Pharmaceutical Supply Chain Networks with Outsourcing Under Price and Quality Competition

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where a full list of references can be found.

Outline

- Background and Motivation
- The Pharmaceutical Supply Chain Network Model with Outsourcing Under Price and Quality Competition
- The Underlying Dynamics
- The Algorithm
- Numerical Examples
- Summary and Conclusions

Background and Motivation

Outsourcing is defined as the behavior of moving some of a firm's responsibilities and/or internal processes, such as product design or manufacturing, to a third party company (Chase, Jacobs, and Aquilano (2004)).

Background and Motivation

There is, currently, a tremendous shift from in-house manufacturing towards outsourcing in US pharmaceutical companies.

- The US market for outsourced pharmaceutical manufacturing is expanding at the rate of **10%** to **12%** annually (Olson and Wu (2011)).
- Up to **40%** of the drugs that Americans take are imported, and more than **80%** of the active ingredients for drugs sold in the United States are outsourced (Economy In Crisis (2010)).
- Pharmaceutical companies are increasingly farming out activities other than production, such as **research and development** activities (Higgins and Rodriguez (2006)).

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Background and Motivation

Quality issues in outsourced products must be of paramount concern.

- In 2006, outsourced cold medicine contained a **toxic substance** used in antifreeze, which can cause death (Bogdanich and Hooker (2007)).

- In 2008, fake heparin made by one Chinese manufacturer not only led to recalls of drugs in over **ten** European countries (Payne (2008)), but also caused the deaths of **81** American citizens (The New York Times (2011)).

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Background and Motivation

A more comprehensive supply chain network model that captures

- **contractor selection**,
- the minimization of the **disrepute** of the pharmaceutical firm, as well as
- the **competition among contractors**,

is an imperative.

Literature Review

Related literature

- Balakrishnan, K., Mohan, U., Seshadri, S., 2008. Outsourcing of front-end business processes: Quality, information, and customer contact. *Journal of Operations Management* 26 (6), 288-302.
- Kaya, M., Özer, Ö., 2009. Quality risk in outsourcing: Noncontractible product quality and private quality cost information. *Naval Research Logistics* 56, 669-685.
- Kaya, O., 2011. Outsourcing vs. in-house production: A comparison of supply chain contracts with effort dependent demand. *Omega* 39, 168-178.

Background and Motivation

We develop a pharmaceutical supply chain network model which takes into account the **quality concerns** in the context of global outsourcing.

- This model captures the behavior of the pharmaceutical firm and its potential contractors.
- The contractors compete by determining the **prices** that they charge the pharmaceutical firm for manufacturing and delivering the product to the demand markets and the **quality levels** of the products to **maximize its total profit**.
- The pharmaceutical firm seeks to **minimize its total cost**, which includes its weighted **disrepute** cost, which is influenced by the **quality** of the product produced by its contractors and the amount of product that is outsourced.

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Definitions

Quality levels and quality cost are quantified and incorporated.

- Quality level is defined as the **"quality conformance level"**, the degree to which a specific product conforms to a design or specification (Juran and Gryna (1988)), and it should be **within 0 and 100 percent** of defects levels.
- Quality costs are defined as "costs incurred in ensuring and assuring quality as well as the loss incurred when quality is not achieved" (ASQC (1971) and BS (1990)), which is widely believed to be a **convex** function of the quality conformance level (see, e.g., Juran and Gryna (1988), Campanella (1990), Feigenhaum (1983), Porter and Rayner (1992), and Shank and Govindarajan (1994)).

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Definitions

The external failure cost, which is described as following, is defined and incorporated as the cost of **disrepute** for the original firm in this work.

- External failure cost, which is one category of quality related cost, is the **compensation cost** incurred when customers are unsatisfied with the quality of the products.(see, e.g., Crosby (1979), Harrington (1987), Plunkett and Dale (1988), Juran and Gryna (1993), and Rapley, Prickett, and Elliot (1999)).

Assumptions

The game theory supply chain network model developed in this paper is based on the following assumptions:

- The original pharmaceutical firm pays the **transaction cost**, which includes the costs of evaluating suppliers and the negotiation costs (see Picot (1991), Franceschini (2003), Heshmati (2003), and Liu and Nagurney (2011a)).

  The production/distribution costs and the quality cost information of the contractors are **known** by the firm through the transaction cost.

- In-house supply chain activities can ensure a **100%** perfect quality conformance level (see Schneiderman (1986) and Kaya (2011)).

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Background and Motivation

We develop both a **static** version of the model (at the equilibrium state) and also a **dynamic** one using the theory of **projected dynamical systems** (cf. Nagurney and Zhang (1996)).

