

**Modeling Generator Power Plant Portfolios
and Pollution Taxes
in
Electric Power Supply Chain Networks:
A Transportation Network Equilibrium
Transformation**

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Electricity is Modernity



Electricity is Big Business

- In US: half a trillion dollars worth of net assets
- Over \$220 billion of annual sales
- Consumes almost 40% of domestic primary energy
- Heavy user of fossil fuels; over a third of total CO₂, NO_x emissions
- Deregulation: from vertically integrated to competitive markets



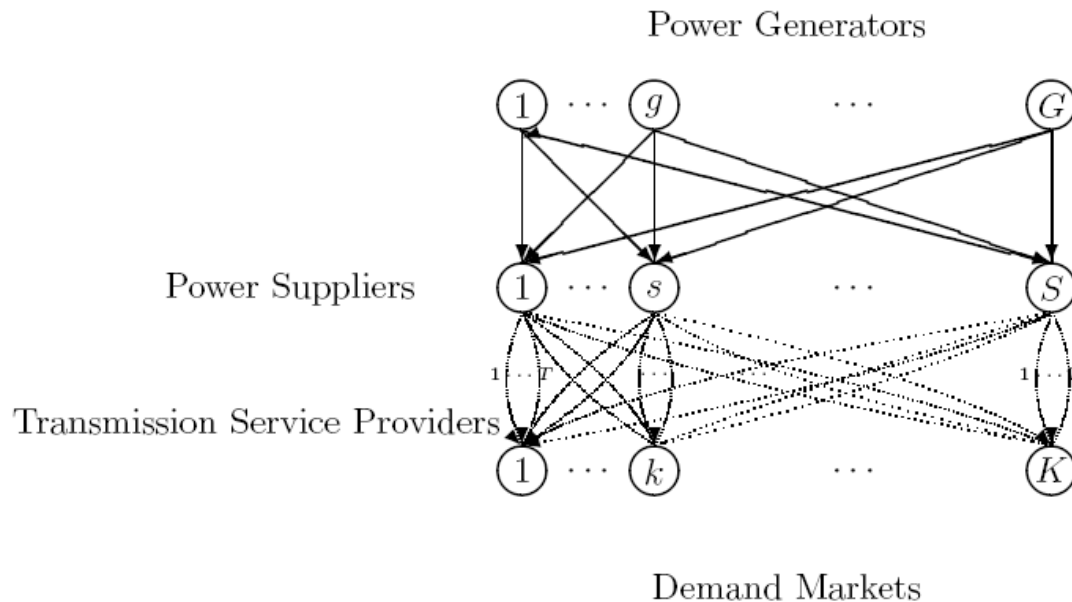
Tax the Bad, Credit the Good

- Climate change poses immense risks
- Market failure: externalities need to be internalized
- Renewables are long-term solution
- Tax on bad emissions is one solution, e.g. CO₂ tax
- Credit for clean energy, e.g. Renewable Portfolio Standards, green certs



Model: Electric Power Supply Chain Network

- 2004 paper by Nagurney, Matsypura
- 3 Players: Gencos, Suppliers, Consumers



The Big VI To Solve

The utility maximization of each player in the network gives the variational inequality:

