Supply Chain Performance Assessment and Supplier and Component Importance Identification in a General Competitive Multitiered Supply Chain Network Model

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Outline

- Background and Motivation
- The Multitiered Supply Chain Network Game Theory Model with Suppliers
- Supply Chain Network Performance Measures and Supplier and Component Importance Identification
- The Algorithm
- Numerical Examples
- Summary and Conclusions

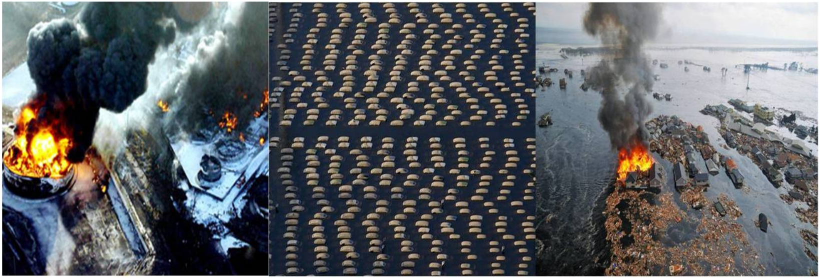
Background and Motivation

Suppliers are critical in providing essential components and resources for finished goods in today's globalized supply chain networks. Even in the case of simpler products, such as bread, ingredients may travel across the globe as inputs into production processes.

Suppliers are also decision-makers and they compete with one another to provide components to downstream manufacturing firms.

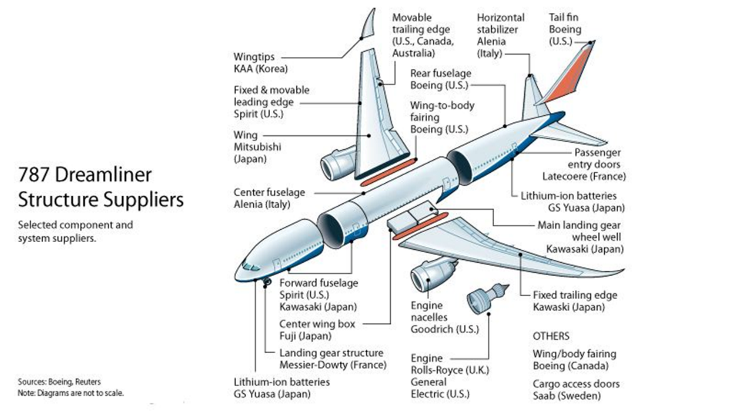
Background and Motivation

When suppliers are faced with disruptions, whether due to man-made activities or errors, natural disasters, unforeseen events, or even terrorist attacks, the ramifications and effects may propagate through a supply chain or multiple supply chains.



Background and Motivation

Boeing, facing challenges with its 787 Dreamliner supply chain design and numerous delays, ended up having to buy two suppliers for $2.4 billion because the units were underperforming in the chain (Tang, Zimmerman, and Nelson (2009)).



Background and Motivation

Hence,

- capturing supplier behavior and the competition among multiple suppliers,
- integrating suppliers and their behavior into general multitiered supply chain network equilibrium frameworks, and
- identifying the importance of a supplier and the components that he provides to the firms

are essential in modeling the full scope of supply chain network competition.

Overview

In this paper, we develop a multitiered competitive supply chain network game theory model, which includes the supplier tier.

- The firms are differentiated by brands and can produce their own components, as reflected by their capacities, and/or obtain components from one or more suppliers, who also are capacitated.
- The firms compete in Cournot-Nash fashion, whereas
- the suppliers compete a la Bertrand.
- All decision-makers seek to maximize their profits.
- Consumers reflect their preferences through the demand price functions associated with the demand markets for the firms' products.

Overview

- We develop a general multitiered competitive supply chain network equilibrium model with suppliers and firms that includes capacities and constraints to capture the production activities.
- We propose supply chain network performance measures, on the full supply chain and on the individual firm levels, that assess the efficiency of the supply chain or firm, respectively, and also allow for the identification and ranking of the importance of suppliers as well as the components of suppliers with respect to the full supply chain or individual firm.
- Our framework adds to the growing literature on supply chain disruptions by providing metrics that allow individual firms, industry overseers or regulators, and/or government policy-makers to identify the importance of suppliers and the components that they produce for various product supply chains.

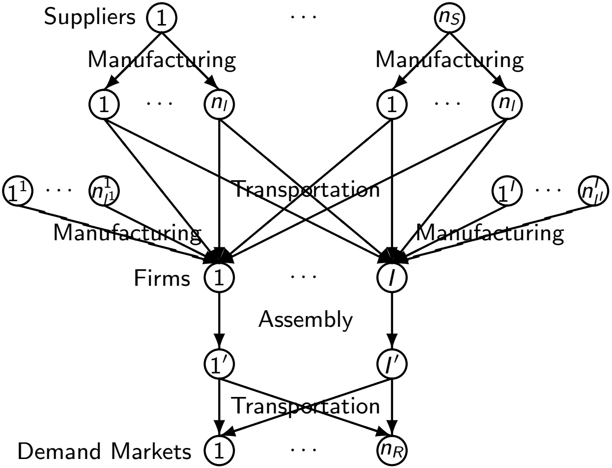


Figure: The Multitiered Supply Chain Network Topology

Notations

Q^S_{jil}: the nonnegative amount of firm i's component l produced by supplier j; $j = 1, \ldots, n_S$; $i = 1, \ldots, I$; $l = 1, \ldots, n_{l^i}$.

Q^F_{il}: the nonnegative amount of firm i's component l produced by firm i itself.

Q_{ik}: the nonnegative shipment of firm i's product from firm i to demand market k; $k = 1, \ldots, n_R$.

π_{jil}: the price charged by supplier j for producing one unit of firm i's component l.

d_{ik}: the demand for firm i's product at demand market k.

θ_{il}: the amount of component l needed by firm i to produce one unit product i.

The Behavior of the Firms

$f_i(Q)$: firm i's cost for assembling its product using the components needed.

$f_{il}^F(Q^F)$: firm i's production cost for producing its component l.

$tc_{ik}^F(Q)$: firm i's transportation cost for shipping its product to demand market k.

