Network Economics

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Geographic Information Systems (GIS)
and Spatial Economics in World Bank Projects

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IBRD & IDA: Working for a World Free of Poverty

Outline of Lecture:

- Background - Overview
- Brief History of the Science of Networks
- Interdisciplinary Impact of Networks
- User-Optimization vs. System-Optimization Behavior
- The Braess Paradox, Toll Policies, and Other Paradoxes
- Some Interesting Applications of Variational Inequalities
- The Time-Dependent (Demand-Varying) Braess Paradox
- A New Network Performance/Efficiency Measure with Applications to a Variety of Network Systems
- Transportation Network Robustness
- Ongoing Research and Questions

What is Network Economics?

Network economics is a scientific approach of using mathematical network models and algorithms to abstract decision-making focusing on several or numerous interacting decision-makers.

It is usually used to *analyze* complex real-world network-based systems in a graphical manner.

We are in a New Era of Decision-Making Characterized by:

- complex interactions among decision-makers in organizations;
- alternative and at times conflicting criteria used in decision-making;
- constraints on resources: natural, human, financial, time, etc.;
- global reach of many decisions;
- high impact of many decisions;
- increasing risk and uncertainty, and
- the *importance of dynamics* and realizing a fast and sound response to evolving events.

This era is ideal for applying the tools of Network Economics.

Network problems are their own class of problems and they come in various forms and formulations, i.e., as optimization (linear or nonlinear) problems or as equilibrium problems and even dynamic network problems.

Network economic problems will be the focus of this lecture with transportation as the unifying application.

Transportation, Communication, and Energy Networks

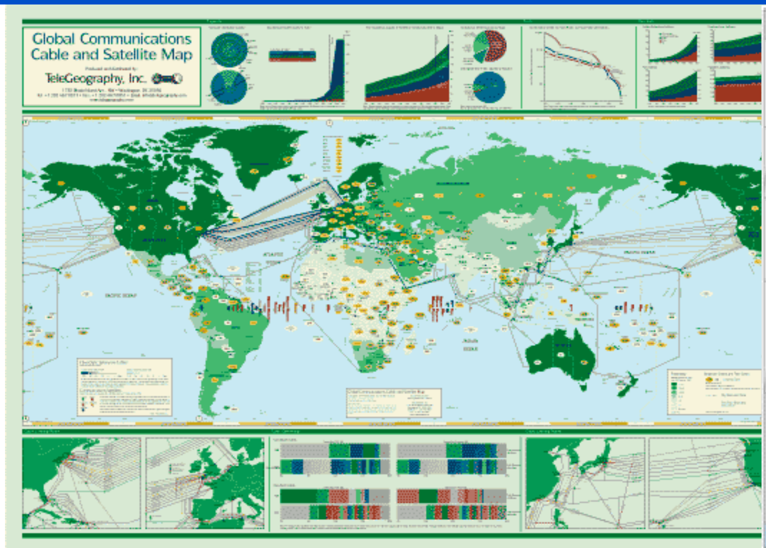
Subway Network

Railroad Network

Iridium Satellite Constellation Network

Satellite and Undersea Cable Networks

Duke Energy Gas Pipeline Network

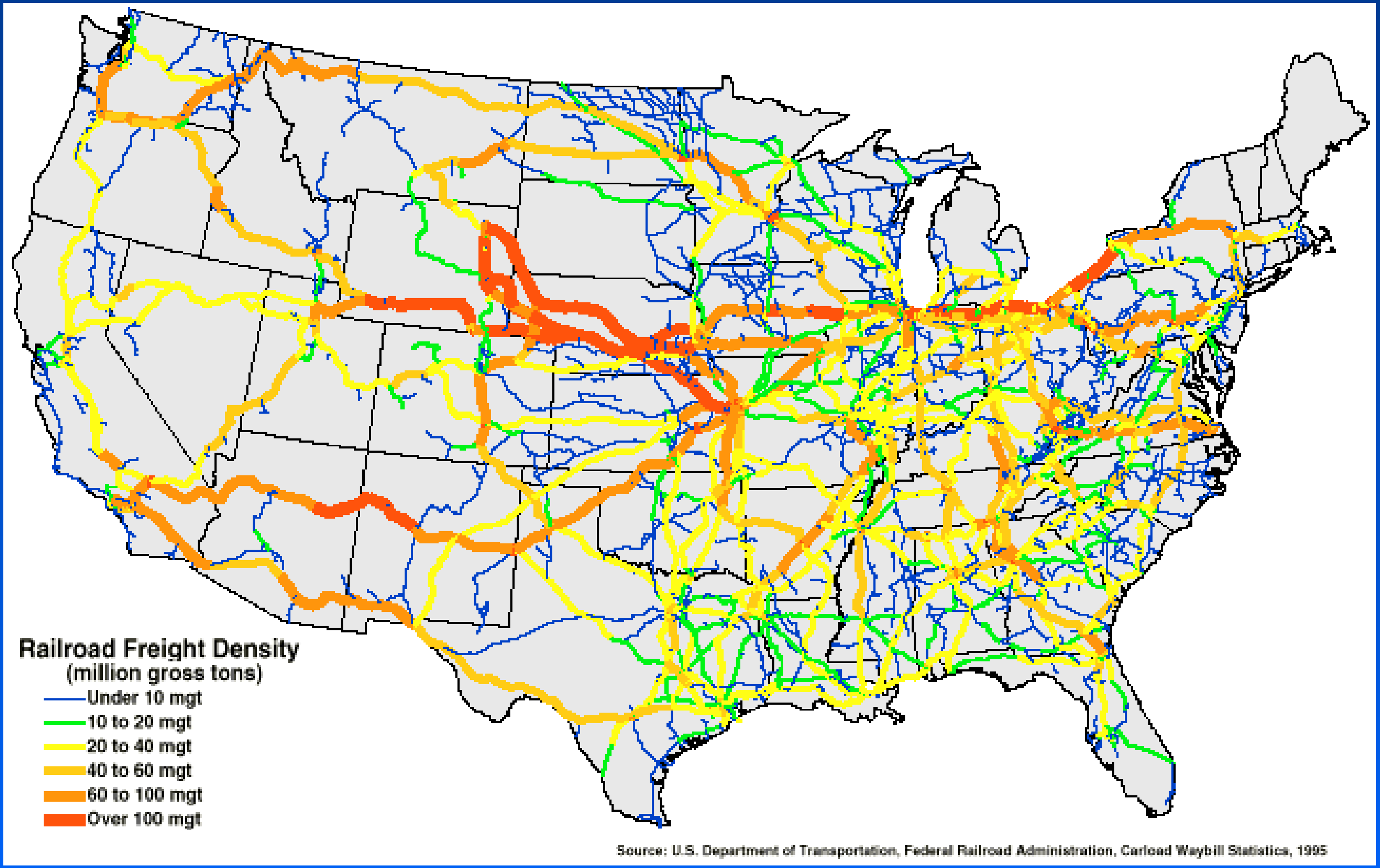


Components of Common Physical Networks

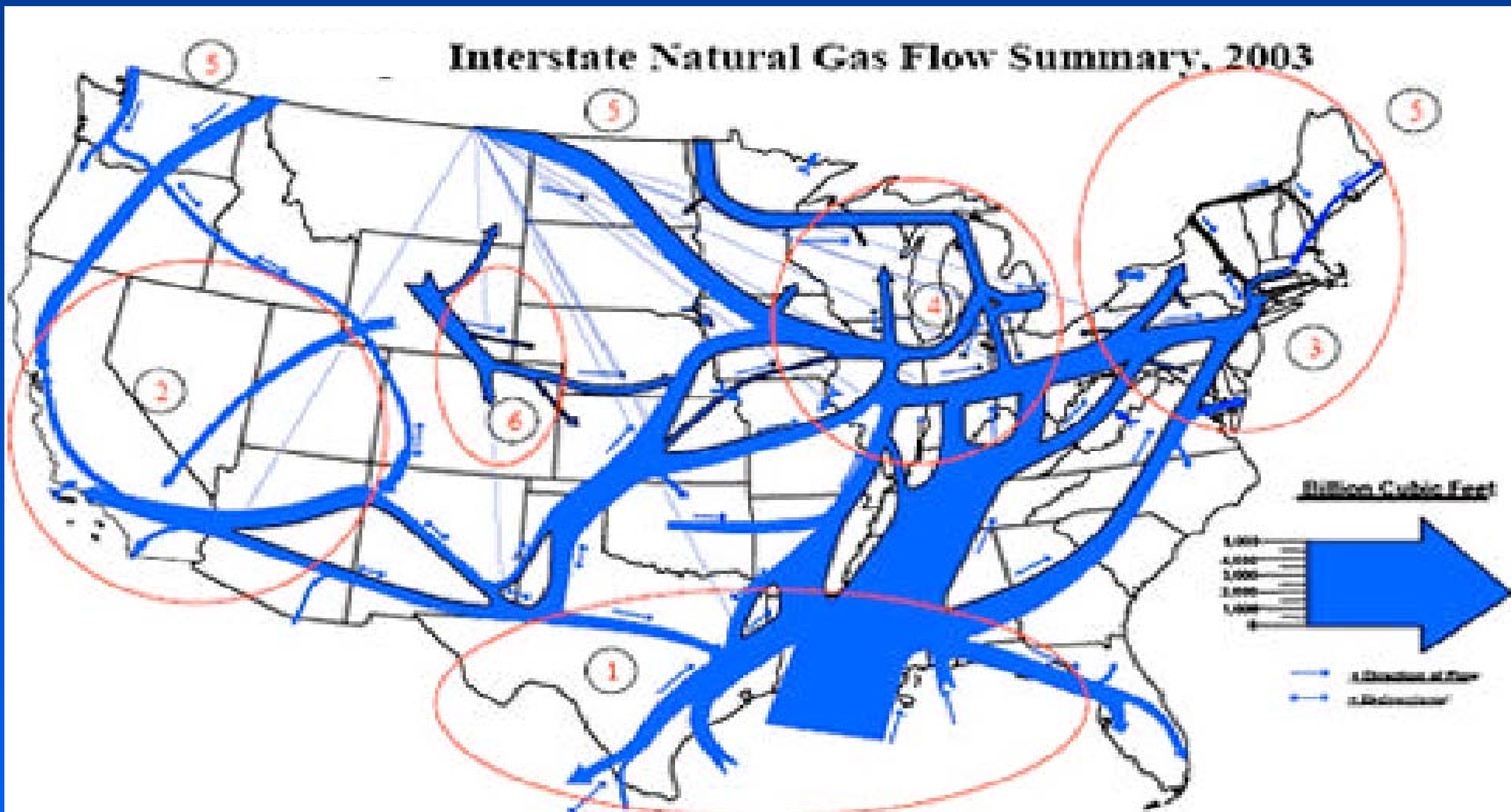
Network System	Nodes	Links	Flows
Transportation	Intersections, Homes, Workplaces, Airports, Railyards	Roads, Airline Routes, Railroad Track	Automobiles, Trains, and Planes,
Manufacturing and logistics	Workstations, Distribution Points	Processing, Shipment	Components, Finished Goods
Communication	Computers, Satellites, Telephone Exchanges	Fiber Optic Cables Radio Links	Voice, Data, Video
Energy	Pumping Stations, Plants	Pipelines, Transmission Lines	Water, Gas, Oil, Electricity

Interstate Highway System

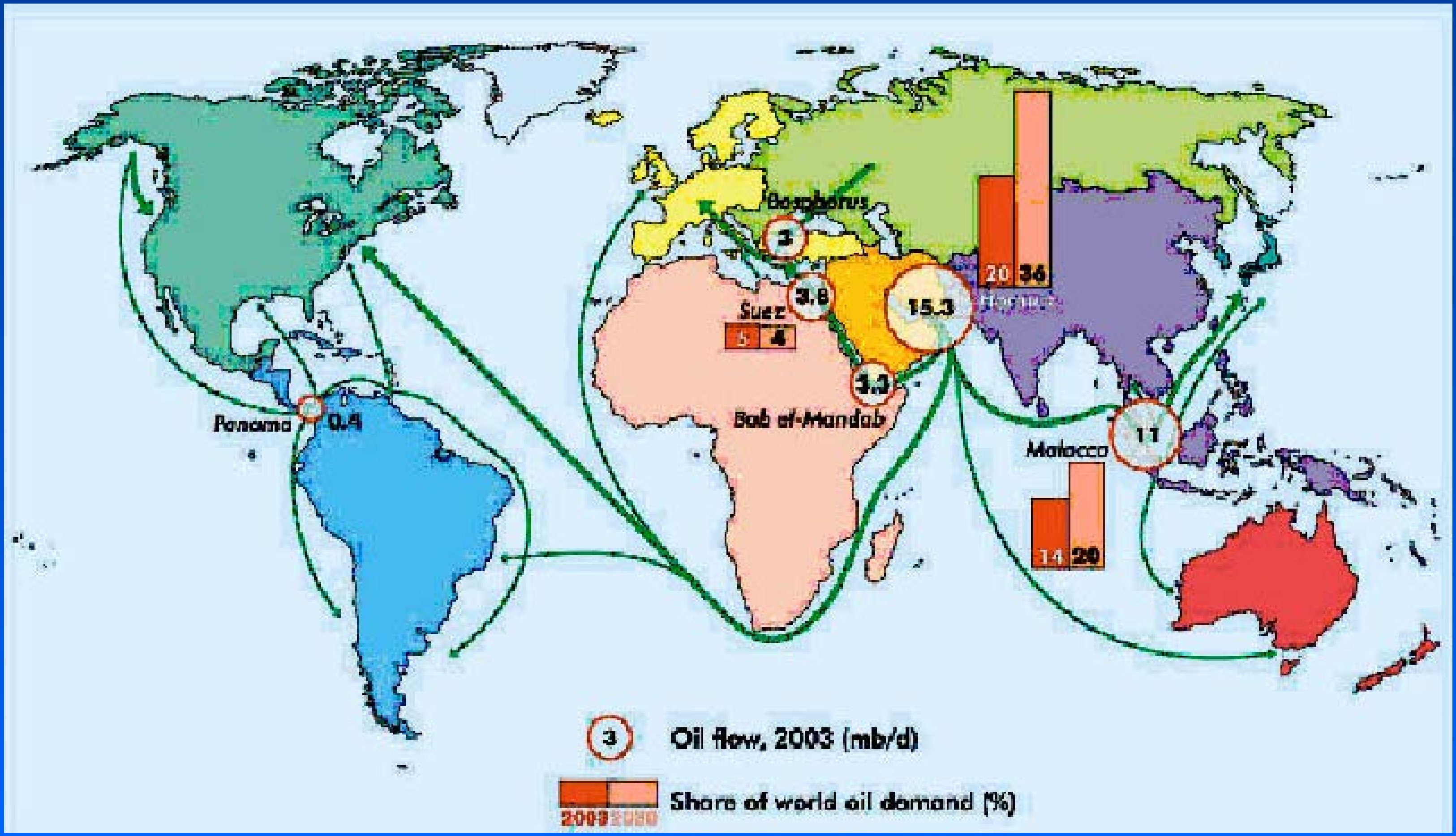
US Railroad Freight Flows



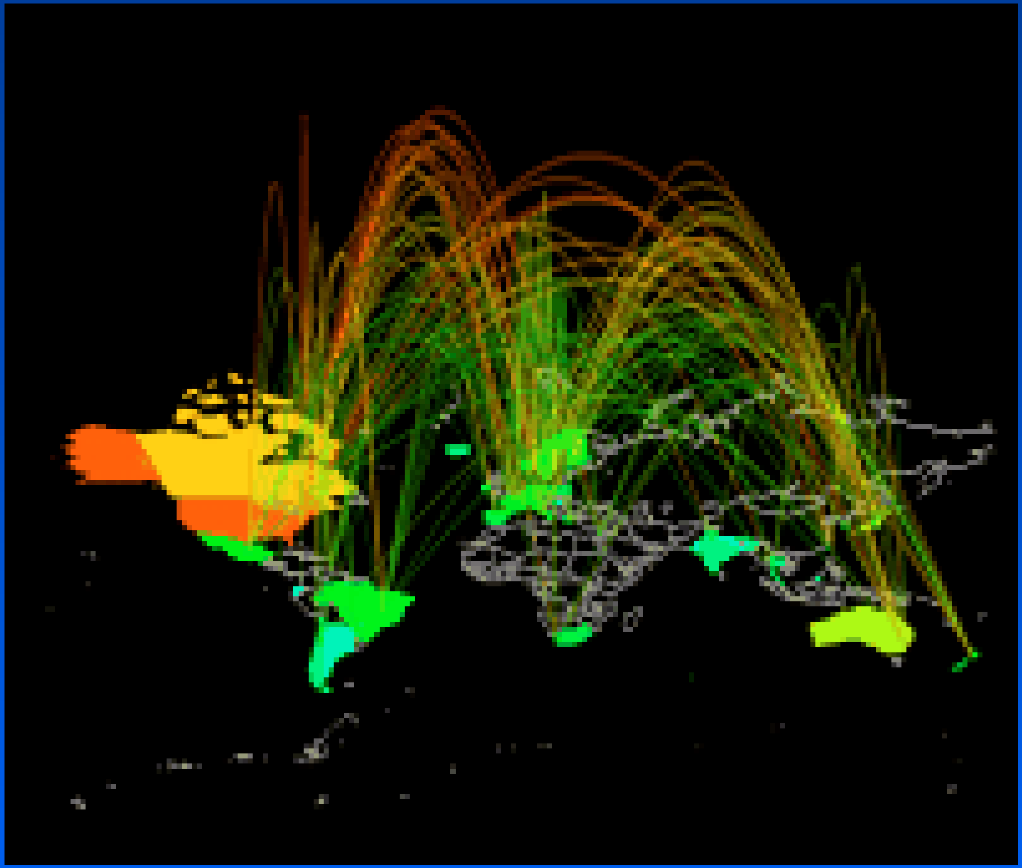
Natural Gas Pipeline Network in the US



World Oil Trading Network

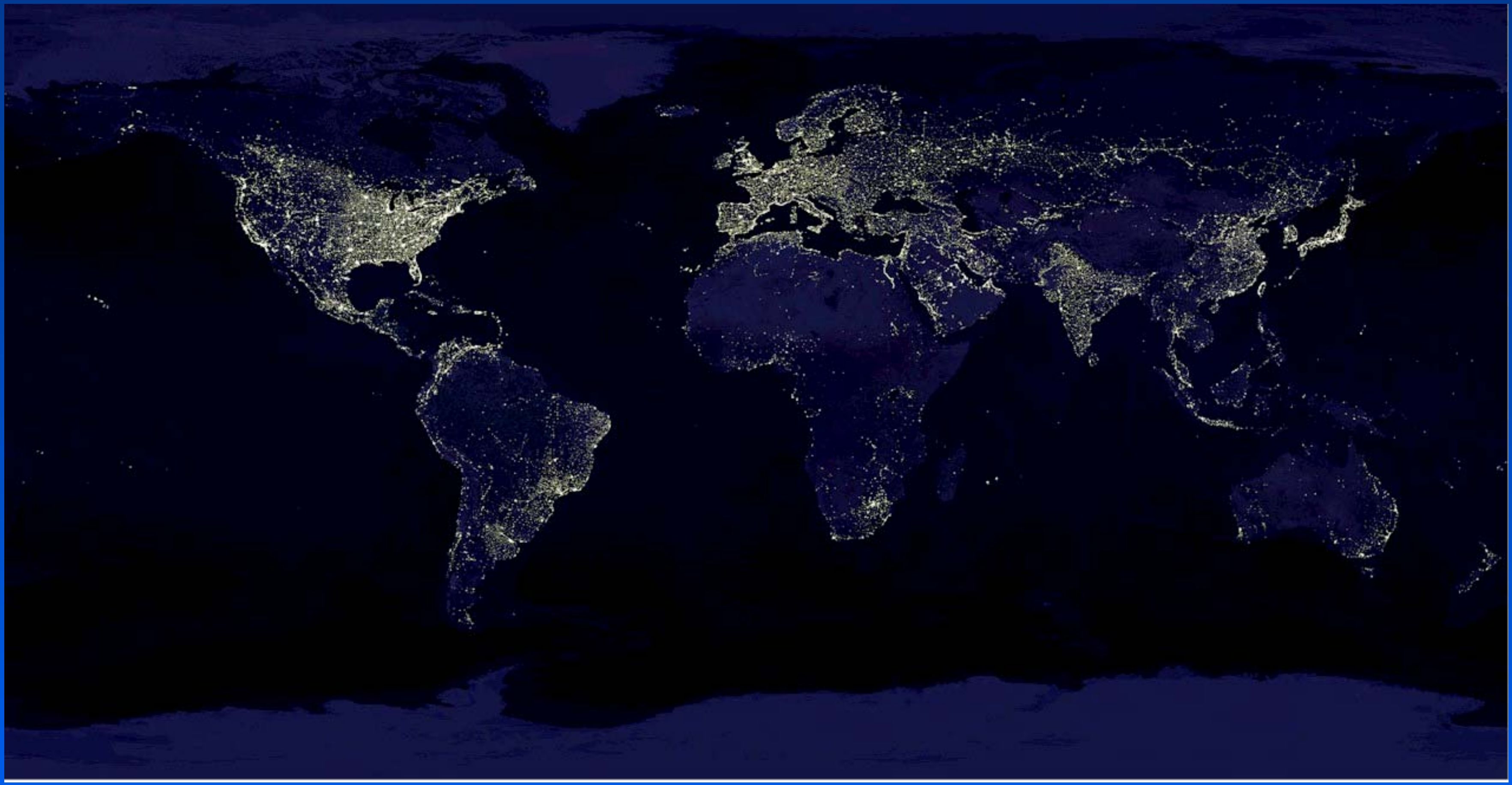


Internet Traffic Flows Over One 2 Hour Period



from Stephen Eick, Visual Insights

Electricity is Modernity

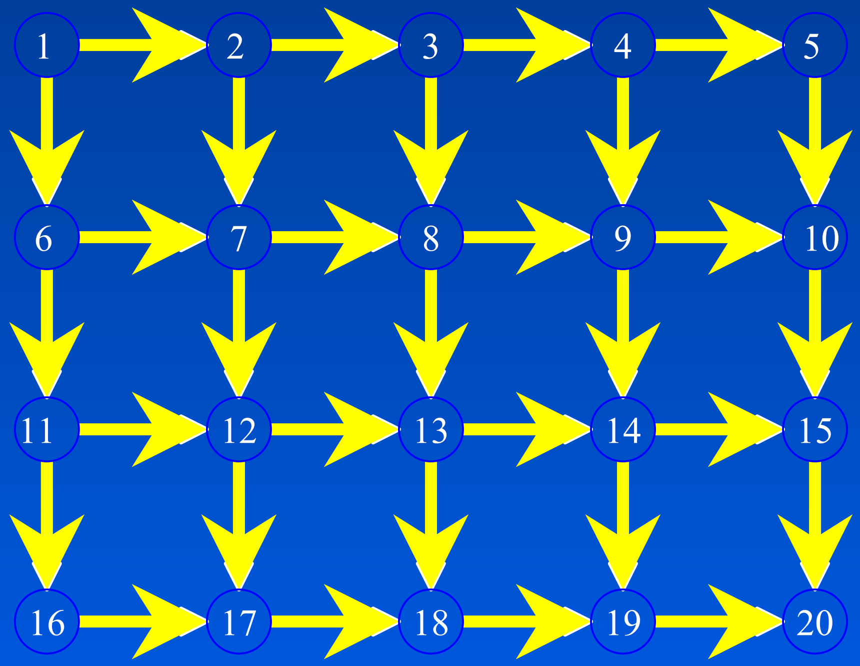


The scientific study of networks involves:

- how to **model** such applications as **mathematical entities**,
- how to **study the models** qualitatively,
- how to design **algorithms** to solve the resulting models.