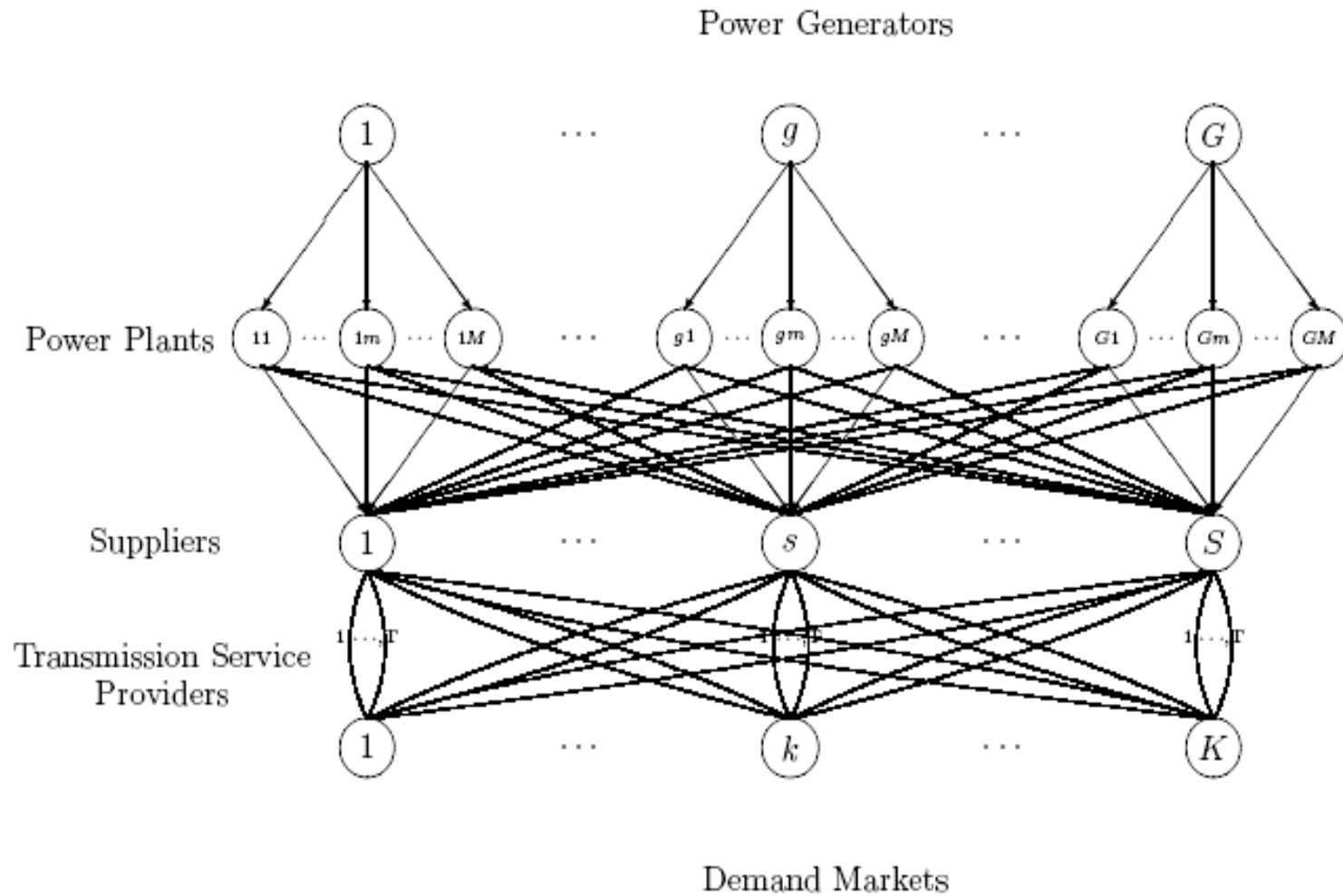
$$\begin{aligned}
 & \sum_{g=1}^G \sum_{s=1}^S \left[\frac{\partial f_g(Q^{1*})}{\partial q_{gs}} + \frac{\partial c_{gs}(Q^{1*})}{\partial q_{gs}} + \frac{\partial c_s(Q^{1*}, Q^{2*})}{\partial q_{gs}} + \frac{\partial \hat{c}_{gs}(Q^{1*})}{\partial q_{gs}} - \gamma_s^* \right] \times [q_{gs} - q_{gs}^*] \\
 & + \sum_{s=1}^S \sum_{k=1}^K \sum_{t=1}^T \left[\frac{\partial c_s(Q^{1*}, Q^{2*})}{\partial q_{sk}^t} + \frac{\partial c_{sk}^t(Q^{2*})}{\partial q_{sk}^t} + \hat{c}_{sk}^t(Q^{2*}) + \gamma_s^* - \rho_{3k}^* \right] \times [q_{sk}^t - q_{sk}^{t*}] \\
 & + \sum_{s=1}^S \left[\sum_{g=1}^G q_{gs}^* - \sum_{k=1}^K \sum_{t=1}^T q_{sk}^{t*} \right] \times [\gamma_s - \gamma_s^*] + \sum_{k=1}^K \left[\sum_{s=1}^S \sum_{t=1}^T q_{sk}^{t*} - d_k(\rho_3^*) \right] \times [\rho_{3k} - \rho_{3k}^*] \geq 0, \\
 & \quad \forall (Q^1, Q^2, \gamma, \rho_3) \in \mathcal{K},
 \end{aligned}$$

