Sustainable Supply Chain Network Design: A Multicriteria Perspective

Anna Nagurney[1] Ladimer S. Nagurney[2]

[1]Isenberg School of Management
University of Massachusetts
Amherst, Massachusetts 01003

[2]Department of Electrical and Computer Engineering
University of Hartford
West Hartford, Connecticut 06117

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Outline

- Background and Motivation
- An Overview of Some of the Relevant Literature
- The Sustainable Supply Chain Network Design Model
- Numerical Examples
- Summary and Conclusions

Background and Motivation

Supply chain networks provide the infrastructure for the production, storage, and distribution of products as varied as pharmaceuticals, vehicles, computers, food products, furniture, and clothing, throughout the globe.

Sustainable Systems

Supply chain networks are components of sustainable systems.

Background and Motivation

The **design of supply chain networks** is a topic of engineering importance since it involves the determination of both the sites and the levels of operation of its relevant facilities.

Background and Motivation

Sustainability of supply chains has emerged as a major theme in both research and practice since the impacts of climate change have made both producers and consumers more cognizant of their decision-making and how their decisions affect the environment.

Photos of oil spill crisis in Gulf of Mexico, May 2010

Environmental Effects of Globalization

Businesses, and, in particular, supply chains, have become increasingly globalized.

However, criticism of globalization has increased, specifically, by those concerned about the environment on the basis that global free trade may result in the growth of global pollution.

For example, free trade may shift pollution-intensive manufacturing processes from countries with strict environmental regulations to those with less restrictive ones.

Environmentally-Friendly Supply Chains

Nevertheless, legal requirements and evolving consumer tastes are placing pressure on manufacturers and distributors to become more environmentally-friendly. (Bloemhof-Ruwaard et al. (1995), Hill (1997), and Ingram (2002))

Poor environmental performance at any stage of the supply chain process may damage what is considered a firm's premier asset, its reputation (Fabian (2000)).

An Overview of Some of the Relevant Literature

- M. J. Beckmann, C. B. McGuire, and C. B. Winsten (1956) *Studies in the Economics of Transportation*. Yale University Press, New Haven, Connecticut.
- S. C. Dafermos and F. T. Sparrow (1969) The Traffic Assignment Problem for a General Network. *Journal of Research of the National Bureau of Standards*. 73B: 91-118.
- D. E. Boyce, H. S. Mahmassani, A. Nagurney (2005) A Retrospective on Beckmann, McGuire, and Winsten's *Studies in the Economics of Transportation*. *Papers in Regional Science* 84: 85-103.
- A. Nagurney and F. Toyasaki (2005) Reverse Supply Chain Management and Electronic Waste Recycling: A Multitiered Network Equilibrium Framework for E-Cycling. *Transportation Research E* 41: 1-28.

- A. Nagurney, Z. Liu, and T. Woolley (2007) Sustainable Supply Chain Networks and Transportation. *International Journal of Sustainable Transportation* 1: 29-51.
- A. Nagurney, Q. Qiang, and L. Nagurney (2010) Environmental Impact Assessment of Transportation Networks with Degradable Links in an Era of Climate Change. *International Journal of Sustainable Transportation* 4: 154-171.
- A. Nagurney and T. Woolley (2010) Environmental and Cost Synergy in Supply Chain Network Integration in Mergers and Acquisitions. Sustainable Energy and Transportation Systems, Proceedings of the 19th International Conference on Multiple Criteria Decision Making, Lecture Notes in Economics and Mathematical Systems, M. Ehrgott, B. Naujoks, T. Stewart, and J. Wallenius, Editors, Springer, Berlin, Germany 51-78.

This talk is based on the paper:

Sustainable Supply Chain Network Design: A Multicriteria Perspective,

A. Nagurney and Ladimer S. Nagurney, to appear in the *International Journal of Sustainable Engineering*,

where additional background information as well as references can be found.

