the pre-merger problem. Furthermore, the projected weekly values of demand for red blood cells at hospitals  $R_1^1$  and  $R_2^1$  (corresponding to Organization 1) as well as hospitals  $R_1^2$  and  $R_2^2$  (corresponding to Organization 2) are shown below:

$$\text{OneBlood, Inc.: } v_{11}^* = 385.03 \text{ units, } v_{12}^* = 241.59 \text{ units, and}$$

$$\text{ITxM: } v_{21}^* = 360.11 \text{ units, } v_{22}^* = 106.05 \text{ units.}$$

Note that for all four demand points, as we would expect, the value of the projected demand falls between the lower and the upper bounds of their respective uniform distribution ranges. In addition, for all four hospitals, this value is closer to the upper bound which

suggests that the decision-makers in both organizations have assigned a much larger shortage penalty as compared to the surplus penalties. It should be mentioned that adjusting the values of these penalties can assist the blood banks with the crucial tradeoff between the shortage of red blood cell units and the expiration of unused units at the hospitals (cf. Masoumi (2013)).

## 4.2 Solution to The Post-Merger Problem in the Case Study

Once/if the merger process between the two organizations is completed, the supply chain network topology for the post-merger problem would evolve to the one depicted in Figure 5. As seen in the new topology, both organizations now operate under a unified management denoted by supernode 0. Newly added links 32 and 33, connecting node 0 to nodes 1 and 2 are assumed to have a total cost of zero in that the proposed mathematical framework represents the operations of the blood banks in the post-merger situation rather than during the process of merger. Furthermore, links 34 through 43 are added to reflect the fact that each of the organizations now has the ability to ship the collected units of blood to the blood centers of the other organization (for testing and processing, and storage) should it make sense demand-wise and cost-wise. Similarly, under the new situation, the hospitals now have an added flexibility to enjoy their demand being satisfied by the storage/distribution centers of the other organization via links 44 through 51, if need be. Adding the new links to the network topology quadruples the number of potential paths (from 20 in the pre-merger problem to 80 in the post-merger problem).

Data corresponding to the multipliers, the capacities, and the cost functions of the new links are displayed in Table 5 where we assume that the respective data regarding the pre-existing links remain the same as in Tables 2 and 3.

Similar to the pre-merger problem, the aforementioned algorithm was applied to the above post-merger problem using Matlab. The optimal values of the link flows, the weekly frequencies of the activities, the Lagrange multipliers as well as the incurred values at optimality of