where $\mathcal{K} \equiv \{(Q^1, Q^2, \gamma, \rho_3) | (Q^1, Q^2, \gamma, \rho_3) \in R_+^{GS+TSK+S+K}\}$.

Modeling Energy Taxes and Credits: The Genco's Choice

- Each Genco has a portfolio of power plants
- Each power plant can have different supply costs and transaction costs
- Supply costs can reflect capital costs, O&M, fuel costs
- Transaction costs reflect possible taxes or credits

The Extended Model



Variational Inequality for Extended Model

The equilibrium conditions governing the electric power supply chain network according to Definition 1 coincide with the solution of the variational inequality given by: determine $(q^*, h^*, Q^{1*}, Q^{2*}, d^*) \in \mathcal{K}^5$ satisfying:

$$\begin{aligned}
 & \sum_{g=1}^G \sum_{m=1}^M \left[\frac{\partial f_{gm}(q_m^*)}{\partial q_{gm}} + \tau_{gm} \right] \times [q_{gm} - q_{gm}^*] + \sum_{s=1}^S \frac{\partial c_s(h^*)}{\partial h_s} \times [h_s - h_s^*] \\
 & + \sum_{g=1}^G \sum_{m=1}^M \sum_{s=1}^S \left[\frac{\partial c_{gms}(q_{gms}^*)}{\partial q_{gms}} + \frac{\partial \hat{c}_{gms}(q_{gms}^*)}{\partial q_{gms}} \right] \times [q_{gms} - q_{gms}^*] \\
 & + \sum_{s=1}^S \sum_{k=1}^K \sum_{t=1}^T \left[\frac{\partial c_{sk}^t(q_{sk}^{t*})}{\partial q_{sk}^t} + \hat{c}_{sk}^t(Q^{2*}) \right] \times [q_{sk}^t - q_{sk}^{t*}] - \sum_{k=1}^K \rho_{3k}(d^*) \times [d_k - d_k^*] \geq 0, \\
 & \forall (q, h, Q^1, Q^2, d) \in \mathcal{K}^5, \tag{18}
 \end{aligned}$$