$c_{ijl}(Q^S)$: the transaction cost paid by firm i for transacting with supplier j for its component l.

$\rho_{ik}(d)$: the demand price for firm i's product at demand market k.

All the $\{Q_{jil}^S\}$ elements are grouped into the vector $Q^S \in R_+^{n_S \sum_{i=1}^I n_{l^i}}$.

All the $\{Q_{il}^F\}$ elements are grouped into the vector $Q^F \in R_+^{\sum_{i=1}^I n_{l^i}}$.

All the $\{Q_{ik}\}$ elements are grouped into the vector $Q \in R_+^{In_R}$.

We group all $\{d_{ik}\}$ elements into the vector $d \in R_+^{In_R}$.

The Behavior of the Firms

$$\text{Maximize}_{Q_i, Q_i^F, Q_i^S} \quad U_i^F = \sum_{k=1}^{n_R} \rho_{ik}(d) d_{ik} - f_i(Q) - \sum_{l=1}^{n_{l^i}} f_{il}^F(Q^F) - \sum_{k=1}^{n_R} tc_{ik}^F(Q)$$

$$- \sum_{j=1}^{n_S} \sum_{l=1}^{n_{l^i}} \pi_{jil}^* Q_{jil}^S - \sum_{j=1}^{n_S} \sum_{l=1}^{n_{l^i}} c_{ijl}(Q^S) \tag{1}$$

subject to:

$$Q_{ik} = d_{ik}, \quad i = 1, \ldots, I; k = 1, \ldots, n_R, \tag{2}$$

$$\sum_{k=1}^{n_R} Q_{ik} \theta_{il} \leq \sum_{j=1}^{n_S} Q_{jil}^S + Q_{il}^F, \quad i = 1, \ldots, I; l = 1, \ldots, n_{l^i}, \tag{3}$$

$$Q_{ik} \geq 0, \quad i = 1, \ldots, I; k = 1, \ldots, n_R, \tag{4}$$

$$CAP_{jil}^S \geq Q_{jil}^S \geq 0, \quad j = 1, \ldots, n_S; i = 1, \ldots, I; l = 1, \ldots, n_{l^i}, \tag{5}$$

$$CAP_{il}^F \geq Q_{il}^F \geq 0, \quad i = 1, \ldots, I; l = 1, \ldots, n_{l^i}. \tag{6}$$

For firm i, we group its $\{Q_{jil}^S\}$ elements into the vector $Q_i^S \in R_+^{n_S n_{l^i}}$, its $\{Q_{il}^F\}$ elements into the vector $Q_i^F \in R_+^{n_{l^i}}$, and its $\{Q_{ik}\}$ elements into the vector $Q_i \in R_+^{n_R}$.

The Behavior of the Firms

We define $\overline{K}_i^F \equiv \{(Q_i, Q_i^F, Q_i^S) | (3)$ - (6) are satisfied$\}$. All $\overline{K}_i^F$; $i = 1, \ldots, I$, are closed and convex. We also define the feasible set $\overline{\mathcal{K}}^F \equiv \Pi_{i=1}^I \overline{K}_i^F$.

Definition 1: A Cournot-Nash Equilibrium

A product shipment, in-house component production, and contracted component production pattern $(Q^*, Q^{F^*}, Q^{S^*}) \in \overline{\mathcal{K}}^F$ is said to constitute a Cournot-Nash equilibrium if for each firm i; $i = 1, \ldots, I$,

$$U_i^F(Q_i^*, \hat{Q}_i^*, Q_i^{F^*}, \hat{Q}_i^{F^*}, Q_i^{S^*}, \hat{Q}_i^{S^*}, \pi^*) \geq U_i^F(Q_i, \hat{Q}_i^*, Q_i^F, \hat{Q}_i^{F^*}, Q_i^S, \hat{Q}_i^{S^*}, \pi^*),$$

$$\forall (Q_i, Q_i^F, Q_i^S) \in \overline{K}_i^F, \tag{7}$$

where

$$\hat{Q}_i^* \equiv (Q_1^*, \ldots, Q_{i-1}^*, Q_{i+1}^*, \ldots, Q_I^*),$$
$$\hat{Q}_i^{F^*} \equiv (Q_1^{F^*}, \ldots, Q_{i-1}^{F^*}, Q_{i+1}^{F^*}, \ldots, Q_I^{F^*}),$$
$$\hat{Q}_i^{S^*} \equiv (Q_1^{S^*}, \ldots, Q_{i-1}^{S^*}, Q_{i+1}^{S^*}, \ldots, Q_I^{S^*}).$$

Theorem 1

Assume that, for each firm i; $i = 1, \ldots, I$, the utility function $U_i^F(Q, Q^F, Q^S, \pi^*)$ is concave with respect to its variables in Q_i, Q_i^F, and Q_i^S, and is continuous and continuously differentiable. Then $(Q^*, Q^{F^*}, Q^{S^*}) \in \overline{\mathcal{K}}^F$ is a Counot-Nash equilibrium according to Definition 1 if and only if it satisfies the variational inequality:

$$-\sum_{i=1}^{I}\sum_{k=1}^{n_R} \frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, \pi^*)}{\partial Q_{ik}} \times (Q_{ik} - Q_{ik}^*)$$

$$-\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}} \frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, \pi^*)}{\partial Q_{il}^F} \times (Q_{il}^F - Q_{il}^{F^*})$$

$$-\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}} \frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, \pi^*)}{\partial Q_{jil}^S} \times (Q_{jil}^S - Q_{jil}^{S^*}) \geq 0, \quad \forall (Q, Q^F, Q^S) \in \overline{\mathcal{K}}^F, \tag{8}$$

