Transportation Science and the Dynamics of Critical Infrastructure Networks

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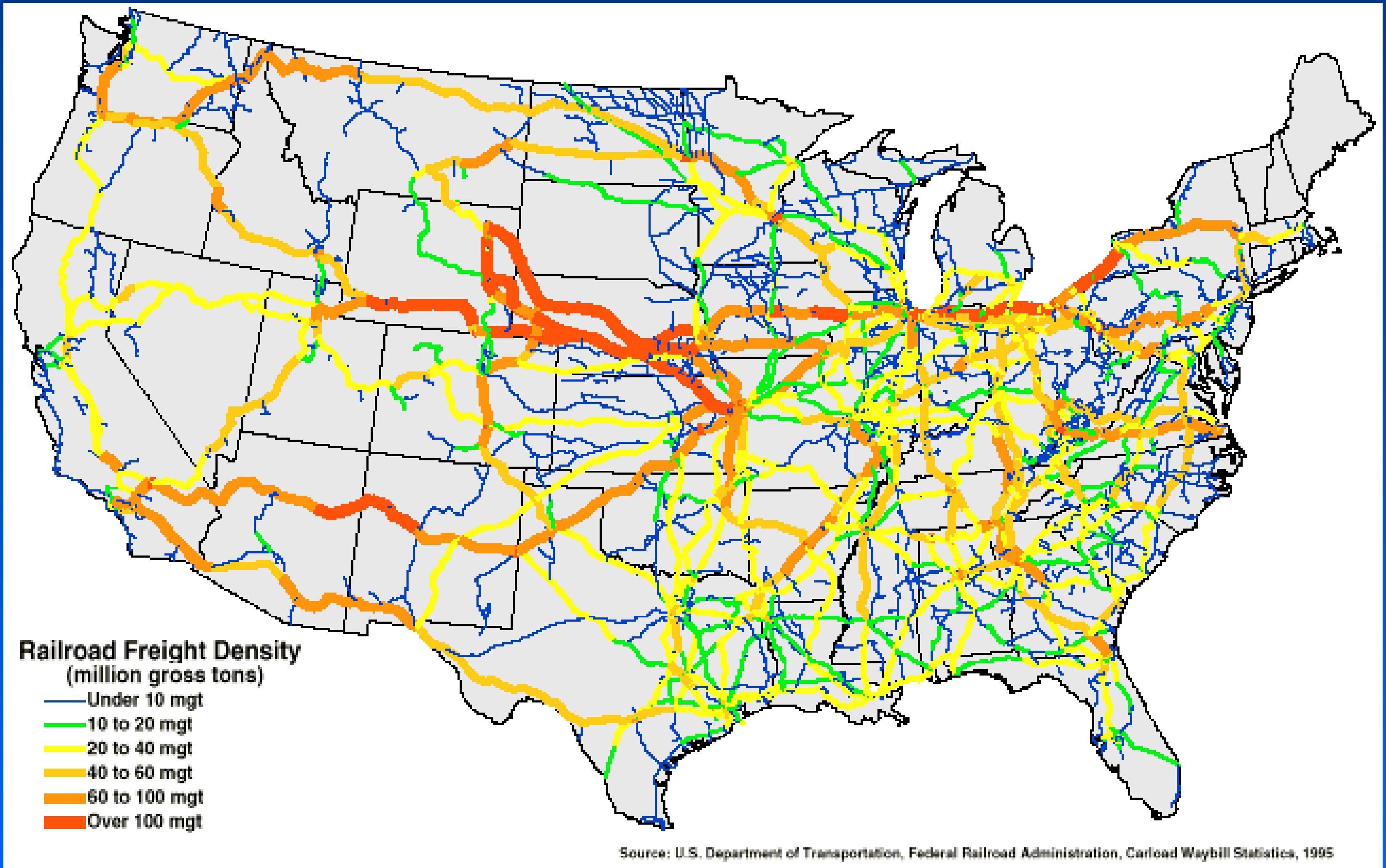
Outline of Presentation:

- Background
- Brief History of the Science of Networks
- Interdisciplinary Impact of Networks
- The Braess Paradox
- Methodological Tools
- Some Interesting Critical Infrastructure Networks
- The Time-Dependent (Demand-Varying) Braess Paradox
- A New Network Performance/Efficiency Measure with Applications to Critical Infrastructure Networks
- New Challenges and Opportunities: Unification of Evolutionary Variational Inequalities and Projected Dynamical Systems

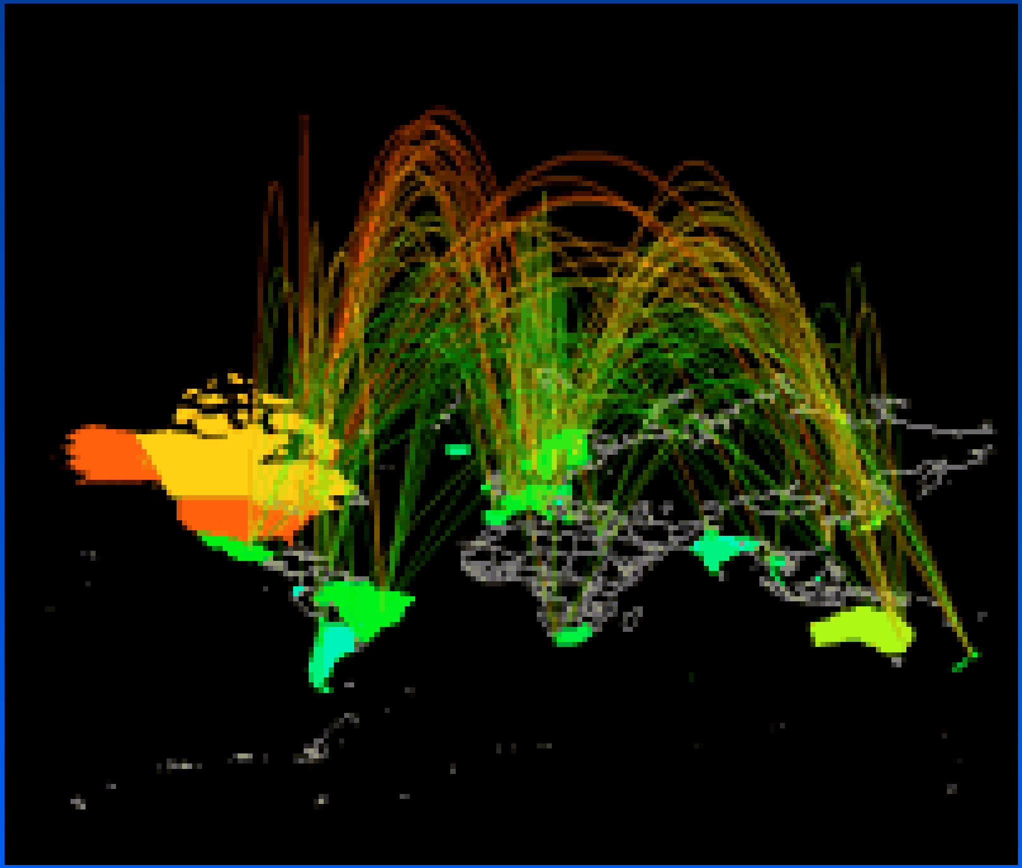
Components of Common Physical Networks

Network System	Nodes	Links	Flows
Transportation	Intersections, Homes, Workplaces, Airports, Railyards	Roads, Airline Routes, Railroad Track	Automobiles, Trains, and Planes,
Manufacturing and logistics	Workstations, Distribution Points	Processing, Shipment	Components, Finished Goods
Communication	Computers, Satellites, Telephone Exchanges	Fiber Optic Cables Radio Links	Voice, Data, Video
Energy	Pumping Stations, Plants	Pipelines, Transmission Lines	Water, Gas, Oil, Electricity

US Railroad Freight Flows

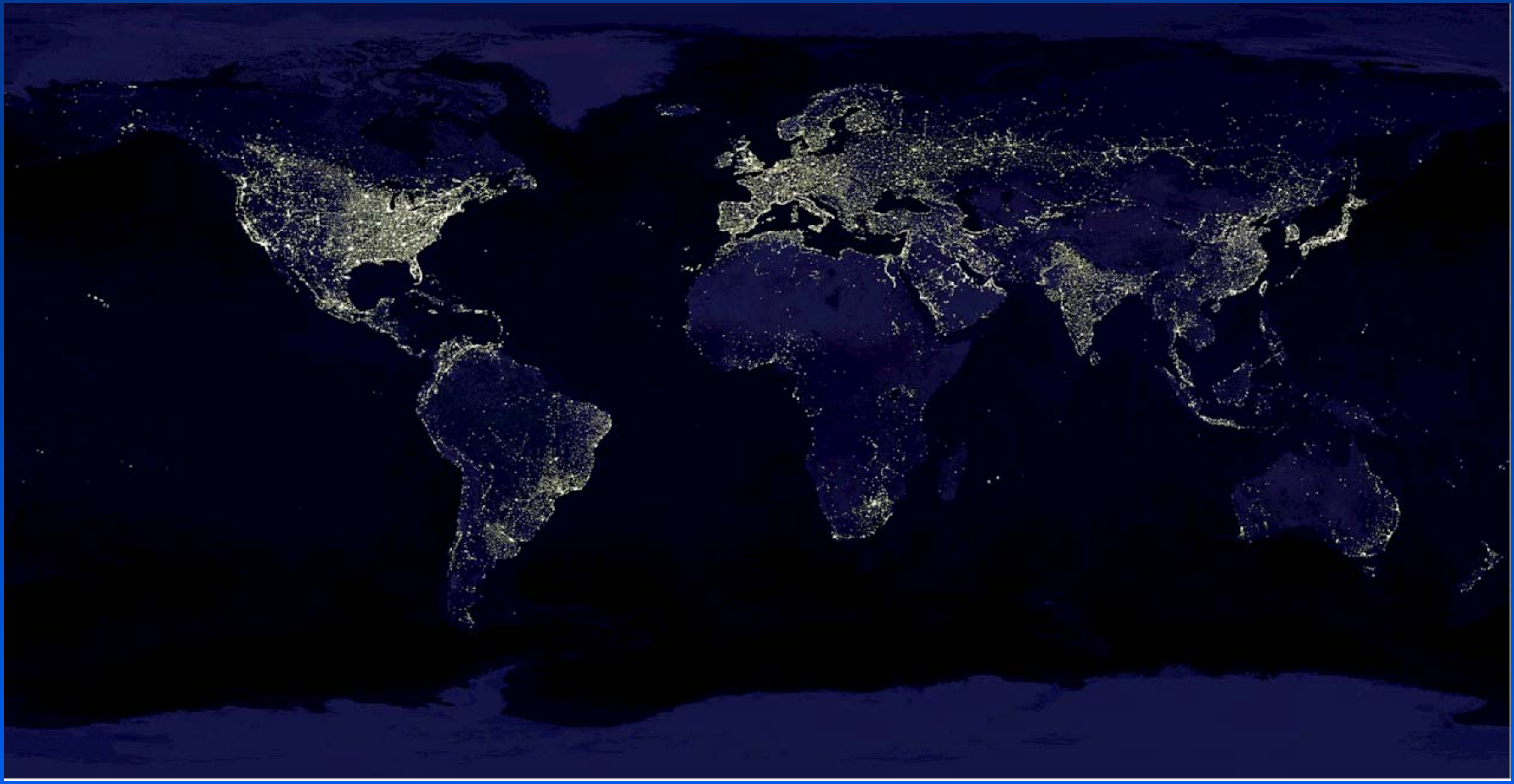


Internet Traffic Flows Over One 2 Hour Period



from Stephen Eick, Visual Insights

Electricity is Modernity



The scientific study of networks involves:

- how to **model** such applications as **mathematical entities**,

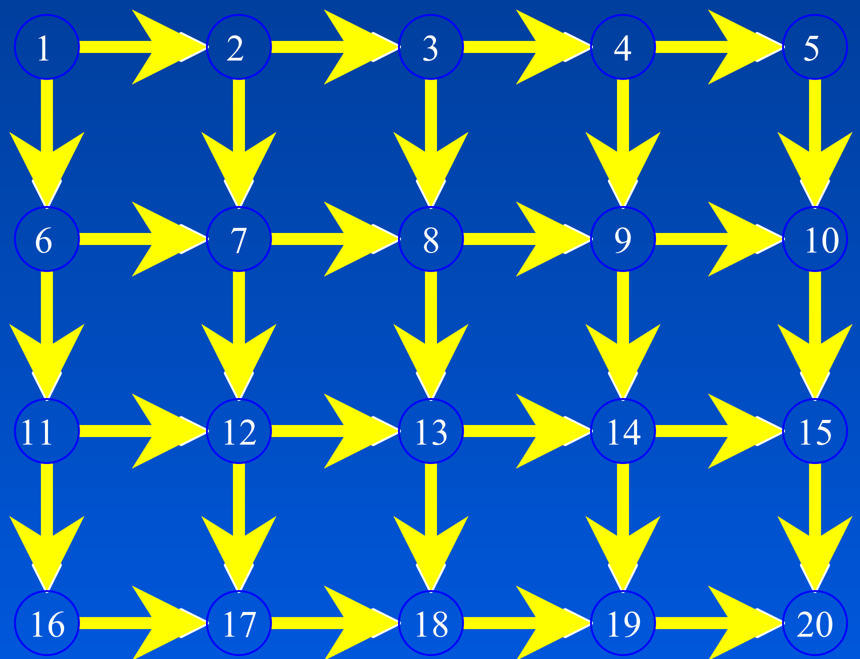
- how to **study the models** qualitatively,

- how to design **algorithms** to solve the resulting models.

The basic components of networks are:

- Nodes
- Links or arcs
- Flows

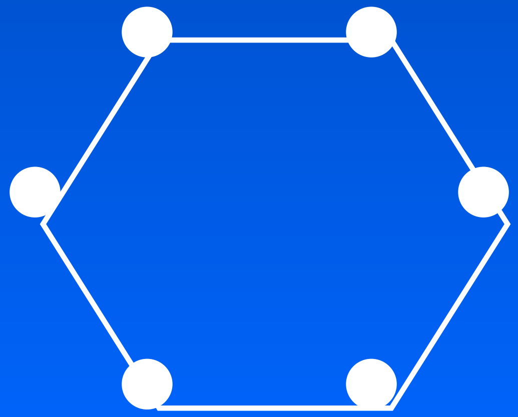
Nodes Links Flows



Brief History of the Science of Networks

1736 - Euler - the earliest paper on graph theory - Konigsberg bridges problem.

1758 - Quesnay in his *Tableau Economique* introduced a graph to depict the circular flow of financial funds in an economy.



1781 - Monge, who had worked under Napoleon Bonaparte, publishes what is probably the first paper on transportation in minimizing cost.

1838 - Cournot states that a competitive price is determined by the intersection of supply and demand curves in the context of spatially separate markets in which transportation costs are included.

1841 - Kohl considered a two node, two route transportation network problem.