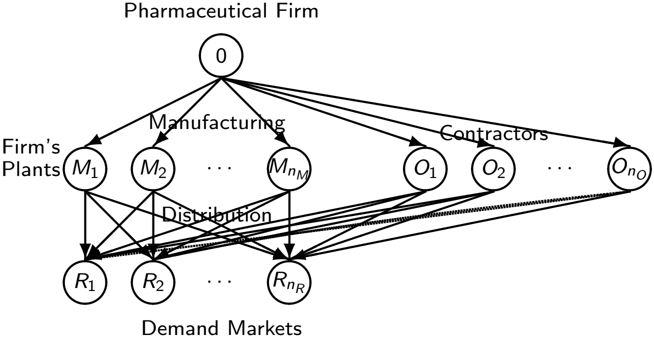

Figure: The Pharmaceutical Supply Chain Network Topology with Outsourcing

Notation for the Game Theoretic Supply Chain Network Model with Outsourcing

n: $n = n_M + n_O$, the number of manufacturing plants, whether in-house or belonging to the contractors'.

Q_{jk}: the nonnegative amount of pharmaceutical product produced at manufacturing plant j and delivered to demand market k. We group the $\{Q_{jk}\}$ elements into the vector $Q \in R_+^{nn_R}$.

d_k: the demand for the product at demand market k, assumed known and fixed.

π_{jk}: the price charged by contractor j for producing and delivering a unit of the product to k. We group the $\{\pi_{jk}\}$ elements for contractor j into the vector $\pi_j \in R_+^{n_R}$ and then group all such vectors for all the contractors into the vector $\pi \in R^{n_O n_R}$.

$dc(q\prime)$: the cost of disrepute, which corresponds to the external failure quality cost.

The Behavior of the Pharmaceutical Firm

We assume that **in-house** activities can ensure a **100%** perfect quality conformance level.

Quality level of contractor j

$$0 \leq q_j \leq q^U, \quad j = 1, \ldots, n_O, \tag{1}$$

where q^U is the value representing perfect quality achieved by the pharmaceutical firm in its in-house manufacturing.

Average quality level of the pharmaceutical firm

$$q\prime = \frac{\sum_{j=n_M+1}^{n} \sum_{k=1}^{n_R} Q_{jk} q_{j-n_M} + (\sum_{j=1}^{n_M} \sum_{k=1}^{n_R} Q_{jk}) q^U}{\sum_{k=1}^{n_R} d_k}. \tag{2}$$

The Behavior of the Pharmaceutical Firm

The total utility maximization objective of the pharmaceutical firm

$$\text{Maximize}_Q \quad U_0(Q,q,\pi) = -\sum_{j=1}^{n_M} f_j(\sum_{k=1}^{n_R} Q_{jk}) - \sum_{j=1}^{n_M}\sum_{k=1}^{n_R} \hat{c}_{jk}(Q_{jk})$$

$$-\sum_{j=1}^{n_O}\sum_{k=1}^{n_R} \pi^*_{jk} Q_{n_M+j,k} - \sum_{j=1}^{n_O} tc_j(\sum_{k=1}^{n_R} Q_{n_M+j,k}) - \omega dc(q\prime). \qquad (3)$$

subject to:

$$\sum_{j=1}^{n} Q_{jk} = d_k, \quad k = 1,\dots,n_R, \qquad (4)$$

$$Q_{jk} \geq 0, \quad j = 1,\dots,n; k = 1,\dots,n_R, \qquad (5)$$

with $q\prime$ in (3) as in (2).

Variational Inequality Formulation

We assume that the utility functions are concave, continuous,, and continuously differentiable.

Theorem 1

Determine $Q^* \in K^0$, such that:

$$-\sum_{h=1}^{n}\sum_{l=1}^{n_R} \frac{\partial U_0(Q^*, q^*, \pi^*)}{\partial Q_{hl}} \times (Q_{hl} - Q^*_{hl}) \geq 0, \quad \forall Q \in K^0, \tag{6}$$

where $K^0 \equiv \{Q | Q \in R_+^{nn_R} \text{ with } (4) \text{ satisfied}\}$,

Variational Inequality Formulation

Theorem 1 (cont'd)

for $h = 1, \ldots, n_M$; $l = 1, \ldots, n_R$:

$$-\frac{\partial U_o}{\partial Q_{hl}} = \left[\frac{\partial f_h(\sum_{k=1}^{n_R} Q_{hk})}{\partial Q_{hl}} + \frac{\partial \hat{c}_{hl}(Q_{hl})}{\partial Q_{hl}} + \omega \frac{\partial dc(q\prime)}{\partial Q_{hl}} \right]$$

$$= \left[\frac{\partial f_h(\sum_{k=1}^{n_R} Q_{hk})}{\partial Q_{hl}} + \frac{\partial \hat{c}_{hl}(Q_{hl})}{\partial Q_{hl}} + \omega \frac{\partial dc(q\prime)}{\partial q\prime} \frac{q^U}{\sum_{k=1}^{n_R} d_k} \right],$$