The Basic Components of Networks

Nodes Links Flows



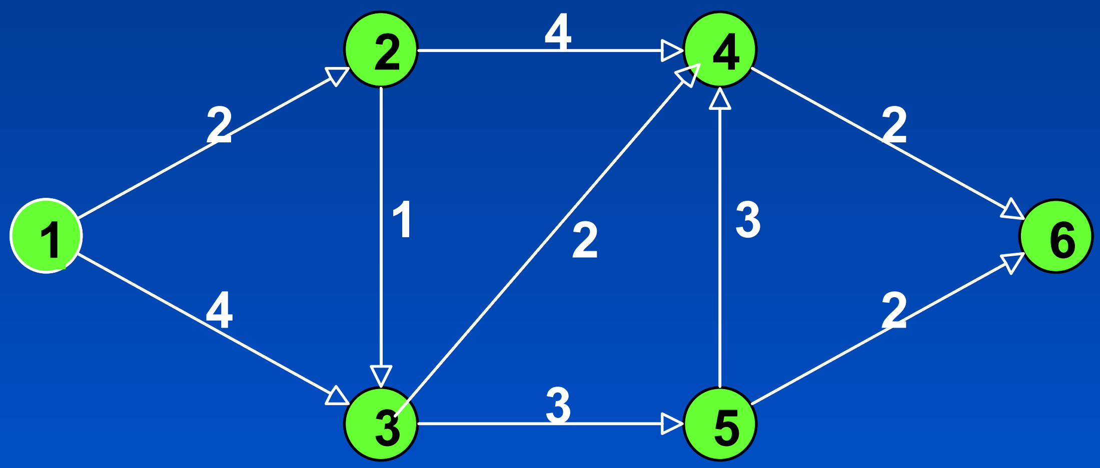
Classic Examples of Network Problems

The Shortest Path Problem

The Maximum Flow Problem

The Minimum Cost Flow Problem.

The Shortest Path Problem



What is the shortest path from 1 to 6?

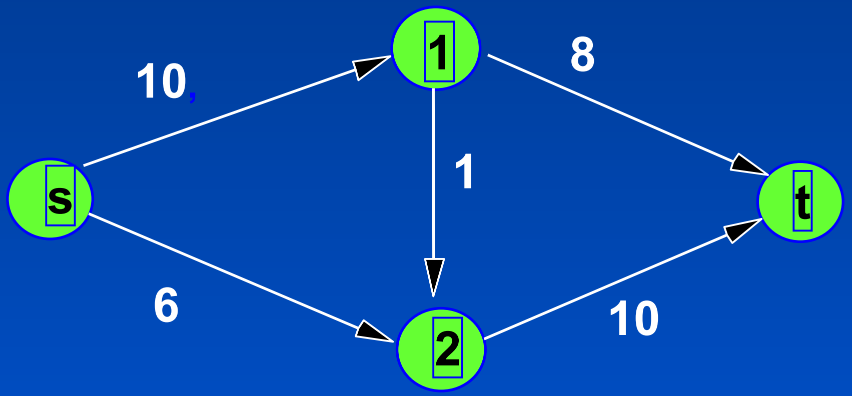
Applications of the Shortest Path Problem

Arise in transportation and telecommunications.

Other applications include:

- simple building evacuation models
- DNA sequence alignment
- assembly line balancing
- compact book storage in libraries.

The Maximum Flow Problem



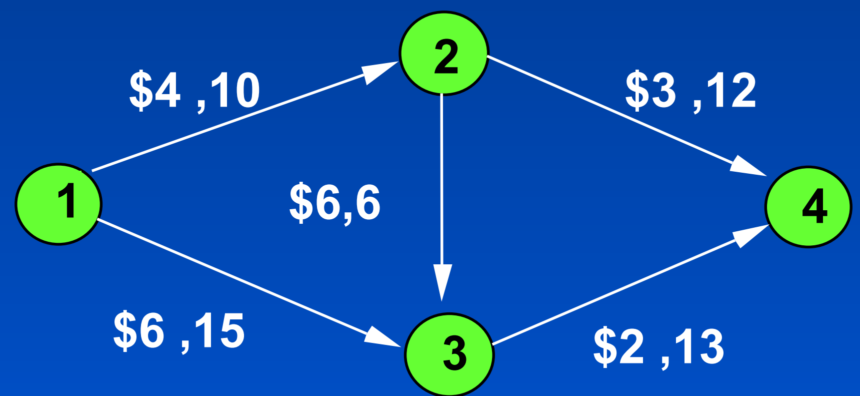
Each link has a maximum capacity.

How does one Maximize the flow from s to t, subject to the link capacities?

Applications of the Maximum Flow Problem

- machine scheduling
- network reliability testing
- building evacuation

The Minimum Cost Flow Problem



Each link has a linear cost and a maximum capacity.

How does one Minimize Cost for a given flow from 1 to 4?

The Optimization Formulation

Flow out of node i - Flow into node i = b(i)

Minimize $\Sigma_{i,j}\, c_{ij}\, x_{ij}$

s.t. $\Sigma_j\, x_{ij} - \Sigma_j\, x_{ji} = b(i)$ for each node i

$0 \preceq x_{ij} \preceq u_{ij}$ for all i,j

$\Sigma_i\, b(i) = 0$

Applications of the Minimum Cost Flow Problem

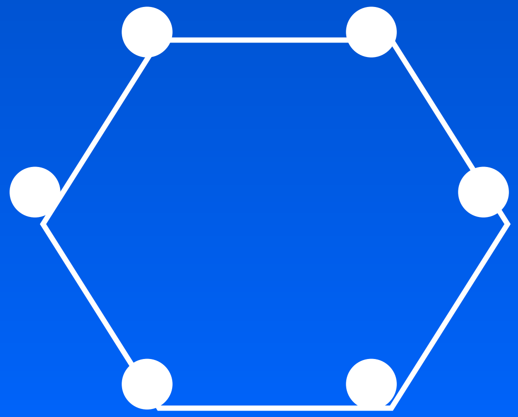
- warehousing and distribution
- vehicle fleet planning
- cash management
- automatic chromosome classification
- satellite scheduling

The study of the efficient operation on transportation networks dates to *ancient Rome* with a classical example being the publicly provided Roman road network and the *time of day chariot policy,* whereby chariots were banned from the ancient city of Rome at particular times of day.

Brief History of the Science of Networks

1736 - Euler - the earliest paper on graph theory - Konigsberg bridges problem.

1758 - Quesnay in his *Tableau Economique* introduced a graph to depict the circular flow of financial funds in an economy.



1781 - Monge, who had worked under Napoleon Bonaparte, publishes what is probably the first paper on transportation in minimizing cost.

1838 - Cournot states that a competitive price is determined by the intersection of supply and demand curves in the context of spatially separate markets in which transportation costs are included.

1841 - Kohl considered a two node, two route transportation network problem.

1845 - Kirchhoff wrote Laws of Closed Electric Circuits.

1920 - Pigou studied a transportation network system of two routes and noted that the decision-making behavior of the users on the network would result in different flow patterns.

1936 - Konig published the first book on graph theory.

1939, 1941, 1947 - Kantorovich, Hitchcock, and Koopmans considered the network flow problem associated with the classical minimum cost transportation problem and provided insights into the special network structure of these problems, which yielded special-purpose algorithms.

1948, 1951 - Dantzig published the simplex method for linear programming and adapted it for the classical transportation problem.

1951 - Enke showed that spatial price equilibrium problems can be solved using electronic circuits

1952 - Copeland in his book asked, *Does money flow like water or electricity?*

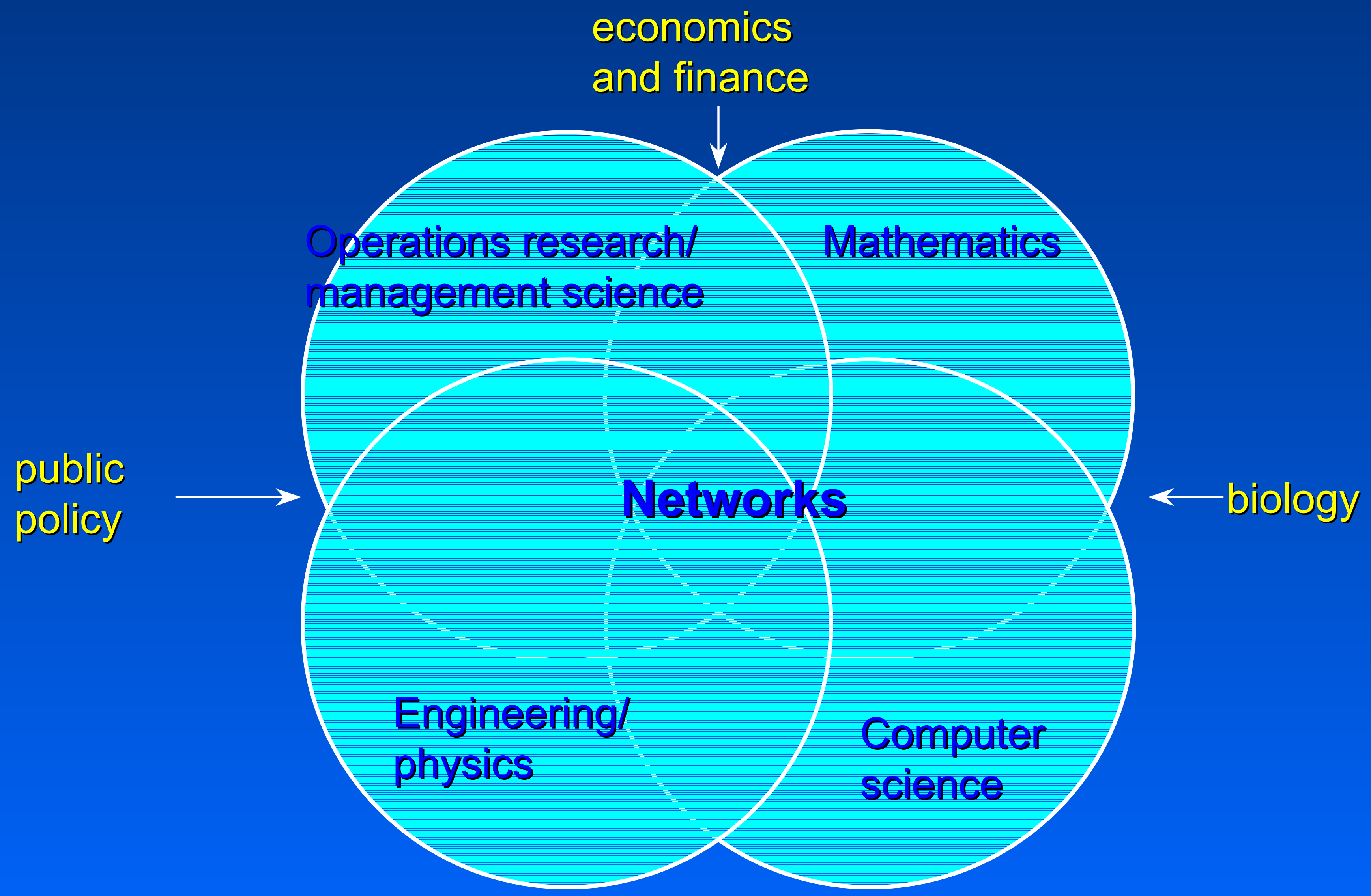
1952 - Samuelson gave a rigorous mathematical formulation of spatial price equilibrium and emphasized the network structure.

1956 - Beckmann, McGuire, and Winsten in their book, *Studies in the Economics of Transportation*, provided a rigorous treatment of congested urban transportation systems under different behavioral mechanisms due to Wardrop (1952).

1962 - Ford and Fulkerson publish *Flows in Networks*.

1969 - Dafermos and Sparrow coined the terms *user-optimization* and system-optimization and develop algorithms for the computation of solutions that exploit the network structure of transportation problems.

Networks in Different Disciplines



Interdisciplinary Impact of Networks

Models and Algorithms

Economics

Interregional Trade

General Equilibrium

Industrial Organization

Portfolio Optimization

Flow of Funds Accounting

Engineering

Energy

Manufacturing

Telecommunications

Transportation

Networks

Sociology

Social Networks

Organizational Theory

Computer Science

Routing Algorithms

Biology

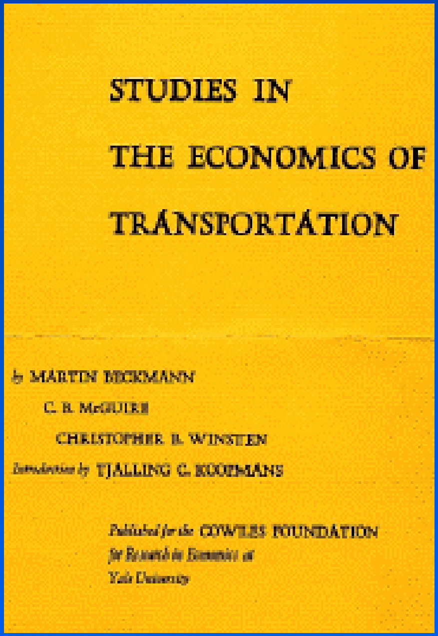
DNA Sequencing

Targeted Cancer Therapy

Characteristics of Networks Today

- *large-scale nature* and complexity of network topology;
- *congestion*;
- alternative behavior of users of the network, which may lead to *paradoxical phenomena*;
- the *interactions among networks* themselves such as in transportation versus telecommunications;
- *policies* surrounding networks today may have a *major impact* not only economically but also *socially, politically, and security-wise*.

Transportation science has historically been the discipline that has pushed the frontiers in terms of methodological developments for such problems (which are often large-scale) beginning with the book, *Studies in the Economics of Transportation*, by Beckmann, McGuire, and Winsten (1956).



The Cowles Foundation has made the book available on its website:
http://cowles.econ.yale.edu/archive/reprints/specpub-BMW.pdf

There are *two fundamental principles of travel behavior*, due to Wardrop (1952), which we refer to as user-optimization (or network equilibrium) or system-optimization. These terms were coined by Dafermos and Sparrow (1969); see also Beckmann, McGuire, and Winsten (1956).

In a *user-optimized (network equilibrium) problem*, each user of a network system seeks to determine his/her cost-minimizing route of travel between an origin/destination pair, until an equilibrium is reached, in which no user can decrease his/her cost of travel by unilateral action.

In a *system-optimized network problem*, users are allocated among the routes so as to minimize the total cost in the system. Both classes of problems, under certain imposed assumptions, possess optimization formulations.

The Transportation Social - Knowledge Network

Professors Beckmann and Dafermos at Anna Nagurney's Post-Ph.D. Defense Party

Professor Beckmann with Professor Michael Florian of Montreal

On the Beach in Mallacoota, Austrailia

INFORMS Honoring the 50th Anniversary of the Publication of ***Studies in the Economics of Transportation***

Professors Beckman and McGuire

Congestion according to the Texas Transportation Institute's *2007 Urban Mobility Report* is costing Americans more than $78 billion dollars a year in the form of 4.2 billion lost hours and 2.9 billion gallons of wasted fuel. Urban travelers are delayed in rush hour traffic nearly 40 hours a year.

The degradation of transportation networks due to poor maintenance, natural disasters, deterioration over time, as well as unforeseen attacks now lead to estimates of $94 billion in the United States in terms of needed repairs for roads alone (cf. American Society of Civil Engineers (2005)).

Poor road conditions in the United States cost US motorists $54 billion in repairs and operating costs annually. (cf. American Society of Civil Engineers (2005)).

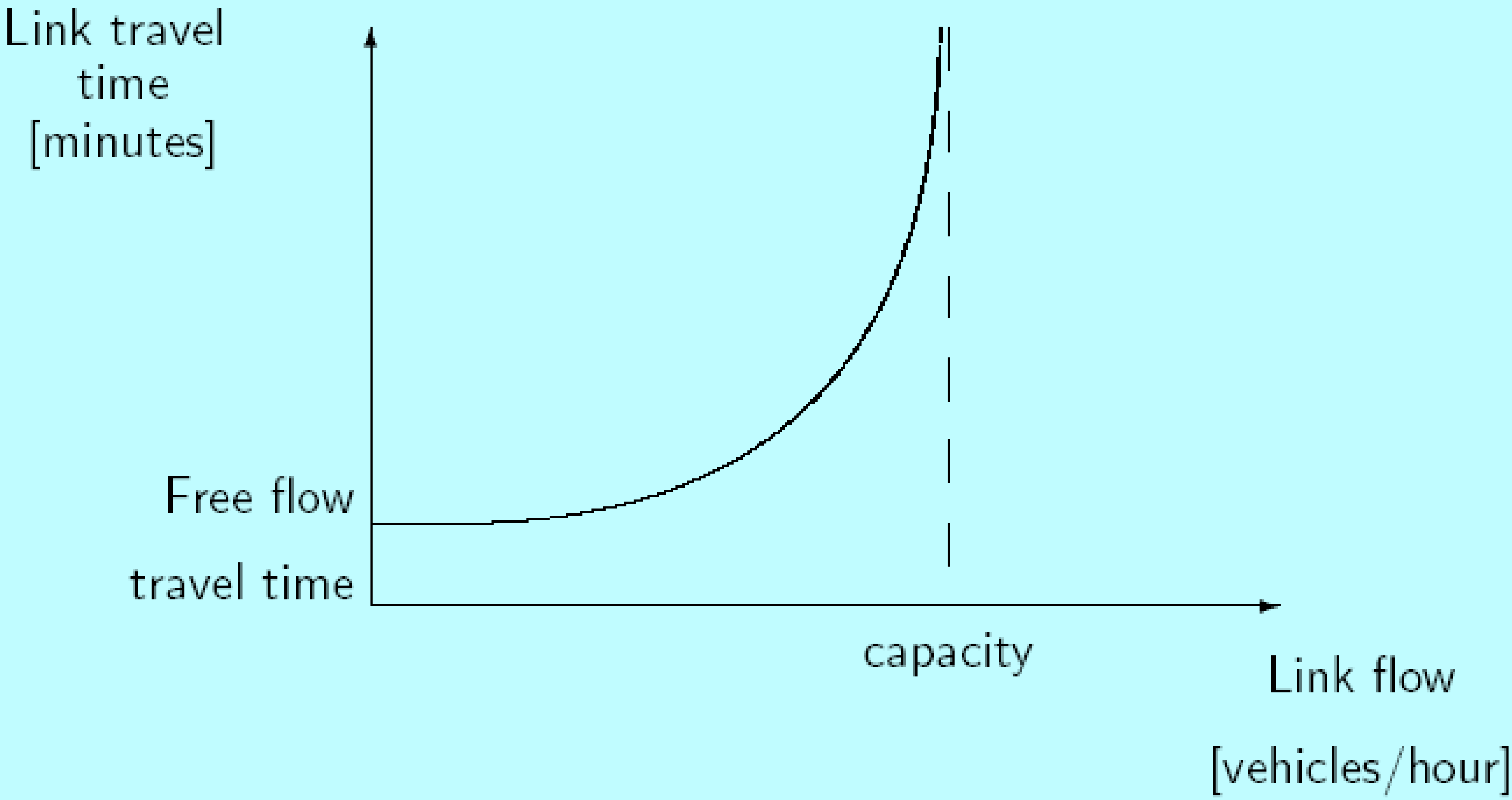
Even worse, over one-quarter of the nation's 590,750 bridges were rated structurally deficient or functionally obsolete (National Bridge Inventory (2005)).

At the same time, a recent report from the Federal Highway Administration (2006) states that the United States is experiencing a freight capacity crisis that threatens the strength and productivity of the US economy.

According to the American Road & Transportation Builders Association (Jeanneret (2006)), nearly 75% of US freight is carried in the US on highways and bottlenecks are causing truckers 243 million hours of delay annually with an estimated associated cost of $8 billion.

Traffic Congestion

Capturing Link Congestion



For a typical user link travel time function, where the free flow travel time refers to the travel time to traverse a link when there is zero flow on the link (or zero vehicles).

BPR Link Cost Function

A common link performance function is the Bureau of Public Roads (BPR) cost function developed in 1964.
This equation is given by

$$c_a = c_a^0 \left[1 + \alpha (\frac{f_a}{t'_a})^{\beta} \right],$$

where, c_a and f_a are the travel time and link flow, respectively, on link a, c_a^0 is the free-flow travel time, and t'_a is the "practical capacity" of link a. The quantities α and β are model parameters, for which the values $\alpha = 0.15$ minutes and $\beta = 4$ are typical values.
For example, these values imply that the practical capacity of a link is the flow at which the travel time is 15% greater than the free-flow travel time.

The User-Optimization (U-O) Problem Transportation Network Equilibrium

Consider a general network $G = [N, L]$, where N denotes the set of nodes, and L the set of directed links. Let a denote a link of the network connecting a pair of nodes, and let p denote an acyclic path consisting of a sequence of links connecting an origin/destination (O/D) pair of nodes. P_w denotes the set of paths connecting the O/D pair of nodes w and P the set of all paths.

Let x_p represent the flow on path p and let f_a denote the flow on link a. The following conservation of flow equations must hold:

$$f_a = \sum_{p \in P} x_p \delta_{ap},$$

where $\delta_{ap} = 1$, if link a is contained in path p, and 0, otherwise. This expression states that the flow on a link a is equal to the sum of all the path flows on paths p that contain (traverse) link a.

Moreover, if we let d_w denote the demand associated with O/D pair w, then we must have that

$$d_w = \sum_{p \in P_w} x_p,$$

where $x_p \geq 0$, $\forall p$, that is, the sum of all the path flows between an origin/destination pair w must be equal to the given demand d_w.