1845 - Kirchhoff wrote Laws of Closed Electric Circuits.

1920 - Pigou studied a transportation network system of two routes and noted that the decision-making behavior of the users on the network would result in different flow patterns.

1936 - Konig published the first book on graph theory.

1939, 1941, 1947 - Kantorovich, Hitchcock, and Koopmans considered the network flow problem associated with the classical minimum cost transportation problem and provided insights into the special network structure of these problems, which yielded special-purpose algorithms.

1948, 1951 - Dantzig published the simplex method for linear programming and adapted it for the classical transportation problem.

1951 - Enke showed that spatial price equilibrium problems can be solved using electronic circuits

1952 - Copeland in his book asked, *Does money flow like water or electricity?*

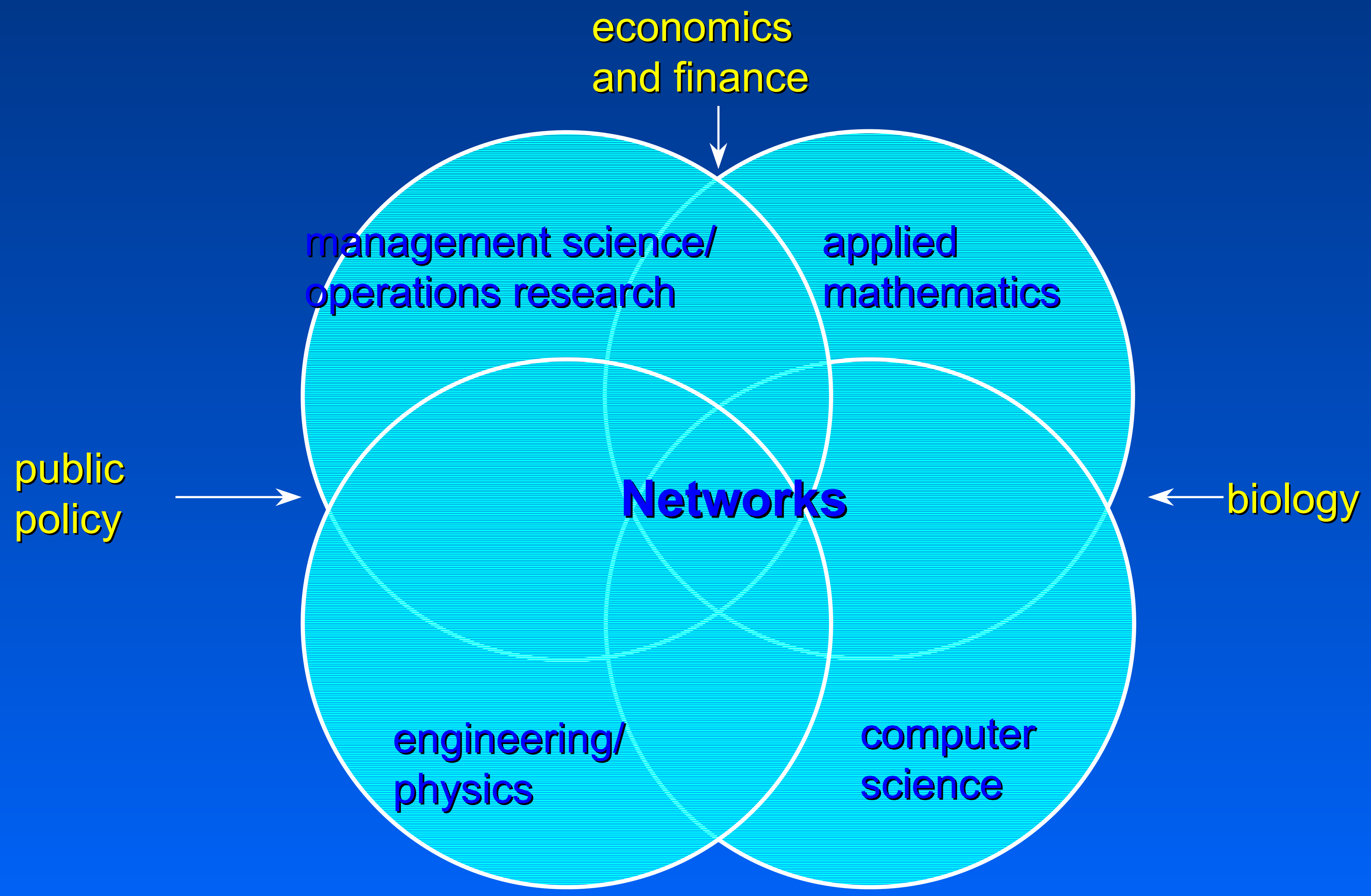
1952 - Samuelson gave a rigorous mathematical formulation of spatial price equilibrium and emphasized the network structure.

1956 - Beckmann, McGuire, and Winsten in their book, *Studies in the Economics of Transportation*, provided a rigorous treatment of congested urban transportation systems under different behavioral mechanisms due to Wardrop (1952).

1962 - Ford and Fulkerson publish *Flows in Networks*.

1969 - Dafermos and Sparrow coined the terms *user-optimization* and system-optimization and develop algorithms for the computation of solutions that exploit the network structure of transportation problems.

Networks in Different Disciplines



Interdisciplinary Impact of Networks

Management Science

Networks

Economics

Interregional Trade

General Equilibrium

Industrial Organization

Portfolio Optimization

Flow of Funds Accounting

Engineering

Energy

Manufacturing

Telecommunications

Transportation

Biology

DNA Sequencing

Targeted Cancer Therapy

Sociology

Social Networks

Organizational Theory

Computer Science

Routing Algorithms

Characteristics of Networks Today

- *large-scale nature* and complexity of network topology;
- *congestion*;
- alternative behavior of users of the network, which may lead to *paradoxical phenomena*;
- the *interactions among networks* themselves such as in transportation versus telecommunications;
- *policies* surrounding networks today may have a *major impact* not only economically but also *socially, politically, and security-wise*.

- *alternative behaviors of the users of the network*

  – system-optimized versus

  – user-optimized (network equilibrium),

  which may lead to

  paradoxical phenomena.

Transportation science has historically been the discipline that has pushed the frontiers in terms of methodological developments for such problems (which are often large-scale) beginning with the work of Beckmann, McGuire, and Winsten (1956).

Definition: Transportation Network Equilibrium

A route flow pattern $x^* \in K$ is said to be a transportation network equilibrium (according to Wardrop's (1952) first principle) if only the minimum cost routes are used (that is, have positive flow) for each O/D pair. The state can be expressed by the following equilibrium conditions which must hold for every O/D pair $w \in W$, every path $p \in P_w$:

$$C_p(x^*) - \lambda_w^* \begin{cases} = 0, & \text{if} \quad x_p^* > 0, \\ \geq 0, & \text{if} \quad x_p^* = 0. \end{cases}$$

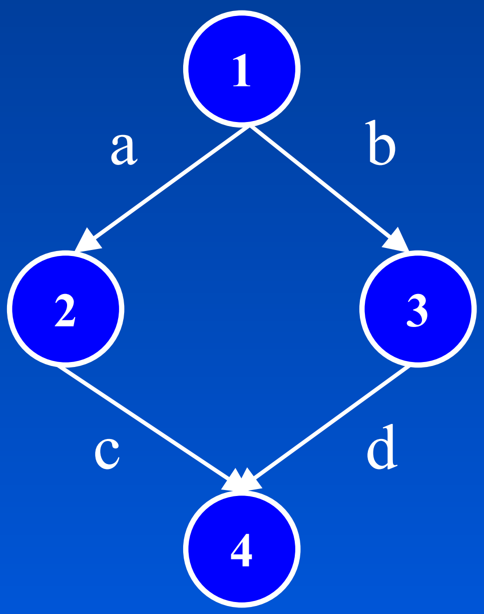
The Braess (1968) Paradox

Assume a network with a single O/D pair (1,4). There are 2 paths available to travelers:
p_1=(a,c) and **p_2=(b,d).**

For a travel demand of **6,** the equilibrium path flows are $\mathbf{x_{p_1}}^*$ $\mathbf{= x_{p_2}{}^* = 3}$ and

The equilibrium path travel cost is

$C_{p_1} = C_{p_2} = 83$.



$$\mathbf{c_a(f_a) = 10\, f_a \quad c_b(f_b) = f_b + 50}$$

$$\mathbf{c_c(f_c) = f_c + 50 \quad c_d(f_d) = 10\, f_d}$$

Adding a Link Increases Travel Cost for All!

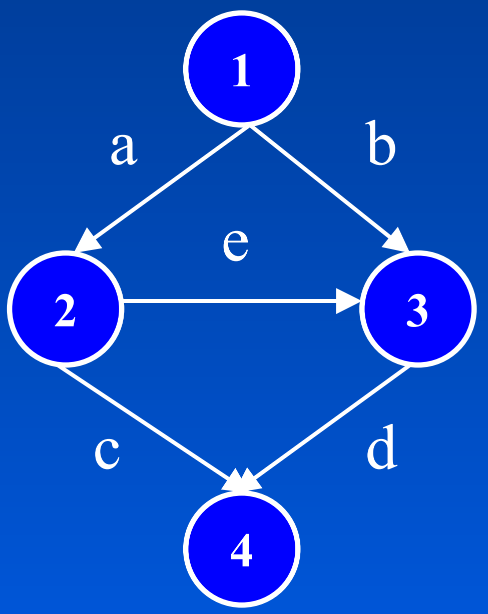
Adding a new link creates a new path **p_3=(a,e,d).**

The original flow distribution pattern is no longer an equilibrium pattern, since at this level of flow the cost on path **p_3, C_{p_3}=70.**

The new equilibrium flow pattern network is

$x_{p_1}^* = x_{p_2}^* = x_{p_3}^* = 2.$

The equilibrium path travel costs: **$C_{p_1} = C_{p_2} = C_{p_3} = 92.$**



$c_e(f_e) = f_e + 10$

The 1968 Braess article has been translated from German to English and appears as

On a Paradox of Traffic Planning

by Braess, Nagurney, Wakolbinger

in the November 2005 issue of *Transportation Science.*

The tools that we are using in our Dynamic Network research include:

- network theory
- optimization theory
- game theory
- variational inequality theory
- evolutionary variational inequality theory
- projected dynamical systems theory
- double-layered dynamics theory
- network visualization tools.

Dafermos (1980) showed that the transportation network equilibrium (also referred to as user-optimization) conditions as formulated by Smith (1979) were a finite-dimensional variational inequality.

In 1993, Dupuis and Nagurney proved that the set of solutions to a variational inequality problem coincided with the set of solutions to a projected dynamical system (PDS) in R^n.

In 1996, Nagurney and Zhang published **Projected Dynamical Systems and Variational Inequalities**.

VI Formulation of Transportation Network Equilibrium (Dafermos (1980), Smith (1979))

A traffic path flow pattern satisfies the above equilibrium conditions if and only if it satisfies the variational inequlity problem: determine $x^* \in K$, such that

$$\sum_p C_p(x^*) \times (x_p - x_p^*) \geq 0, \quad \forall x \in K.$$

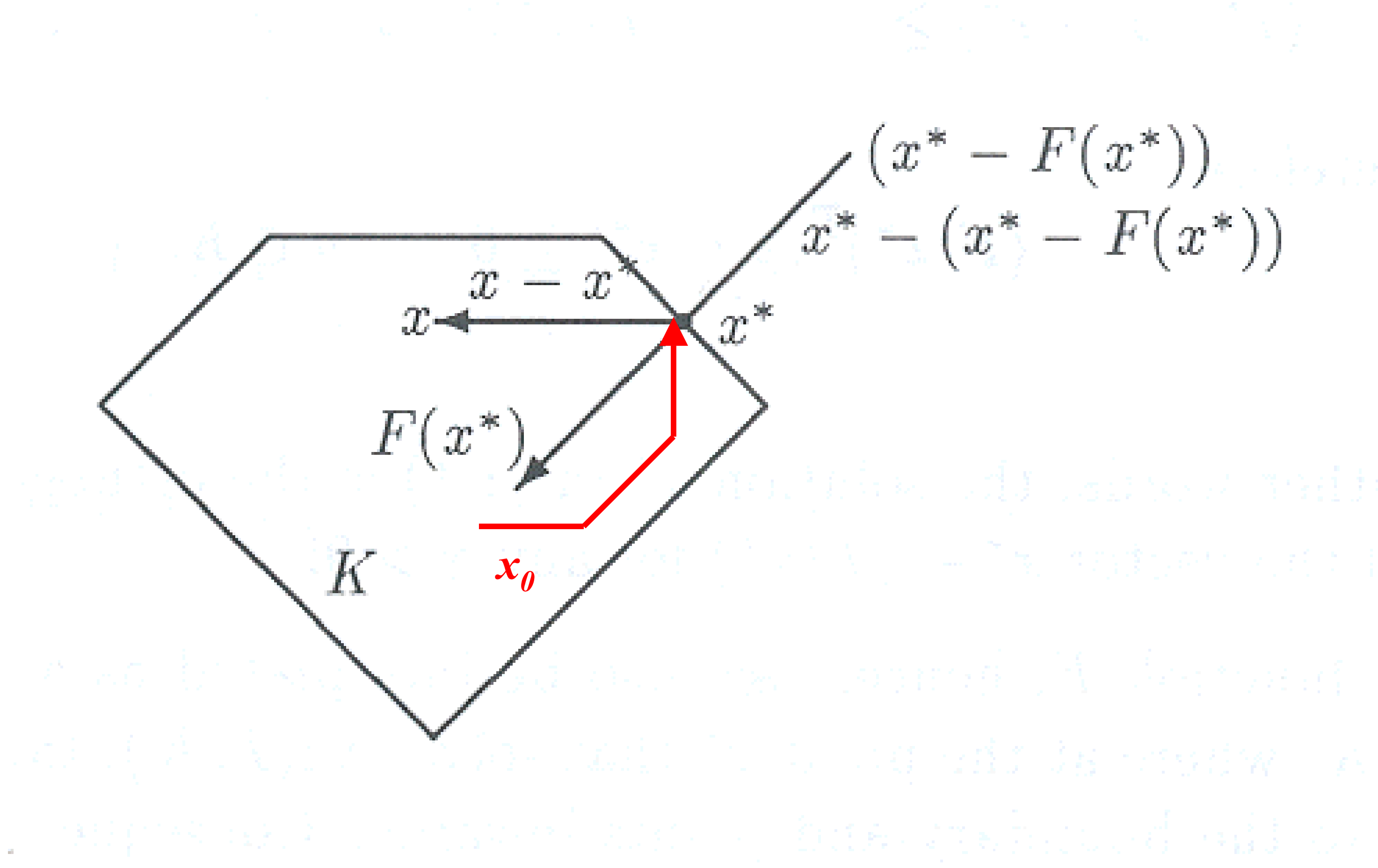
Finite-dimensional variational inequality theory has been applied to-date to the wide range of equilibrium problems noted above.