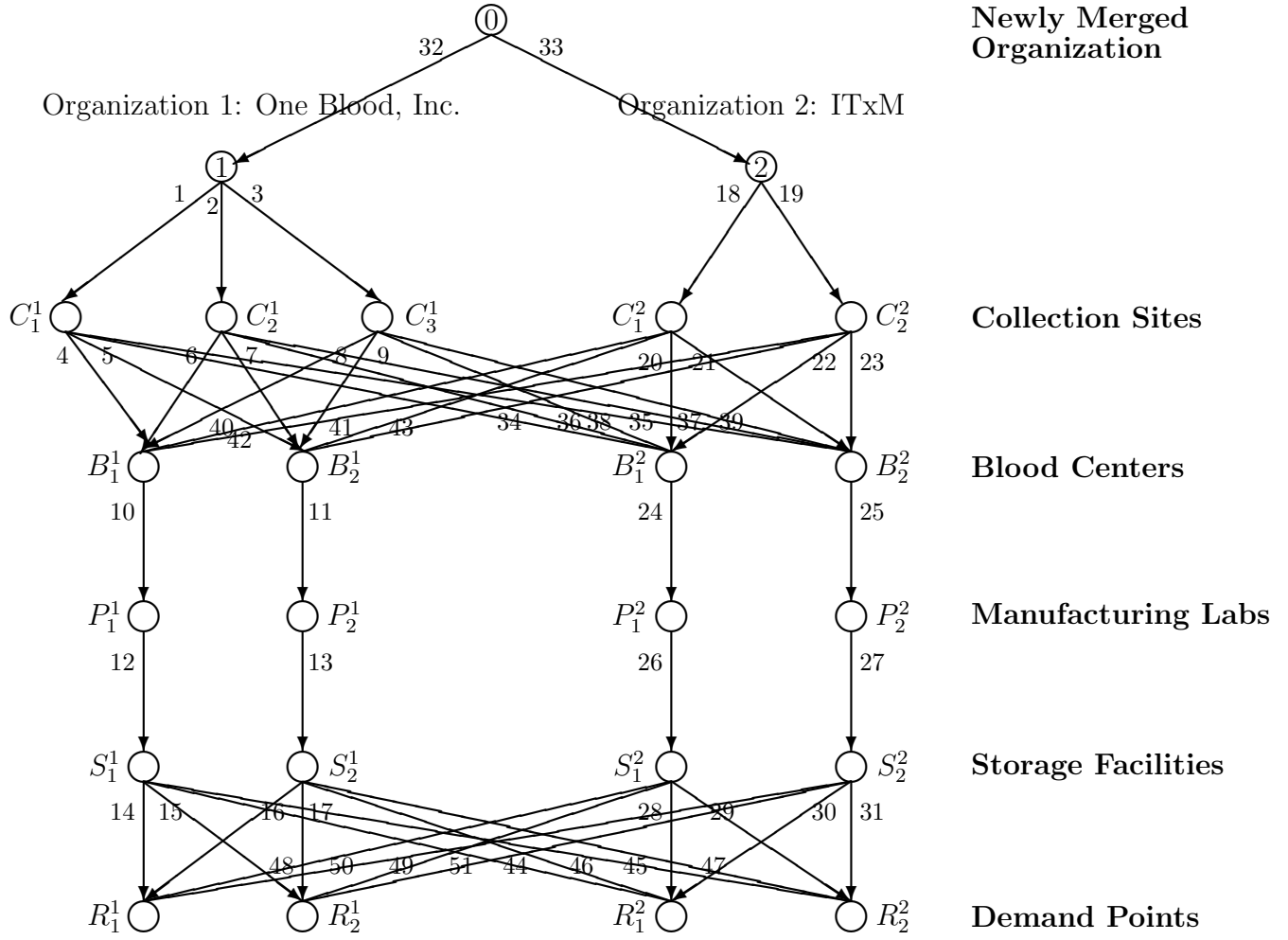


Figure 5: Supply Chain Network Topology for the Post-Merger Problem in the Case Study

the total operational and discarding cost functions for the newly merged organization can be seen in Table 6.

Comparing the optimal link flows of the blood organizations in the pre-merger problem with those of the newly-merged organization reveals the following results. For both organizations, the majority of links in the post-merger problem have maintained similar flows to the pre-merger problem. However, a few links (mostly of distribution type, including Links 16, 17, and 30) demonstrate some notable changes in the optimal weekly amount of blood to be distributed to their respective hospitals. On the other hand, apart from the non-operational Links 32 and 33 (representing the merger), only 3 out of the 18 newly added links play a

Table 5: Throughput Multipliers, Weekly Capacities, Total Operational Cost and Total Discarding Cost Functions for the New Links (32-51) in the Post-Merger Problem

Link $a$	$\alpha_a$	$\bar{u}_a$	$\hat{c}_a(f_a, \gamma_a)$	$\hat{z}_a(f_a)$
32	1.00	$n/a$	0	0
33	1.00	$n/a$	0	0
34	.99	30	$.4(f_{34})^2 + 1f_{34} + 9(\gamma_{34})^2 + 4\gamma_{34}$	$.75f_{34}$
35	.98	30	$.15(f_{35})^2 + 1.4f_{35} + 10(\gamma_{35})^2 + 5\gamma_{35}$	$.75f_{35}$
36	.99	30	$.5(f_{36})^2 + 2f_{36} + 8(\gamma_{36})^2 + 7\gamma_{36}$	$.9f_{36}$
37	.98	30	$.25(f_{37})^2 + 1f_{37} + 12(\gamma_{37})^2 + 6.5\gamma_{37}$	$.8f_{37}$
38	.98	30	$.3(f_{38})^2 + 1.5f_{38} + 9(\gamma_{38})^2 + 4\gamma_{38}$	$.75f_{38}$
39	.99	30	$.55(f_{39})^2 + 2f_{39} + 10(\gamma_{39})^2 + 4.5\gamma_{39}$	$.8f_{39}$
40	.99	25	$.17(f_{40})^2 + 1.5f_{40} + 7(\gamma_{40})^2 + 6\gamma_{40}$	$.75f_{40}$
41	1.00	25	$.15(f_{41})^2 + 2f_{41} + 7.5(\gamma_{41})^2 + 4\gamma_{41}$	$.6f_{41}$
42	.99	25	$.2(f_{42})^2 + 2f_{42} + 6(\gamma_{42})^2 + 6\gamma_{42}$	$.55f_{42}$
43	.98	25	$.25(f_{43})^2 + 2.5f_{43} + 8(\gamma_{43})^2 + 6\gamma_{43}$	$.7f_{43}$
44	.99	35	$.15(f_{44})^2 + 3f_{44} + 4.5(\gamma_{44})^2 + 6\gamma_{44}$	$.55f_{44}$
45	1.00	35	$.17(f_{45})^2 + 3f_{45} + 7(\gamma_{45})^2 + 5\gamma_{45}$	$.6f_{45}$
46	1.00	35	$.18(f_{46})^2 + 2.5f_{46} + 5(\gamma_{46})^2 + 4.5\gamma_{46}$	$.65f_{46}$
47	.99	35	$.16(f_{47})^2 + 3.5f_{47} + 4(\gamma_{47})^2 + 7\gamma_{47}$	$.65f_{47}$
48	1.00	32	$.35(f_{48})^2 + 3f_{48} + 2.5(\gamma_{48})^2 + 5\gamma_{48}$	$.55f_{48}$
49	.99	32	$.38(f_{49})^2 + 4f_{49} + 2(\gamma_{49})^2 + 6\gamma_{49}$	$.65f_{49}$
50	.98	32	$.4(f_{50})^2 + 1.8f_{50} + 3(\gamma_{50})^2 + 5.5\gamma_{50}$	$.5f_{50}$
51	1.00	32	$.35(f_{51})^2 + 1.5f_{51} + 2.5(\gamma_{51})^2 + 7\gamma_{51}$	$.55f_{51}$

