Supply Chain Network Competition Among Blood Service Organizations: A Generalized Nash Equilibrium Framework

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December 2017; revised August 2018

Annals of Operations Research (2019), **275(2)**, pp 551-586.

Abstract

In this paper we present a Generalized Nash Equilibrium model of supply chain network competition among blood service organizations which compete not only for blood donors but also for business from hospitals and medical centers. The model incorporates not only link capacities and associated arc multipliers to capture perishability, but also bounds on the number of donors in regions as well as lower and upper bounds on the demands at the demand points in order to ensure needed amounts for surgeries, treatments, etc., while reducing wastage. The concept of a variational equilibrium is utilized to transform the problem into a variational inequality problem, and alternative formulations are given. A Lagrange analysis yields economic insights. The proposed algorithmic procedure is then applied to a series of numerical examples in order to illustrate the impacts of disruptions in the form of a reduction on the number of donors as well as that of decreases in capacities of critical links such as testing and processing on RBC prices, demands, net revenues of the blood service organizations, and their overall utilities.

Keywords: game theory, blood supply chains, supply chain competition, Generalized Nash Equilibrium, variational inequalities, healthcare

1. Introduction

The efficient and effective management of blood supply chains is crucial as well as challenging for blood service organizations. Several inherent characteristics of this life-saving product such as its short shelf life, and the fact that it cannot be manufactured but must be donated, create difficulties and risks for stakeholders in this healthcare sector. Shortages in the supply can lead to societal loss through deaths and, at the same time, excess blood collected from donors can create loss through wastage.

The blood banking industry in the United States is at a crossroads and is trying to adapt to the changes in the market and to sustain itself economically (cf. Nagurney (2017)). According to the American Red Cross, the leading supplier of blood in the US, with about 40% of the market, there was a 33% decrease in blood transfusions in the period 2010-2014 (Wald (2014)). Faced with decreased demand, resulting, for example, from medical advances associated with minimally invasive technologies, a rise in competition among blood banks, and an increase in the cost of testing of collected blood, many blood service organizations have been running their operations at negative margins (see Mulcahy et al. (2016)). Furthermore, the new economic landscape for this industry has been accompanied by an increasing number of mergers and acquisitions (cf. Masoumi, Yu, and Nagurney (2017)) with the goal of identifying and exploiting various synergies, including cost-based ones. Blood banks need to be prepared financially in order to meet the demand with a steady supply of safe blood.

The supply of blood in the United States, however, depends on voluntary donations and donors cannot be paid for providing red blood cells (RBCs). Interestingly, according to the World Health Organization (2017), globally, 74 countries obtain more than 90% of their blood supply from voluntary unpaid blood donors, whereas 71 countries collect more than 50% of their blood supply from family/replacement or paid donors. On one hand, it may be difficult to invoke altruism in people and to motivate them to donate blood, and, on the other hand, individuals who are motivated to donate blood might not qualify to do so. According to the Food and Drug Administration regulations, in the US, blood donors have to go through a strict screening procedure. In addition to meeting the age, weight, and hemoglobin level requirements, as well as the time period between donations, donors are also screened for disease risk factors and may get deferred for reasons such as exhibiting signs and symptoms of colds or the flu, and/or relevant transfusion-transmitted infections, i.e., HIV, viral hepatitis, etc. (cf. American Association of Blood Banks (2017)). As a result, 38% of the US population is eligible to donate blood at any given time. However, less than 10% actually donates blood in a year (cf. American Red Cross (2017)). Issues of seasonality place additional pressures on obtaining blood donations since donors may be preoccupied with holidays and/or weather-related issues. Hence, since the blood banking industry has to rely on voluntary donations from altruistic donors, it, despite an overall decrease in demand, nevertheless, faces major challenges in terms of maintaining a sufficient supply of blood and will continue to do so, given the aging population.

Industry experts are noting that the blood supply chain is becoming more and more similar to traditional commercial supply chains. Consequently, although the blood banking industry is characteristically not for profit, it is not surprising to find the prevalence of competition among blood service organizations. Competition exists for blood donations as well as for supply contracts with hospitals and other medical facilities (Snyder (2001), Hart (2011)). Since 2011, a small Sarasota-based blood bank, SunCoast Communities Blood Bank, had been competing for blood donations with a much larger organization, Florida Blood Services, that served hospitals in Tampa and neighboring areas (Smith (2011)). In 2012 the plight of SunCoast Communities Blood Bank in conducting its operations became more evident when it urged the state of Florida to stop the merger between three blood service organizations; namely, Orlando-based Florida's Blood Centers, the Community Blood Centers of Lauderhill, and Florida Blood Services of St. Petersburg on grounds of antitrust issues (Smith (2012)). In 2013 Eastern Maine Medical Center ended its contract with American Red Cross to do business with Puget Sound Blood Center, a Seattle-based community blood bank (Barber (2013)). This trend is visible all across the country. Stone (2015) notes that "loyal blood donors will no longer see the iconic red cross on the side of the blood mobile next time they give blood at one of Mission Health's 17 facilities in Western North Carolina" because of a switch of supplier to a regional nonprofit blood bank of South Carolina, The Blood Connection, from American Red Cross, ending a 30 year old contractual relationship. Prior to this new three year contract during which The Blood Connection will be the sole blood provider to Mission Health, it had been providing only a supplemental blood supply to the Mission hospitals. More recently, at the end of 2016, the American Red Cross lost its business in Central Arkansas to Arkansas Blood Institute, an affiliate of the Oklahoma Blood Institute. This resulted in a layoff of 44 Red Cross employees at two blood centers (Brantley (2017)).

It is evident from the above examples that hospitals and medical centers may have several options for suppliers not only within the community and region, but also from out of state blood banks that may offer lower prices. In order to mitigate the risk of shortages due to possible supply shortfalls, hospitals and trauma medical centers may try to diversify their supplier base and contract with multiple blood service organizations (Merola (2017)). The contracts vary in terms of price, quantity, product mix and duration (Merola (2017)). There have been cases where reduced prices have led hospitals to switch blood suppliers (Schwartz (2012)). However, hospitals and medical centers are most sensitive to service failures such as the inability to meet demand in a timely manner (Wellis (2017)).

The changing dynamics in the blood industry is forcing blood banks (also referred to as blood service organizations) to be more innovative in conducting their business. Blood banks need to price their products competitively based on the supplied quantity in order to recover costs of their operations and to generate revenue for activities such as research and development required for providing a steady supply of safe blood. Hence, there is a need for a change in the approach towards blood supply chain management, which should take into consideration not only the well-defined problems of perishability, outdating, shortage, and wastage (see, e.g., Nagurney, Masoumi, and Yu (2012)), but also limits on supply capacity and competition among blood banks.

In this paper, we formulate a supply chain network competition model for blood banks in which each blood service organization is faced with the objective of utility maximization, with components of each blood service organization's utility function comprised of revenue, cost, and also a weighted altruism component, since they are nonprofits. The blood service organizations compete noncooperatively for blood donations and on the network economic activities associated with their blood supply chains, while aiming to be selected by the hospitals and other medical centers to supply the required quantity of blood to them, with these institutions expressing their preferences through the prices that they are willing to pay. The competing blood service organizations face common supply capacity constraints in terms of number of available donors in the collection regions. In addition, there are lower and upper bounds associated with the demand for blood at the hospitals and medical centers at the demand points over the time horizon since the healthcare facilities need to have a sufficient amount in stock due to scheduled surgeries, possible additional demand due to trauma and accidents, etc., while, at the same time, they do not want to have an oversupply resulting in wastage and adding to the costs of storage and upkeep. The competitive supply chain network model, due to the common, that is, the shared, constraints on the supply side and on the demand side, is governed by a Generalized Nash Equilibrium (GNE) (cf. von Heusinger (2009) and Fischer, Herrich, and Schonefeld (2014)) rather than a Nash equilibrium (see Nash (1950, 1951)), since not only do the utility functions of the blood service organizations depend on the strategies of the other blood service organizations, but their feasible sets do as well. We remark that Rosen (1965), in his foundational paper, studied a class of Generalized Nash Equilibrium problems.

We derive alternative variational inequality formulations of the governing Generalized

Nash Equilibrium using the concept of a *Variational Equilibrium*, and also conduct economic analysis using Lagrange theory. In addition, we propose an effective algorithmic scheme which is then applied to numerical examples illustrating the generality and flexibility of the framework.

2. Literature Review

In this section we present a review of the existing literature on blood supply chain optimization and that on supply chain competition and game theory, as well as on capacity constraints and Generalized Nash Equilibrium, with relevance to our competitive blood service organizations supply chain network model. We identify the gaps in some of the areas and also note how our work contributes to bridging those gaps.

2.1 Blood Supply Chain Optimization

The body of literature on blood supply optimization has been growing steadily over the years with some of the fundamental literature including that of Nahmias (1982) on perishable product inventory and that of Cohen and Pierskalla (1979) targeted at hospital and regional blood banks. Beliën and Forcé (2012) provide a comprehensive review of the supply chain management of blood products. The authors classify the works according to various categories such as the type of blood product, the solution method utilized, etc. Given the uncertainty in demand and supply of blood and the complex nature of blood supply chains, some authors have used simulation techniques to optimize the inventory levels (see, e.g., Rytilä and Spens (2006), Kopach (2008)) while others (see, e.g., Pierskalla (2005), Hemmelmayr et al. (2009)) have used mathematical programming to solve associated facility location and routing problems. Sarhangian et al. (2017) studied the performance of threshold-based allocation policies for optimizing blood inventory taking into consideration the trade-off between age of the blood and availability. Ramezanian and Behboodi (2017) used mixed integer linear programming (MILP) to solve a deterministic location/allocation problem for blood collection facilities that takes into consideration the utility of blood donors in order to motivate them. They further utilized a robust optimization method to account for the issue of uncertainty in demand for blood. Fortsch and Khapalova (2016) tackle some of the issues of blood supply chain management by using various forecasting techniques to better predict the demand for blood at the blood centers; thereby, reducing the uncertainty regarding the demand for blood. Nagurney, Masoumi, and Yu (2012) take into account uncertainty in the demand for blood and construct the full associated supply chain network of a blood service organization. El-Amine, Bish, and Bish (2017) focus on blood screening and consider that the budget-constrained blood center's goal is to construct a "robust" postdonation blood-screening scheme that minimizes the risk of an infectious donation being released into the blood supply. Ayer et al. (2017), in turn, consider when and from which mobile collection sites to collect blood for cryo production, such that the weekly collection target is met while the collection costs are minimized. Recent interesting work has also considered the design of a blood supply chain network in a crisis via a robust stochastic model (see Salehi, Mahootchi, and Husseini (2017). Overall, there are few studies that address the supply side in blood banking (Fahimnia et al. (2017), Ramezanian and Behboodi (2017)) and especially in the context of a competitive environment. In this paper, we incorporate supply side capacity limits that arise due to the dependence on donations from voluntary donors since not only do blood service organizations have physical capacities associated with collection, and other network economic activities in their supply chains, but they also vie for donations from a limited donor pool.

Perishability of blood and wastage due to outdating is another common issue in blood supply chains and, hence, has been incorporated into some studies (Chazan and Gal (1977), Nagurney, Masoumi, and Yu (2012), Duan and Liao (2014), Wang and Ma (2015)). Much of the recent work on blood supply chains focuses on the optimization of the inventory at the hospital and blood bank levels as well as on the optimization of the shipment of blood from the blood banks to the hospitals (Gunpinar and Centeno (2015), Wang and Ma (2015)). However, there is a dearth of research on optimization of the entire blood supply chain network due to its complexity. In their literature review paper, focusing on quantitative models in blood supply chain management, Osorio, Brailsford, and Smith (2015) mention a few studies on integrated models or models that include all stages of the blood supply chain (see also, e.g., Katsaliaki and Brailsford (2007), Delen, Erraguntla, and Mayer (2011), Nagurney, Masoumi, and Yu (2012)). For example, Nagurney, Masoumi, and Yu (2012) develop a multicriteria optimization model for a regional blood banking system while capturing the myriad associated supply chain network activities. Similarly, in this paper we take a holistic approach and include in our network structure and model the different stages of the blood supply chain along with the associated activities, such as collection, testing, processing, distribution, etc., of the multiple blood service organizations.

2.2 Supply Chain Competition Among Nonprofits and Game Theory

A major contribution of our paper is the modeling of the supply chain competition among blood service organizations using game theory. We identify the network structure in a manner that it can reflect competition among blood service organizations on a regional basis as well as on a national level, if need be. To the best of our knowledge, this is the first attempt of its kind. While there are some studies on the effects of competition among nonprofit organizations (see Muggy and Heier Stamm (2014), Nagurney, Alvarez Flores, and Soylu (2016), Nagurney and Li (2017), and the references therein), a thorough literature search failed to identify any work on the modeling of competition among blood service organizations in the framework of their supply chains.