Let c_a denote the user cost associated with traversing link a, and C_p the user cost associated with traversing the path p. Then

$$C_p = \sum_{a \in L} c_a \delta_{ap}.$$

In other words, the cost of a path is equal to the sum of the costs on the links comprising the path. In the classical model, $c_a = c_a(f_a)$, $\forall a \in L$. In the most general case, $c_a = c_a(f), \forall a \in L$, where f is the vector of link flows.

Transportation Network Equilibrium Conditions

The network equilibrium conditions are then given by: For each path $p \in P_w$ and every O/D pair w:

$$C_p \begin{cases} = \lambda_w, & \text{if} \quad x_p^* > 0 \\ \geq \lambda_w, & \text{if} \quad x_p^* = 0 \end{cases}$$

where λ_w is an indicator, whose value is not known a priori. The equilibrium conditions state that the user costs on all used paths connecting a given O/D pair will be minimal and equalized. This is Wardrop's first principle of travel behavior.

As shown by Beckmann, McGuire, and Winsten (1956) and Dafermos and Sparrow (1969), if the user link cost functions satisfy the symmetry property that $[\frac{\partial c_a}{\partial c_b} = \frac{\partial c_b}{\partial c_a}]$, for all links a, b in the network then the solution to the above U-O problem can be reformulated as the solution to an associated optimization problem. For example, if we have that $c_a = c_a(f_a)$, for all links $a \in L$, then the solution to the U-O problem can be obtained by solving:

$$\text{Minimize} \quad \sum_{a \in L} \int_0^{f_a} c_a(y)dy$$

subject to:

$$d_w = \sum_{p \in P_w} x_p, \quad \forall w \in W,$$

$$f_a = \sum_{p \in P} x_p, \quad \forall a \in L,$$

$$x_p \geq 0, \quad \forall p \in P.$$

The System-Optimization (S-O) Problem

The above discussion focused on the user-optimized (U-O) problem. We now turn to the system-optimized (S-O) problem in which a central controller, say, seeks to minimize the total cost in the network system, where the total cost is expressed as

$$\sum_{a \in L} \hat{c}_a(f_a)$$

where it is assumed that the total cost function on a link a is defined as:

$$\hat{c}_a(f_a) \equiv c_a(f_a) \times f_a,$$

subject to the conservation of flow constraints, and the nonnegativity assumption on the path flows. Here separable link costs have been assumed, for simplicity, and other total cost expressions may be used, as mandated by the particular application.

The S-O Optimality Conditions

Under the assumption of strictly increasing user link cost functions, the optimality conditions are: For each path $p \in P_w$, and every O/D pair w:

$$\hat{C}'_p \begin{cases} = \mu_w, & \text{if} \quad x_p > 0 \\ \geq \mu_w, & \text{if} \quad x_p = 0, \end{cases}$$

where $\hat{C}'_p$ denotes the marginal total cost on path p, given by:

$$\hat{C}'_p = \sum_{a \in L} \frac{\partial \hat{c}_a(f_a)}{\partial f_a} \delta_{ap}.$$

The above conditions correspond to Wardrop's second principle of travel behavior.

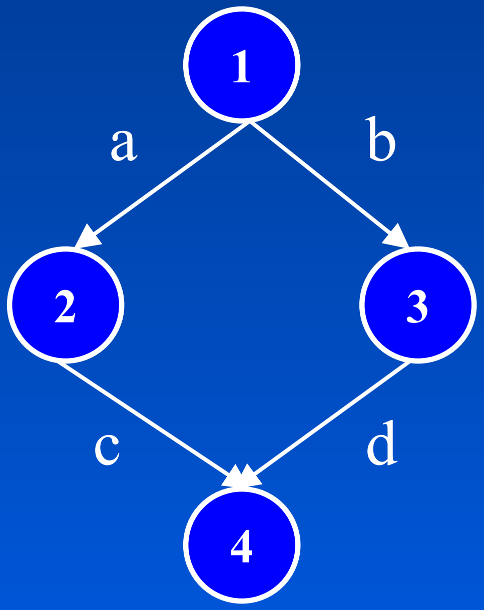
The Braess (1968) Paradox

Assume a network with a single O/D pair (1,4). There are 2 paths available to travelers: $\mathbf{p_1=(a,c)}$ and $\mathbf{p_2=(b,d)}$.

For a travel demand of **6,** the equilibrium path flows are $\mathbf{x_{p_1}^* = x_{p_2}^* = 3}$ and

The equilibrium path travel cost is

$\mathbf{C_{p_1} = C_{p_2} = 83}$.



$\mathbf{c_a(f_a)=10\,f_a \quad c_b(f_b) = f_b+50}$

$\mathbf{c_c(f_c) = f_c+50 \quad c_d(f_d) = 10\,f_d}$

Adding a Link Increases Travel Cost for All!

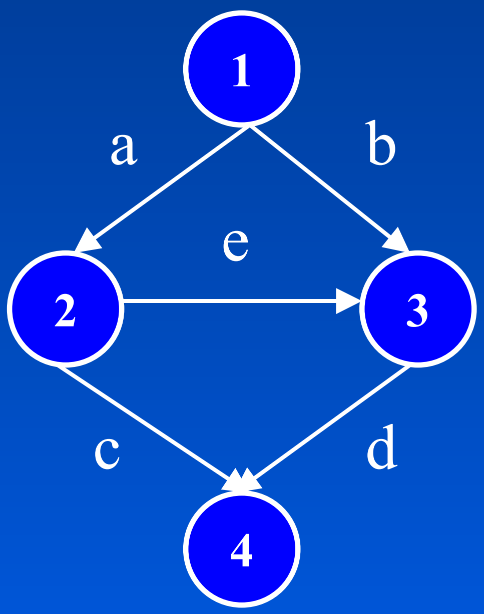
Adding a new link creates a new path **p_3=(a,e,d).**

The original flow distribution pattern is no longer an equilibrium pattern, since at this level of flow the cost on path **p_3, C_{p_3}=70.**

The new equilibrium flow pattern network is

$x_{p_1}^* = x_{p_2}^* = x_{p_3}^* = 2.$

The equilibrium path travel costs: **$C_{p_1} = C_{p_2} = C_{p_3} = 92.$**



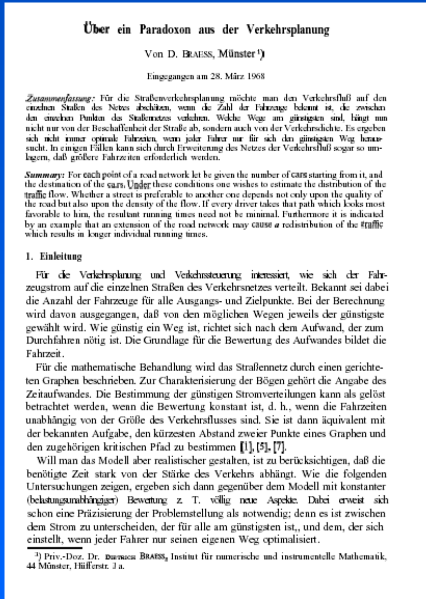
$c_e(f_e) = f_e + 10$

The 1968 Braess article has been translated from German to English and appears as

On a Paradox of Traffic Planning

by Braess, Nagurney, Wakolbinger

in the November 2005 issue of *Transportation Science.*



Über ein Paradoxon aus der Verkehrsplanung

Von D. BRAESS, Münster [1]

Eingegangen am 28. März 1968

Zusammenfassung: Für die Straßenverkehrsplanung möchte man den Verkehrsfluß auf den einzelnen Straßen des Netzes abschätzen, wenn die Zahl der Fahrzeuge bekannt ist, die zwischen den einzelnen Punkten des Straßennetzes verkehren. Welche Wege am günstigsten sind, hängt nun nicht nur von der Beschaffenheit der Straße ab, sondern auch von der Verkehrsdichte. Es ergeben sich nicht immer optimale Fahrzeiten, wenn jeder Fahrer nur für sich den günstigsten Weg heraussucht. In einigen Fällen kann sich durch Erweiterung des Netzes der Verkehrsfluß sogar so umlagern, daß größere Fahrzeiten erforderlich werden.

Summary: For each point of a road network let be given the number of cars starting from it, and the destination of the cars. Under these conditions one wishes to estimate the distribution of the traffic flow. Whether a street is preferable to another one depends not only upon the quality of the road but also upon the density of the flow. If every driver takes that path which looks most favorable to him, the resultant running times need not be minimal. Furthermore it is indicated by an example that an extension of the road network may cause a redistribution of the traffic which results in longer individual running times.

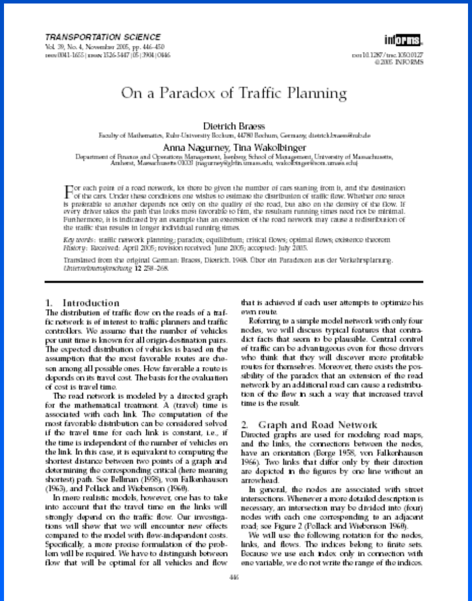
1. Einleitung

Für die Verkehrsplanung und Verkehrssteuerung interessiert, wie sich der Fahrzeugstrom auf die einzelnen Straßen des Verkehrsnetzes verteilt. Bekannt sei dabei die Anzahl der Fahrzeuge für alle Ausgangs- und Zielpunkte. Bei der Berechnung wird davon ausgegangen, daß von den möglichen Wegen jeweils der günstigste gewählt wird. Wie günstig ein Weg ist, richtet sich nach dem Aufwand, der zum Durchfahren nötig ist. Die Grundlage für die Bewertung des Aufwandes bildet die Fahrzeit.

Für die mathematische Behandlung wird das Straßennetz durch einen gerichteten Graphen beschrieben. Zur Charakterisierung der Bögen gehört die Angabe des Zeitaufwandes. Die Bestimmung der günstigen Stromverteilungen kann als gelöst betrachtet werden, wenn die Bewertung konstant ist, d. h., wenn die Fahrzeiten unabhängig von der Größe des Verkehrsflusses sind. Sie ist dann äquivalent mit der bekannten Aufgabe, den kürzesten Abstand zweier Punkte eines Graphen und den zugehörigen kritischen Pfad zu bestimmen [1], [5], [7].

Will man das Modell aber realistischer gestalten, ist zu berücksichtigen, daß die benötigte Zeit stark von der Stärke des Verkehrs abhängt. Wie die folgenden Untersuchungen zeigen, ergeben sich dann gegenüber dem Modell mit konstanter (belastungsunabhängiger) Bewertung z. T. völlig neue Aspekte. Dabei erweist sich schon eine Präzisierung der Problemstellung als notwendig; denn es ist zwischen dem Strom zu unterscheiden, der für alle am günstigsten ist,, und dem, der sich einstellt, wenn jeder Fahrer nur seinen eigenen Weg optimalisiert.

[1]) Priv.-Doz. Dr. DIETRICH BRAESS, Institut für numerische und instrumentelle Mathematik, 44 Münster, Hüfferstr. 3 a.



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On a Paradox of Traffic Planning

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For each point of a road network, let there be given the number of cars starting from it, and the destination of the cars. Under these conditions one wishes to estimate the distribution of traffic flow. Whether one street is preferable to another depends not only on the quality of the road, but also on the density of the flow. If every driver takes the path that looks most favorable to him, the resultant running times need not be minimal. Furthermore, it is indicated by an example that an extension of the road network may cause a redistribution of the traffic that results in longer individual running times.

Key words: traffic network planning; paradox; equilibrium; critical flows; optimal flows; existence theorem
History: Received: April 2005; revision received: June 2005; accepted: July 2005.

Translated from the original German: Braess, Dietrich. 1968. Über ein Paradoxon aus der Verkehrsplanung. *Unternehmensforschung* **12** 258–268.

1. Introduction

The distribution of traffic flow on the roads of a traffic network is of interest to traffic planners and traffic controllers. We assume that the number of vehicles per unit time is known for all origin-destination pairs. The expected distribution of vehicles is based on the assumption that the most favorable routes are chosen among all possible ones. How favorable a route is depends on its travel cost. The basis for the evaluation of cost is travel time.

The road network is modeled by a directed graph for the mathematical treatment. A (travel) time is associated with each link. The computation of the most favorable distribution can be considered solved if the travel time for each link is constant, i.e., if the time is independent of the number of vehicles on the link. In this case, it is equivalent to computing the shortest distance between two points of a graph and determining the corresponding critical (here meaning shortest) path. See Bellman (1958), von Falkenhausen (1963), and Pollack and Wiebenson (1960).

In more realistic models, however, one has to take into account that the travel time on the links will strongly depend on the traffic flow. Our investigations will show that we will encounter new effects compared to the model with flow-independent costs. Specifically, a more precise formulation of the problem will be required. We have to distinguish between flow that will be optimal for all vehicles and flow that is achieved if each user attempts to optimize his own route.

Referring to a simple model network with only four nodes, we will discuss typical features that contradict facts that seem to be plausible. Central control of traffic can be advantageous even for those drivers who think that they will discover more profitable routes for themselves. Moreover, there exists the possibility of the paradox that an extension of the road network by an additional road can cause a redistribution of the flow in such a way that increased travel time is the result.

2. Graph and Road Network

Directed graphs are used for modeling road maps, and the links, the connections between the nodes, have an orientation (Berge 1958, von Falkenhausen 1966). Two links that differ only by their direction are depicted in the figures by one line without an arrowhead.

In general, the nodes are associated with street intersections. Whenever a more detailed description is necessary, an intersection may be divided into (four) nodes with each one corresponding to an adjacent road; see Figure 2 (Pollack and Wiebenson 1960).

We will use the following notation for the nodes, links, and flows. The indices belong to finite sets. Because we use each index only in connection with one variable, we do not write the range of the indices.

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What is the S-O solution for the two Braess networks (before and after the addition of a new link e)?

Before the addition of the link e, we may write:

$$\hat{c}'_a = 20f_a, \quad \hat{c}'_b = 2f_b + 50,$$

$$\hat{c}'_c = 2f_c + 50, \quad \hat{c}'_d = 20f_d.$$

It is easy to see that, in this case, the S-O solution is identical to the U-O solution with $x_{p_1} = x_{p_2} = 3$ and $\hat{C}'_{p_1} = \hat{C}'_{p_2} = 116$. Furthermore, after the addition of link e, we have that $\hat{c}'_e = 2f_e + 10$. The new path p_3 is not used in the S-O solution, since with zero flow on path p_3, we have that $\hat{C}'_{p_3} = 170$ and $\hat{C}'_{p_1} = \hat{C}'_{p_2}$ remains at 116.

Toll Policies

The system-optimizing flow pattern is one that minimizes the total travel cost over the entire network, whereas the user-optimized flow pattern has the property that no user has any incentive to make a unilateral decision to alter his/her travel path.

The latter solution typically results in a higher total system cost and, in a sense, is an underutilization of the transportation network. In order to remedy this situation tolls can be applied with the recognition that imposing tolls will not change the travel cost as perceived by society since tolls are not lost.

Tolls can be collected on a *link basis*, that is, every traveler on a link will be charged the same toll, irrespective of origin or final destination, or on a *path basis*, in which every traveler traveling from an origin to a destination on a particular path will be charged the same toll.

Note that one can construct multimodal versions of such a toll policy in which the pricing on links or paths is according to mode; see Dafermos (1973) and Nagurney (1999).

Let t_a denote a toll associated with link a in the link-toll collection policy.

Observe that, after the imposition of tolls, the travel cost as perceived by society remains $c_a(f_a)$, for all links $a \in L$. The travel cost on a path p as perceived by the individual, however, is modified to

$$\bar{C}_p = C_p(f) + t_p, \quad \forall p \in P.$$

Consequently, a system-optimizing flow pattern is still defined as before, that is, it is one that solves the problem

$$\text{Minimize}_{f \in K} \sum_{a \in L} \hat{c}_a(f_a).$$

Solution of the Link-Toll Collection Policy

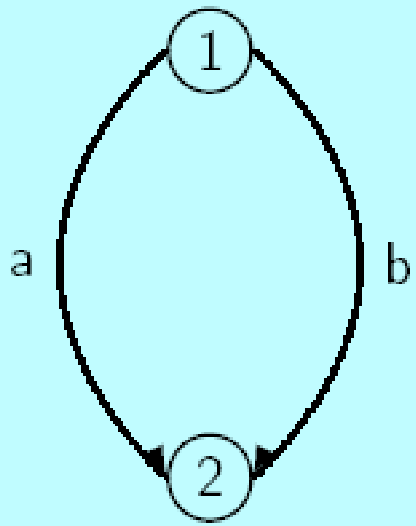
The link-toll collection policy is determined by

$$t_a = \frac{\partial \hat{c}_a(f_a)}{\partial f_a} - c_a(f_a), \quad \forall a \in L,$$

where both the first and the second terms on the right-hand side of the expression are evaluated at the system-optimizing solution f.

A Link Toll Policy Example

Consider the network depicted below in which there are two nodes: 1, 2; two links: a, b; and a single O/D pair $w_1 = (1, 2)$. Let path $p_1 = a$ and path $p_2 = b$.



Assume that the user link travel cost functions are:

$$c_a(f_a) = 2f_a + 5, \quad c_b(f_b) = f_b + 10,$$

and the travel demand: $d_{w_1} = 10$.

In the absence of any policies, travelers operating in a user-optimized manner will select the paths as follows: $x_{p_1} = 5$, and $x_{p_2} = 5$ with induced link flow patterns of: $f_a = 5$ and $f_b = 5$. The incurred user travel costs on the paths under this user-optimized flow pattern will be:

$$C_{p_1} = c_a = 15, \quad C_{p_2} = c_b = 15,$$

which satisfies the traffic equilibrium conditions. This path flow pattern, in turn, will yield a total cost on the network given by $c_a \times f_a + c_b \times f_b = 75 + 75 = 150.$

The system-optimized flow pattern is, however, given by: $x_{p_1} = 4\frac{1}{6}$, $x_{p_2} = 5\frac{5}{6}$, which induces the link flow pattern: $f_a = 4\frac{1}{6}$, $f_b = 5\frac{5}{6}$ and the marginals of the total travel costs on the paths are:

$$\hat{C}'_{p_1} = \hat{c}'_a = 21\frac{2}{3}, \quad \hat{C}'_{p_2} = \hat{c}'_b = 21\frac{2}{3},$$

with a total cost in the network under the S-O pattern equal to $131\frac{7}{18}$, which is clearly lower than the total cost under the U-O flow pattern above, which was 150.

The link-toll policy that renders the system-optimizing flow pattern also user-optimized is given by:

$$t_a = 8\frac{1}{3}, \quad t_b = 5\frac{5}{6},$$

with the induced user costs with the imposed tolls being: $\bar{C}_{p_1} = \bar{C}_{p_2} = 21\frac{2}{3}$.

Extensions of the fixed demand models in which the cost on a link depends on the flow on a link have been made to capture multiple modes of transportation as well as elastic demands.

- For example, one may have that the cost on a link as experienced by a mode of transportation (or a class of user) depends, in general, on the flow of all the modes (or classes) on all the links on the network.

- To handle elastic demand associated with travel between origin/destination pairs, we introduce a travel disutility associated with traveling between each O/D pair which can be a function of the travel demands associated with all the O/D pairs (and all modes in a multimodal case).

- The U-O and the S-O conditions are then generalized to include the multiple modes/classes of transportation as well as the travel disutilities, which are now functions and are associated with the different modes/classes.

- For a variety of such models, along with references see the books by Nagurney (1999, 2000).

