# A Supply Chain Network Economy


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# Literature

- Most SCM research is on intra chain *coordination* problems:
  - Distribution network design
  - Inventory management
  - Production coordination
  - Supplier selection
- Optimization framework

# Reality

- It is no longer a firm vs. a firm competition.
- But a supply chain vs. a supply chain competition in many marketplaces today.

# Supply Chain Economy

- **Supply Chain Economy (SCE)**
  - ***a network of <u>interrelated</u> activities of***
  - ***procurement, production, distribution, vendition, and consumption of***
  - ***one or many related products or services,***
  - ***conducted by several coalitions of business entities who act collectively within a coalition.***
- **In short, SCE is a network of interrelated SCs**

# Objectives

We desire a model to address the following questions

- How do supply chains compete each other?
- Which supply chain(s) will win through the competition?
- How much market share will a winner supply chain get?
- What will be the output level of a firm participating in a winner supply chain?
- How do business agents compete to form a supply chain?

# Novelty and Characteristics

- The model studies a supply chain economy
  - Comprising heterogeneous supply chains
  - Involving in production, distribution, vendition and consumption
  - Multiple markets (geographic areas, Internet)
- The model captures
  - intra chain cooperation
  - Inter chain competition
- The model deals with
  - product flows
  - information flows

# Network Model

A connected network *G* =[*N*, *L*] for a SCE

*N* - set of nodes, *L* - set of directed links

All the nodes can therefore be classified as:

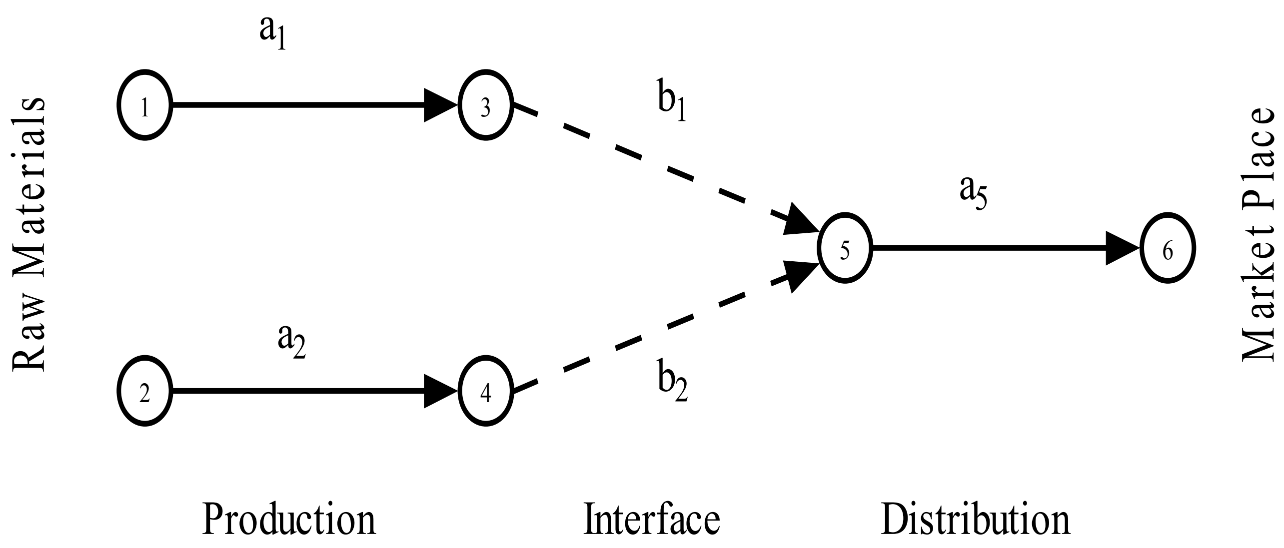
- Origin node (always a head node)
  - source of raw material or other resource
- Intermediate node (both a head node and tail node)
  - connection of SC links
- Destination node (always a tail node)
  - destination of the material flow, end product marketplace