In particular, the finite-dimensional variational inequality problem is to determine $x^* \in K \subset R^n$ such that

$$\langle F(x^*), x - x^* \rangle \geq 0, \quad \forall x \in K,$$

where $\langle \cdot, \cdot \rangle$ denoted the inner product in R^n and K is closed and convex.

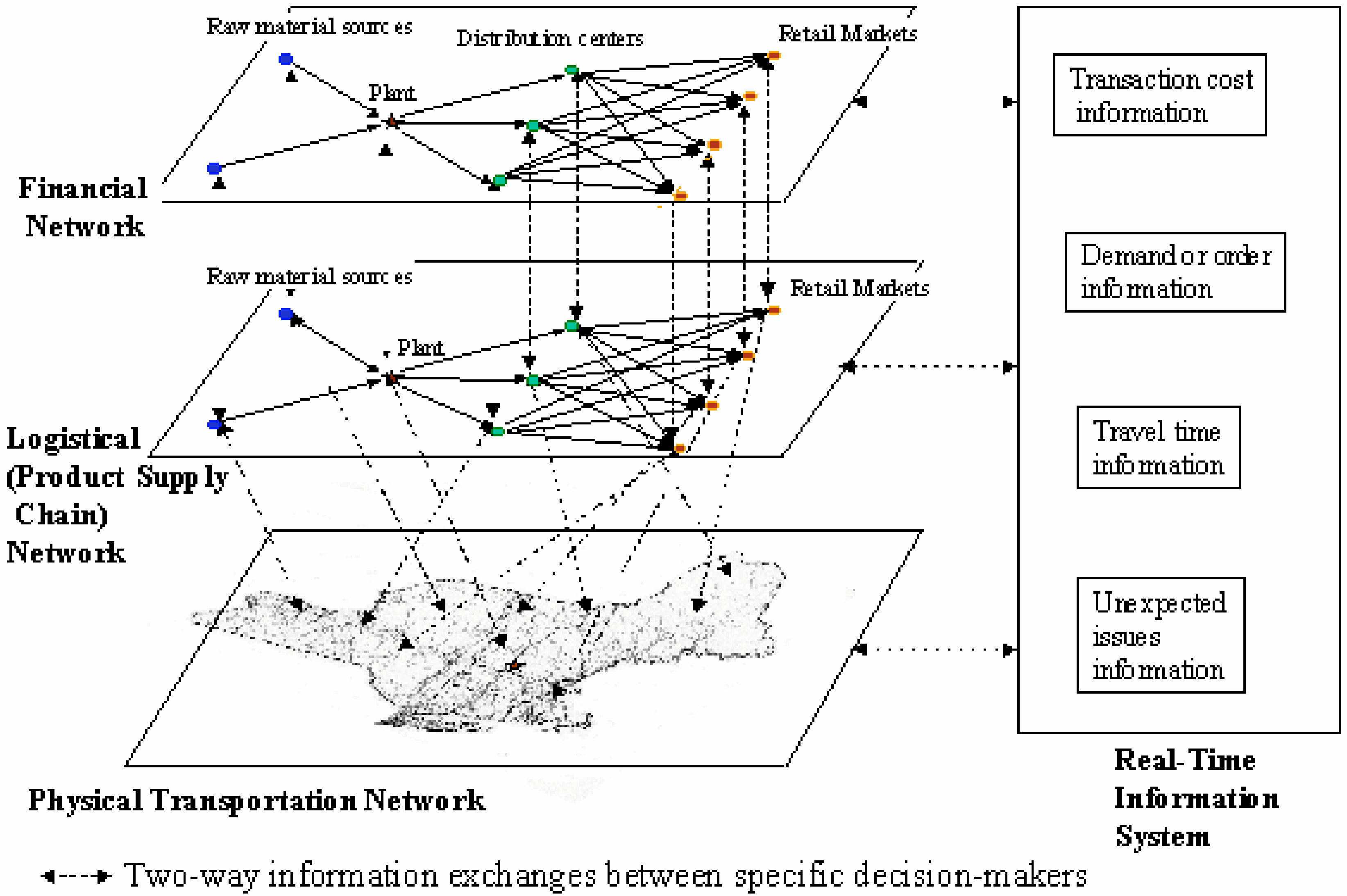
A Geometric Interpretation of a Variational Inequality and a Projected Dynamical System (Dupuis and Nagurney (1993), Nagurney and Zhang (1996))



Some Interesting Applications

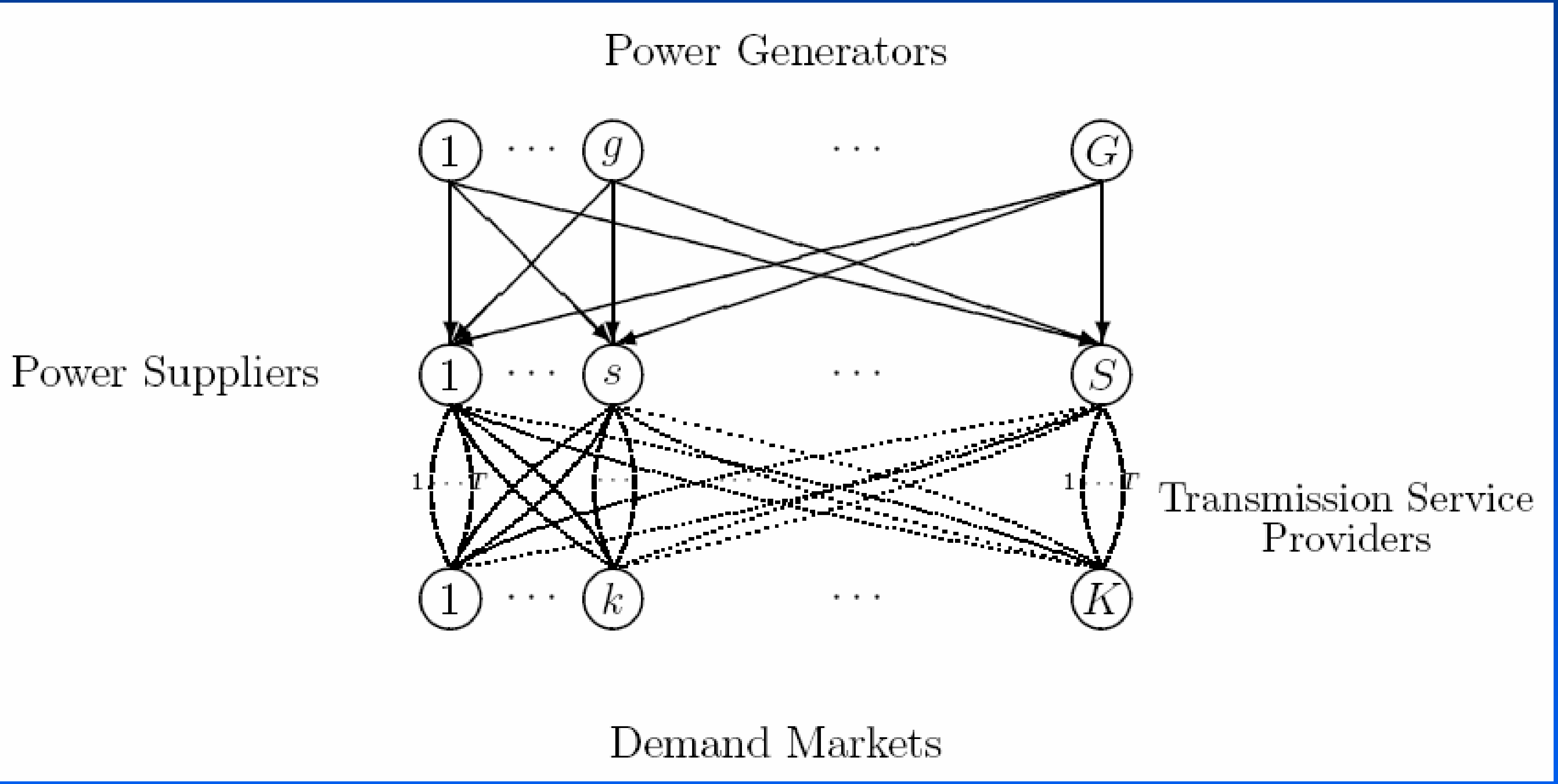
- Telecommuting/Commuting Decision-Making
- Teleshopping/Shopping Decision-Making
- Supply Chain Networks with Electronic Commerce
- Financial Networks with Electronic Transactions
- Reverse Supply Chains with E-Cycling
- Knowledge Networks
- Energy Networks/Power Grids
- Social Networks integrated with Economic Networks

Supply Chain -Transportation Supernetwork Representation



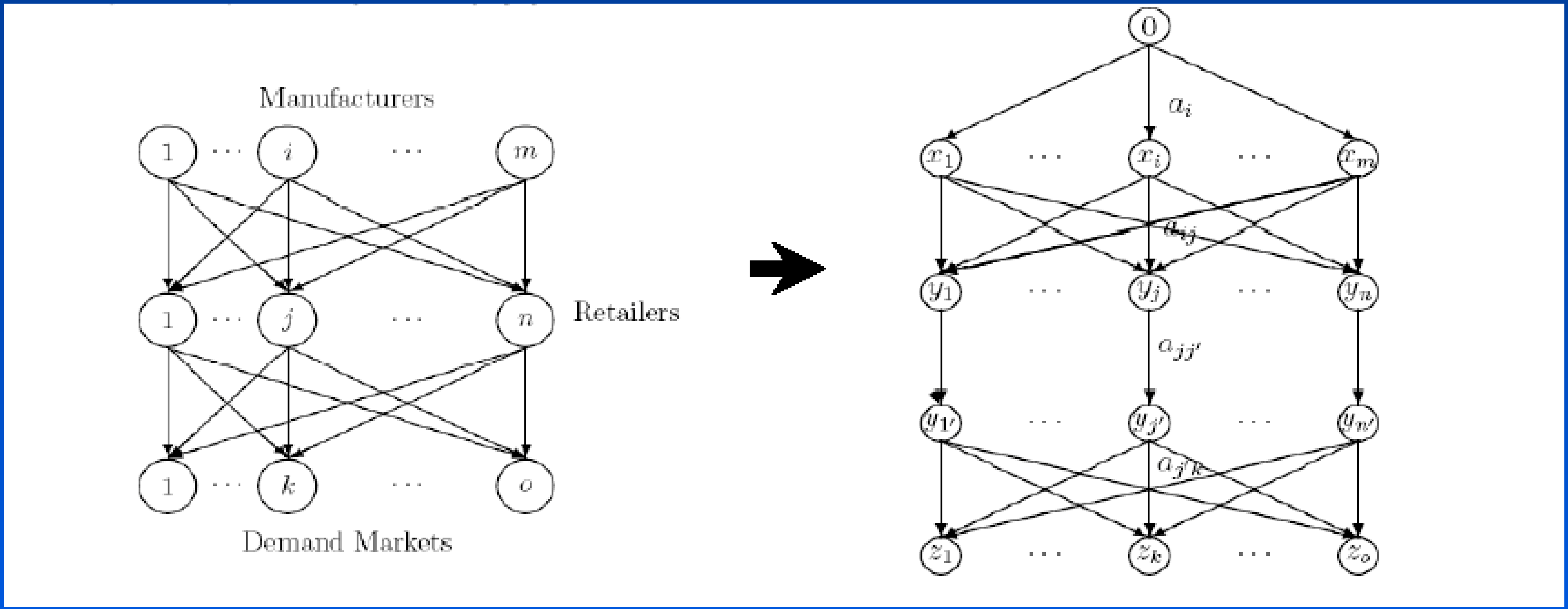
Nagurney, Ke, Cruz, Hancock, Southworth, *Environment and Planning B* (2002)

The Electric Power Supply Chain Network



Nagurney and Matsypura, Proceedings of the *CCCT* (2004)

The Equivalence of Supply Chain Networks and Transportation Networks

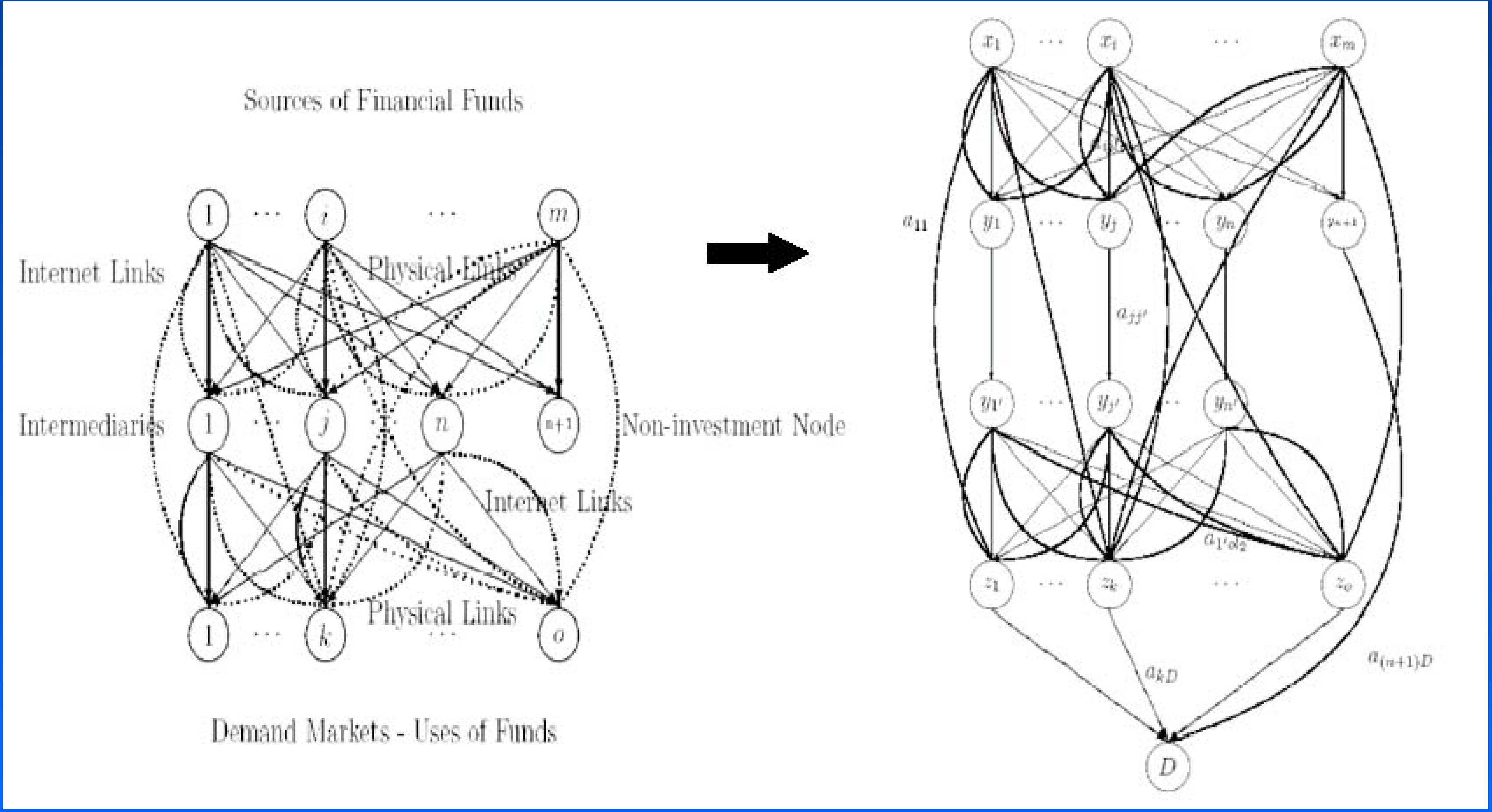


Nagurney, *Transportation Research E* (2006)

Copeland (1952) wondered whether money flows like water or electricity.