where

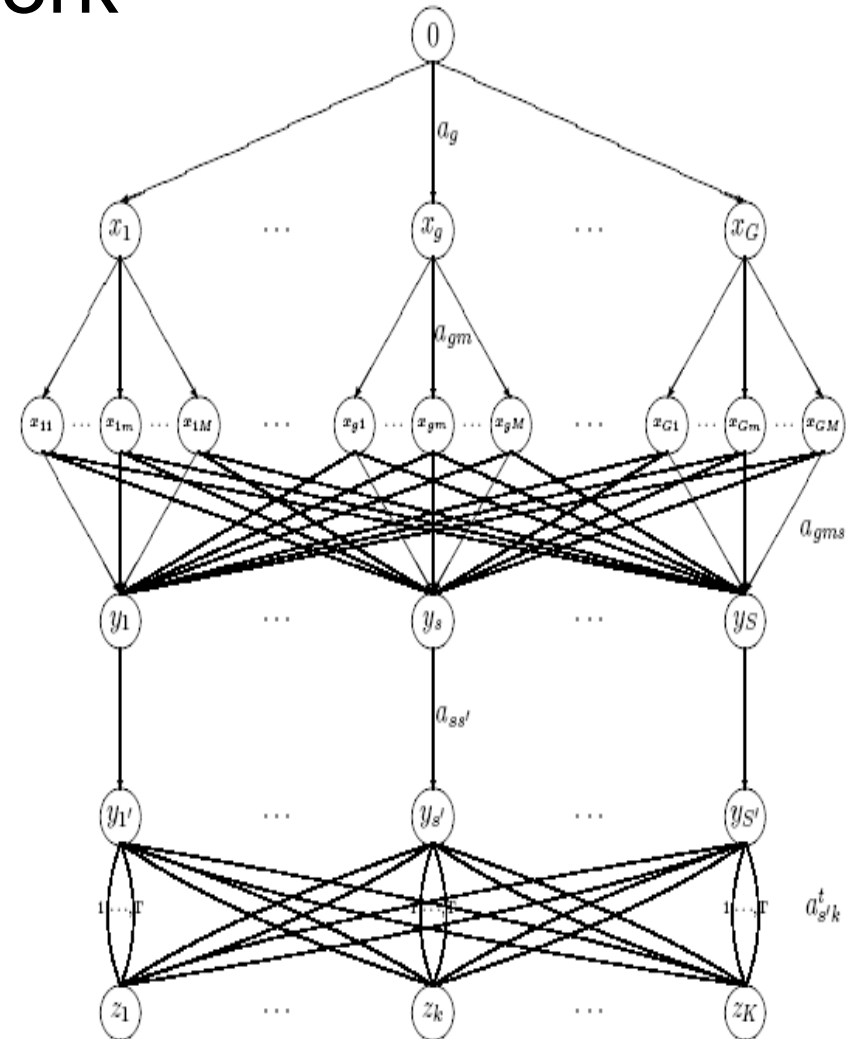
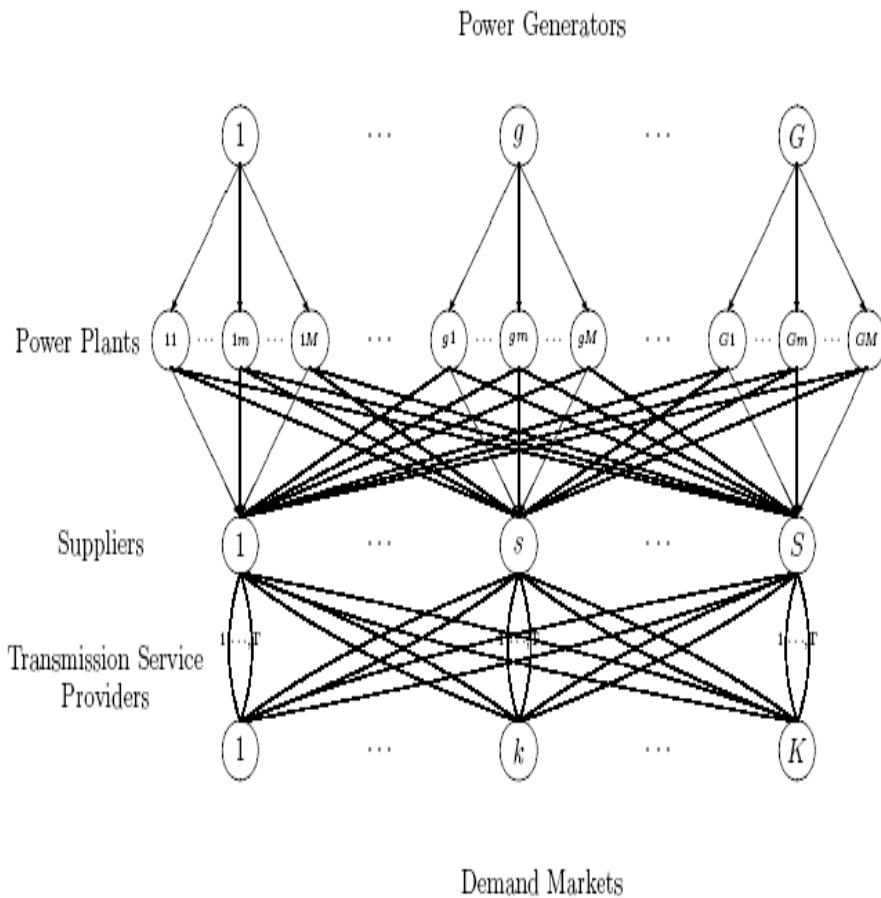
$$\mathcal{K}^5 \equiv \{(q, h, Q^1, Q^2, d) | (q, h, Q^1, Q^2, d) \in R_+^{GM+S+GMS+TSK+K}$$

and (2), (5), (11), and (15) hold\}.