The Behavior of the Firms

Theorem 1

with notice that: for $i = 1, \ldots, I$; $k = 1, \ldots, n_R$:

$$-\frac{\partial U_i^F}{\partial Q_{ik}} = \left[\frac{\partial f_i(Q)}{\partial Q_{ik}} + \sum_{h=1}^{n_R} \frac{\partial tc_{ih}^F(Q)}{\partial Q_{ik}} - \sum_{h=1}^{n_R} \frac{\partial \hat{\rho}_{ih}(Q)}{\partial Q_{ik}} Q_{ih} - \hat{\rho}_{ik}(Q)\right],$$

for $i = 1, \ldots, I$; $l = 1, \ldots, n_{l^i}$:

$$-\frac{\partial U_i^F}{\partial Q_{il}^F} = \left[\sum_{m=1}^{n_{l^i}} \frac{\partial f_{im}^F(Q^F)}{\partial Q_{il}^F}\right],$$

for $j = 1, \ldots, n_S$; $i = 1, \ldots, I$; $l = 1, \ldots, n_{l^i}$:

$$-\frac{\partial U_i^F}{\partial Q_{jil}^S} = \left[\pi_{jil}^* + \sum_{g=1}^{n_S} \sum_{m=1}^{n_{l^i}} \frac{\partial c_{igm}(Q^S)}{\partial Q_{jil}^S}\right].$$

The Behavior of the Firms

Theorem 1

Equivalently, $(Q^*, Q^{F^*}, Q^{S^*}, \lambda^*) \in \mathcal{K}^F$ *is a vector of the equilibrium product shipment, in-house component production, contracted component production pattern, and Lagrange multipliers if and only if it satisfies the variational inequality*

$$\sum_{i=1}^{I}\sum_{k=1}^{n_R}\left[\frac{\partial f_i(Q^*)}{\partial Q_{ik}}+\sum_{h=1}^{n_R}\frac{\partial tc^F_{ih}(Q^*)}{\partial Q_{ik}}-\sum_{h=1}^{n_R}\frac{\partial \hat{\rho}_{ih}(Q^*)}{\partial Q_{ik}}Q^*_{ih}-\hat{\rho}_{ik}(Q^*)+\sum_{l=1}^{n_{l^i}}\lambda^*_{il}\theta_{il}\right]$$

$$\times(Q_{ik}-Q^*_{ik})+\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\left[\sum_{m=1}^{n_{l^i}}\frac{\partial f^F_{im}(Q^{F^*})}{\partial Q^F_{il}}-\lambda^*_{il}\right]\times(Q^F_{il}-Q^{F^*}_{il})$$

$$+\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\left[\pi^*_{jil}+\sum_{g=1}^{n_S}\sum_{m=1}^{n_{l^i}}\frac{\partial c_{igm}(Q^{S^*})}{\partial Q^S_{jil}}-\lambda^*_{il}\right]\times(Q^S_{jil}-Q^{S^*}_{jil})$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\left[\sum_{j=1}^{n_S}Q^{S^*}_{jil}+Q^{F^*}_{il}-\sum_{k=1}^{n_R}Q^*_{ik}\theta_{il}\right]\times(\lambda_{il}-\lambda^*_{il})\geq 0,\quad \forall (Q,Q^F,Q^S,\lambda)\in\mathcal{K}^F, \tag{9}$$

where $\mathcal{K}^F \equiv \Pi^I_{i=1}K^F_i$ *and* $K^F_i \equiv \{(Q_i, Q^F_i, Q^S_i, \lambda_i) | \lambda_i \geq 0$ *with (4) - (6) satisfied*$\}$.

The Behavior of the Suppliers

$f^S_{jl}(Q^S)$: supplier j's production cost for producing component l; $l = 1, \ldots, n_l$.

$tc^S_{jil}(Q^S)$: supplier j's transportation cost for shipping firm i's component l.

$oc_j(\pi)$: supplier j's opportunity cost.

We group all the $\{\pi_{jil}\}$ elements into the vector $\pi \in R_+^{n_S \sum_{i=1}^{I} n_{l^i}}$.

The Behavior of the Suppliers

$$\text{Maximize}_{\pi_j} \quad U_j^S = \sum_{i=1}^{I} \sum_{l=1}^{n_{l^i}} \pi_{jil} Q_{jil}^{S^*} - \sum_{l=1}^{n_l} f_{jl}^S(Q^{S^*}) - \sum_{i=1}^{I} \sum_{l=1}^{n_{l^i}} tc_{jil}^S(Q^{S^*}) - oc_j(\pi) \tag{11}$$

subject to:

$$\pi_{jil} \geq 0, \quad j = 1, \ldots, n_S; i = 1, \ldots, I; l = 1, \ldots, n_{l^i}. \tag{12}$$

For supplier j, we group its $\{\pi_{jil}\}$ elements into the vector $\pi_j \in R_+^{\sum_{i=1}^{I} n_{l^i}}$.

The Behavior of the Suppliers

We define the feasible sets $K_j^S \equiv \{\pi_j | \pi_j \in R_+^{\sum_{i=1}^{I} n_{I^i}}\}$, $\mathcal{K}^S \equiv \Pi_{j=1}^{n_S} K_j^S$, and $\overline{\mathcal{K}} \equiv \overline{\mathcal{K}}^F \times \mathcal{K}^S$.

Definition 2: A Bertrand-Nash Equilibrium

A price pattern $\pi^* \in \mathcal{K}^S$ is said to constitute a Bertrand-Nash equilibrium if for each supplier j; $j = 1, \ldots, n_S$,

$$U_j^S(Q^{S^*}, \pi_j^*, \hat{\pi}_j^*) \geq U_j^S(Q^{S^*}, \pi_j, \hat{\pi}_j^*), \quad \forall \pi_j \in K_j^S, \tag{13}$$

where

$$\hat{\pi}_j^* \equiv (\pi_1^*, \ldots, \pi_{j-1}^*, \pi_{j+1}^*, \ldots, \pi_{n_S}^*).$$

The Behavior of the Suppliers

Theorem 2

Assume that, for each supplier j; $j = 1, \ldots, n_S$, the profit function $U_j^S(Q^{S^*}, \pi)$ is concave with respect to the variables in π_j, and is continuous and continuously differentiable. Then $\pi^* \in \mathcal{K}^S$ is a Bertrand-Nash equilibrium according to Definition 2 if and only if it satisfies the variational inequality:

$$-\sum_{j=1}^{n_S} \sum_{i=1}^{I} \sum_{l=1}^{n_{l^i}} \frac{\partial U_j^S(Q^{S^*}, \pi^*)}{\partial \pi_{jil}} \times (\pi_{jil} - \pi_{jil}^*) \geq 0,$$

$$\forall \pi \in \mathcal{K}^S, \quad (14)$$

with notice that: for $j = 1, \ldots, n_S$; $i = 1, \ldots, I$; $l = 1, \ldots, n_{l^i}$:

$$-\frac{\partial U_j^S}{\partial \pi_{jil}} = \frac{\partial oc_j(\pi)}{\partial \pi_{jil}} - Q_{jil}^{S^*}.$$

Definition 3: Multitiered Supply Chain Network Equilibrium with Suppliers

The equilibrium state of the multitiered supply chain network with suppliers is one where both variational inequalities (8) (or (9)) and (14) hold simultaneously.