and for $h = n_M + 1, \ldots, n$; $l = 1, \ldots, n_R$:

$$-\frac{\partial U_o}{\partial Q_{hl}} = \left[\pi_{h-n_M,l} + \frac{\partial tc_{h-n_M}(\sum_{k=1}^{n_R} Q_{hk})}{\partial Q_{hl}} + \omega \frac{\partial dc(q\prime)}{\partial Q_{hl}} \right]$$

$$= \left[\pi_{h-n_M,l} + \frac{\partial tc_{h-n_M}(\sum_{k=1}^{n_R} Q_{hk})}{\partial Q_{hl}} + \omega \frac{\partial dc(q\prime)}{\partial q\prime} \frac{q_h}{\sum_{k=1}^{n_R} d_k} \right].$$

The total utility maximization objective of contractor j

$$\text{Maximize}_{q_j,\pi_j} \quad U_j(Q,q,\pi)=\sum_{k=1}^{n_R}\pi_{jk}Q^*_{n_M+j,k}-\sum_{k=1}^{n_R}\hat{sc}_{jk}(Q^*,q)-\hat{qc}_j(q)$$
$$-\sum_{k=1}^{n_R}oc_{jk}(\pi_{jk}) \quad (7)$$

subject to:

$$\pi_{jk}\geq 0, \quad k=1,\ldots,n_R, \quad (8)$$

and (1) for each j.

We assume that the utility functions are **continuous**, **continuously differentiable**, and **convex**.

Opportunity Cost

- "the **loss of potential gain** from other alternatives when one alternative is chosen" (New Oxford American Dictionary (2010)).
- Opportunity cost functions include both **explicit and implicit** costs (Mankiw (2011)).
- Nobel laureate Akerlof (1970): "The Market for Lemons: Quality Uncertainty and the Market Mechanism".
- It has been emphasized in **pharmaceutical firm** competition by Grabowski and Vernon (1990), Palmer and Raftery (1999), and Cockburn (2004).
- Gan and Litvinov (2003) also constructed opportunity cost functions that are functions of **prices** but for energy.

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Definition: A Nash-Bertrand Equilibrium

Let K^j denote the feasible set according to contractor j, where $K^j \equiv \{(q_j, \pi_j) | \pi_j$ satisfies (8) and q_j satisfies (1) for $j\}$, and $\mathcal{K}^1 \equiv \prod_{j=1}^{n_O} K^j$. They are **closed and convex**.

Definition 1

A quality level and price pattern $(q^*, \pi^*) \in \mathcal{K}^1$ is said to constitute a Bertrand-Nash equilibrium if for each contractor j; $j = 1, \ldots, n_O$

$$U_j(Q^*, q_j^*, \hat{q}_j^*, \pi_j^*, \hat{\pi}_j^*) \geq U_j(Q^*, q_j, \hat{q}_j^*, \pi_j, \hat{\pi}_j^*), \quad \forall (q_j, \pi_j) \in K^j, \quad (9)$$

where

$$\hat{q}_j^* \equiv (q_1^*, \ldots, q_{j-1}^*, q_{j+1}^*, \ldots, q_{n_O}^*), \quad (10)$$

$$\hat{\pi}_j^* \equiv (\pi_1^*, \ldots, \pi_{j-1}^*, \pi_{j+1}^*, \ldots, \pi_{n_O}^*). \quad (11)$$

Variational Inequality Formulation

Theorem 2

$(q^*, \pi^*) \in \mathcal{K}^1$ *is a Bertrand-Nash equilibrium according to Definition 1 if and only if it satisfies the variational inequality:*

$$-\sum_{j=1}^{n_O} \frac{\partial U_j(Q^*, q^*, \pi^*)}{\partial q_j} \times (q_j - q_j^*) - \sum_{j=1}^{n_O} \sum_{k=1}^{n_R} \frac{\partial U_j(Q^*, q^*, \pi^*)}{\partial \pi_{jk}} \times (\pi_{jk} - \pi_{jk}^*)$$

$$\geq 0, \quad \forall (q, \pi) \in \mathcal{K}^1, \qquad (12)$$

Variational Inequality Formulation

Theorem 2 (cont'd)

with notice that: for $j = 1, \ldots, n_O$*:*

$$-\frac{\partial U_j}{\partial q_j} = \sum_{k=1}^{n_R} \frac{\partial \hat{sc}_{jk}(Q,q)}{\partial q_j} + \frac{\partial \hat{qc}_j(q)}{\partial q_j}, \tag{13}$$

and for $j = 1, \ldots, n_O; k = 1, \ldots, n_R$*:*

$$-\frac{\partial U_J}{\partial \pi_{jk}} = \frac{\partial oc_{jk}(\pi_{jk})}{\partial \pi_{jk}} - Q_{n_M+j,k}. \tag{14}$$

The Equilibrium Conditions for the Supply Chain Network with Outsourcing and with Price and Quality Competition

Definition 2

The equilibrium state of the pharmaceutical supply chain network with outsourcing is one where both variational inequalities (6) and (12) hold **simultaneously**.