Noncooperative game theory is a powerful tool that is used extensively for formulating and solving problems where there is competition. While it is primarily used in the case of profit-making entities, several studies have used game theory to model competition among nonprofit organizations; see, e.g., Ortmann (1996), Tuckman (1998), Castaneda, Garen, and Thornton (2008), Bose (2015), and Nagurney and Li (2017). There is, however, a limited number of works applying game theory in the realm of nonprofit supply chains and even fewer in the context of blood services. Saxton and Zhuang (2013) argued for the relevance of game theory in markets for charitable contributions and presented a model consisting of an organization and a donor. Zhuang, Saxton, and Wu (2011) provided a sequential game theoretical model of disclosure-donation interactions with one nonprofit organization and multiple donors. Nagurney, Alvarez Flores, and Soylu (2016) developed a Generalized Nash Equilibrium network model in which nonprofit organizations are competing for financial funds for post-disaster relief operations, while minimizing costs associated with relief item distribution. Nagurney and Dutta (2018) built on that paper and constructed a game theoretical model that captures the competition among blood banks for voluntary donations. In this paper, we move forward to develop the competition model for the blood supply chain networks associated with multiple blood service organizations, which also yields pricing information, upon solution.

2.3 Supply Capacity Constraints and Generalized Nash Equilibrium

There exists a rich body of literature on capacity constraints in supply chains (Gavirneni (2002), Lee and Kim (2002), Choi, Dai, and Song (2004), Goh, Lim, and Meng (2007), Jung et al. (2008), Nagurney and Li (2016), Nagurney, Yu, and Besik (2017), and Nagurney (2018)). However, the capacities considered in those supply chains pertain to physical capacities of production plants, distribution channels, freight service providers, etc., which vary from one firm to another. Similar to commercial firms, blood service organizations have limited resources in terms of space for collection, processing, and storage, and access to transportation vehicles, etc. Masoumi, Yu, and Nagurney (2017) introduce upper bounds on the capacity volume of various activities in the blood supply chain network consisting of collection, processing, shipment, storage, and distribution along with frequencies of supply chain activities. In keeping with their work, we include explicit physical capacities on all links pertaining to activities, from the collection of blood to the distribution to demand markets; however, since our work is more on the tactical and strategic levels, we exclude frequencies. Moreover, we include multiple, competing blood service organizations as well as novel supply and demand constraints.

A segment of the blood supply literature is focused on donor motivation. Theoretical works such as those of Andreoni (1990), Mellström and Johanesson (2008), and Evans and Ferguson (2013) discuss the effect of altruism on blood donor motivation. Several papers (see Gillespie and Hillyer (2002), Schreiber et al. (2006), Yuan et al. (2011)) study the impact of operational aspects of blood collection that might motivate or deter blood donation such as the wait time, the location of the collection center, and the treatment by staff. Based on the empirical evidence provided in such studies, Nagurney and Dutta (2018) formulated a game theoretical model that captures the competition for blood donations among blood service organizations that compete on the quality of service provided. The voluntary donation aspect, as mentioned earlier, poses another challenge to blood banks since only a segment of the population can donate blood. Hence, we can say that there is an upper bound on the quantity of blood that can be collected, imposing additional but, now, common/shared, constraints for the competing blood service organizations. Furthermore, we include common/shared constraints on the demand side at the demand points. Nagurney, Yu, and Besik (2017) utilize the concept of a Variational Equilibrium to formulate and solve the Generalized Nash Equilibrium problem in the case of commercial supply chain network capacity competition with outsourcing with capacities associated with shared links of storage and freight service provision. For a detailed discussion of GNE, in the context of a nonprofit, disaster relief framework, and the associated challenges of formulation and solution, we direct the readers also to Nagurney, Alvarez Flores, and Soylu (2016) and the references therein.

2.4 Our Contributions

Since Generalized Nash Equilibrium problems are challenging to solve when formulated as quasivariational inequality problems (Bensoussan (1974)), for which the state-of-the-art in terms of algorithmic procedures is not as advanced as that for variational inequality problems (cf. Nagurney (2006) for background on supply chains and variational inequalities), here we utilize the concept of *Variational Equilibrium*. In particular, a variational equilibrium, which is a specialized type of GNE, allows us to formulate and solve our model as a variational inequality problem. According to Luna (2013), the associated Lagrange multipliers of the common constraints are then equal to the different players in the game, which provides us with an elegant economic interpretation in terms of fairness.

To-date, there has been only a limited literature in terms of supply chain network competition with shared constraints in terms of either commercial applications (see Nagurney, Yu, and Besik (2017)) or nonprofit ones (cf. Nagurney, Alvarez Flores, and Soylu (2016), Nagurney et al. (2017)). Moreover, in terms of Lagrange analysis and supply chain network problems, which are Generalized Nash Equilibrium problems, to-date, the work of Nagurney et al. (2017) in competitive supply chain models for disaster relief is relevant. However, that model is a pure network flow model, whereas the model in this paper is a generalized network model to capture perishability. For Lagrange theory applied to other variational inequality network models, but not Generalized Nash Equilibrium ones, see: Daniele (2001) (spatial economic models), Barbagallo, Daniele, and Maugeri (2012) (financial networks), Toyasaki, Daniele, and Wakolbinger (2014) (end-of-life products networks), Daniele and Giuffrè (2015) (random traffic networks), Caruso and Daniele (2018) (transplant networks), and Colajanni et al. (2018) (supply chains and cybersecurity investments).

The supply chain competition model for blood service organizations (BSOs) in this paper bridges gaps in the literature through the following contributions:

1. We include multiple, competing blood service organizations in which the link cost functions are not assumed to be separable; that is, the cost on a link may, in general, depend not only on the flow on that link but also on flows on other links of the specific BSO's supply chain network as well as on the flows on links of other BSOs' supply chain networks. In the case of multiple blood supply chain networks, in contrast, Masoumi, Yu, and Nagurney (2017) considered link cost functions that were separable and in the case of cost minimization. The generality of our cost functions enables the modeling of supply chain network competition for resources among the BSOs.

2. Common/shared capacities are incorporated on the supply side in terms of blood donations, and common/shared constraints on the demand side due to demand point constraints consisting of lower and upper bounds on the blood needed. No model, to-date, considers such features with the former uniquely relevant to blood supply chain network competition, not considered until this paper, and the latter also very relevant due to the need to meet the demand for blood while also minimizing wastage.

3. The utility functions of the blood service organizations contain revenue as well as altruism/benefit components with the latter being weighted. Nagurney and Li (2017) also considered nonprofit competition with revenue and altruism features but in the case of hospital competition on a simpler, bipartite network and with the altruism component of an

entirely different construct than herein.

4. Not only is the equilibrium blood product flow in terms of RBCs determined for each blood service organization, given the competition and the constraints, and also perishability, but also differentiated prices revealed. Hence, our model enables the incorporation of different factors that would affect prices that distinct hospitals and medical centers would be willing to pay for RBCs and that different blood service organizations would, therefore, be able to charge them.

5. We provide alternative variational inequality formulations of the Generalized Nash Equilibrium problem, along with economic analysis utilizing Lagrange theory associated with the various capacity constraints as well as the demand constraints.

6. Finally, we propose an effective computational scheme and then apply it to compute the equilibrium solutions in numerical examples comprising our case study.

The remainder of this paper is organized as follows. In Section 3, the model is constructed, and alternative variational inequality formulations provided. In addition, Lagrange analysis is conducted to gain insights into the economic meaning associated with the supply and demand constraints. In Section 4, the algorithmic scheme is proposed, along with explicit formulae, at each iteration, for the RBC path flows, and the Lagrange multipliers associated with the blood collection links, the physical capacity link bounds, and the demand point upper and lower bounds for RBCs. We then demonstrate the applicability of the framework through a case study consisting of a series of numerical examples. We summarize our results in Section 5 and provide suggestions for future research.

3. The Multiple Blood Service Organizations Supply Chain Network Competition Model

Blood service organizations (blood banks) collect blood periodically through blood drives at collection facilities and/or through blood mobile units. Once whole blood is collected at the collection sites it is sent to component laboratories for processing and testing for disease markers. The processing involves separation of the whole blood into components such as red blood cells, plasma, and platelets. Different blood products have distinct shelf lives with RBCs having to be used within 42 days and platelets within 5 days. Each type of product also needs to be stored at specific temperatures. Hence, supply chain management strategies for blood need to be component-specific. We focus on RBCs in our model since these are the most common type of blood product and are used for transfusions in surgeries, treatments for cancer and other diseases, etc.

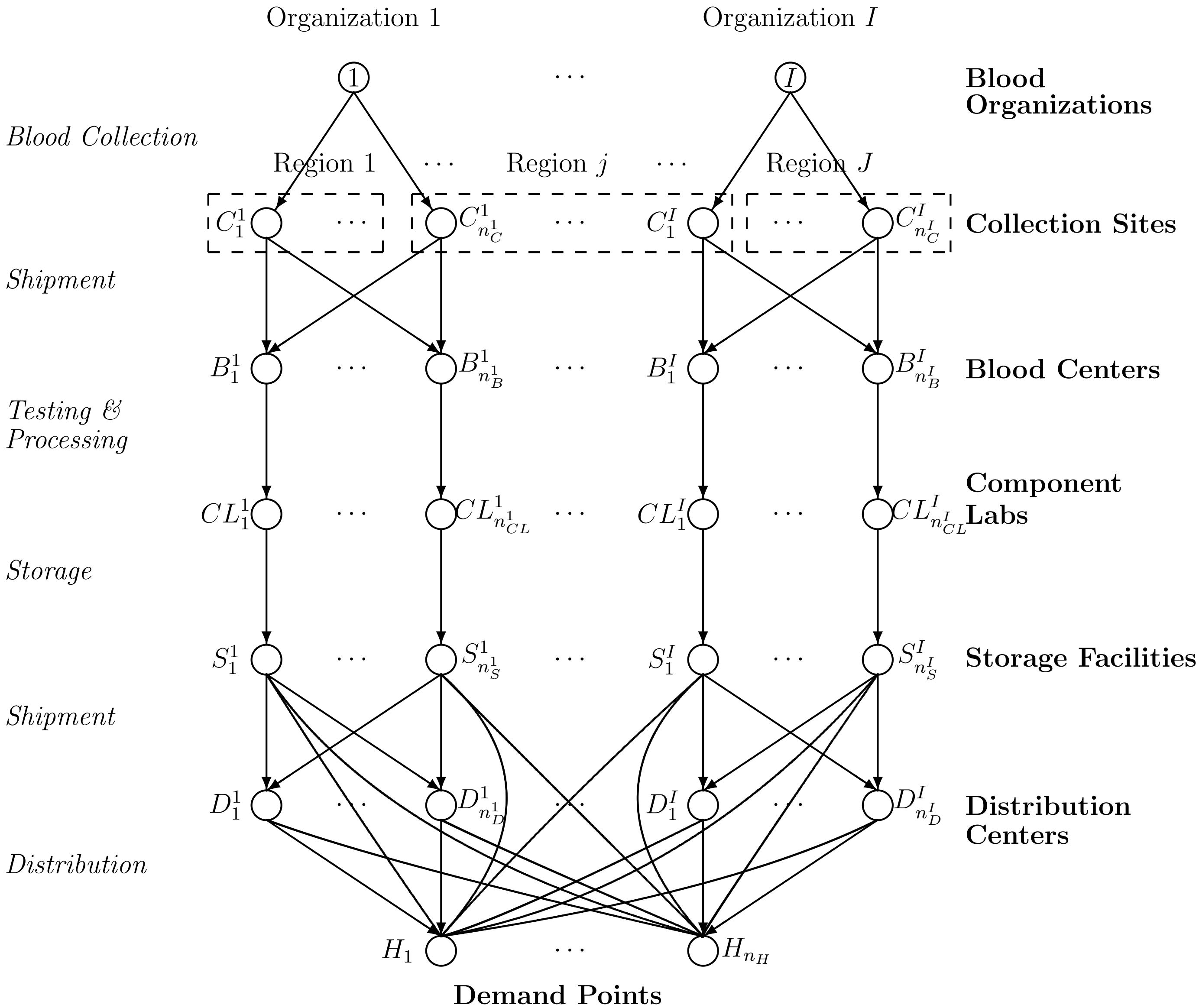

Figure 1: Supply Chain Network Topology for I Blood Organizations

As depicted in Figure 1, there are I blood service organizations competing with each other. Each blood service organization i can collect blood at n_C^i collection sites. We assume that there are J regions in which blood banks can set up collection sites or send blood mobiles to. Each of the $n_C^i; i = 1, .., I$, collection sites belongs to a region $j; j = 1, \ldots, J$. Collected blood by i is then shipped to n_B^i blood centers. From there, blood is sent to n_{CL}^i component laboratories for testing and processing and, subsequently, shipped to n_S^i storage facilities. The component laboratories may not be separate physical entities but may exist within the blood centers (cf. Nagurney, Masoumi, and Yu (2012)). The subsequent tier of the supply chain network for i; $i = 1, \ldots, I$, in Figure 1, is comprised of n_D^i distribution centers. The blood banks may serve the same n_H demand points consisting of hospitals, medical centers, etc., and denoted by the bottom nodes: $H_1, \ldots, H_{n_H}$ in Figure 1. These "demand markets" may be served by multiple blood banks since this is the case in reality. For example, Baystate Health in Massachusetts procures blood from the American Red Cross and from the Rhode Island Blood Bank in addition to having in-house blood collection (Merola (2017)).

Each link between a pair of nodes denotes an activity along the supply chain. The links from the blood service organizations to the collection sites represent the collection procedure. The next set of links to the component labs represent the processing and testing of blood. The successive sets of links denote, respectively, storage, shipment, and distribution to demand points. There are also some direct links from storage facilities to demand points since in some cases blood banks work closely with the hospitals and monitor their inventory levels and ship the required amount of blood directly to reduce cost (Wellis (2017)). Hence, the network topology corresponding to even a single blood service organization, as depicted in Figure 1, is more general than those constructed in Nagurney, Masoumi, and Yu (2012) and in Masoumi, Yu, and Nagurney (2017).