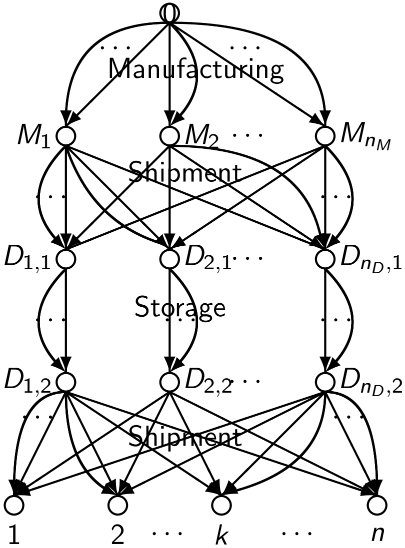
The Sustainable Supply Chain Network Design Model

We assume a network topology where the top level (origin) node 0 corresponds to the firm and the bottom level (destination) nodes correspond to the demand sites, which can correspond, for example, to retailers or consumers.

The paths joining the origin node to the destination nodes represent sequences of supply chain network activities that ensure that the product is produced and, ultimately, delivered to the demand sites.

There are n_M possible manufacturing facilities/plants, n_D possible distribution centers to serve the n possible demand locations with the respective demands given by: d_1, d_2, . . ., d_n.

The Baseline Supply Chain Network Topology



The Sustainable Supply Chain Network Design Model

The links from the top-tiered node 0 are connected to the possible manufacturing nodes of the firm, which are denoted, respectively, by: $M_1, \ldots, M_{n_M}$, and these links represent the manufacturing links.

The links from the manufacturing nodes, in turn, are connected to the possible distribution center nodes of the firm which are denoted by $D_{1,1}, \ldots, D_{n_D,1}$. These links correspond to the possible shipment links.

The links joining nodes $D_{1,1}, \ldots, D_{n_D,1}$ with nodes $D_{1,2}, \ldots, D_{n_D,2}$ correspond to the possible storage links. Finally, there are possible shipment links joining the nodes $D_{1,2}, \ldots, D_{n_D,2}$ with the demand nodes: $1, \ldots, n$.

The Sustainable Supply Chain Network Design Model

Multiple alternative manufacturing links correspond to different possible technologies associated with a given manufacturing plant, which may also result in different levels of environmental emissions.

Similarly, for shipments and storage, we may have multiple alternative links which represent different modes of transportation and different storage technologies, respectively.

The Sustainable Supply Chain Network Design Model

The supply chain network consists of the graph $G = [N, L]$, where N denotes the set of nodes and L the set of directed links.

The formalism is that of optimization, where the firm wishes to determine which manufacturing plants it should operate (and the corresponding technologies) and at what level; the same for the distribution centers. In addition, the firm seeks to determine the capacity levels of the shipment links and the mode of transportation/shipment used.

The firm seeks to minimize the total costs associated with its production, storage, and distribution activities, along with the total capital outlays, plus the total cost of environmental emissions in order to achieve the activity levels as given by the capacities on its various links, subject to the demand being satisfied at the demand sites.

The Notation

- $\hat{c}_a$: the total cost on link a.
- f_a: the flow of the product on link a.
- p: a path in the network joining the origin node to a destination node representing the activities and their sequence associated with producing the product and having it, ultimately, delivered.
- w_k: the pair of origin/destination (O/D) nodes $(0, k)$
- P_{w_k}: the set of paths, which represent alternative associated possible supply chain network processes, joining $(0, k)$.
- P: the set of all paths joining node 0 to the demand nodes.
- n_P: the number of paths from the origin to the demand markets.
- x_p: the flow of the product on path p.
- d_k: the demand, which is assumed to be known and fixed, for the product at demand location k.

Demand satisfaction constraint

$$d_k \equiv \sum_{p \in P_{w_k}} x_p, \quad k = 1, \ldots, n, \tag{1}$$

that is, the demand must be satisfied at each demand site.

Conservation of flow between path flows and link flows

$$f_a = \sum_{p \in P} x_p \delta_{ap}, \quad \forall a \in L, \tag{2}$$

where $\delta_{ap} = 1$, if link a is contained in path p, and $\delta_{ap} = 0$, otherwise.

Path flow nonnegativity constraint

$$x_p \geq 0, \quad \forall p \in P, \tag{3}$$

Formulation

The total cost functions on links

$$\hat{c}_a = \hat{c}_a(f_a), \quad \forall a \in L. \tag{4}$$

The total cost on a link is assumed to be a function of the flow of the product on the link, which is assumed to be convex and is continuously differentiable.

