Dynamic Networks: Recent Results and Applications

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Outline of Presentation:

- Background
- Brief History of the Science of Networks
- Interdisciplinary Impact of Networks
- Characteristics of Networks Today
- The Braess Paradox
- Some Interesting Applications
- New Tools
- The Time-Dependent Braess Paradox
- New Challenges and Opportunities: Unification of Evolutionary Variational Inequalities and Projected Dynamical Systems
- The Internet as a Dynamic Network

We are in a New Era of Decision-Making Characterized by:

- complex interactions among decision-makers in organizations;
- alternative and at times conflicting criteria used in decision-making;
- constraints on resources: natural, human, financial, time, etc.;
- global reach of many decisions;
- high impact of many decisions;
- increasing risk and uncertainty, and
- the *importance of dynamics* and realizing a fast and sound response to evolving events.

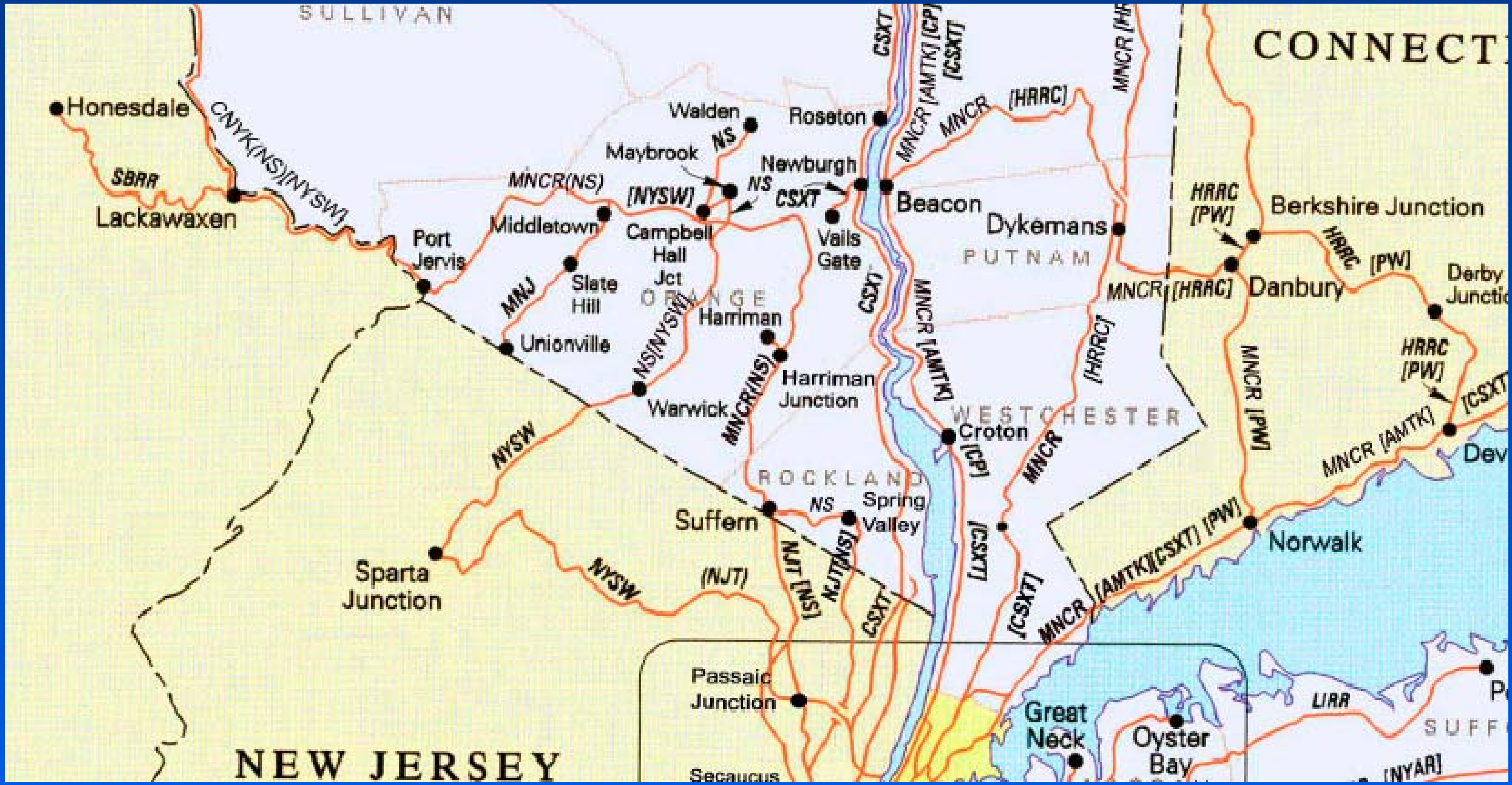
Transportation Networks

provide us with the means to cross distance in order to conduct our work, and to see our colleagues, students, friends, and family members.

They provide us with access to food and consumer products.

Subway Network

Railroad Network



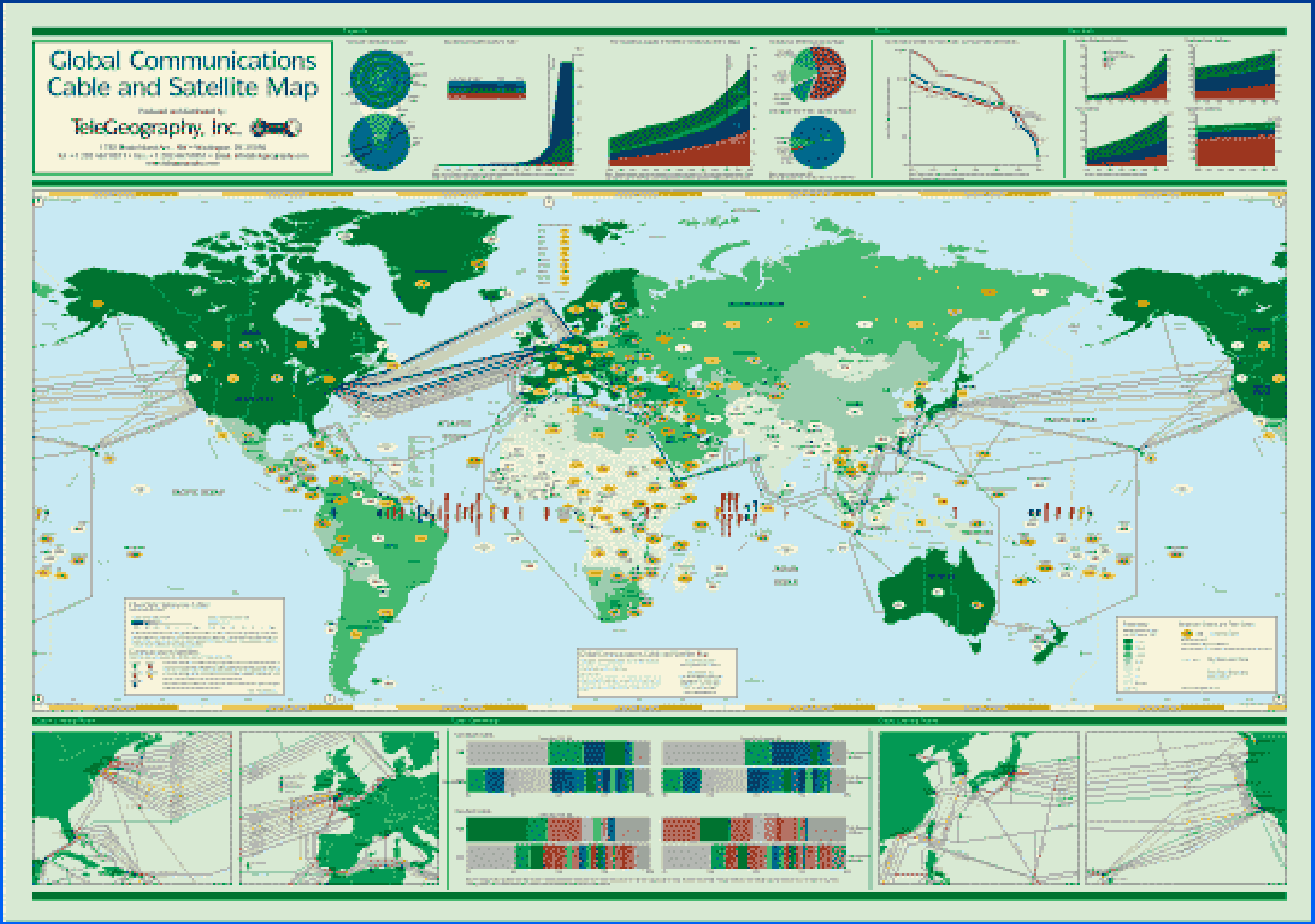
Communication Networks

allow us to communicate within our own communities and across regions and national boundaries,

and have transformed the way we live, work, and conduct business.

Iridium Satellite Constellation Network

Satellite and Undersea Cable Networks



Energy Networks

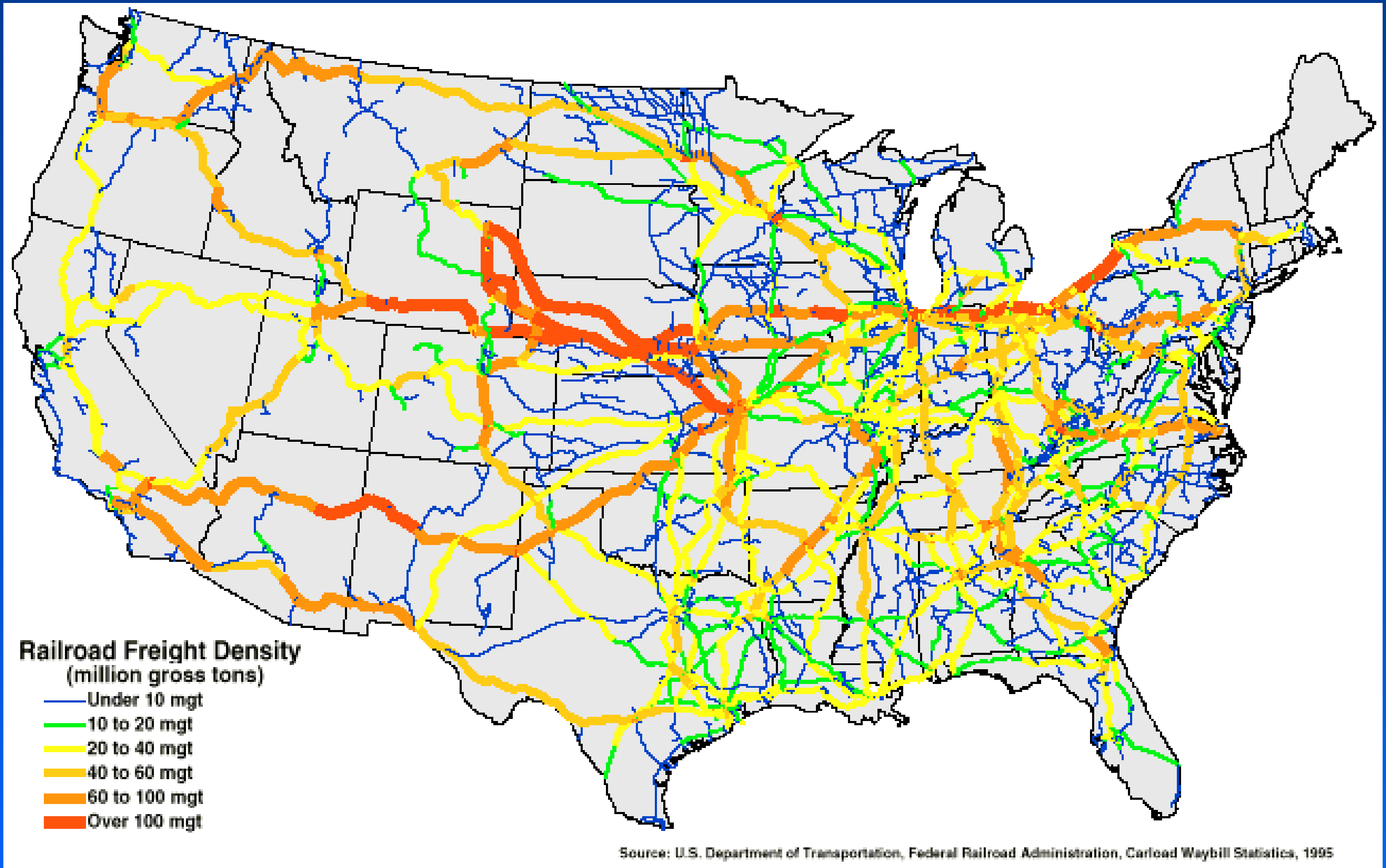
provide the energy for our homes, schools, and businesses, and to run our vehicles.

Duke Energy Gas Pipeline Network

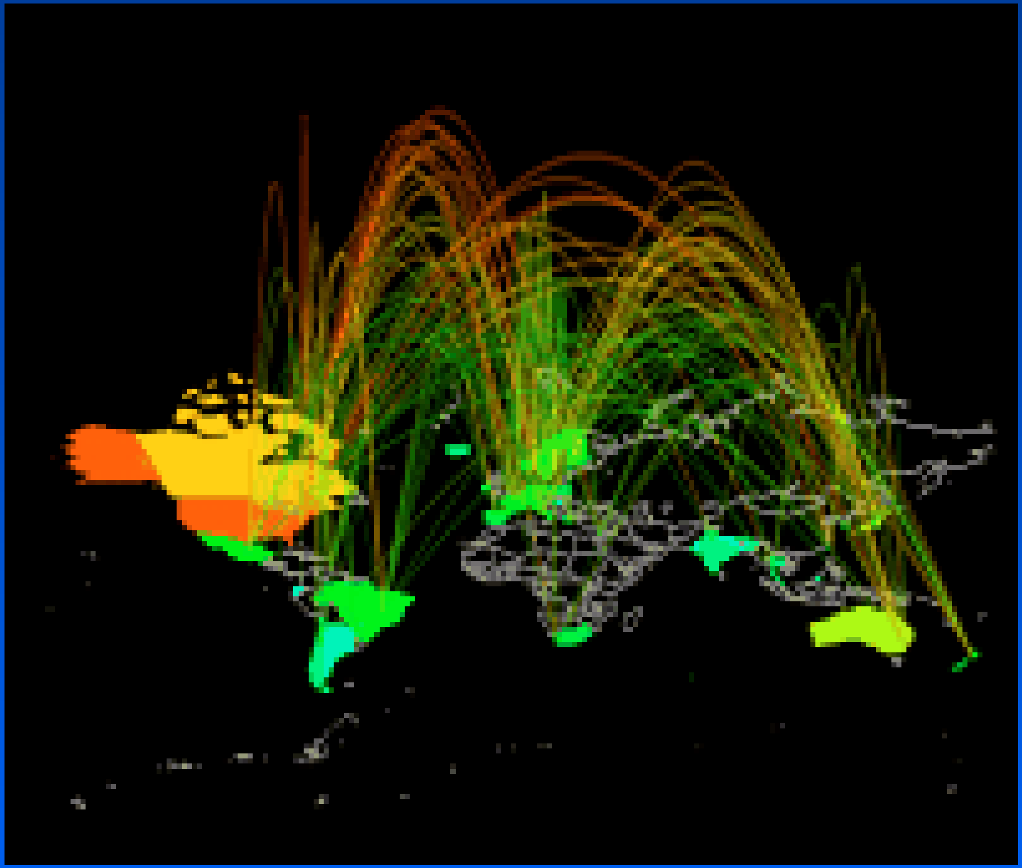
Components of Common Physical Networks

Network System	Nodes	Links	Flows
Transportation	Intersections, Homes, Workplaces, Airports, Railyards	Roads, Airline Routes, Railroad Track	Automobiles, Trains, and Planes,
Manufacturing and logistics	Workstations, Distribution Points	Processing, Shipment	Components, Finished Goods
Communication	Computers, Satellites, Telephone Exchanges	Fiber Optic Cables Radio Links	Voice, Data, Video
Energy	Pumping Stations, Plants	Pipelines, Transmission Lines	Water, Gas, Oil, Electricity

US Railroad Freight Flows

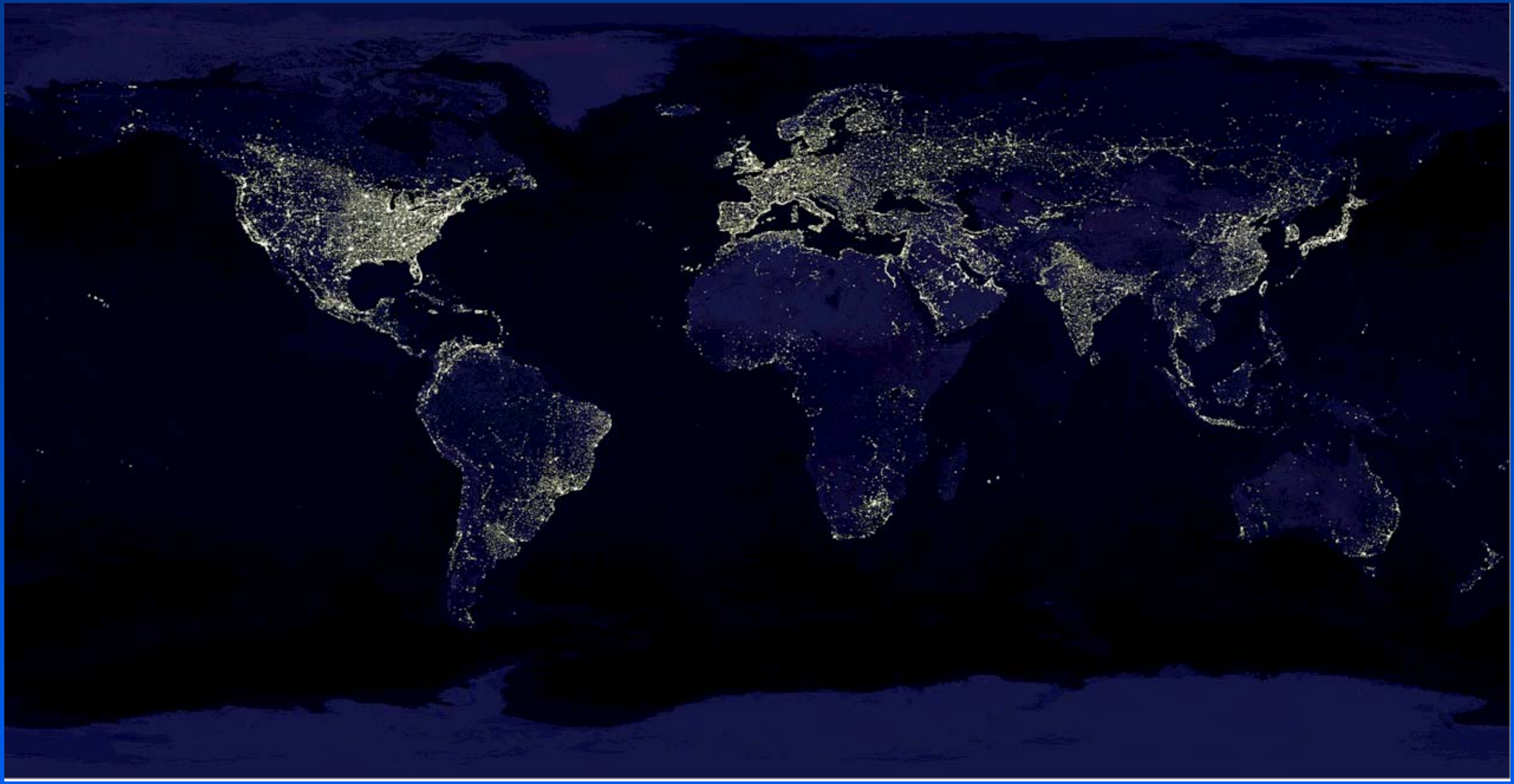


Internet Traffic Flows Over One 2 Hour Period



from Stephen Eick, Visual Insights

Electricity is Modernity



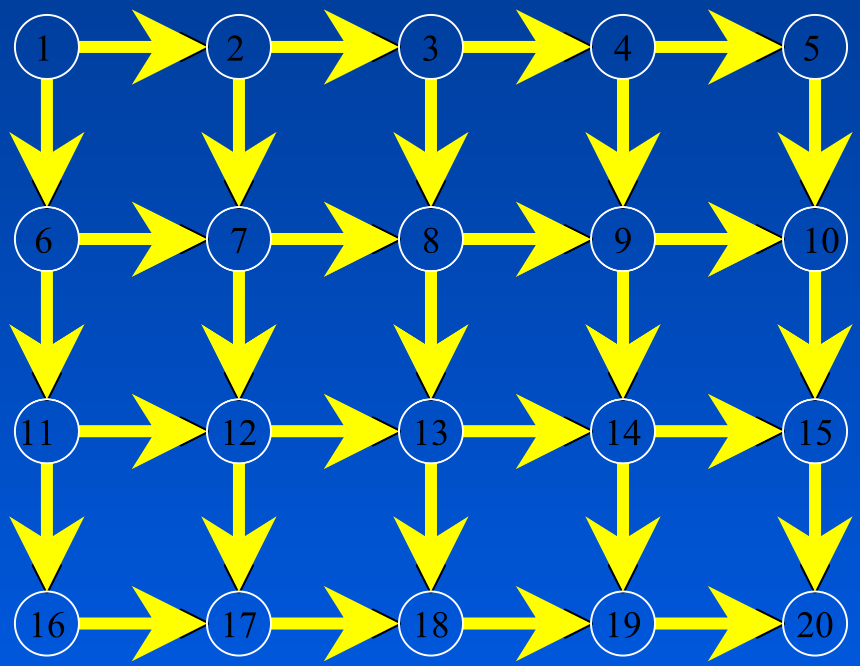
The scientific study of networks involves:

- how to **model** such applications as **mathematical entities**,
- how to **study the models** qualitatively,
- how to design **algorithms** to solve the resulting models.