# Operation Link and Interface Link

- An operation link represents a substantial business function performed by a firm
  - E.g. manufacturing, transportation, storage, service operation, etc.
- An operation link can participate in multiple SCs
  - E.g. a third party logistic company
- An interface link represents a coordination function between two successive operation links in a SC
  - E.g. information sharing, production coordination, etc.

$$L = A \bigcup B$$

# Illustrative Example



# Chain Flow and Link Flow

$X_s$ **chain flow on chain *s***

$x_a$ **link flow on operation link *a***

$x_b$ **link flow on interface link *b***

The link flow of an operation link is the total amount of work processed on this link in the SCE network.

The link flow on an interface link indicates the amount of integration work, coordination effort, and information processed.

*Units measuring link flows may vary*

# of fabricated parts, machine hours, labor hours, truck loads, square footage space…

# Chain-Link Process Rate

$\lambda_{as}$ the process rate of link $a$ on supply chain $s$

The amount of resource utilized (work conducted or information processed) by link $a$ necessary to make one unit of the end product of supply chain $s$

The process rate of a tire manufacturer operation link

- for a participating family car chain is 4
- for a participating ten-wheel truck chain is 10

# Illustrative Example

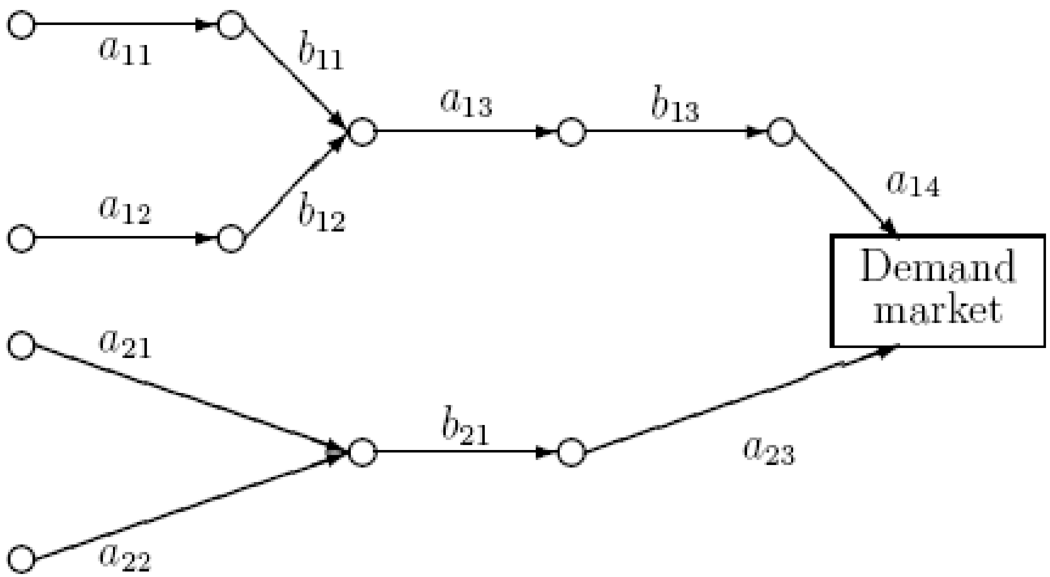



Figure 2: An example of two supply chains competing at a demand market

# Link Chain Flow Conservation

Mathematically, one has the following flow conservation equation of (operation) link flow variables and chain flow variables:

$$x_a = \lambda_{as} X_s, \quad \forall a \in A$$

Assume that the market demand at a market place is *elastic* and can be characterized by its market value.

$$q_j = \sum_{s \in S_j} X_s, \; j = 1, \ldots, n.$$

$$v_j = v_j(q), \; j = 1, \ldots, n.$$

The link flow of an interface link is determined by the chain flow of its governing supply chain, that is,

$$x_b = x_b(X_s), \quad \forall b \in B, \forall s \in S$$

# Operation Link Cost

$$\overline{c}_a = \overline{c}_a(x_a), \qquad \forall a \in A$$

- It measures an appropriate combination of monetary cost and ineffectiveness in performing the corresponding task – *total cost*
  - The ineffectiveness reflects important factors such as time and quality in conforming the task
  - The generic cost function can take a weighted majority form,
    - with the monetary cost being assigned a weight of one
    - the other factors of ineffectiveness being assigned relative weights that can convert to equivalent dollar amount