The total capital cost

$\hat{\pi}_a$: total capital cost of adding capacity u_a on link a:

$$\hat{\pi}_a = \hat{\pi}_a(u_a), \quad \forall a \in L. \tag{5}$$

These functions are also assumed to be convex and continuously differentiable and to have bounded second order partial derivatives.

The First Objective Function

The firm seeks to minimize the total cost including the total cost of operating the various links and the total cost of capacity capital investments:

The total cost minimization objective function

$$\text{Minimize} \quad \sum_{a \in L} \hat{c}_a(f_a) + \hat{\pi}_a(u_a). \tag{6}$$

The Second Objective Function

- $e_a(f_a)$: emission-generation function associated with link a in the operation phase, which is assumed to be a function of the amount of product flow on the link.
- $\hat{e}_a(u_a)$: emission-generation function associated with link a in the capital investment phase.

These functions are also assumed to be convex and continuously differentiable and to have bounded second order partial derivatives.

The minimization of emissions objective function

$$\text{Minimize} \quad \sum_{a \in L} e_a(f_a) + \hat{e}_a(u_a). \tag{7}$$

The Multicriteria Optimization Formulation

ω: constant nonnegative weight assigned by the firm to the emission-generation criterion (7) or, equivalently, a price per unit of emissions that the firm is willing to pay. It can also be interpreted as a tax imposed by the governmental authority.

The multicriteria optimization formulation

$$\text{Minimize} \quad \sum_{a\in L} \hat{c}_a(f_a) + \hat{\pi}_a(u_a) + \omega \left(\sum_{a\in L} e_a(f_a) + \hat{e}_a(u_a) \right), \quad (8)$$

subject to the constraints: (1), (2), (3), and

$$f_a \leq u_a, \quad \forall a \in L, \quad (9)$$

$$0 \leq u_a, \quad \forall a \in L. \quad (10)$$

The Variational Inequality Formulation

Our optimization problem is characterized, under our assumptions, by a convex objective function and a convex feasible set.

β_a: the Lagrange multiplier associated with constraint (9) for each link $a \in L$.

β_a^*: the associated optimal Lagrange multiplier.

These terms may also be interpreted as the price or value of an additional unit of capacity on link a. We group these Lagrange multipliers into the respective vectors β and β^*.

The Variational Inequality Formulation

Theorem 1

The optimization problem (8), subject to the constraints (1) – (3), and (9), (10), is equivalent to the variational inequality problem: determine the vectors of link flows, link capacities, and Lagrange multipliers $(f^*, u^*, \beta^*) \in K$, such that:

$$\sum_{a\in L}\left[\frac{\partial \hat{c}_a(f_a^*)}{\partial f_a} + \omega \frac{\partial e_a(f_a^*)}{\partial f_a} + \beta_a^*\right] \times [f_a - f_a^*]$$

$$+\sum_{a\in L}\left[\frac{\partial \hat{\pi}_a(u_a^*)}{\partial u_a} + \omega \frac{\partial \hat{e}_a(u_a^*)}{\partial u_a} - \beta_a^*\right] \times [u_a - u_a^*]$$

$$+\sum_{a\in L}[u_a^* - f_a^*] \times [\beta_a - \beta_a^*] \geq 0, \quad \forall (f, u, \beta) \in K, \tag{11}$$

$K \equiv \{(f, u, \beta) | \exists x \geq 0, \text{ and } (1), (2), (3) \text{ and } (10) \text{ hold, and } \beta \geq 0\}$, where f is the vector of link flows, u is the vector of link capacities, and x is the vector of path flows.

The Standard Variational Inequality Form

Determine $X^* \in \mathcal{K}$ such that:

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{12}$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in $\mathcal{N}$-dimensional Euclidean space.

If we define the column vectors: $X \equiv (f, u, \beta)$ and $F(X) \equiv (F_1(X), F_2(X), F_3(X))$, such that

$$F_1(X) \equiv \left[\frac{\partial \hat{c}_a(f_a)}{\partial f_a} + \omega \frac{\partial e_a(f_a)}{\partial f_a} + \beta_a; \quad a \in L \right], \tag{13}$$

$$F_2(X) \equiv \left[\frac{\partial \hat{\pi}_a(u_a)}{\partial u_a} + \omega \frac{\partial \hat{e}_a(u_a)}{\partial u_a} - \beta_a; a \in L \right], \tag{14}$$

$$F_3(X) \equiv [u_a - f_a; \quad a \in L], \tag{15}$$

and define $\mathcal{K} \equiv K$, then (11) can be re-expressed as (12).