The basic components of networks are:

- Nodes
- Links or arcs
- Flows

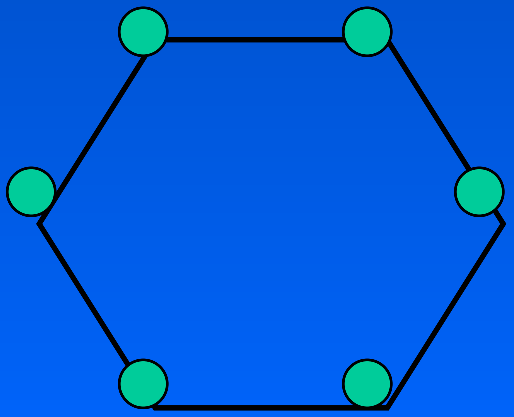
Nodes Links Flows



Brief History of the Science of Networks

1736 - Euler - the earliest paper on graph theory - Konigsberg bridges problem.

1758 - Quesnay in his *Tableau Economique* introduced a graph to depict the circular flow of financial funds in an economy.



1781 - Monge, who had worked under Napoleon Bonaparte, publishes what is probably the first paper on transportation in minimizing cost.

1838 - Cournot states that a competitive price is determined by the intersection of supply and demand curves in the context of spatially separate markets in which transportation costs are included.

1841 - Kohl considered a two node, two route transportation network problem.

1845 - Kirchhoff wrote *Laws of Closed Electric Circuits.*

1920 - Pigou studied a transportation network system of two routes and noted that the decision-making behavior of the users on the network would result in different flow patterns.

1936 - Konig published the first book on graph theory.

1939, 1941, 1947 - Kantorovich, Hitchcock, and Koopmans considered the network flow problem associated with the classical minimum cost transportation problem and provided insights into the special network structure of these problems, which yielded special-purpose algorithms.

1948, 1951 - Dantzig published the simplex method for linear programming and adapted it for the classical transportation problem.

1951 - Enke showed that spatial price equilibrium problems can be solved using electronic circuits

1952 - Copeland in his book asked, *Does money flow like water or electricity?*

1952 - Samuelson gave a rigorous mathematical formulation of spatial price equilibrium and emphasized the network structure.

1956 - Beckmann, McGuire, and Winsten in their book, *Studies in the Economics of Transportation*, provided a rigorous treatment of congested urban transportation systems under different behavioral mechanisms due to Wardrop (1952).

1962 - Ford and Fulkerson publish *Flows in Networks*.

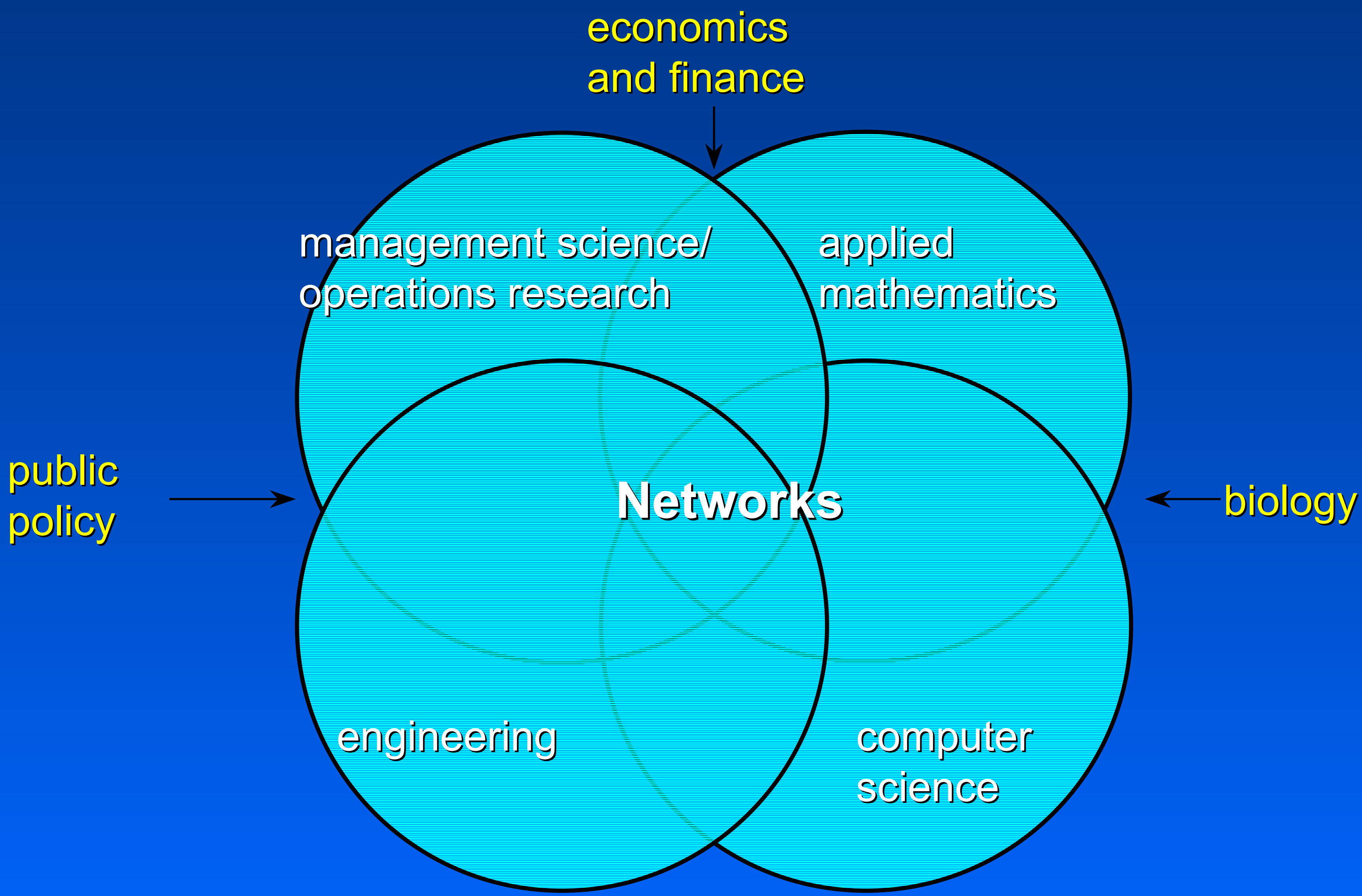
1969 - Dafermos and Sparrow coined the terms *user-optimization* and system-optimization and develop algorithms for the computation of solutions that exploit the network structure of transportation problems.

The advantages of a scientific network formalism:

- many present-day problems are concerned with flows over space and time and, hence, ideally suited as an application domain for network theory;

- provides a graphical or visual depiction of different problems;

- helps to identify similarities and differences in distinct problems through their underlying network structure;
- enables the application of efficient network algorithms;
- allows for the study of disparate problems through a unifying methodology.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computer
science

Interdisciplinary Impact of Networks

Management Science

Networks

Economics

Interregional Trade

General Equilibrium

Industrial Organization

Portfolio Optimization

Flow of Funds Accounting

Engineering

Energy

Manufacturing

Telecommunications

Transportation

Sociology

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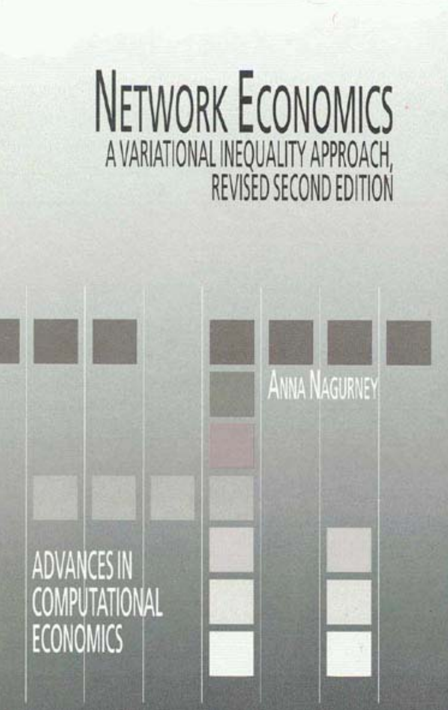
Biology

DNA Sequencing

Targeted Cancer Therapy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A VARIATIONAL INEQUALITY APPROACH,
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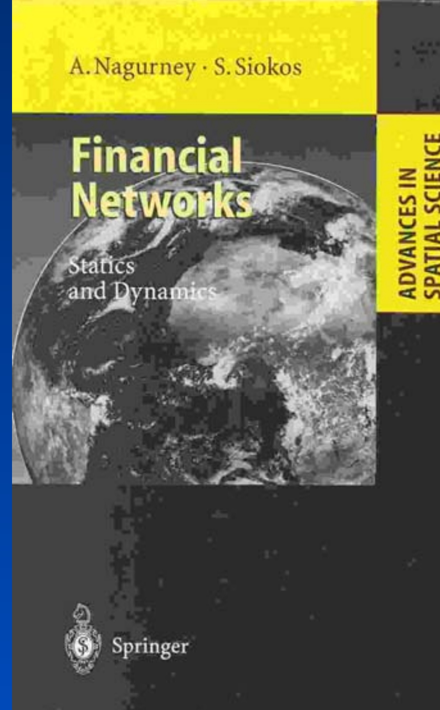
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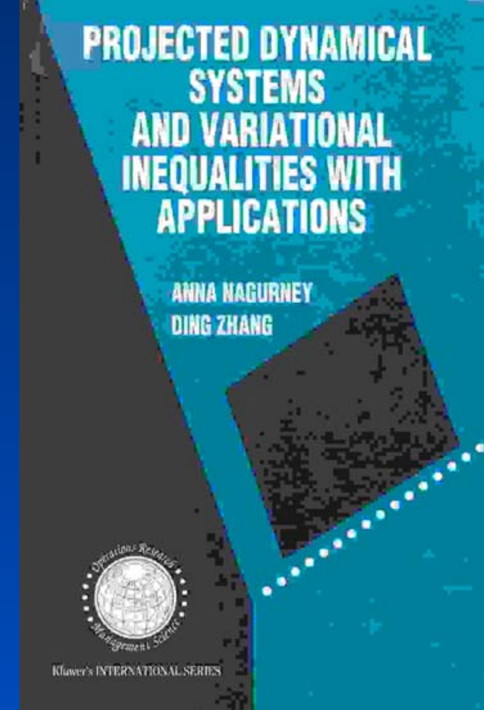
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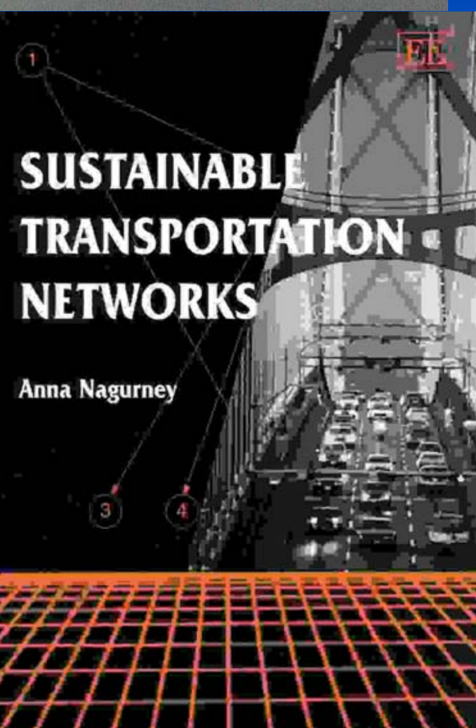
Kluwer's INTERNATIONAL SERIES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NETWORKS
Anna Nagurney
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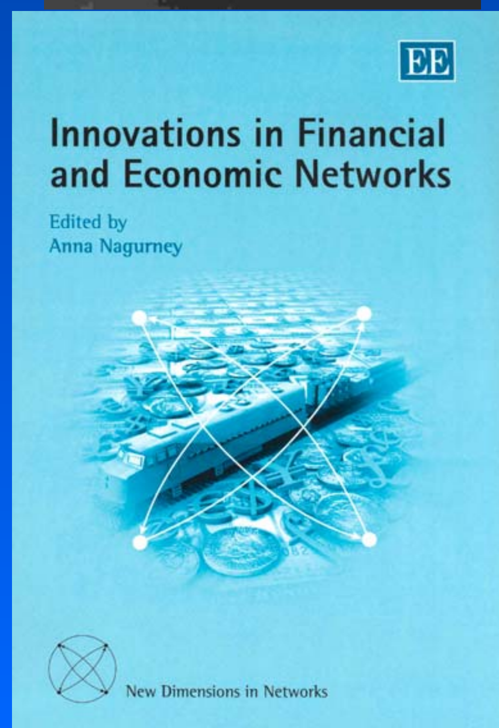
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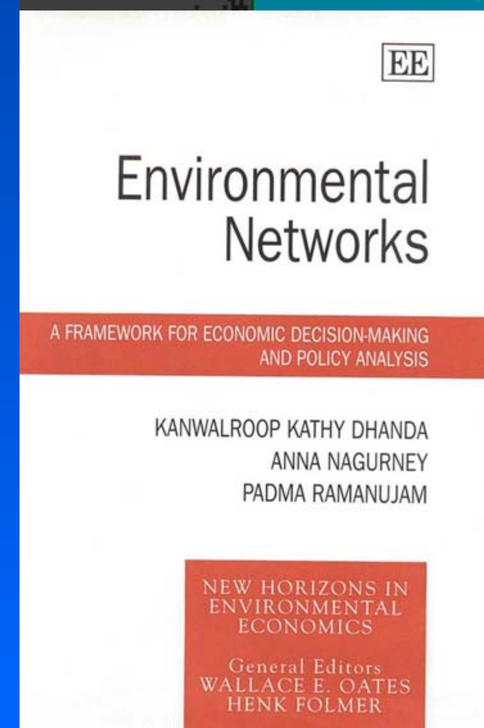
and Economic Networks
Edited by
Anna Nagurney
New Dimensions in Networks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Environmental
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A FRAMEWORK FOR ECONOMIC DECISION-MAKING
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Characteristics of Networks Today

- *large-scale nature* and complexity of network topology;
- *congestion*;
- alternative behavior of users of the network, which may lead to *paradoxical phenomena*;
- the *interactions among networks* themselves such as in transportation versus telecommunications networks;
- *policies* surrounding networks today may have a *major impact* not only economically but also *socially, politically, and security-wise*.

- *alternative behaviors of the users of the network*

  – system-optimized versus

  – user-optimized (network equilibrium),

  which may lead to

  paradoxical phenomena.

Transportation science has historically been the discipline that has pushed the frontiers in terms of methodological developments for such problems (which are often large-scale) beginning with the work of Beckmann, McGuire, and Winsten (1956).

Definition: Transportation Network Equilibrium

A route flow pattern $x^* \in K$ is said to be a transportation network equilibrium (according to Wardrop's (1952) first principle) if only the minimum cost routes are used (that is, have positive flow) for each O/D pair. The state can be expressed by the following equilibrium conditions which must hold for every O/D pair $w \in W$, every path $p \in P_w$:

$$C_p(x^*) - \lambda_w^* \begin{cases} = 0, & \textit{if} \quad x_p^* > 0, \\ \geq 0, & \textit{if} \quad x_p^* = 0. \end{cases}$$

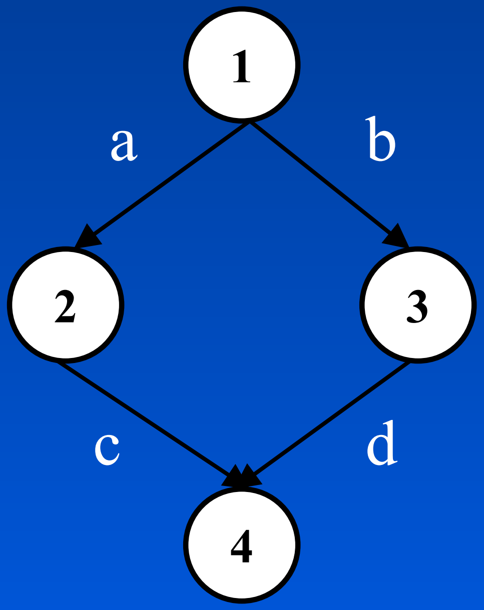
The Braess Paradox

Assume a network with a single O/D pair (1,4). There are 2 paths available to travelers: $\mathbf{p_1=(a,c)}$ and $\mathbf{p_2=(b,d)}$.

For a travel demand of **6,** the equilibrium path flows are $\mathbf{x_{p_1}}^*$ $= \mathbf{x_{p_2}}^* = \mathbf{3}$ and

The equilibrium path travel cost is

$\mathbf{C_{p_1} = C_{p_2} = 83}$.



$$\mathbf{c_a(f_a)=10\ f_a \quad c_b(f_b) = f_b+50}$$

$$\mathbf{c_c(f_c) = f_c+50 \quad c_d(f_d) = 10\ f_d}$$

Adding a Link Increases Travel Cost for All!

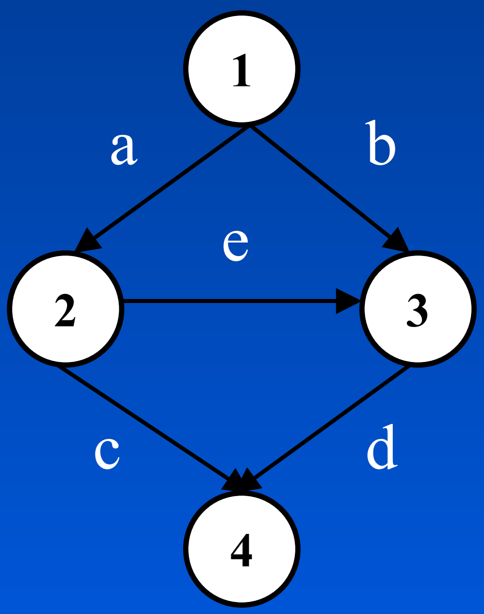
Adding a new link creates a new path **p_3=(a,e,d).**

The original flow distribution pattern is no longer an equilibrium pattern, since at this level of flow the cost on path **p_3, C_{p_3}=70.**

The new equilibrium flow pattern network is

$x_{p_1}^* = x_{p_2}^* = x_{p_3}^* = 2$.

The equilibrium path travel costs: **$C_{p_1} = C_{p_2} = C_{p_3} = 92$.**



$c_e(f_e) = f_e + 10$

The 1968 Braess article has been translated from German to English and appears as

On a Paradox of Traffic Planning

by Braess, Nagurney, Wakolbinger

in the November 2005 issue of *Transportation Science.*

VI Formulation of Transportation Network Equilibrium

A traffic path flow pattern satisfies the above equilibrium conditions if and only if it satisfies the variational inequlity problem: determine $x^* \in K$, such that

$$\sum_p C_p(x^*) \times (x_p - x_p^*) \geq 0, \quad \forall x \in K.$$

Finite-dimensional variational inequality theory has been applied to-date to the wide range of equilibrium problems noted above.