Variational Inequality Formulation

Theorem 3

Determine $(Q^*, q^*, \pi^*) \in \mathcal{K}$*, such that:*

$$-\sum_{h=1}^{n}\sum_{l=1}^{n_R} \frac{\partial U_0(Q^*, q^*, \pi^*)}{\partial Q_{hl}} \times (Q_{hl} - Q_{hl}^*) - \sum_{j=1}^{n_O} \frac{\partial U_j(Q^*, q^*, \pi^*)}{\partial q_j} \times (q_j - q_j^*)$$

$$-\sum_{j=1}^{n_O}\sum_{k=1}^{n_R} \frac{\partial U_j(Q^*, q^*, \pi^*)}{\partial \pi_{jk}} \times (\pi_{jk} - \pi_{jk}^*) \geq 0, \quad \forall (Q, q, \pi) \in \mathcal{K}, \quad (15)$$

where $\mathcal{K} \equiv K^0 \times \mathcal{K}^1$.

Standard Form of the Variational Inequality

Standard Form

Determine $X^* \in \mathcal{K}$ *where* X *is a vector in* R^N, $F(X)$ *is a continuous function such that* $F(X) : X \mapsto \mathcal{K} \subset R^N$, *and*

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{16}$$

where $\langle \cdot, \cdot \rangle$ *is the inner product in the N-dimensional Euclidean space, and* $\mathcal{K}$ *is closed and convex.*

Standard Form of the Variational Inequality

Standard Form

We define the vector $X \equiv (Q, q, \pi)$. *Also,* $N = nn_R + n_O + n_o n_R$. *Note that (16) may be rewritten as:*

$$\sum_{i=1}^{N} F_i(X^*) \times (X_i - X_i^*) \geq 0, \quad \forall X \in \mathcal{K}, \tag{17}$$

where first nn_R *components of* F *are given by:*
$-\frac{\partial U_0(Q,q,\pi)}{\partial Q_{hl}}$,*for* $h = 1, \ldots, n$; $l = 1, \ldots, n_R$;
the next n_O *components of* F *are given by:*
$-\frac{\partial U_j(Q,q,\pi)}{\partial q_j}$,*for* $j = 1, \ldots, n_O$;
and the subsequent $n_O n_R$ *components of* F *are given by:*
$-\frac{\partial U_j(Q,q,\pi)}{\partial \pi_{jk}}$, *for* $j = 1, \ldots, n_O$; $k = 1, \ldots, n_R$.

The Underlying Dynamics

Recall the **pertinent ordinary differential equation** (ODE):

$$\dot{X} = \Pi_{\mathcal{K}}(X, -F(X)), \tag{18}$$

where, since $\mathcal{K}$ is a convex polyhedron, according to Dupuis and Nagurney (1993), $\Pi_{\mathcal{K}}(X, -F(X))$ is the projection, with respect to $\mathcal{K}$, of the vector $-F(X)$ at X defined as

$$\Pi_{\mathcal{K}}(X, -F(X)) = \lim_{\delta \to 0} \frac{P_{\mathcal{K}}(X - \delta F(X)) - X}{\delta} \tag{19}$$

with $P_{\mathcal{K}}$ denoting the **projection map**:

$$P(X) = \text{argmin}_{x \in \mathcal{K}} \|X - z\|, \tag{20}$$

where $\| \cdot \| = \langle x^T, x \rangle$, and $F(X) = -\nabla U(Q, q, \pi)$.

Theorem

Theorem 4

X^* solves the variational inequality problem (15) equivalently, (16), if and only if it is a **stationary point** of the ODE (18), that is,

$$\dot{X} = 0 = \Pi_{\mathcal{K}}(X^*, -F(X^*)). \tag{21}$$

Existence and Uniqueness Results of the Equilibrium Pattern

Theorem 5

Suppose that F is **strongly monotone**. *Then there* **exists a unique** *solution to variational inequality (16); equivalently, to variational inequality (15).*

Theorem 6

- *If $F(X)$ is* **monotone***, then every supply chain network equilibrium, as defined in Definition 2, provided its existence, is a global monotone attractor for the utility gradient processes.*
- *If $F(X)$ is* **strictly monotone***, then there exists at most one supply chain network equilibrium. Furthermore, given existence, the unique equilibrium is a strictly global monotone attractor for the utility gradient processes.*
- *If $F(X)$ is* **strongly monotone***, then the unique supply chain network equilibrium, which is guaranteed to exist, is also globally exponentially stable for the utility gradient processes.*

Stability Under Monotonicity

The monotonicity of a function F is closely related to the **positive-definiteness** of its Jacobian ∇F (cf. Nagurney (1999)).