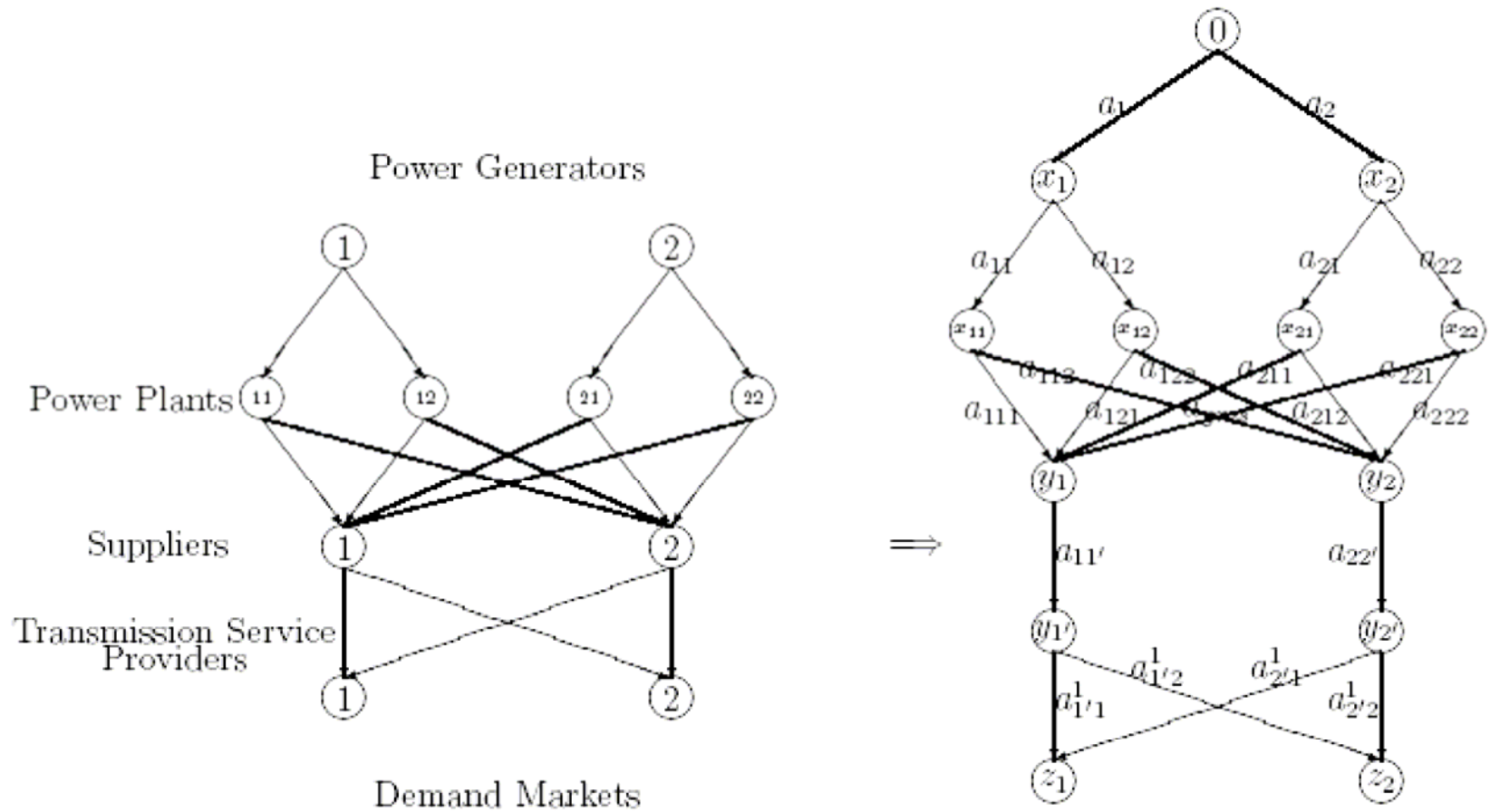
Transportation Network Equilibrium Isomorphism