Theorem 3

The equilibrium conditions governing the multitiered supply chain network model with suppliers are equivalent to the solution of the variational inequality problem: determine $(Q^*, Q^{F^*}, Q^{S^*}, \pi^*) \in \overline{\mathcal{K}}$, *such that:*

$$-\sum_{i=1}^{I}\sum_{k=1}^{n_R}\frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, \pi^*)}{\partial Q_{ik}} \times (Q_{ik} - Q_{ik}^*) - \sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, \pi^*)}{\partial Q_{il}^F}$$

$$\times (Q_{il}^F - Q_{il}^{F^*}) - \sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, \pi^*)}{\partial Q_{jil}^S} \times (Q_{jil}^S - Q_{jil}^{S^*})$$

$$-\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\frac{\partial U_j^S(Q^{S^*}, \pi^*)}{\partial \pi_{jil}} \times (\pi_{jil} - \pi_{jil}^*) \geq 0, \quad \forall (Q, Q^F, Q^S, \pi) \in \overline{\mathcal{K}}. \quad (15)$$

Theorem 3

Equivalently: determine $(Q^*, Q^{F^*}, Q^{S^*}, \lambda^*, \pi^*) \in \mathcal{K}$, *such that:*

$$\sum_{i=1}^{I}\sum_{k=1}^{n_R}\left[\frac{\partial f_i(Q^*)}{\partial Q_{ik}} + \sum_{h=1}^{n_R}\frac{\partial tc^F_{ih}(Q^*)}{\partial Q_{ik}} - \sum_{h=1}^{n_R}\frac{\partial \hat{\rho}_{ih}(Q^*)}{\partial Q_{ik}} Q^*_{ih} - \hat{\rho}_{ik}(Q^*) + \sum_{l=1}^{n_{l^i}}\lambda^*_{il}\theta_{il}\right]$$

$$\times (Q_{ik} - Q^*_{ik}) + \sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\left[\sum_{m=1}^{n_{l^i}}\frac{\partial f^F_{im}(Q^{F^*})}{\partial Q^F_{il}} - \lambda^*_{il}\right] \times (Q^F_{il} - Q^{F^*}_{il})$$

$$+ \sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\left[\pi^*_{jil} + \sum_{g=1}^{n_S}\sum_{m=1}^{n_{l^i}}\frac{\partial c_{igm}(Q^{S^*})}{\partial Q^S_{jil}} - \lambda^*_{il}\right] \times (Q^S_{jil} - Q^{S^*}_{jil})$$

$$+ \sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\left[\sum_{j=1}^{n_S} Q^{S^*}_{jil} + Q^{F^*}_{il} - \sum_{k=1}^{n_R} Q^*_{ik}\theta_{il}\right] \times (\lambda_{il} - \lambda^*_{il})$$

$$+ \sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\left[\frac{\partial oc_j(\pi^*)}{\partial \pi_{jil}} - Q^{S^*}_{jil}\right] \times (\pi_{jil} - \pi^*_{jil}) \geq 0, \quad \forall (Q, Q^F, Q^S, \lambda, \pi) \in \mathcal{K}, \tag{16}$$

where $\mathcal{K} \equiv \mathcal{K}^F \times \mathcal{K}^S$.

Standard Form

Determine $X^* \in \mathcal{K}$ where X is a vector in R^N, $F(X)$ is a continuous function such that $F(X) : X \mapsto \mathcal{K} \subset R^N$, and

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{17}$$

where $\langle \cdot, \cdot \rangle$ is the inner product in the N-dimensional Euclidean space, $N = In_R + 2n_S \sum_{i=1}^{I} n_{l^i} + 2 \sum_{i=1}^{I} n_{l^i}$, and $\mathcal{K}$ is closed and convex. We define the vector $X \equiv (Q, Q^F, Q^S, \lambda, \pi)$ and the vector $F(X) \equiv (F^1(X), F^2(X), F^3(X), F^4(X), F^5(X))$,

Standard Form

such that:

$$F^1(X) = \left[\frac{\partial f_i(Q)}{\partial Q_{ik}} + \sum_{h=1}^{n_R} \frac{\partial tc^F_{ih}(Q)}{\partial Q_{ik}} - \sum_{h=1}^{n_R} \frac{\partial \hat{\rho}_{ih}(Q)}{\partial Q_{ik}} Q_{ih} - \hat{\rho}_{ik}(Q) + \sum_{l=1}^{n_{l^i}} \lambda_{il}\theta_{il}; \right.$$

$$\left. i = 1, \ldots, I; k = 1, \ldots, n_R \right], \quad (18a)$$

$$F^2(X) = \left[\sum_{m=1}^{n_{l^i}} \frac{\partial f^F_{im}(Q^F)}{\partial Q^F_{il}} - \lambda_{il}; i = 1, \ldots, I; l = 1, \ldots, n_{l^i} \right], \quad (18b)$$

$$F^3(X) = \left[\pi_{jil} + \sum_{g=1}^{n_S} \sum_{m=1}^{n_{l^i}} \frac{\partial c_{igm}(Q^S)}{\partial Q^S_{jil}} - \lambda_{il}; \right.$$

$$\left. j = 1, \ldots, n_S; i = 1, \ldots, I; l = 1, \ldots, n_{l^i} \right], \quad (18c)$$

$$F^4(X) = \left[\sum_{j=1}^{n_S} Q^S_{jil} + Q^F_{il} - \sum_{k=1}^{n_R} Q_{ik}\theta_{il}; i = 1, \ldots, I; l = 1, \ldots, n_{l^i} \right], \quad (18d)$$

$$F^5(X) = \left[\frac{\partial oc_j(\pi)}{\partial \pi_{jil}} - Q^S_{jil}; j = 1, \ldots, n_S; i = 1, \ldots, I; l = 1, \ldots, n_{l^i} \right]. \quad (18e)$$

It is reasonable to expect that the price charged by each supplier j for producing one unit of firm i's component l, π_{jil}, is bounded by a sufficiently large value, since, in practice, each supplier cannot charge unbounded prices to the firms.