As mentioned earlier, the new model can be used to capture regional as well as nationwide competition. Moreover, large blood service organizations such as the American Red Cross and the New York Blood Center have multiple component labs, storage, and distribution centers, whereas smaller community ones might have one each. In our model we assume the time horizon in which all the activities are occurring to be one week.

The network topology of the blood service organizations' supply chains is represented by $G = [N, L]$ where N and L denote the sets of nodes and links, respectively. We also define L^i as the set of all the directed links corresponding to the sequence of activities pertaining to the supply chain network of blood service organization i; $i = 1, \ldots, I$. Associated with each link a is a total operational cost function, denoted by $\hat{c}_a$ $\forall a \in L$, representing the cost for each activity corresponding to collection, processing and testing, storage and distribution. A path p consists of a sequence of links originating at one of the top origin nodes in Figure 1, ranging from node 1 through node I, and ending at a destination node, corresponding to one of the demand points: $H_1, \ldots, H_{n_H}$.

In order to capture perishability, we utilize a generalized network approach with appropriate arc and path multipliers (see also, e.g., Nagurney et al. (2013)) as defined in Table 1. Moreover, since we are dealing with RBCs those paths that would have a time length greater than 42 days are explicitly removed from the network(s) in Figure 1 since they would, in

Table 1: Multiplier Notation for Perishability

Notation	**Definition**
α_a	The arc multiplier associated with link a, which represents the percentage of throughput on link a. $\alpha_a \in (0,1]$; $a \in L$.
α_{ap}	The arc-path multiplier, which is the product of the multipliers of the links on path p that precede link a; $a \in L$ and $p \in P$; that is, $$\alpha_{ap} \equiv \begin{cases} \delta_{ap} \prod_{b \in \{a' < a\}_p} \alpha_b, & \text{if } \{a' < a\}_p \neq \emptyset, \\ \delta_{ap}, & \text{if } \{a' < a\}_p = \emptyset, \end{cases}$$ where $\{a' < a\}_p$ denotes the set of the links preceding link a in path p and $\delta_{ap} = 1$, if link a is contained in path p, and 0, otherwise.
μ_p	The multiplier corresponding to the percentage of throughput on path p; that is, $\mu_p \equiv \prod_{a \in p} \alpha_a$; $p \in P$.

effect, be infeasible (and against Food and Drug Administration regulations).

Let x_p denote the nonnegative flow of blood on path p. Let the weekly demand for blood from blood service organization i at demand point k be denoted by $d_{ik}; i = 1, ..., I$; $k = H_1, \ldots, H_{n_H}$. Let P_k^i denote the set of all paths joining blood service organization node i with destination node H_k. We group the demands into the vector $d \in R_+^{In_H}$.

The conservation of flow equation that has to hold for each blood service organization i; $i = 1, \ldots, I$, at each demand point k; $k = H_1, \ldots, H_{n_H}$, is

$$\sum_{p \in P_k^i} \mu_p x_p = d_{ik}, \tag{1}$$

that is, the demand for blood at each demand point from each blood service organization has to be satisfied. Observe that, according to (1) the amount of blood product flow along a path that arrives at a destination node is equal to the path multiplier times the initial flow on the path since there may be losses due to testing, etc.

Moreover, the path flows must be nonnegative, that is:

$$x_p \geq 0, \quad \forall p \in P, \tag{2}$$

where P denotes the set of all paths in the network in Figure 1 from origin nodes corresponding to the organizations to the destination nodes corresponding to the demand points.

Let f_a denote the flow of blood on link a. Then, the following conservation of flow equations must also hold:

$$f_a = \sum_{p \in P} x_p \alpha_{ap}, \quad \forall a \in L. \tag{3}$$

Note that, according to (3), the initial product flow on link a is the sum of the product flows along paths that contain that link, taking into account possible losses in the preceding activities.

As mentioned earlier, the raw material in the supply chain for blood products cannot be manufactured but has to be collected from voluntary donors. Hence, the total amount of blood that can be collected is restricted in terms of the percentage of population that is eligible to donate blood in a particular region in a given week. An eligible donor, say, in Sarasota County in Florida, is unlikely to travel to a distant region in the state or to another state, unless it is in proximity, to donate blood. Therefore, we specify region-based populations and recall that, typically, a donor donates one pint of blood at a time. We let L_1^j denote the set of top-tier links in the network in Figure 1 representing blood collection in region j. Then we have the following constraint for each region j; $j = 1, \ldots, J$:

$$\sum_{a \in L_1^j} f_a \leq S^j, \tag{4}$$

where S^j represents the total population eligible to donate blood in a given week in region j; $j = 1, \ldots, J$. Unlike commercial product supply chains with capacity constraints, in this case, the constraint is not on the physical capacity of the production or collection facilities but on the actual supply of the raw material. Observe that (4) is a common, that is, a shared constraint among the blood service organizations if a given region includes collection links of multiple blood service organizations.

In addition, we incorporate explicit link capacities on all the network links in Figure 1, which represent the actual physical capacities. Hence, for each blood service organization i; $i = 1, \ldots, I$, each link $a \in L^i$ has a positive associated capacity denoted by u_a. Then, the following constraints must also be satisfied:

$$f_a \leq u_a, \quad \forall a \in L^i, \quad i = 1, \ldots, I. \tag{5}$$

We group all the link flows in the network into the vector $f \in R^{n_L}$ where n_L is the total number of elements in L, the set of all links.

Finally, hospitals and medical centers, that is, the demand points, have constraints, which may be included in the contracts with the blood service organizations. In particular, they contract for a lower bound for the weekly deliveries of blood, while also dealing with upper bounds on the amounts that they can safely store in order to also reduce wastage and associated costs. These constraints are as follows:

$$\sum_{i=1}^{I}\sum_{p\in P_k^i}\mu_p x_p \geq \underline{d}_k, \quad k=H_1,\ldots,H_{n_H}, \tag{6}$$

$$\sum_{i=1}^{I}\sum_{p\in P_k^i}\mu_p x_p \leq \bar{d}_k, \quad k=H_1,\ldots,H_{n_H}, \tag{7}$$

where $\underline{d}_k$ denotes the lower bound for units of RBCs at demand point k and $\bar{d}_k$ denotes the upper bound at k. Observe that these are common/shared constraints for the blood service organizations and, hence, will affect their feasible sets, as they compete to serve the hospitals and medical centers with blood.

The total link cost on link a, denoted by $\hat{c}_a$, $\forall a \in L$, may, in general, be a function of all the link flows in the network. This is to enable the modeling of competition for resources across the blood service organizations' supply chain networks. Hence, we have that

$$\hat{c}_a = \hat{c}_a(f), \quad \forall a \in L. \tag{8}$$

For example, blood service organizations may compete for staff to conduct the various supply chain network activities; moreover, they may compete for freight services for distribution purposes, etc.

The price that demand point k is willing to pay for a unit of RBCs from blood service organization i is denoted by ρ_{ik} for $i=1,\ldots,I$; $k=H_1,\ldots H_{n_H}$ and is given by the function:

$$\rho_{ik} = \rho_{ik}(d), \quad i=1,\ldots,I; \quad k=H_1\ldots,H_{n_H}. \tag{9}$$

Hence, the price charged per unit of RBCs may, in general, depend on the vector of demands, due to the competition among the hospitals and medical centers for blood. The prices represent the value that a hospital or medical center places on a unit of RBC from a specific blood service organization and that it is willing to pay. These price functions may incorporate parameters reflecting the duration of the contract, if the BSO is selected by the particular hospital, as well as historical information as to the reliability of the former.

In addition, since the majority of blood banks in the US are nonprofits, there is a utility associated with the service that they provide (cf. Nagurney, Alvarez Flores, and Soylu (2016)). Let γ_{ik} correspond to a measurement of the satisfaction that blood service organization i derives from supplying blood to demand point k. The overall such "service" utility of blood service organization i associated with all the demand points is then given by $\sum_{k=H_1}^{H_{n_H}} \gamma_{ik} d_{ik}$. This service utility also represents altruism (cf. Nagurney, Alvarez Flores, and Soylu (2016)). In addition, each blood service organization i associates a weight ω_i with its service utility, which monetizes it. According to the function $\omega_i \sum_{k=H_1}^{H_{n_H}} \gamma_{ik} d_{ik}$, the greater the amount made available, the more patients that can benefit and, therefore, the greater the good that can be accomplished.

By synthesizing the above revenue and cost terms as well as what may be considered to be a weighted altruism function, the utility function of blood service organization i; $i = 1, \ldots, I$, denoted by U_i, can be expressed as:

$$U_i = \sum_{k=H_1}^{H_{n_H}} \rho_{ik}(d) d_{ik} + \omega_i \sum_{k=H_1}^{H_{n_H}} \gamma_{ik} d_{ik} - \sum_{a \in L^i} \hat{c}_a(f). \tag{10}$$

The utility function (10) is assumed to be concave and continuously differentiable. It is to be noted that this is the utility of each blood service organization over a time horizon of a week.

In our model the blood service organizations are trying to maximize their utility, subject to constraints (1)-(7), while competing for the quantity of blood to be obtained and to be supplied to the hospitals and medical centers. Hence, each blood service organization has, as its strategies, its vector of path flows, X_i, such that

$$X_i \equiv \{\{x_p\} | p \in P^i\} \in R_+^{n_{P^i}}, \tag{11}$$

where P^i denotes the set of all paths associated with i and n_{P^i} denotes the number of paths from i to the demand points. Then, X is the vector of all the blood banks' path flows, that is, $X \equiv \{\{X_i\} | i = 1, \ldots, I\}$. We, also, for simplicity of notation, use $x \equiv X$.

Using the conservation of flow equations (3), we can rewrite the shared constraint (4), for each region $j = 1, \ldots, J$, in terms of the strategic variables, i.e., the path flows, as:

$$\sum_{a \in L_1^j} \sum_{p \in P} x_p \delta_{ap} \leq S^j, \quad j = 1, \ldots, J. \tag{12}$$

Since collection of blood is the first activity in the network and there are no preceding links, from the definition of the arc-path multiplier we have $\alpha_{ap} = \delta_{ap}$.

Similarly, the individual blood bank's capacity constraints for all activities can be rewritten as follows:

$$\sum_{p \in P} x_p \alpha_{ap} \leq u_a, \quad \forall a \in L^i, \quad i = 1, \ldots, I. \tag{13}$$

We define the i-th blood bank's individual feasible set, K_i, as

$$K_i \equiv \{X_i | (2) \, \text{and} \, (13) \, \text{hold for} \, i\}. \tag{14}$$

Further, we define the feasible set consisting of the shared constraints, $\mathcal{S}$, as:

$$\mathcal{S} \equiv \{X| \, (12), \, (6), \, \text{and} \, (7) \, \text{hold}\}. \tag{15}$$

Also, in view of (1), we may reexpress the demand price functions (9) as:

$$\hat{\rho}_{ik} = \hat{\rho}_{ik}(x) \equiv \rho_{ik}(d), \quad i = 1, \ldots, I; k = H_1, \ldots, H_{n_H}. \tag{16}$$

Using the conservation of flow equations (1) through (3), and, given the form of the total link cost functions, the demand price functions, and the weighted altruism functions, we can define each blood service organization utility function in terms of path flows only, that is, $\hat{U}_i(X) \equiv U_i; i = 1, \ldots I$. We then group these utilities into an I-dimensional vector $\hat{U}$, where

$$\hat{U} = \hat{U}(X). \tag{17}$$

In our model we assume that the blood service organizations compete noncooperatively in an oligopolistic market framework in which each blood service organization selects its blood product flows to maximize its utility, until an equilibrium is achieved, according to the definition below.

Definition 1: Blood Supply Chain Network Generalized Nash Equilibrium

A blood product path flow pattern $X^* \in K \equiv \prod_{i=1}^I K^i, X^* \in \mathcal{S}$, constitutes a blood supply chain network Generalized Nash Equilibrium if for each blood service organization i; $i = 1, ..., I$:

$$U_i(X_i^*, \hat{X}_i^*) \geq U_i(X_i, \hat{X}_i^*), \quad \forall X_i \in K^i, \forall X \in \mathcal{S}, \tag{18}$$

where

$$\hat{X}_i^* \equiv (X_1^*, \ldots, X_{i-1}^*, X_{i+1}^*, \ldots, X_I^*).$$

According to (18) an equilibrium is established if no blood service organization can unilaterally improve upon its utility by selecting an alternative vector of blood product flows, given the blood product flows of the other blood service organizations, and subject to the capacity constraints, both individual and shared ones, the shared demand constraints, and the nonnegativity constraints. It is to be noted that K and $\mathcal{S}$ are both convex sets.