In addition, in the book *Sustainable Transportation Networks* published in 2000 I illustrate, through specific examples, the following counterintuitive/paradoxical phenomena:

1. the addition of a road may result in an increase in total emissions with no change in travel demand;

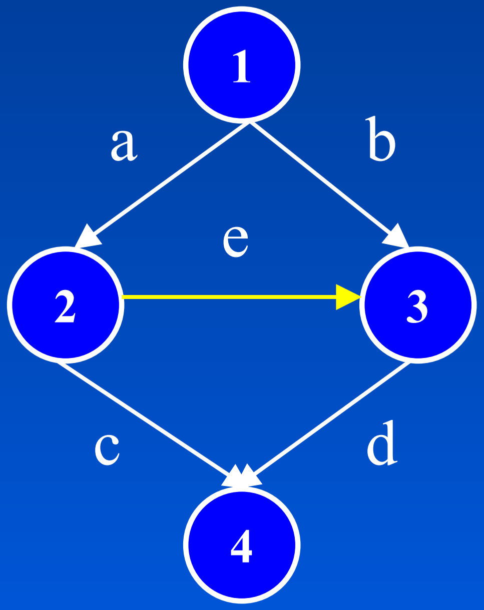
2. a decrease in the travel demand may result in an increase in total emissions;

3. the improvement of a road in terms of travel cost reduction may result in an increase in total emissions without a change in the travel demand;

4. a transfer of travel demand from a mode with higher total emissions to a mode with lower total emissions on a network may result in an increase in the total emissions; and

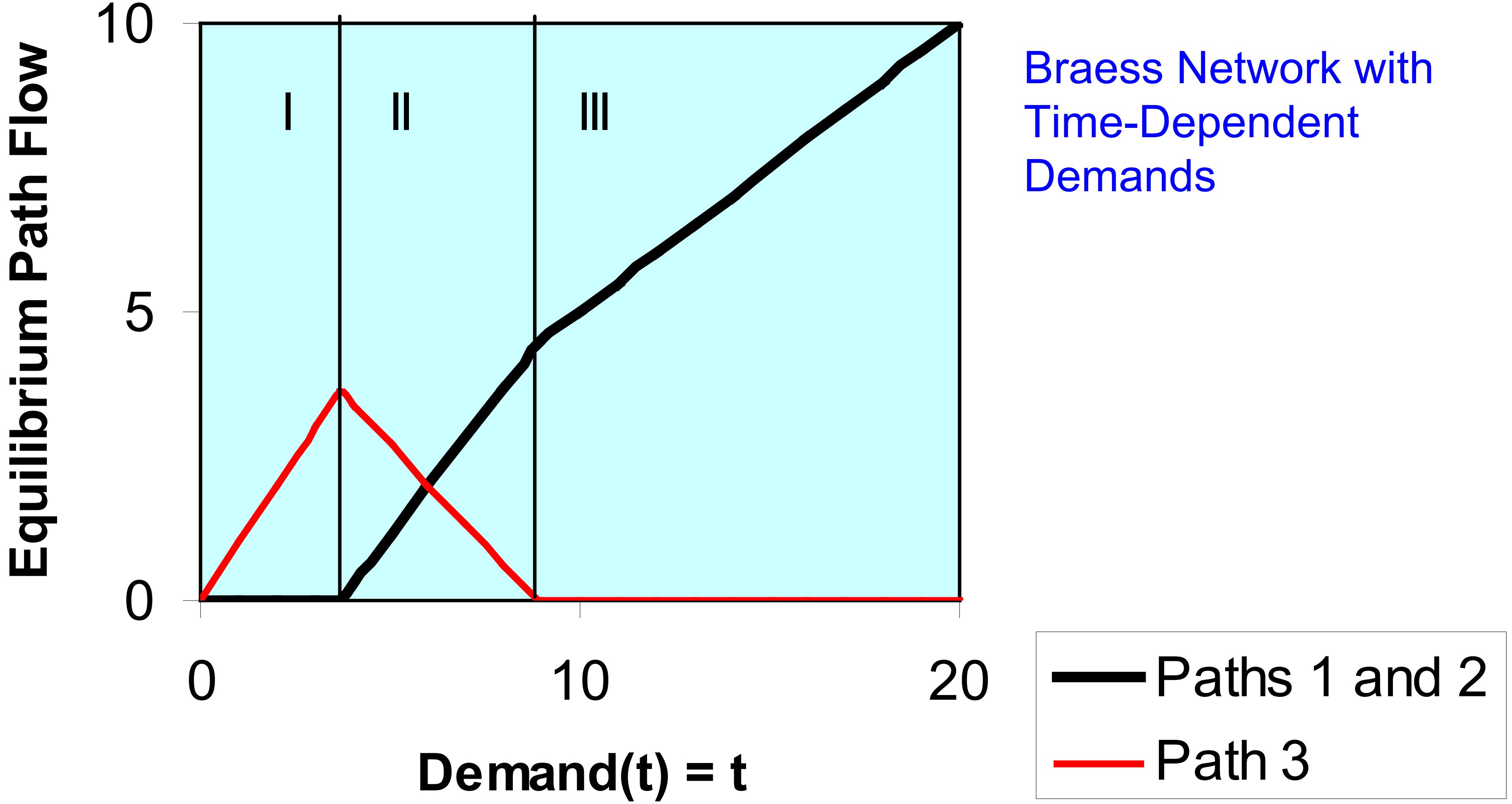
5. making travel less attractive between an origin/destination pair as revealed through its travel disutility function may result in an increase in total emissions.

Recall the Braess Network
where we add the link e.



What happens if the demand varies over time?

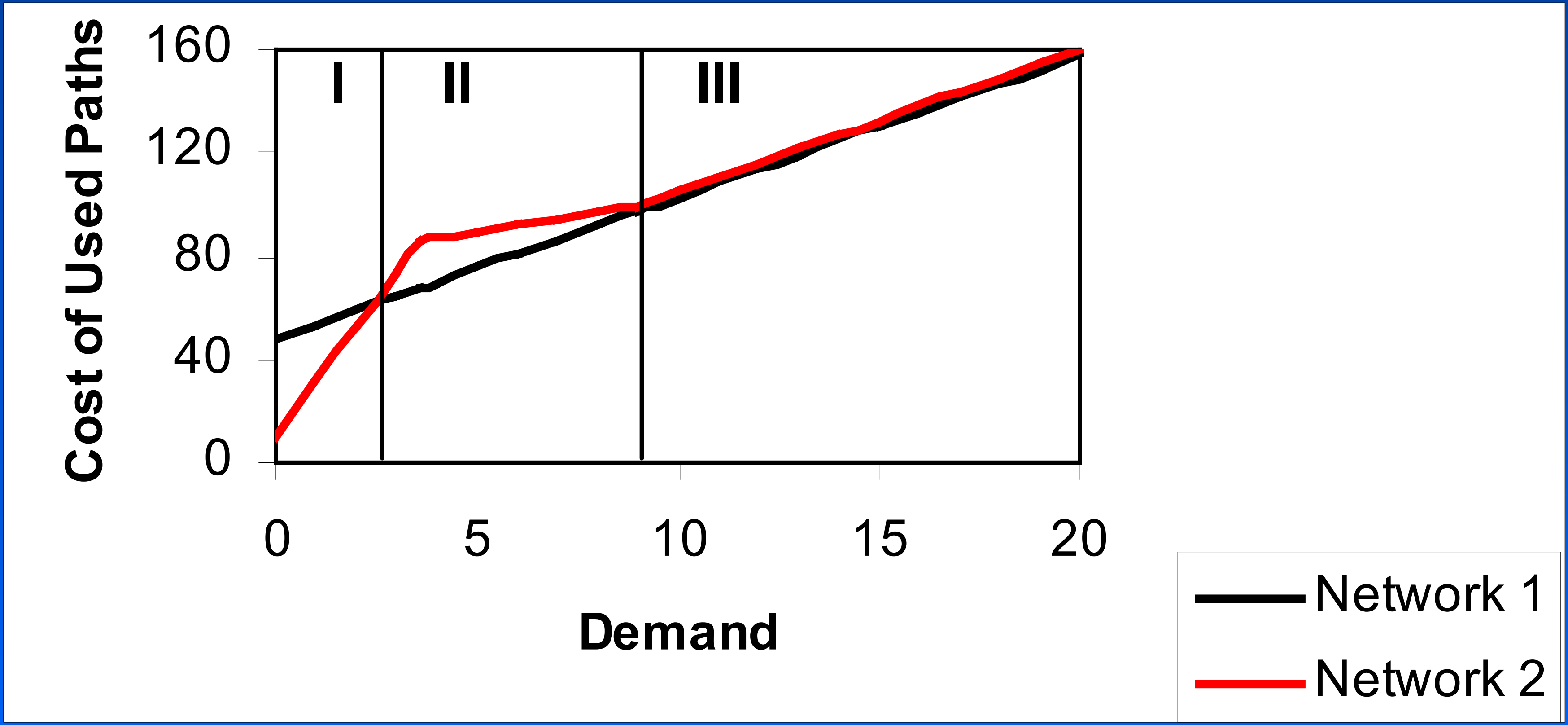
The Solution of the Braess Network with Added Link (Path) and Time-Varying Demands



In Demand Regime I, only the new path is used.

In Demand Regime II, the Addition of a New Link (Path) Makes Everyone Worse Off!

In Demand Regime III, only the original paths are used.



Network 1 is the Original Braess Network - Network 2 has the added link.

The new link is NEVER used after a certain demand is reached even if the demand approaches infinity.

Hence, in general, except for a limited range of demand, building the new link is a complete waste!

If the symmetry assumption does not hold for the user link costs functions, then the equilibrium conditions can no longer be reformulated as an associated optimization problem and the equilibrium conditions are formulated and solved as a *variational inequality problem!*

Smith (1979), Dafermos (1980)

VI Formulation of Transportation Network Equilibrium (Dafermos (1980), Smith (1979))

A traffic path flow pattern satisfies the above equilibrium conditions if and only if it satisfies the variational inequlity problem: determine $x^* \in K$, such that

$$\sum_p C_p(x^*) \times (x_p - x_p^*) \geq 0, \quad \forall x \in K.$$

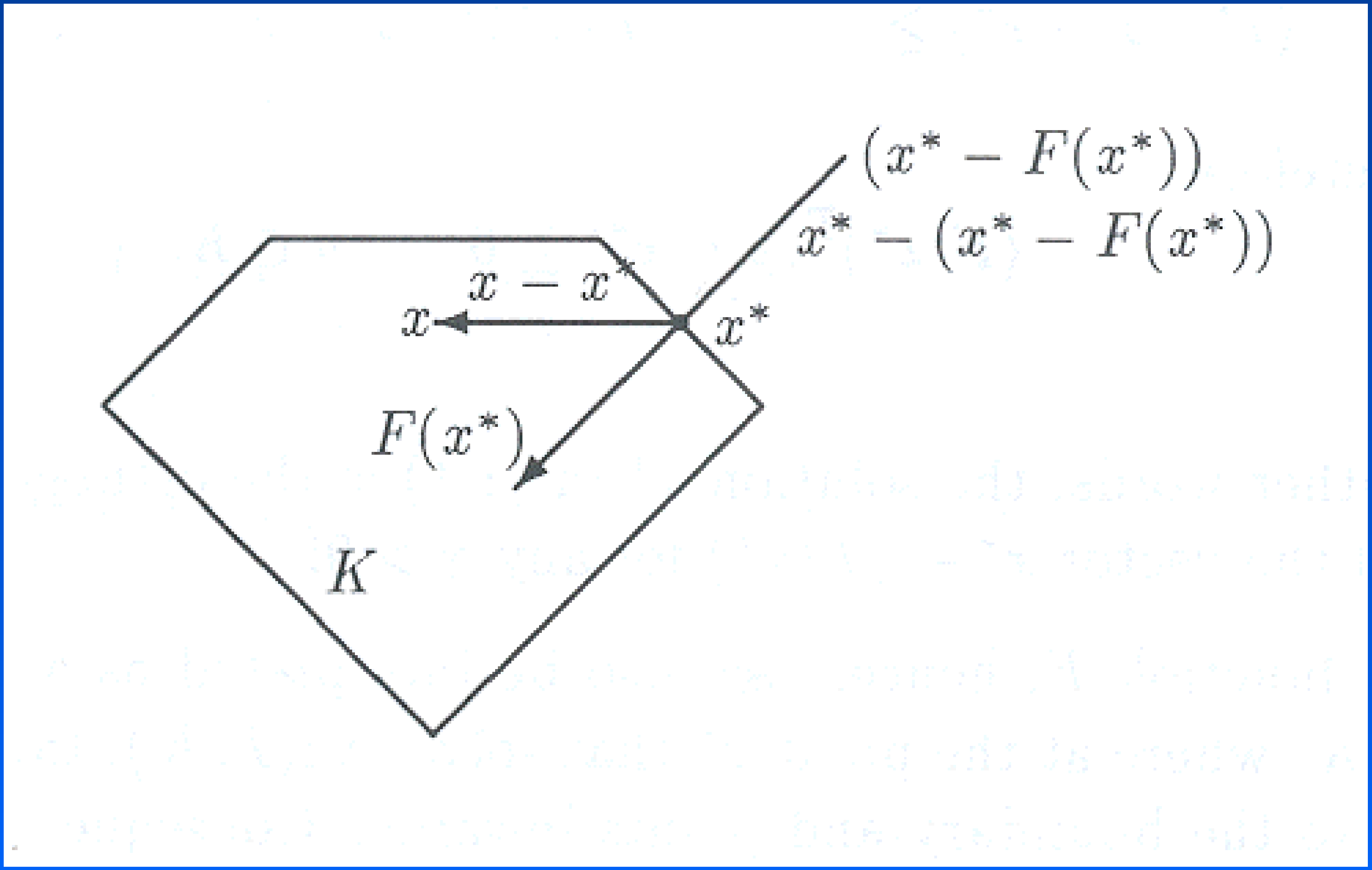
Finite-dimensional variational inequality theory has been applied to-date to the wide range of equilibrium problems noted above.

In particular, the finite-dimensional variational inequality problem is to determine $x^* \in K \subset R^n$ such that

$$\langle F(x^*), x - x^* \rangle \geq 0, \quad \forall x \in K,$$

where $\langle \cdot, \cdot \rangle$ denoted the inner product in R^n and K is closed and convex.

A Geometric Interpretation of a Variational Inequality



The variational inequality problem, contains, as special cases, such classical problems as:

- systems of equations
- optimization problems
- complementarity problems

and is also closely related to fixed point problems.

Hence, it is a unifying mathematical formulation for a variety of mathematical programming problems.

In particular, variational inequalities have been used to formulate such equilibrium problems as:

- transportation network equilibrium problems
- spatial price equilibrium problems
- oligopolistic market equilibrium problems operating under Nash equilibrium
- migration equilibrium problems
- a variety of financial equilibrium problems.

Moreover, all such problems have network structure, which can be further exploited for computational purposes.

Some Other Interesting Applications

- Telecommuting/Commuting Decision-Making
- Teleshopping/Shopping Decision-Making
- Supply Chain Networks with Electronic Commerce
- Financial Networks with Electronic Transactions
- Reverse Supply Chains with E-Cycling
- Knowledge Networks
- Energy Networks/Power Grids
- Social Networks integrated with Economic Networks

Transportation and Other Network Systems

The TNE Paradigm is the Unifying Paradigm for a Variety of Network Systems:

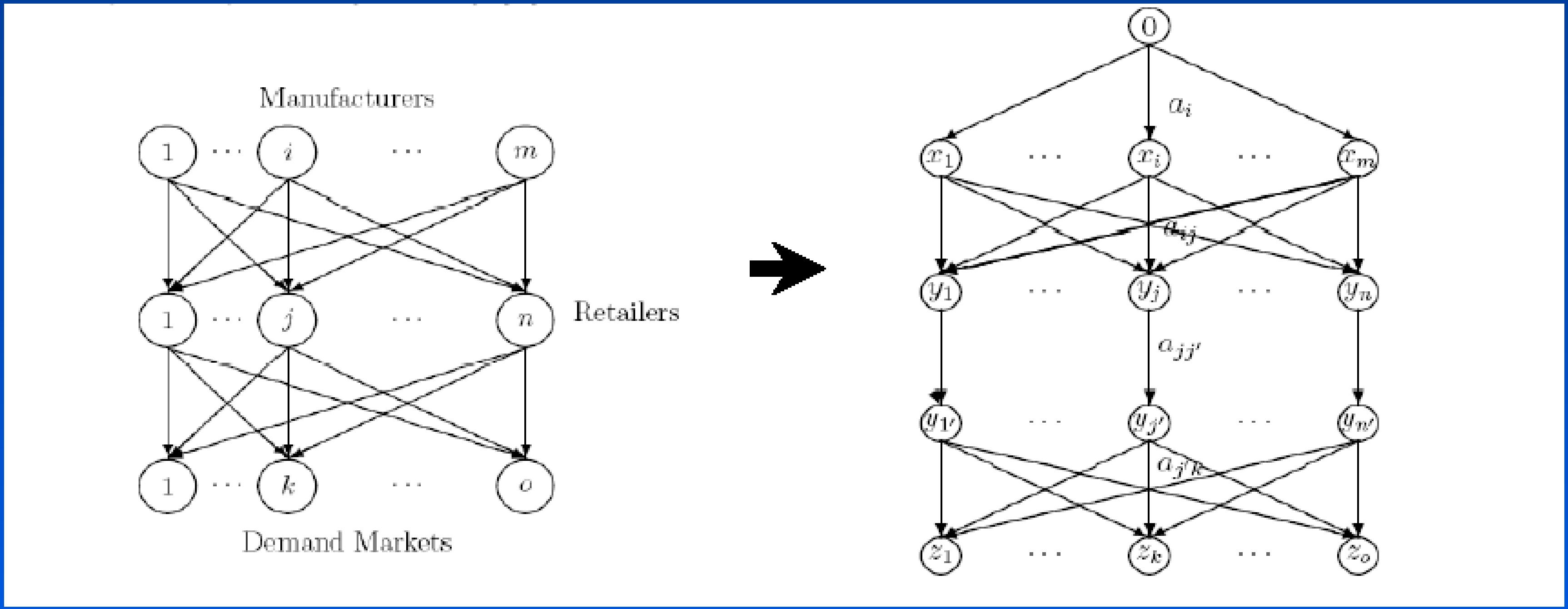
- Transportation Networks
- the Internet
- Financial Networks
- Supply Chains
- Electric Power Generation and Distribution Networks.

The TNE Paradigm can also capture multicriteria decision-making. Decision-makers (manufacturers, retailers, and/or consumers) in multitiered networks may seek to:

- maximize profits
- minimize pollution (emissions/waste)
- minimize risk

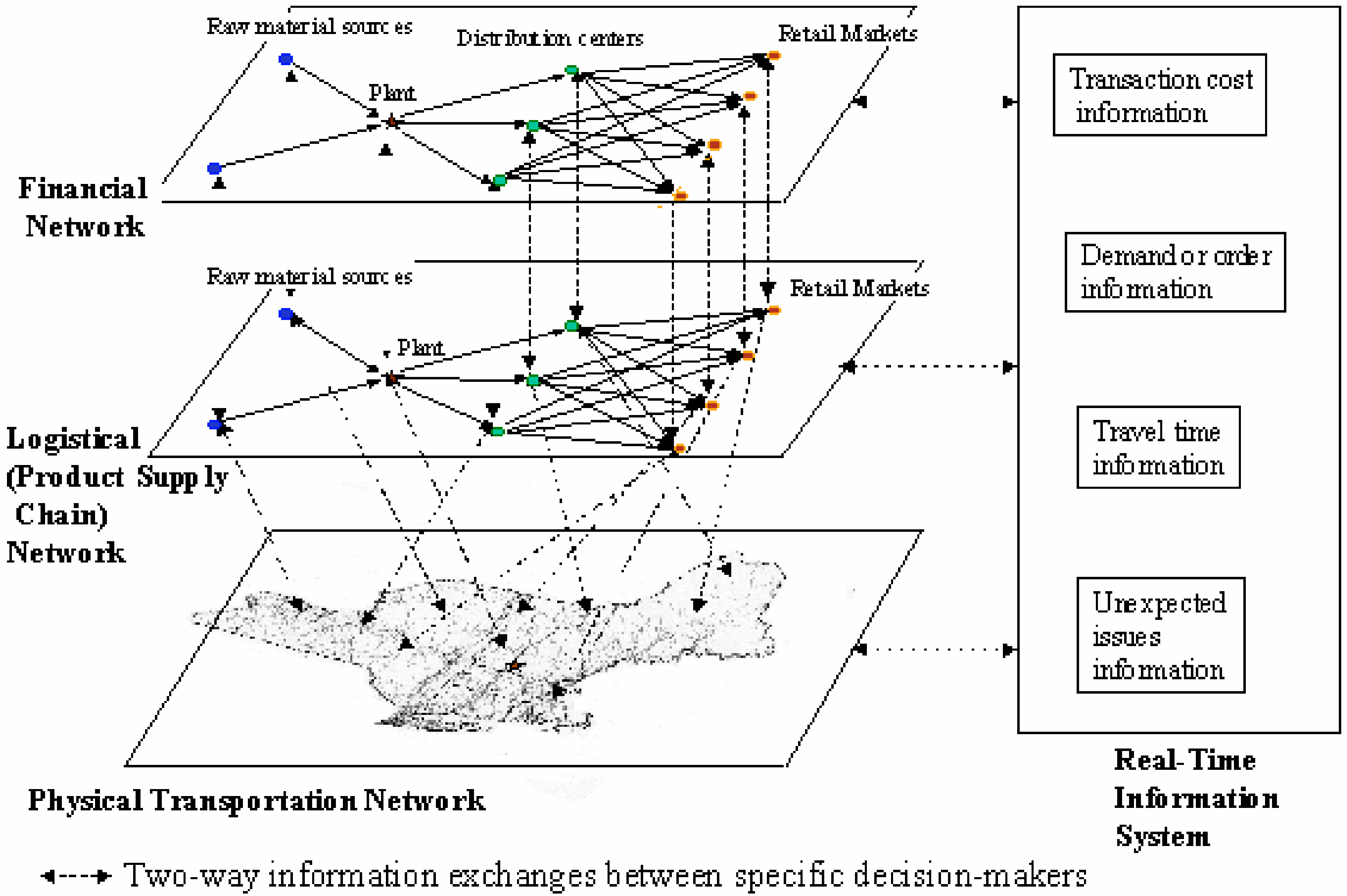
with individual weights associated with the different criteria.

The Equivalence of Supply Chains and Transportation Networks



Nagurney, *Transportation Research E* (2006).

Supply Chain -Transportation Supernetwork Representation



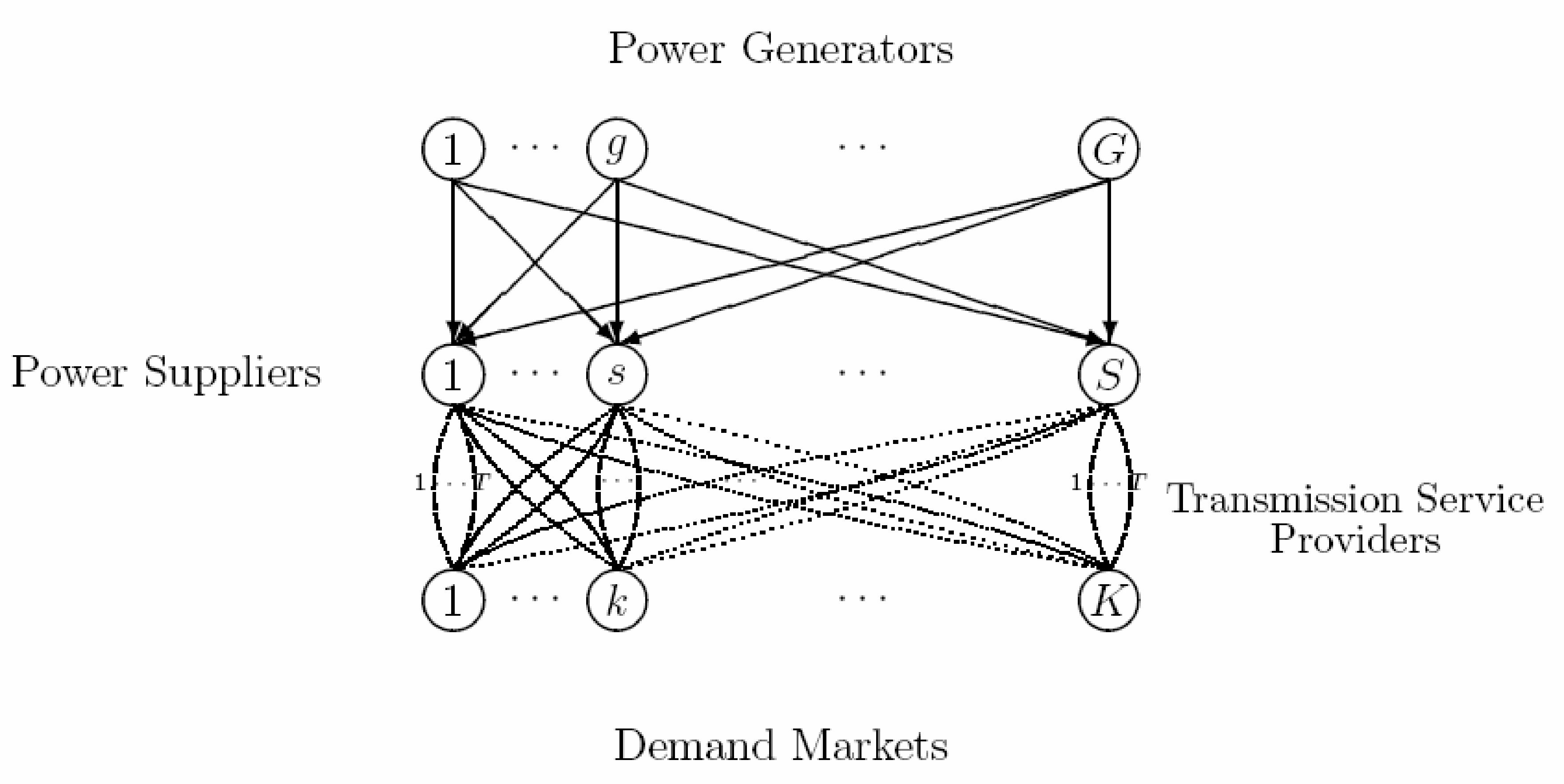
Nagurney, Ke, Cruz, Hancock, Southworth, *Environment and Planning B* (2002).