In particular, the finite-dimensional variational inequality problem is to determine $x^* \in K \subset R^n$ such that

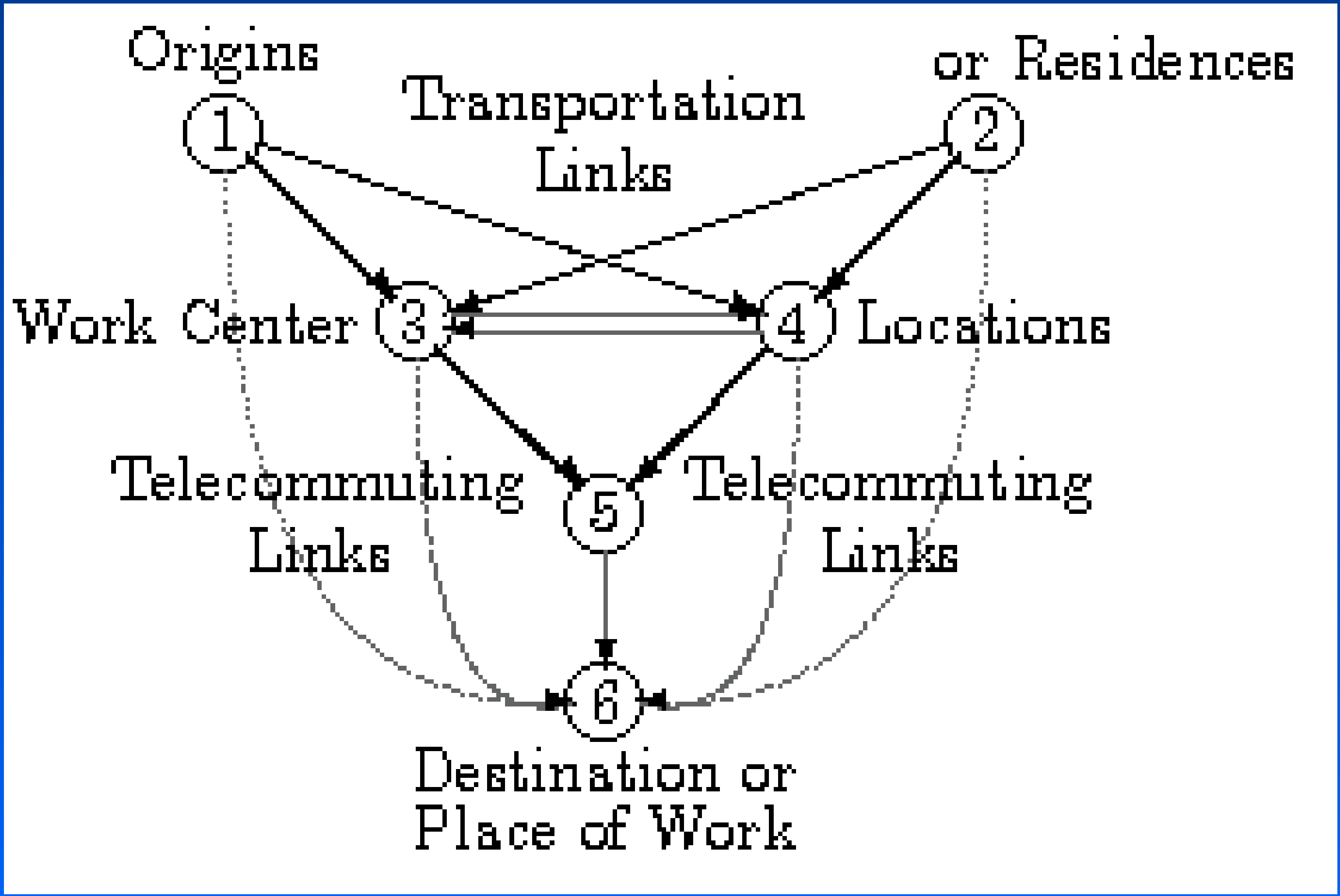
$$\langle F(x^*), x - x^* \rangle \geq 0, \quad \forall x \in K,$$

where $\langle \cdot, \cdot \rangle$ denoted the inner product in R^n and K is closed and convex.

Some Interesting Applications

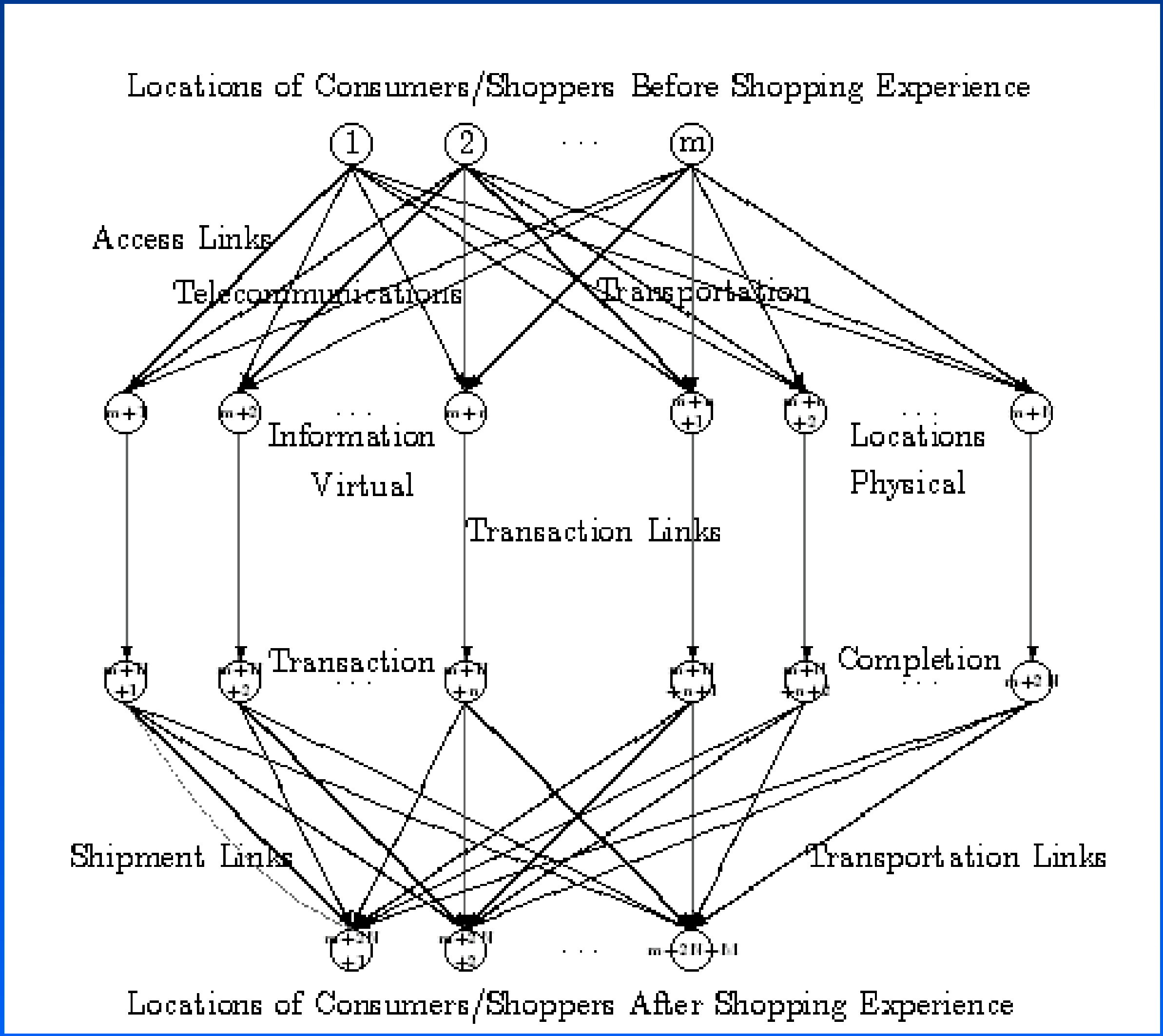
- Telecommuting/Commuting Decision-Making
- Teleshopping/Shopping Decision-Making
- Supply Chain Networks with Electronic Commerce
- Financial Networks with Electronic Transactions
- Reverse Supply Chains with E-Cycling
- Knowledge Networks
- Energy Networks/Power Grids
- Social Networks integrated with Economic Networks

A Conceptualization of Commuting versus Telecommuting



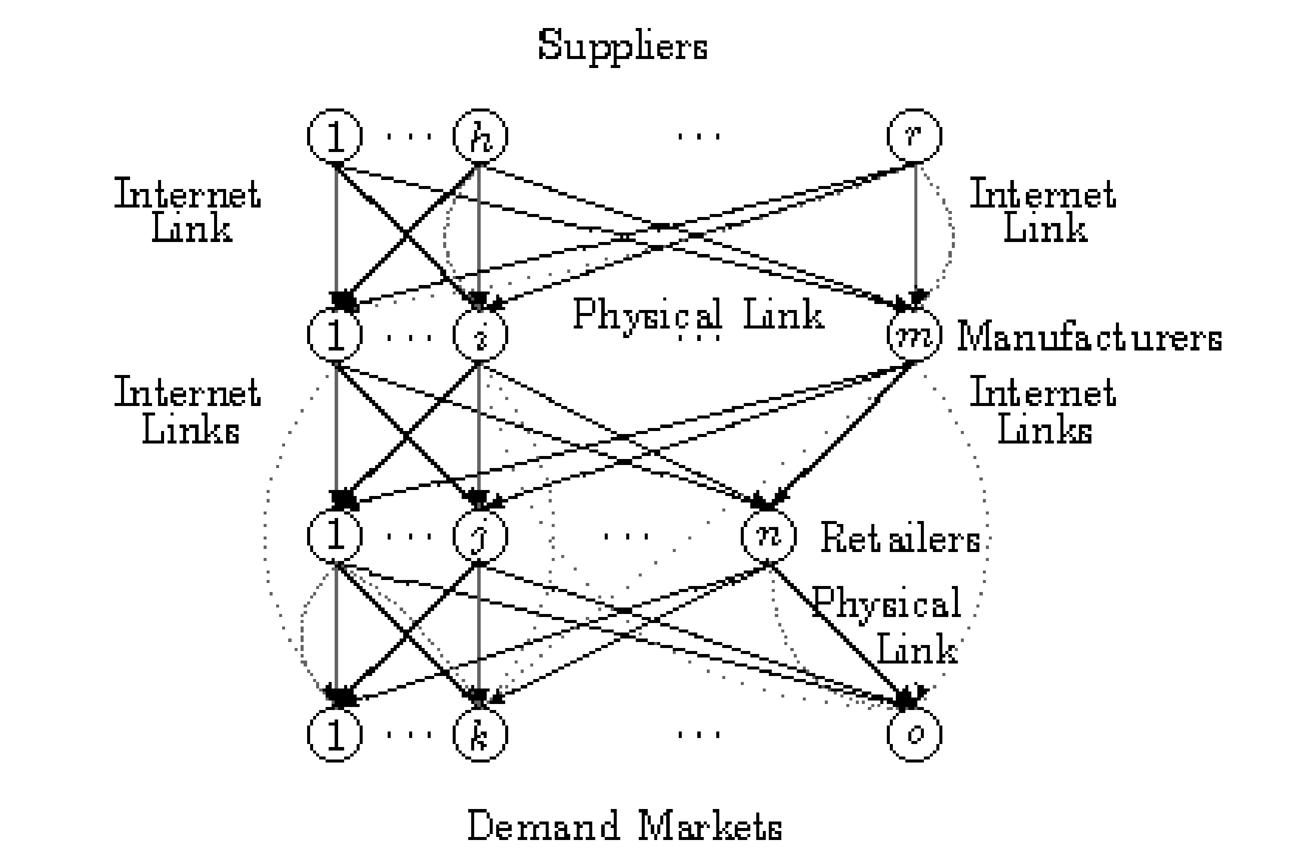
Nagurney, Dong, and Mokhtarian, *JEDC* (2002)

A Framework for Teleshopping versus Shopping



Nagurney, Dong, and Mokhtarian, *JEDC* (2002)

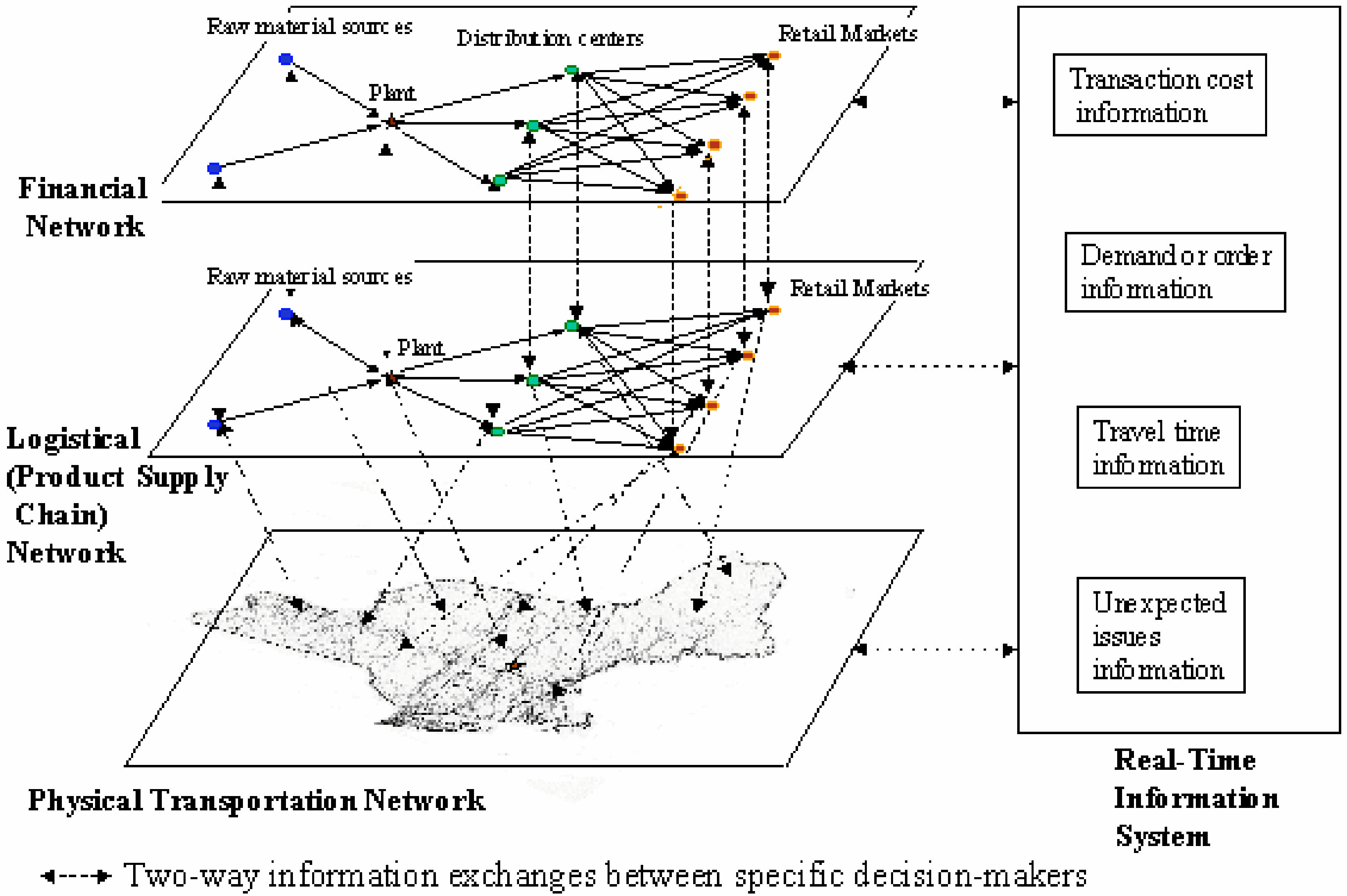
The Structure of a Supply Chain Network



Nagurney, Dong, and Zhang, *Transportation Research E* (2002)

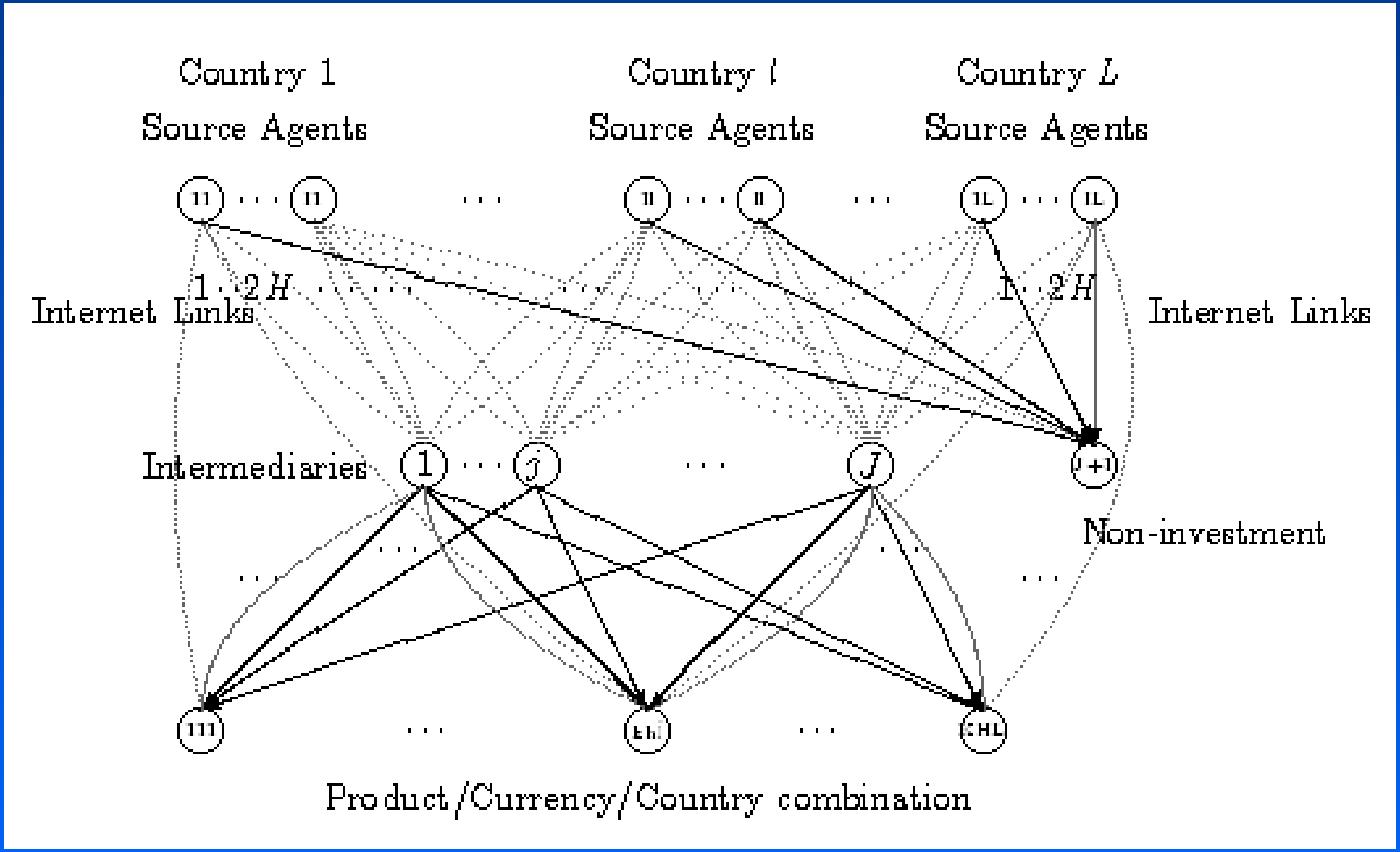
Nagurney, Loo, Dong, and Zhang, *Netnomics* (2002)