# Interface Link Cost

$$\overline{c}_b = \overline{c}_b(X_s), \quad \forall b \in B, \forall s \in S$$

- **The cost of an interface link reflects the ineffectiveness of coordination and integration of the two operation links that it bridges**
- **Major factors affecting interface link cost include:**
  - **Geographic distance,**
  - **Past cooperation experience,**
  - **Level of information integration,**
  - **Compatibility between the two operation links**
- **The cost of an interface link may in general depend on the flows of all the interface links belonging to the same market chain**

# Chain Cost

$$\bar{C}_s = \bar{C}_s(X) = \sum_{a \in A} \bar{c}_{as} + \sum_{b \in B_s} \bar{c}_b$$

- **The cost of a supply chain $s$ is the sum of the link costs of its operation and interface links.**

# Link Marginal Cost

$$c_a = \begin{cases} \bar{c}_a(x_a)/x_a, & \text{if} \quad x_a \geq \lambda_{as}, \\ \bar{c}_a(\lambda_{as})/\lambda_{as}, & \text{if} \quad x_a \leq 1. \end{cases}$$

$$c_b = \begin{cases} \bar{c}_b(X_s)/X_s, & \text{if} \quad X_s \geq 1, \\ \bar{c}_b(1), & \text{if} \quad X_s \leq 1. \end{cases}$$

# Link Marginal Cost

- Any active operation link in a SCE must carry a minimum of one unit link flow in practice.
- Therefore, the link marginal cost so defined is the unit processing cost for an active link.
- On the other hand, if the link flow turns out to be less than one unit in the model, then
  - it would imply that the link is not actually being used.
  - In this case, the link marginal cost can be seen as the *triggering cost*, i.e., the cost necessary to activate this link

# Chain Marginal Cost

$$C_s = \begin{cases} \bar{C}_s(X)/X_s, & if \quad X_s \geq 1 \\ \bar{C}_s(1), & if \quad X_s < 1 \end{cases}$$

$$C_s = C_s(X) = \sum_{a \in A_s} \lambda_{as} c_{as}(x_a) + \sum_{b \in B_s} c_b(X_s)$$

# Wardrop's First Principle

***The travel costs of all paths actually used are equal and less than those which would be experienced by a single traveler on any unused path.***

# Differences

## Supply Chain

- a connected *subgraph* connected the raw materials sources and the end market, (simplest topology would be a tree)
- flow evolves from raw materials, parts, subassemblies, to finished goods

## Transportation

- a *path* connecting an O/D pair
- travelers (in vehicles) arrive at their destinations in the same form as they leave their origins

# Similarity

*The economic rationale that an inferior or less cost-effective supply chain will lose to its rivals is the same spirit as Wardrop's first principle.*

# Equilibrium of a Supply Chain Network Economy

$$C_s(X)\begin{cases} = v_j(q^*), \text{ if } X_s^* > 0 \\ \geq v_j(q^*), \text{ if } X_s^* = 0, \end{cases} \quad \forall s \in S_j,$$

*for all end markets j,; j=1,…n.*

# Variational Inequality Formulation

**Theorem 1:** ***$(X^*, q^*)$ is a supply chain network equilibrium if and only if***

***$(X^*, q^*)$ solves the following variational inequality problem:***

$$\sum_{j=1}^{n}\left[\sum_{s\in S_j} C_s(X^*)(X_s - X_s^*) - v_j(q^*)(q_j - q_j^*)\right] \geq 0, (X, q) \in \Omega_{Xq}.$$

# Existence

**Theorem 2:** ***Suppose that the link cost functions for all the operation and interface links are continuous. Then there exists an equilibrium*** $(X^*, q)$.