The fifth chapter of Beckmann, McGuire, and Winsten's book, ***Studies in the Economics of Transportation*** (1956) describes some *unsolved problems* including a single commodity network equilibrium problem that the authors imply could be generalized to capture electric power networks.

Specifically, they asked whether electric power generation and distribution networks can be reformulated as transportation network equilibrium problems.

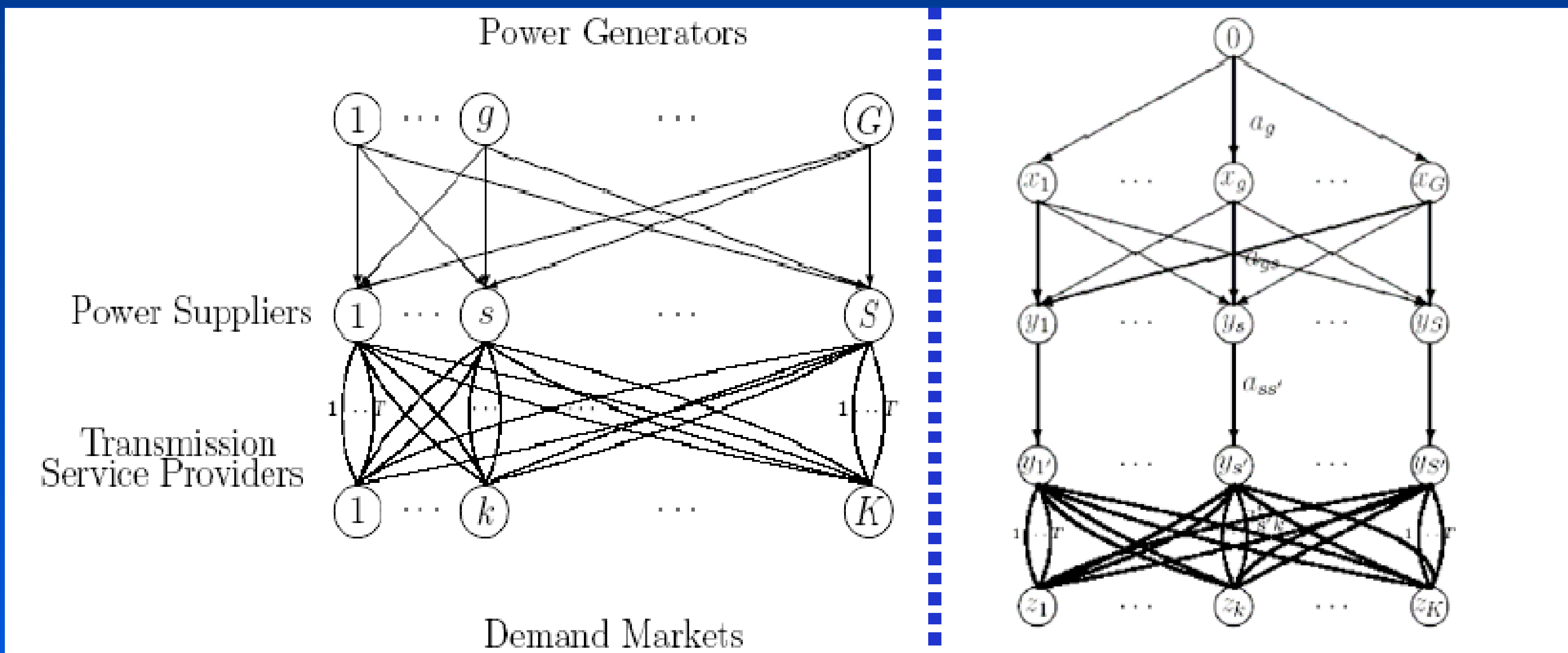
Electric Power Supply Chains

The Electric Power Supply Chain Network



Nagurney and Matsypura, *Proceedings of the CCCT* (2004).

The Transportation Network Equilibrium Reformulation of Electric Power Supply Chain Networks

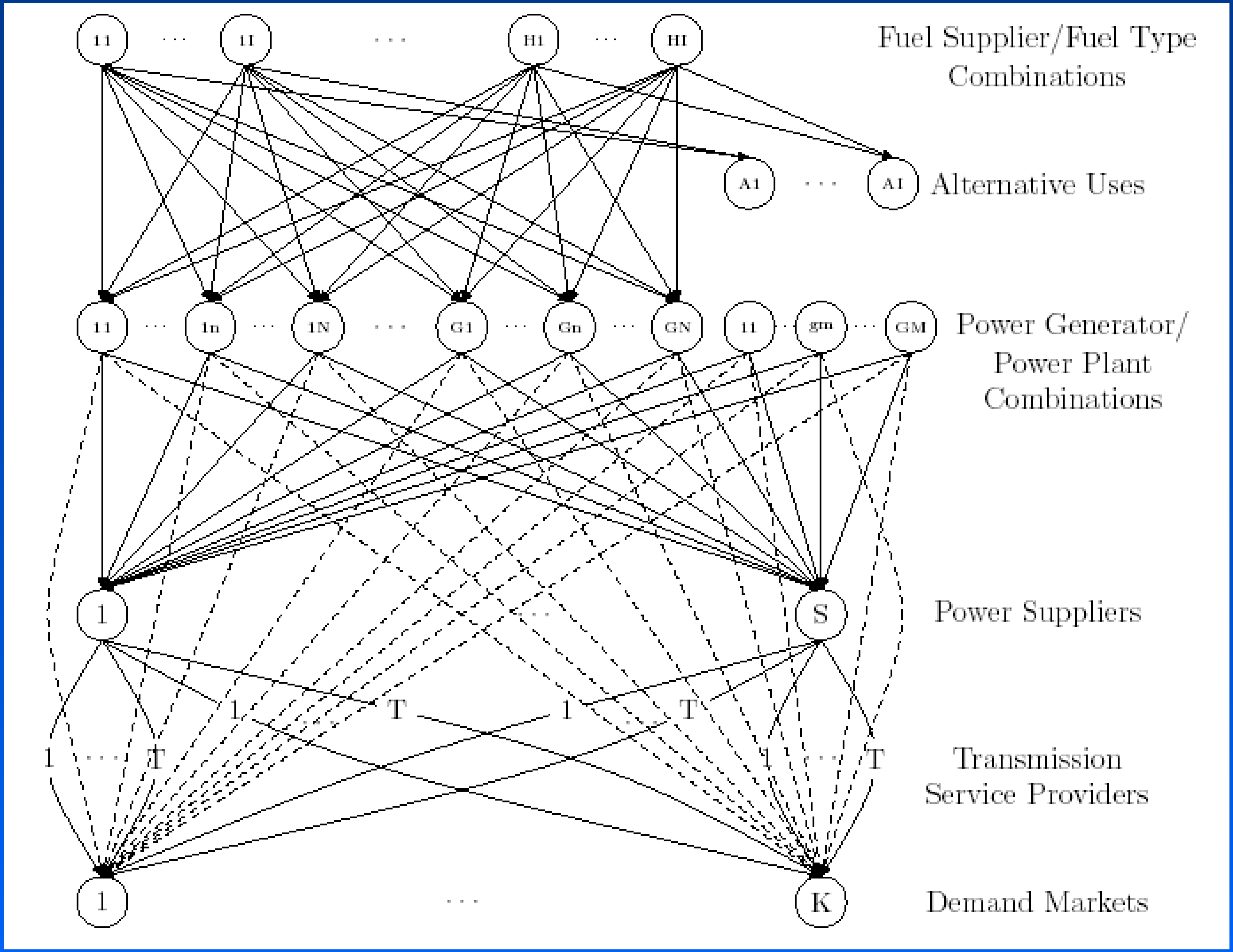


Electric Power Supply Network

Transportation Chain Network

Nagurney, Liu, Cojocaru, and Daniele, *Transportation Research E* (2007).

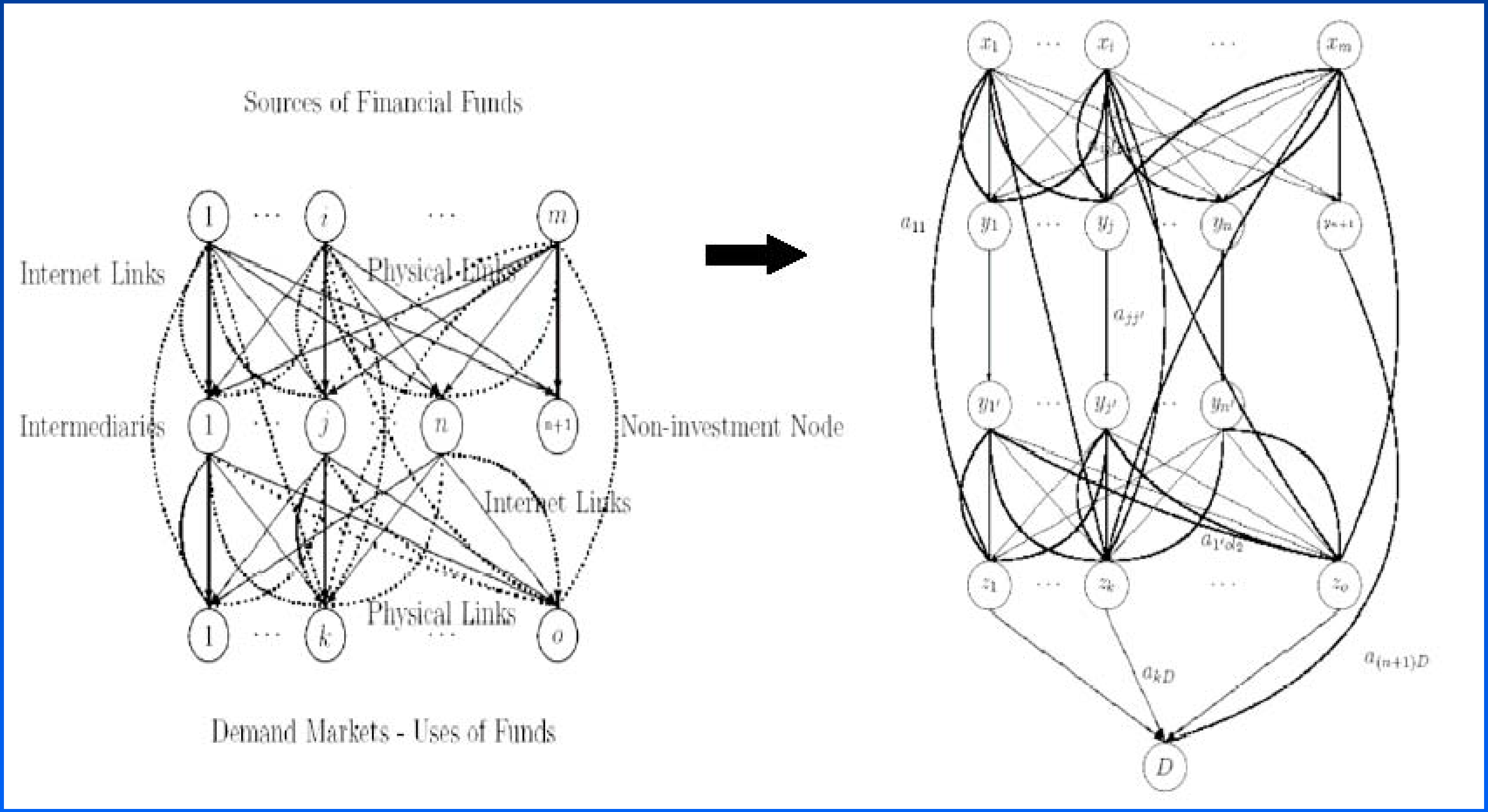
Electric Power Supply Chain Network with Fuel Suppliers



Matsypura, Nagurney, and Liu, *International Journal of Emerging Power Systems* (2007).

In 1952, Copeland wondered whether money flows like water or electricity.

The Transportation Network Equilibrium Reformulation of the Financial Network Equilibrium Model with Intermediation



Liu and Nagurney, *Computational Management Science* (2007).

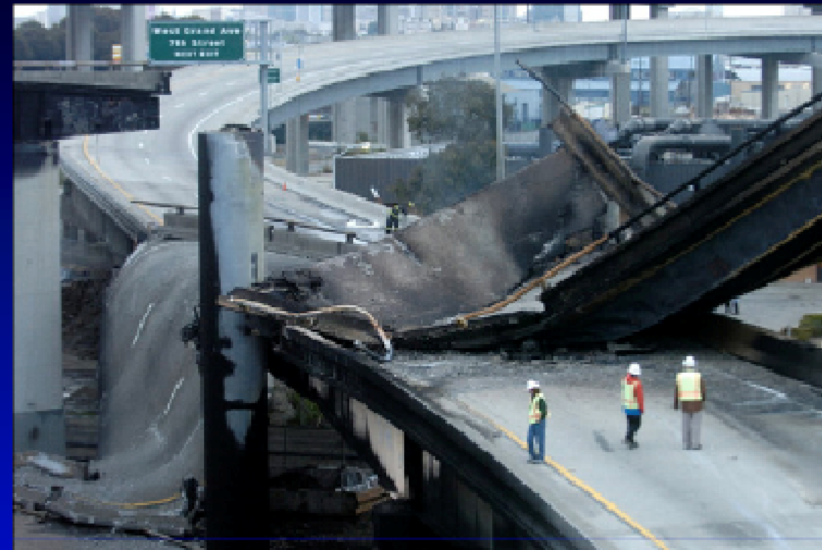
We have shown that *money* as well as *electricity* flow like *transportation* and have answered questions posed fifty years ago by Copeland and by Beckmann, McGuire, and Winsten!

Recent disasters have demonstrated the importance and the vulnerability of network systems.

Examples:

- 9/11 Terrorist Attacks, September 11, 2001;
- The biggest blackout in North America, August 14, 2003;
- Two significant power outages in September 2003 -- one in the UK and the other in Italy and Switzerland;
- Hurricane Katrina, August 23, 2005;
- The Minneapolis I35 Bridge Collapse, August 1, 2007;
- The cable destruction in the Mediterranean, January 30, 2008.

Disasters in Transportation Networks



www.salem-news.com

www.boston.com

Disasters in Electric Power Networks

media.collegepublisher.com

www.cellar.org

www.crh.noaa.gov

Disasters in Communication Networks

www.tx.mb21.co.uk

www.w5jgv.com

www.wirelessestimator.com

Recent Literature on Network Vulnerability

- Latora and Marchiori (2001, 2002, 2004)
- Holme, Kim, Yoon and Han (2002)
- Taylor and D'este (2004)
- Murray-Tuite and Mahmassani (2004)
- Chassin and Posse (2005)
- Barrat, Barthélemy and Vespignani (2005)
- Sheffi (2005)
- Dall'Asta, Barrat, Barthélemy and Vespignani (2006)
- Jenelius, Petersen and Mattson (2006)
- Taylor and D'Este (2007)

Our Research on Network Efficiency, Vulnerability, and Robustness

A Network Efficiency Measure for Congested Networks, Nagurney and Qiang, *Europhysics Letters,* **79**, August (2007).

A Transportation Network Efficiency Measure that Captures Flows, Behavior, and Costs with Applications to Network Component Importance Identification and Vulnerability, Nagurney and Qiang, *Proceedings of the POMS 18th Annual Conference,* Dallas, Texas (2007).

A Network Efficiency Measure with Application to Critical Infrastructure Networks, Nagurney and Qiang, *Journal of Global Optimization,* ***40*** (2008).

Robustness of Transportation Networks Subject to Degradable Links, Nagurney and Qiang, *Europhysics Letters,* **80**, December (2007).

A Unified Network Performance Measure with Importance Identification and the Ranking of Network Components, Qiang and Nagurney, *Optimization Letters,* ***2*** (2008).

A New Network Performance/Efficiency Measure with Applications to a Variety of Network Systems

The Nagurney and Qiang (N-Q) Network Efficiency Measure

The network performance/efficiency measure $\mathcal{E}(G,d)$, for a given network topology G and fixed demand vector d, is defined as

$$\mathcal{E}(G,d) = \frac{\sum_{w \in W} \frac{d_w}{\lambda_w}}{n_W},$$

where n_w is the number of O/D pairs in the network and λ_w is the equilibrium disutility for O/D pair w.

Nagurney and Qiang, *Europhysics Letters,* **79** (2007).

Importance of a Network Component

Definition: Importance of a Network Component

The importance, *I(g),* of a network component *gεG* is measured by the relative network efficiency drop after *g* is removed from the network:

$$I(g) = \frac{\triangle \mathcal{E}}{\mathcal{E}} = \frac{\mathcal{E}(G,d) - \mathcal{E}(G-g,d)}{\mathcal{E}(G,d)},$$

where *G-g* is the resulting network after component g is removed.

The Latora and Marchiori (L-M) Network Efficiency Measure

Definition: The L-M Measure

The network performance/efficiency measure, *E(G)* for a given network topology, *G*, is defined as:

$$E(G) = \frac{1}{n(n-1)} \sum_{i \neq j \in G} \frac{1}{d_{ij}}$$

where *n* is the number of nodes in the network and d_{ij} is the shortest path length between node *i* and node *j*.

The L-M Measure vs. the N-Q Measure

Theorem:

If positive demands exist for all pairs of nodes in the network, *G*, and each of demands is equal to 1, and if d_{ij} is set equal to λ_w, where $w=(i,j)$, for all $w \varepsilon W$, then the N-Q and L-M network efficiency measures are one and the same.

The Approach to Study the Importance of Network Components

The elimination of a link is treated in the N-Q network efficiency measure by removing that link while the removal of a node is managed by removing the links entering and exiting that node.

In the case that the removal results in no path connecting an O/D pair, we simply assign the demand for that O/D pair to an abstract path with a cost of infinity. Hence, our measure is well-defined even in the case of disconnected networks.

The measure generalizes the Latora and Marchiori network measure for complex networks.

Example 1

Assume a network with two O/D pairs:
$\boldsymbol{w_1}$=(1,2) and $\boldsymbol{w_2}$=(1,3) with demands:
$\boldsymbol{d_{w_1}}$=100 and $\boldsymbol{d_{w_2}}$=20.

The paths are:

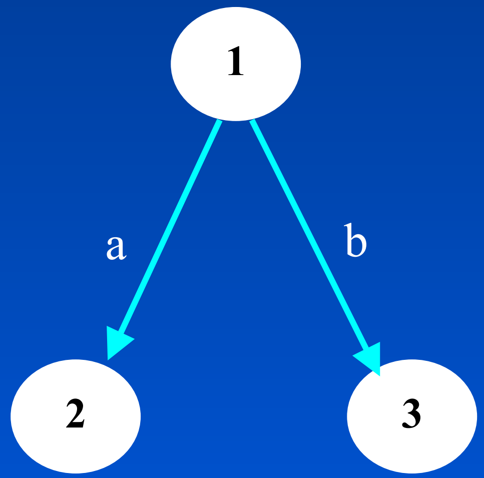
for $\boldsymbol{w_1}$, $\boldsymbol{p_1=a}$; for $\boldsymbol{w_2}$, $\boldsymbol{p_2=b}$.

The equilibrium path flows are:

$\boldsymbol{x_{p_1}^*}$= 100, $\boldsymbol{x_{p_2}^*}$=20.

The equilibrium path travel costs are:

$\boldsymbol{C_{p_1}=C_{p_2}}$=20.



$c_a(f_a)=0.01f_a+19$
$c_b(f_b)=0.05f_b+19$

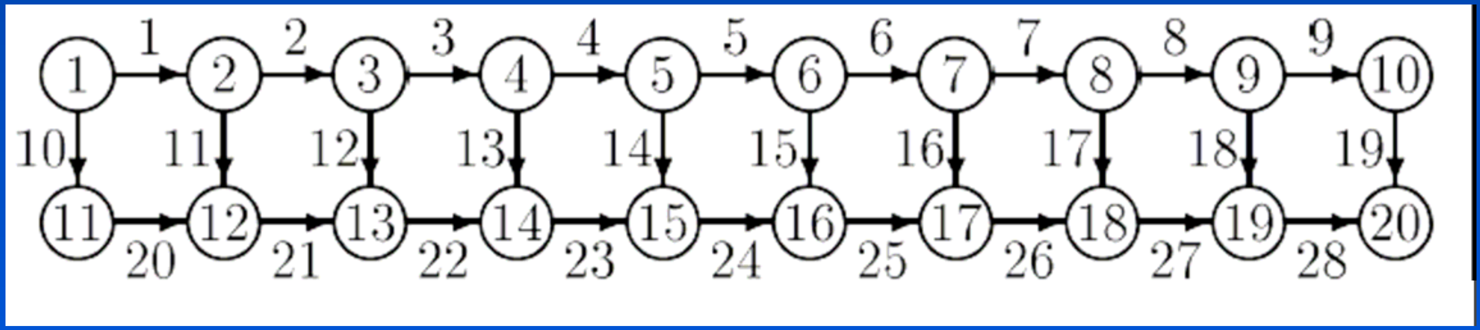
Importance and Ranking of Links and Nodes

Link	Importance Value from Our Measure	Importance Ranking from Our Measure
a	0.8333	1
b	0.1667	2

Node	Importance Value from Our Measure	Importance Ranking from Our Measure
1	1	1
2	0.8333	2
3	0.1667	3

Example 2

The network is given by:



From: Nagurney,

Transportation Research B (1984)

w_1=(1,20) w_2=(1,19)

d_{w_1} = 100 d_{w_2} = 100

Example 2: Link Cost Functions

Link a	Link Cost Function $c_a(f_a)$
1	$.00005 f_1^4 + 5 f_1 + 500$
2	$.00003 f_2^4 + 4 f_2 + 200$
3	$.00005 f_3^4 + 3 f_3 + 350$
4	$.00003 f_4^4 + 6 f_4 + 400$
5	$.00006 f_5^4 + 6 f_5 + 600$
6	$7 f_6 + 500$
7	$.00008 f_7^4 + 8 f_7 + 400$
8	$.00004 f_8^4 + 5 f_8 + 650$
9	$.00001 f_9^4 + 6 f_9 + 700$
10	$4 f_{10} + 800$
11	$.00007 f_{11}^4 + 7 f_{11} + 650$
12	$8 f_{12} + 700$
13	$.00001 f_{13}^4 + 7 f_{13} + 600$
14	$8 f_{14} + 500$

Link a	Link Cost Function $c_a(f_a)$
15	$.00003 f_{15}^4 + 9 f_{15} + 200$
16	$8 f_{16} + 300$
17	$.00003 f_{17}^4 + 7 f_{17} + 450$
18	$5 f_{18} + 300$
19	$8 f_{19} + 600$
20	$.00003 f_{20}^4 + 6 f_{20} + 300$
21	$.00004 f_{21}^4 + 4 f_{21} + 400$
22	$.00002 f_{22}^4 + 6 f_{22} + 500$
23	$.00003 f_{23}^4 + 9 f_{23} + 350$
24	$.00002 f_{24}^4 + 8 f_{24} + 400$
25	$.00003 f_{25}^4 + 9 f_{25} + 450$
26	$.00006 f_{26}^4 + 7 f_{26} + 300$
27	$.00003 f_{27}^4 + 8 f_{27} + 500$
28	$.00003 f_{28}^4 + 7 f_{28} + 650$

Algorithms for Solution

The projection method (cf. Dafermos (1980) and Nagurney (1999)) embedded with the equilibration algorithm of Dafermos and Sparrow (1969) was used for the computations.