- Why do we need the transportation network equilibrium isomorphism?
- We have proved the supernetwork equivalence of properly configured transportation networks with
 - Supply Chain Networks (A. Nagurney)
 - Financial Networks (Z. Liu and A. Nagurney)
 - Electrical Power Networks (A. Nagurney and Z. Liu)

Supernetwork Equivalence of the Electrical Power Network with the Transportation Network



A Numerical Example



The Cost Functions

- Generating cost functions

$$f_{11}(q_1) = 2.5q_{11}^2 + q_{11}q_{21} + 2q_{11}, \quad f_{12}(q_2) = 2.5q_{12}^2 + q_{11}q_{12} + 2q_{22}, \quad f_{21}(q_1) = .5q_{21}^2 + .5q_{11}q_{21} + 2q_{21},$$

$$f_{22}(q_2) = .5q_{22}^2 + q_{12} + 2.$$

- Transaction cost between power generators and the suppliers

$$c_{111}(q_{111}) = .5q_{111}^2 + 3.5q_{111}, \quad c_{112}(q_{112}) = .5q_{112}^2 + 3.5q_{112}, \quad c_{121}(q_{121}) = .5q_{121}^2 + 3.5q_{121},$$

$$c_{122}(q_{122}) = .5q_{122}^2 + 3.5q_{122},$$

$$c_{211}(q_{211}) = .5q_{211}^2 + 2q_{211}, \quad c_{212}(q_{212}) = .5q_{212}^2 + 2q_{212}, \quad c_{221}(q_{221}) = .5q_{221}^2 + 2q_{221},$$

$$c_{222}(q_{222}) = .5q_{222}^2 + 2q_{222}.$$

The Cost Functions and Price Functions

- The operating cost of the power generators

$$c_1(Q^1) = .5\left(\sum_{i=1}^2 q_{i1}\right)^2, \quad c_2(Q^1) = .5\left(\sum_{i=1}^2 q_{i2}\right)^2.$$

- The demand market price function

$$\rho_{31}(d) = -1.33d_1 + 366.6, \quad \rho_{32} = -1.33d_2 + 366.6,$$

- The transaction cost between the suppliers and the customers

$$\hat{c}_{sk}^1(q_{sk}^1) = q_{sk}^1 + 5, \quad s = 1, 2; k = 1, 2.$$

The Solution

- Equilibrium pattern for the transportation network

$$f_{a_1}^* = 32.53, \quad f_{a_2}^* = 115.22,$$

$$f_{a_{11}}^* = 22.57, \quad f_{a_{12}}^* = 9.96, \quad f_{a_{21}}^* = 22.90, \quad f_{a_{22}}^* = 92.32,$$

$$f_{a_{11'}}^* = f_{a_{22'}}^* = 73.87,$$

$$f_{a_{111}}^* = 11.29, \quad f_{a_{112}}^* = 11.29, \quad f_{a_{121}}^* = 4.98, \quad f_{a_{222}}^* = 4.98,$$

$$f_{a_{211}}^* = 11.45, \quad f_{a_{212}}^* = 11.45, \quad f_{a_{221}}^* = 46.16, \quad f_{a_{222}}^* = 46.16,$$

$$f_{a_{1'1}}^* = f_{a_{1'2}}^* = f_{a_{2'1}}^* = f_{a_{2'2}}^* = 36.94,$$

The Solution (cont)

- The equilibrium pattern for the electrical power network

$$q_1^* = 32.53, \quad q_2^* = 115.22,$$

$$q_{11}^* = 22.57, \quad q_{12}^* = 9.96, \quad q_{21}^* = 22.90, \quad q_{22}^* = 9.96,$$