# Uniqueness

***Theorem 3: Suppose that the marginal link cost functions are strictly monotone increasing, and the market value functions are strictly monotone decreasing, with respect to their arguments. Then there exists a unique $(X^*,q)$.***

# Example

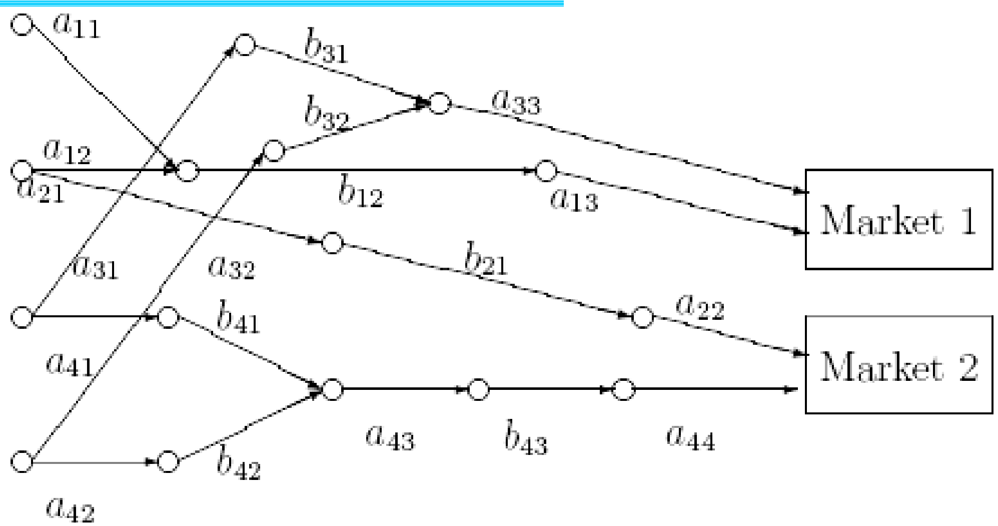



Figure 3: An example of a Supply Chain Network Economy

| | | |
|---|---|---|
| S1 | a11, a12, a13, b11 | Mkt1 |
| S2 | a21, a22, b21 | Mkt2 |
| S3 | a31, a32, a33, b31, b32 | Mkt1 |
| S4 | a41, a42, a43, b41, b42, b43 | Mkt2 |

$$\lambda_{11} = 2, \quad \lambda_{12} = 3, \quad \lambda_{13} = 1,$$

$$\lambda_{21} = 4, \quad \lambda_{22} = 1,$$

$$\lambda_{31} = 2, \quad \lambda_{32} = 2, \quad \lambda_{33} = 1,$$

$$\lambda_{41} = 2, \quad \lambda_{42} = 4, \quad \lambda_{43} = 2, \quad \lambda_{44} = 1.$$

The link marginal cost functions of the operation and the interface links, for each supply chain, are as given below.

$$c_{a11} = 0.5x_{a11} + 1, \quad c_{a12} = 0.1x_{a12},$$

$$c_{b11} = 2.1x_{b11} + 1, \quad c_{a13} = 5x_{a13} + 3,$$

$$c_{a21} = 0.5x_{a21} + 0.5, \quad c_{b21} = X_2, \quad c_{a22} = x_{a22} + 2,$$

$$c_{a31} = x_{a31} + 4, \quad c_{a32} = x_{a32} + 4, \quad c_{b31} = X_3 + 4,$$

$$c_{b32} = X_3 + 4, \quad c_{a33} = x_{a33} + 4,$$

$$c_{a41} = c_{a42} = c_{a43} = 0.5,$$

$$c_{a44} = 2x_{a44} + 1, \quad c_{b41} = c_{b42} = c_{b43} = X_4.$$

$$C_{S_1} = 10X_1 + 6, \quad C_{S_2} = 10X_2 + 4, \quad C_{S_3} = 11X_3 + 28, \quad C_{S_4} = 5X_4 + 5.$$