If there are no coupling, that is, shared, constraints in this problem then X and X^* in Definition 1 need only lie in the feasible set K, and, under the assumption of concavity of the utility functions and that they are continuously differentiable, we know that (cf. Gabay and Moulin (1980) and Nagurney (1999)) the solution to what would then be a Nash Equilibrium problem (see Nash (1950, 1951)) would coincide with the solution to the following variational inequality problem: determine $X^* \in K$, such that

$$-\sum_{i=1}^{I} \langle \nabla_{X_i} \hat{U}_i(X^*), X_i - X_i^* \rangle \geq 0, \quad \forall X \in K, \tag{19}$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in the corresponding Euclidean space and $\nabla_{X_i} \hat{U}_i(X)$ denotes the gradient of $\hat{U}_i(X)$ with respect to X_i.

However, as mentioned earlier, since here the blood service organizations have common constraints on the amount of blood that can be collected, and on the amounts to be delivered, the strategies of each BSO affect both the objective functions as well as the feasible sets of the other BSOs. Consequently, this is a Generalized Nash Equilibrium (GNE) which cannot be directly formulated as variational inequality problem, but may be formulated as a quasi-variational inequality.

We now define the variational equilibrium which, as emphasized in Nagurney, Yu, and Besik (2017) and Nagurney et al. (2017), is a refinement of the Generalized Nash Equilibrium and is a specific type of GNE (see Kulkarni and Shahbhang (2012)). In a GNE defined by a variational equilibrium, the Lagrange multipliers associated with the shared constraints are all the same which provides a fairness interpretation and makes sense from an economic standpoint. Specifically, we have:

Definition 2: Variational Equilibrium

A strategy vector X^* is said to be a variational equilibrium of the above Generalized Nash Equilibrium game if $X^* \in K, X^* \in \mathcal{S}$ is a solution of the variational inequality:

$$-\sum_{i=1}^{I} \langle \nabla_{X_i} \hat{U}_i(X^*), X_i - X_i^* \rangle \geq 0, \quad \forall X \in K, \forall X \in \mathcal{S}. \tag{20}$$

Hence, we can take advantage of the well-developed theory of variational inequalities, including algorithms (see Nagurney (1999) and the references therein), which is in a more advanced state of development and application than the algorithms for quasivariational inequality problems.

We now expand the terms in the variational inequality (20).

From the definition of a gradient, we know that

$$-\nabla_{X_i}\hat{U}_i(X) = \left[-\frac{\partial \hat{U}_i}{\partial x_p}; p \in P_k^i; k = H_1, \ldots, H_{n_H}\right]. \tag{21}$$

We also know that, in view of (1) and (10), that for paths $p \in P_k^i$:

$$-\frac{\partial \hat{U}_i}{\partial x_p} = -\frac{\partial(\sum_{l=H_1}^{H_{n_H}} \rho_{il}(d) \sum_{q\in P_l^i} \mu_q x_q + \omega_i \sum_{l=H_1}^{H_{n_H}} \gamma_{il} \sum_{q\in P_l^i} \mu_q x_q - \sum_{b\in L^i} \hat{c}_b(f))}{\partial x_p}. \tag{22}$$

Then, making use of (1) and (3) and the expression (16), we have that for $p \in P_k^i$:

$$\frac{\partial \hat{C}_p(x)}{\partial x_p} \equiv \sum_{a\in L^i}\sum_{b\in L^i} \frac{\partial \hat{c}_b(f)}{\partial f_a}\alpha_{ap}, \tag{23a}$$

$$\frac{\partial \hat{\rho}_{il}(x)}{\partial x_p} \equiv \frac{\partial \rho_{il}(d)}{\partial d_{ik}}\mu_p, \tag{23b}$$

and we obtain for $p \in P_k^i$:

$$-\frac{\partial \hat{U}_i}{\partial x_p} = \left[\frac{\partial \hat{C}_p(x)}{\partial x_p} - \omega_i\gamma_{ik}\mu_p - \hat{\rho}_{ik}(x)\mu_p - \sum_{l=H_1}^{H_{n_H}} \frac{\partial \hat{\rho}_{il}(x)}{\partial x_p} \sum_{q\in P_l^i} \mu_q x_q\right]. \tag{24}$$

Hence, (20) is equivalent to the variational inequality: determine $x^* \in K, x^* \in \mathcal{S}$ such that:

$$\sum_{i=1}^{I}\sum_{k=H_1}^{H_{n_H}}\sum_{p\in P_k^i}\left[\frac{\partial \hat{C}_p(x^*)}{\partial x_p} - \omega_i\gamma_{ik}\mu_p - \hat{\rho}_{ik}(x^*)\mu_p - \sum_{l=H_1}^{H_{n_H}} \frac{\partial \hat{\rho}_{il}(x^*)}{\partial x_p}\sum_{q\in P_l^i}\mu_q x_q^*\right] \times [x_p - x_p^*] \geq 0,$$

$$\forall x \in K, x \in \mathcal{S}. \tag{25}$$

For simplicity, we refer to $\frac{\partial \hat{C}_p(x)}{\partial x_p}$ as the *marginal total cost of path* p.

Variational inequality (25) is now put into standard form (cf. Nagurney (1999)): determine $Y^* \in \mathcal{K} \subset R^N$, such that

$$\langle F(Y^*), Y - Y^*\rangle \geq 0, \quad \forall Y \in \mathcal{K}, \tag{26}$$

where F is a given continuous function from $\mathcal{K}$ to R^N, $\mathcal{K}$ is a closed convex set, and here $N = n_P$ with Y, Y^*, and $F(Y^*)$ all being column vectors.

We define $Y \equiv X \equiv x$, and let the p-th component of $F(Y)$ for a given i, k, $p \in P_k^i$, $\forall i, k$, be

$$\left[\frac{\partial \hat{C}_p(x)}{\partial x_p} - \omega_i \gamma_{ik} \mu_p - \hat{\rho}_{ik}(x)\mu_p - \sum_{l=H_1}^{H_{n_H}} \frac{\partial \hat{\rho}_{il}(x)}{\partial x_p} \sum_{q \in P_l^i} \mu_q x_q\right], \tag{27}$$

with $\mathcal{K} \equiv \mathcal{K}^1 \equiv K \cap \mathcal{S}$, then variational inequality (25) can be put into the form (26).

Remark: Existence of an Equilibrium Solution

We assume that the feasible set $\mathcal{K}$ is nonempty, which will be the case if the capacities on the links and blood donor regions are sufficient to satisfy the sum of the demands for blood at the demand points. An equilibrium blood flow pattern $X^* = x^* \in \mathcal{K}$ satisfying variational inequality (26); equivalently, variational inequality (25), is guaranteed to exist since the function $F(X)$ is continuous under the imposed assumptions and the feasible set $\mathcal{K}$ is compact, due to the nonnegative assumption on the blood path flows and the link and blood donor regional upper bound capacities.

3.1 Alternative Variational Inequality Formulations and Lagrange Analysis with Economic Interpretation

In this subsection, we first present an alternative variational inequality formulation to the one in (25), again, in path flows, but using Lagrange multipliers. We then conduct an economic analysis using Lagrange theory and conclude with, yet, another variational inequality, which we utilize for computational purposes in Section 4.

Let η_j, $\forall j$, and $\theta_a; a \in L$, denote the Lagrange multipliers associated with constraints (12) and (13), respectively. In addition, let σ_k; $\forall k$, denote the Lagrange multiplier associated with the k-th lower bound demand constraint (6) and let ϵ_k; $\forall k$, denote the Lagrange multiplier associated with the k-th upper bound demand constraint (7). We group the above Lagrange multipliers into the respective vectors: $\eta \in R_+^J$, $\theta \in R_+^{n_L}$, $\sigma \in R_+^{n_H}$, and $\epsilon \in R_+^{n_H}$. Also, we let β_p; $\forall p \in P$, denote the Lagrange multiplier associated with each path p nonnegativity constraint (2) and we group these Lagrange multipliers into the vector $\beta \in R_+^{n_P}$. We define the feasible set $\mathcal{K}^2 \equiv \{(x, \beta, \eta, \theta, \sigma, \epsilon) | x \in R_+^{n_P}, \beta \in R_+^{n_P}, \eta \in R_+^J, \theta \in R_+^{n_L}, \sigma \in R_+^{n_H}, \epsilon \in R_+^{n_H}\}$. Then, we have the following result:

Theorem 1: Alternative Variational Inequality Formulation of the Variational Equilibrium in Path Flows

The variational inequlity (25) is equivalent to the variational inequality: determine the vector of equilibrium path flows and Lagrange multipliers, $(x^*, \beta^*, \eta^*, \theta^*, \sigma^*, \epsilon^*) \in \mathcal{K}^2$*, such that:*

$$\sum_{i=1}^{I} \sum_{k=H_1}^{H_{n_H}} \sum_{p \in P_k^i} \Bigg[\frac{\partial \hat{C}_p(x^*)}{\partial x_p} - \beta_p^* + \sum_{j=1}^{J} \sum_{a \in L_1^j} \eta_j^* \delta_{ap} + \sum_{a \in L^i} \theta_a^* \alpha_{ap} - \omega_i \gamma_{ik} \mu_p - \sigma_k^* \mu_p + \epsilon_k^* \mu_p - \hat{\rho}_{ik}(x^*) \mu_p$$
$$- \sum_{l=H_1}^{H_{n_H}} \frac{\partial \hat{\rho}_{il}(x^*)}{\partial x_p} \sum_{q \in P_l^i} \mu_q x_q^* \Bigg] \times [x_p - x_p^*]$$
$$+ \sum_{p \in P} x_p^* \times [\beta_p - \beta_p^*] + \sum_{j=1}^{J} \Bigg[S^j - \sum_{a \in L_1^j} \sum_{p \in P} x_p^* \delta_{ap} \Bigg] \times [\eta_j - \eta_j^*] + \sum_{i=1}^{I} \sum_{a \in L^i} \Bigg[u_a - \sum_{p \in P} x_p^* \alpha_{ap} \Bigg] \times [\theta_a - \theta_a^*]$$
$$+ \sum_{k=H_1}^{H_{n_H}} \Big(\sum_{i=1}^{I} \sum_{p \in P_k^i} \mu_p x_p^* - \underline{d}_k \Big) \times (\sigma_k - \sigma_k^*) + \sum_{k=H_1}^{H_{n_H}} \Big(\bar{d}_k - \sum_{i=1}^{I} \sum_{p \in P_k^i} \mu_p x_p^* \Big) \times (\epsilon_k - \epsilon_k^*) \geq 0, \quad \forall (x, \beta, \eta, \theta, \sigma, \epsilon) \in \mathcal{K}^2. \tag{28}$$

Proof: By setting:

$$V(x) = \sum_{i=1}^{I} \sum_{k=H_1}^{H_{n_H}} \sum_{p \in P_k^i} \Bigg[\frac{\partial \hat{C}_p(x^*)}{\partial x_p} - \omega_i \gamma_{ik} \mu_p - \hat{\rho}_{ik}(x^*) \mu_p - \sum_{l=H_1}^{H_{n_H}} \frac{\partial \hat{\rho}_{il}(x^*)}{\partial x_p} \sum_{q \in P_l^i} \mu_q x_q^* \Bigg] \times [x_p - x_p^*], \tag{29}$$

variational inequality (25) can be rewritten as:

$$\text{Min}_{\mathcal{K}} V(x) = V(x^*) = 0. \tag{30}$$

Under the previously imposed assumptions we know that all the underlying functions in (30) are continuously differentiable and convex.

We then let:

$$\begin{aligned}
b_p &= -x_p \leq 0, && \forall p, \\
e_j &= \sum_{a \in L_1^j} \sum_{p \in P} x_p \delta_{ap} - S^j \leq 0, && \forall j, \\
g_a &= \sum_{p \in P} x_p \alpha_{ap} - u_a \leq 0, && \forall a, \\
h_k &= \underline{d}_k - \sum_{i=1}^{I} \sum_{p \in P_k^i} \mu_p x_p \leq 0, && \forall k, \\
r_k &= \sum_{i=1}^{I} \sum_{p \in P_k^i} \mu_p x_p - \bar{d}_k \leq 0, && \forall k,
\end{aligned} \tag{31}$$

and

$$\Gamma(x) = (b_p, e_j, g_a, h_k, r_k)_{p\in P; j=1,\ldots,J; a\in L; k=H_1,\ldots,H_{n_H}}. \tag{32}$$

Hence, we can rewrite the feasible set $\mathcal{K}$ as

$$\mathcal{K} = \{x \in R_+^{n_P} : \Gamma(x) \leq 0\}. \tag{33}$$

We now construct the Lagrange function:

$$\mathcal{L}(x,\beta,\eta,\theta,\sigma,\epsilon) = \sum_{i=1}^{I}(-\sum_{k=H_1}^{H_{n_H}} \hat{\rho}_{ik}(x) \sum_{p\in P_k^i} \mu_p x_p - \omega_i \sum_{k=H_1}^{H_{n_H}} \gamma_{ik} \sum_{p\in P_k^i} \mu_p x_p + \sum_{a\in L^i} \hat{c}_a(Ax))$$

$$+\sum_{p\in P} \beta_p b_p + \sum_{j=1}^{J} \eta_j e_j + \sum_{a\in L} \theta_a g_a + \sum_{k=H_1}^{H_{n_H}} \sigma_k h_k + \sum_{k=H_1}^{H_{n_H}} \epsilon_k r_k, \tag{34}$$

$$\forall x \in R_+^{n_P}, \forall \beta \in R_+^{n_P}, \forall \eta \in R_+^{J}, \forall \theta \in R_+^{n_L}, \forall \sigma \in R_+^{n_H}, \forall \epsilon \in R_+^{n_H},$$

where A is the arc-path incidence matrix with component $ap = 1$, if link a is contained in path p and 0, otherwise; β is the vector with components: $\{\beta_p, \forall p \in P\}$, with η and the other vectors of Lagrange multipliers as defined above.