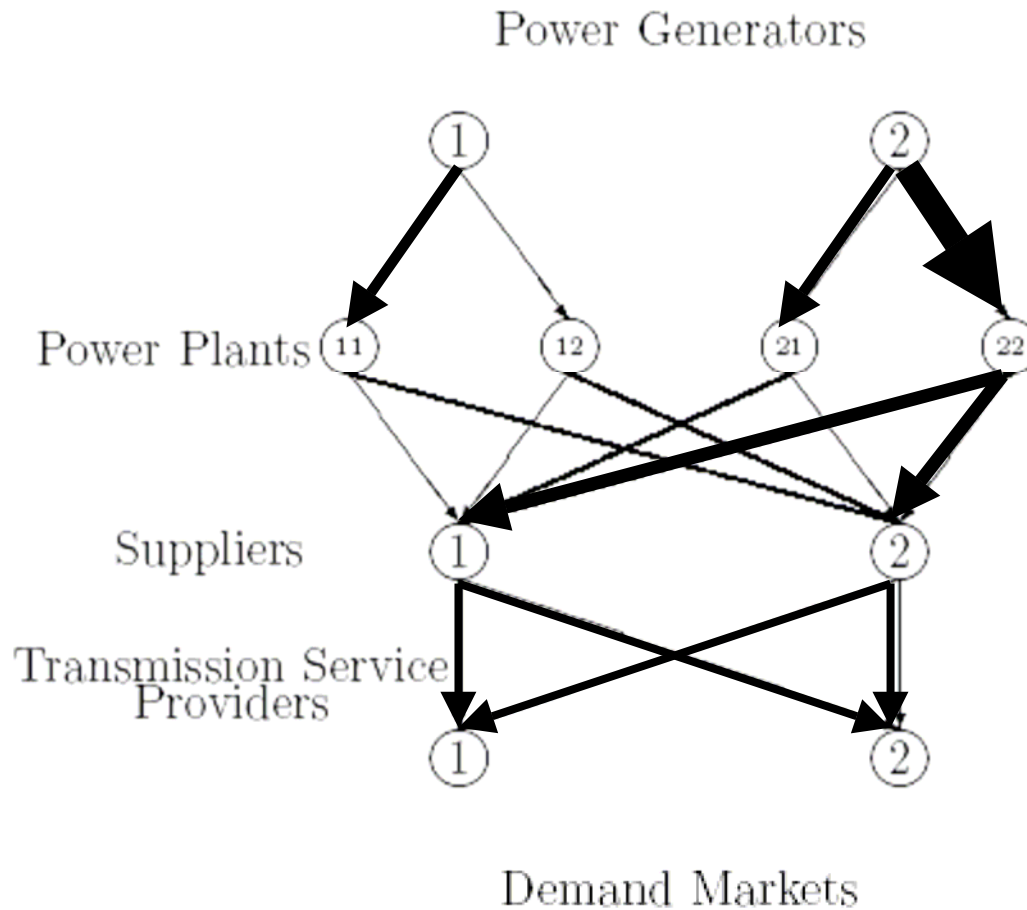
$$h_1^* = h_2^* = 73.87,$$

$$q_{111}^* = 11.29, \quad q_{112}^* = 11.29, \quad q_{121}^* = 4.98, \quad q_{222}^* = 4.98,$$

$$q_{211}^* = 11.45, \quad q_{212}^* = 11.45, \quad q_{221}^* = 46.16, \quad q_{222}^* = 46.16,$$

$$q_{1'1}^{1*} = q_{1'2}^{1*} = q_{2'1}^{1*} = q_{2'2}^{1*} = 36.94.$$

Solution of Example 1



Example 2

- Same network configuration except that we added tax 133 to the first plant of genco 1 which is highly polluting.