relatively significant role in the post-merger problem. Interestingly, all three are distribution links, representing potential shipments of blood between the following origin-destination pairs:

- Link 46: from the storage facility in Tampa to the large-size hospital in Chicago with a weekly amount of 49.11 units;
- Link 50: from the storage facility in Pittsburgh to the large-size hospital in Florida with a weekly amount of 30.25 units; and
- Link 51: from the storage facility in Pittsburgh to the medium-size hospital in Florida with a weekly amount of 18.73 units.

Table 6: Optimal Values of the Weekly Link Flows, Weekly Frequencies of Activities, Lagrange Multipliers, as well as the Incurred Values of the Total Operational and Discarding Cost Functions for the New Organization in the Post-Merger Problem

Link $a$	$f_a^*$	$\gamma_a^*$	$\eta_a^*$	$\hat{c}_a(f_a^*, \gamma_a^*) + \hat{z}_a(f_a^*)$	Link $a$	$f_a^*$	$\gamma_a^*$	$\eta_a^*$	$\hat{c}_a(f_a^*, \gamma_a^*) + \hat{z}_a(f_a^*)$
1	208.21	1.04	.01	\$5,844.91	27	190.68	.09	.00	\$2,391.24
2	182.24	.81	.01	\$5,164.86	28	275.86	9.10	.04	\$201.94
3	300.78	1.34	.01	\$9,334.38	29	.00	.15	.11	\$0.00
4	191.06	5.44	.00	\$393.00	30	35.78	1.02	.29	\$591.98
5	4.74	.19	.26	\$16.33	31	105.91	2.70	.03	\$59.71
6	125.61	3.07	.03	\$188.13	32	691.23	$n/a$	.00	\$0.00
7	49.21	1.64	.11	\$163.17	33	491.01	$n/a$	.00	\$0.00
8	.00	.00	.08	\$0.00	34	6.16	.21	.26	\$27.16
9	297.77	7.50	.09	\$271.85	35	.00	.00	.17	\$0.00
10	315.42	.17	.00	\$55,602.90	36	3.77	.13	.30	\$19.02
11	352.62	.22	.00	\$57,080.43	37	.00	.00	.22	\$0.00
12	302.80	.15	.00	\$6,812.00	38	.00	.00	.13	\$0.00
13	331.46	.21	.00	\$6,956.56	39	.00	.00	.15	\$0.00
14	299.78	8.39	.03	\$299.18	40	.00	.00	.24	\$0.00
15	.00	.00	.09	\$0.00	41	.00	.00	.16	\$0.00
16	56.24	1.41	.41	\$585.76	42	.00	.00	.24	\$0.00
17	222.79	6.36	.00	\$260.96	43	.96	.04	.26	\$3.54
18	275.74	1.06	.02	\$8,613.85	44	.00	.00	.17	\$0.00
19	215.27	.92	.00	\$6,218.84	45	.00	.00	.13	\$0.00
20	270.22	10.81	.59	\$371.87	46	49.11	1.40	.53	\$605.07
21	.00	.00	.11	\$0.00	47	.00	.00	.18	\$0.00
22	13.53	.39	.25	\$51.10	48	.00	.00	.16	\$0.00
23	198.62	5.73	.02	\$176.08	49	.00	.00	.17	\$0.00
24	293.32	.13	.00	\$52,119.02	50	30.25	.95	.35	\$443.56
25	198.62	.10	.00	\$33,970.34	51	18.73	.59	.31	\$166.11
26	278.65	.11	.00	\$4,160.94					