Assumption 1

Suppose that in our supply chain network model with suppliers there exists a sufficiently large Π, *such that,*

$$\pi_{jil} \leq \Pi, \quad j = 1, \ldots, n_S; i = 1, \ldots, I; l = 1, \ldots, n_{l^i}. \tag{19}$$

Theorem 4: Existence

With Assumption 1 satisfied, there exists at least one solution to variational inequalities (17); equivalently, (16) and (15).

Theorem 5: Uniqueness

If Assumption 1 is satisfied, the equilibrium product shipment, in-house component production, contracted component production, and suppliers' price pattern $(Q^*, Q^{F^*}, Q^{S^*}, \pi^*)$ *in variational inequality (17), is unique under the following conditions:*

(i) one of the two families of convex functions $f_i(Q)$; $i = 1, \ldots, I$, *and* $tc_{ik}^F(Q)$; $k = 1 \ldots . n_R$, *is strictly convex in* Q_{ik};
(ii) the $f_{il}^F(Q^F)$; $i = 1, \ldots, I, l = 1, \ldots . n_{l^i}$, *are strictly convex in* Q_{il}^F;
(iii) the $c_{ijl}(Q^S)$; $j = 1, \ldots, n_S$, $i = 1, \ldots, I, l = 1, \ldots . n_{l^i}$, *are strictly convex in* Q_{jil}^S;
(iv) the $oc_j(\pi)$; $j = 1, \ldots, n_S$, *are strictly convex in* π_{jil};
(v) the $\rho_{ik}(d)$; $i = 1, \ldots, I, k = 1, \ldots . n_R$, *are strictly monotone decreasing of* d_{ik}.

Supply Chain Network Performance Measures

We now present the supply chain network performance measure for the whole competitive supply chain network G and that for the supply chain network of each individual firm i; $i = 1, \ldots, I$, under competition.

- Such measures capture the efficiency of the supply chains in that the higher the demand to price ratios normalized over associated firm and demand market pairs, the higher the efficiency.
- Hence, a supply chain network is deemed to perform better if it can satisfy higher demands, on the average, relative to the product prices.

Supply Chain Network Performance Measures

Definition 4.1: The Supply Chain Network Performance Measure for the Whole Competitive Supply Chain Network *G*

The supply chain network performance/efficiency measure, $\mathcal{E}(G)$, for a given competitive supply chain network topology G and the equilibrium demand vector d^*, is defined as follows:

$$\mathcal{E} = \mathcal{E}(G) = \frac{\sum_{i=1}^{I} \sum_{k=1}^{n_R} \frac{d_{ik}^*}{\rho_{ik}(d^*)}}{I \times n_R}. \tag{20}$$

Definition 4.2: The Supply Chain Network Performance Measure for an Individual Firm under Competition

The supply chain network performance/efficiency measure, $\mathcal{E}_i(G_i)$, for the supply chain network topology of a given firm i, G_i, under competition and the equilibrium demand vector d^*, is defined as:

$$\mathcal{E}_i = \mathcal{E}_i(G_i) = \frac{\sum_{k=1}^{n_R} \frac{d_{ik}^*}{\rho_{ik}(d^*)}}{n_R}, \quad i = 1, \ldots, I. \tag{21}$$

Definition 5.1: Importance of a Supplier for the Whole Competitive Supply Chain Network G

The importance of a supplier j, corresponding to a supplier node $j \in G$, $I(j)$, for the whole competitive supply chain network, is measured by the relative supply chain network efficiency drop after j is removed from the whole supply chain:

$$I(j) = \frac{\triangle \mathcal{E}}{\mathcal{E}} = \frac{\mathcal{E}(G) - \mathcal{E}(G - j)}{\mathcal{E}(G)}, \quad j = 1, \ldots, n_S, \tag{22}$$

where $G - j$ is the resulting supply chain after supplier j is removed from the competitive supply chain network G.

We also can construct using an adaptation of (22) a robustness-type measure for the whole competitive supply chain by evaluating how the supply chain is impacted if all the suppliers are eliminated due to a major disruption. Specifically, we let:

$$I(\sum_{j=1}^{n_S} j) = \frac{\triangle \mathcal{E}}{\mathcal{E}} = \frac{\mathcal{E}(G) - \mathcal{E}(G - \sum_{j=1}^{n_S} j)}{\mathcal{E}(G)}, \tag{23}$$

measure how the whole supply chain can respond if all of its suppliers are unavailable.

Definition 5.2: Importance of a Supplier for the Supply Chain Network of an Individual Firm under Competition

The importance of a supplier j, corresponding to a supplier node $j \in G_i$, $I_i(j)$, for the supply chain network of a given firm i under competition, is measured by the relative supply chain network efficiency drop after j is removed from G_i:

$$I_i(j) = \frac{\triangle \mathcal{E}_i}{\mathcal{E}_i} = \frac{\mathcal{E}_i(G_i) - \mathcal{E}_i(G_i - j)}{\mathcal{E}_i(G_i)}, \quad i = 1, \ldots, I; j = 1, \ldots, n_S. \tag{24}$$

The corresponding robustness measure for the supply chain of firm i if all the suppliers are eliminated is:

$$I_i(\sum_{j=1}^{n_S} j) = \frac{\triangle \mathcal{E}_i}{\mathcal{E}_i} = \frac{\mathcal{E}_i(G_i) - \mathcal{E}_i(G_i - \sum_{j=1}^{n_S} j)}{\mathcal{E}_i(G_i)}, \quad i = 1, \ldots, I. \tag{25}$$

Definition 5.3: Importance of a Supplier's Component for the Whole Competitive Supply Chain Network G

The importance of a supplier j's component l_j; $l_j = 1_j, \ldots, n_{l_j}$, corresponding to j's component node $l_j \in G$, $I(l_j)$, for the whole competitive supply chain network, is measured by the relative supply chain network efficiency drop after l_j is removed from G:

$$I(l_j) = \frac{\triangle \mathcal{E}}{\mathcal{E}} = \frac{\mathcal{E}(G) - \mathcal{E}(G - l_j)}{\mathcal{E}(G)}, \quad j = 1, \ldots, n_S; l_j = 1_j, \ldots, n_{l_j}. \qquad (26)$$

where $G - l_j$ is the resulting supply chain after supplier j's component l_j is removed from the whole competitive supply chain network.