Particularly,

- if ∇F is **positive-semidefinite**, F is monotone;
- if ∇F is **positive-definite**, F is strictly monotone;
- and, if ∇F is **strongly positive definite**, in the sense that the symmetric part of $\nabla F, (\nabla F^T + \nabla F)/2$, has only positive eigenvalues, then F is strongly monotone.

The Algorithm - The Euler Method

Iteration τ of the Euler method (see also Nagurney and Zhang (1996))

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - a_{\tau} F(X^{\tau})), \tag{29}$$

where $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$ and F is the function that enters the variational inequality problem (16).

For convergence of the general iterative scheme, which induces the Euler method, the sequence $\{a_{\tau}\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_{\tau} = \infty$, $a_{\tau} > 0$, $a_{\tau} \to 0$, as $\tau \to \infty$.

The Algorithm - The Euler Method

We emphasize that this is the first time that this algorithm is being adapted and applied for the solution of supply chain network game theory problems under **Nash-Bertrand** competition and with **outsourcing**.

The Algorithm - The Euler Method

The strictly convex quadratic programming problem

$$X^{\tau+1} = \text{Minimize}_{X \in \mathcal{K}} \quad \frac{1}{2}\langle X, X \rangle - \langle X^{\tau} - a_{\tau} F(X^{\tau}), X \rangle. \quad (30)$$

In order to obtain the values of the **product flows** at each iteration, we can apply the **exact equilibration algorithm**, originated by Dafermos and Sparrow (1969) and applied to many different applications of networks with special structure (cf. Nagurney (1999) and Nagurney and Zhang (1996)).

Explicit Formulae for Quality Levels and Contractor Prices

$$q_j^{\tau+1} = \min\{q^U, \max\{0, q_j^\tau + a_\tau(-\sum_{k=1}^{n_R} \frac{\partial \hat{sc}_{jk}(Q^\tau, q^\tau)}{\partial q_j} - \frac{\partial \hat{qc}_j(q^\tau)}{\partial q_j})\}\},$$

$$j = 1, \ldots, n_O, \quad (31)$$

$$\pi_{jk}^{\tau+1} = \max\{0, \pi_{jk}^\tau + a_\tau(-\frac{\partial oc_{jk}(\pi_{jk}^\tau)}{\partial \pi_{jk}} + Q_{n_M+j,k}^\tau)\},$$

$$j = 1, \ldots, n_O; k = 1, \ldots, n_R. \quad (32)$$

The Convergence Result

Theorem 7

In the supply chain network model with outsourcing, let $F(X) = -\nabla U(Q, q, \pi)$ *be* **strongly monotone**. *Also, assume that* F *is* **uniformly Lipschitz continuous**. *Then there exists a* **unique** *equilibrium product flow, quality level, and price pattern* $(Q^*, q^*, \pi^*) \in \mathcal{K}$ *and any sequence generated by the Euler method as given by (29) above, where* $\{a_\tau\}$ *satisfies* $\sum_{\tau=0}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \to 0$, *as* $\tau \to \infty$ *converges to* (Q^*, q^*, π^*).

Note that convergence also holds if $F(X)$ is **monotone** (cf. Theorem 8.6 in Nagurney and Zhang (1996)) provided that the price iterates are **bounded**.

An Illustrative Example

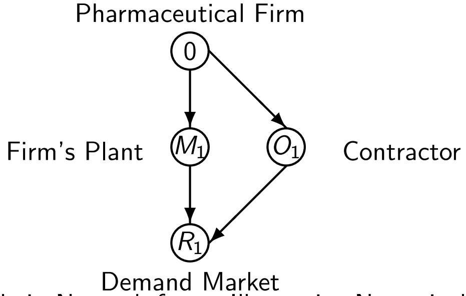

Figure: Supply Chain Network for an Illustrative Numerical Example

An Illustrative Example

The **demand** at demand market R_1: 1,000, q^U=100, and ω=1.

The firm's **production** cost:

$$f_1(Q_{11}) = Q_{11}^2 + Q_{11}.$$

Total **transportation** cost:

$$\hat{c}_{11}(Q_{11}) = .5Q_{11}^2 + Q_{11}.$$

Transaction cost:

$$tc_1(Q_{21}) = .05Q_{21}^2 + Q_{21}.$$

An Illustrative Example

The contractor's total cost of **production and distribution**:

$$\hat{s}c_{11}(Q_{21}, q_1) = Q_{21} q_1.$$

Total **quality** cost:

$$\hat{q}c_1(q_1) = 10(q_1 - 100)^2.$$

Opportunity cost:

$$oc_{11}(\pi_{11}) = .5(\pi_{11} - 10)^2.$$

Cost of **disrepute**:

$$dc(q\prime) = 100 - q\prime,$$

where $q\prime = \frac{Q_{21} q_1 + Q_{11} 100}{1000}$.