This shows that, under normal demand scenarios, it makes sense for the newly-merged organization to allow merely between 10-20% of the demand of selected under-contract hospitals to be satisfied via a blood center located in the further location (belonging to the other organization in the pre-merger problem). This result suggests that satisfaction of the remaining hospital demand via the further blood center can not be economically justified due to a significantly higher transportation cost (This will be further discussed in Section 4.4). In addition, the fact that Links 34 through 43 (connecting the collection sites of one organization to the blood centers of the other organization) have negligible flows suggests that two merging blood centers geographically located far apart will not benefit from the possibility of having their collected blood processed and stored in the other blood center. Furthermore, the projected weekly values of demand for red blood cells at the 4 hospitals in the post-merger problem remain more-or-less unchanged:

$$v_{11}^* = 385.10 \text{ units}, \quad v_{12}^* = 241.52 \text{ units}, \quad v_{21}^* = 360.40 \text{ units}, \quad v_{22}^* = 105.91 \text{ units}.$$

It should be noted here that the above results; specifically, the reported percentages of demand to be satisfied by the other merging organization, to some extent depend on the choice of the shortage and surplus penalties by the blood organization(s) in addition to the parameters of the merger case (including but not limited to the operational cost functions as well as the probability distribution of the demands). Nevertheless, the overall conclusion can be generalized to a vast majority of mergers in the blood banking industry due to the high cost of storage and transportation of this highly perishable product over long distances. In the next section, we will analyze the synergy of a merger between two blood banks in response to an unexpected demand surge such as a natural disaster.

### 4.3 Investigation of the Impact of a Surge in Demand on a Merger

Terrorist attacks, natural disasters, as well as pandemics can result in a surge in demand for blood (cf. Mulcahy et al. (2016)). In this section, we investigate potential efficiencies

that can be garnered in the case of such an event, realizing also that in the case of climate change and even urbanization as well as increased global mobility, disaster preparedness, in which a sustainable blood supply plays a key role, is of critical importance.

In this section, we use the same merger case described earlier in this section to compare the resiliency of blood banks in the pre- and post-merger problems during the response phase of a disaster, which leads to a surge in demand. Assume a relatively large-scale natural disaster has hit parts of Florida, resulting in an abrupt surge in the demand for red blood cells in the two aforementioned Floridian hospitals immediately after the occurrence of the disaster. We assume the demand at  $R_1^1$  and  $R_2^1$  during that week follows a continuous uniform distribution on the intervals  $[2000,4000]$  and  $[1500,2500]$ , respectively; i.e., an increase by a factor of 10 as compared to the base scenario. The demand at the other two hospitals (located in Chicago and Pittsburgh) remains unchanged; i.e.,  $[220,370]$  for  $R_1^2$  and  $[80,110]$  for  $R_2^2$ . Furthermore, we assume the structure of the two blood organizations as well as all parameters of the pre- and post-merger problems (including the operational and discarding cost functions, arc multipliers, capacities on links, and unit shortage and surplus penalties) stay the same as their respective values.

We used the same computational framework to calculate the optimal solution (link flows, frequencies of activities as well as Lagrange multipliers) for the pre-merger problem and the post-merger one. Here, we report the highlights of the results for the demand surge case.

#### 4.3.1 Pre-Merger Problem Under a Demand Surge

The projected values of demand for red blood cells at the 4 hospitals during the week following the disaster are as follows. For OneBlood, Inc., we have:  $v_{11}^* = 2,951.65$  and  $v_{12}^* = 1,919.88$ . For ITxM, we have:  $v_{21}^* = 360.11$ , and  $v_{22}^* = 106.55$ . As expected, the projected demand for the hospitals in Florida has significantly risen while the other two stayed unchanged. The values of the projected demand for  $R_1^1$  and  $R_2^1$  still fall between the lower and upper bounds of the uniform distribution, although they are a bit closer to the

lower bound as a result of the choice of the unit shortage penalties by the decision-maker given that the blood bank is utilizing the same network structure to respond to the abrupt demand surge.