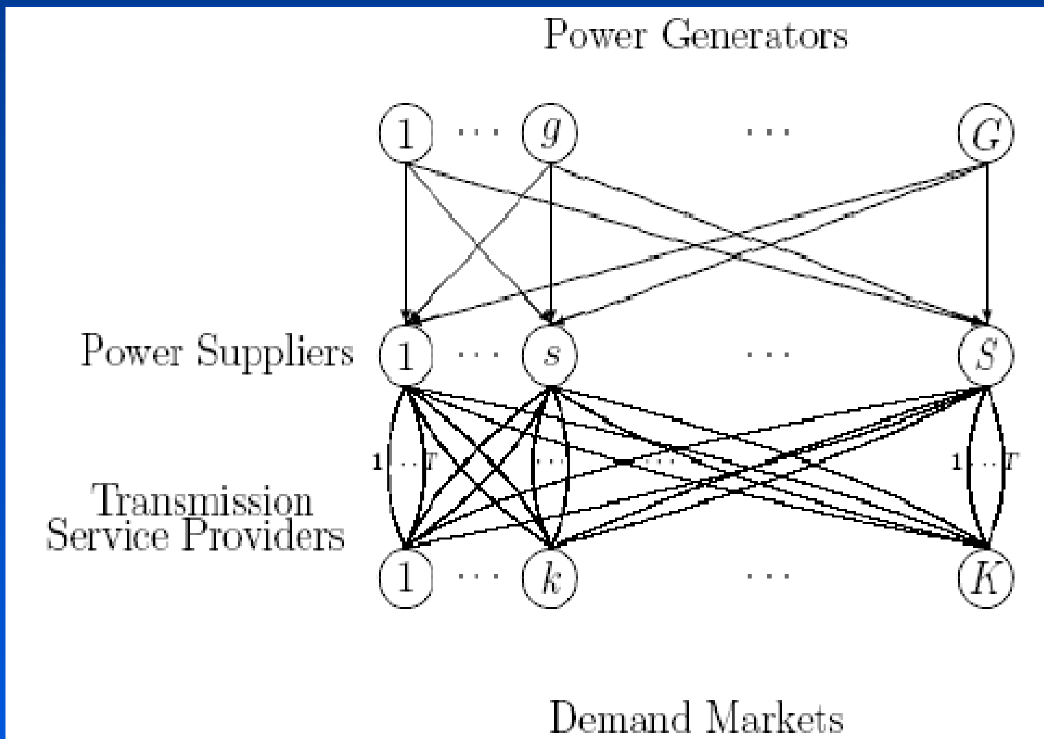
Liu and Nagurney have shown that money and electricity flow like transportation network flows (*Computational Management Science* (2006)).

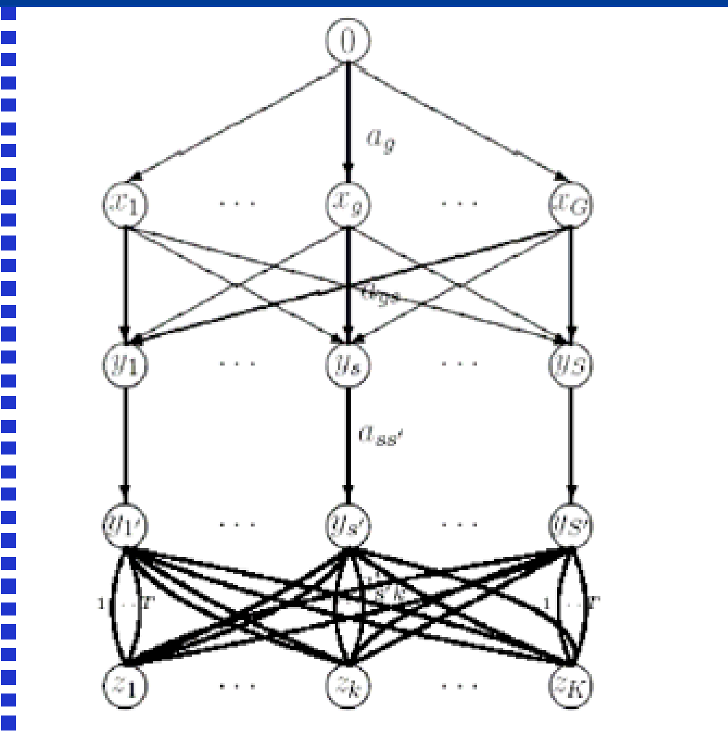
The Transportation Network Equilibrium Reformulation of the Financial Network Equilibrium Model with Intermediation



The fifth chapter of Beckmann, McGuire, and Winsten’s book, **Studies in the Economics of Transportation** (1956) describes some *unsolved problems* including a single commodity network equilibrium problem that the authors imply could be generalized to capture electric power networks.

The Transportation Network Equilibrium Reformulation of Electric Power Supply Chain Networks





Electric Power Supply Chain Network

Transportation Network

Nagurney et al, to appear in *Transportation Research E*

We have, hence, shown that money as well as electricity flow like *transportation* and have answered questions posed fifty years ago by Copeland and Beckmann, McGuire, and Winsten, respectively.

We are using evolutionary variational inequalities to model dynamic networks with:

- *dynamic (time-dependent)* supplies and demands
- *dynamic (time-dependent)* capacities
- *structural changes* in the networks themselves.

Such issues are important for robustness, resiliency, and reliability of networks (including supply chains and the Internet).

What happens if the demand is varied in the Braess Network?

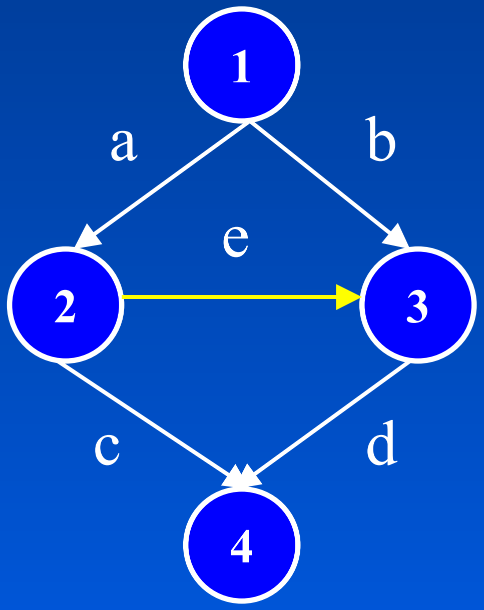
The answer lies in the solution of an Evolutionary (Time-Dependent) Variational Inequality.

Find $x^* \in K$, such that

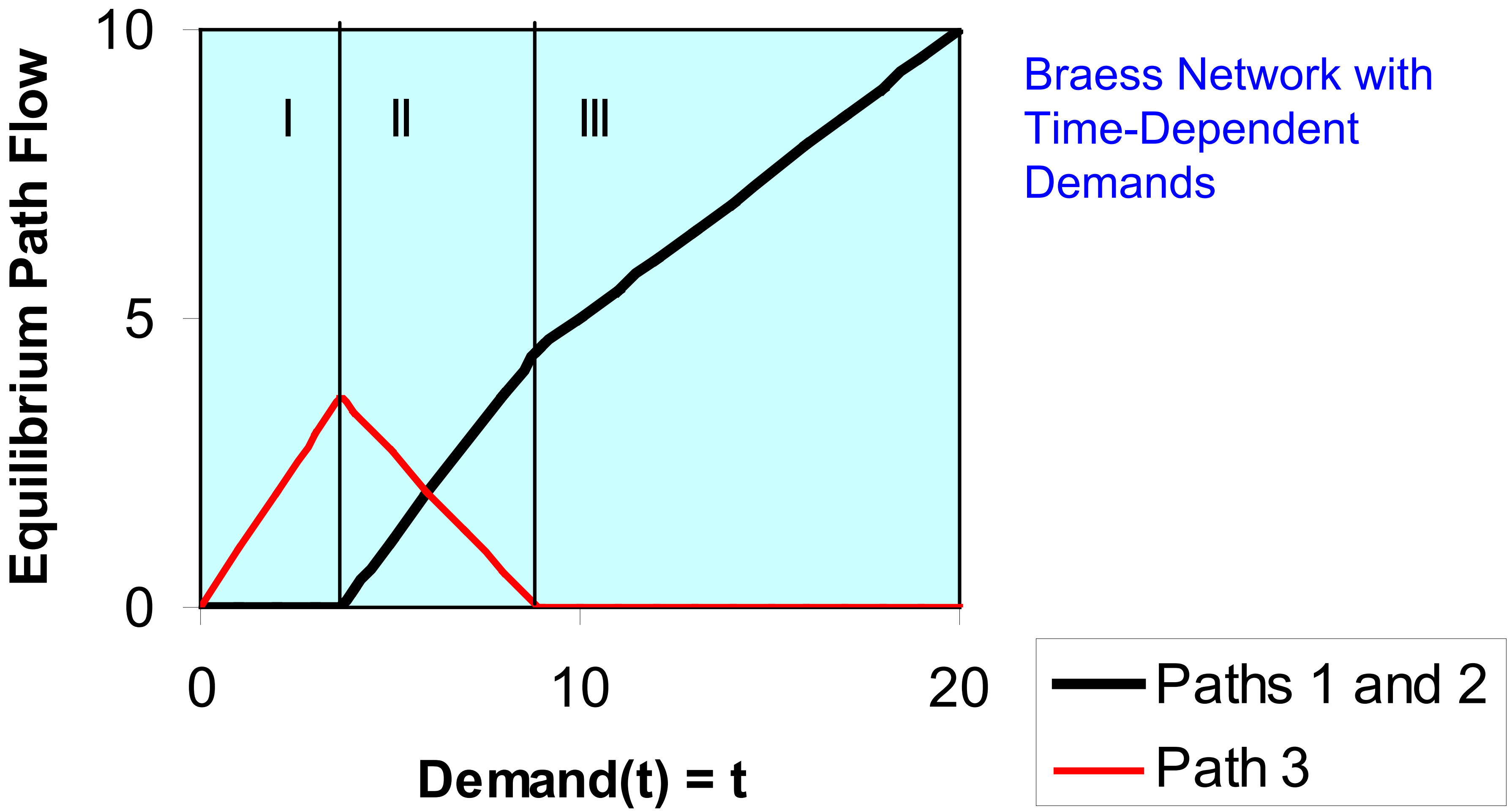
$$\int_0^T \langle C(x^*(t)), x(t) - x^*(t) \rangle \, dt \geq 0 \qquad \forall x \in K$$

Nagurney, Parkes, and Daniele, *Computational Management Science* (2006)

Recall the Braess Network
where we add the link e.

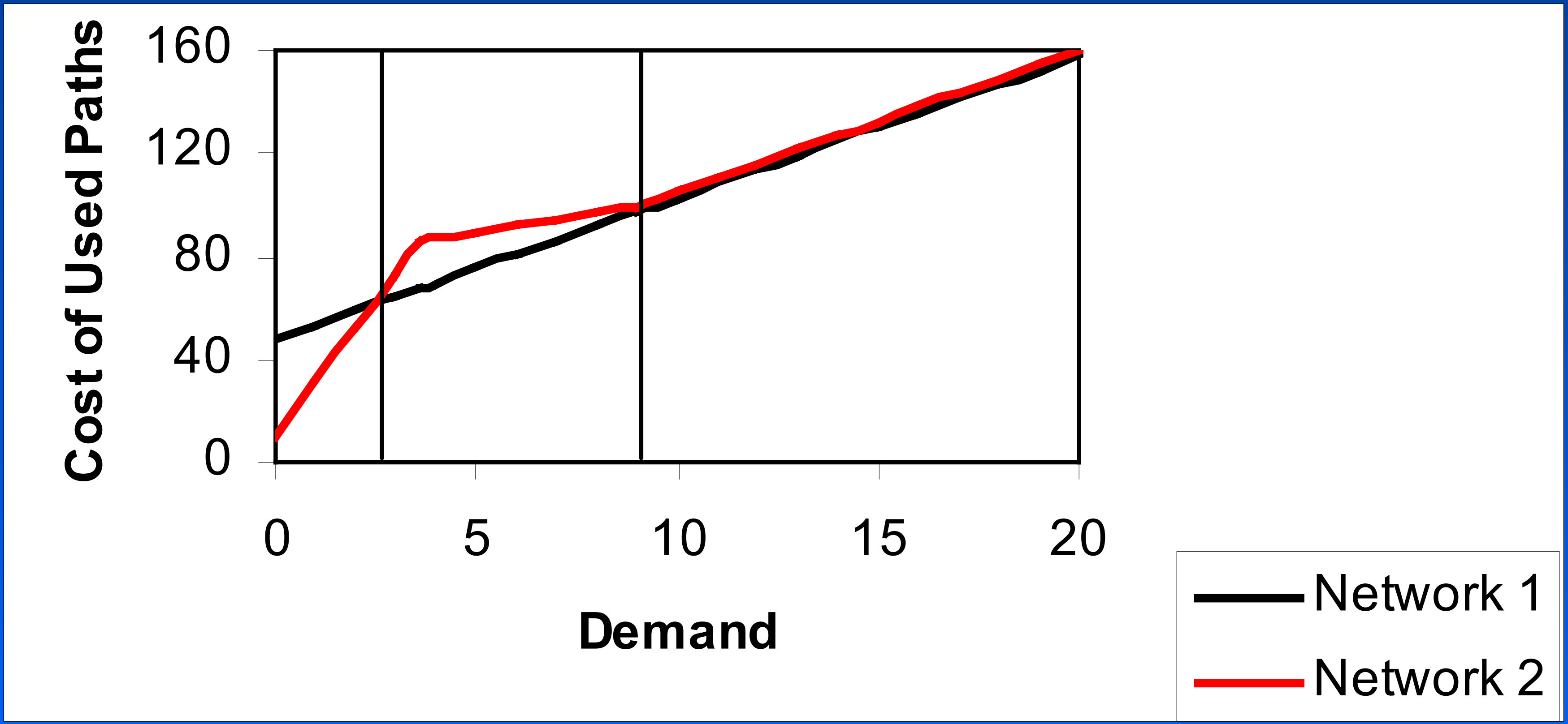


The Solution of an Evolutionary (Time-Dependent) Variational Inequality for the Braess Network with Added Link (Path)



Braess Network with Time-Dependent Demands

In Demand Regime I, only the new path is used.
In Demand Regime II, the Addition of a New Link (Path) Makes Everyone Worse Off!
In Demand Regime III, only the original paths are used.