The corresponding robustness measure for the whole competitive supply chain network if all suppliers' component l_j; $l_j = 1_j, \ldots, n_{l_j}$, are eliminated is:

$$I(\sum_{j=1}^{n_S} l_j) = \frac{\triangle \mathcal{E}}{\mathcal{E}} = \frac{\mathcal{E}(G) - \mathcal{E}(G - \sum_{j=1}^{n_S} l_j)}{\mathcal{E}(G)}, \quad l_j = 1_j, \ldots, n_{l_j}. \qquad (27)$$

Definition 5.4: Importance of a Supplier's Component for the Supply Chain Network of an Individual Firm under Competition

The importance of supplier j's component l_j; $l_j = 1_j, \ldots, n_{l_j}$, corresponding to a component node $l_j \in G_i$, $I_i(l_j)$, for the supply chain network of a given firm i under competition, is measured by the relative supply chain network efficiency drop after l_j is removed from G_i:

$$I_i(l_j) = \frac{\triangle \mathcal{E}_i}{\mathcal{E}_i} = \frac{\mathcal{E}_i(G_i) - \mathcal{E}_i(G_i - l_j)}{\mathcal{E}_i(G_i)}, \quad i = 1, \ldots, I; j = 1, \ldots, n_S; l_j = 1_j, \ldots, n_{l_j}. \tag{28}$$

The corresponding robustness measure for the supply chain network of firm i if all suppliers' component l_j, $l_j = 1_j, \ldots, n_{l_j}$, are eliminated is:

$$I_i(\sum_{j=1}^{n_S} l_j) = \frac{\triangle \mathcal{E}_i}{\mathcal{E}_i} = \frac{\mathcal{E}_i(G_i) - \mathcal{E}_i(G_i - \sum_{j=1}^{n_S} l_j)}{\mathcal{E}_i(G_i)}, \quad i = 1, \ldots, I; l_j = 1_j, \ldots, n_{l_j}. \tag{29}$$

The Algorithm - The Euler Method

Iteration τ of the Euler method

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - a_{\tau} F(X^{\tau})), \tag{30}$$

where $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$ and F is the function that enters the variational inequality problem (17).

For convergence of the general iterative scheme, which induces the Euler method, the sequence $\{a_{\tau}\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_{\tau} = \infty$, $a_{\tau} > 0$, $a_{\tau} \to 0$, as $\tau \to \infty$.

$$Q_{ik}^{\tau+1} = \max\{0, Q_{ik}^{\tau} + a_{\tau}(-\frac{\partial f_i(Q^{\tau})}{\partial Q_{ik}} - \sum_{h=1}^{n_R} \frac{\partial tc_{ih}^F(Q^{\tau})}{\partial Q_{ik}} + \sum_{h=1}^{n_R} \frac{\partial \hat{\rho}_{ih}(Q^{\tau})}{\partial Q_{ik}} Q_{ih}^{\tau} + \hat{\rho}_{ik}(Q^{\tau})$$

$$- \sum_{l=1}^{n_{l^i}} \lambda_{il}^{\tau} \theta_{il})\}; i = 1, \ldots, I; k = 1, \ldots, n_R. \quad (31a)$$

$$Q_{il}^{F^{\tau+1}} = \min\{CAP_{il}^F, \max\{0, Q_{il}^{F^{\tau}} + a_{\tau}(-\sum_{m=1}^{n_{l^i}} \frac{\partial f_{im}^F(Q^{F^{\tau}})}{\partial Q_{il}^F} + \lambda_{il}^{\tau})\}\};$$

$$i = 1, \ldots, I; l = 1, \ldots, n_{l^i}. \quad (31b)$$

$$Q_{jil}^{S^{\tau+1}} = \min\{CAP_{jil}^S, \max\{0, Q_{jil}^{S^{\tau}} + a_{\tau}(-\pi_{jil}^{\tau} - \sum_{g=1}^{n_S} \sum_{m=1}^{n_{l^i}} \frac{\partial c_{igm}(Q^{S^{\tau}})}{\partial Q_{jil}^S} + \lambda_{il}^{\tau})\}\};$$

$$j = 1, \ldots, n_S; i = 1, \ldots, I; l = 1, \ldots, n_{l^i}. \quad (31c)$$

$$\lambda_{il}^{\tau+1} = \max\{0, \lambda_{il}^{\tau} + a_\tau(-\sum_{j=1}^{n_S} Q_{jil}^{S^\tau} - Q_{il}^{F^\tau} + \sum_{k=1}^{n_R} Q_{ik}^{\tau}\theta_{il})\}; i = 1, \ldots, I; l = 1, \ldots, n_{l^i}. \quad (31d)$$

$$\pi_{jil}^{\tau+1} = \max\{0, \pi_{jil}^{\tau} + a_\tau(-\frac{\partial oc_j(\pi^\tau)}{\partial \pi_{jil}} + Q_{jil}^{S^\tau})\}; j = 1, \ldots, n_S; i = 1, \ldots, I; l = 1, \ldots, n_{l^i}. \quad (31e)$$

Numerical Examples

We implemented the Euler method using Matlab on a Lenovo Z580. The convergence tolerance is 10^{-6}, so that the algorithm is deemed to have converged when the absolute value of the difference between each successive quantities, prices, and Lagrange multipliers is less than or equal to 10^{-6}. The sequence $\{a_\tau\}$ is set to: $\{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \ldots\}$. We initialize the algorithm by setting the product and component quantities equal to 50 and the prices and the Lagrange multipliers equal to 0.