Supply Chain -Transportation Supernetwork Representation



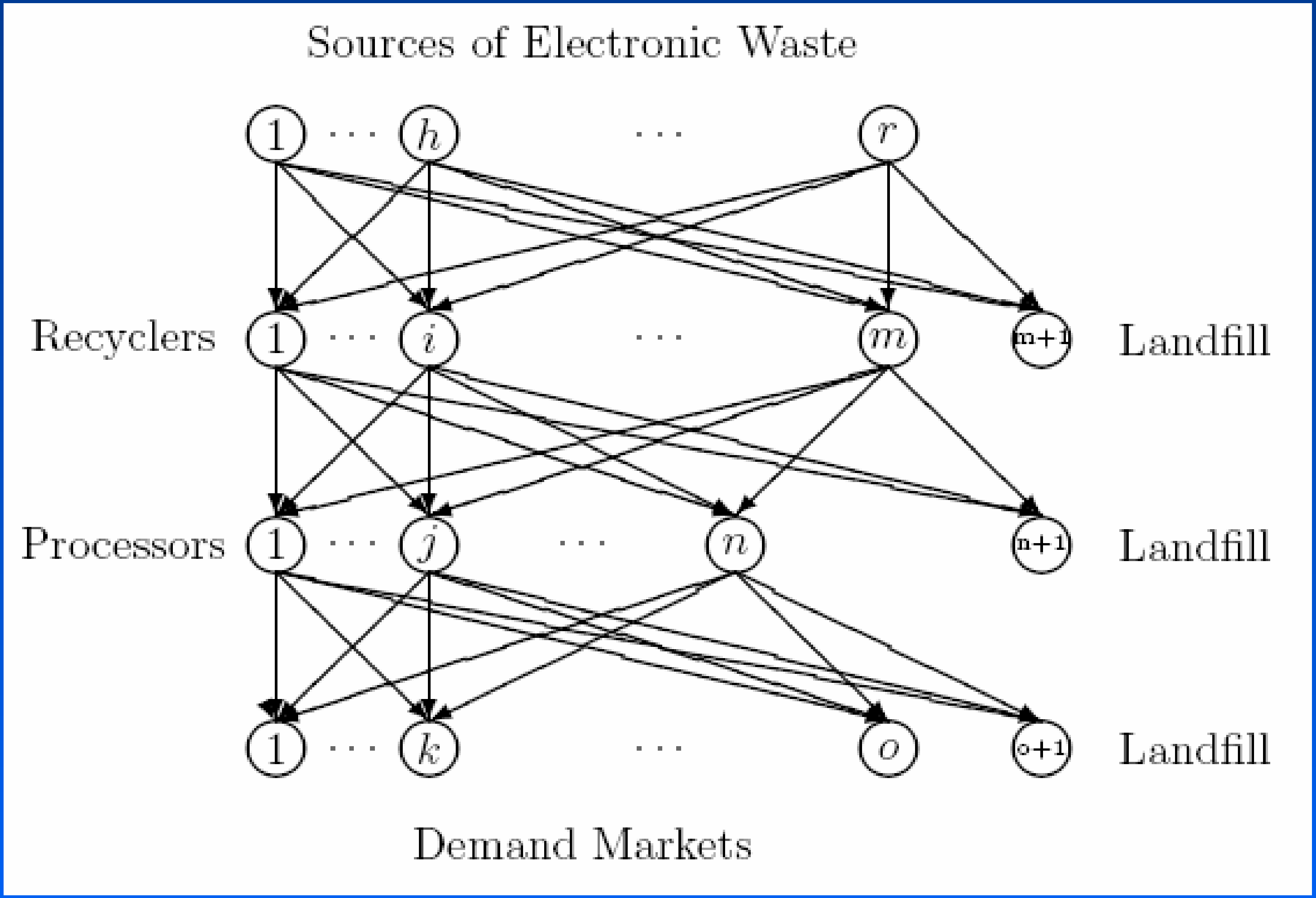
Nagurney, Ke, Cruz, Hancock, Southworth, *Environment and Planning B* (2002)

International Financial Networks with Electronic Transactions



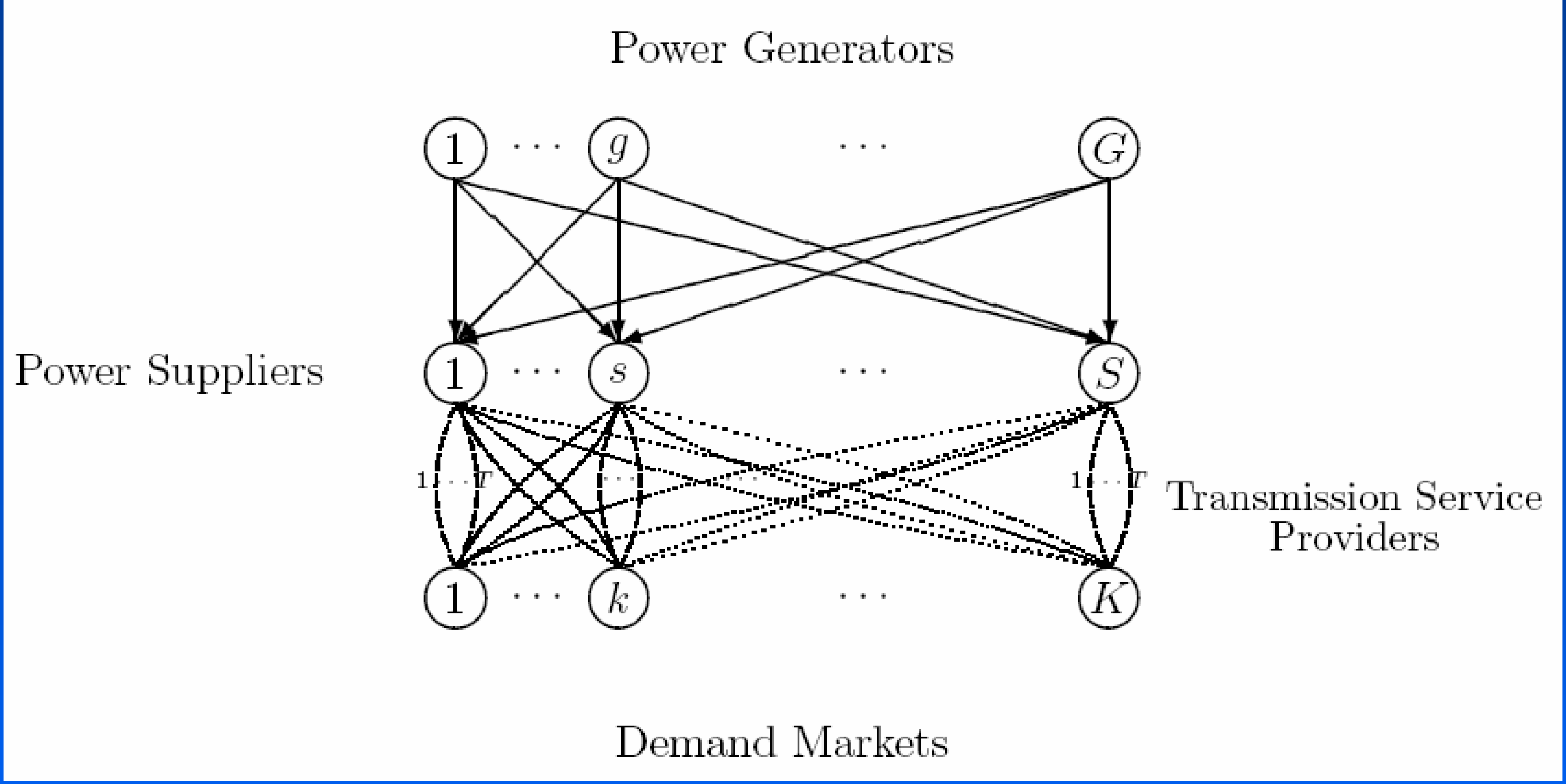
Nagurney and Cruz, in **Innovations in Financial and Economics Networks** (2003)

The 4-Tiered E-Cycling Network



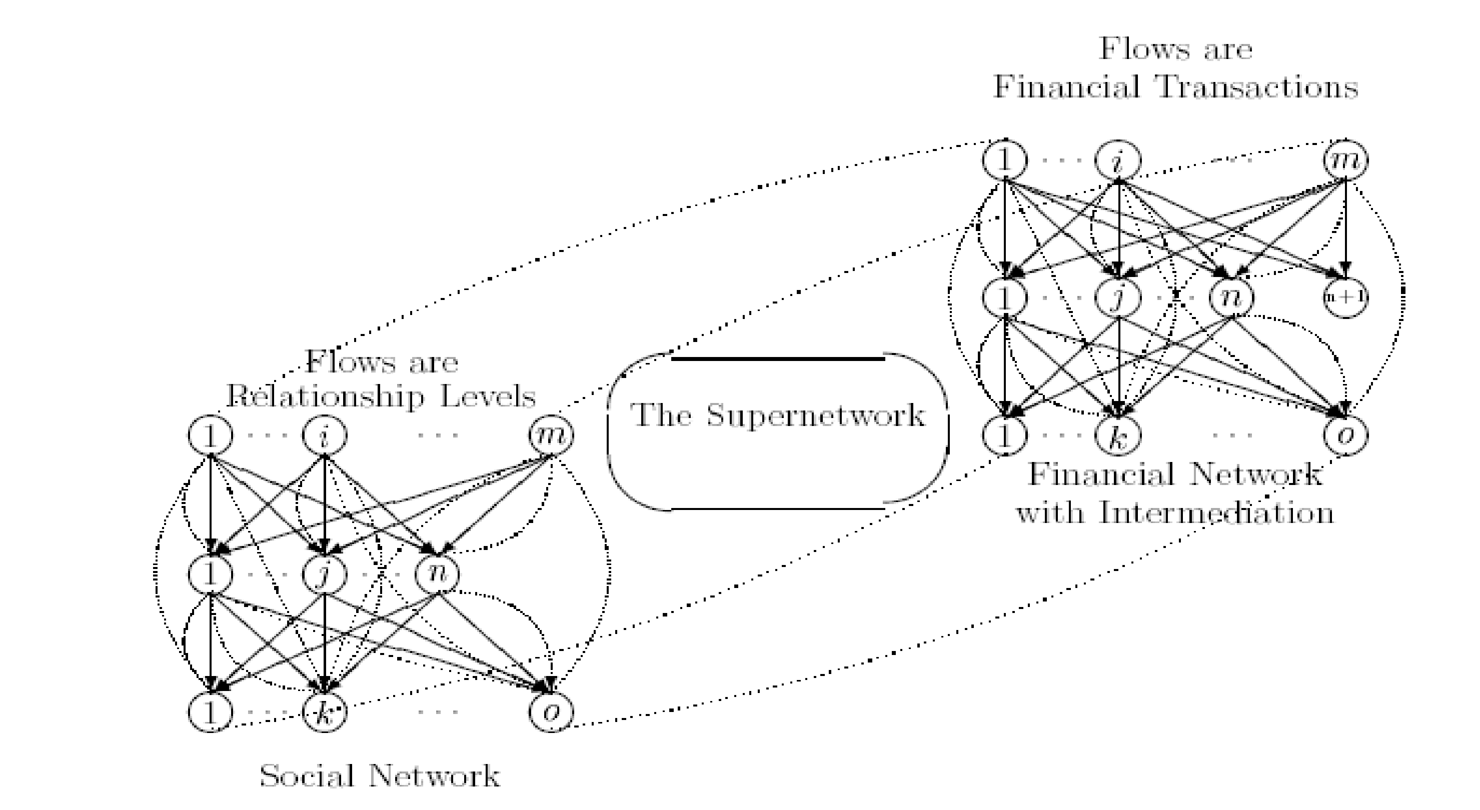
Nagurney and Toyasaki, *Transportation Research E* (2005)

The Electric Power Supply Chain Network



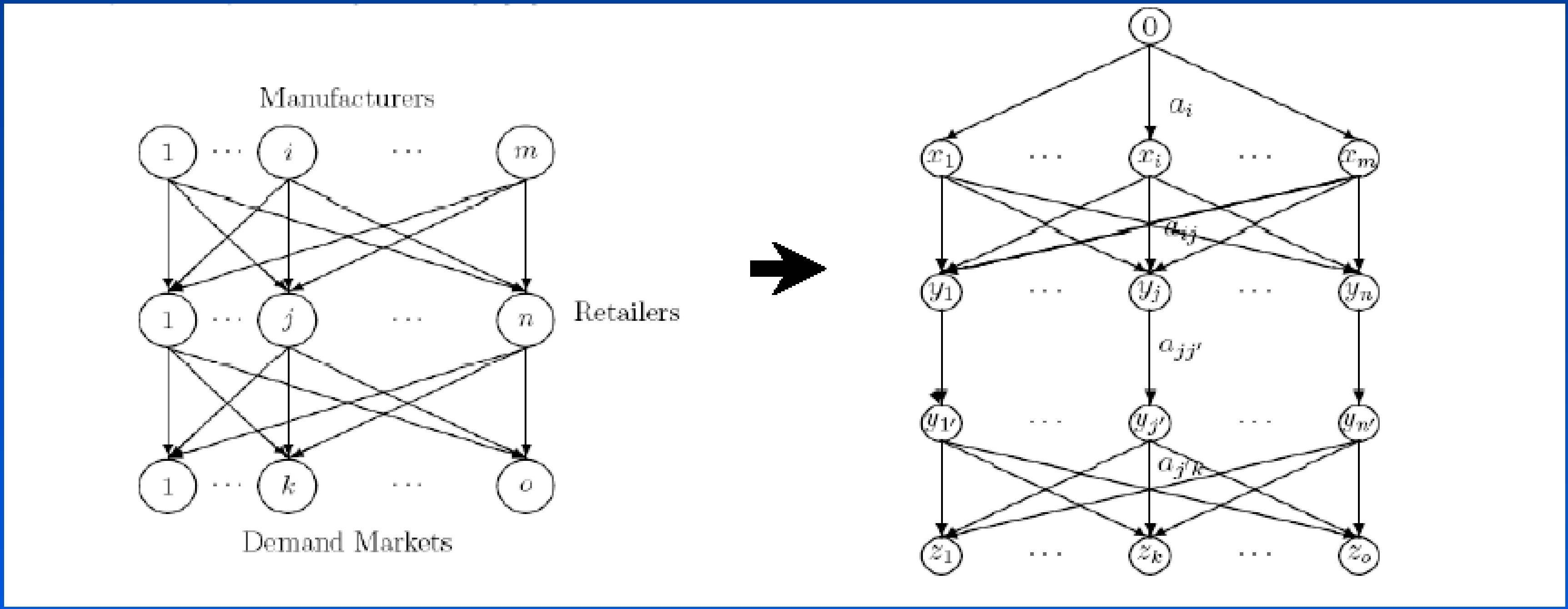
Nagurney and Matsypura, Proceedings of the *CCCT* (2004)

The Integrated Financial/Social Network System



Cruz, Nagurney, and Wakolbinger, *Naval Research Logistics* (2006)

The Equivalence of Supply Chain Networks and Transportation Networks

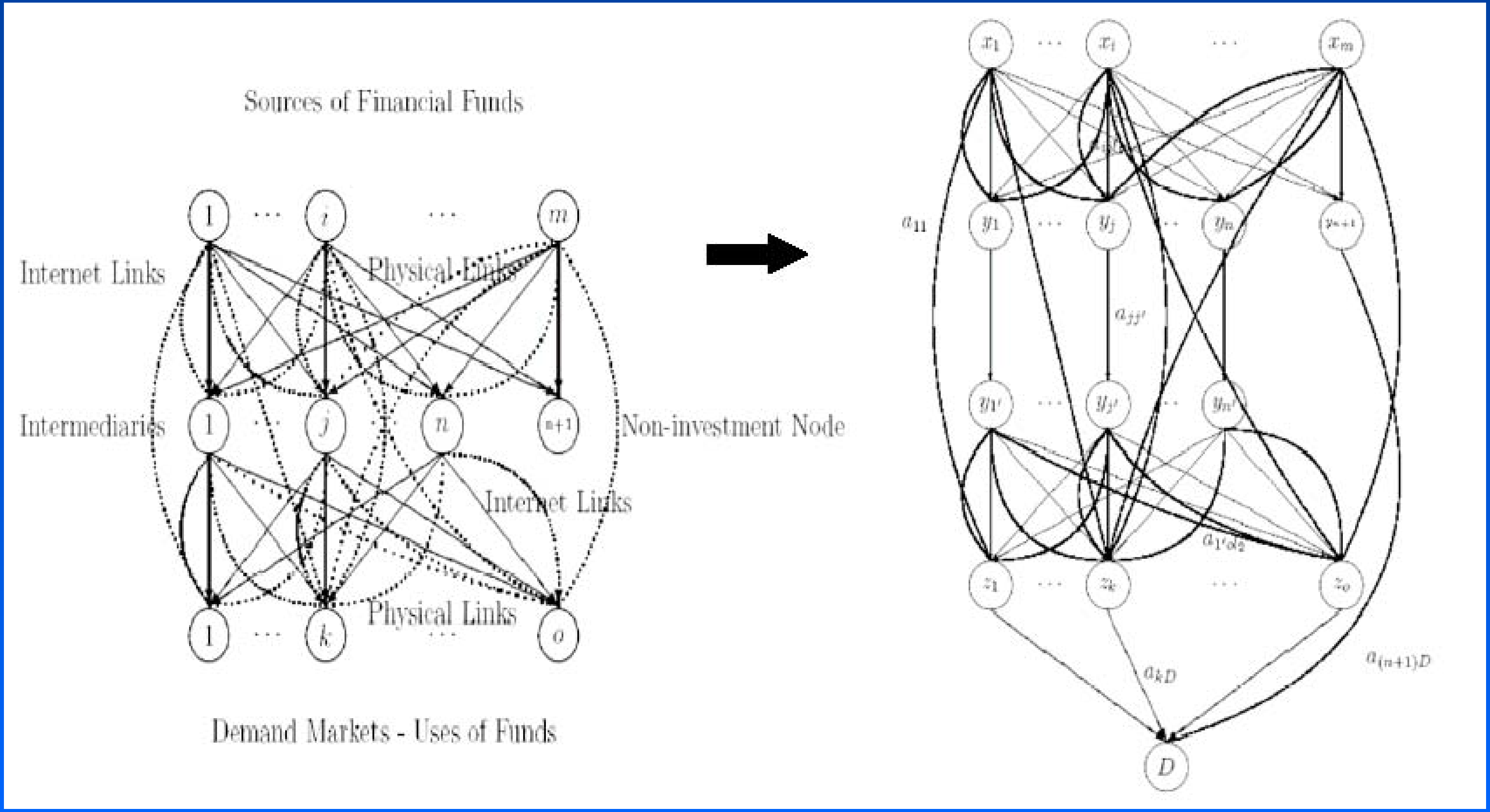


Nagurney, *Transportation Research E* (2006)

Copeland (1952) wondered whether money flows like water or electricity.

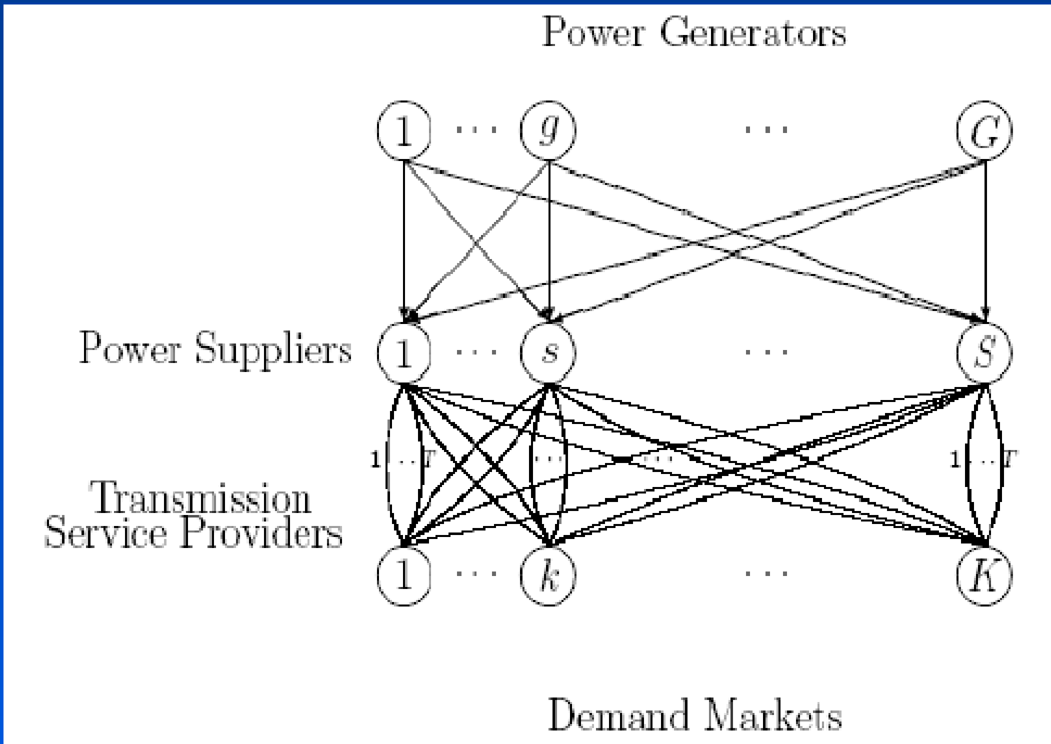
Liu and Nagurney have shown that money and electricity flow like transportation network flows (*Computational Management Science* (2006)).