The optimal link flows and frequencies on links corresponding to Organization 1 (OneBlood) have largely increased to be able to respond to the demand surge in the hospitals. The other organization experiences no changes, not surprisingly.

### 4.3.2 Post-Merger Problem Under a Demand Surge

We now have the following projected values of demand for red blood cells during the week of the disaster:  $v_{11}^* = 3,239.64$ ,  $v_{12}^* = 2,080.32$ ,  $v_{21}^* = 328.26$ , and  $v_{22}^* = 91.93$ . As seen here, values of the projected demand for the hospitals located in the disaster-impacted region have now shifted closer to the upper bound of their uniform distributions. Interestingly, the other two hospitals (located over a thousand miles away from the region) have suffered drops of 8.8% and 13.7% in their projected demand values. This is because the new organization now minimizes the overall objective value of the entire network which results in the shortage being distributed across the network.

Several links across the supply chain network experience increased flows and frequencies in the optimal solution. For example, the optimal flow on blood collection Links 18 and 19 (located in Chicago and Pittsburgh) have increased to 1,378.23 and 1,115.24 units; i.e., increases of 381.28% and 419.97% as compared to the pre-merger problem, respectively.

## 4.4 Discussion: Synergy of the Mergers

We have summarized the comparison of the two blood banks in the pre- and post-merger problems under status quo versus the increased demand scenario from a cost perspective in Table 7.

As seen in Table 7, under status quo demand scenario, the total weekly value of objective

Table 7: Comparison of the Optimal Values of the Weekly Projected Demand, Total Objective Value, Expected Total Shortage and Surplus Costs under Status Quo and Disaster Scenarios corresponding to Pre-Merger and Post-Merger Problems, as well as the Total Cost Efficiency (Synergy), Shortage and Surplus Penalty Measures

		Status Quo Scenario			Disaster Scenario		
		Pre-Merger		Post-Merger	Pre-Merger		Post-Merger
		OneBlood, Inc.	ITxM		OneBlood, Inc.	ITxM	
Projected	$v_{11}^*$	385.03	–	385.10	2,951.65	–	3,239.64
	$v_{12}^*$	241.59	–	241.52	1,919.88	–	2,080.32
Demand	$v_{21}^*$	–	360.11	360.40	–	360.11	328.26
	$v_{22}^*$	–	106.05	105.91	–	106.55	91.93
Total Objective Value		\$161,753.22	\$119,078.72	\$278,957.84	\$11,666,944.68	\$119,079.06	\$8,966,685.81
Expected Total Shortage Cost		\$6,044.32	\$3,570.41	\$9,528.83	\$2,932,952.43	\$3,570.03	\$1,606,741.21
Expected Total Surplus Cost		\$6,796.02	\$3,465.61	\$10,263.21	\$16,609.31	\$3,465.64	\$31,052.66
$\mathcal{E}^{TC}$		0.67%			23.92%		
$\mathcal{S}^-$		0.67%			45.28%		
$\mathcal{S}^+$		-0.02%			-54.68%		

function (5) including the total shortage and surplus penalties for Organizations 1 and 2 (i.e., OneBlood, Inc., and ITxM, as shown in Figure 4) in the pre-merger problem are \$161,753.22, and \$119,078.72, respectively.

In the post-merger problem, the total weekly value of objective function (13) is \$278,957.84. Utilizing the introduced total cost synergy measure in (14) while taking into account this objective value along with the ones reported above for Organizations 1 and 2 yields a paltry  $\mathcal{E}^{TC}$  value of 0.67% for the pending merger between the two blood banks. In addition, the values of the supply shortage synergy measure,  $\mathcal{S}^-$ , and the surplus synergy measure,  $\mathcal{S}^+$ , are 0.67% and -0.02%, respectively (see equations (15) and (16)).

Under the demand surge scenario, in the pre-merger problem, the total objective value of (5), corresponding to OneBlood, Inc., has risen to \$11,666,944.68 including \$8.7M in operational costs as well as a \$2.9M shortage penalty. The total objective value of ITxM remains unchanged.