An Illustrative Example

We set the convergence tolerance to 10^{-3}. The Euler method was initialized with $Q_{11}^0 = Q_{21}^0 = 500$, $q_1^0 = 1$, and $\pi_{11}^0 = 0$. The sequence $\{a_\tau\}$ was set to: $\{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \ldots\}$.

The Euler method converged in 87 iterations and yielded the following product flow, quality level, and price pattern:

$$Q_{11}^* = 270.50, \quad Q_{21}^* = 729.50, \quad q_1^* = 63.52, \quad \pi_{11}^* = 739.50.$$

The total cost incurred by the pharmaceutical firm was 677,128.65 with the contractor earning a profit of 213,786.67. The value of q' was 73.39.

An Illustrative Example

The **Jacobian matrix** of $F(X) = -\nabla U(Q, q, \pi)$, for this example, is

$$J(Q_{11}, Q_{21}, q_1, \pi_{11}) = \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & .1 & -.001 & 1 \\ 0 & 1 & 20 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}.$$

Both the **existence and the uniqueness** of the solution to variational inequality (15) with respect to this example are guaranteed.

A Variant

The **transportation cost function** was reduced by a factor of 10:

$$\hat{c}_{11}(Q_{11}) = .05Q_{11}^2 + .1Q_{11}.$$

The Euler method again required 87 iterations for convergence and yielded the following equilibrium solution:

$$Q_{11}^* = 346.86,\ Q_{21}^* = 653.14, \quad q_1^* = 67.34, \quad \pi_{11}^* = 663.15.$$

The pharmaceutical firm's total costs were now 581,840.07 and the contractor's profits were now 165,230.62. The value of $q\prime$ was now 78.67.

The **Jacobian of F** for the variant is also strongly positive-definite with the only change in the Jacobian matrix above being that the 3 is replaced by 2.1.

Sensitivity Analysis

We returned to the original example and increased the demand for the pharmaceutical product at R_1 in increments of 1,000.

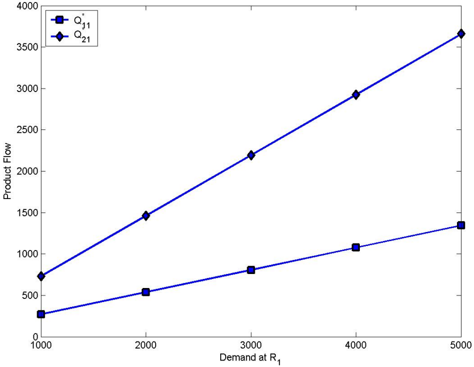

Figure: Equilibrium Product Flows as the Demand Increases for the Illustrative Example

Sensitivity Analysis

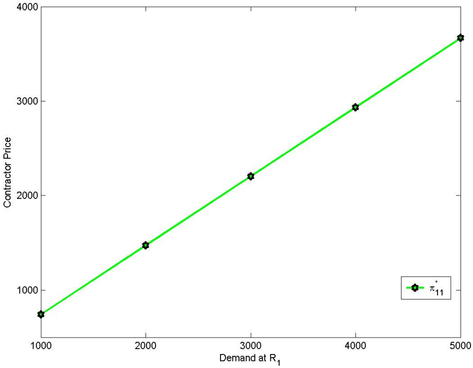

Figure: Equilibrium Contractor Prices as the Demand Increases for the Illustrative Example

Sensitivity Analysis

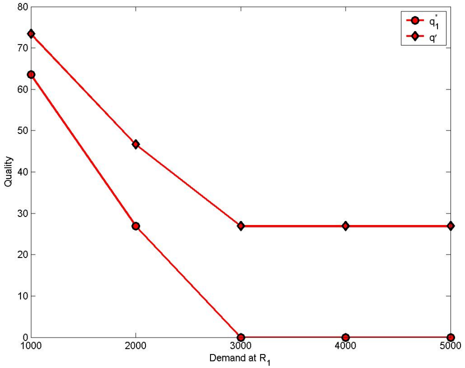

Figure: Equilibrium Contractor Quality Level and the Average Quality as the Demand Increases for the Illustrative Example

Additional Numerical Examples - Example 1

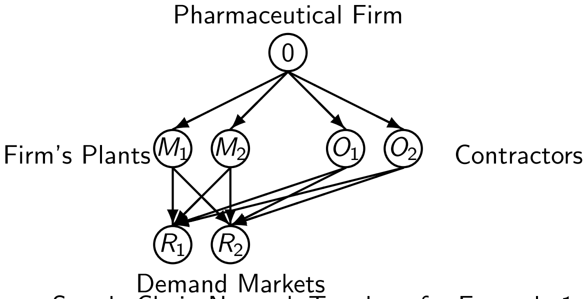

Figure: Supply Chain Network Topology for Example 1

Additional Numerical Examples - Example 1

The **demand** at R_1 was 1,000 and it was 500 at R_2. q^U=100, and ω=1.