Network 1 is the Original Braess Network - Network 2 has the added link.

The new link is NEVER used after a certain demand is reached even if the demand approaches infinity.

Hence, in general, except for a limited range of demand, building the new link is a complete waste!

- Recent disasters have demonstrated the importance as well as the vulnerability of critical infrastructure networks.
- For example:
  - Hurricane Katrina, August 23, 2005
  - The biggest blackout in North America, August 14, 2003
  - 9/11 Terrorist Attacks, September 11, 2001

An Urgent Need for a Network Efficiency/Performance Measure

In order to be able to assess the performance/efficiency of a network, it is imperative that appropriate measures be devised.

Appropriate network measures can assist in the identification of the importance of network components, that is, nodes and links, and their rankings. Such rankings can be very helpful in the case of the determination of network vulnerabilities as well as when to reinforce/enhance security.

The Network Efficiency Measure of Latora and Marchiori (2001)

- Latora and Marchiori (2001) proposed a network efficiency measure (the L-M measure) as follows:

Definition 2: The L-M Measure

The network performance/efficiency measure, $E(G)$, according to Latora and Marchiori (2001) for a given network topology G, is defined as:

$$E(G) = \frac{1}{n(n-1)} \sum_{i \neq j \in G} \frac{1}{d_{ij}}$$

where n is the number of nodes in the network and d_{ij} is the shortest path length between node i and node j.

The Network Efficiency Measure of Nagurney and Qiang (2006)

- Nagurney and Qiang (2006) proposed a network efficiency measure (the N-Q measure) which captures the demand and flow information under the network equilibrium. It is defined as follows:

Definition 3: The N-Q Measure

The network performance/efficiency measure, $\mathcal{E}(G,d)$, according to Nagurney and Qiang (2006), for a given network topology G and fixed demand vector d, is defined as:

$$\mathcal{E}(G,d) = \frac{\sum_{w \in W} \frac{d_w}{\lambda_w}}{n_W},$$

where recall that n_W is the number of O/D pairs in the network and λ_w is the equilibrium disutility for O/D pair w (cf. (6)).

Importance of a Network Component

Definition 4: Importance of a Network Component According to the L-M Measure

The importance of a network component $g \in G$, $\bar{I}(g)$, is measured by the network efficiency drop, determined by the L-M measure, after g is removed from the network:

$$\bar{I}(g) = \frac{\triangle E}{E(G)} = \frac{E(G) - E(G-g)}{E(G)}. \tag{11}$$

where $G - g$ is the resulting network after component g is removed from network G.

Definition 5: Importance of a Network Component According to the N-Q Measure

The importance of a network component $g \in G$, $I(g)$, is measured by the relative network efficiency drop, determined by the N-Q measure, after g is removed from the network:

$$I(g) = \frac{\triangle \mathcal{E}}{\mathcal{E}} = \frac{\mathcal{E}(G,d) - \mathcal{E}(G-g,d)}{\mathcal{E}(G,d)}, \tag{12}$$

where $G - g$ is the resulting network after component g is removed from network G.

The Approach to Study the Importance of Network Components

The elimination of a link is treated in the N-Q measure by removing that link while the removal of a node is managed by removing the links entering and exiting that node. In the case that the removal results in no path connecting an O/D pair, we simply assign the demand for that O/D pair to an abstract path with a cost of infinity.

Hence, our measure is well-defined even in the case of disconnected networks.

The L-M Measure vs. the N-Q Measure

Theorem 2

For a network with a single O/D pair (and fixed demand), the importance of a network component according to the L-M measure (defined in (11)) is equal to that obtained via the N-Q measure (defined in (12)).

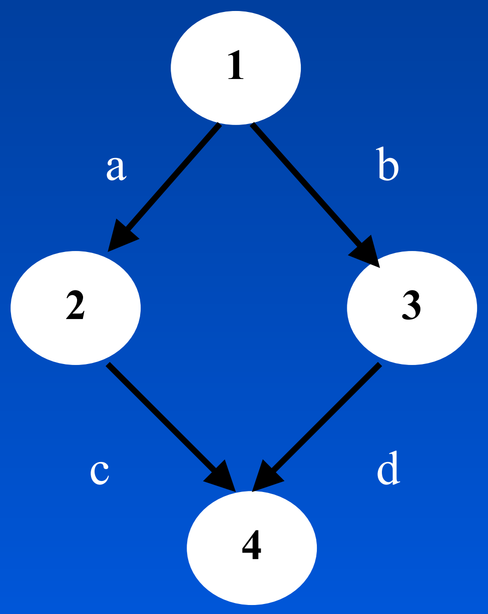
Application I: the Braess (1968) Network

Assume a network with a single O/D pair (1,4). There are 2 paths available to travelers: $\mathbf{p_1=(a,c)}$ and $\mathbf{p_2=(b,d)}$.

For a travel demand of 6, the equilibrium path flows are $\mathbf{x_{p_1}^*= x_{p_2}^*=3}$.

The equilibrium path travel cost is

$\mathbf{C_{p_1}=C_{p_2}=83}$.



$c_a(f_a)=10f_a$ $\quad$ $c_b(f_b)= f_b+50$

$c_c(f_c)=f_c+50$ $\quad$ $c_d(f_d)= 10f_d$

Adding a Link Increases Travel Cost for All!

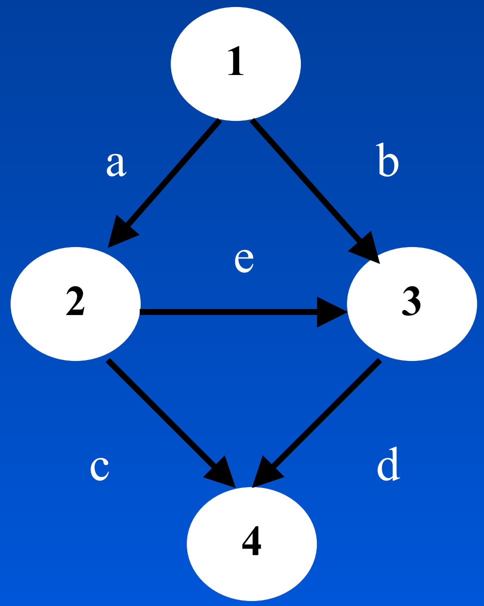
Adding a new link creates a new path **p_3=(a,e,d).**

The original flow distribution pattern is no longer an equilibrium pattern, since at this level of flow, the cost on path **p_3, C_{p3}=70.**

The new equilibrium flow pattern network is **$x_{p_1}^* = x_{p_2}^* = x_{p_3}^* = 2$.**

The equilibrium path travel cost is

$C_{p_1} = C_{p_2} = C_{p_3} = 92$.



$c_e(f_e) = f_e + 10$

Four Demand Ranges

- Demand Range I: $d_w \in [0, 80/31)$
  - Only $\mathbf{p_1}$ and $\mathbf{p_2}$ are used and the Braess Paradox does not occur
- Demand Range II: $d_w \in [80/31, 40/11]$
  - Only $\mathbf{p_1}$ and $\mathbf{p_2}$ are used and the Braess Paradox occurs
- Demand Range III: $d_w \in (40/11, 80/9]$
  - All paths are used and the Braess Paradox still occurs
- Demand Range IV: $d_w \in (80/9, \infty)$
  - Only $\mathbf{p_1}$ and $\mathbf{p_2}$ are used and the Braess Paradox vanishes

Importance and Ranking of Links and Nodes in Demand Range I

Table 1: Importance and Ranking of Links in Demand Range I: $d_w \in [0, 2\frac{18}{31})$

Link	Importance Value from the N-Q Measure	Importance Ranking from the N-Q Measure	Importance Value from the L-M Measure	Importance Ranking from the L-M Measure
a	$\frac{10(4-d_w)}{11d_w+50}$	1	$\frac{10(4-d_w)}{11d_w+50}$	1
b	0.00	3	0.00	3
c	0.00	3	0.00	3
d	$\frac{10(4-d_w)}{11d_w+50}$	1	$\frac{10(4-d_w)}{11d_w+50}$	1
e	$\frac{(80-31d_w)}{(11d_w+100)}$	2	$\frac{(80-31d_w)}{(11d_w+100)}$	2

Table 2: Importance and Ranking of Nodes in Demand Range I: $d_w \in [0, 2\frac{18}{31})$

Node	Importance Value from the N-Q Measure	Importance Ranking from the N-Q Measure	Importance Value from the L-M Measure	Importance Ranking from the L-M Measure
1	1.00	1	1.00	1
2	$\frac{10(4-d_w)}{11d_w+50}$	2	$\frac{10(4-d_w)}{11d_w+50}$	2
3	$\frac{10(4-d_w)}{11d_w+50}$	2	$\frac{10(4-d_w)}{11d_w+50}$	2
4	1.00	1	1.00	1

Importance and Ranking of Links and Nodes in Demand Range II

Table 3: Importance and Ranking of Links in Demand Range II: $d_w \in [2\frac{18}{31}, 3\frac{7}{11}]$

Link	Importance Value from the N-Q Measure	Importance Ranking from the N-Q Measure	Importance Value from the L-M Measure	Importance Ranking from the L-M Measure
a	$\frac{10(4-d_w)}{11d_w+50}$	1	$\frac{10(4-d_w)}{11d_w+50}$	1
b	0.00	2	0.00	2
c	0.00	2	0.00	2
d	$\frac{10(4-d_w)}{11d_w+50}$	1	$\frac{10(4-d_w)}{11d_w+50}$	1
e	$\frac{(80-31d_w)}{(11d_w+100)}$	3	$\frac{(80-31d_w)}{(11d_w+100)}$	3

Table 4: Importance and Ranking of Nodes in Demand Range II: $d_w \in [2\frac{18}{31}, 3\frac{7}{11}]$

Node	Importance Value from the N-Q Measure	Importance Ranking from the N-Q Measure	Importance Value from the L-M Measure	Importance Ranking from the L-M Measure
1	1.00	1	1.00	1
2	$\frac{10(4-d_w)}{11d_w+50}$	2	$\frac{10(4-d_w)}{11d_w+50}$	2
3	$\frac{10(4-d_w)}{11d_w+50}$	2	$\frac{10(4-d_w)}{11d_w+50}$	2
4	1.00	1	1.00	1

Importance and Ranking of Links and Nodes in Demand Range III

Table 5: Importance and Ranking of Links in Demand Range III: $d_w \in (3\frac{7}{11}, 8\frac{8}{9}]$

Link	Importance Value from the N-Q Measure	Importance Ranking from the N-Q Measure	Importance Value from the L-M Measure	Importance Ranking from the L-M Measure
a	$\frac{8(14d_w-45)}{13(11d_w+50)}$	1	$\frac{8(14d_w-45)}{13(11d_w+50)}$	1
b	$\frac{121(11d_w-40)}{13(131d_w+560)}$	2	$\frac{121(11d_w-40)}{13(131d_w+560)}$	2
c	$\frac{121(11d_w-40)}{13(131d_w+560)}$	2	$\frac{121(11d_w-40)}{13(131d_w+560)}$	2
d	$\frac{8(14d_w-45)}{13(11d_w+50)}$	1	$\frac{8(14d_w-45)}{13(11d_w+50)}$	1
e	$\frac{9(9d_w-80)}{13(11d_w+100)}$	3	$\frac{9(9d_w-80)}{13(11d_w+100)}$	3

Table 6: Importance and Ranking of Nodes in Demand Range III: $d_w \in (3\frac{7}{11}, 8\frac{8}{9}]$

Node	Importance Value from the N-Q Measure	Importance Ranking from the N-Q Measure	Importance Value from the L-M Measure	Importance Ranking from the L-M Measure
1	1.00	1	1.00	1
2	$\frac{8(14d_w-45)}{13(11d_w+50)}$	2	$\frac{8(14d_w-45)}{13(11d_w+50)}$	2
3	$\frac{8(14d_w-45)}{13(11d_w+50)}$	2	$\frac{8(14d_w-45)}{13(11d_w+50)}$	2
4	1.00	1	1.00	1

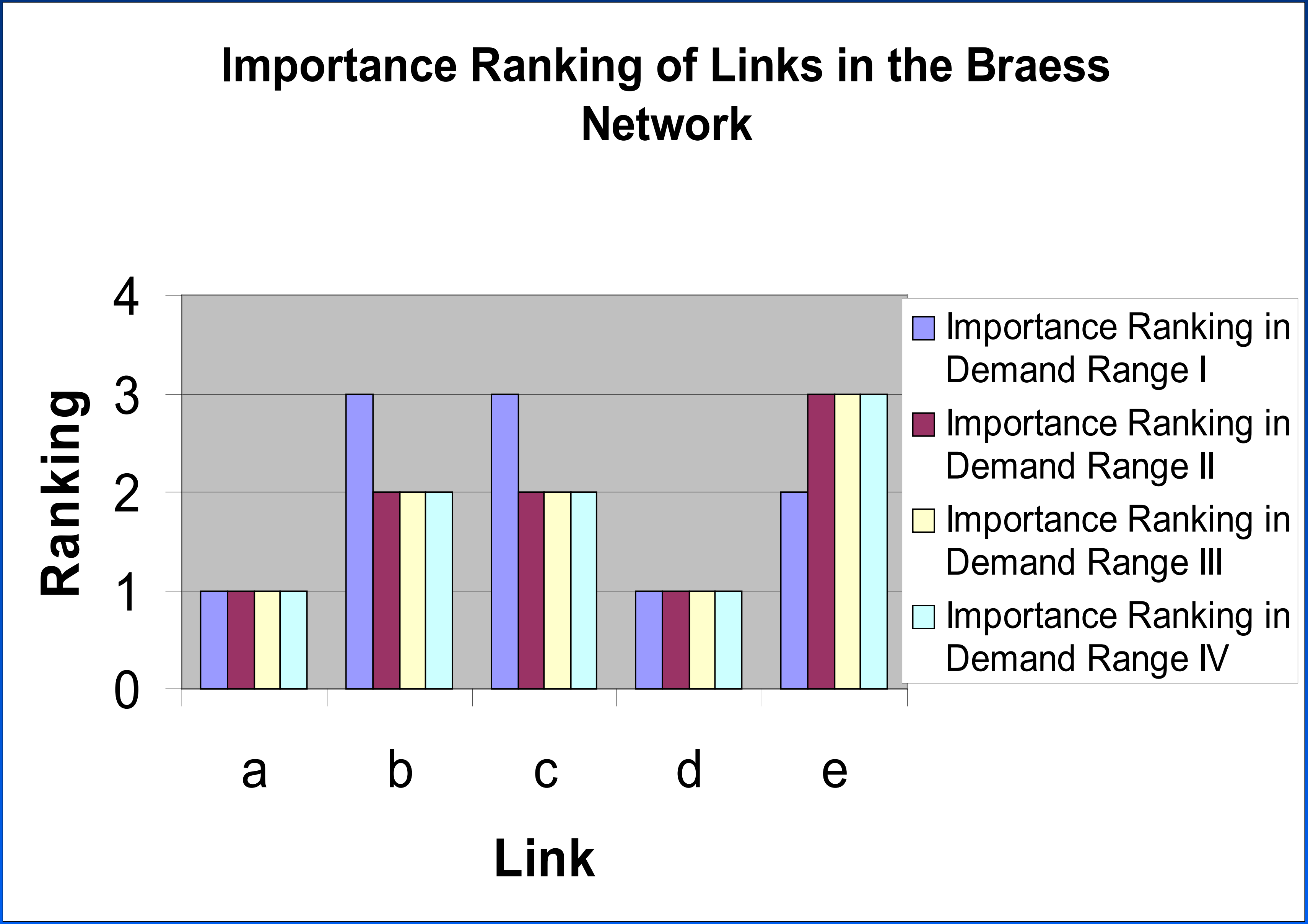
Importance and Ranking of Links and Nodes in Demand Range IV