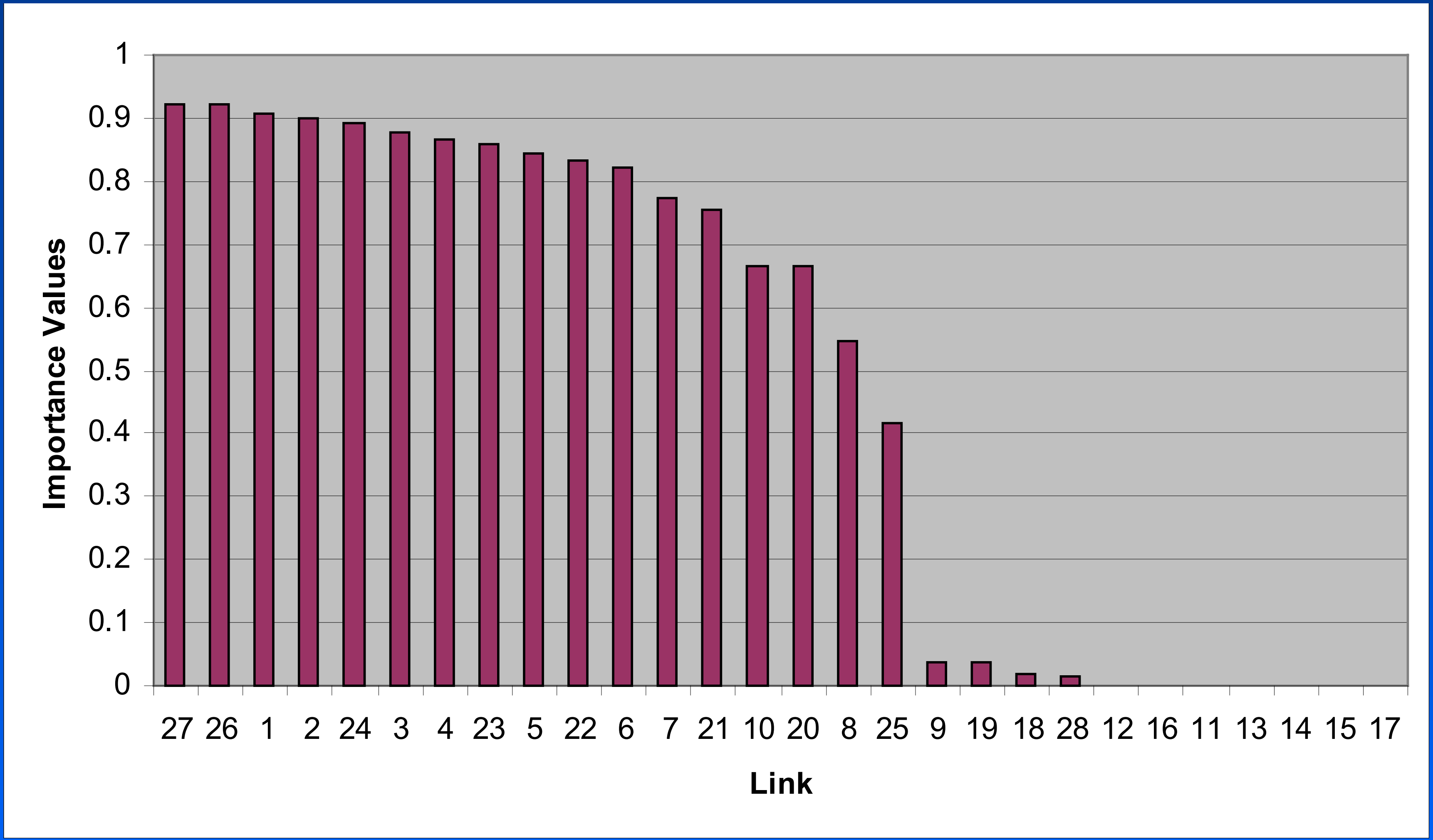
In addition, the column generation method of Leventhal, Nemhauser, and Trotter (1973) was implemented to generate paths, as needed, in the case of the large-scale Sioux Falls network example.

Example 2: Importance and Ranking of Links

Link a	Importance Value	Importance Ranking
1	0.9086	3
2	0.8984	4
3	0.8791	6
4	0.8672	7
5	0.8430	9
6	0.8226	11
7	0.7750	12
8	0.5483	15
9	0.0362	17
10	0.6641	14
11	0.0000	22
12	0.0006	20
13	0.0000	22
14	0.0000	22

Link a	Importance Value	Importance Ranking
15	0.0000	22
16	0.0001	21
17	0.0000	22
18	0.0175	18
19	0.0362	17
20	0.6641	14
21	0.7537	13
22	0.8333	10
23	0.8598	8
24	0.8939	5
25	0.4162	16
26	0.9203	2
27	0.9213	1
28	0.0155	19

Example 2: Link Importance Rankings

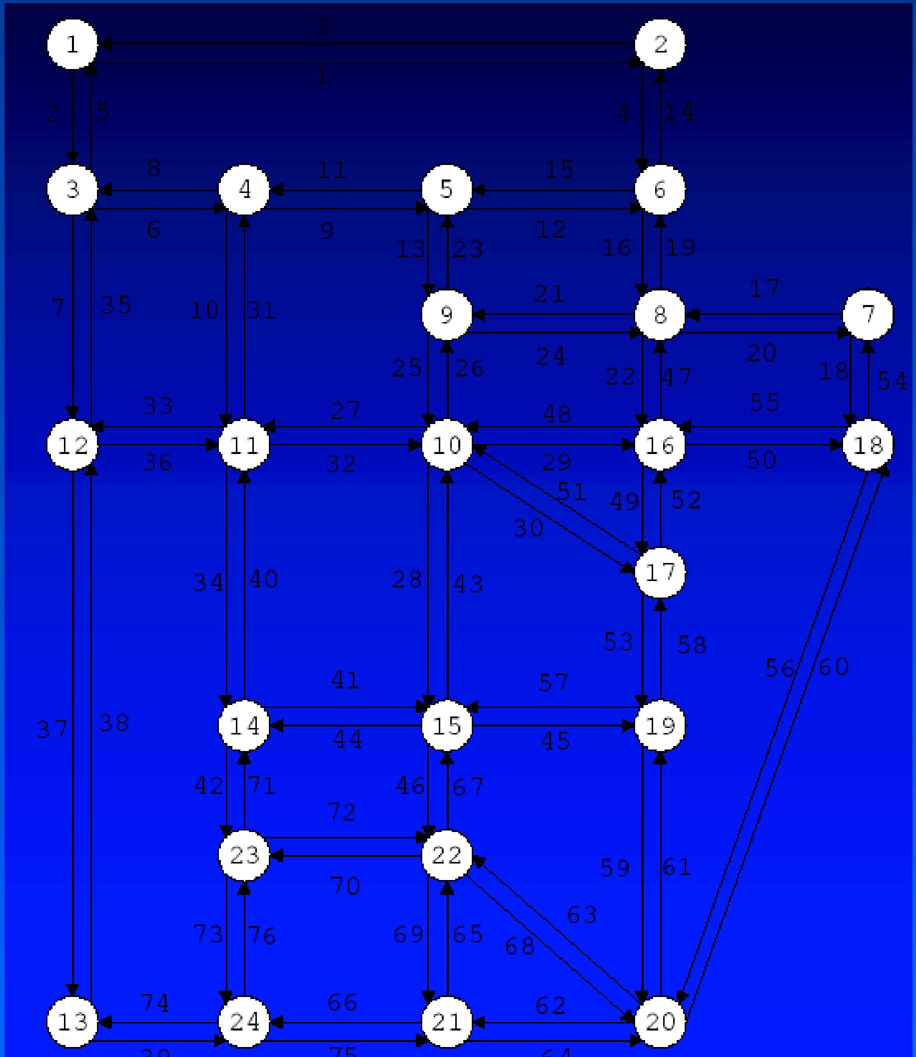


Example 3 - Sioux Falls Network

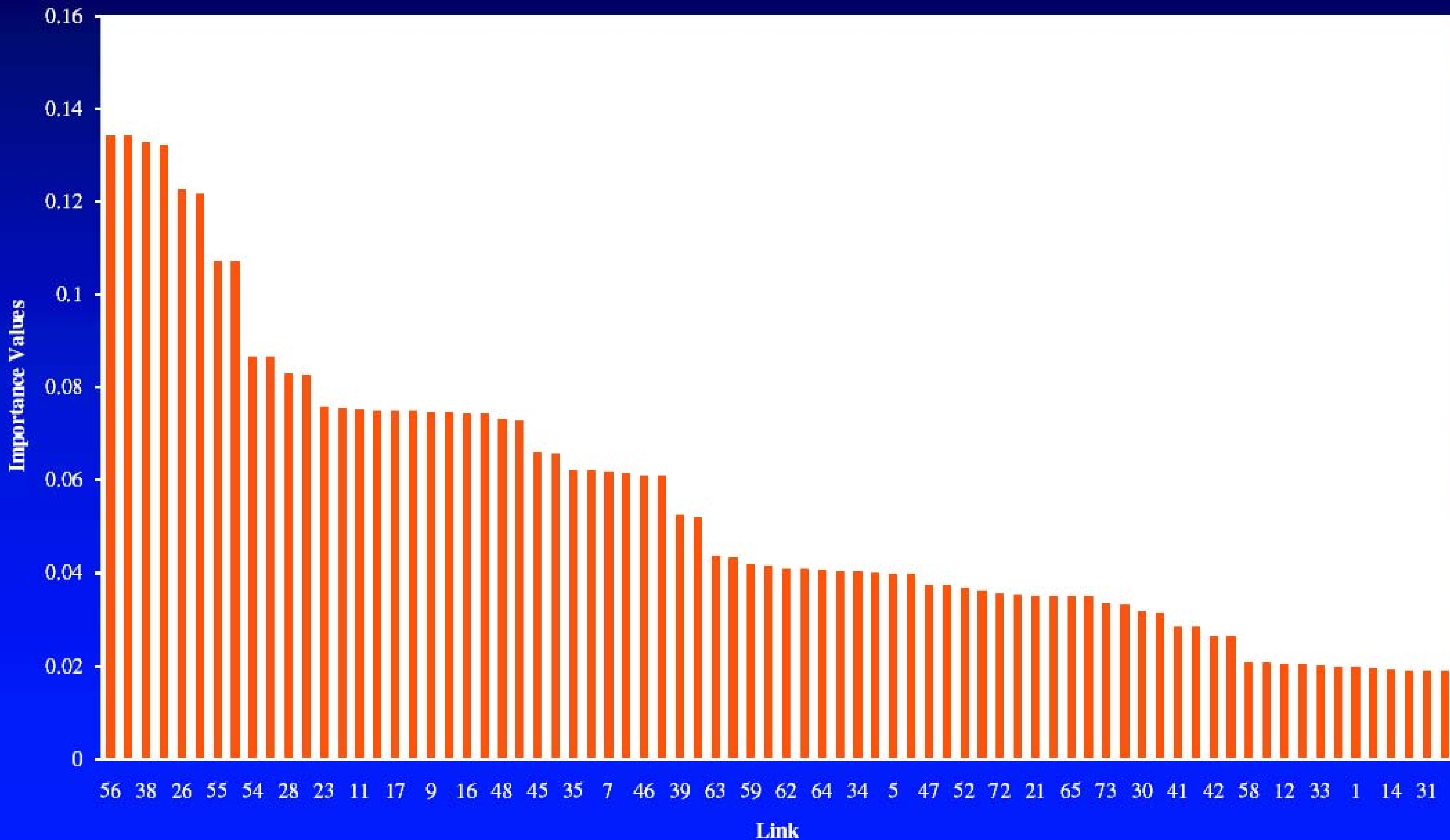
The network data are from LeBlanc, Morlok, and Pierskalla (1975).

The network has 528 O/D pairs, 24 nodes, and 76 links.

The user link cost functions are of Bureau of Public Roads (BPR) form.



Example 3 - Sioux Falls Network Link Importance Rankings



The Advantages of the N-Q Network Efficiency Measure

- The measure captures demands, flows, costs, and behavior of users, in addition to network topology;
- The resulting importance definition of network components is applicable and well-defined even in the case of disconnected networks;
- It can be used to identify the importance (and ranking) of either nodes, or links, or both; and
- It can be applied to assess the efficiency/performance of a wide range of network systems.
- It is applicable also to elastic demand networks (Qiang and Nagurney, *Optimization Letters* (2008)).
- It has been extended to dynamic networks (Nagurney and Qiang, *Netnomics*, in press).

Motivation for Research on Transportation Network Robustness

According to the American Society of Civil Engineering:

Poor maintenance, natural disasters, deterioration over time, as well as unforeseen attacks now lead to estimates of $94 billion in the US in terms of needed repairs for roads alone.

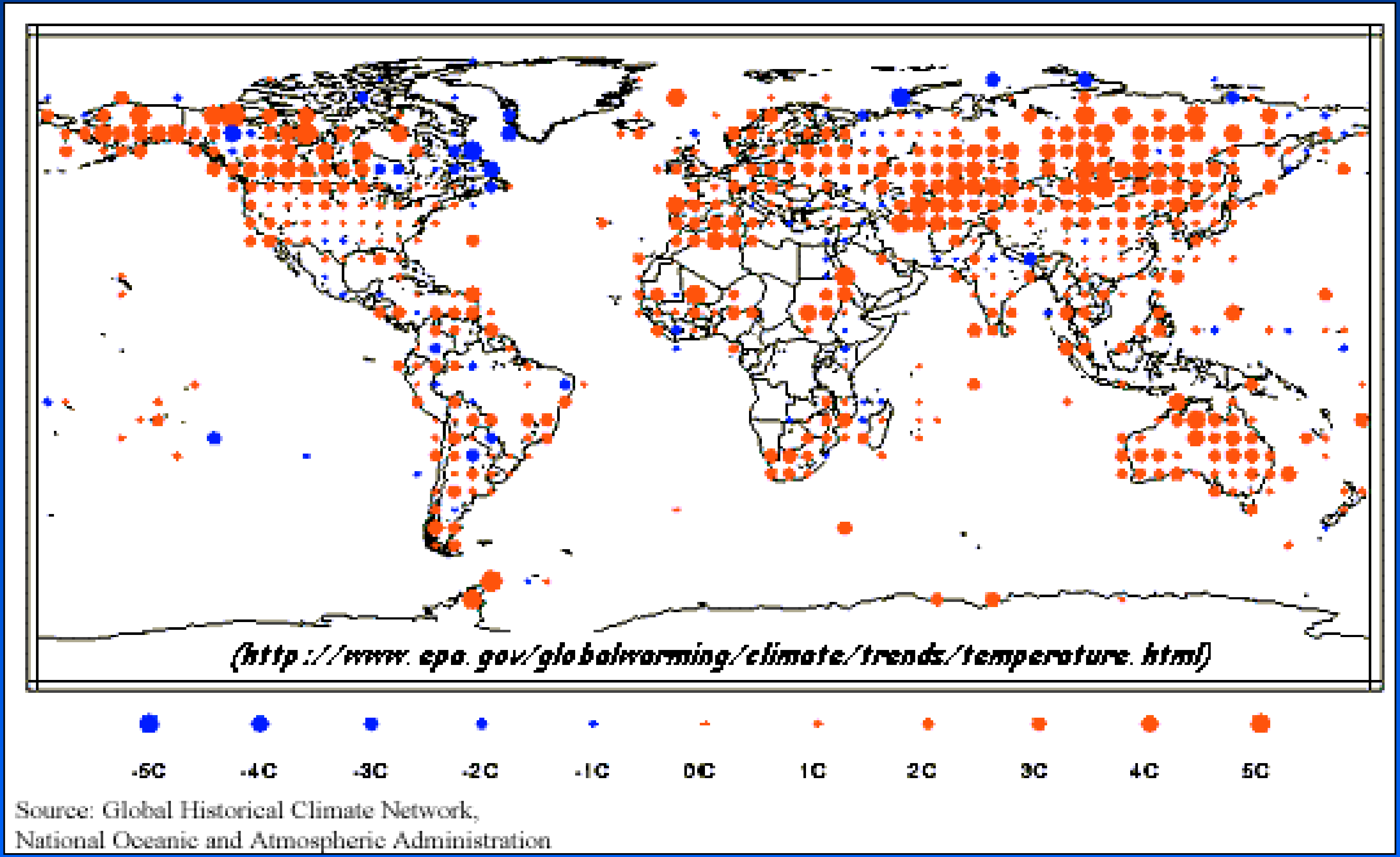
Poor road conditions in the United States cost US motorists $54 billion in repairs and operating costs annually.

Transportation Network Robustness

The focus of the robustness of networks (and complex networks) has been on the impact of different network measures when facing the removal of nodes on networks.

We focus on the *degradation of links through reductions in their capacities* and the effects on the induced travel costs in the presence of known travel demands and different functional forms for the links.

Global Annual Mean Temperature Trend 1950-1999



Impacts of Climate Change on Transportation Infrastructure



Examples from Alaska (Smith and Lavasseur)

According to the European Environment Agency (2004), since 1990 the *annual number of extreme weather and climate related events has doubled*, in comparison to the previous decade. These events account for approximately 80% of all economic losses caused by catastrophic events. In the course of climate change, catastrophic events are projected to occur more frequently (see Schulz (2007)).

Schulz (2007) applied the Nagurney and Qiang (2007) network efficiency measure to a German highway system in order to identify the critical road elements and found that this measure provided more reasonable results than the measure of Taylor and D'Este (2007).

Robustness in Engineering and Computer Science

IEEE (1990) defined robustness as *the degree to which a system of component can function correctly in the presence of invalid inputs or stressful environmental conditions*.

Gribble (2001) defined system robustness as *the ability of a system to continue to operate correctly across a wide range of operational conditions, and to fail gracefully outside of that range*.

Schilllo et al. (2001) argued that robustness has to be studied *in relation to some definition of the performance measure*.

“Robustness” in Transportation

Sakakibara et al. (2004) proposed a topological index. The authors considered a transportation network to be robust if it is “dispersed” in terms of the number of links connected to each node.

Scott et al. (2005) examined transportation network robustness by analyzing the increase in the total network cost after removal of certain network components.

A New Approach to Transportation Network Robustness

The Transportation Network Robustness Measure

Nagurney and Qiang, Europhysics Letters, 80, December (2007)

The robustness measure $\mathcal{R}^\gamma$ for a transportation network G with the vector of demands d, the vector of user link cost functions c, and the vector of link capacities u is defined as the relative performance retained under a given uniform capacity retention ratio γ ($\gamma \in (0,1]$) so that the new capacities are given by γu. Its mathematical definition is given as:

$$\mathcal{R}^\gamma = \mathcal{R}(G, c, d, \gamma, u) = \frac{\mathcal{E}^\gamma}{\mathcal{E}} \times 100\%$$

where $\mathcal{E}$ and $\mathcal{E}^\gamma$ are the network performance measures with the original capacities and the remaining capacities, respectively.

We utilize BPR functions user link cost functions c for the robustness analysis.

Simple Example

Assume a network with one O/D pair: w_1=(1,2) with demand given by d_{w1}=10.

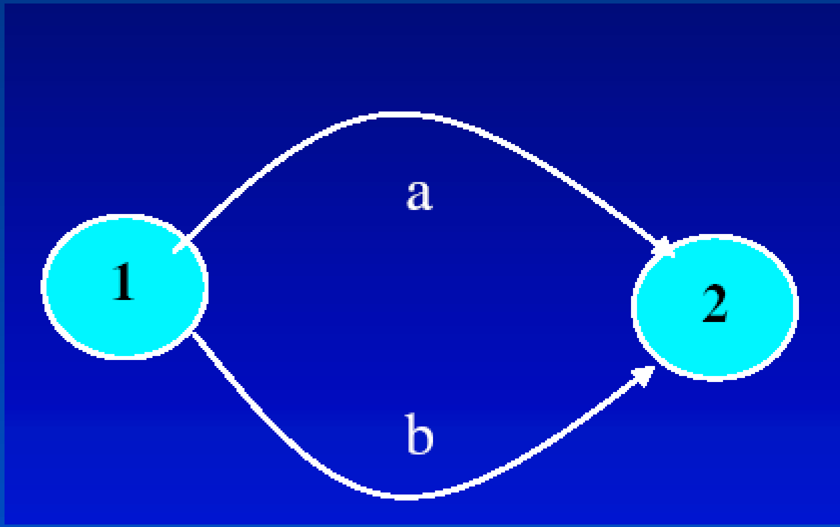
The paths are: $p_1=a$ and $p_2=b$.

In the BPR link cost function, k=1 and β=4; t_a^0=10 and t_a^0=1.