It is straightforward to establish that the feasible set $\mathcal{K}$ is convex and that the Slater condition holds. Then, if x^* is the minimal solution to problem (30), there exist $\beta^* \in R_+^{n_P}$, $\eta^* \in R_+^J$, $\theta^* \in R_+^{n_L}$, $\sigma^* \in R_+^{n_H}$, and $\epsilon^* \in R_+^{n_H}$ such that the vector $(x^*, \beta^*, \eta^*, \theta^*, \sigma^*, \epsilon^*)$ is a saddle point of the Lagrange function (34), that is:

$$\mathcal{L}(x^*,\beta,\eta,\theta,\sigma,\epsilon) \leq \mathcal{L}(x^*,\beta^*,\eta^*,\theta^*,\sigma^*,\epsilon^*) \leq \mathcal{L}(x,\beta^*,\eta^*,\theta^*,\sigma^*,\epsilon^*), \tag{35}$$

$$\forall x \in R_+^{n_P}, \forall \beta \in R_+^{n_P}, \forall \eta \in R_+^{J}, \forall \theta \in R_+^{n_L}, \forall \sigma \in R_+^{n_H}, \forall \epsilon \in R_+^{n_H},$$

and

$$\beta_p^* b_p^* = 0, \quad \forall p \in P,$$

$$\eta_j^* e_j^* = 0, \quad \forall j,$$

$$\theta_a^* g_a^* = 0, \quad \forall a \in L,$$

$$\sigma_k^* h_k^* = 0, \quad \epsilon_k^* r_k^* = 0, \quad \forall k. \tag{36}$$

From the right-hand side of (35) it follows that $x^* \in R_+^{n_P}$ is a minimal point of $\mathcal{L}(x,\beta^*,\eta^*,\theta^*,\sigma^*,\epsilon^*)$ in the entire space R^{n_P} and, therefore, we have that for all $p \in P_k^i$, $\forall i$, $\forall k$:

$$\frac{\partial \mathcal{L}(x^*,\beta^*,\eta^*,\theta^*,\sigma^*,\epsilon^*)}{\partial x_p}$$

$$= \frac{\partial \hat{C}_p(x^*)}{\partial x_p} - \omega_i \gamma_{ik} \mu_p - \hat{\rho}_{ik}(x^*)\mu_p - \sum_{l=H_1}^{H_{n_H}} \frac{\partial \hat{\rho}_{il}(x^*)}{\partial x_p} \sum_{q \in P_l^i} \mu_q x_q^* - \beta_p^* + \sum_{j=1}^{J} \sum_{a \in L_1^j} \eta_j^* \delta_{ap} + \sum_{a \in L^i} \theta_a^* \alpha_{ap} - \sigma_k^* \mu_p + \epsilon_k^* \mu_p = 0, \tag{37}$$

together with conditions (36).

Conditions (36) and (37) correspond to an equivalent variational inequality to that in (25). For example, if we multiply (37) by $(x_p - x_p^*)$ and sum with respect to $p \in P_k^i$, $\forall i$, $\forall k$, we obtain:

$$\sum_{i=1}^{I} \sum_{k=H_1}^{H_{n_H}} \sum_{p \in P_k^i} \left[\frac{\partial \hat{C}_p(x^*)}{\partial x_p} - \omega_i \gamma_{ik} \mu_p - \hat{\rho}_{ik}(x^*)\mu_p - \sum_{l=H_1}^{H_{n_H}} \frac{\partial \hat{\rho}_{il}(x^*)}{\partial x_p} \sum_{q \in P_l^i} \mu_q x_q^* \right] \times (x_p - x_p^*)$$

$$= \sum_{i=1}^{I} \sum_{k=H_1}^{H_{n_H}} \sum_{p \in P_k^i} \beta_p^* x_p - \sum_{i=1}^{I} \sum_{k=H_1}^{H_{n_H}} \sum_{p \in P_k^i} \beta_p^* x_p^*$$

$$- \sum_{i=1}^{I} \sum_{k=H_1}^{H_{n_H}} \sum_{p \in P_k^i} \sum_{j=1}^{J} \sum_{a \in L_j^i} \eta_j^* x_p \delta_{ap} + \sum_{i=1}^{I} \sum_{k=H_1}^{H_{n_H}} \sum_{p \in P_k^i} \sum_{j=1}^{J} \sum_{a \in L_1^j} \eta_j^* x_p^* \delta_{ap}$$

$$- \sum_{i=1}^{I} \sum_{k=H_1}^{H_{n_H}} \sum_{p \in P_k^i} \sum_{a \in L^i} \theta_a^* \alpha_{ap} x_p + \sum_{i=1}^{I} \sum_{k=H_1}^{H_{n_H}} \sum_{p \in P_k^i} \sum_{a \in L^i} \theta_a^* \alpha_{ap} x_p^*$$

$$+ \sum_{i=1}^{I} \sum_{k=H_1}^{H_{n_H}} \sum_{p \in P_k^i} \sigma_k^* \mu_p x_p - \sum_{i=1}^{I} \sum_{k=H_1}^{H_{n_H}} \sum_{p \in P_k^i} \sigma_k^* \mu_p x_p^* - \sum_{i=1}^{I} \sum_{k=H_1}^{H_{n_H}} \sum_{p \in P_k^i} \epsilon_k^* \mu_p x_p + \sum_{i=1}^{I} \sum_{k=H_1}^{H_{n_H}} \sum_{p \in P_k^i} \epsilon_k^* \mu_p x_p^*. \tag{38}$$

Examining the expressions on the right-hand side of the equal sign in (38) we know that for $j = 1, \ldots, J$, $\forall a \in L$, and for $k; k = H_1, \ldots, H_{n_H}$:

$$\sum_{i=1}^{I} \sum_{k=H_1}^{H_{n_H}} \sum_{p \in P_k^i} \beta_p x_p^* = 0,$$

$$\sum_{i=1}^{I} \sum_{k=H_1}^{H_{n_H}} \sum_{p \in P_k^i} \sum_{a \in L_1^j} \eta_j^* x_p^* \delta_{ap} = \eta_j^* S_j,$$

$$\sum_{i=1}^{I} \sum_{k=H_1}^{H_{n_H}} \sum_{p \in P_k^i} \theta_a^* \alpha_{ap} x_p^* = \theta_a^* u_a,$$

$$\sum_{i=1}^{I} \sum_{p \in P_k^i} \sigma_k^* \mu_p x_p^* = \sigma_k^* \underline{d}_k, \quad \sum_{i=1}^{I} \sum_{p \in P_k^i} \epsilon_k^* \mu_p x_p^* = \epsilon_k^* \bar{d}_k. \tag{39}$$

Hence, the right-hand side of (38) simplifies to:

$$\sum_{i=1}^{I}\sum_{k=H_1}^{H_{n_H}}\sum_{p\in P_k^i}\beta_p^* x_p-\sum_{j=1}^{J}\eta_j^*(\sum_{i=1}^{I}\sum_{a\in L_j^i}\sum_{k=H_1}^{H_{n_H}}\sum_{p\in P_k^i}x_p\delta_{ap}-S^j)-\sum_{a\in L}\theta_a^*(f_a-u_a)$$

$$+\sum_{k=H_1}^{H_{n_H}}\sigma_k^*(\sum_{i=1}^{I}\sum_{p\in P_k^i}\mu_p x_p-\underline{d}_k)-\sum_{k=H_1}^{H_{n_H}}\epsilon_k^*(\sum_{i=1}^{I}\sum_{p\in P_k^i}\mu_p x_p-\bar{d}_k)\geq 0, \quad (40)$$

and the conclusion follows. □

We now provide an economic interpretation of the Lagrange multipliers. We consider a path $p\in P_k^i$ for a fixed i and k where $x_p^*>0$, that is, the equilibrium blood flow on the path is positive. Then, from the first line of (36) we know that $\beta_p^*=0$. In particular, we consider multiple distinct cases.

Case I: None of the Associated Constraints are Active

We first consider the case when the associated path capacity and demand constraints are not active, that is, in equality (37) we have that, in addition to $\beta_p^*=0$, the corresponding $\eta_j^*=0$, as well as the corresponding $\theta_a^*=0$, with also $\sigma_k^*=0$ and $\epsilon_k^*=0$. Hence, we then have that (37) satisfies

$$\frac{\partial \hat{C}_p(x^*)}{\partial x_p}-\omega_i\gamma_{ik}\mu_p-\hat{\rho}_{ik}(x^*)\mu_p-\sum_{l=H_1}^{H_{n_H}}\frac{\partial\hat{\rho}_{il}(x^*)}{\partial x_p}\sum_{q\in P_l^i}\mu_q x_q^*-\beta_p^*+\sum_{j=1}^{J}\sum_{a\in L_1^j}\eta_j^*\delta_{ap}$$

$$+\sum_{a\in L^i}\theta_a^*\alpha_{ap}-\mu_p\sigma_k^*+\mu_p\epsilon_k^*=0$$

$$\Longleftrightarrow\quad \frac{\partial \hat{C}_p(x^*)}{\partial x_p}=\omega_i\gamma_{ik}\mu_p+\hat{\rho}_{ik}(x^*)\mu_p+\sum_{l=H_1}^{H_{n_H}}\frac{\partial\hat{\rho}_{il}(x^*)}{\partial x_p}\sum_{q\in P_l^i}\mu_q x_q^*, \quad (41)$$

which means that, in this case, the marginal total cost on path p is equal to the marginal utility associated with weighted altruism of the pair (i,k) plus the marginal revenue associated with the path p.

Case II: The Associated Donor Supply Constraints Are Active but Other Capacity and Demand Constraints Associated with the Path Are Not

We now consider the situation in which the blood collected in the regions that link a of path p is contained in is equal to the available supply, in which case the corresponding η_j^* of those regions j will be positive. Also, the other capacity and demand constraints relevant to path p are not at their bounds. Hence, we then get from (37) that

$$\frac{\partial \hat{C}_p(x^*)}{\partial x_p} = \omega_i \gamma_{ik} \mu_p + \hat{\rho}_{ik}(x^*)\mu_p + \sum_{l=H_1}^{H_{n_H}} \frac{\partial \hat{\rho}_{il}(x^*)}{\partial x_p} \sum_{q \in P_l^i} \mu_q x_q^* - \sum_{j=1}^{J} \sum_{a \in L_1^j} \eta_j^* \delta_{ap}, \tag{42}$$

and, therefore,

$$\frac{\partial \hat{C}_p(x^*)}{\partial x_p} < \omega_i \gamma_{ik} \mu_p + \hat{\rho}_{ik}(x^*)\mu_p + \sum_{l=H_1}^{H_{n_H}} \frac{\partial \hat{\rho}_{il}(x^*)}{\partial x_p} \sum_{q \in P_l^i} \mu_q x_q^*. \tag{43}$$

This result is quite intuitive, since it implies that the marginal total cost on path p is less than the marginal utility associated with the weighted altruism plus the marginal revenue associated with the path p. This situation is beneficial for BSO i.

Case III: One or More Links on the Path Are at Their Capacities But No Other Associated Capacity or Demand Constraints Are Active

In this case we know that (37) yields:

$$\frac{\partial \hat{C}_p(x^*)}{\partial x_p} = \omega_i \gamma_{ik} \mu_p + \hat{\rho}_{ik}(x^*)\mu_p + \sum_{l=H_1}^{H_{n_H}} \frac{\partial \hat{\rho}_{il}(x^*)}{\partial x_p} \sum_{q \in P_l^i} \mu_q x_q^* - \sum_{a \in L^i} \theta_a^* \alpha_{ap}, \tag{44}$$

and, therefore,

$$\frac{\partial \hat{C}_p(x^*)}{\partial x_p} < \omega_i \gamma_{ik} \mu_p + \hat{\rho}_{ik}(x^*)\mu_p + \sum_{l=H_1}^{H_{n_H}} \frac{\partial \hat{\rho}_{il}(x^*)}{\partial x_p} \sum_{q \in P_l^i} \mu_q x_q^*. \tag{45}$$

This is also reasonable, since if the path p has one or more links at their capacities, then one would expect that the marginal total cost of that path would be less that the marginal utility associated with the weighted altruism/benefit function plus the marginal revenue associated with the path p.

Case IV: The Demand Point That the Path Is Destined to Has Its Demand at the Lower Bound Whereas No Other Associated Constraints Are Active

In this case we know that $\sigma_k^* > 0$ and all other relevant Lagrange multipliers are zero so that expression (37) now yields:

$$\frac{\partial \hat{C}_p(x^*)}{\partial x_p} = \omega_i \gamma_{ik} \mu_p + \hat{\rho}_{ik}(x^*)\mu_p + \sum_{l=H_1}^{H_{n_H}} \frac{\partial \hat{\rho}_{il}(x^*)}{\partial x_p} \sum_{q \in P_l^i} \mu_q x_q^* + \mu_p \sigma_k^*, \tag{46}$$

and, hence,

$$\frac{\partial \hat{C}_p(x^*)}{\partial x_p} > \omega_i \gamma_{ik} \mu_p + \hat{\rho}_{ik}(x^*)\mu_p + \sum_{l=H_1}^{H_{n_H}} \frac{\partial \hat{\rho}_{il}(x^*)}{\partial x_p} \sum_{q \in P_l^i} \mu_q x_q^*. \tag{47}$$

This is not a desirable situation since the marginal total cost on the path p now exceeds the marginal utility associated with the weighted altruism/benefit function plus the marginal revenue associated with the path p.