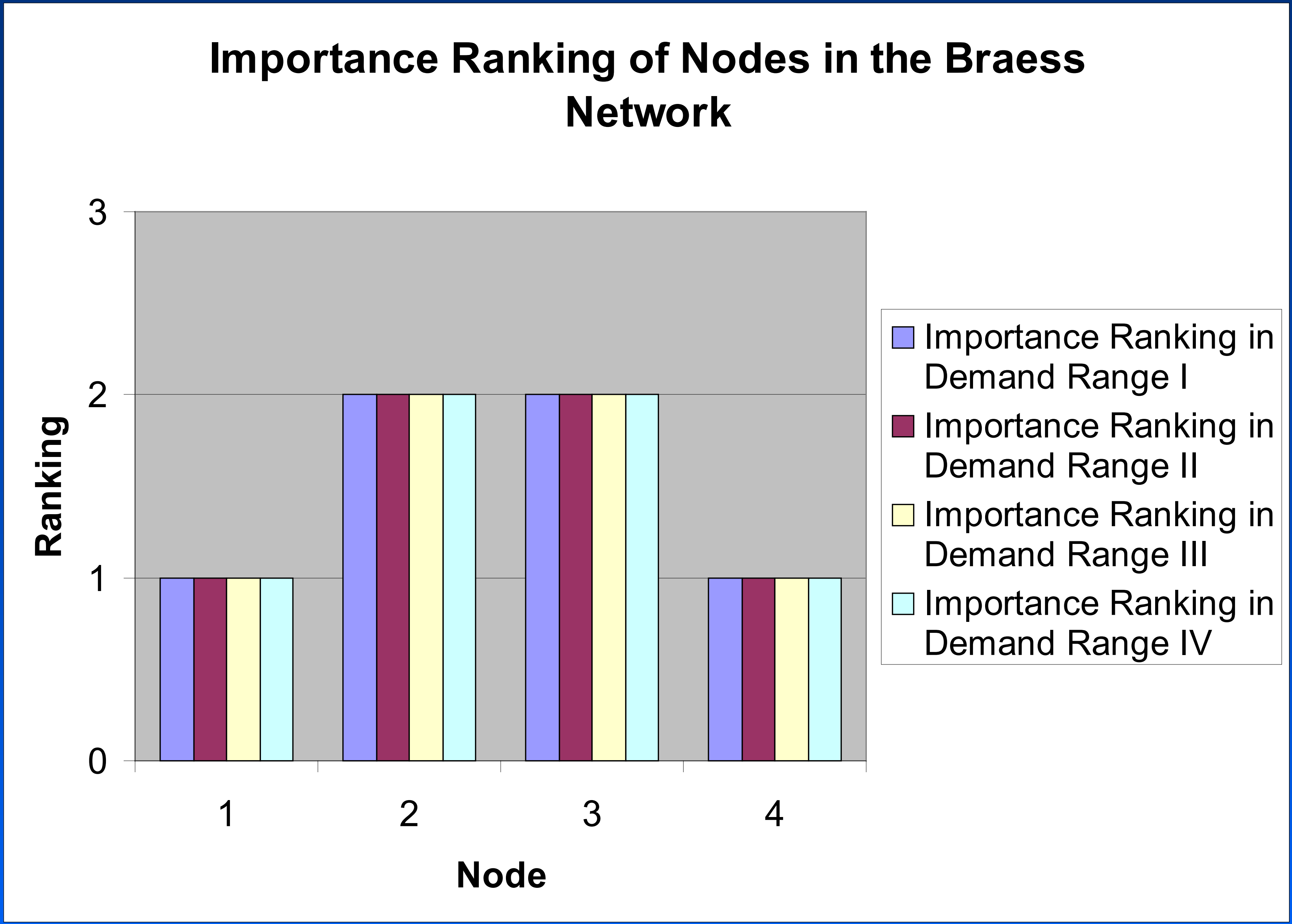
Table 7: Importance and Ranking of Links in Demand Range IV: $d_w \in (8\frac{8}{9}, \infty)$

Link	Importance Value from the N-Q Measure	Importance Ranking from the N-Q Measure	Importance Value from the L-M Measure	Importance Ranking from the L-M Measure
a	$\frac{11d_w}{2(11d_w+50)}$	1	$\frac{11d_w}{2(11d_w+50)}$	1
b	$\frac{5(13d_w-8)}{(131d_w+560)}$	2	$\frac{5(13d_w-8)}{(131d_w+560)}$	2
c	$\frac{5(13d_w-8)}{(131d_w+560)}$	2	$\frac{5(13d_w-8)}{(131d_w+560)}$	2
d	$\frac{11d_w}{2(11d_w+50)}$	1	$\frac{11d_w}{2(11d_w+50)}$	1
e	0.00	3	0.00	3

Table 8: Importance and Ranking of Nodes in Demand Range IV: $d_w \in (8\frac{8}{9}, \infty)$

Node	Importance Value from the N-Q Measure	Importance Ranking from the N-Q Measure	Importance Value from the L-M Measure	Importance Ranking from the L-M Measure
1	1.00	1	1.00	1
2	$\frac{11d_w}{2(11d_w+50)}$	2	$\frac{11d_w}{2(11d_w+50)}$	2
3	$\frac{11d_w}{2(11d_w+50)}$	2	$\frac{11d_w}{2(11d_w+50)}$	2
4	1.00	1	1.00	1

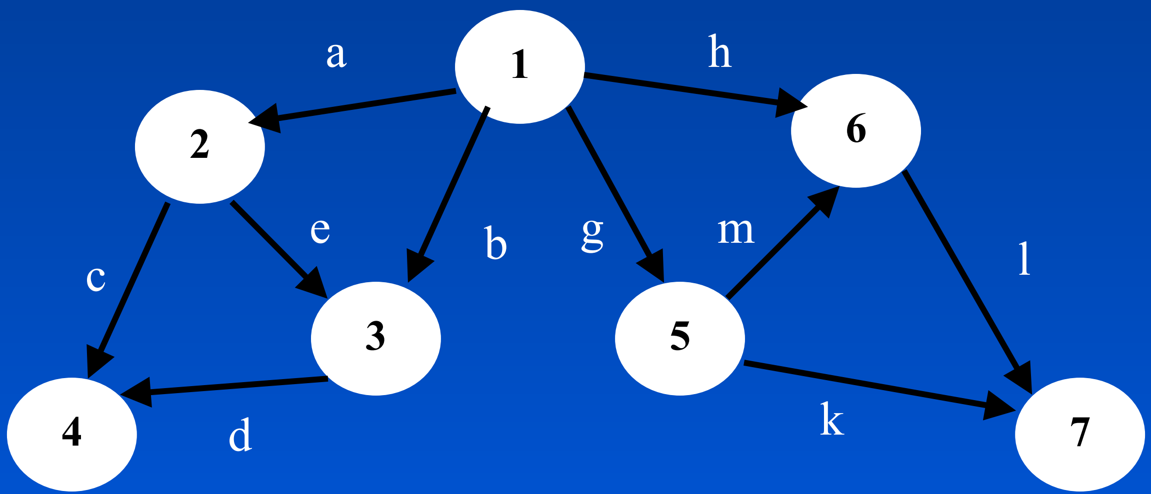

Importance Ranking of Links in the Braess Network
Ranking
4
3
2
1
0
a
b
c
d
e
Link
Importance Ranking in Demand Range I
Importance Ranking in Demand Range II
Importance Ranking in Demand Range III
Importance Ranking in Demand Range IV


Importance Ranking of Nodes in the Braess Network
Ranking
3
2
1
0
1
2
3
4
Node
Importance Ranking in Demand Range I
Importance Ranking in Demand Range II
Importance Ranking in Demand Range III
Importance Ranking in Demand Range IV

Discussion

Links *b* and *c* are less important in Demand Range I than Demand Range II, III and IV because they carry zero flow in Demand Range I

Application II: the "Coupled" Braess Network



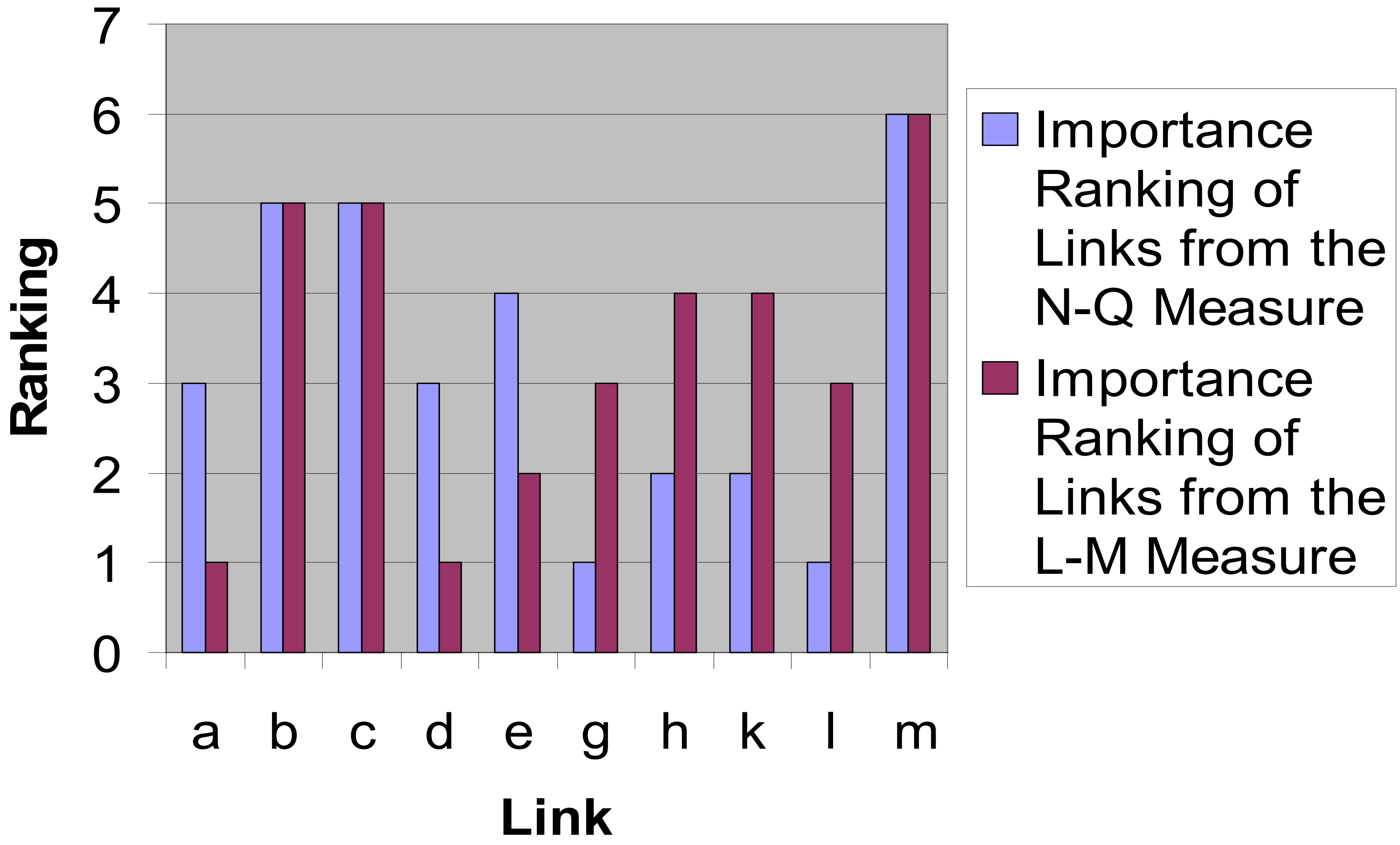
$d_{w_1}=2$ $d_{w_2}=6$

Importance and Ranking of Links

Table 9: Importance and the Ranking of Links in the Coupled Braess Example

Link	Importance Value from the N-Q Measure	Importance Ranking from the N-Q Measure	Importance Value from the L-M Measure	Importance Ranking from the L-M Measure
a	0.10	3	0.18	1
b	0.00	5	0.00	5
c	0.00	5	0.00	5
d	0.10	3	0.18	1
e	0.05	4	0.09	2
g	0.13	1	0.07	3
h	0.11	2	0.06	4
k	0.11	2	0.06	4
l	0.13	1	0.07	3
m	-0.07	6	-0.04	6