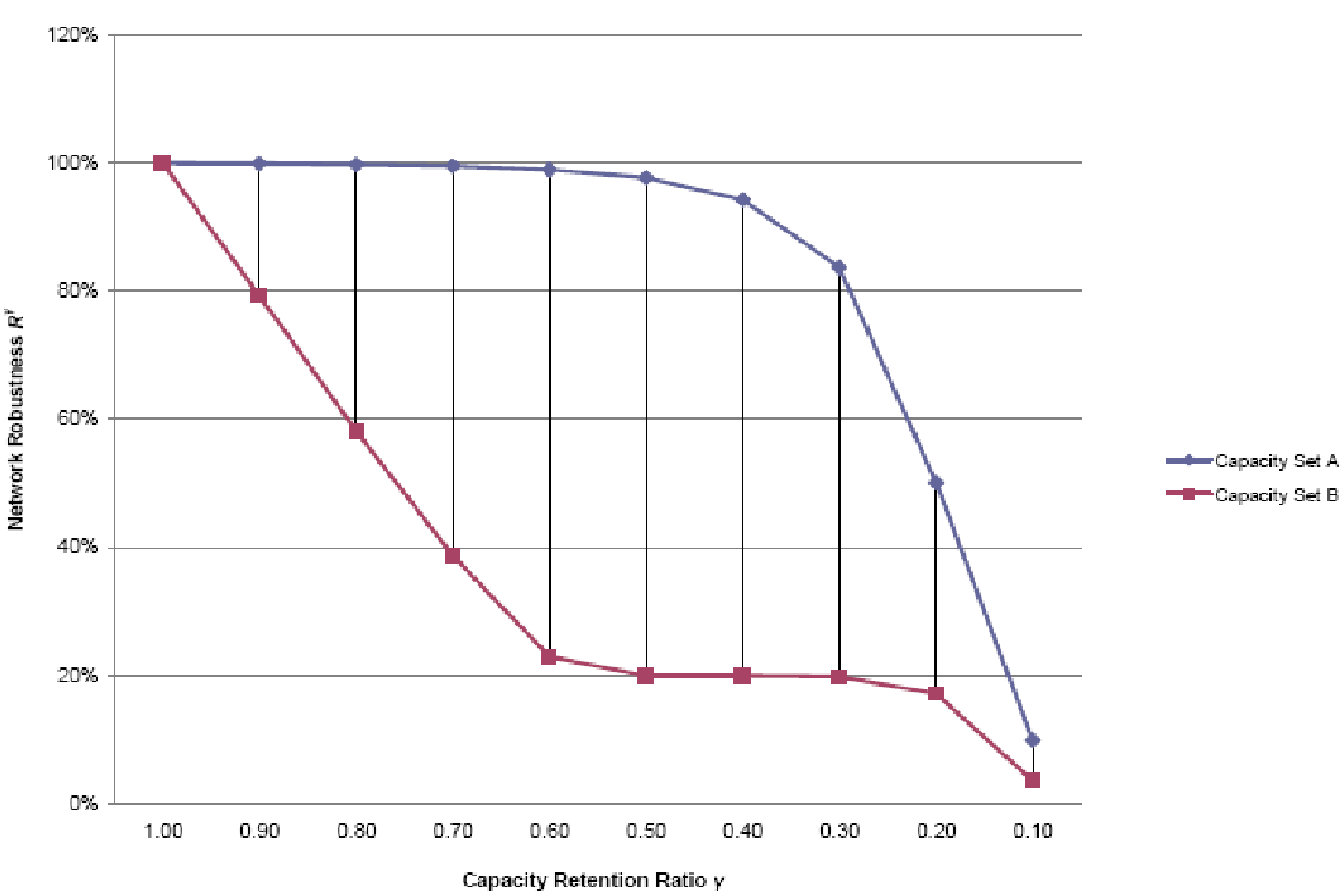
Assume that there are two sets of capacities:

Capacity Set A, where $u_a=u_b$=50;

Capacity Set B, where u_a=50 and u_b=10.



Robustness of the Simple Network



Example: Braess Network with BPR Functions

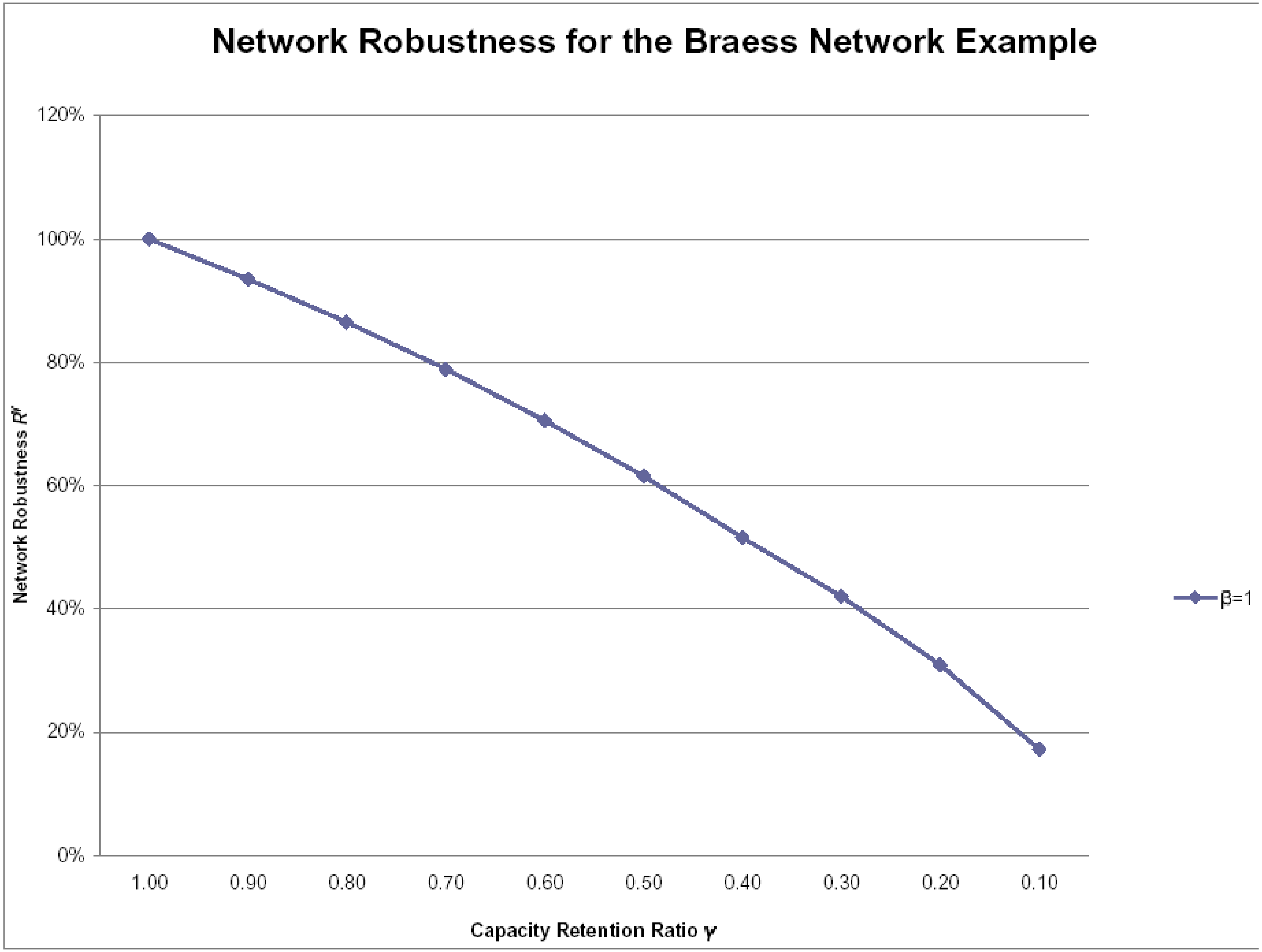
Instead of using the original cost functions, we construct a set of BPR functions as below under which the Braess Paradox still occurs. The new demand is 110.

$$c_a(f_a) = 1 + (\frac{f_a}{20})^\beta, \quad c_b(f_b) = 50(1 + (\frac{f_b}{50})^\beta),$$

$$c_c(f_c) = 50(1 + (\frac{f_b}{50})^\beta), \quad c_d(f_d) = 1 + (\frac{f_d}{20})^\beta,$$

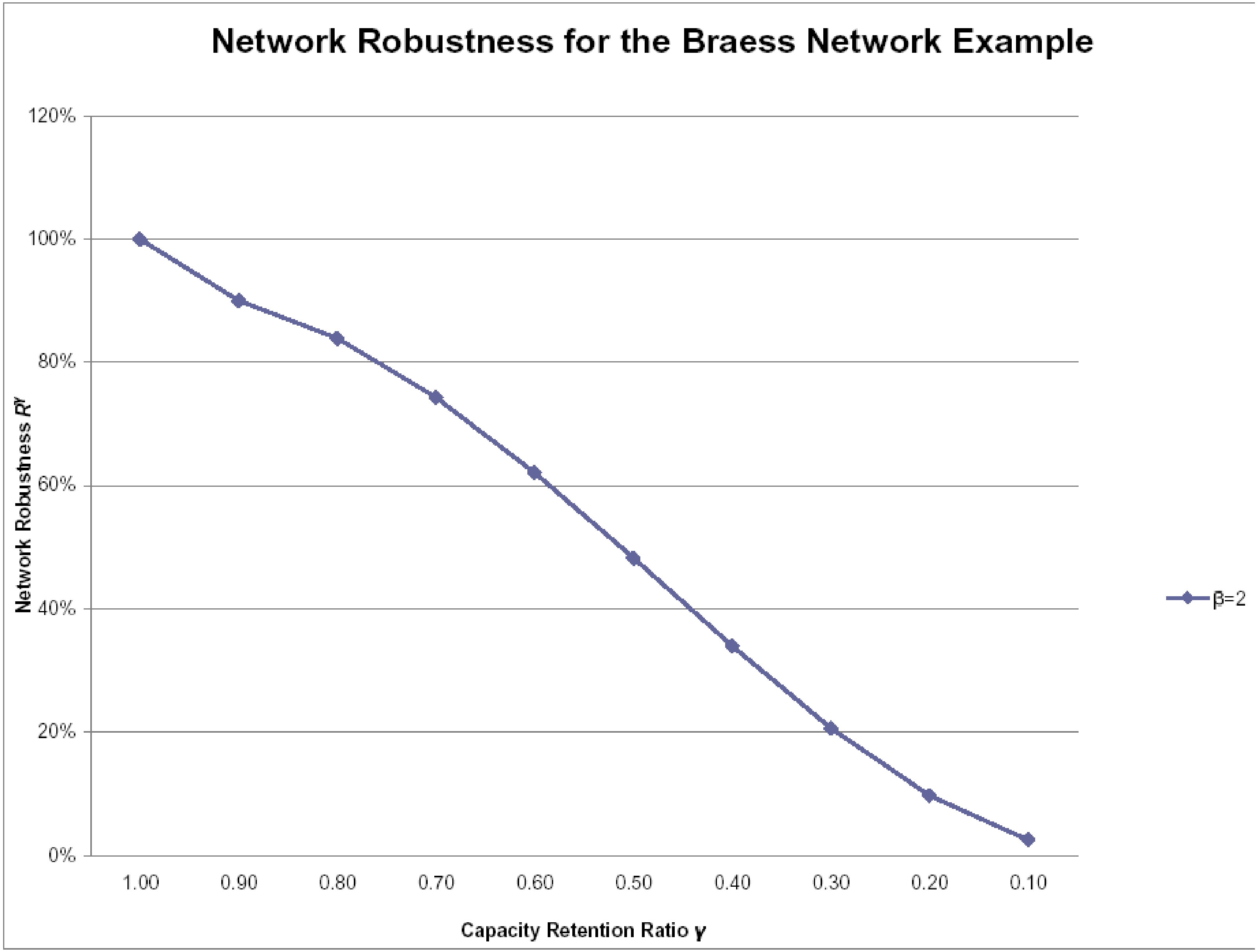
$$c_e(f_e) = 10(1 + (\frac{f_e}{100})^\beta).$$

β= 1

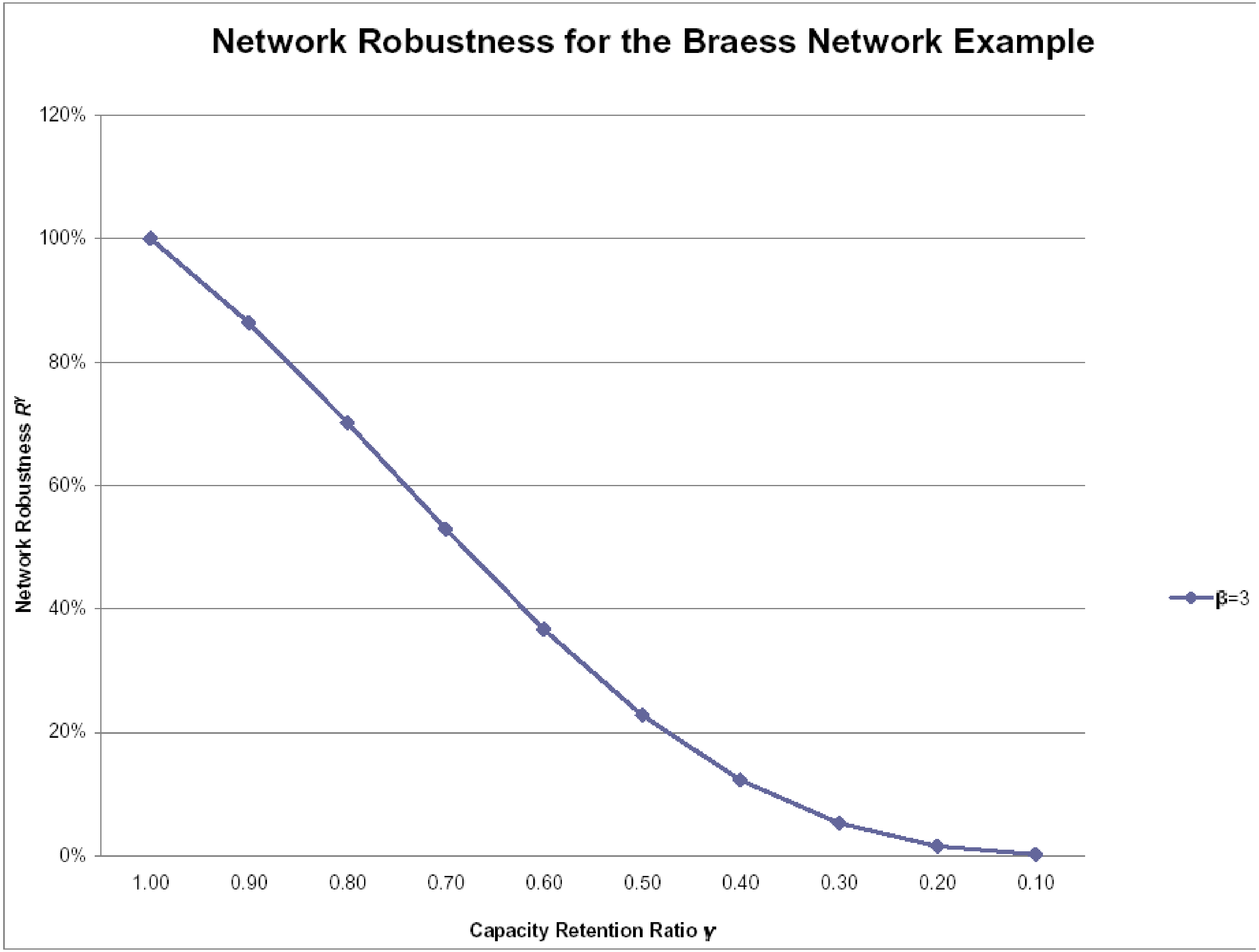


Network Robustness for the Braess Network Example
120%
100%
80%
60%
40%
20%
0%
Network Robustness R^{γ}
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
Capacity Retention Ratio γ
β=1

$\beta = 2$

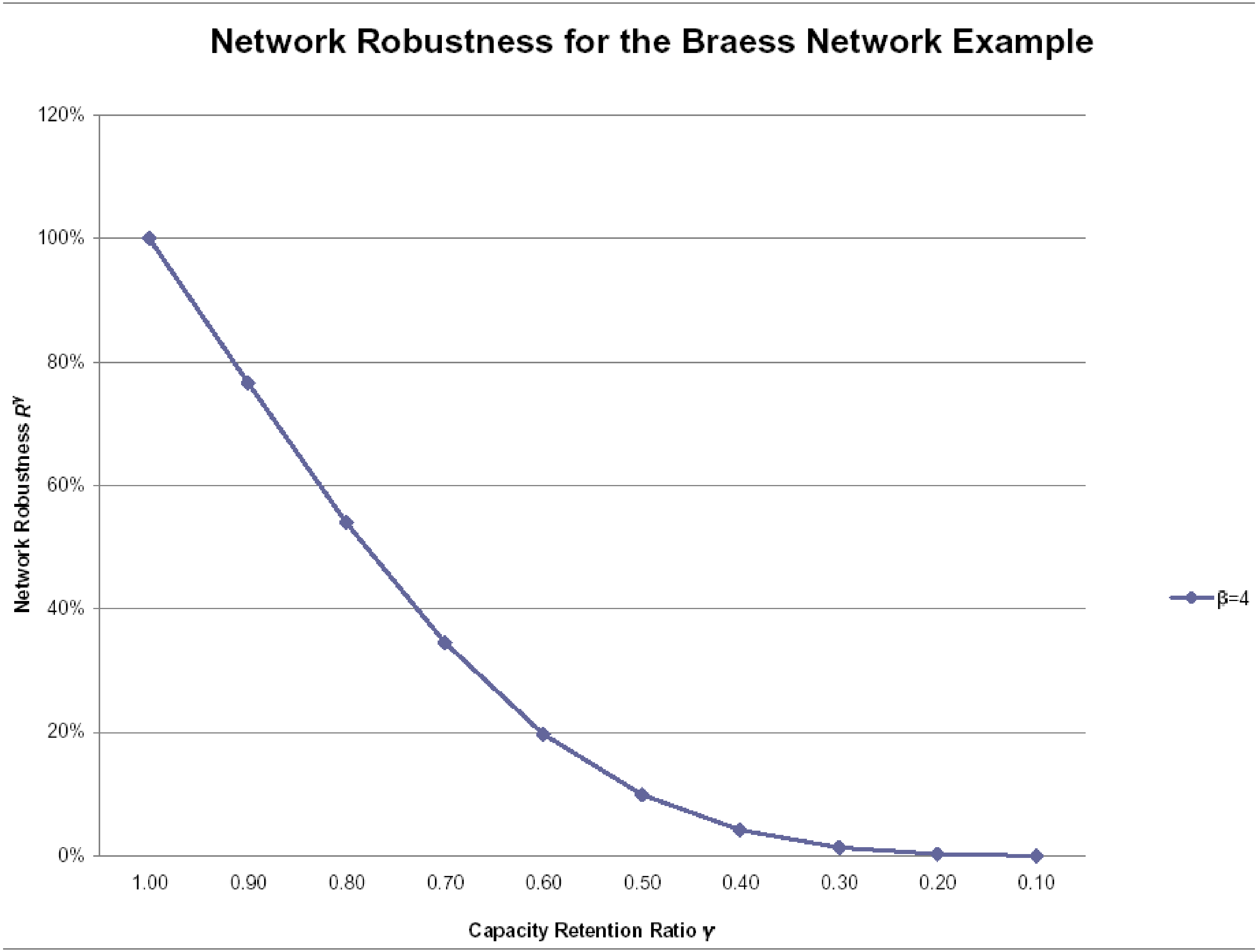

Network Robustness for the Braess Network Example
120%
100%
80%
60%
40%
20%
0%
Network Robustness R^{γ}
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
Capacity Retention Ratio γ
β=2

$\beta = 3$



Network Robustness for the Braess Network Example
120%
100%
80%
60%
40%
20%
0%
Network Robustness R^γ
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
Capacity Retention Ratio γ
β=3

$\beta = 4$



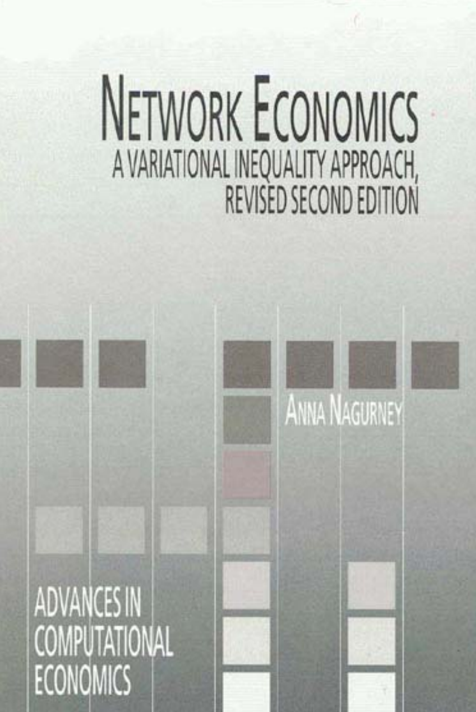
Network Robustness for the Braess Network Example
120%
100%
80%
60%
40%
20%
0%
Network Robustness R^{γ}
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
Capacity Retention Ratio γ
β=4

Importance of Nodes and Links in the Dynamic Braess Network Using the N-Q Measure when T=10

Link	Importance Value	Importance Ranking
a	0.2604	1
b	0.1784	2
c	0.1784	2
d	0.2604	1
e	-0.1341	3

Node	Importance Value	Importance Ranking
1	1.0000	1
2	0.2604	2
3	0.2604	2
4	1.0000	1