The Solution Method

Variational inequality (11) can be easily solved using the modified projection method.

For the solution of the embedded network problems, we utilize the well-known equilibration algorithm (system-optimization version) of Dafermos and Sparrow (1969), which has been widely applied. Recall that the modified projection method (cf. Korpelevich (1977)) is guaranteed to converge to a solution of a variational inequality problem, provided that the function that enters the variational inequality problem is monotone and Lipschitz continuous and that a solution exists.

The solution (f^*, u^*) to (11) minimizes the objective function (8) associated with the design of the sustainable supply chain network.

Monotonicity and Lipschitz Continuity

Theorem 2

The function $F(X)$ as aforementioned is monotone, that is,

$$\langle (F(X^1) - F(X^2))^T, X^1 - X^2 \rangle \geq 0, \quad \forall X^1, X^2 \in \mathcal{K}. \tag{16}$$

Theorem 3

The function $F(X)$ as defined is Lipschitz continuous, that is,

$$\|F(X^1) - F(X^2)\| \leq \|X^1 - X^2\|, \quad \forall X^1, X^2 \in \mathcal{K}. \tag{17}$$

Convergence

Theorem 4

Assume that the function that enters the variational inequality (11) (or its standard form equivalent (12)) has at least one solution and satisfies the conditions in Theorem 2 and in Theorem 3. Then the modified projection method converges to the solution of the variational inequality (11) (or (12)).

For proofs of Theorems 2, 3, and 4, see the paper.

Numerical Examples

The baseline supply chain network topology for all the examples is as depicted next with the links defined by numbers.

The numerical examples consisted of a firm faced with 3 possible manufacturing plants, each of which had 2 possible technologies; 2 distribution centers, each of which also had 2 distinct technologies, and the firm had to supply the 3 demand points. There was only a single mode of transportation/shipment available between each manufacturing plant and each distribution center and between each distribution center at a given demand point.

The common input data for the three examples is reported in Table 1.

The Baseline Supply Chain Network Topology for the Examples

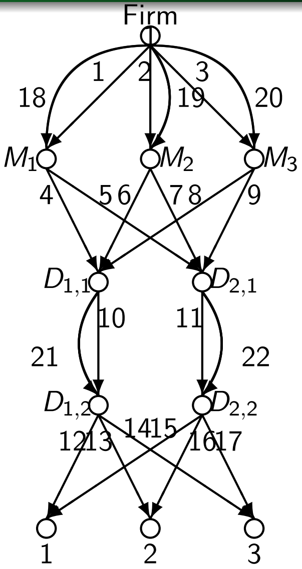


Table 1: Total Cost and Emission Functions for the Numerical Examples

Link a	$\hat{c}_a(f_a)$	$\hat{\pi}_a(u_a)$	$e_a(f_a)$	$\hat{e}_a(u_a)$
1	$f_1^2 + 2f_1$	$.5u_1^2 + u_1$	$.05f_1^2 + f_1$	$1.5u_1^2 + 2u_1$
2	$.5f_2^2 + f_2$	$2.5u_2^2 + u_2$	$.1f_2^2 + f_2$	$2u_2^2 + 2u_2$
3	$.5f_3^2 + f_3$	$u_3^2 + 2u_3$	$.15f_3^2 + 2f_3$	$2.5u_3^2 + u_3$
4	$1.5f_4^2 + 2f_4$	$u_4^2 + u_4$	$.05f_4^2 + .1f_4$	$.1u_4^2 + .2u_4$
5	$f_5^2 + 3f_5$	$2.5u_5^2 + 2u_5$	$.05f_5^2 + .1f_5$	$.05u_5^2 + .1u_5$
6	$f_6^2 + 2f_6$	$.5u_6^2 + u_6$	$.1f_6^2 + .1f_6$	$.05u_6^2 + .1u_6$
7	$.5f_7^2 + 2f_7$	$.5u_7^2 + u_7$	$.05f_7^2 + .2f_7$	$.1u_7^2 + .2u_7$
8	$.5f_8^2 + 2f_8$	$1.5u_8^2 + u_8$	$.05f_8^2 + .1f_8$	$.1u_8^2 + .3u_8$
9	$f_9^2 + 5f_9$	$2u_9^2 + 3u_9$	$.05f_9^2 + .1f_9$	$.1u_9^2 + .2u_9$
10	$.5f_{10}^2 + 2f_{10}$	$u_{10}^2 + 5u_{10}$	$.2f_{10}^2 + f_{10}$	$1.5u_{10}^2 + 3u_{10}$
11	$f_{11}^2 + f_{11}$	$.5u_{11}^2 + 3u_{11}$	$.25f_{11}^2 + 3f_{11}$	$2u_{11}^2 + 3u_{11}$