In the post-merger problem, the total objective value of the new organization, (13), is \$8,966,685.81 resulting in a remarkable 23.92% total cost synergy ( $\mathcal{E}^{Tc}$ ) during the time of disaster (see (14)). More importantly, the supply shortage synergy,  $\mathcal{S}^-$ , see (15), reaches a significant 45.28%, contributing to the resiliency of the organization during the disaster as a result of the merger. In contrast, the surplus synergy measure,  $\mathcal{S}^+$ , see (16), shows a major decrease of -54.68%. It should be noted that the surplus penalty is of least importance during the response phase of a disaster. In addition, the dollar value of the surplus penalty in the post-merger problem is merely \$31,052.66 as compared to the \$1,606,741.21 shortage penalty justifying the relative importance of the shortage when compared to the surplus penalty under the disaster-related demand surge scenario.

Comparing the values in the last three rows of Table 7 corresponding to status quo and the demand surge, specifically the total cost synergy measure,  $\mathcal{E}^{Tc}$ , as well as the expected total shortage synergy measure,  $\mathcal{S}^-$ , reveals the following conclusion. When the two merging blood banks are geographically distant, the synergy gained from the merger is not significant under normal demand situations. In contrast, under a demand surge scenario, the merged organization will experience a significant synergy, both in terms of the total operational cost as well as the expected shortage penalty despite the distance between the blood banks. The two aforementioned outcomes (improved operational efficiency and enhanced stability of the blood supply) are the major motives for blood organizations to merge. Thus, blood banks should take into careful consideration the intensity of irregular demand scenarios, in addition to other operational and organizational factors, when facing the crucial decision of a merger, acquisition, or any other form of alliance.

## 5. Conclusions

The blood banking industry, critical to healthcare, is at a crossroads in the United States, with medical and technological advances, as well as economic pressures, resulting in a downward demand trend for blood products. As a consequence, the past decade has seen a



doubling in the number of mergers and acquisitions in this sector, in comparison to the decade prior, with a big driver being cost reduction.

In this paper, we develop a methodological framework that allows for the quantifiable assessment of a merger or acquisition concerning blood banks. Blood is a unique product in that it cannot be produced but must be donated and, at the same time, it is a perishable product with red blood cells having a lifetime of only 42 days. In particular, we construct network optimization pre-merger and post-merger models reflecting blood supply chains of different blood banks, which have the following features:

1. The models capture perishability of blood through link multipliers, which reflect the throughput on a link, be it a collection, processing and testing, storage, or distribution link, along with the capacity on the link (over a time horizon, such as a week).

2. The models, when solved, yield the cost-minimizing blood flows on paths and links in the network(s), along with the frequencies of the link activities, and the Lagrange multipliers associated with the capacity constraints.

3. Uncertainty on the demand side is captured along with discarding costs associated with waste on the links, and costs associated with expected shortages or surpluses at the demand points.

4. A total cost efficiency measure is introduced, which quantifies the potential synergy associated with the merger or acquisition, along with measures capturing the expected supply shortage and surplus.

5. A case study, focusing on a real-world pending merger is then presented, under two different scenarios - one considering the status quo and the other - a time of a disaster.

The proposed methodological framework can handle the assessment of a pending merger or acquisition of two or more blood bank organizations and can serve as a valuable tool for such organizations in these uncertain and volatile times for the industry.

Nevertheless, in this paper, we focused on vertically integrated blood service organizations. We did not consider a decentralized blood supply chain network, involving different blood collection organizations and individual hospitals. Moreover, we did not consider uncertainties on the supply side, as well as possible disruptions in blood supply chain operations. Finally, it would also be interesting to capture a combination of for-profit as well as non-profit actors in blood supply chains.

In addition, in future research, the time component can be explicitly captured in an extension of the model(s). Furthermore, the time consumption associated with each blood supply chain activity may depend on not only the amount of blood products but also on the activity frequency.

Although there are directions for future research, our framework fills a gap in the literature since it provides a methodology to assess mergers and acquisitions in a healthcare industry of a unique, highly perishable product, that of blood.