Link e is never used after $t = 8.89$ and in the range $t \in [2.58, 8.89]$, it increases the cost, so the fact that link e has a negative importance value makes sense; over time, its removal would, on the average, improve the network efficiency!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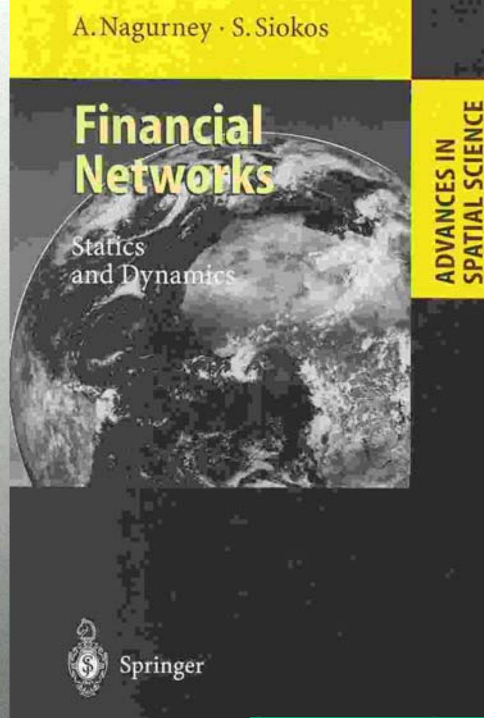
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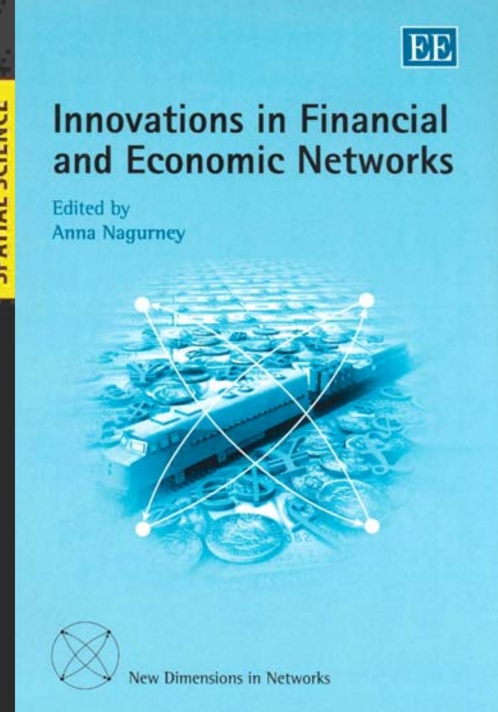
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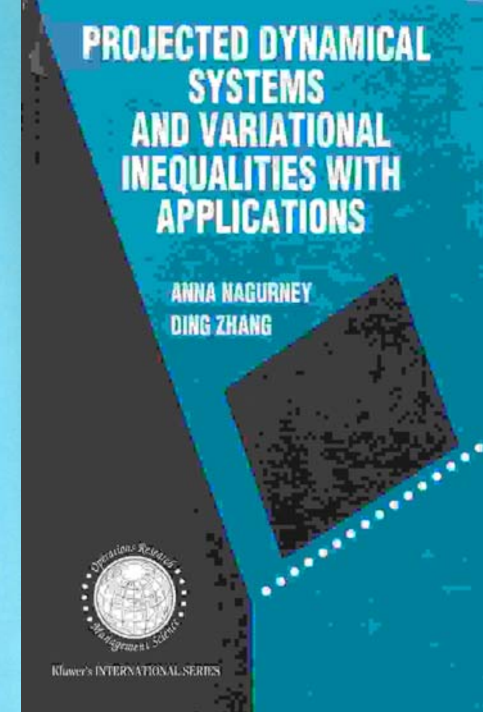
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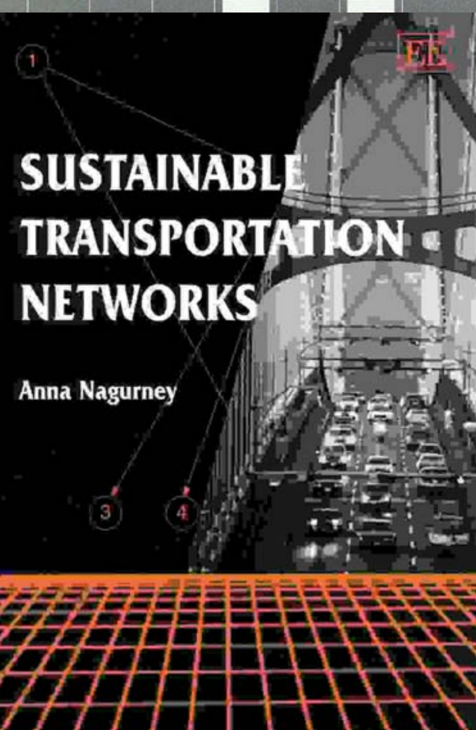
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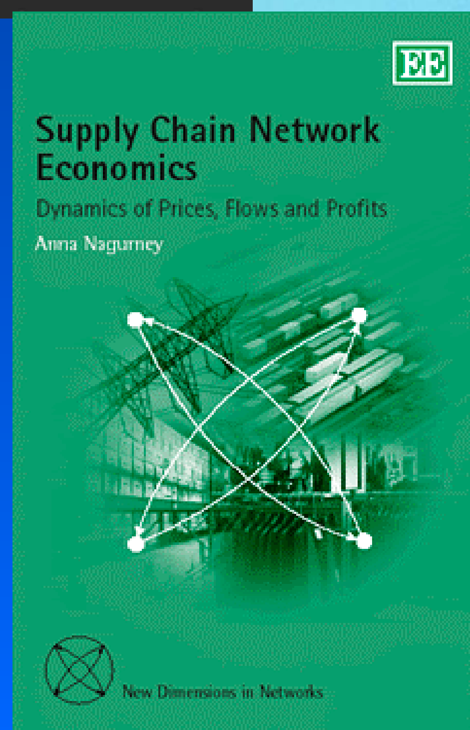
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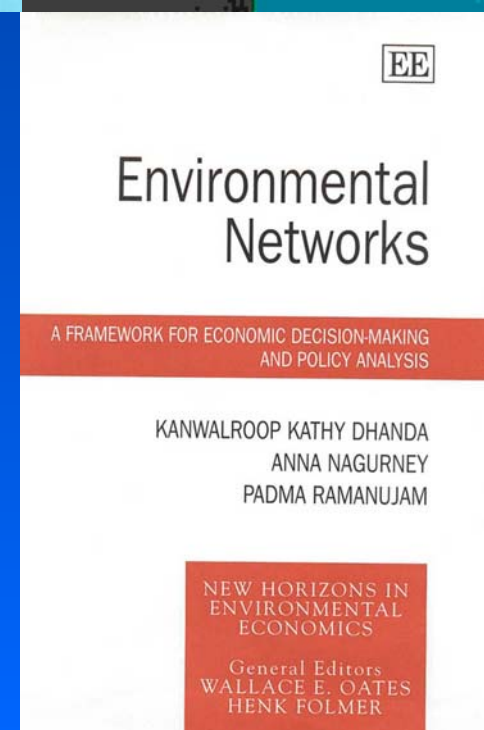
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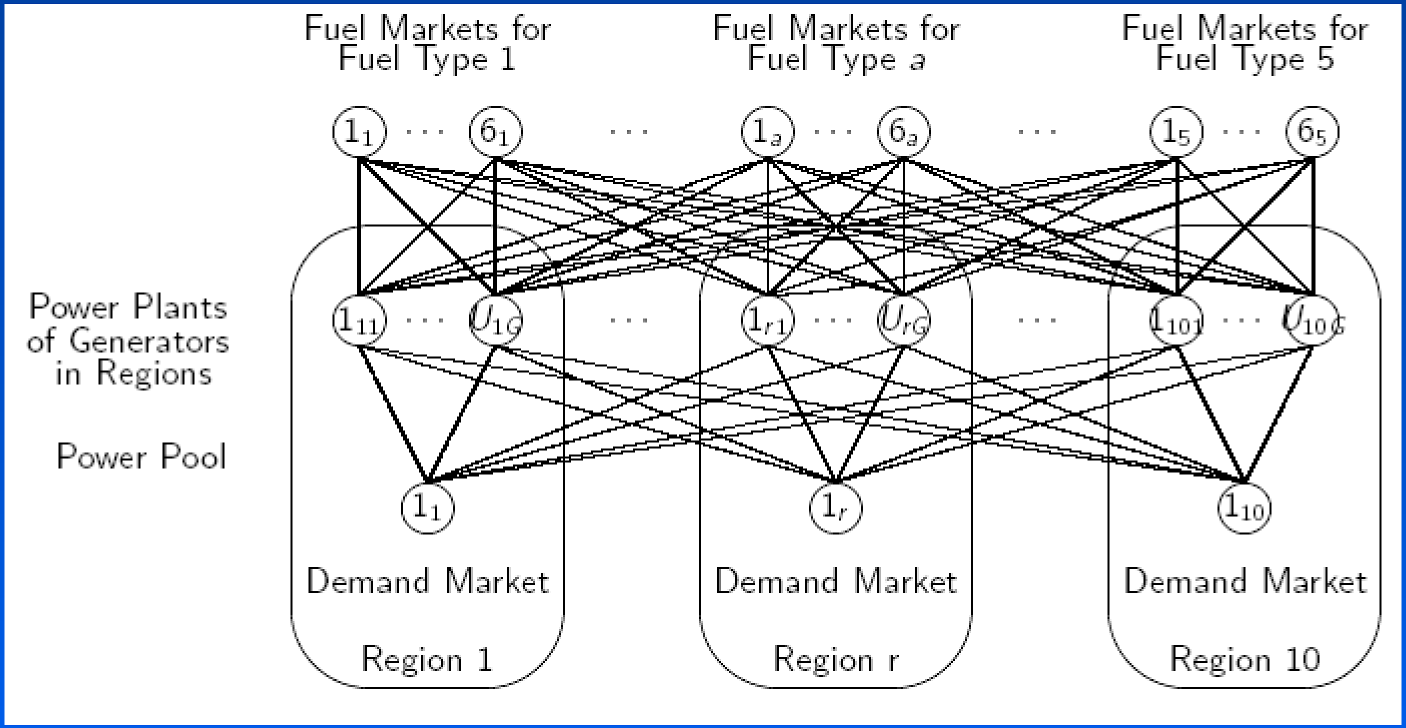
Where Are We Now?

An Integrated Electric Power Supply Chain and Fuel Market Network Framework: Theoretical Modeling with Empirical Analysis for New England, Liu and Nagurney (2007).

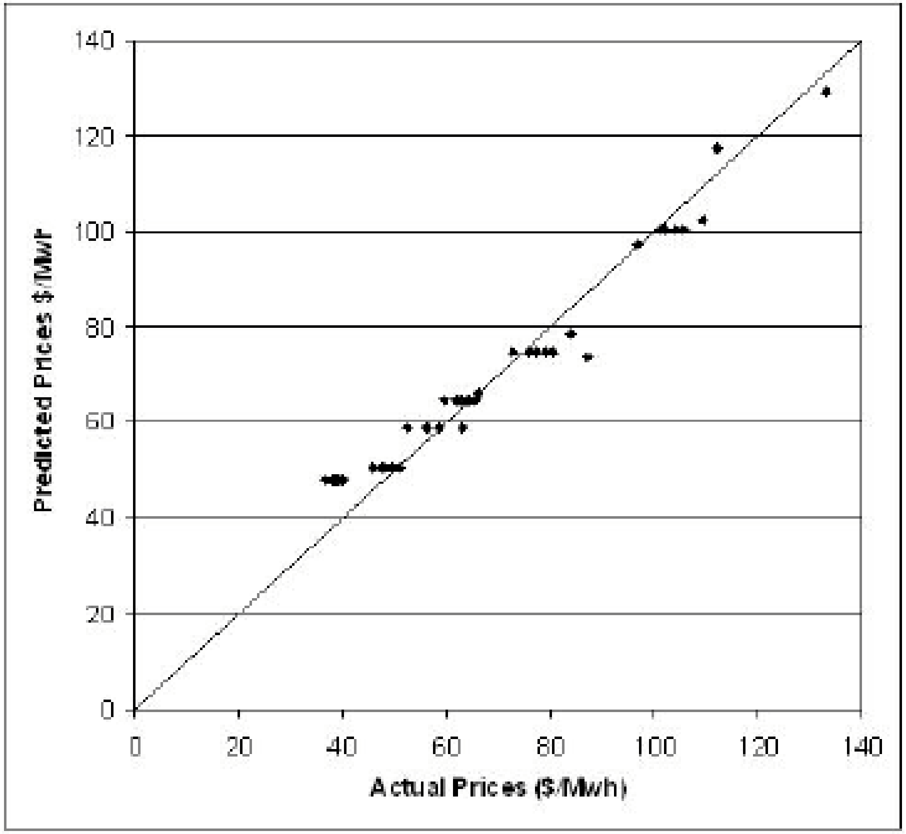
Empirical Case Study

- New England electric power market and fuel markets
- 82 generators who own and operate 573 power plants
- 5 types of fuels: natural gas, residual fuel oil, distillate fuel oil, jet fuel, and coal
- Ten regions (R=10): 1. Maine, 2. New Hampshire, 3. Vermont, 4. Connecticut(excluding Southwest Connecticut), 5. Southwest Connecticut(excluding Norwalk-Stamford area), 6. Norwalk-Stamford area, 7. Rhode Island, 8. Southeast Massachusetts, 9. West and Central Massachusetts, 10. Boston/Northeast Massachusetts
- Hourly demand/price data of July 2006 ($24 \times 31 = 744$ scenarios)
- 6 blocks ($L1 = 94$ hours, and $Lw = 130$ hours; $w = 2, \ldots, 6$)

The New England Electric Power Supply Chain Network with Fuel Suppliers



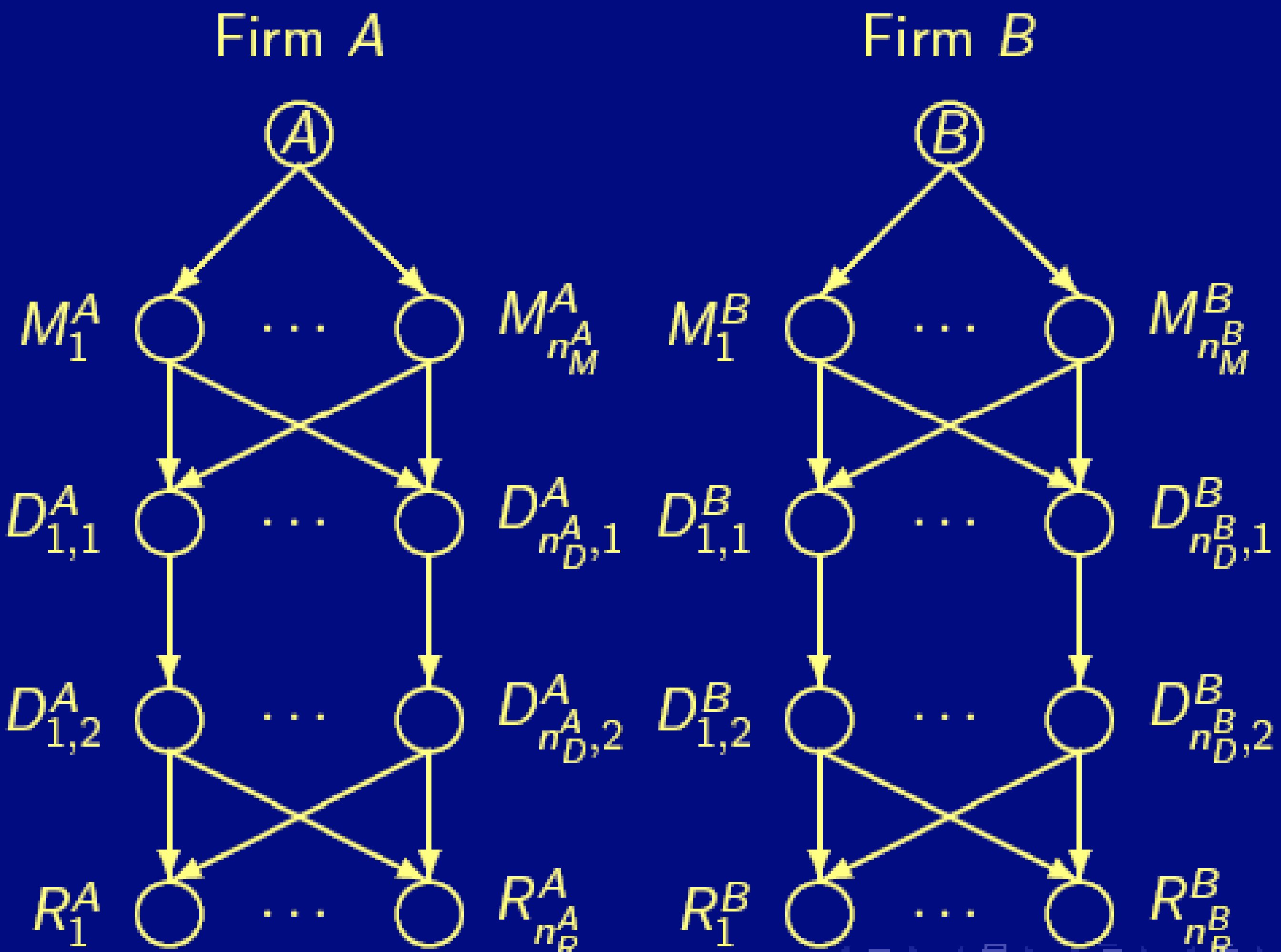
Predicted Prices vs. Actual Prices ($/Mwh)



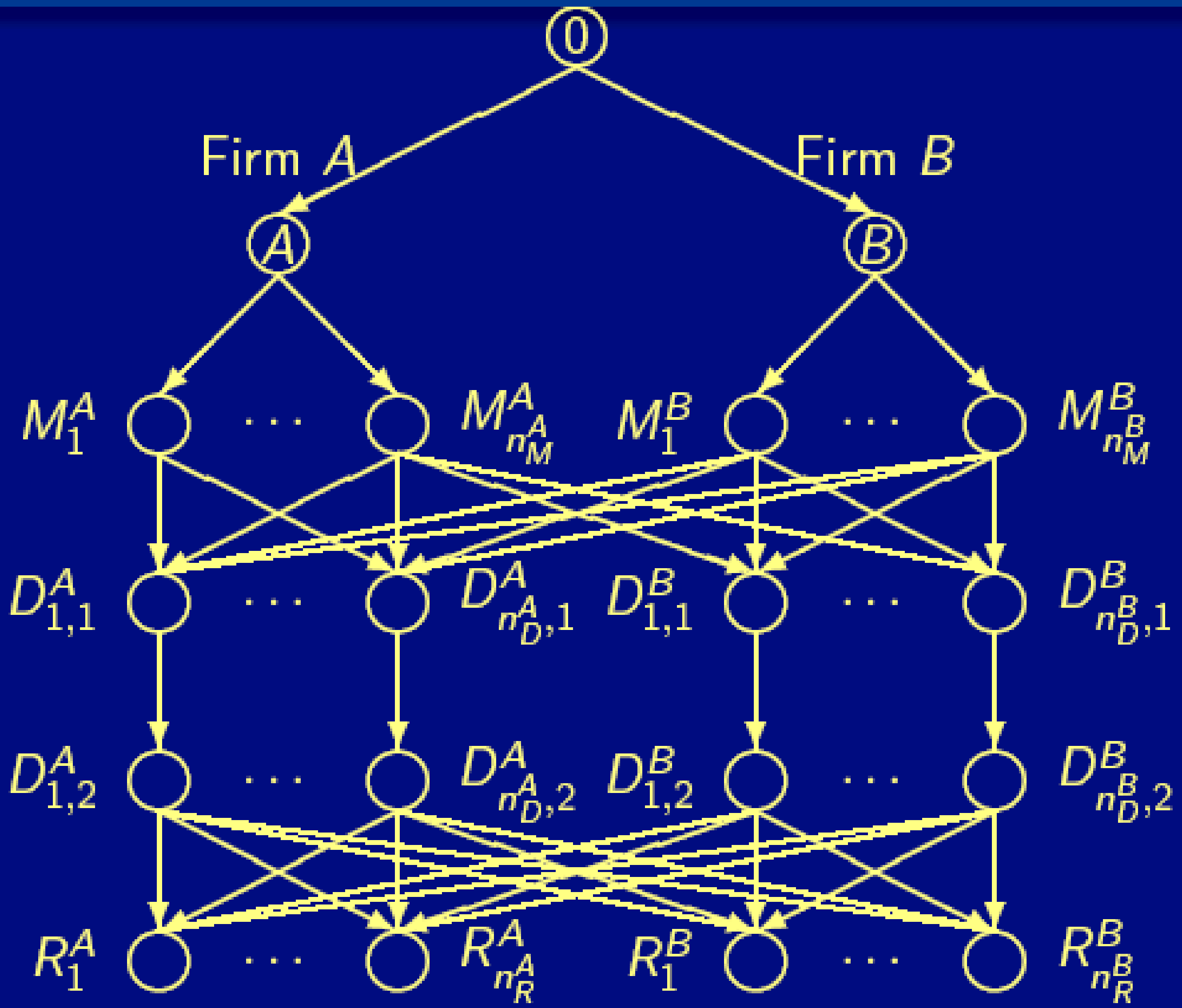
We have been focusing on network vulnerability and robustness analysis. We also have results in terms of *synergy* in the case of supply chain network integration as would occur in mergers and acquisitions.

In this framework we model the economic activities of each firm as a S-O problem on a network.

Supply Chain Prior to the Merger



Supply Chain Post Merger



Quantifying the Synergy of the Merger

The synergy associated with the total generalized costs which captures both the total costs and the weighted total emissions is defined as:

$$S^{TGC} \equiv [\frac{TGC^0 - TGC^1}{TGC^0}] \times 100\%$$

This framework can also be applied to teaming of humanitarian organizations in the case of humanitarian logistics operations.

http://hlogistics.som.umass.edu

Ongoing Research and Questions

- How can time delays be incorporated into the measure?
- How do we capture multiclass user behavior; equivalently, behavior in multimodal networks?
- Can the framework be generalized to capture multicriteria decision-making?
- What happens if either system-optimizing (S-O) or user-optimizing (U-O) behavior needs to be assessed from a network system performance angle? We have some results in this dimension in terms of vulnerability and robustness analysis as well as from an environmental (emissions generated) perspective.

- We also have results for synergy associated with supply chain network integration in the case of multiple products using an S-O formulation of the economic activities of firms.

References - for Further Reading

Link to Network Economics course materials as well as several other related courses conducted by Professor Anna Nagurney on her Fulbright in Austria: http://supernet.som.umass.edu/austria_lectures/fulmain.html

Overview article on Network Economics by Professor Nagurney: http://supernet.som.umass.edu/articles/NetworkEconomics.pdf

Background article on the importance of the Beckmann, McGuire, and Winsten book, *Studies in the Economics of Transportation*: http://tsap.civil.northwestern.edu/boyce_pubs/retrospective_on_beckmann.pdf

Preface to the translation of the Braess (1968) article and the translation: http://tsap.civil.northwestern.edu/bouce_pubs/preface_to.pdf http://homepage.rub.de/Dietrich.Braess/Paradox-BNW.pdf

Link to numerous articles on network modeling and applications, **vulnerability and robustness analysis, as well as network synergy**: http://supernet.som.umass.edu/dart.html

Link to books of interest: http://supernet.som.umass.edu/bookser.html

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Thank you!

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