Table 1 (continued)

Link a	$\hat{c}_a(f_a)$	$\hat{\pi}_a(u_a)$	$e_a(f_a)$	$\hat{e}_a(u_a)$
12	$.5f_{12}^2 + 2f_{12}$	$.5u_{12}^2 + u_{12}$	$.05f_{12}^2 + .1f_{12}$	$.1u_{12}^2 + .2u_{12}$
13	$.5f_{13}^2 + 5f_{13}$	$.5u_{13}^2 + u_{13}$	$.1f_{13}^2 + .1f_{13}$	$.05u_{13}^2 + .1u_{13}$
14	$f_{14}^2 + 7f_{14}$	$2u_{14}^2 + 5u_{14}$	$.15f_{14}^2 + .2f_{14}$	$.1u_{14}^2 + .1u_{14}$
15	$f_{15}^2 + 2f_{15}$	$.5u_{15}^2 + u_{15}$	$.05f_{15}^2 + .3f_{15}$	$.1u_{15}^2 + .2u_{15}$
16	$.5f_{16}^2 + 3f_{16}$	$u_{16}^2 + u_{16}$	$.05f_{16}^2 + .1f_{16}$	$.1u_{16}^2 + .1u_{16}$
17	$.5f_{17}^2 + 2f_{17}$	$.5u_{17}^2 + u_{17}$	$.15f_{17}^2 + .3f_{17}$	$.05u_{17}^2 + .1u_{17}$
18	$.5f_{18}^2 + f_{18}$	$u_{18}^2 + 2u_{18}$	$.2f_{18}^2 + 2f_{18}$	$2u_{18}^2 + 3u_{18}$
19	$.5f_{19}^2 + 2f_{19}$	$u_{19}^2 + u_{19}$	$.25f_{19}^2 + 3f_{19}$	$3u_{19}^2 + 4u_{19}$
20	$1.5f_{20}^2 + f_{20}$	$u_{20}^2 + u_{20}$	$.3f_{20}^2 + 3f_{20}$	$2.5u_{20}^2 + 5u_{20}$
21	$.5f_{21}^2 + 2f_{21}$	$u_{21}^2 + 3u_{21}$	$.1f_{21}^2 + 3f_{21}$	$1.5u_{21}^2 + 4u_{21}$
22	$f_{22}^2 + 3f_{22}$	$.5u_{22}^2 + 2u_{22}$	$.2f_{22}^2 + 4f_{22}$	$2.5u_{22}^2 + 4u_{22}$

The Solution Method for the Numerical Examples

The modified projected method was implemented in FORTRAN, and a Unix system at the University of Massachusetts Amherst was used for all the computations.

We initialized the algorithm by equally distributing the demand at each demand site among all the paths joining the firm node 0 to the demand node. All other variables, that is, the link capacities and the Lagrange multipliers, were initialized to zero.

Example 1

Demands at destination points:

$$d_1 = 45, \quad d_2 = 35, \quad d_3 = 5.$$

In this example, we assumed that the firm did not care about the emissions generated in its supply chain design, thus, $\omega = 0$. The computed solution is reported in Table 2.

The total cost was: 10,716.33.

The total emissions generated were: 8,630.45.

The value of the objective function (8) was 10,716.33.