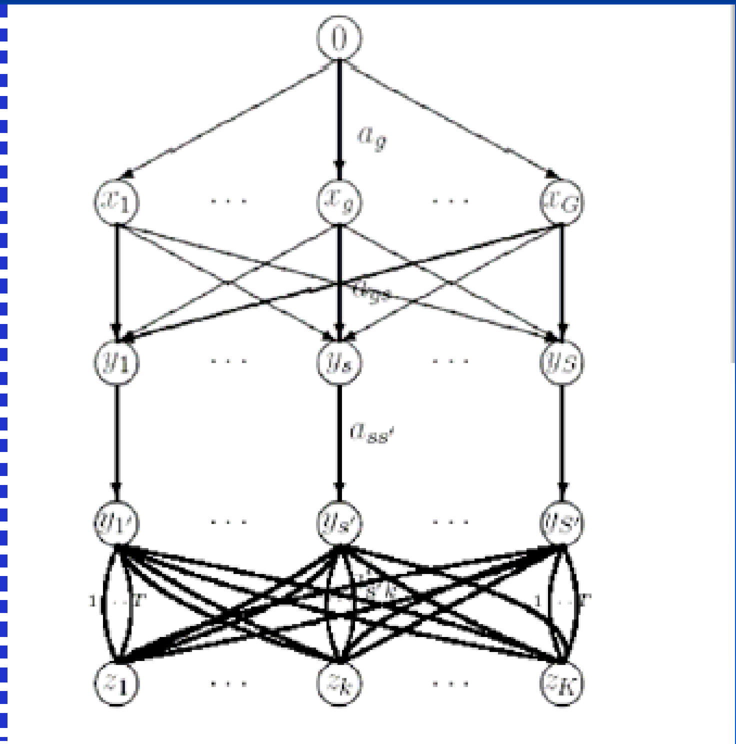
The Transportation Network Equilibrium Reformulation of the Financial Network Equilibrium Model with Intermediation



The fifth chapter of Beckmann, McGuire, and Winsten's book, **Studies in the Economics of Transportation** (1956) describes some *unsolved problems* including a single commodity network equilibrium problem that the authors imply could be generalized to capture electric power networks.

The Transportation Network Equilibrium Reformulation of Electric Power Supply Chain Networks





Electric Power Supply Chain Network

Transportation Network

Nagurney et al, to appear in *Transportation Research E*

We have, hence, shown that money as well as electricity flow like *transportation* and have answered questions posed fifty years ago by Copeland and Beckmann, McGuire, and Winsten, respectively.

New Tools

The tools that we are using in our Dynamic Network research include:

- network theory
- optimization theory
- game theory
- variational inequality theory
- evolutionary variational inequality theory
- projected dynamical systems theory
- double-layered dynamics theory
- network visualization tools.

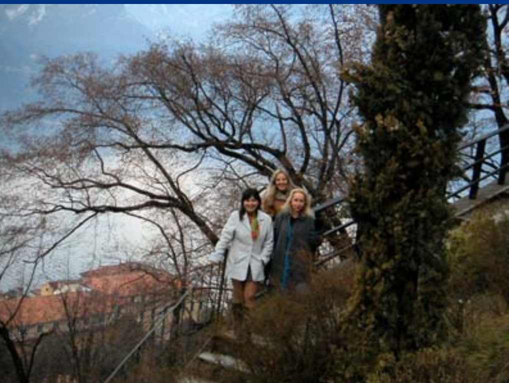
Dafermos (1980) showed that the traffic network equilibrium (also referred to as user-optimization) conditions as formulated by Smith (1979) were a finite-dimensional variational inequality.

In 1993, Dupuis and Nagurney proved that the set of solutions to a variational inequality problem coincided with the set of solutions to a projected dynamical system (PDS) in R^n.

In 1996, Nagurney and Zhang published **Projected Dynamical Systems and Variational Inequalities**.

In 2002, Cojocaru proved the 1993 result for Hilbert Spaces.

Bellagio Research Team Residency March 2004



We are working with Cojocaru and Daniele on infinite-dimensional projected dynamical systems and evolutionary variational inequalities and their relationships and unification.

This allows us to model dynamic networks with:

- *dynamic (time-dependent)* supplies and demands
- *dynamic (time-dependent)* capacities
- *structural changes* in the networks themselves.

Such issues are important for robustness, resiliency, and reliability of networks (including supply chains).

What happens if the demand is varied in the Braess Network?

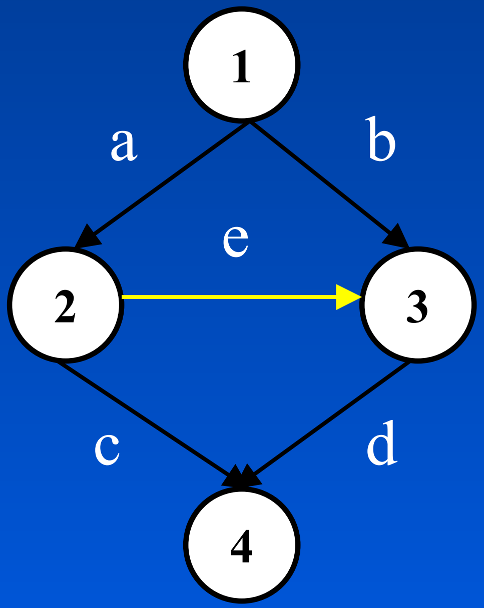
The answer lies in the solution of an Evolutionary (Time-Dependent) Variational Inequality.

Find $x^* \in K$, such that

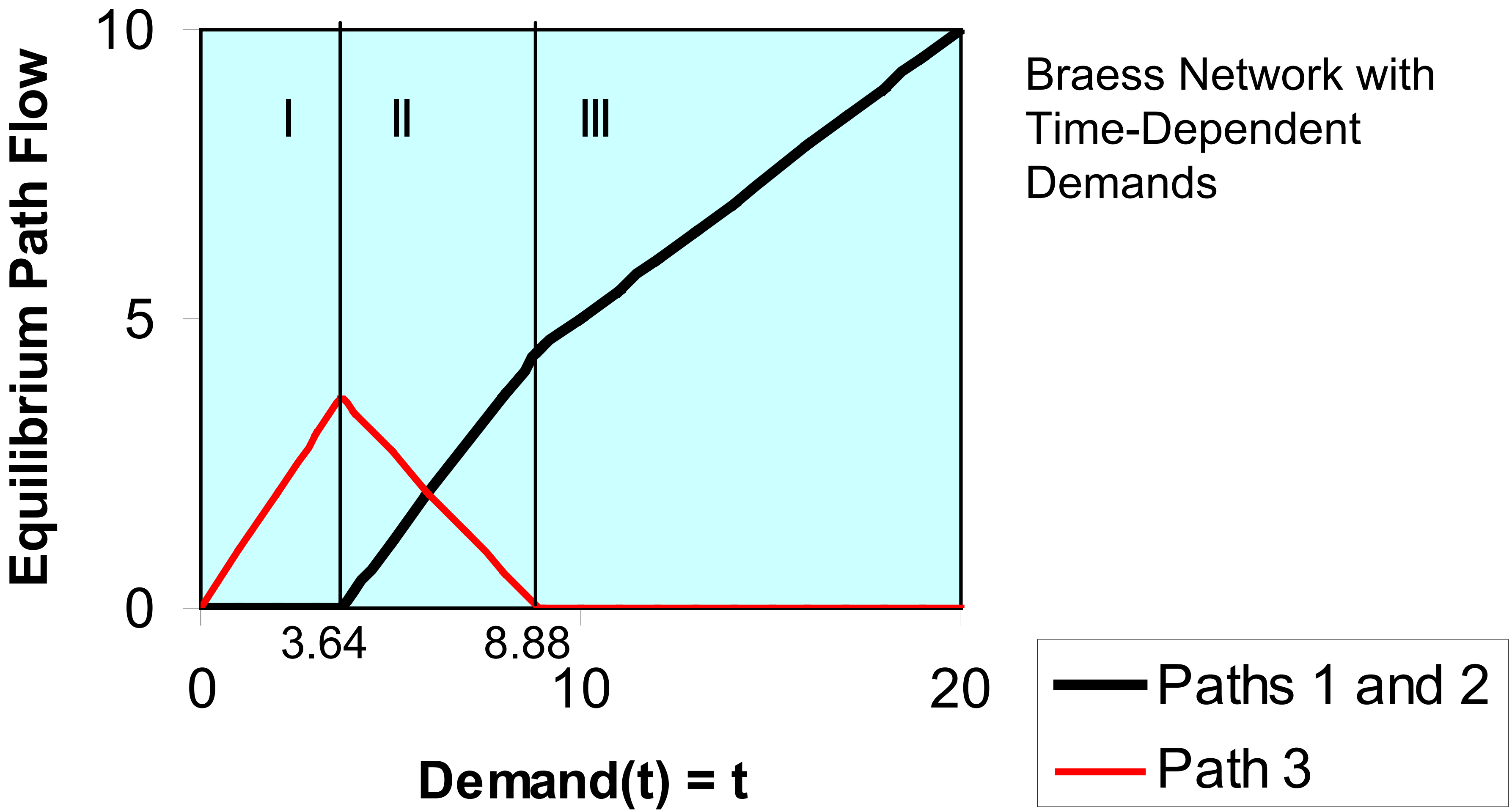
$$\int_0^T \langle C(x^*(t)), x(t) - x^*(t) \rangle \, dt \geq 0 \qquad \forall x \in K$$

Nagurney, Parkes, and Daniele, *Computational Management Science* (2006)

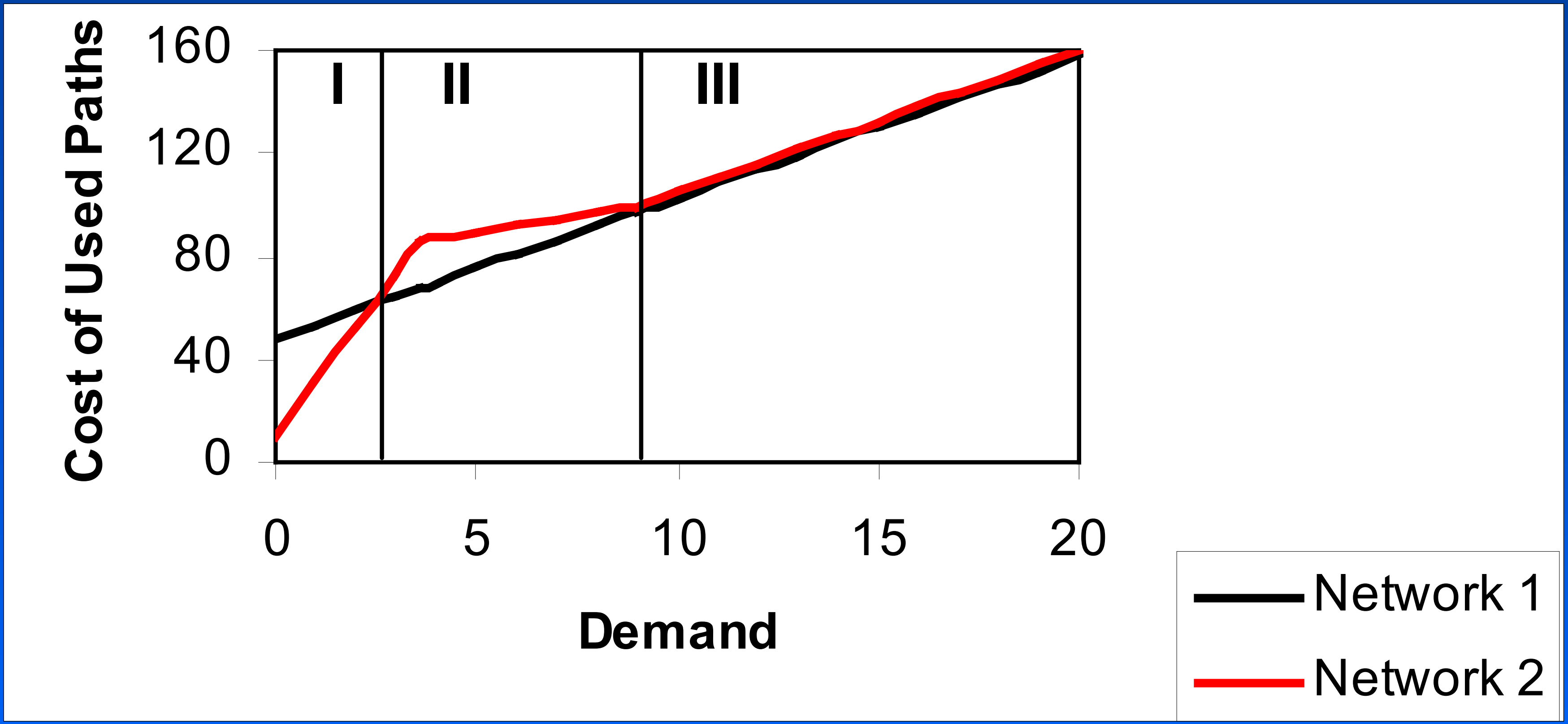
Recall the Braess Network
where we add the link e.



The Solution of an Evolutionary (Time-Dependent) Variational Inequality for the Braess Network with Added Link (Path)



In Demand Regime I, only the new path is used.
In Demand Regime II, the Addition of a New Link (Path) Makes Everyone Worse Off!
In Demand Regime III, only the original paths are used.



Network 1 is the Original Braess Network - Network 2 has the added link.

The new link is NEVER used after a certain demand is reached even if the demand approaches infinity.

Hence, in general, except for a limited range of demand, building the new link is a complete waste!

New Challenges and Opportunities: The Unification of EVIs and PDSs

Double-Layered Dynamics

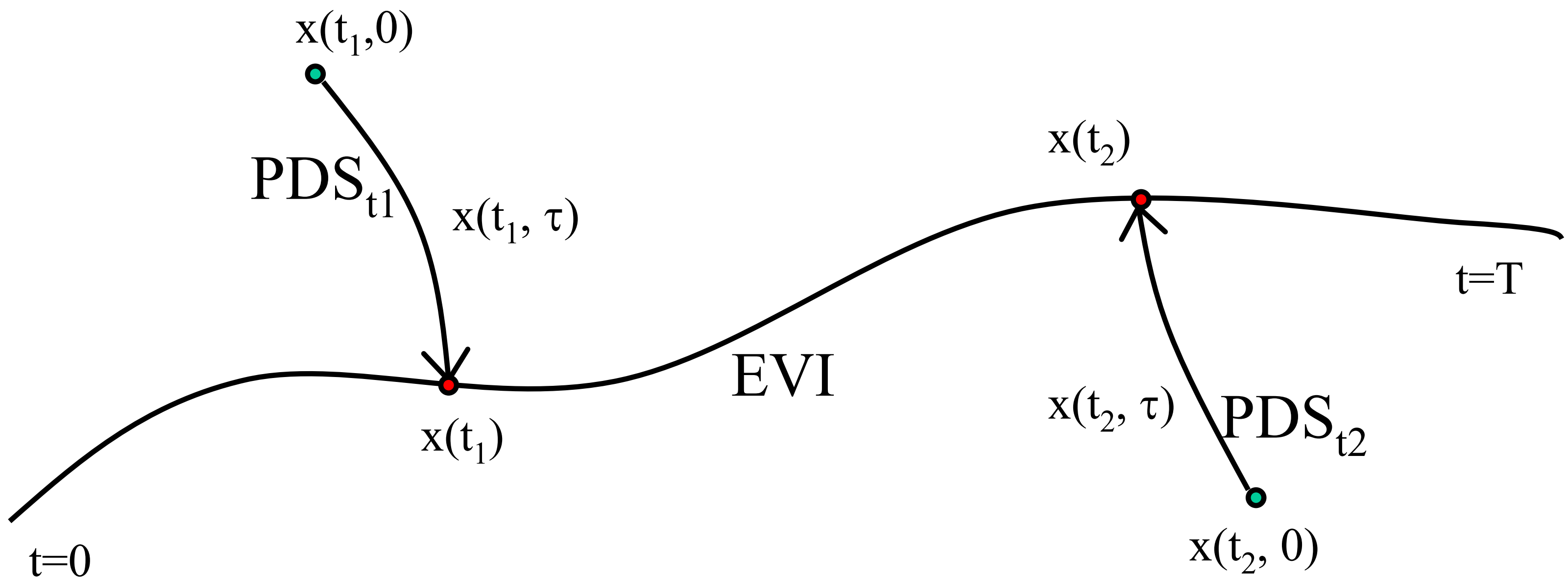
The unification of EVIs and PDSs allows the modeling of dynamic networks over *different time scales*.

Papers:

Projected Dynamical Systems and Evolutionary Variational Inequalities via Hilbert Spaces with Applications (Cojocaru, Daniele, and Nagurney), *Journal of Optimization Theory and Applications*, vol. 127, no. 3, pp. 1-15, December 2005.

Double-Layered Dynamics: A Unified Theory of Projected Dynamical Systems and Evolutionary Variational Inequalities (Cojocaru, Daniele, and Nagurney), *European Journal of Operational Research,* in press.

A Pictorial of the Double-Layered Dynamics



Cojocaru, Daniele, and Nagurney (2005) built the basis for merging the theory of projected dynamical systems (PDS) and that of evolutionary variational inequalities (EVI), in order to further develop the theoretical analysis and computation of solutions to applied problems in which dynamics plays a central role.

The intriguing feature of the merger is that it allows for the modeling of problems that present two (theoretically) distinct timeframes, most simply put, a *big scale time* and a *small scale time*.

The existing literature has focused on understanding human decision-making for a specific timescale rather than viewing decision-making over multiple timescales. The ability to capture multiple timescales can also further support combined strategic and operational decision-making and planning.

There are new exciting questions, both theoretical and computational, arising from this multiple time structure.

In the course of answering these questions, a new theory is taking shape from the synthesis of PDS and EVI, and, as such, it deserves a name of its own; we call it **double-layered dynamics.**

PrDE and PDS - I

The most general mathematical context to date in which we can define a projected differential equation (PrDE) and, consequently, a projected dynamical system (PDS), is that of a Hilbert space *X* of arbitrary (finite or infinite) dimension.

Suppose that we have $K \subset X$, a nonempty, closed, convex subset in a Hilbert space X. Let $F : K \to X$ be a Lipschitz continuous mapping. It is well-known that the ODE:

$$\frac{\partial x(t)}{\partial t} = -F(x(t)), \quad x(0) \in K$$

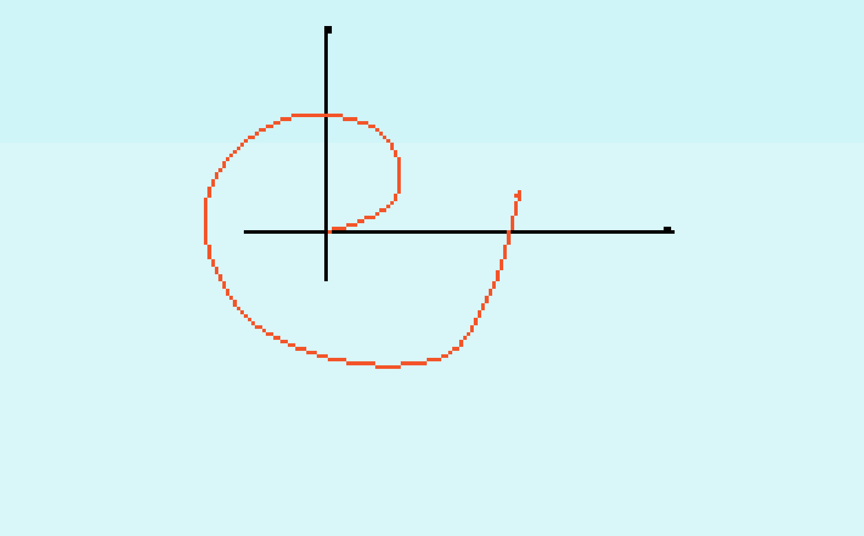
has solutions in a suitable class of functions; here that class will be that of absolutely continuous functions $\mathrm{AC}([0,\infty), X)$.