Case V: The Demand Point That the Path Is Destined to Has Its Demand at the Upper Bound Whereas No Other Associated Constraints Are Active

We now consider the case when the demand at point k is at its upper bound and no other associated constraints are active (and, therefore, all other associated Lagrange multipliers are equal to zero). We know that then $\epsilon_k^* > 0$ and we have that, according to (37):

$$\frac{\partial \hat{C}_p(x^*)}{\partial x_p} = \omega_i \gamma_{ik} \mu_p + \hat{\rho}_{ik}(x^*)\mu_p + \sum_{l=H_1}^{H_{n_H}} \frac{\partial \hat{\rho}_{il}(x^*)}{\partial x_p} \sum_{q \in P_l^i} \mu_q x_q^* - \mu_p \epsilon_k^*, \tag{48}$$

and, consequently,

$$\frac{\partial \hat{C}_p(x^*)}{\partial x_p} < \omega_i \gamma_{ik} \mu_p + \hat{\rho}_{ik}(x^*)\mu_p + \sum_{l=H_1}^{H_{n_H}} \frac{\partial \hat{\rho}_{il}(x^*)}{\partial x_p} \sum_{q \in P_l^i} \mu_q x_q^*. \tag{49}$$

According to (49), the marginal total cost on path p, in this case, is less than the marginal utility associated with the weighted altruism/benefit function plus the marginal revenue of the BSO and demand point pair (i, k). This is clearly another desirable situation.

Taking into account the Lagrange multipliers, an equivalent variational formulation to variational inequality (28) is the following: determine the vector of equilibrium path flows and Lagrange multipliers, $(x^*, \eta^*, \theta^*, \sigma^*, \epsilon^*) \in \mathcal{K}^3$, such that:

$$\sum_{i=1}^{I} \sum_{k=H_1}^{H_{n_H}} \sum_{p \in P_k^i} \Bigg[\frac{\partial \hat{C}_p(x^*)}{\partial x_p} + \sum_{j=1}^{J} \sum_{a \in L_1^j} \eta_j^* \delta_{ap} + \sum_{a \in L^i} \theta_a^* \alpha_{ap} - \omega_i \gamma_{ik} \mu_p - \sigma_k^* \mu_p + \epsilon_k^* \mu_p - \hat{\rho}_{ik}(x^*)\mu_p$$

$$- \sum_{l=H_1}^{H_{n_H}} \frac{\partial \hat{\rho}_{il}(x^*)}{\partial x_p} \sum_{q \in P_l^i} \mu_q x_q^* \Bigg] \times [x_p - x_p^*]$$

$$+ \sum_{j=1}^{J} \Bigg[S^j - \sum_{a \in L_1^j} \sum_{p \in P} x_p^* \delta_{ap} \Bigg] \times [\eta_j - \eta_j^*] + \sum_{i=1}^{I} \sum_{a \in L^i} \Bigg[u_a - \sum_{p \in P} x_p^* \alpha_{ap} \Bigg] \times [\theta_a - \theta_a^*]$$

$$+\sum_{k=H_1}^{H_{n_H}}(\sum_{i=1}^{I}\sum_{p\in P_k^i}\mu_p x_p^* - \underline{d}_k)\times(\sigma_k-\sigma_k^*)+\sum_{k=H_1}^{H_{n_H}}(\bar{d}_k-\sum_{i=1}^{I}\sum_{p\in P_k^i}\mu_p x_p^*)\times(\epsilon_k-\epsilon_k^*)\geq 0, \quad \forall (x,\eta,\theta,\sigma,\epsilon)\in\mathcal{K}^3, \tag{50}$$

where $\mathcal{K}^3 \equiv \{(x,\eta,\theta,\sigma,\epsilon)|x\in R_+^{n_P},\eta\in R_+^J,\theta\in R_+^{n_L},\sigma\in R_+^{n_H},\epsilon\in R_+^{n_H}\}$.

For our case study, we will utilize variational inequality (50). We note that variational inequality (50) can also be put into standard form (26).

4. Algorithm and Numerical Examples

In this section, before presenting the numerical examples, we outline the algorithm that is used for the computations, notably, the Euler method which is induced by the general iterative scheme of Dupuis and Nagurney (1993). Specifically, an iteration $\tau+1$ of the Euler method (see also Nagurney and Zhang (1996)) is given by:

$$Y^{\tau+1} = P_{\mathcal{K}}(Y^\tau - a_\tau F(Y^\tau)), \tag{51}$$

where $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$ and F is the utility function that enters the variation inequality problem (19). As shown in Dupuis and Nagurney (1993) and Nagurney and Zhang (1996), for convergence of the general iterative scheme, the sequence $\{a_\tau\}$ must satisfy: $\sum_{\tau=0}^{\infty}=\infty, a_\tau>0, a_\tau\to 0$, as $\tau\to\infty$. Convergence conditions specific to various applications for the solutions of network-based problems can be found in Nagurney and Zhang (1996), Nagurney (2006), Nagurney, Masoumi and Yu (2012).

Specifically, the notable feature of this algorithm, when applied to the blood supply chain network competition model, is that it yields closed form expressions for the variables at each iteration, resulting in an elegant procedure for computations and solution.

4.1 Explicit Formulae for the Euler Method Applied to the Alternative Variational Inequality Formulation (50)

In particular, for this problem, we have the following closed form expressions for the path flows at iteration $\tau+1$. For each path $p\in P_k^i, \forall i,k$, we have:

$$x_p^{\tau+1}=\max\{0, x_p^\tau + a_\tau(\hat{\rho}_{ik}(x^\tau)\mu_p + \sum_{l=H_1}^{H_{n_H}}\frac{\partial\hat{\rho}_{il}(x^\tau)}{\partial x_p}\sum_{q\in P_l^i}x_q^\tau\mu_q + \omega_i\gamma_{ik}\mu_p - \frac{\partial\hat{C}_p(x^\tau)}{\partial x_p} - \sum_{j=1}^{J}\sum_{a\in L_1^j}\eta_j^\tau\delta_{ap}$$
$$-\sum_{a\in L^i}\theta_a^\tau\alpha_{ap}+\sigma_k^\tau\mu_p-\epsilon_k^\tau\mu_p)\}. \tag{52}$$

The Lagrange multipliers associated with blood collection links $a \in L_1^j$; $j = 1, \ldots, J$, are computed according to:

$$\eta_j^{\tau+1} = \max\{0, \eta_j^{\tau} + a_\tau(\sum_{a \in L_1^j} \sum_{p \in P} x_p^\tau \delta_{ap} - S^j)\}. \tag{53}$$

The closed form expression for the Lagrange multipliers for the capacity constraint on link $a \in L^i$; $i = 1, \ldots, I$ is:

$$\theta_a^{\tau+1} = \max\{0, \theta_a^{\tau} + a_\tau(\sum_{p \in P} x_p^\tau \alpha_{ap} - u_a)\}. \tag{54}$$

Next, we provide the closed form expressions for the Lagrange multipliers associated with the upper and lower bounds on the demands. The explicit formulae for the Lagrange multipliers associated with the lower bounds on the demands at demand points: $k = H_1, \ldots, H_{n_H}$, are:

$$\sigma_k^{\tau+1} = \max\{0, \sigma_k^{\tau} + a_\tau(\underline{d}_k - \sum_{i=1}^{I} \sum_{p \in P_k^i} \mu_p x_p^\tau)\}. \tag{55}$$

The Lagrange multipliers associated with the upper bounds on the demands at the demand points: $k = H_1, \ldots, H_{n_H}$, in turn, are computed according to:

$$\epsilon_k^{\tau+1} = \max\{0, \epsilon_k^{\tau} + a_\tau(\sum_{i=1}^{I} \sum_{p \in P_k^i} \mu_p x_p^\tau - \bar{d}_k)\}. \tag{56}$$

The algorithm is assumed to have converged when the absolute value of successive iterates is less than or equal to the imposed convergence tolerance ϵ.

4.2 Numerical Examples

The numerical examples are inspired by a particular region of New England in which there is growing competition between blood service organizations. The examples are stylized but capture the features of the game theory model and demonstrate the types of insights that can be revealed.

The Euler method was implemented in FORTRAN and a Linux system at the University of Massachusetts Amherst was used for the computations. The Euler method was initialized with all variables identically equal to 0.00. The $\{a_\tau\}$ sequence utilized was: $.1\{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \ldots\}$. The convergence tolerance utilized was 10^{-6}; in other words, the algorithm was terminated when the absolute value of successive computed variable iterates was less than or equal to this value. The numerical examples below contain explicit input and output data.

Example 1

We consider two blood service organizations, with Organization 1 being a local one, and Organization 2 being an iconic national one. Please refer to Figure 2 for the supply chain network topology for all the numerical examples. Organization 1 has two collection sites, a single blood center for processing and testing as well as a single component lab and storage facility, similar to, for example, the Rhode Island Blood Center, which is based in Providence, Rhode Island. Organization 2, in turn, has three collection sites, two blood centers for testing and processing, two component labs and storage facilities, as well as distribution centers.

There are four demand points with the first and third, denoted, respectively, by H_1 and H_3 denoting major trauma hospitals and the other two: H_2 and H_4 corresponding to smaller hospitals.

Also, there are three regions, as depicted in Figure 2, with Region 2 being common (that is, in proximity) to a collection site of each organization. Here regions correspond to counties.

We now provide the data for this example. Example 1 serves as the baseline from which other examples are constructed.

The number of people eligible to donate blood in each of the regions are:

$$S^1 = 6000, \quad S^2 = 2400, \quad S^3 = 3000.$$

Such values are reasonable since the Northeast has a higher percentage of senior citizens (65 years and above) (Wilson (2013)) and older people are found to be less likely to donate or retire from donating after a certain age (Shaz et al. (2011), Aleccia (2017),).

The weekly upper and lower bounds on the demand at each hospital are given below:

$$\underline{d}_{H_1} = 200, \quad \bar{d}_{H_1} = 350,$$

$$\underline{d}_{H_2} = 60, \quad \bar{d}_{H_2} = 150,$$

$$\underline{d}_{H_3} = 200, \quad \bar{d}_{H_3} = 300,$$

$$\underline{d}_{H_4} = 100, \quad \bar{d}_{H_4} = 120.$$

The link definitions, associated link capacities, arc multipliers, total cost functions, and computed equilibrium link flows and associated link Lagrange multipliers are provided in

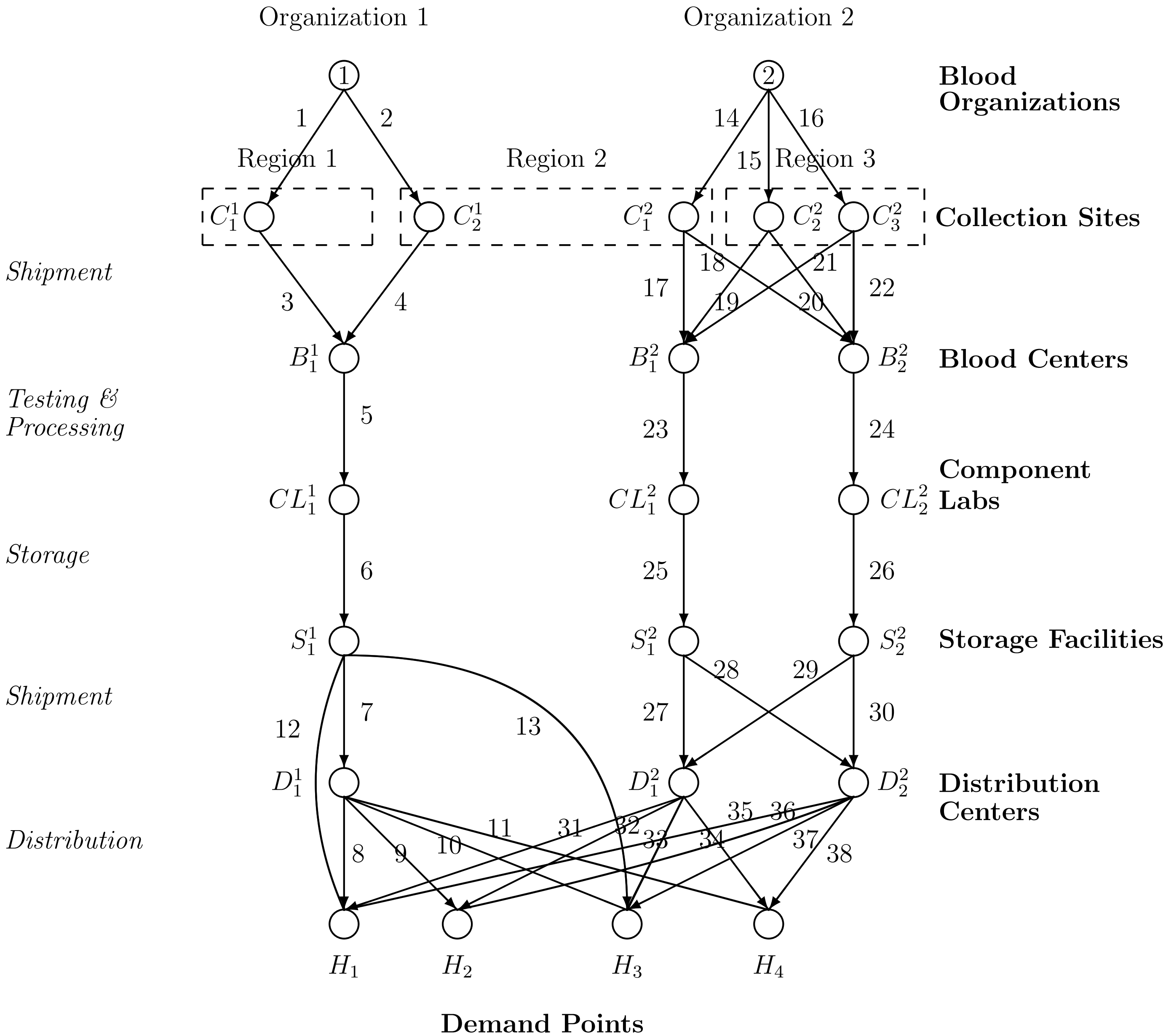

Figure 2: The Supply Chain Network Topology for Examples 1, 2, 3, and 4

Table 2. Since Organization 2 operates on a national level, it has more resources than Organization 1 which is reflected in many of the link capacities. The cost functions and demand price functions are constructed using information obtained from Tracy (2010), Carlyle (2012), Gunpinar and Centeno (2015), and Masoumi, Yu and Nagurney (2017). Also, we have losses on links associated with testing and processing, and, hence, those arc multipliers are less than 1.