Importance Ranking of Links in the Coupled Braess Network

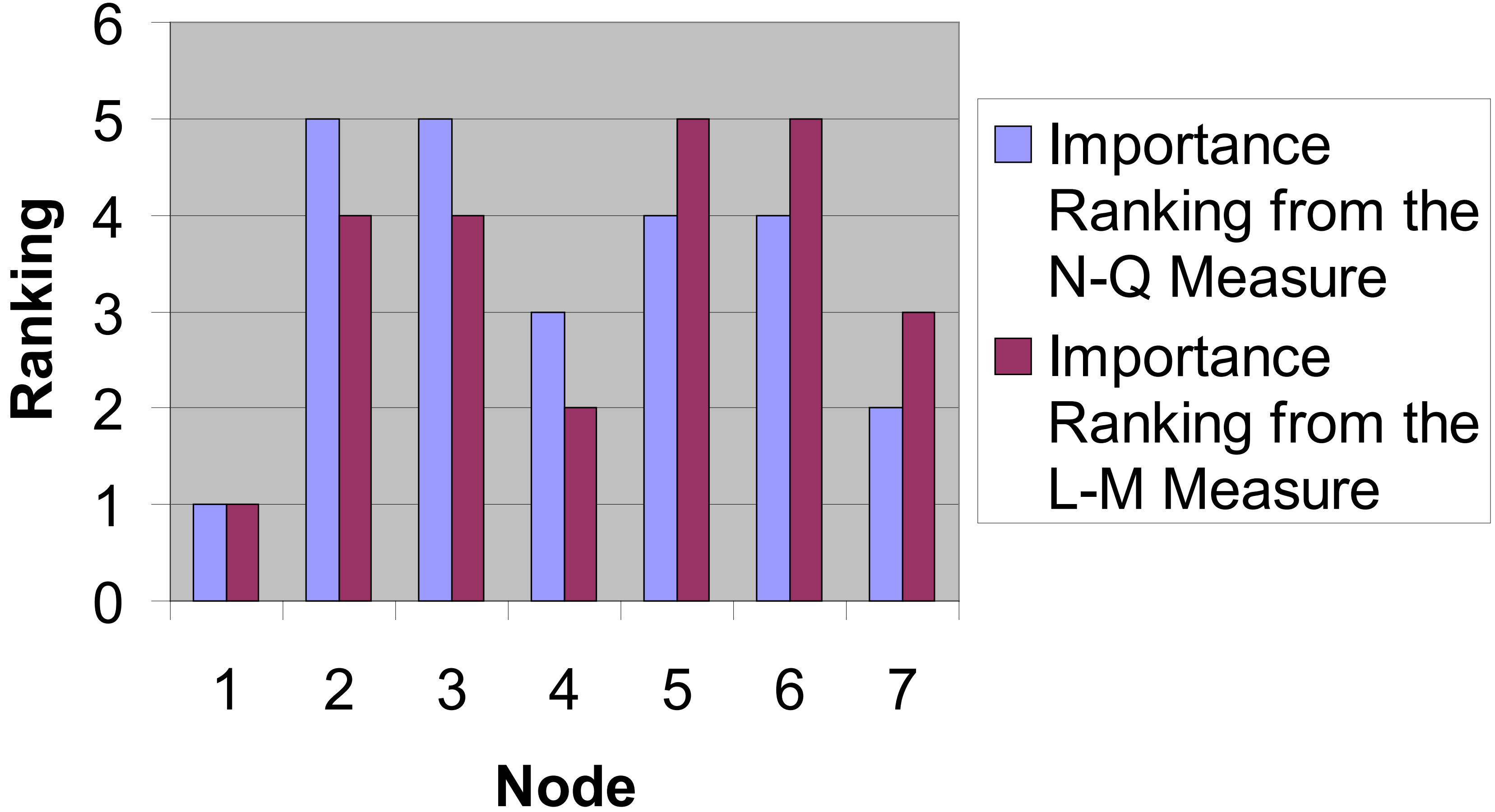


Importance and Ranking of Nodes

Table 10: Importance and Ranking of Nodes in the Coupled Braess Example

Node	Importance Value from the N-Q Measure	Importance Ranking from the N-Q Measure	Importance Value from the L-M Measure	Importance Ranking from the L-M Measure
1	1.00	1	1.00	1
2	0.10	5	0.18	4
3	0.10	5	0.18	4
4	0.37	3	0.64	2
5	0.13	4	0.07	5
6	0.13	4	0.07	5
7	0.63	2	0.36	3

Importance Ranking of Nodes in the Coupled Braess Network



Discussion

- Links *l* and *g* are the most important links from the N-Q measure while they are only in third place from the L-M measure because they carry a larger amount of flow.

- Links *a* and *d* are the most important links from the L-M measure while they are only in third place from the N-Q measure because they carry a less amount of flow.

Application III: An Electric Power Supply Chain Network

Nagurney and Liu (2006) and Nagurney, Liu, Cojocaru and Daniele (2005) have shown that an electric power supply chain network can be transformed into an equivalent transportation network problem.

Supernetwork Transformation

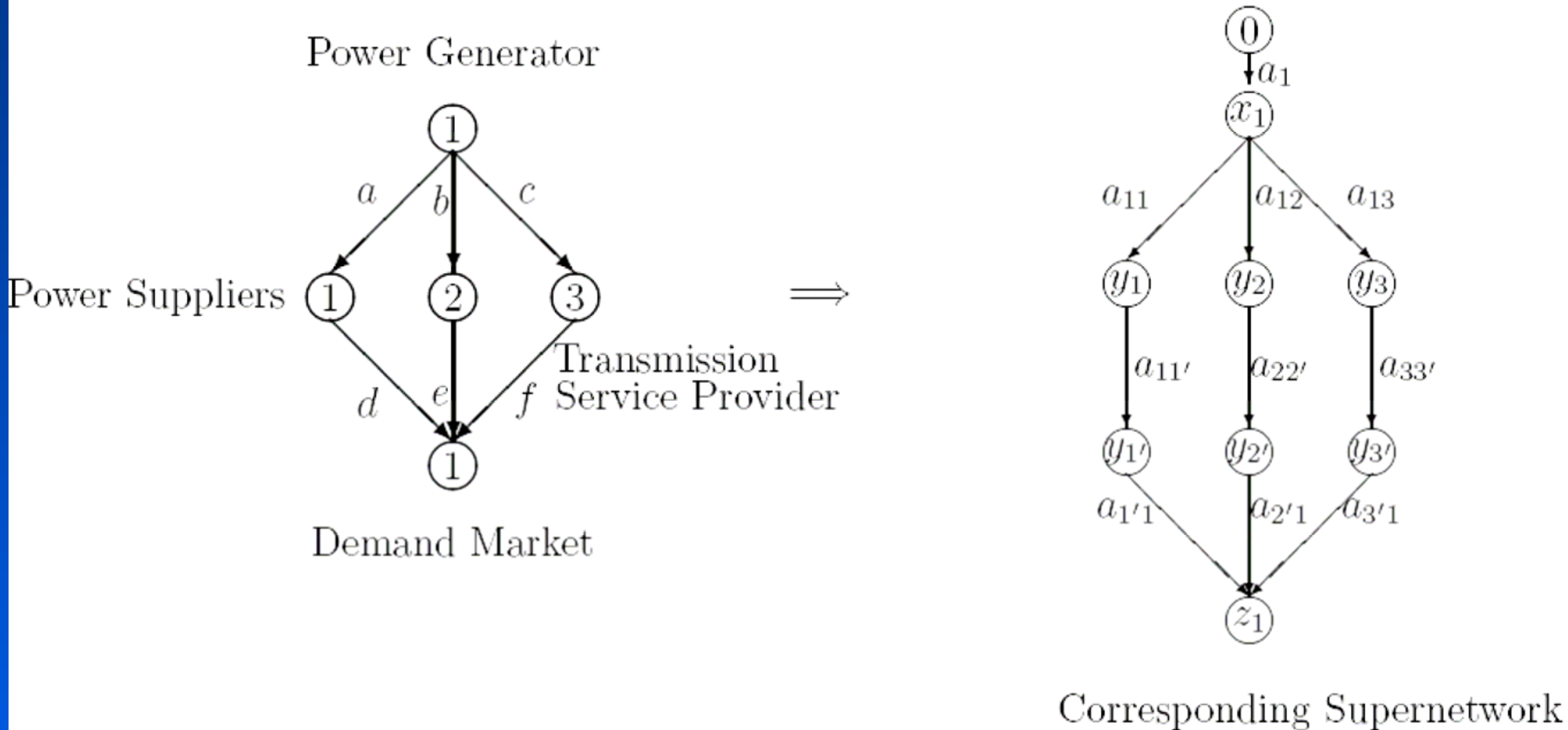


Figure 3: Electric Power Supply Chain Network and the Corresponding Supernetwork

Example 1 from Nagurney, Liu, Cojocaru and Daniele, *TRE* (2005)

Five Demand Ranges

- Demand Range I: $d_w \in [0, 1]$
- Demand Range II: $d_w \in (1, 4/3]$
- Demand Range III: $d_w \in (4/3, 7/3]$
- Demand Range IV: $d_w \in (7/3, 11/3]$
- Demand Range V: $d_w \in (11/3, \infty)$

Importance and Ranking of Links and Nodes in Demand Range I

Table 11: Importance and Ranking of Links in Demand Range I: $d_{w_1} \in [0, 1]$

Link	Importance Value from the N-Q Measure	Importance Ranking from the N-Q Measure	Importance Value from the L-M Measure	Importance Ranking from the L-M Measure
a	$\frac{6}{16d_{w_1}+19}$	1	$\frac{6}{16d_{w_1}+19}$	1
b	0.00	2	0.00	2
c	0.00	2	0.00	2
d	$\frac{6}{16d_{w_1}+19}$	1	$\frac{6}{16d_{w_1}+19}$	1
e	0.00	2	0.00	2
f	0.00	2	0.00	2

Table 12: Importance and Ranking of Nodes in Demand Range I: $d_{w_1} \in [0, 1]$

Node	Importance Value from the N-Q Measure	Importance Ranking from the N-Q Measure	Importance Value from the L-M Measure	Importance Ranking from the L-M Measure
Power Generator 1	1.00	1	1.00	1
Power Supplier 1	$\frac{6}{16d_{w_1}+19}$	2	$\frac{6}{16d_{w_1}+19}$	2
Power Supplier 2	0.00	3	0.00	3
Power Supplier 3	0.00	3	0.00	3
Demand Market 1	1.00	1	1.00	1

Importance and Ranking of Links and Nodes in Demand Range II

Table 13: Importance and Ranking of Links in Demand Range II: $d_{w_1} \in (1, \frac{4}{3}]$

Link	Importance Value from the N-Q Measure	Importance Ranking from the N-Q Measure	Importance Value from the L-M Measure	Importance Ranking from the L-M Measure
a	$\frac{3(d_{w_1}+1)}{16d_{w_1}+19}$	1	$\frac{3(d_{w_1}+1)}{16d_{w_1}+19}$	1
b	$\frac{3(d_{w_1}-1)}{16d_{w_1}+13}$	2	$\frac{3(d_{w_1}-1)}{16d_{w_1}+13}$	2
c	0.00	3	0.00	3
d	$\frac{3(d_{w_1}+1)}{16d_{w_1}+19}$	1	$\frac{3(d_{w_1}+1)}{16d_{w_1}+19}$	1
e	$\frac{3(d_{w_1}-1)}{16d_{w_1}+13}$	2	$\frac{3(d_{w_1}-1)}{16d_{w_1}+13}$	2
f	0.00	3	0.00	3

Table 14: Importance and Ranking of Nodes in Demand Range II: $d_{w_1} \in (1, \frac{4}{3}]$

Node	Importance Value from the N-Q Measure	Importance Ranking from the N-Q Measure	Importance Value from the L-M Measure	Importance Ranking from the L-M Measure
Power Generator 1	1.00	1	1.00	1
Power Supplier 1	$\frac{3(d_{w_1}+1)}{16d_{w_1}+19}$	2	$\frac{3(d_{w_1}+1)}{16d_{w_1}+19}$	2
Power Supplier 2	$\frac{3(d_{w_1}-1)}{16d_{w_1}+13}$	3	$\frac{3(d_{w_1}-1)}{16d_{w_1}+13}$	3
Power Supplier 3	0.00	4	0.00	4
Demand Market 1	1.00	1	1.00	1

Importance and Ranking of Links and Nodes in Demand Range III

Table 15: Importance and Ranking of Links in Demand Range III: $d_{w_1} \in (\frac{4}{3}, \frac{7}{3}]$

Link	Importance Value from the N-Q Measure	Importance Ranking from the N-Q Measure	Importance Value from the L-M Measure	Importance Ranking from the L-M Measure
a	$\frac{7}{13d_{w_1}+23}$	1	$\frac{7}{13d_{w_1}+23}$	1
b	$\frac{3(d_{w_1}-1)}{16d_{w_1}+13}$	2	$\frac{3(d_{w_1}-1)}{16d_{w_1}+13}$	2
c	0.00	3	0.00	3
d	$\frac{7}{13d_{w_1}+23}$	1	$\frac{7}{13d_{w_1}+23}$	1
e	$\frac{3(d_{w_1}-1)}{16d_{w_1}+13}$	2	$\frac{3(d_{w_1}-1)}{16d_{w_1}+13}$	2
f	0.00	3	0.00	3

Table 16: Importance and Ranking of Nodes in Demand Range III: $d_{w_1} \in (\frac{4}{3}, \frac{7}{3}]$

Node	Importance Value from the N-Q Measure	Importance Ranking from the N-Q Measure	Importance Value from the L-M Measure	Importance Ranking from the L-M Measure
Power Generator 1	1.00	1	1.00	1
Power Supplier 1	$\frac{7}{13d_{w_1}+23}$	2	$\frac{7}{13d_{w_1}+23}$	2
Power Supplier 2	$\frac{3(d_{w_1}-1)}{16d_{w_1}+13}$	3	$\frac{3(d_{w_1}-1)}{16d_{w_1}+13}$	3
Power Supplier 3	0.00	4	0.00	4
Demand Market 1	1.00	1	1.00	1

Importance and Ranking of Links and Nodes in Demand Range IV

Table 17: Importance and Ranking of Links in Demand Range IV: $d_{w_1} \in (\frac{7}{3}, \frac{11}{3}]$

Link	Importance Value from the N-Q Measure	Importance Ranking from the N-Q Measure	Importance Value from the L-M Measure	Importance Ranking from the L-M Measure
a	$\frac{7}{13d_{w_1}+23}$	1	$\frac{7}{13d_{w_1}+23}$	1
b	$\frac{4}{13d_{w_1}+20}$	2	$\frac{4}{13d_{w_1}+20}$	2
c	0.00	3	0.00	3
d	$\frac{7}{13d_{w_1}+23}$	1	$\frac{7}{13d_{w_1}+23}$	1
e	$\frac{4}{13d_{w_1}+20}$	2	$\frac{4}{13d_{w_1}+20}$	2
f	0.00	3	0.00	3

Table 18: Importance and Ranking of Nodes in Demand Range IV: $d_{w_1} \in (\frac{7}{3}, \frac{11}{3}]$

Node	Importance Value from the N-Q Measure	Importance Ranking from the N-Q Measure	Importance Value from the L-M Measure	Importance Ranking from the L-M Measure
Power Generator 1	1.00	1	1.00	1
Power Supplier 1	$\frac{7}{13d_{w_1}+23}$	2	$\frac{7}{13d_{w_1}+23}$	2
Power Supplier 2	$\frac{4}{13d_{w_1}+20}$	3	$\frac{4}{13d_{w_1}+20}$	3
Power Supplier 3	0.00	4	0.00	4
Demand Market 1	1.00	1	1.00	1