Let us define a PrDE on an example, *with drawings*

Suppose $X := R^2$, $K := R^2_+$, and suppose that the image below represents a trajectory of the equation

$$\frac{\partial x(t)}{\partial t} = -F(x(t)),$$

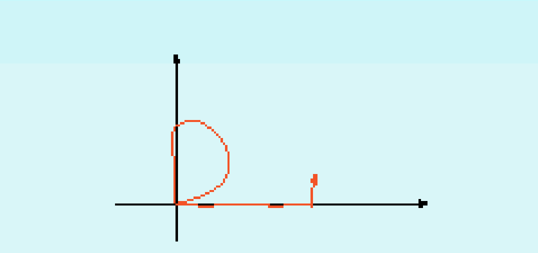
starting in R^2_+.



A PrDE describes the control problem:

$$\frac{\partial}{\partial t}(x(t)) = -F(x(t)), \quad x(0) \in R^2_+$$

such that $x(t) \in R^2_+$, as shown in the figure below:



In other words, a trajectory of a projected differential equation is always "trapped" in the constraint set $K = R^2_+$ and the velocity field along any such trajectory is not continuous.

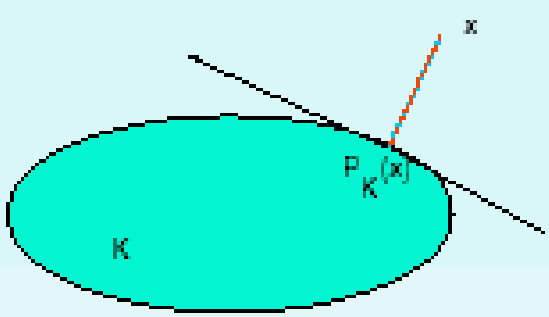
PrDE and PDS - II

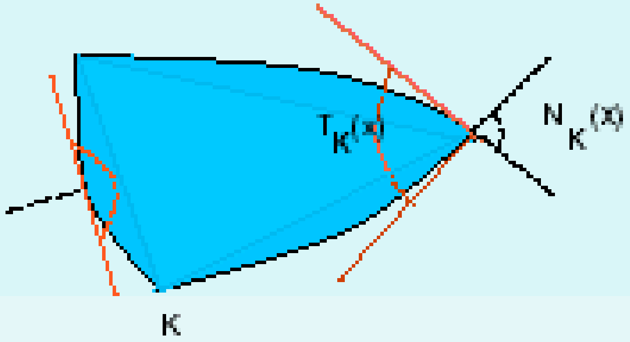
To rigorously define the two notions, we recall the following:

1). the projection of X onto K by $P_K : X \to K$, with

$$\|P_K(x) - x\| = \inf_{z \in K} \|z - x\|, \quad \forall z \in X,$$

2. the tangent cone $T_K(x) = \overline{\cup_{h>0} \frac{1}{h}(K - x)}$.





PrDE and PDS - III

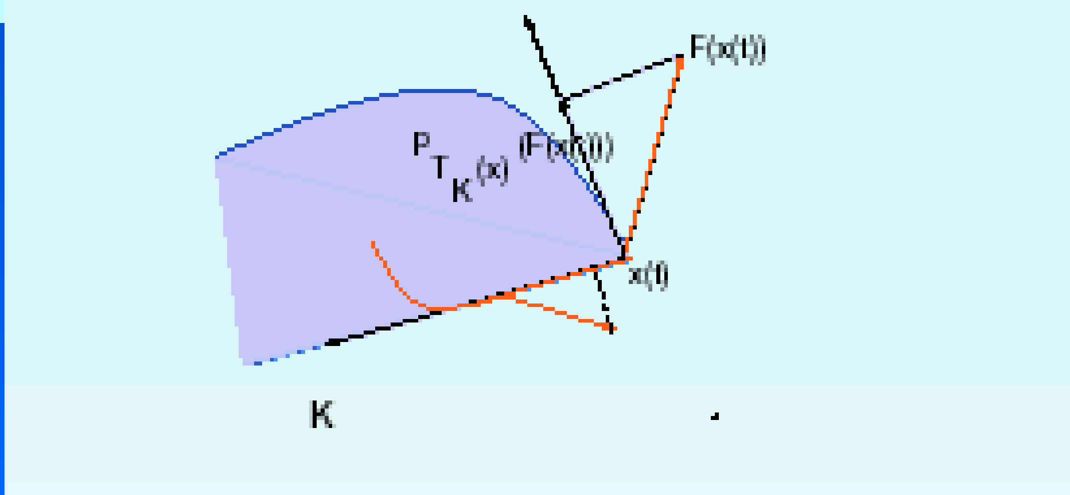
Let X, $K \subset X$, and $F : K \to X$ as before. Then a PrDE is defined by:

$$\frac{\partial}{\partial t}(x(t)) = \Pi_K(x(t), -F(x(t))), \quad x(0) = x_0,$$

where

$$\Pi_K(x, -F(x)) = \lim_{\delta \to 0^+} \frac{P_K(x - \delta F(x)) - x}{\delta} =: P_{T_K(x)}(-F(x)),$$

where $T_K(x)$ is the tangent cone to the set K at x and $N_K(x)$ is the normal cone to K at the same point x.



The right-hand side of any PrDE is nonlinear and discontinuous.

An existence result for such equations was obtained by Dupuis and Nagurney (1993) for $X:=R^n$, and by Cojocaru (2002) for general Hilbert spaces.

Theorem 1

Let X be a Hilbert space of arbitrary dimension and let $K \subset X$ be a non-empty, closed, and convex subset. Let $F : K \to X$ be a Lipschitz continuous vector field on K with $x_0 \in K$. Then the initial value problem

$$\frac{dx(t)}{dt} = \Pi_K(x(t), -F(x(t))), x(0) = x_0$$

has a unique solution in $AC([0, \infty), K)$.

A projected dynamical system (PDS) is the dynamical system given by the set of trajectories of a PrDE

EQUILIBRIA of PDS and VARIATIONAL INEQUALITIES

An important feature of any PDS is that it is intimately related to a variational inequality problem (VI).

The starting point of VI theory: 1966 (Hartman and Stampacchia); 1967 (Lions and Stampacchia); it is now part of the calculus of variations; it has been used to show existence of equilibrium in a plethora of equilibrium problems and free boundary problems.

The following relation between a PDS and a VI was shown by Dupuis and Nagurney (1993) for $X := R^n$ and by Cojocaru (2002) for any Hilbert space. Here $F : K \to X$.

Theorem 2

The equilibria of a PDS:

$$\frac{\partial}{\partial t}(x(t)) = \Pi_K(x(t), -F(x(t))),$$

that is, $x^* \in K$ *such that*

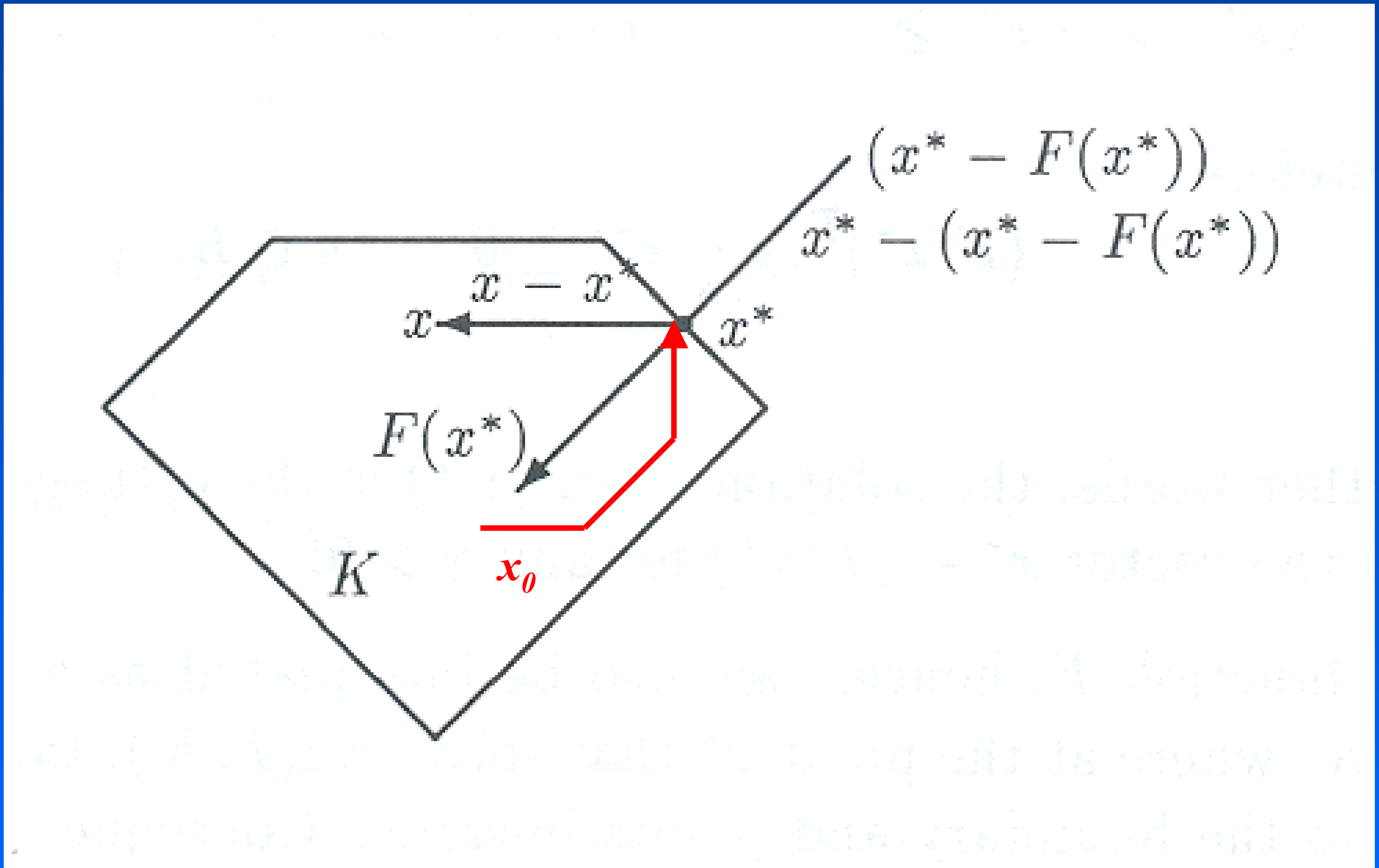
$$\Pi_K(x^*, -F(x^*)) = 0$$

are solutions to the $\mathrm{VI}(F, K)$*: find* $x^* \in K$ *such that*

$$\langle F(x^*), x - x^* \rangle \geq 0, \quad \forall x \in K,$$

and vice-versa, where $\langle \cdot, \cdot \rangle$ *denotes the inner product on* X*.*

A Geometric Interpretation of a Variational Inequality and a Projected Dynamical System



EVOLUTIONARY VARIATIONAL INEQUALITIES

Evolutionary variational inequalities, which are infinite dimensional, were originally introduced by Lions and Stampacchia (1967) and by Brezis (1967) in order to study problems arising principally from mechanics. They provided a theory for the existence and uniqueness of the solution of such problems.

Steinbach (1998) studied an obstacle problem with a memory term as a variational inequality problem and established existence and uniqueness results under suitable assumptions on the time-dependent conductivity.

Daniele, Maugeri, and Oettli (1998, 1999), motivated by dynamic traffic network problems, introduced evolutionary (time-dependent) variational inequalities to this application domain and to several others.

See also Ran and Boyce(1996).

A UNIFIED FEASIBLE SET and EVI FORMULATON (Cojocaru, Daniele, and Nagurney (2005))

We consider a nonempty, convex, closed, bounded subset of the reflexive Banach space $L^p([0,T],R^q)$ given by:

$$K = \bigcup_{t\in[0,T]} \{x \in L^p([0,T],R^q) \,|\, \lambda(t) \le x(t) \le \mu(t) \text{ a.e. in } [0,T];$$

$$\sum_{i=1}^{q} \xi_{ji} x_i(t) = \rho_j(t) \text{ a.e. in } [0,T], \xi_{ji} \in \{0,1\}, i \in \{1,..,q\},$$

$$j \in \{1,\ldots,l\}\}.$$

Let $\lambda, \mu \in L^p([0,T],R^q)$, $\rho \in L^p([0,T],R^l)$ be convex functions in the above definition. For chosen values of the scalars ξ_{ji}, of the dimensions q and l, and of the boundaries λ, μ, we obtain each of the previous above-cited model constraint set formulations as follows:

- for the traffic network problem (see Daniele, Maugeri, and Oettli (1998, 1999)) we let $\xi_{ji} \in \{0,1\}$, $i \in \{1,..,q\}$, $j \in \{1,\ldots,l\}$, and $\lambda(t) \ge 0$ for all $t \in [0,T]$;
- for the quantity formulation of spatial price equilibrium (see Daniele (2004)) we let $q = n + m + nm$, $l = n + m$, $\xi_{ji} \in \{0,1\}$, $i \in \{1,..,q\}$, $j \in \{1,\ldots,l\}$; $\mu(t)$ large and $\lambda(t) = 0$, for any $t \in [0,T]$;

- for the price formulation of spatial price equilibrium (see Daniele (2003)) we let $q = n + m + mn$, $l = 1$, $\xi_{ji} = 0$, $i \in \{1, .., q\}$, $j \in \{1, \dots, l\}$, and $\lambda(t) \geq 0$ for all $t \in [0, T]$;

- for the financial equilibrium problem (cf. Daniele (2003)) we let $q = 2mn + n$, $l = 2m$, $\xi_{ji} = \{0, 1\}$ for $i \in \{1, .., n\}$, $j \in \{1, \dots, l\}$; $\mu(t)$ large and $\lambda(t) = 0$, for any $t \in [0, T]$.

STANDARD EVI FORM (TIME-DEPENDENT) VARIATIONAL INEQUALITIES

Recall that $\ll \phi, u \gg := \int_0^T \langle \phi(t), u(t) \rangle dt$ is the duality mapping on $L^p([0,T], R^q)$, where $\phi \in (L^p([0,T], R^q))^*$ and $u \in L^p([0,T], R^q)$. Let $F : K \to (L^p([0,T], R^q))^*$.

The standard form of the evolutionary variational inequality (EVI) that we work with is:

find $x^* \in K$ such that $\ll F(x^*), x - x^* \gg \geq 0, \ \forall x \in K,$

or, equivalently, find $x^* \in K$ such that

$$\int_0^T \langle F(x^*(t)), x(t) - x^*(t) \rangle dt \geq 0, \quad \forall x \in K.$$

SOME PRELIMINARIES AND DEFINITIONS

In the general theory of variational inequalities, of which EVI are a part, as well as in Nonlinear Analysis and Optimization, the concept of monotone mappings and its extensions have been extensively used in existence uniqueness-type results.

From among the extensions of monotonicity, we recall here definitions of pseudomonotonicity, which are used throughout the analysis.

Definition

Let E be a reflexive Banach space with dual E^*, $\ll \cdot, \cdot \gg$ the duality map between E^* and E, K a non-empty closed, convex subset of E and $F : K \to E^*$. Then:

(1) A map F is called **pseudo-monotone** *on K if, for every pair of points $x, y \in K$, we have*

$$\langle F(x), y - x \rangle \geq 0 \to \langle F(y), y - x \rangle \geq 0.$$

(2) A map F is **strictly pseudo-monotone** *on K if, for every pair of distinct points x, y, we have*

$$\langle F(x), y - x \rangle \geq 0 \to \langle F(y), y - x \rangle > 0.$$

(3) A map F is **strongly pseudo-monotone** *on K if, there exists $\eta > 0$ such that, for every pair of distinct points x, y, we have*

$$\langle F(x), y - x \rangle \geq 0 \to \langle F(y), y - x \rangle \geq \eta ||y - x||^2.$$

Daniele, Maugeri, and Oettli (1998) gave an existence result for an EVI as above:

Theorem 3

If F satisfies either of the following conditions:

1. *F is hemicontinuous with respect to the strong topology on K, and there exist $A \subseteq K$ nonempty, compact, and $B \subseteq K$ compact such that, for every $v \in K \setminus A$, there exists $v \in B$ with $\ll F(u), v-u \gg \geq 0$;*

2. *F is hemicontinuous with respect to the weak topology on K;*

3. *F is pseudomonotone and hemicontinuous along line segments,*

then the EVI problem above admits a solution over the constraint set K.

DOUBLE-LAYERED DYNAMICS: MERGING PDSs and EVIs

The theory of EVI and that of PDS can be intertwined for the purpose of deepening the analysis of many dynamic applied problems arising in different disciplines. The fundamental theoretical ideas, together with an example of such problems, specifically, a dynamic traffic network problem, were given in Cojocaru, Daniele, and Nagurney (2005). However, the implications of one theory over the other have to be further studied.

Here we continue to develop and consolidate the mathematical formalism of this new emerging theory which we call double-layered dynamics, thus opening up new questions as topics for future work.

First and foremost, we have seen that the EVI considered involves a constraint set of a Banach space, but to be used in conjunction with PDS theory, we need to limit ourselves to Hilbert spaces; therefore, we set $p := 2$ and consider only constraint sets $K \in L^2([0,T], R^q)$, as given.

By definition, such sets are closed and convex.

Also note that the elements in the set K vary with time, but K is fixed in the space of functions $L^2([0,T], R^q)$, $T > 0$ given.

Double-Layered Dynamics

Consider the above (EVI), where F is pseudomonotone and Lipschitz continuous and $K \in L^2([0,T], R^q)$ is given as above.

Lipschitz continuity implies hemicontinuity, which, in turn, implies hemicontinuity on line segments, so according to Theorem 3, the EVI problem has solutions.

We are also in the scope of Theorem 1, and, therefore, we can consider the PDS defined on the closed and convex set K by the PrDE:

$$\frac{dx(\cdot,\tau)}{d\tau} = \Pi_K(x(\cdot,\tau), -F(x(\cdot,\tau))),$$

$$x(\cdot,0) = x(\cdot) \in K,$$

where time τ is different than time t in the EVI. In general, the PDS has solutions in the set of absolutely continuous functions in the τ variable, $AC([0,\infty), K)$. However, we will limit ourselves to finite intervals for τ, i.e., with $\tau \in [0,l]$, $l > 0$, given.

Theorem 4 (Cojocaru, Daniele, and Nagurney (2005))

The solutions to the EVI problem are the same as the critical points of the PDS and vice versa, that is, the critical points of the PDS are the solutions to the EVI.

Hence, by choosing the Hilbert space to be $L^2([0,T], R^q)$, we find that the solutions to the evolutionary variational inequality: find $x^* \in K$ such that

$$\int_0^T \langle F(x^*(t)), x(t) - x^*(t) \rangle dt \geq 0, \quad \forall x \in K$$

are the same as the critical points of the equation:

$$\frac{\partial x(t,\tau)}{\partial \tau} = \Pi_K(x(t,\tau), -F(x(t,\tau))),$$

that is, the points such that

$$\Pi_K(x(t,\tau), -F(x(t,\tau))) \equiv 0 \quad a.e. in \ [0,T],$$

which are obviously stationary with respect to τ.

This result is the most important feature in merging the two theories and in computing and interpreting problems ranging from spatial price (quantity and price formulations), traffic network equilibrium problems, and general financial equilibrium problems.

Now we are ready to answer the question of uniqueness of solutions to the EVI. It is known that, in general, strict monotonicity implies uniqueness of solutions for a variational inequality (Stampacchia (1968)) and, hence, if F is strictly monotone, then it is pseudomonotone and the solution to the EVI is unique.

This is not so in the PDS theory, where it is easy to show that if F is only strictly pseudomonotone, but not strictly monotone, the PDS still has a unique equilibrium.

Proposition 1 (Cojocaru, Daniele, and Nagurney (2005b))

Assume that F is strictly pseudomonotone and Lipschitz on K. Then the PDS has at most one equilibrium point.

Here is a direct, important consequence of the new theory of double-layered dynamics:

Proposition 2

Assume either one of the hypotheses (2) or (3) of Theorem 3, where F is strictly pseudomonotone on K and assume DLDH. Then the EVI has at most one solution.