Numerical Examples - Example 1

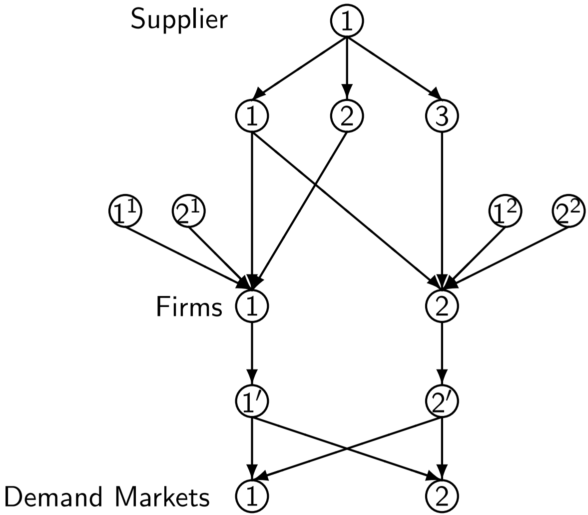


Figure: Example 1

Numerical Examples - Example 1

The capacities of the suppliers are:

$$CAP^S_{111} = 80, \quad CAP^S_{112} = 90, \quad CAP^S_{121} = 80, \quad CAP^S_{122} = 50,$$

The firms are not capable of producing components 1^1 or 1^2, so their capacities are:

$$CAP^F_{11} = 0, \quad CAP^F_{12} = 20, \quad CAP^F_{21} = 0, \quad CAP^F_{22} = 30.$$

The supplier's production costs are:

$$f^S_{11}(Q^S_{111}, Q^S_{121}) = 2(Q^S_{111} + Q^S_{121}), \quad f^S_{12}(Q^S_{112}) = 3Q^S_{112}, \quad f^S_{13}(Q^S_{122}) = Q^S_{122}.$$

The supplier's transportation costs are:

$$tc^S_{111}(Q^S_{111}, Q^S_{112}) = 0.75Q^S_{111} + 0.1Q^S_{112}, \quad tc^S_{112}(Q^S_{112}, Q^S_{111}) = 0.1Q^S_{112} + 0.05Q^S_{111},$$

$$tc^S_{121}(Q^S_{121}, Q^S_{122}) = Q^S_{121} + 0.2Q^S_{122}, \quad tc^S_{122}(Q^S_{122}, Q^S_{121}) = 0.6Q^S_{122} + 0.25Q^S_{121}.$$

The opportunity cost of the supplier is:

$$oc_1(\pi_{111}, \pi_{112}, \pi_{121}, \pi_{122}) = 0.5(\pi_{111}-10)^2+(\pi_{112}-5)^2+0.5(\pi_{121}-10)^2+0.75(\pi_{122}-7)^2.$$

Numerical Examples - Example 1

The firms' assembly costs are:

$$f_1(Q_{11}, Q_{12}, Q_{21}, Q_{22}) = 2(Q_{11} + Q_{12})^2 + 2(Q_{11} + Q_{12}) + (Q_{11} + Q_{12})(Q_{21} + Q_{22}),$$

$$f_2(Q_{11}, Q_{12}, Q_{21}, Q_{22}) = 1.5(Q_{21} + Q_{22})^2 + 2(Q_{21} + Q_{22}) + (Q_{11} + Q_{12})(Q_{21} + Q_{22}).$$

The firms' production costs for producing their components are:

$$f_{11}^F(Q_{11}^F, Q_{21}^F) = 3Q_{11}^{F^2} + Q_{11}^F + 0.5Q_{11}^F Q_{21}^F, \quad f_{12}^F(Q_{12}^F) = 2Q_{12}^{F^2} + 1.5Q_{12}^F,$$

$$f_{21}^F(Q_{11}^F, Q_{21}^F) = 3Q_{21}^{F^2} + 2Q_{21}^F + 0.75Q_{11}^F Q_{21}^F, \quad f_{22}^F(Q_{22}^F) = 1.5Q_{22}^{F^2} + Q_{22}^F.$$

The firms' transportation costs for shipping their products to the demand markets are:

$$tc_{11}^F(Q_{11}, Q_{21}) = Q_{11}^2 + Q_{11} + 0.5Q_{11}Q_{21}, \quad tc_{12}^F(Q_{12}, Q_{22}) = 2Q_{12}^2 + Q_{12} + 0.5Q_{12}Q_{22},$$

$$tc_{21}^F(Q_{21}, Q_{11}) = 1.5Q_{21}^2 + Q_{21} + 0.25Q_{11}Q_{21}, \quad tc_{22}^F(Q_{12}, Q_{22}) = Q_{22}^2 + 0.5Q_{22} + 0.25Q_{12}Q_{22}.$$

Numerical Examples - Example 1

The transaction costs of the firms are:

$$c_{111}(Q^S_{111}) = 0.5 Q^{S^2}_{111} + 0.25 Q^S_{111}, \quad c_{112}(Q^S_{112}) = 0.25 Q^{S^2}_{112} + 0.3 Q^S_{112},$$

$$c_{211}(Q^S_{121}) = 0.3 Q^{S^2}_{121} + 0.2 Q^S_{121}, \quad c_{212}(Q^S_{122}) = 0.2 Q^{S^2}_{122} + 0.1 Q^S_{122}.$$

The demand price functions are:

$$\rho_{11}(d_{11}, d_{21}) = -1.5 d_{11} - d_{21} + 500, \quad \rho_{12}(d_{12}, d_{22}) = -2 d_{12} - d_{22} + 450,$$

$$\rho_{21}(d_{11}, d_{21}) = -2 d_{21} - 0.5 d_{11} + 500, \quad \rho_{22}(d_{12}, d_{22}) = -2 d_{22} - d_{12} + 400.$$

Numerical Examples - Example 1

The Euler method converges in 380 iterations.