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## Appendix

### Proof of Theorem 1:

Let  $TC_i^0$  denote that part of the objective function (5) corresponding to blood service organization  $i$ ;  $i = 1, \dots, I$ . Since the functions in (5) are convex and continuously differentiable and the feasible set  $K$  is closed and convex, based on the classical theory of variational inequalities (see Kinderlehrer and Stampacchia (1980) and Nagurney (1999)), we know that the optimal solution to (5) coincides with the solution to the following variational inequality: determine  $(x^*, \gamma^*, \eta^*) \in K$ , such that:

$$\begin{aligned} & \sum_{i=1}^I \sum_{k=1}^{n_R^i} \sum_{p \in \mathcal{P}_{R_k^i}^0} \left[ \frac{\partial TC_i^0}{\partial x_p} + \sum_{a \in L_i} \eta_a^* \alpha_{ap} \right] \times [x_p - x_p^*] + \sum_{i=1}^I \sum_{a \in L_i} \left[ \frac{\partial TC_i^0}{\partial \gamma_a} - \bar{u}_a \eta_a^* \right] \times [\gamma_a - \gamma_a^*] \\ & + \sum_{i=1}^I \sum_{a \in L_i} \left[ \bar{u}_a \gamma_a^* - \sum_{q \in \mathcal{P}^0} x_q^* \alpha_{aq} \right] \times [\eta_a - \eta_a^*] \geq 0, \quad \forall (x, \gamma, \eta) \in K. \end{aligned} \quad (\text{A1})$$

For each path  $p$ ;  $p \in \mathcal{P}_{R_k^i}^0$ , the partial derivative of the objective function with respect to the flow on that path is calculated as follows:

$$\begin{aligned} \frac{\partial TC_i^0}{\partial x_p} &= \frac{\partial [\sum_{a \in L_i} \hat{c}_a(f_a, \gamma_a) + \sum_{a \in L_i} \hat{z}_a(f_a) + \sum_{k=1}^{n_R^i} (\lambda_{ik}^- E(\Delta_{ik}^-) + \lambda_{ik}^+ E(\Delta_{ik}^+))]}{\partial x_p} \\ &= \sum_{a \in L_i} \frac{\partial \hat{c}_a(f_a, \gamma_a)}{\partial x_p} + \sum_{a \in L_i} \frac{\partial \hat{z}_a(f_a)}{\partial x_p} + \sum_{k=1}^{n_R^i} \lambda_{ik}^- \frac{\partial E(\Delta_{ik}^-)}{\partial x_p} + \sum_{k=1}^{n_R^i} \lambda_{ik}^+ \frac{\partial E(\Delta_{ik}^+)}{\partial x_p} \\ &= \sum_{a \in L_i} \frac{\partial \hat{c}_a(f_a, \gamma_a)}{\partial f_a} \frac{\partial f_a}{\partial x_p} + \sum_{a \in L_i} \frac{\partial \hat{z}_a(f_a)}{\partial f_a} \frac{\partial f_a}{\partial x_p} + \sum_{k=1}^{n_R^i} \lambda_{ik}^- \frac{\partial E(\Delta_{ik}^-)}{\partial v_{ik}} \frac{\partial v_{ik}}{\partial x_p} + \sum_{k=1}^{n_R^i} \lambda_{ik}^+ \frac{\partial E(\Delta_{ik}^+)}{\partial v_{ik}} \frac{\partial v_{ik}}{\partial x_p} \end{aligned}$$

$$= \sum_{a \in L_i} \frac{\partial \hat{c}_a(f_a, \gamma_a)}{\partial f_a} \alpha_{ap} + \sum_{a \in L_i} \frac{\hat{z}_a(f_a)}{\partial f_a} \alpha_{ap} + \sum_{k=1}^{n_R^i} \lambda_{ik}^- \frac{\partial E(\Delta_{ik}^-)}{\partial v_{ik}} \mu_p + \sum_{k=1}^{n_R^i} \lambda_{ik}^+ \frac{E(\Delta_{ik}^+)}{\partial v_{ik}} \mu_p. \quad (\text{A2})$$

Note that following Leibniz's integral rule, we have

$$\frac{\partial E(\Delta_{ik}^-)}{\partial v_{ik}} = P_{ik}(v_{ik}) - 1 = P_{ik} \left( \sum_{q \in \mathcal{P}_{R_k^i}^0} x_q \mu_q \right) - 1, \quad i = 1, \dots, I; \quad k = 1, \dots, n_R^i, \quad (\text{A3a})$$

$$\frac{\partial E(\Delta_{ik}^+)}{\partial v_{ik}} = P_{ik}(v_{ik}) = P_{ik} \left( \sum_{q \in \mathcal{P}_{R_k^i}^0} x_q \mu_q \right), \quad i = 1, \dots, I; \quad k = 1, \dots, n_R^i. \quad (\text{A3b})$$