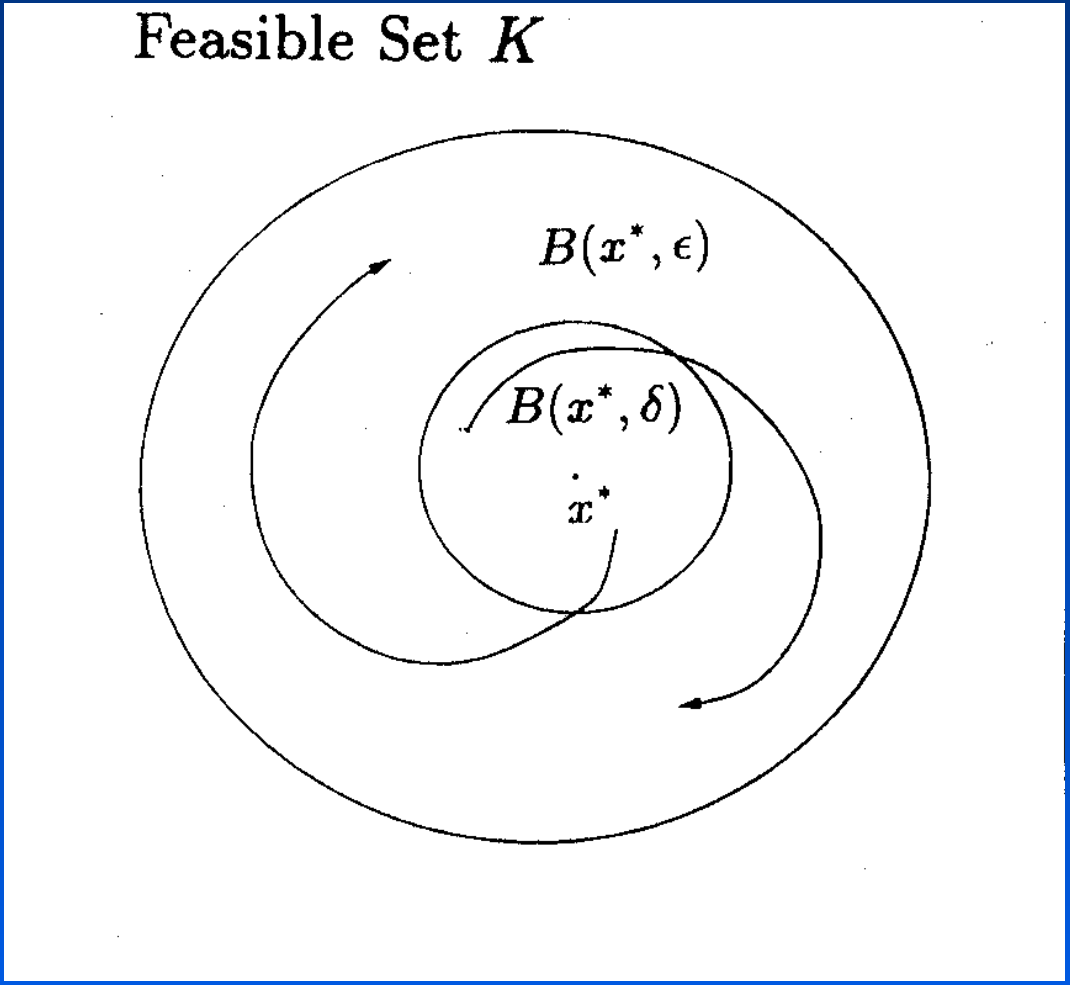
STABILITY PROPERTIES of the CURVE of EQUILIBRIA; the RELATION BETWEEN the TWO TIME-FRAMES

We now address the stability properties of solution(s) to the EVI, viewed as curves of equilibria for PDS. We also make precise the relation between PDS time and EVI time, together with its meaning in applications.

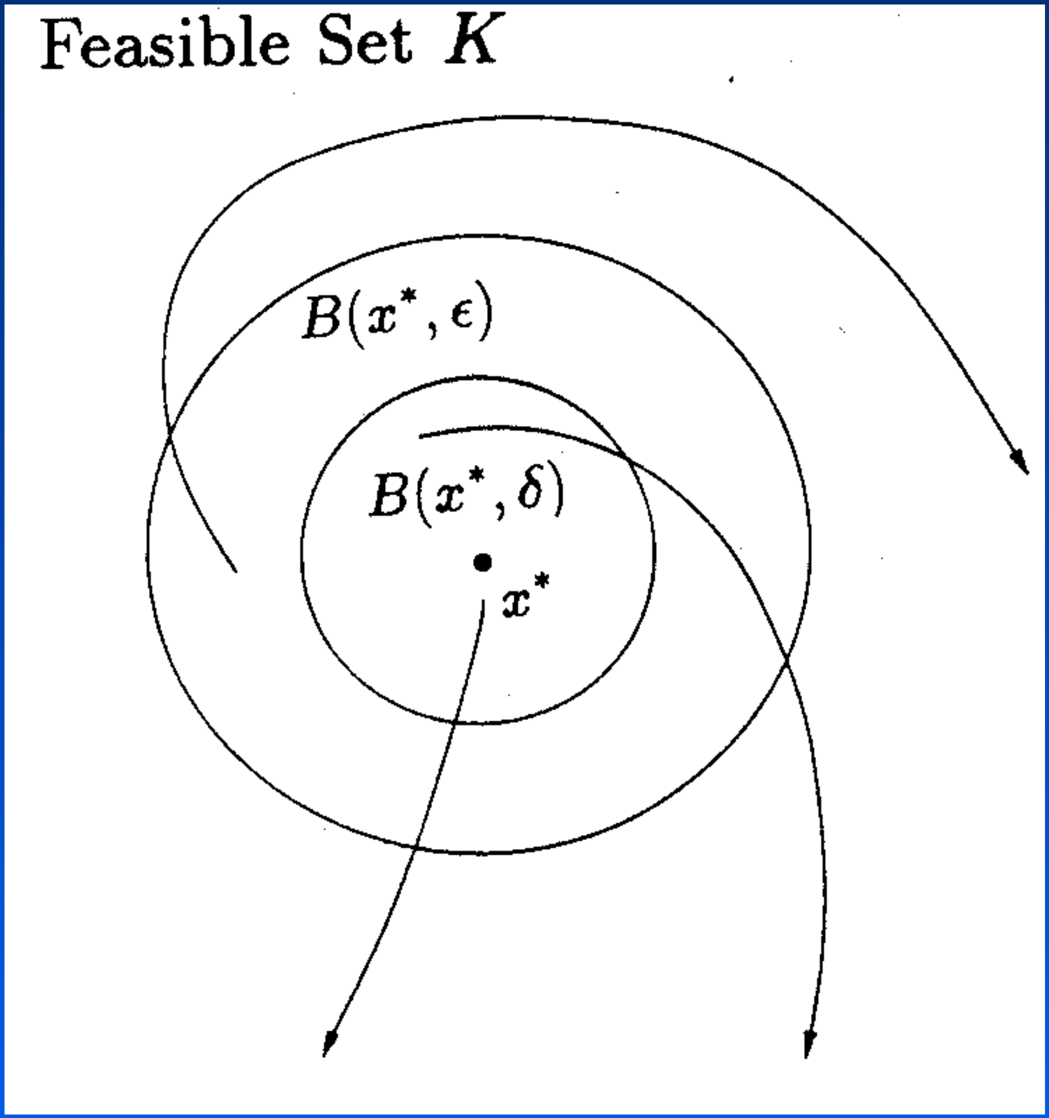
The assumption of pseudomonotonicity is vital to the existence of EVI solutions, but not so for solutions to PDS.

However, it plays a very important role in the stability study of perturbed equilibria of PDS, more precisely, in the study of the local/global properties of the projected systems around these equilibria.

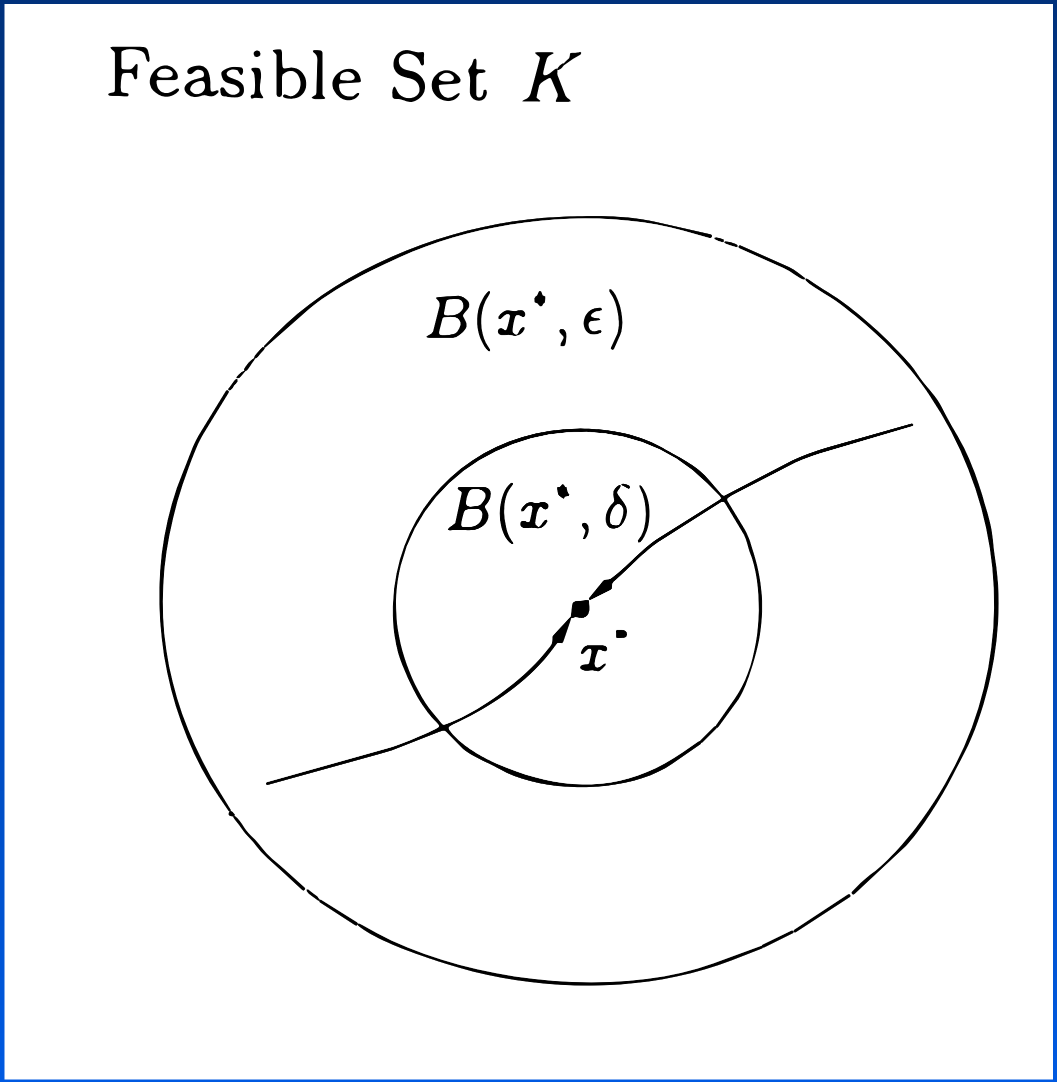
This stability question remains meaningful in the double-layered dynamics theory, where we seek to unravel the behavior of perturbations of the curve(s) of equilibria.



A Stable Equilibrium Point



An Unstable Equilibrium Point



A Finite Time Attractor

Three Important Questions

We see next that pseudomonotonicity-type conditions fully answer three important questions along the lines of our remarks above:

1. Is it accurate to expect that for almost all $t \in [0, T]$ given, the trajectories of the PDS at t (which we denote by PDS_t) evolve towards the curve of equilibria?

2. What is the relation between an arbitrarily chosen $t \in [0, T]$ and the time it takes for solutions to PDS_t to actually reach the curve of equilibria?

3. What is the interpretation of the double-layered dynamics for applications?

Definition

Let X be a Hilbert space, $K \subset X$ closed, convex subset.
(1) A point $x^* \in K$ is called a **local monotone attractor for the PDS** *if there exists a neighborhood V of x^* such that the function $d(t) := ||x(t) - x^*||$ is a non-increasing function of t, for any solution $x(t)$ of the PDS, starting in the neighborhood V.*
(2) A point $x^* \in K$ is a **local strict monotone attractor** *if the function $d(t)$ is decreasing.*

A point $x^* \in K$ is a **global** *monotone attractor (respectively a* **global** *strict monotone attractor) if conditions (1) and (2) are satisfied for solutions starting at any point of K.*

It is not difficult to see that the notion of monotone attractor and that of an attractor are different. For example, a monotone attractor is not necessarily an attractor, if say $d(x,t)$ decreases for $t \in [0, t_1]$ and remains constant in time for $t \geq t_1$, for some $t_1 \in R_+$. In the same way, an attractor is not necessarily a monotone attractor, unless $d(x,t)$ is monotonically decreasing to zero.

Answer to Question 1

Theorem 5 (Cojucaru, Daniele, and Nagurney (2005b))

Assume $F : K \rightarrow L^2([0,T], R^q)$ is Lipschitz continuous on K and consider the EVI and the PDS. Then the following hold:

1. *if F is (locally) pseudomonotone on K, then the curve(s) of equilibria (solution(s) of EVI) is(are) a (local) monotone attractor;*
2. *if F is (locally) strictly pseudomonotone on K, then the unique curve of equilibria is a (local) strict monotone attractor;*
3. *if F is (locally) strongly pseudomonotone on K, then the unique curve of equilibria is exponentially stable and a (local) attractor.*

Answer to Question 2

Answer to question (2). The stability properties of the curve of equilibria as a whole, given by Theorem 5, show that the curve is attracting solutions of almost all PDS_t and that it is possible for the curve to be reached for some of the moments $t \in [0, T]$.

To answer question (2), we start first by noticing that for almost all $t \in [0, T]$, arbitrarily fixed, we can identify a closed and convex subset $K_t \in R^q$, given by

$$K_t := \{x(t) \in R^q \,|\, \lambda(t) \le x(t) \le \mu(t);\ \lambda(t), \mu(t) \text{ given;}$$

$$\sum_{i=1}^{q} \xi_{ji} x_i(t) = \rho_j(t),\ \xi_{ji} \in \{0, 1\}, i \in \{1, .., q\}, j \in \{1, \ldots, l\}\}.$$

Evidently, to each such fixed t, we have a PDS_t given by

$$\frac{dx(t, \tau)}{d\tau} = \Pi_{K_t}(x(t, \tau), -F(x(t, \tau))),\ x(t, 0) = x_0^t \in K_t.$$

We recall the following definition (Zhang and Nagurney (1995)).

Definition

A map F is called **strongly pseudo-monotone with degree** α *on* K *if, there exists* $\eta > 0$ *such that, for every pair of distinct points* x, y, *we have*

$$\langle F(x), y - x \rangle \geq 0 \rightarrow \langle F(y), y - x \rangle \geq \eta ||y - x||^{\alpha}.$$

Evidently, if *F* is strongly pseudo-monotone with degree α, it is strictly pseudo-monotone. Hence the EVI gives a unique curve of equilibria.

The Answer to Question 2

Theorem 6 (Cojocaru, Daniele, and Nagurney (2005b))

Consider the above EVI with F Lipschitz continuous and strongly pseudo-monotone with degree $\alpha < 2$ on K, for almost all fixed $t \in [0, T]$, there exists $l_t > 0$, finite, such that the unique equilibrium $x^* := x^*(t)$ of the PDS_t is reached by the (unique) solution $x(t, \tau)$ of the PDS_t, starting at the initial point $x_0^t \in K_t$. The time l_t depends upon η, α and $||x_0^t - x^*||$.

We have proved that for each $x_0^* \in K_t$, there exists $l_t < \infty$, depending on $\eta, \alpha, ||x_0^t - x^*||$, given by

$$l_t := \frac{||x_0^t - x^*||^{2-\alpha}}{(2-\alpha)\eta},$$

such that whenever $\alpha < 2$,

$$D(\tau) > 0 \text{ when } \tau < l_t \text{ and } D(\tau) = 0 \text{ when } \tau \geq l_t.$$

In other words, x^* is a globally finite-time attractor for the unique solution of PDS_t starting at x_0^t and it will be reached in l_t units of time.

Answer to Question 3

Answer to question (3). In real life, there is only one concept of time in terms of a timeline. Therefore, in applications it is important to have a clear, easy way to estimate if, under what conditions, and, when, the curve of equilibria is reached. Theorem 6 provides exactly the desired answer: for almost any $t \in [0, T]$, we can estimate that the equilibrium on the curve corresponding to t will be reached in the time l_t if and only if

$$t \geq l_t := \frac{||x_0^t - x^*||^{2-\alpha}}{(2-\alpha)\eta}.$$

Otherwise, although the equilibrium can be computed, the solution to PDS_t does not have enough time to reach the curve.

But l_t depends intrinsically upon three parameters,

$$\eta, \quad \alpha, \quad ||x_0^t - x^*||,$$

two of which are given by F. Hence, we have, in fact, only one that we can manipulate, and that is $||x_0^t - x^*||$, i.e., the distance between the initial point of the trajectory and the equilibrium x^* at t.

Naturally, if we want to find/compute those solutions that will be arriving on the curve of equilibria at a fixed moment t, all we have to do is to make sure that we choose a trajectory of the PDS_t starting at a distance $||x_0^t - x^*||$ from the curve, so that the above is satisfied.

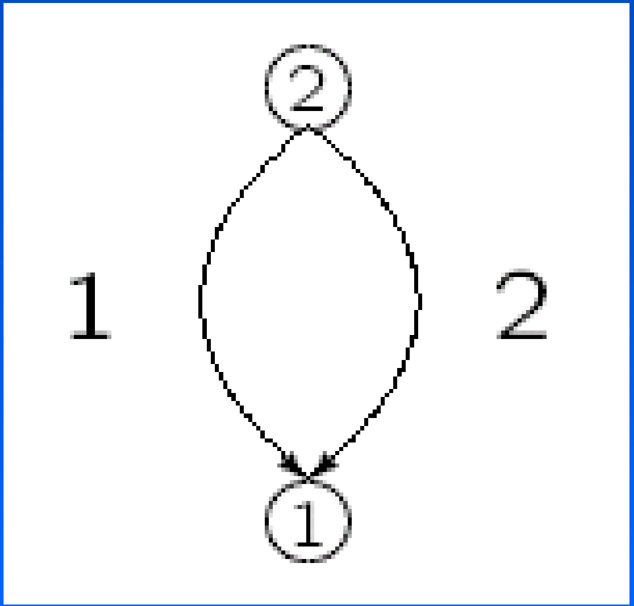
To solve the associated evolutionary variational inequality, we discretize the time horizon T and the corresponding variational inequality (or, equivalently, projected dynamical system) at each discrete point in time is then solved.

Obviously, this procedure is correct if the continuity of the solution is guaranteed.

Continuity results for solutions to evolutionary variational inequalities, in the case where $F(x(t)) = A(t)x(t) + B(t)$ is a linear operator, $A(t)$ is a continuous and positive definite matrix in $[0, T]$, and $B(t)$ is a continuous vector can be found in Barbagallo (2005).

A Dynamic Network Example with Time-Varying Demand and Capacities

We consider a network consisting of a single origin/destination pair of nodes and two paths connecting these nodes.



Let cost on path **1** be: $\mathbf{2x_1(t)-1.5}$ and cost on path **2** be: $\mathbf{x_2(t)-1}$.