Suppose now that the two demand markets are interrelated and that their market value functions are:

$$v_1(q) = 40 - 5q_1 - q_2, \quad v_2(q) = 60 - q_1 - 10q_2.$$

The supply chain network equilibrium in this example is given by

$$X^* = (X_1^*, X_2^*, X_3^*, X_4^*) = (2, 1.4, 0, 2.6),$$

which generates the equilibrium supply pattern

$$q^* = (q_1^*, q_2^*) = (2, 4).$$

$$\text{Mkt1}: v_1 = 26$$

$$C_{S_1} = 26, \qquad C_{S_3} = 28$$

$$\text{Mkt2}: v_2 = 18$$

$$C_{S_2} = 18, \qquad C_{S_4} = 18$$

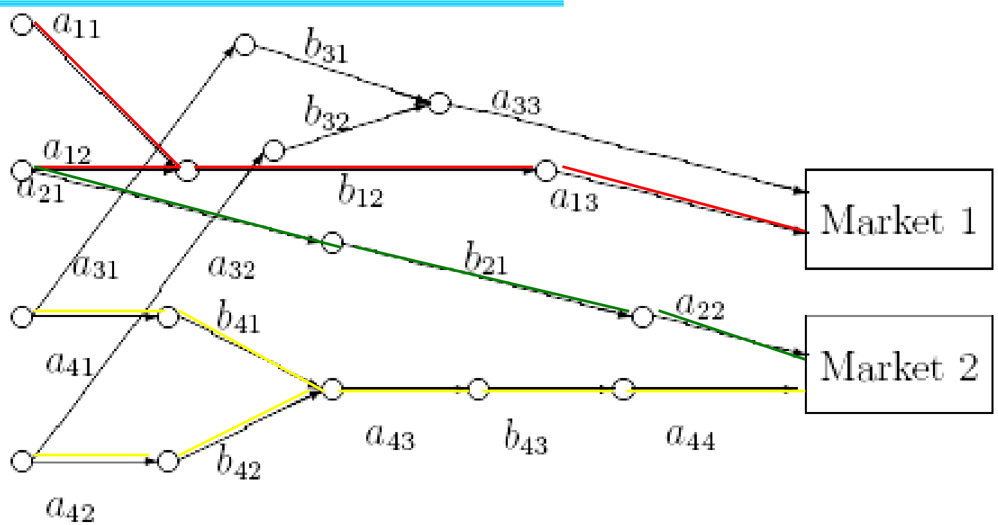



Figure 3: An example of a Supply Chain Network Economy

$$\text{Mkt1: } v_1 = 26$$

$$C_{S_1} = 26, \qquad C_{S_3} = 28$$

$$\text{Mkt2: } v_2 = 18$$

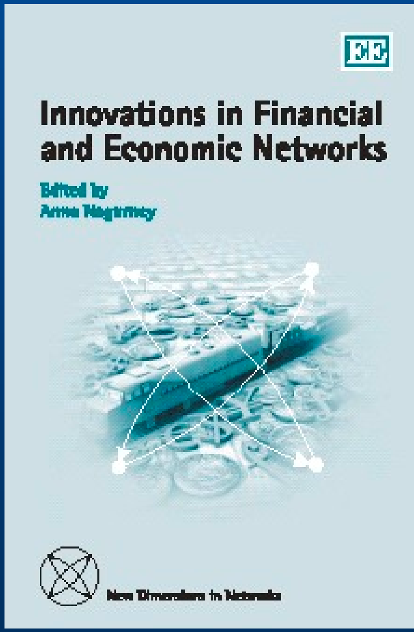
$$C_{S_2} = 18, \qquad C_{S_4} = 18$$

# Further Information

Chapter appears in the book

*Innovations in Financial and Economic Networks* (New Dimensions in Networks), Anna Nagurney, Editor

Edward Elgar Publishing Inc, 2003



# Future Research

- Multi-market, multi-products with common operations links
- Stochastic demand
- Role of core players
- Dynamic chain evolution

# Thank you!