The demand price functions are as follows:

Organization 1:

$$\rho_{1H_1}(d) = -0.07d_{1H_1} - 0.02d_{2H_1} + 300, \quad \rho_{1H_2}(d) = -0.08d_{1H_2} - 0.03d_{2H_2} + 310,$$

$$\rho_{1H_3}(d) = -0.05d_{1H_3} - 0.01d_{2H_3} + 300, \quad \rho_{1H_4}(d) = -0.04d_{1H_4} - 0.02d_{2H_4} + 280.$$

Organization 2:

$$\rho_{2H_1}(d) = -0.05d_{2H_1} - 0.01d_{1H_1} + 280, \quad \rho_{2H_2}(d) = -0.07d_{2H_2} - 0.04d_{1H_2} + 290,$$

$$\rho_{2H_3}(d) = -0.03d_{2H_3} - 0.01d_{1H_3} + 280, \quad \rho_{2H_4}(d) = -0.05d_{2H_4} - 0.02d_{1H_4} + 270.$$

We report the equilibrium link solution in Table 2 since the number of paths is quite large - equal to 60, whereas the number of links is 38.

In addition, we assume that the weights associated with the altruism component of the BSOs' objective functions are both equal to 1 so that $\omega_1 = \omega_2 = 1$. Furthermore, we have that $\gamma_{1H_1} = 2$, $\gamma_{1H_2} = 1$, $\gamma_{1H_3} = 2$, and $\gamma_{1H_4} = 1$, whereas $\gamma_{2H_1} = 2$, $\gamma_{2H_2} = 1$, $\gamma_{2H_3} = 2$, and $\gamma_{2H_4} = 1$. Hence, both BSOs assign a higher value to servicing the larger hospitals.

It can be seen from Table 2 that four of the links are at their capacities and these are links: 13, 34, 36, and 38. All these links are shipment links. Link 13 is associated with BSO 1, whereas the other links are in BSO 2's supply chain network. The BSOs are advised to invest in enhancing the capacities in these links.

We also report the additional equilibrium Lagrange multipliers for this example. In particular, we have that: $\eta_1^* = \eta_2^* = \eta_3^* = 0.00$, since none of the supply/donor upper bound constraints in the three regions are binding.

The equilibrium demands for the RBCs at the demand points from the BSOs are:

$$d^*_{1H_1} = 55.09, \quad d^*_{1H_2} = 28.39, \quad d^*_{1H_3} = 96.64, \quad d^*_{1H_4} = 40.00,$$

$$d^*_{2H_1} = 144.91, \quad d^*_{2H_2} = 71.65, \quad d^*_{2H_3} = 103.36, \quad d^*_{2H_4} = 60.00,$$

and the associated demand prices for the RBCs at the equilibrium demand solution are:

$$\rho_{1H_1}(d^*) = 293.25, \quad \rho_{1H_2}(d^*) = 305.58, \quad \rho_{1H_3}(d^*) = 294.13, \quad \rho_{1H_4}(d^*) = 277.20,$$

$$\rho_{2H_1}(d^*) = 272.20, \quad \rho_{2H_2}(d^*) = 283.85, \quad \rho_{2H_3}(d^*) = 275.93, \quad \rho_{2H_4}(d^*) = 266.20.$$

Table 2: Definition of Links, Associated Weekly Link Capacities, Arc Multipliers, Total Operational Link Cost Functions, Equilibrium Link Solution, and Link Capacity Equilibrium Lagrange Multipliers for Example 1

Link a	From Node	To Node	u_a	α_a	$\hat{c}_a(f)$	f_a^*	θ_a^*
1	1	C_1^1	250	1.00	$0.24f_1^2+0.6f_1$	139.33	0.00
2	1	C_2^1	200	1.00	$0.4f_2^2+0.9f_2$	87.59	0.00
3	C_1^1	B_1^1	300	1.00	$0.06f_3^2+0.1f_3$	139.33	0.00
4	C_2^1	B_1^1	250	1.00	$0.07f_4^2+0.16f_4$	87.59	0.00
5	B_1^1	CL_1^1	500	0.97	$0.36f_5^2+0.45f_5$	226.92	0.00
6	CL_1^1	S_1^1	500	1.00	$0.02f_6^2+0.04f_6$	220.11	0.00
7	S_1^1	D_1^1	500	1.00	$0.03f_7^2+0.09f_7$	166.40	0.00
8	D_1^1	H_1	50	1.00	$0.4f_8^2+0.7f_8$	21.37	0.00
9	D_1^1	H_2	50	1.00	$0.5f_9^2+0.9f_9$	28.38	0.00
10	D_1^1	H_3	100	1.00	$0.15f_{10}^2+0.8f_{10}$	76.64	0.00
11	D_1^1	H_4	60	1.00	$0.35f_{11}^2+0.6f_{11}$	40.00	0.00
12	S_1^1	H_1	50	1.00	$0.4f_{12}^2+0.9f_{12}$	33.71	0.00
13	S_1^1	H_3	20	1.00	$0.7f_{13}^2+1f_{13}$	20.00	5.02
14	2	C_1^2	250	1.00	$0.25f_{14}^2+0.7f_{14}$	130.81	0.00
15	2	C_2^2	300	1.00	$0.2f_{15}^2+0.8f_{15}$	148.27	0.00
16	2	C_3^2	200	1.00	$0.3f_{16}^2+0.5f_{16}$	112.99	0.00
17	C_1^2	B_1^2	100	1.00	$0.12f_{17}^2+0.3f_{17}$	70.11	0.00
18	C_1^2	B_2^2	150	1.00	$0.08f_{18}^2+0.27f_{18}$	60.71	0.00
19	C_2^2	B_1^2	100	1.00	$0.16f_{19}^2+0.45f_{19}$	70.86	0.00
20	C_2^2	B_2^2	200	1.00	$0.1f_{20}^2+0.5f_{20}$	77.41	0.00
21	C_3^2	B_1^2	100	1.00	$0.2f_{21}^2+0.6f_{21}$	35.85	0.00
22	C_3^2	B_2^2	100	1.00	$0.05f_{22}^2+0.08f_{22}$	77.14	0.00
23	B_1^2	CL_1^2	600	0.98	$0.36f_{23}^2+0.8f_{23}$	176.81	0.00
24	B_2^2	CL_2^2	500	0.96	$0.3f_{24}^2+0.7f_{24}$	215.25	0.00
25	CL_1^2	S_1^2	500	1.00	$0.02f_{25}^2+0.05f_{25}$	173.28	0.00
26	CL_2^2	S_2^2	500	1.00	$0.03f_{26}^2+0.04f_{26}$	206.64	0.00
27	S_1^2	D_1^2	150	1.00	$0.15f_{27}^2+0.4f_{27}$	88.02	0.00
28	S_1^2	D_2^2	150	1.00	$0.18f_{28}^2+0.65f_{28}$	85.25	0.00
29	S_2^2	D_1^2	200	1.00	$0.09f_{29}^2+0.12f_{29}$	116.35	0.00
30	S_2^2	D_2^2	150	1.00	$0.14f_{30}^2+0.5f_{30}$	90.30	0.00
31	D_1^2	H_1	100	1.00	$0.24f_{31}^2+0.8f_{31}$	48.90	0.00
32	D_1^2	H_2	80	1.00	$0.32f_{32}^2+0.9f_{32}$	51.65	0.00
33	D_1^2	H_3	100	1.00	$0.25f_{33}^2+f_{33}$	63.82	0.00
34	D_1^2	H_4	40	1.00	$0.5f_{34}^2+0.8f_{34}$	40.00	3.02
35	D_2^2	H_1	150	1.00	$0.1f_{35}^2+0.35f_{35}$	96.01	0.00
36	D_2^2	H_2	20	1.00	$0.5f_{36}^2+0.8f_{36}$	20.00	8.80
37	D_2^2	H_3	80	1.00	$0.35f_{37}^2+0.7f_{37}$	39.53	0.00
38	D_2^2	H_4	20	1.00	$0.4f_{38}^2+0.9f_{38}$	20.00	22.84

Hence, none of the demands are at the imposed upper bounds and, consequently, all the associated Lagrange multipliers ϵ_k^*; $k = 1, \ldots, 4$, are equal to 0.00. On the other hand, three of the demands are at the imposed lower bounds, at demand points: H_1, H_3, and H_4, and, therefore, the associated Lagrange multipliers: $\sigma_{H_1}^*$, $\sigma_{H_3}^*$, and $\sigma_{H_4}^*$, are all positive. In particular, these Lagrange multipliers, at equilibrium, have the following computed values:

$$\sigma_{H_1}^* = 0.55, \quad \sigma_{H_2}^* = 0.00, \quad \sigma_{H_3}^* = 5.80, \quad \sigma_{H_4}^* = 27.63.$$

We now report, for completeness, components of the objective function (cf. (10)) for BSO 1 and for BSO 2. For BSO 1 we have that, at the equilibrium solution, the revenue is equal to: 64,341.70; the altruism component of the utility function is: 371.84, and the total cost associated with its supply chain is: 33,099.85, resulting in a net revenue of: 31,241.85 and a utility of: 31,613.70. As for BSO 2, we have that its revenue is equal to: 104,275.07; its altruism component of its utility function is: 628.19, and the total cost associated with its supply chain network is: 86,525.06, yielding a net revenue of: 17,750.01 and a utility of: 18,378.20.

Example 2

Example 2 is constructed from Example 1 and has the identical data except that the demand lower and upper bound constraints at the four hospital demand points are removed. In Example 2 we are interested in exploring what the potential impacts of removing such constraints are in terms of the RBC deliveries and the associated prices as well as the BSOs' net revenues and utilities. The variational inequality (50) was adapted accordingly to remove the terms and variables associated with the demand lower and upper bounds and the Euler method was, as well. The computed equilibrium link flow pattern and link capacity Lagrange multipliers are reported in Table 3.

The Lagrange multipliers associated with the three regions remain as in Example 1, that is, we have that $\eta_1^* = \eta_2^* = \eta_3^* = 0.00$, since none of the supply/donor upper bound constraints in the three regions are binding. The link capacities at links 13 and 38 are now at their upper bounds. These were also at their upper bounds in Example 1 but links 34 and 36 are no longer at their upper bounds.

The equilibrium demands for the RBCs at the demand points from the BSOs are now:

$$d_{1H_1}^* = 67.08, \quad d_{1H_2}^* = 34.48, \quad d_{1H_3}^* = 99.99, \quad d_{1H_4}^* = 13.32,$$

$$d_{2H_1}^* = 158.41, \quad d_{2H_2}^* = 77.42, \quad d_{2H_3}^* = 97.0, \quad d_{2H_4}^* = 41.32,$$

Table 3: Link, Equilibrium Link Solution, and Link Capacity Equilibrium Lagrange Multipliers for Example 2

Link a	f_a^*	θ_a^*
1	136.03	0.00
2	85.49	0.00
3	136.03	0.00
4	85.49	0.00
5	221.51	0.00
6	214.87	0.00
7	155.56	0.00
8	27.78	0.00
9	34.48	0.00
10	79.99	0.00
11	13.32	0.00
12	39.30	0.00
13	20.00	13.10
14	128.84	0.00
15	146.03	0.00
16	111.29	0.00
17	69.08	0.00
18	59.76	0.00
19	69.81	0.00
20	76.22	0.00
21	35.31	0.00
22	75.98	0.00
23	174.20	0.00
24	211.96	0.00
25	170.71	0.00
26	203.48	0.00
27	84.32	0.00
28	86.39	0.00
29	111.31	0.00
30	92.17	0.00
31	54.97	0.00
32	57.42	0.00
33	61.40	0.00
34	21.84	0.00
35	103.44	0.00
36	20.00	0.00
37	35.64	0.00
38	19.84	13.60

and the associated demand prices for the RBCs at the equilibrium demand solution are:

$$\rho_{1H_1}(d^*) = 292.14, \quad \rho_{1H_2}(d^*) = 304.92, \quad \rho_{1H_3}(d^*) = 294.03, \quad \rho_{1H_4}(d^*) = 278.64,$$

$$\rho_{2H_1}(d^*) = 271.41, \quad \rho_{2H_2}(d^*) = 283.20, \quad \rho_{2H_3}(d^*) = 276.09, \quad \rho_{2H_4}(d^*) = 267.67.$$

Without the imposition of demand bounds at the hospital demand points, the total equilibrium demand at $H_1 = 225.49$; the total demand at $H_2 = 111.90$; the total demand at $H_3 = 197.03$, and that at $H_4 = 54.64$.