- Solution: $q_1^* = 10.77, \quad q_2^* = 128.71,$

$$q_{11}^* = 0.00, \quad q_{12}^* = 10.77, \quad q_{21}^* = 29.14, \quad q_{22}^* = 99.58,$$

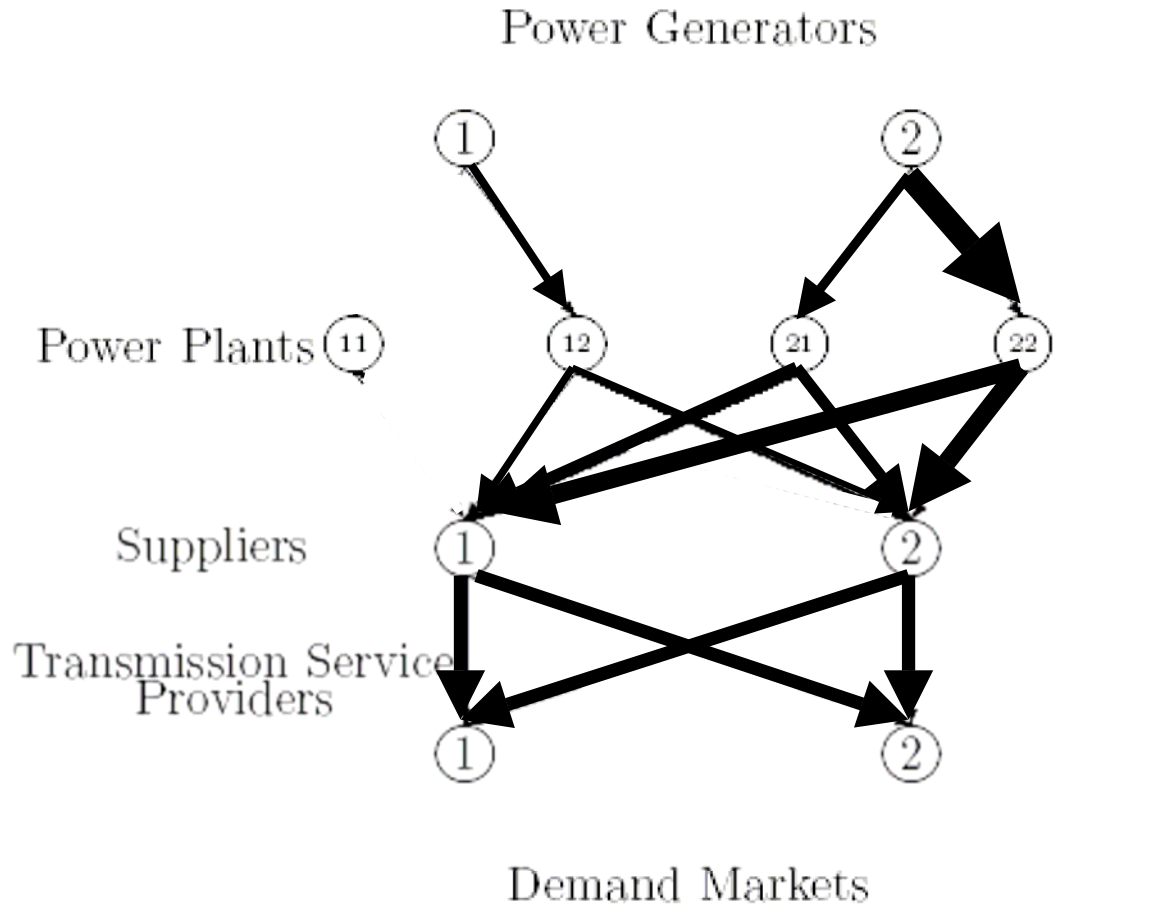
$$h_1^* = h_2^* = 69.74,$$

$$q_{111}^* = 0.00, \quad q_{112}^* = 0.00, \quad q_{121}^* = 5.38, \quad q_{222}^* = 5.38,$$

$$q_{211}^* = 11.45, \quad q_{212}^* = 14.57, \quad q_{221}^* = 49.79, \quad q_{222}^* = 49.79,$$

$$q_{1'1}^{1*} = q_{1'2}^{1*} = q_{2'1}^{1*} = q_{2'2}^{1*} = 34.87.$$

Solution to Example 2



Thank You!

Questions?