The **production** cost functions:

$$f_1(\sum_{k=1}^{2} Q_{1k}) = (Q_{11} + Q_{12})^2 + 2(Q_{11} + Q_{12}),$$

$$f_2(\sum_{k=1}^{2} Q_{2k}) = 1.5(Q_{21} + Q_{22})^2 + 2(Q_{21} + Q_{22}).$$

The **transportation** cost functions:

$$\hat{c}_{11}(Q_{11}) = 1.5Q_{11}^2 + 10Q_{11}, \quad \hat{c}_{12}(Q_{12}) = 1Q_{12}^2 + 25Q_{12},$$

$$\hat{c}_{21}(Q_{21}) = 1Q_{21}^2 + 5Q_{21}, \quad \hat{c}_{22}(Q_{22}) = 2.5Q_{22}^2 + 40Q_{22}.$$

Transaction cost functions:

$$tc_1(Q_{31} + Q_{32}) = .5(Q_{31} + Q_{32})^2 + .1(Q_{31} + Q_{32}),$$

$$tc_2(Q_{41} + Q_{42}) = .25(Q_{41} + Q_{42})^2 + .2(Q_{41} + Q_{42}).$$

Additional Numerical Examples - Example 1

The contractors' total cost of **production and distribution**:

$$\hat{s}c_{11}(Q_{31}, q_1) = Q_{31}q_1, \quad \hat{s}c_{12}(Q_{32}, q_1) = Q_{32}q_1,$$

$$\hat{s}c_{21}(Q_{41}, q_2) = 2Q_{41}q_2, \quad \hat{s}c_{22}(Q_{42}, q_2) = 2Q_{42}q_2.$$

Total **quality** cost functions:

$$\hat{q}c_1(q_1) = 5(q_1 - 100)^2, \quad \hat{q}c_2(q_2) = 10(q_1 - 100)^2.$$

Opportunity cost functions:

$$oc_{11}(\pi_{11}) = .5(\pi_{11} - 10)^2, \quad oc_{12}(\pi_{12}) = (\pi_{12} - 10)^2,$$

$$oc_{21}(\pi_{21}) = (\pi_{21} - 5)^2, \quad oc_{22}(\pi_{22}) = .5(\pi_{22} - 20)^2.$$

Cost of **disrepute**:

$$dc(q\prime) = 100 - q\prime$$

where $q\prime = \frac{Q_{31}q_1 + Q_{32}q_1 + Q_{41}q_2 + Q_{42}q_2 + Q_{11}100 + Q_{12}100 + Q_{21}100 + Q_{22}100}{1500}$.

Additional Numerical Examples - Example 1

The Euler method converged in 153 iterations and yielded the following equilibrium solution.

$$Q_{11}^* = 95.77, \quad Q_{12}^* = 85.51, \quad Q_{21}^* = 118.82, \quad Q_{22}^* = 20.27,$$

$$Q_{31}^* = 213.59, \quad Q_{32}^* = 224.59, \quad Q_{41}^* = 571.83, \quad Q_{42}^* = 169.63.$$

$$q_1^* = 56.18, \quad q_2^* = 25.85,$$

$$\pi_{11}^* = 223.57, \quad \pi_{12}^* = 122.30, \quad \pi_{21}^* = 290.92, \quad \pi_{22}^* = 189.61.$$

The total cost of the pharmaceutical firm was 610,643.26 and the profits of the contractors' were: 5,733.83 and 9,294.44. The value of $q\prime$ was 50.55.

Additional Numerical Examples - Example 1

The **Jacobian matrix** of $F(X) = -\nabla U(Q, q, \pi)$ is

$$J(Q_{11}, Q_{12}, Q_{21}, Q_{22}, Q_{31}, Q_{32}, Q_{41}, Q_{42}, q_1, q_2, \pi_{11}, \pi_{12}, \pi_{21}, \pi_{22})$$

$$= \begin{pmatrix}
5 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 5 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 3 & 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & -6.67 \times 10^{-4} & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & -6.67 \times 10^{-4} & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0.5 & 0 & -6.67 \times 10^{-4} & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0.5 & 0 & -6.67 \times 10^{-4} & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 20 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