For BSO 1 we have that, at the equilibrium solution, the revenue is equal to: 63,221.34; the altruism component of the utility function is: 381.94, and the total cost associated with its supply chain is: 31,685.55, leading to a net revenue of: 31,535.79 and a utility of: 31,917.73. As for BSO 2, we have that its revenue is now equal to: 102,772.48; its altruism component of its utility function is: 629.65, and the total cost associated with its supply chain network is now: 83,461.70, yielding a net revenue of: 19,310.78 and a utility of: 19,940.43.

Both blood service organizations now enjoy higher net revenues, as well as higher utilities, without the demand constraints. However, observe that, without those constraints, both hospitals H_3 and H_4 may suffer serious shortfalls in terms on needed RBCs since $\underline{d}_{H_3} = 200$ and $\underline{d}_{H_4} = 100$ and the total deliveries are only, respectively 197.03 and 54.64. This example illustrates the merits of imposing lower demand bounds, which can be part of the contracts between the hospital(s) and the BSO(s).

Also, another interesting result is regarding the altruism component of the BSO utility functions. In Example 1, BSO 1 enjoyed an altruism component value of 371.84, whereas now, in Example 2, it enjoys an altruism component value of 381.94. BSO 2 enjoyed an altruism component value of 628.19, whereas, in Example 2, the corresponding value is 629.65. Hence, the respective BSO altruism component values have increased.

Example 3

Example 3 is also constructed from Example 1 and has the same data except for the following. We now consider a major disruption in the form of a disease so that the number of those eligible to donate blood drops considerably. In particular, we now have that:

$$S^1 = 500, \quad S^2 = 220, \quad S^3 = 120.$$

The new computed equilibrium link flow solution and corresponding Lagrange multipliers associated with the link capacity constraints are reported in Table 4.

Table 4: Link, Equilibrium Link Solution, and Link Capacity Equilibrium Lagrange Multipliers for Example 3

Link a	f_a^*	θ_a^*
1	237.60	0.00
2	33.49	0.00
3	237.60	0.00
4	33.49	0.00
5	271.09	0.00
6	262.96	0.00
7	196.94	0.00
8	3.38	0.00
9	23.21	0.00
10	96.67	0.00
11	45.68	0.00
12	46.01	0.00
13	20.00	13.10
14	186.51	0.00
15	68.06	0.00
16	51.94	0.00
17	86.42	0.00
18	100.09	0.00
19	35.42	0.00
20	32.64	0.00
21	18.86	0.00
22	33.07	0.00
23	140.70	0.00
24	165.80	0.00
25	137.89	0.00
26	159.17	0.00
27	68.17	0.00
28	9.72	0.00
29	86.81	0.00
30	72.36	0.00
31	42.78	0.00
32	25.43	0.00
33	52.44	0.00
34	34.33	0.00
35	79.83	0.00
36	11.36	0.00
37	30.89	0.00
38	20.00	13.60

Observe that now, unlike in Example 1, and due to a much decreased volume of possible donations, we now have that the constraints for both Regions 2 and 3 are tight and the associated Lagrange multipliers are now: $\eta_1^* = 0.00$, $\eta_2^* = 109.82$, and $\eta_3^* = 85.00$.

The equilibrium demands for the RBCs at the demand points in Example 3 are:

$$d^*_{1H_1} = 77.39, \quad d^*_{1H_2} = 23.21, \quad d^*_{1H_3} = 116.17, \quad d^*_{1H_4} = 45.68,$$

$$d^*_{2H_1} = 122.61, \quad d^*_{2H_2} = 36.79, \quad d^*_{2H_3} = 83.33, \quad d^*_{2H_4} = 54.33.$$

The associated demand prices for the RBCs at the equilibrium demand solution are:

$$\rho_{1H_1}(d^*) = 292.13, \quad \rho_{1H_2}(d^*) = 307.04, \quad \rho_{1H_3}(d^*) = 293.33, \quad \rho_{1H_4}(d^*) = 277.09,$$

$$\rho_{2H_1}(d^*) = 273.10, \quad \rho_{2H_2}(d^*) = 286.50, \quad \rho_{2H_3}(d^*) = 276.33, \quad \rho_{2H_4}(d^*) = 266.37.$$

The equilibrium total demands at the four hospital demand points are at their respective lower bounds. The Lagrange multipliers, at the equilibrium, associated with the lower and upper bounds on the demands at the four demand points are now:

$$\sigma^*_{H_1} = 107.14, \quad \sigma^*_{H_2} = 90.43, \quad \sigma^*_{H_3} = 110.02, \quad \sigma^*_{H_4} = 129.07,$$

and

$$\epsilon^*_{H_1} = 0.00, \quad \epsilon^*_{H_2} = 0.00, \quad \epsilon^*_{H_3} = 0.00, \quad \epsilon^*_{H_4} = 0.00.$$

We now report the values, at the computed equilibrium, of the components of the objective function (cf. (10)) for BSO 1 and for BSO 2. For BSO 1 we have that the revenue is now equal to: 76,616.49; the altruism component of the utility function is: 574.02, and the total costs associated with its supply chain is: 50,978.78 resulting in a net revenue of: 25,637.71 and a utility of: 26,094.73. As for BSO 2, we have that its revenue is now equal to: 81,522.08; its altruism component of its utility function is: 503.00, and the total cost associated with its supply chain network is: 87,042.11, yielding a net revenue of: -5,520.03 and a utility of: -5,017,03.

With a much reduced donor base, the two BSOs still manage to meet their delivery obligations. However, BSO 1 suffers a reduction in net revenue and utility of approximately 20%, as compared to the corresponding values in Example 1. BSO 2, on the other hand, experiences not only a significant reduction in net revenue and utility, but these attain negative values and, hence, BSO 2 incurs a financial loss. This example illustrates that blood service organizations need to maintain a sufficiently large donor base for the life-saving product that is blood, which cannot be manufactured, but must be voluntarily donated. This is especially essential in times such as disease outbreaks as well as during different times of various seasons when donors may not be available due to holidays or inclement weather.

Example 4

Example 4 is also constructed from Example 1 but in Example 4 we explore the impacts of decreased capacity associated with BSO 2's testing and processing and storage links 24 and 26 due to a natural disaster. The data is the same as in Example 1 except that now the link upper bounds $u_{24} = 200$ and $u_{26} = 200$. The computed equilibrium link flow pattern and associated Lagrange multiplier pattern are reported in Table 5.

Observe that links: 13, 24, 34, 36, and 38 are now at their capacities.

The equilibrium Lagrange multipliers associated with the bounds on donors in the three regions are: $\eta_1^* = \eta_2^* = \eta_3^* = 0.00$ since these constraints are not binding.

The equilibrium demands for the RBCs at the demand points in Example 4 are:

$$d^*_{1H_1} = 57.31, \quad d^*_{1H_2} = 26.22, \quad d^*_{1H_3} = 98.68, \quad d^*_{1H_4} = 40.00,$$

$$d^*_{2H_1} = 142.69, \quad d^*_{2H_2} = 66.50, \quad d^*_{2H_3} = 101.32, \quad d^*_{2H_4} = 60.00.$$

The associated demand prices for the RBCs, in turn, at the equilibrium demand solution are:

$$\rho_{1H_1}(d^*) = 293.13, \quad \rho_{1H_2}(d^*) = 305.91, \quad \rho_{1H_3}(d^*) = 294.05, \quad \rho_{1H_4}(d^*) = 277.20,$$

$$\rho_{2H_1}(d^*) = 272.29, \quad \rho_{2H_2}(d^*) = 284.30, \quad \rho_{2H_3}(d^*) = 275.97, \quad \rho_{2H_4}(d^*) = 266.20.$$

The equilibrium total demands at the hospital demand points H_1, H_3, and H_4 are at their respective lower bounds. The Lagrange multipliers, at the equilibrium, associated with the lower and upper bounds on the demands at the four demand points are now:

$$\sigma^*_{H_1} = 4.11, \quad \sigma^*_{H_2} = 0.00, \quad \sigma^*_{H_3} = 9.08, \quad \sigma^*_{H_4} = 30.20,$$

and

$$\epsilon^*_{H_1} = 0.00, \quad \epsilon^*_{H_2} = 0.00, \quad \epsilon^*_{H_3} = 0.00, \quad \epsilon^*_{H_4} = 0.00.$$

BSO 1's revenue is now equal to: 64,925.04; the altruism component of the utility function is: 78.20, and the total costs associated with its supply chain is: 33,706.32. Hence, the net revenue is: 31,218.71 and the utility is: 31,596.91. BSO 2's revenue is now equal to: 101,693.17; its altruism component of its utility function is: 614.52, and the total cost associated with its supply chain network is: 84,635.16, resulting in a net revenue of: 17,058.01 and a utility of: 17,672.53.

Table 5: Link, Equilibrium Link Solution, and Link Capacity Equilibrium Lagrange Multipliers for Example 4

Link a	f_a^*	θ_a^*
1	140.65	0.00
2	88.43	0.00
3	140.65	0.00
4	88.43	0.00
5	229.08	0.00
6	222.21	0.00
7	167.35	0.00
8	22.45	0.00
9	26.22	0.00
10	78.68	0.00
11	40.00	0.00
12	34.86	0.00
13	20.00	5.71
14	127.67	0.00
15	144.65	0.00
16	109.84	0.00
17	72.21	0.00
18	55.46	0.00
19	72.05	0.00
20	72.60	0.00
21	37.80	0.00
22	71.94	0.00
23	182.15	0.00
24	200.00	17.81
25	178.51	0.00
26	192.00	0.00
27	90.63	0.00
28	87.88	0.00
29	107.11	0.00
30	84.89	0.00
31	48.47	0.00
32	46.50	0.00
33	62.77	0.00
34	40.00	1.74
35	94.22	0.00
36	20.00	5.23
37	38.55	0.00
38	20.00	21.38

Note that, with decreased capacity on critical links, BSO 2's net revenue as well as utility decrease relative to their respective values in Example 1. Interestingly, the reduced capacity of BSO 2 also affects BSO 1 and, although it now has higher revenues, it also incurs higher costs, resulting in a reduced value of net revenue (31,218.71 versus 31,241.85). This suggests that the blood service organizations may gain by cooperating rather than competing.

5. Summary and Suggestions for Future Research

The blood services industry in the United States is undergoing major changes, which include increasing competition among blood banks, that is, blood service organizations. In this paper, we presented the first game theory model for competitive supply chain networks associated with blood service organizations that includes not only perishability but also an altruism component in their objective functions since they are nonprofit organizations. In addition to capacities on the links representing the network economic activities associated with such supply chain networks we also incorporated upper bounds reflecting donations in different regions as well as lower bounds and upper bounds associated with the demand for RBCs at the various demand points, which correspond to hospitals and medical centers. Such demand constraints ensure that each hospital or medical center has the minimum amount needed for a given week while also guaranteeing that waste will be reduced because of the upper bounds. The novel features of the competitive supply chain network game theory model result in a Generalized Nash Equilibrium (GNE), rather than just a Nash Equilibrium, since the utility function of each blood service organization depends on its own strategies in the form of blood path flows, as well as those of the other BSOs, and the feasible sets do as well.

We utilized the concept of a variational equilibrium to transform the problem into a variational inequality problem in which the Lagrange multipliers corresponding to the shared / common constraints are equal among the competitors. This provides a nice economic fairness interpretation.

We also provided alternative variational inequality formulations and presented a Lagrange analysis with economic interpretations. An algorithm was outlined which resolves the problem into closed form expressions at each iteration in terms of path flows and the various Lagrange multipliers. The algorithm was then applied to compute solutions to a series of numerical examples for which full input and output data are reported. The examples illustrated the impacts of disruptions as in a reduction in the number of donors as well as that of decreases in capacities of critical links such as testing and processing on RBC prices, demands, net revenues of the blood service organizations, and their overall utilities. The framework here focused on competition among blood service organizations not only in terms of blood donations but also for business with hospitals as well as along their supply chain networks through the generality of the link total cost functions. Future work of interest includes that of modeling cooperation among blood banks in terms of their various supply chain network activities.

Acknowledgments

The authors acknowledge helpful conversations with Dr. Louis Katz, Michael Merola, Dr. David Wellis, Beau Tompkins and Professor Amir H. Masoumi. The authors also thank the anonymous reviewer and the editor for taking the time to read the paper.

The first author also acknowledges support from the Radcliffe Institute for Advanced Study at Harvard University, where she was a Summer Fellow in 2018, and from the John F. Smith Memorial Foundation at the University of Massachusetts Amherst.

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