Importance and Ranking of Links and Nodes in Demand Range V

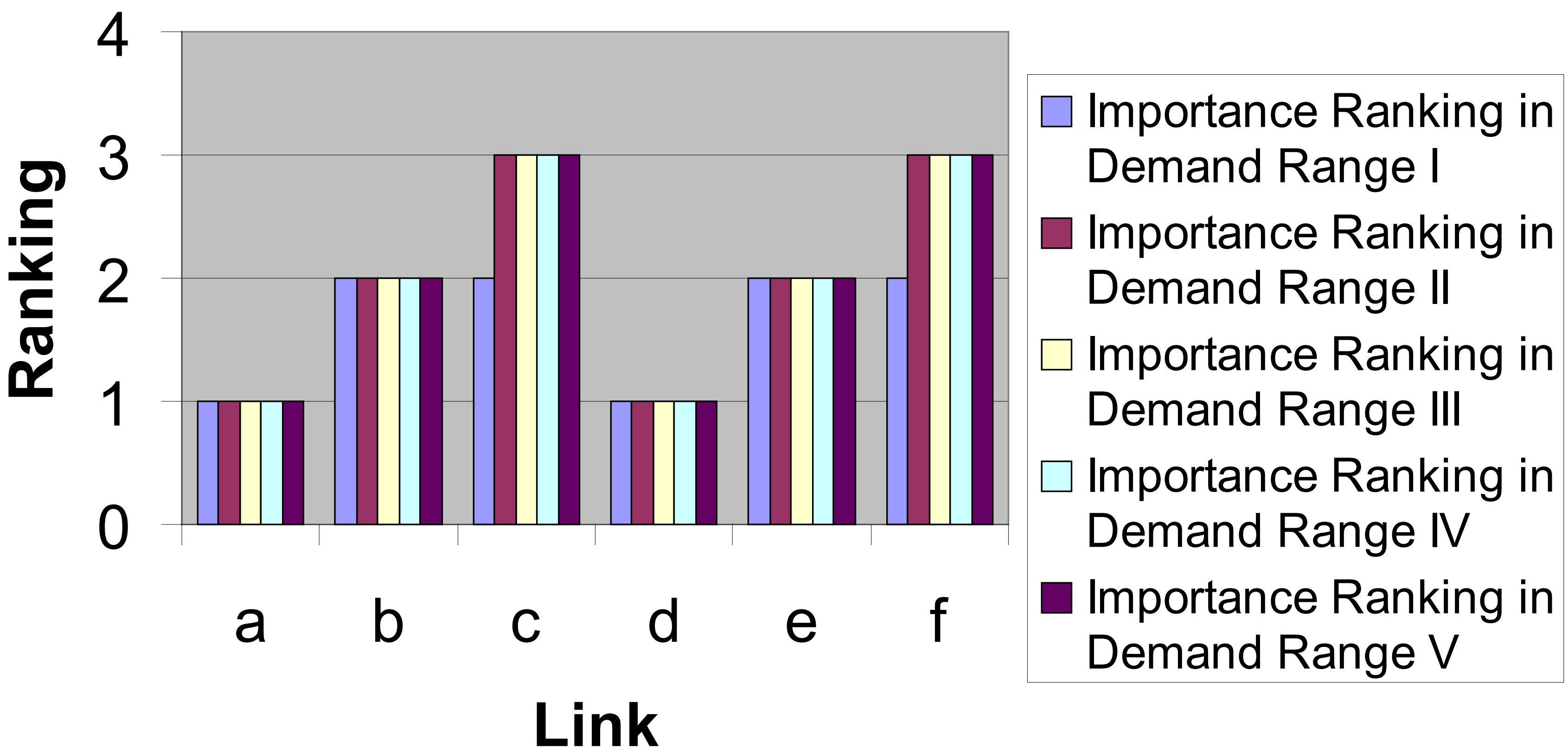
Table 19: Importance and Ranking of Links in Demand Range V: $d_{w_1} \in (\frac{11}{3}, \infty)$

Link	Importance Value from the N-Q Measure	Importance Ranking from the N-Q Measure	Importance Value from the L-M Measure	Importance Ranking from the L-M Measure
a	$\frac{3d_{w_1}+10}{3(13d_{w_1}+23)}$	1	$\frac{3d_{w_1}+10}{3(13d_{w_1}+23)}$	1
b	$\frac{3d_{w_1}+1}{3(13d_{w_1}+20)}$	2	$\frac{3d_{w_1}+1}{3(13d_{w_1}+20)}$	2
c	$\frac{3d_{w_1}-11}{3(13d_{w_1}+16)}$	3	$\frac{3d_{w_1}-11}{3(13d_{w_1}+16)}$	3
d	$\frac{3d_{w_1}+10}{3(13d_{w_1}+23)}$	1	$\frac{3d_{w_1}+10}{3(13d_{w_1}+23)}$	1
e	$\frac{3d_{w_1}+1}{3(13d_{w_1}+20)}$	2	$\frac{3d_{w_1}+1}{3(13d_{w_1}+20)}$	2
f	$\frac{3d_{w_1}-11}{3(13d_{w_1}+16)}$	3	$\frac{3d_{w_1}-11}{3(13d_{w_1}+16)}$	3

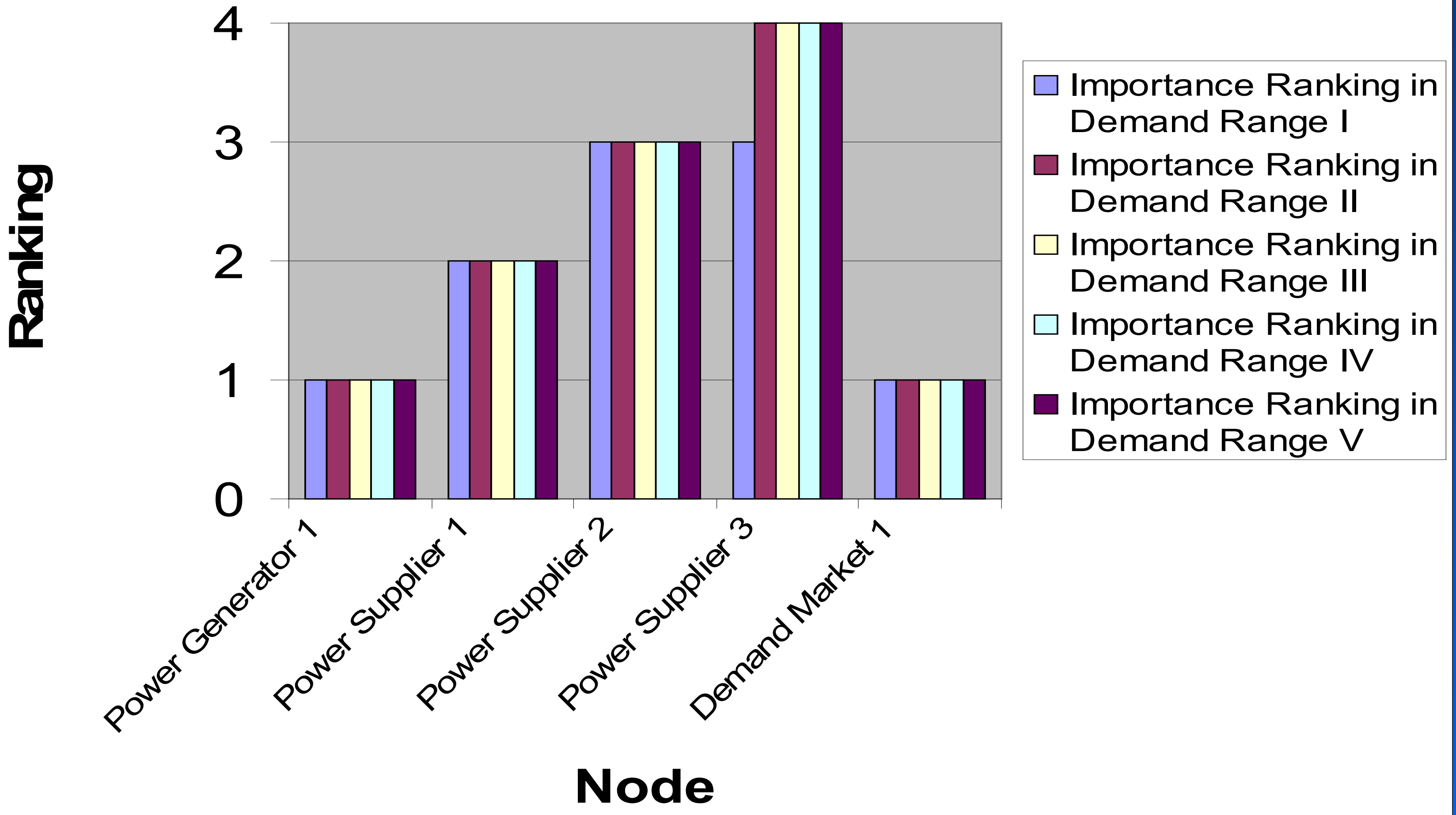
Table 20: Importance and Ranking of Nodes in Demand Range V: $d_{w_1} \in (\frac{11}{3}, \infty)$

Node	Importance Value from the N-Q Measure	Importance Ranking from the N-Q Measure	Importance Value from the L-M Measure	Importance Ranking from the L-M Measure
Power Generator 1	1.00	1	1.00	1
Power Supplier 1	$\frac{3d_{w_1}+10}{3(13d_{w_1}+23)}$	2	$\frac{3d_{w_1}+10}{3(13d_{w_1}+23)}$	2
Power Supplier 2	$\frac{3d_{w_1}+1}{3(13d_{w_1}+20)}$	3	$\frac{3d_{w_1}+1}{3(13d_{w_1}+20)}$	3
Power Supplier 3	$\frac{3d_{w_1}-11}{3(13d_{w_1}+16)}$	4	$\frac{3d_{w_1}-11}{3(13d_{w_1}+16)}$	4
Demand Market 1	1.00	1	1.00	1

Importance Ranking of Links in the Electric Power Supply Chain Network



Importance Ranking of Nodes in the Electric Power Supply Chain Network



Discussion

Links ***a*** and ***d*** are the most important links and power supplier 1 is ranked the second due to the fact that path $\boldsymbol{p_1}$, which consists of links ***a*** and ***d*** and power supplier 1 carry the largest amount of flow.

New Challenges and Opportunities: The Unification of EVIs and PDSs

Bellagio Research Team Residency March 2004

Double-Layered Dynamics

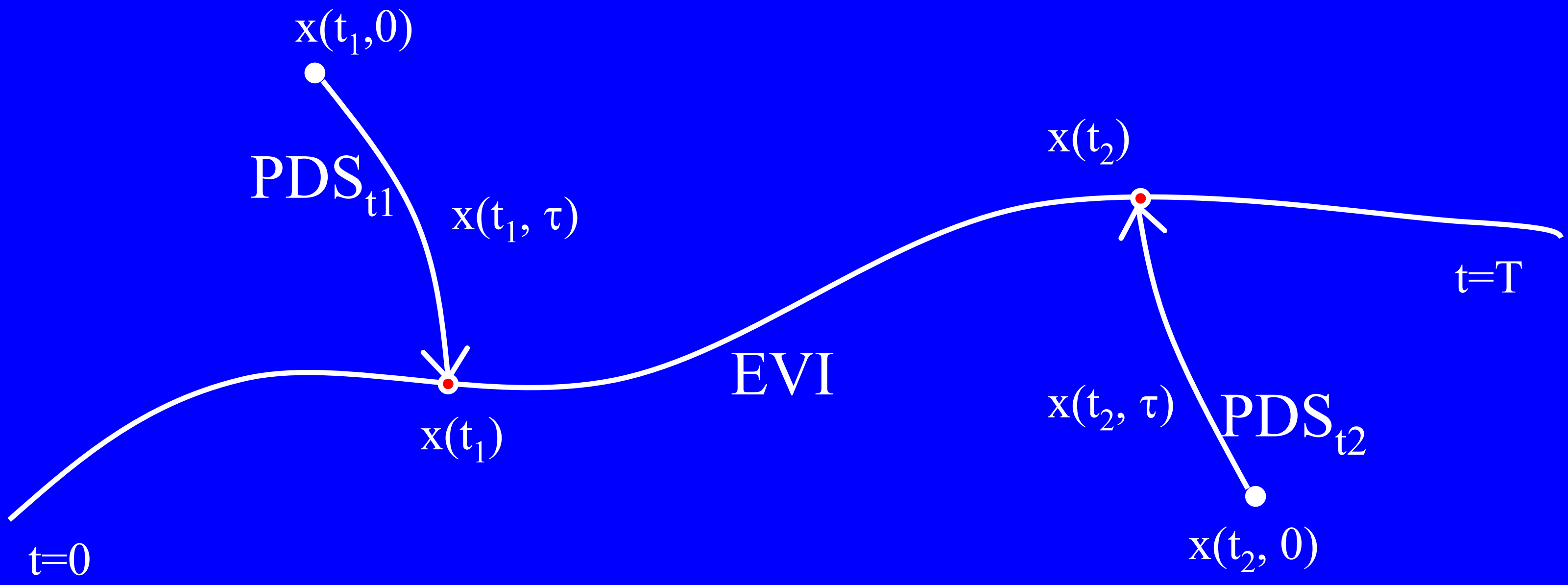
The unification of EVIs and PDSs allows the modeling of dynamic networks over *different time scales*.

Papers:

Projected Dynamical Systems and Evolutionary Variational Inequalities via Hilbert Spaces with Applications (Cojocaru, Daniele, and Nagurney), *Journal of Optimization Theory and Applications*, vol. 127, no. 3, pp. 1-15, December 2005.

Double-Layered Dynamics: A Unified Theory of Projected Dynamical Systems and Evolutionary Variational Inequalities (Cojocaru, Daniele, and Nagurney), *European Journal of Operational Research,* in press.

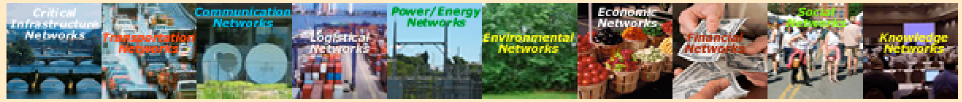
A Pictorial of the Double-Layered Dynamics



There are new exciting questions, both theoretical and computational, arising from this multiple time structure.

In the course of answering these questions, a new theory is taking shape from the synthesis of PDS and EVI, and, as such, it deserves a name of its own; we call it **double-layered dynamics.**

The Virtual Center for Supernetworks



Supernetworks for Optimal Decision-Making and Improving the Global Quality of Life

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Mission: The mission of the Virtual Center for Supernetworks is to foster the study and application of supernetworks and to serve as a resource to academia, industry, and government on networks ranging from transportation, logistical, telecommunication, and power networks to economic, environmental, financial, knowledge and social networks.

The applications of Supernetworks include: transportation, logistics, critical infrastructure, telecommunications, power and energy, electronic commerce, supply chain management, environment, economics, finance, knowledge and social networks, and decision-making.

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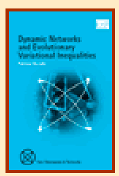

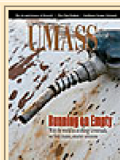

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The route to sustainable transportation is paved with knowledge and creativity
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Professor Dietrich Braess visits the Virtual Center for Supernetworks April 7, 2006

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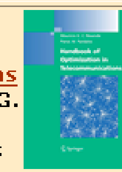

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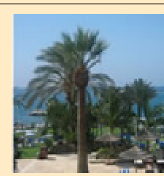

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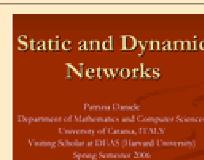

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