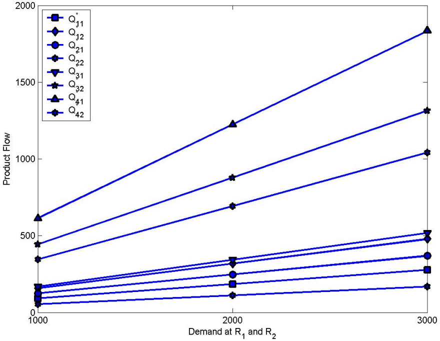

Figure: Equilibrium Product Flows as the Demand Increases for Example 1

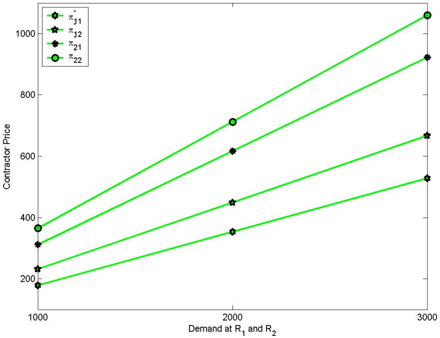

Figure: Equilibrium Contractor Prices as the Demand Increases for Example 1

Additional Numerical Examples - Example 1 - Sensitivity Analysis

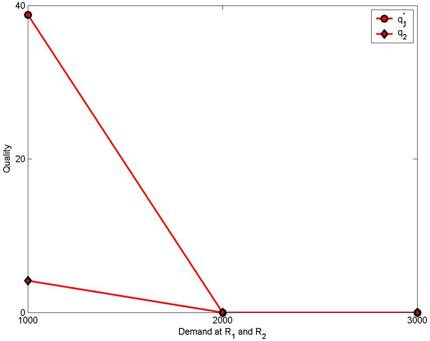

Figure: Equilibrium Quality Levels as the Demand Increases for Example 1

Additional Numerical Examples - Example 2

We considered the following **disruption**. The data were as in **Example 1** but contractor O_2 was not able to provide any production and distribution services. This could arise due to a natural disaster, adulteration in its production process, and/or an inability to procure an ingredient.

Additional Numerical Examples - Example 2

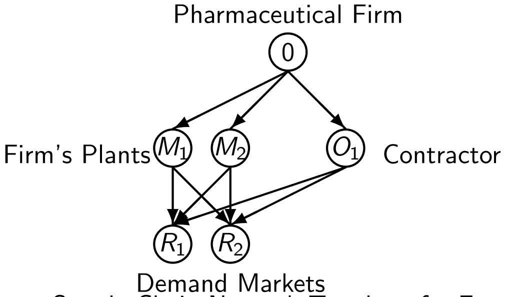

Figure: Supply Chain Network Topology for Example 2

Additional Numerical Examples - Example 2

The Euler method converged in **73** iterations and yielded the following new equilibrium solution.

$$Q^*_{11} = 218.06, \quad Q^*_{12} = 141.79, \quad Q^*_{21} = 260.20, \quad Q^*_{22} = 25.96,$$

$$Q^*_{31} = 521.74, \quad Q^*_{32} = 332.25.$$

$$q^*_1 = 14.60,$$

$$\pi^*_{11} = 531.74, \quad \pi^*_{12} = 176.12.$$

The new average quality level was $q\prime$=51.38. The total cost of the pharmaceutical firm was now 1,123,226.62 whereas the profit of the first contractor was now 123,460.67.

Summary and Conclusions

We developed a supply chain network game theory model, in both **equilibrium and dynamic** versions, to capture contractor selection, based on the competition among the contractors in the **prices** that they charge as well as the **quality levels** of the pharmaceutical products that they produce.

Summary and Conclusions

- We introduced a **disrepute** cost associated with the average quality at the demand markets.
- We utilized **variational inequality** theory for the formulation of the equilibrium conditions.
- We provided stability analysis results as well as an algorithm that can be interpreted as a **discretization** of the continuous-time adjustment processes.
- Our numerical studies included **sensitivity analysis** results as well as a **disruption** to the supply chain network in that a contractor is no longer available for production and distribution.

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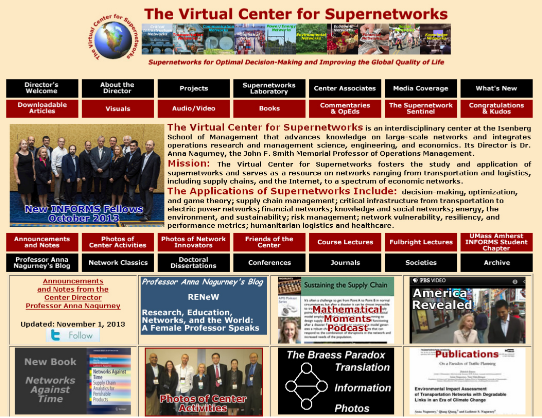
Summary and Conclusions

This paper is a contribution to the literature on outsourcing with a focus on **quality** with an emphasis on the pharmaceutical industry which is a prime example of an industry where the quality of a product is paramount. It also is an interesting application of game theory and associated methodologies.

The ideas in this paper may be adapted, with appropriate modifications, to **other industries**.

Thank you!



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