The demand is **t** in the interval **[0,2].**

Suppose that we also have capacities**:**

$\mathbf{(0,0) \leq (x_1(t), x_2(t)) \leq (t, 3/2\, t).}$

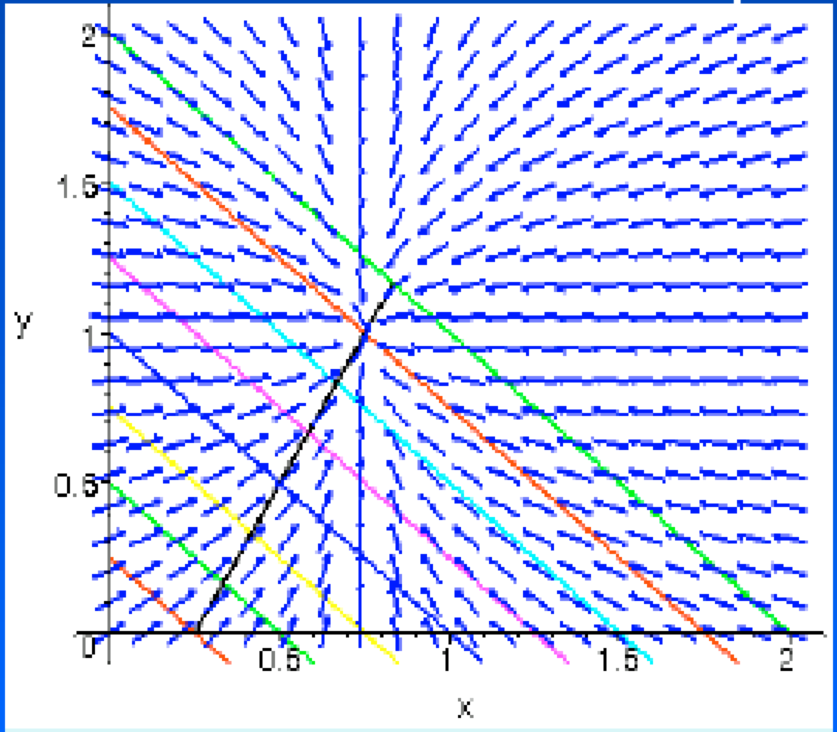
With the help of PDS theory, we can compute an approximate curve of equilibrium by choosing

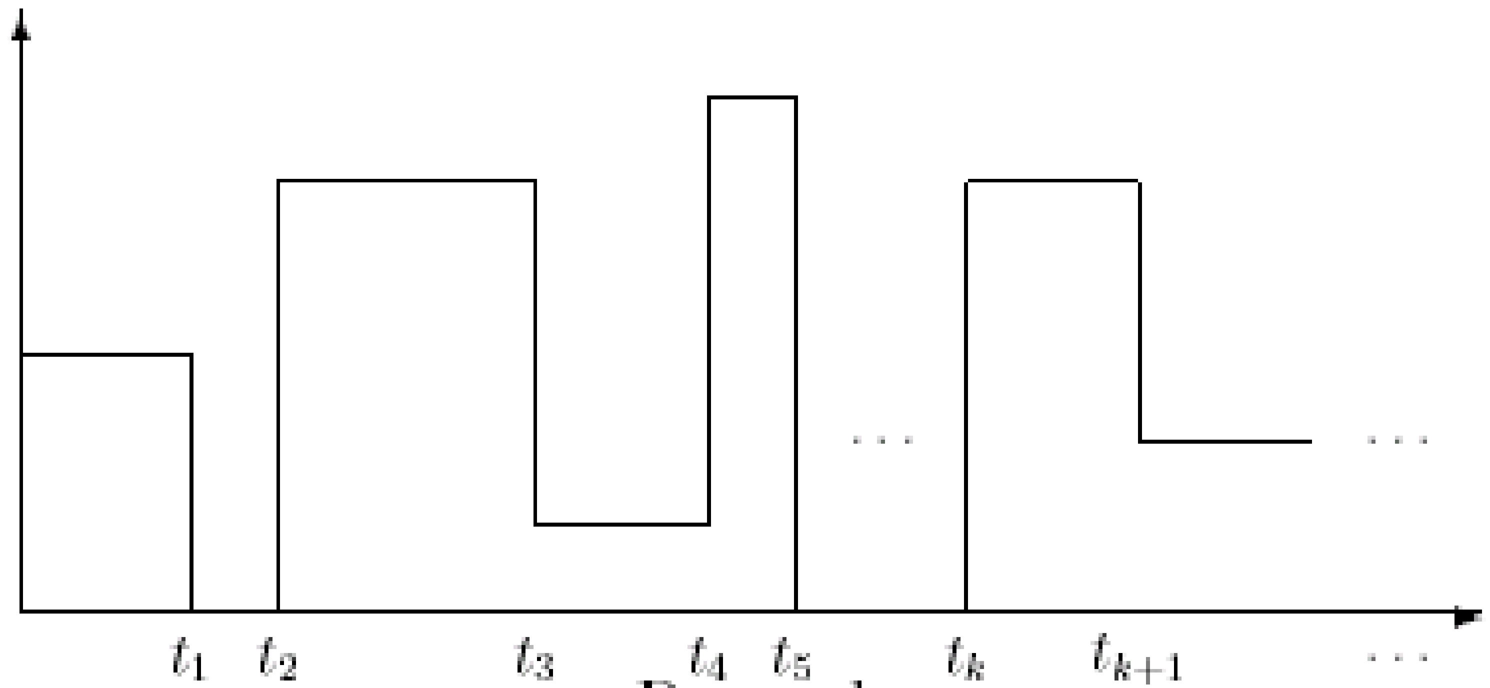
$$t_0 \in \left\{ \frac{k}{4} | k \in \{0, \ldots, 8\} \right\}.$$

Using a simple MAPLE computation, we obtain that the equilibria are the points:

$$\left\{(0,0),\left(\frac{1}{4},0\right),\left(\frac{1}{3},\frac{1}{6}\right),\left(\frac{5}{12},\frac{1}{3}\right),\left(\frac{1}{2},\frac{1}{2}\right),\left(\frac{7}{12},\frac{2}{3}\right),\left(\frac{2}{3},\frac{5}{6}\right),\left(\frac{3}{4},1\right),\left(\frac{5}{6},\frac{7}{6}\right)\right\}.$$

Interpolating these points, we obtain the approximate curve of network equilibria:





If the demand is a step function, the solution to the EVI has the structure:

$$x^*(t) = \begin{cases} x_1^* & \text{if } 0 \leq t \leq t_1 \\ x_2^* & \text{if } t_1 < t \leq t_2 \\ \vdots & \vdots \\ x_{k+1}^* & \text{if } t_k < t \leq t_{k+1} \\ \vdots & \vdots \end{cases}$$

2005-2006 Radcliffe Institute for Advanced Study Fellowship Year at Harvard Collaboration with Professor David Parkes and Professor Patrizia Daniele (Visiting from Italy)

The Internet -- A Dynamic Network

The Internet has revolutionized the way in which we work, interact, and conduct our daily activities. It has affected the young and the old as they gather information and communicate and has transformed business processes, financial investing and decision-making, and global supply chains. The Internet has evolved into a network that underpins our developed societies and economies.

The motivation for this research comes from several directions:

1. The need to develop a dynamic, that is, time-dependent, model of the Internet, as argued by computer scientists.

Indeed, as noted on page 11 of Roughgarden (2005), *A network like the Internet is volatile. Its traffic patterns can change quicky and dramatically ... The assumption of a static model is therefore particularly suspect in such networks*.

2. Analogues have been identified between transportation networks and telecommunication networks and, in particular, the Internet, in terms of decentralized decision-making, flows and costs, and even the Braess paradox, which allows us to take advantage of such a connection:

Beckmann, McGuire, and Winsten (1956), Beckmann (1967), Braess (1968) Dafermos and Sparrow (1969), Dafermos (1972), Cantor and Gerla (1974), Gallager (1977), Bertsekas and Tsitsiklis (1989), Bertsekas and Gallager (1992), Korilis, Lazar, and Orda (1999), and Boyce, Mahmassani, and Nagurney (2005).

3. The development of a fundamental dynamic model of the Internet will allow for the exploration and development of different incentive mechanisms, including dynamic tolls and pricing mechanisms in order to reduce congestion and also aid in the design of a better Internet, a dynamic network, par excellence.

It has been shown that distributed routing, which is common in computer networks and, in particular, the Internet, and *selfish* (or *source* routing in computer networks) routing, as occurs in the case of user-optimized transportation networks, in which travelers select the minimum cost route between an origin and destination, are one and the same if the cost functions associated with the links that make up the paths/routes coincide with the lengths used to define the shortest paths.

We assume that the costs on the links are congestion-dependent, that is, they depend on the volume of the flow on the link.

Note that the cost on a link may represent travel delay but we utilize *cost* functions since these are more general conceptually than delay functions and they can include, for example, tolls associated with pricing, etc.

It is important to also emphasize that, in the case of transportation networks, it is travelers that make the decisions as to the route selection between origin/destination (O/D) pairs of nodes, whereas in the case of the Internet, it is algorithms, implemented in software, that determine the shortest paths.

We can expect that a variety of time-dependent demand structures will occur on the Internet as individuals seek information and news online in response to major events or simply go about their daily activities whether at work or at home. Hence, the development of this dynamic network model of the Internet is timely.

The costs on routes are related to costs on links through the following equations:

$$C_r^k(x(t)) = \sum_{a \in L} c_a^k(x(t))\delta_{ar}, \quad \forall r \in P, \forall k,$$

that is, the cost on a route of class k at a time t is equal to the sum of costs of the class on links that make up the route at time t. We group the path costs at time t into the vector $C(t)$, which is of dimension Kn_P.

We define the feasible set $\mathcal{K}$. We consider the Hilbert space $\mathcal{L} = L^2([0,T], R^{Kn_P})$ (where T denotes the time interval under consideration) given by

$$\mathcal{K} = \left\{ \begin{array}{r} x \in L^2([0,T], R^{Kn_P}) : 0 \leq x(t) \leq \mu(t) \text{ a.e. in } [0,T]; \\ \sum_{p \in P_w} x_p^k(t) = d_w^k(t), \forall w, \forall k, \text{ a.e. in } [0,T] \end{array} \right\}.$$

We assume that the capacities $\mu_r^k(t)$, for all r and k, are in $\mathcal{L}$ and that the demands, $d_w^k \geq 0$, for all w and k, are also in $\mathcal{L}$. Further, we assume that

$$0 \leq d(t) \leq \Phi\mu(t), \text{a.e. on } [0,T],$$

where Φ is the $Kn_W \times Kn_P$-dimensional O/D pair-route incidence matrix, with element (kw, kr) equal to 1 if route r is contained in P_w, and 0, otherwise. Hence, the feasible set $\mathcal{K}$ is nonempty. It is easily seen that $\mathcal{K}$ is also convex, closed, and bounded. Note that we are not restricted as to the form that the time-varying demand for the O/D pair takes since convexity is guaranteed even if the demands have a step-wise structure, or are piecewise continuous.

The dual space of $\mathcal{L}$ will be denoted by $\mathcal{L}^*$. On $\mathcal{L} \times \mathcal{L}^*$ we define the canonical bilinear form by

$$\langle\langle G, x \rangle\rangle := \int_0^T \langle G(t), x(t) \rangle dt, \quad G \in \mathcal{L}^*, \quad x \in \mathcal{L}.$$

Furthermore, the cost mapping $C : \mathcal{K} \to \mathcal{L}^*$, assigns to each flow trajectory $x(\cdot) \in \mathcal{K}$ the cost trajectory $C(x(\cdot)) \in \mathcal{L}^*$.

Definition: Dynamic Multiclass Network Equilibrium

A multiclass route flow pattern $x^* \in \mathcal{K}$ is said to be a dynamic network equilibrium (according to the generalization of Wardrop's first principle (cf. Wardrop (1952) and Beckmann, McGuire, and Winsten (1956))), if, at each time t, only the minimum cost routes for each class not at their capacities are used (that is, have positive flow) for each O/D pair unless the flow of that class on a route is at its upper bound (in which case those class routes' costs can be lower than those on the routes not at their capacities). The state can be expressed by the following equilibrium conditions which must hold for every O/D pair $w \in W$, every path $r \in P_w$, every class k; $k = 1, \ldots, K$, and a.e. on $[0, T]$:

$$C_r^k(x^*(t)) - \lambda_w^{k*}(t) \begin{cases} \leq 0, & \text{if} \quad x_r^{k*}(t) = \mu_r^k(t), \\ = 0, & \text{if} \quad 0 < x_r^{k*}(t) < \mu_r^k(t), \\ \geq 0, & \text{if} \quad x_r^{k*}(t) = 0. \end{cases}$$

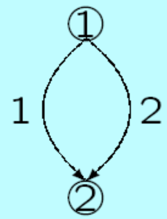
Theorem (Nagurney, Parkes, and Daniele (2006))

$x^* \in \mathcal{K}$ is an equilibrium flow according to the Definition if and only if it satisfies the evolutionary variational inequality:

$$\int_0^T \langle C(x^*(t)), x(t) - x^*(t) \rangle dt \geq 0, \quad \forall x \in \mathcal{K}.$$

A Multiclass Numerical Example

Consider a network (small subnetwork of the Internet) consisting of two nodes and two links. There is a single O/D pair w = (1, 2). Since the routes connecting the O/D pair consist of single links we work with the routes r_1 and r_2 directly:



Network Structure of the Multiclass Numerical Example

There are assumed to be two classes/jobs and the route costs are:

for Class 1:

$$C^1_{r_1}(x(t)) = 2x^1_{r_1}(t) + x^2_{r_1}(t) + 5, \quad C^1_{r_2}(x(t)) = 2x^2_{r_2}(t) + 2x^1_{r_2}(t) + 10,$$

for Class 2:

$$C^2_{r_1}(x(t)) = x^2_{r_1}(t) + x^1_{r_1}(t) + 5, \quad C^2_{r_2}(x(t)) = x^1_{r_2}(t) + 2x^2_{r_2}(t) + 5.$$

The time horizon is $[0, 10]$. The demands for the O/D pair are:

$$d^1_w(t) = 10 - t, \quad d^2_w(t) = t.$$

The upper bounds are: $\mu^1_{r_1} = \mu^1_{r_2} = \mu^1_{r_1} = \mu^2_{r_2} = \infty$.

Equilibrium Route Flows for the Multiclass Numerical Example

Equilibrium Multiclass Route Flows at time t					
Flow	$t=0$	$t=2.5$	$t=5$	$t=7.5$	$t=10$
$x^{1*}_{r_1}(t)$	6.25	6.25	5.00	2.50	0.00
$x^{1*}_{r_2}(t)$	3.75	1.25	0.00	0.00	0.00
$x^{2*}_{r_1}(t)$	0.00	0.00	$1.6\bar{6}$	$4.16\bar{6}$	$6.6\bar{6}$
$x^{2*}_{r_2}(t)$	0.00	2.50	$3.3\bar{3}$	$3.3\bar{3}$	$3.3\bar{3}$

For completeness, we also provide the following class O/D pair minimum costs at times $t = 0, 2.5, 5, 7.5$ and 10:

$$\lambda^{1*}_w(0) = 17.50, \quad \lambda^{1*}_w(2.5) = 17.50,$$

$$\lambda^{1*}_w(5) = 16.6\bar{6}, \quad \lambda^{1*}_w(7.5) = 14.16\bar{6}, \quad \lambda^{1*}_w(10) = 11.6\bar{6}$$

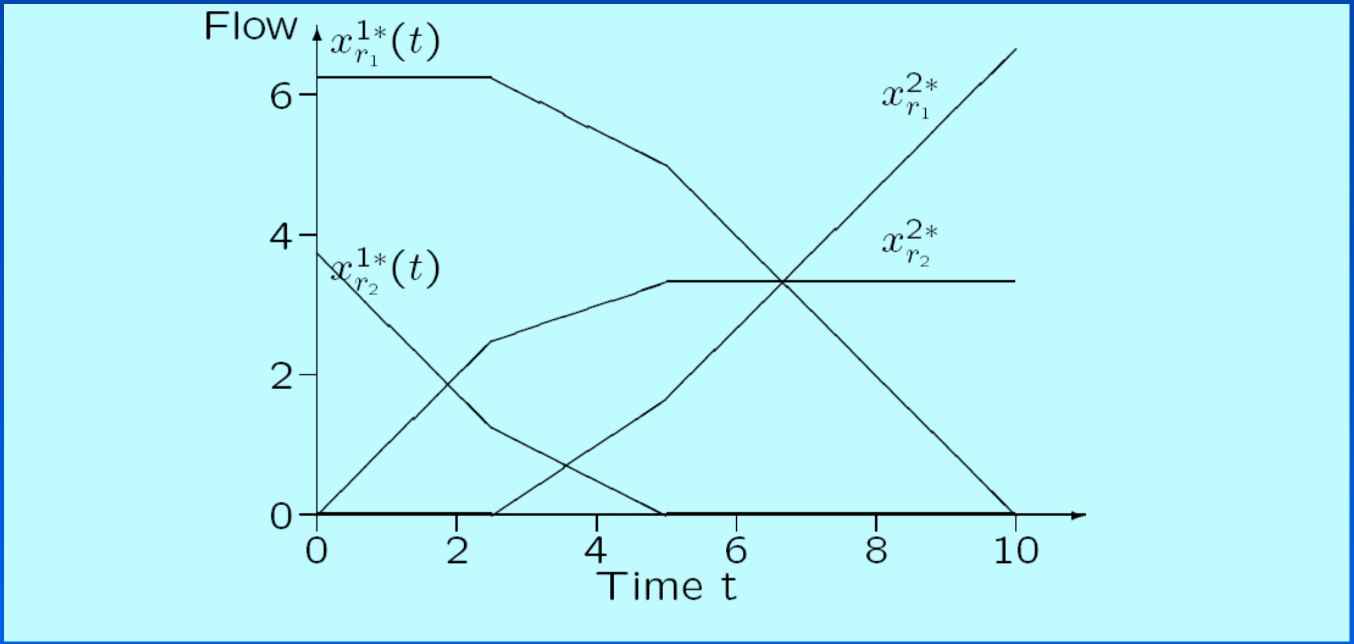
and

$$\lambda^{2*}_w(0) = 8.75, \quad \lambda^{2*}_w(2.5) = 11.25,$$

$$\lambda^{2*}_w(5) = 11.6\bar{6}, \quad \lambda^{2*}_w(7.5) = 11.6\bar{6}, \quad \lambda^{2*}_w(10) = 11.6\bar{6}.$$

We provide a graph of the equilibrium route trajectories, where we display also the interpolations between the discrete solutions. Since the route cost functions are strictly monotone over the time horizon [0, 10] we know that the equilibrium trajectories are unique.

As the theory predicts, the trajectories are also continuous for this example. It is interesting to see that after time t = 5 route r_2 is never used by class 1, whereas route r_1 is not utilized for class 2 traffic until after t = 2.



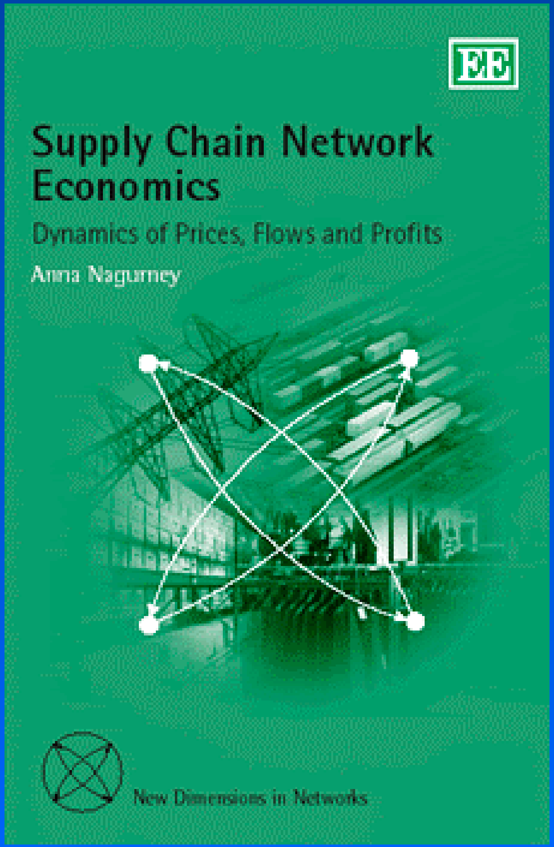
Equilibrium Trajectories for the Multiclass Numerical Example

Evolutionary variational inequalities have now been used to model dynamic:

- *transportation networks,*
- *supply chains,*
- *financial networks*,
- *electric power supply chains, and the Internet*.

For additional background and new applications see:

Supply Chain Network Economics



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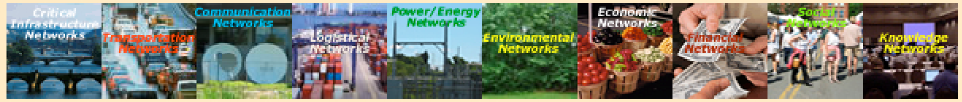

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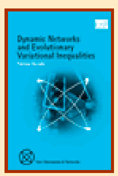

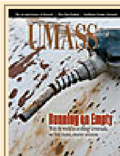

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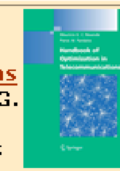

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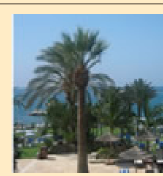

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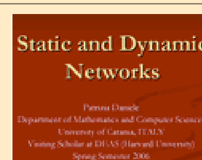

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