Similarly, for each link  $a$ ;  $a \in L_i$ , the partial derivative of the objective function with respect to the frequency on that link is:

$$\begin{aligned} \frac{\partial TC_i^0}{\partial \gamma_a} &= \frac{\partial [\sum_{a \in L_i} \hat{c}_a(f_a, \gamma_a) + \sum_{a \in L_i} \hat{z}_a(f_a) + \sum_{k=1}^{n_R^i} (\lambda_{ik}^- E(\Delta_{ik}^-) + \lambda_{ik}^+ E(\Delta_{ik}^+))]}{\partial \gamma_a} \\ &= \sum_{a \in L_i} \frac{\partial \hat{c}_a(f_a, \gamma_a)}{\partial \gamma_a} + \sum_{a \in L_i} \frac{\hat{z}_a(f_a)}{\partial \gamma_a} + \sum_{k=1}^{n_R^i} \lambda_{ik}^- \frac{\partial E(\Delta_{ik}^-)}{\partial \gamma_a} + \sum_{k=1}^{n_R^i} \lambda_{ik}^+ \frac{E(\Delta_{ik}^+)}{\partial \gamma_a} \\ &= \sum_{a \in L_i} \frac{\partial \hat{c}_a(f_a, \gamma_a)}{\partial \gamma_a}. \end{aligned} \quad (\text{A4})$$

By making use of the definitions in (7a)–(7c) and the results in (A3a) and (A3b), variational inequality (6) is immediate. In addition, the equivalence between variational inequalities (6) and (8) can be proved with (1)–(3).  $\square$

## Explicit Formulae for the Euler Method Applied to Blood Supply Chain Network Variational Inequality (6)

We now present a framework for the computation of the optimal solutions to the pre- and post-merger blood banking problems. The below explicit formulae are the realization of the Euler method, which is based on the general iterative scheme introduced by Dupuis and

Nagurney (1993), for our model and iteratively yield the solution to the pre-merger blood banking problem governed by variational inequality (6) (see also (8)).

Specifically, the closed form expression for the path flow on path  $p \in \mathcal{P}_{R_k^i}^0$ ;  $i = 1, \dots, I$ ;  $k = 1, \dots, n_R^i$ , at iteration  $\tau + 1$  is:

$$x_p^{\tau+1} = \max\left\{0, x_p^\tau + a_\tau \left[ \lambda_{ik}^- \mu_p \left( 1 - P_{ik} \left( \sum_{q \in \mathcal{P}_{R_k^i}^0} x_q^\tau \mu_q \right) \right) - \lambda_{ik}^+ \mu_p P_{ik} \left( \sum_{q \in \mathcal{P}_{R_k^i}^0} x_q^\tau \mu_q \right) - \frac{\partial \hat{C}_p(x^\tau, \gamma^\tau)}{\partial x_p} - \frac{\partial \hat{Z}_p(x^\tau)}{\partial x_p} - \sum_{a \in L_i} \eta_a^\tau \alpha_{ap} \right] \right\}. \quad (\text{A5})$$

The closed form expression for the activity frequency  $\gamma_a$ ;  $a \in L_i$ ;  $i = 1, \dots, I$ , is:

$$\gamma_a^{\tau+1} = \max\left\{0, \gamma_a^\tau + a_\tau \left[ \bar{u}_a \eta_a^\tau - \frac{\partial \hat{C}_p(x^\tau, \gamma^\tau)}{\partial \gamma_a} \right] \right\}, \quad (\text{A6})$$

with the Lagrange multipliers being computed for  $a \in L^0$  according to:

$$\eta_a^{\tau+1} = \max\left\{0, \eta_a^\tau + a_\tau \left[ \sum_{q \in \mathcal{P}^0} x_q^\tau \alpha_{aq} - \bar{u}_a \gamma_a^\tau \right] \right\}. \quad (\text{A7})$$

As shown in Dupuis and Nagurney (1993), as well as in Nagurney and Zhang (1996), convergence of the iterative scheme requires that the sequence  $\{a_\tau\}$  satisfies:  $\sum_{\tau=0}^{\infty} a_\tau = \infty$ ,  $a_\tau > 0$ ,  $a_\tau \rightarrow 0$ , as  $\tau \rightarrow \infty$ . The aforementioned references contain additional conditions for convergence as well as other network applications.

Note that the explicit formulae for the post-merger model are akin to the respective expressions (A5) – (A7) where  $a \in L^1$ ,  $p \in \mathcal{P}_{n_k^i}^1$ , for  $i = 1, \dots, I$ , and  $k = 1, \dots, n_R^i$ .