Table 2: Example 1 Solution

Link a	f_a^*	u_a^*	β_a^*
1	12.43	12.43	13.43
2	11.67	11.67	59.33
3	15.81	15.81	33.62
4	14.69	14.69	30.39
5	10.16	10.16	52.82
6	13.94	13.94	14.94
7	20.70	20.70	12.70
8	15.83	15.83	48.50
9	9.66	9.66	41.66
10	21.90	21.90	48.80
11	20.43	20.43	23.43

Link a	f_a^*	u_a^*	β_a^*
12	25.44	25.44	26.44
13	19.03	19.03	20.03
14	0.00	0.00	3.85
15	19.56	19.56	20.56
16	15.97	15.97	32.93
17	5.00	5.00	6.00
18	12.43	12.43	26.86
19	22.98	22.98	46.95
20	9.69	9.69	20.37
21	22.57	22.57	48.14
22	20.10	20.10	22.10

Note that link 14 has zero capacity and, thus, zero flow. Therefore, the final optimal sustainable supply chain network topology for this problem is the baseline network but with link 14 removed.

Example 2

Identical data as in Example 1 except that the firm now was more concerned about the environment with $\omega = 5$. The new computed solution is given in Table 3.

The total cost was now: 11,285.04.

The total emissions generated were now: 7,759.35.

The value of the objective function (8) was 50,081.77.

The number of emissions decreased due to the higher weight on the total emissions generated; however, at a higher total cost. Note that now all the links have positive capacity and positive flows.

Table 3: Example 2 Solution

Link a	f_a^*	u_a^*	β_a^*
1	19.32	19.32	320.17
2	15.69	15.69	403.10
3	13.45	13.45	370.17
4	19.43	19.43	60.29
5	13.80	13.80	78.38
6	13.75	13.75	22.12
7	13.28	13.28	28.55
8	15.73	15.73	65.43
9	9.02	9.02	49.09
10	24.03	24.03	428.53
11	19.71	19.71	431.81

Link a	f_a^*	u_a^*	β_a^*
12	26.62	26.62	55.23
13	20.62	20.62	32.43
14	1.67	1.67	13.87
15	18.38	18.38	38.77
16	14.38	14.38	44.65
17	3.33	3.33	6.49
18	13.90	13.90	322.79
19	11.34	11.34	383.79
20	11.30	11.30	331.01
21	24.88	24.88	445.88
22	16.38	16.38	447.95

Observe that whereas links 1 and 18 had similar amounts of product flows in Example 1, in Example 2, the production shifted from link 18 to link 1 at about a 50% increase, since link 1 corresponded to a more environmentally-friendly technology.

Similar behavior occurred with links 2 and 19.

Example 3

The same data as Examples 1 and 2 but now the firm was even more concerned about the environment with $\omega = 10$. The new solution is given in Table 4.

The total cost was: 11,414.07.

The total emissions generated were now: 7,739.32.

The value of the objective function (8) was 88,807.30.

As in Example 2, all links have positive capacity and positive product flow at the optimal solution.

Table 4: Example 3 Solution

Link a	f_a^*	u_a^*	β_a^*
1	20.16	20.16	645.86
2	15.80	15.80	731.98
3	13.10	13.10	693.10
4	19.66	19.66	81.62
5	14.66	14.66	90.97
6	14.37	14.37	30.74
7	11.99	11.99	38.96
8	15.45	15.45	81.25
9	8.88	8.88	58.25
10	24.30	24.30	812.50
11	19.49	19.49	831.78

Link a	f_a^*	u_a^*	β_a^*
12	26.44	26.44	82.33
13	20.63	20.63	43.25
14	2.40	2.40	20.43
15	18.56	18.56	58.67
16	14.37	14.37	59.50
17	2.60	2.60	7.19
18	14.16	14.16	626.55
19	10.55	10.55	695.13
20	11.22	11.22	634.49
21	25.17	25.17	848.48
22	16.04	16.04	859.93

Summary and Conclusions

- We developed a sustainable supply chain network design model that allows for the evaluation of environmental multicriteria decision-making. Using the presented formalism, a firm may engineer its supply chain to be not only fiscally cost effective, but also environmentally responsible.
- The model utilizes cost minimization within a system-optimization perspective while taking into account the capital investment cost, the operational cost, as well as the total emissions generated.
- The supply chain network design model allows us to determine the optimal capacity for every single activity of the supply chain network along with the optimal operational product flows and yields emission information.

Thank You!

For more information, see: http://supernet.som.umass.edu