$$Q_{11}^* = 13.39, \quad Q_{12}^* = 4.51, \quad Q_{21}^* = 18.62, \quad Q_{22}^* = 5.87.$$

$$d_{11}^* = 13.39, \quad d_{12}^* = 4.51, \quad d_{21}^* = 18.62, \quad d_{22}^* = 5.87.$$

$$\rho_{11} = 461.30, \quad \rho_{12} = 435.11, \quad \rho_{21} = 456.07, \quad \rho_{22} = 383.75.$$

$$Q_{11}^{F^*} = 0.00, \quad Q_{12}^{F^*} = 11.50, \quad Q_{21}^{F^*} = 0.00, \quad Q_{22}^{F^*} = 14.35.$$

$$Q_{111}^{S^*} = 35.78, \quad Q_{112}^{S^*} = 42.18, \quad Q_{121}^{S^*} = 48.99, \quad Q_{122}^{S^*} = 34.64.$$

$$\lambda_{11}^* = 81.82, \quad \lambda_{12}^* = 47.48, \quad \lambda_{21}^* = 88.58 \quad \lambda_{22}^* = 44.05.$$

$$\pi_{11}^* = 45.78, \quad \pi_{12}^* = 26.09, \quad \pi_{21}^* = 58.99, \quad \pi_{22}^* = 30.09.$$

The profits of the firms are, respectively, 2,518.77 and 3,485.51. The profit of the supplier is 3,529.19.

Numerical Examples - Example 1 - Performance Measures

Table: Supply Chain Network Performance Measure values for Example 1

	$\mathcal{E}(G)$	$\mathcal{E}(G-1)$	$\mathcal{E}(G-1_1)$	$\mathcal{E}(G-2_1)$	$\mathcal{E}(G-3_1)$
Whole Supply Chain	0.0239	0	0	0.0181	0.0183

	$\mathcal{E}_i(G_i)$	$\mathcal{E}_i(G_i-1)$	$\mathcal{E}_i(G_i-1_1)$	$\mathcal{E}_i(G_i-2_1)$	$\mathcal{E}_i(G_i-3_1)$
Firm 1's Supply Chain	0.0197	0	0	0.0071	0.0203
Firm 2's Supply Chain	0.0281	0	0	0.0292	0.0163

Numerical Examples - Example 1 - Importance Measures

Table: Importance and Rankings of Supplier 1's Components 1, 2, and 3 for Example 1

	Importance for the Whole Supply Chain	Ranking
Supplier 1	1	
Component 1	1	1
Component 2	0.2412	2
Component 3	0.2331	3

	Importance for Firm 1's Supply Chain	Ranking	Importance for Firm 2's Supply Chain	Ranking
Supplier 1	1		1	
Component 1	1	1	1	1
Component 2	0.6401	2	−0.0387	3
Component 3	−0.0329	3	0.4197	2

	Importance for the Whole Supply Chain	Importance for Firm 1's Supply Chain	Importance for Firm 2's Supply Chain
Supplier 1	1	1	1
Ranking	1	1	1
Component 1	1	1	1
Ranking	1	1	1
Component 2	0.2412	0.6401	−0.0387
Ranking	2	1	3
Component 3	0.2331	−0.0329	0.4197
Ranking	2	3	1

Numerical Examples - Example 2

Example 2 is the same as Example 1 except that supplier 1 is no longer the only entity that can produce components 1^1 and 1^2.

The capacities of the firms are now:

$$CAP^F_{11} = 20, \quad CAP^F_{12} = 20, \quad CAP^F_{21} = 20, \quad CAP^F_{22} = 30.$$

Table: Equilibrium Solution and Incurred Demand Prices for Example 2

Q^*	$Q^*_{11} = 14.43$	$Q^*_{121} = 5.13$	$Q^*_{21} = 19.60$	$Q^*_{22} = 7.02$
Q^{F^*}	$Q^{F^*}_{11} = 10.23$	$Q^{F^*}_{12} = 12.50$	$Q^{F^*}_{21} = 11.28$	$Q^{F^*}_{22} = 15.47$
Q^{S^*}	$Q^{S^*}_{111} = 28.89$	$Q^{S^*}_{112} = 46.19$	$Q^{S^*}_{121} = 41.97$	$Q^{S^*}_{122} = 37.78$
λ^*	$\lambda^*_{11} = 68.04$	$\lambda^*_{12} = 51.49$	$\lambda^*_{21} = 77.35$	$\lambda^*_{22} = 47.40$
π^*	$\pi^*_{111} = 38.89$	$\pi^*_{112} = 28.10$	$\pi^*_{121} = 51.97$	$\pi^*_{122} = 32.19$
d^*	$d^*_{11} = 14.43$	$d^*_{12} = 5.13$	$d^*_{21} = 19.60$	$d^*_{22} = 7.02$
ρ	$\rho_{11} = 458.75$	$\rho_{12} = 432.72$	$\rho_{21} = 453.58$	$\rho_{22} = 380.83$

The profits of the firms are now 2,968.88 and 4,110.89, and the profit of the supplier is now 3,078.45.

Numerical Examples - Example 2 - Performance Measures

Table: Supply Chain Network Performance Measure Values for Example 2

	$\mathcal{E}(G)$	$\mathcal{E}(G-1)$	$\mathcal{E}(G-1_1)$	$\mathcal{E}(G-2_1)$	$\mathcal{E}(G-3_1)$
Whole Supply Chain	0.0262	0.0086	0.0105	0.0197	0.0195
	$\mathcal{E}_i(G_i)$	$\mathcal{E}_i(G_i-1)$	$\mathcal{E}_i(G_i-1_1)$	$\mathcal{E}_i(G_i-2_1)$	$\mathcal{E}_i(G_i-3_1)$
Firm 1's Supply Chain	0.0217	0.0067	0.0106	0.0071	0.0226
Firm 2's Supply Chain	0.0308	0.0105	0.0105	0.0324	0.0163

Numerical Examples - Example 2 - Importance Measures

Table: Importance and Rankings of Supplier 1 and its Components 1, 2, and 3 for Example 2

	Importance for the Whole Supply Chain	Ranking
Supplier 1	0.6721	
Component 1	0.5984	1
Component 2	0.2476	3
Component 3	0.2586	2

	Importance for Firm 1's Supply Chain	Ranking	Importance for Firm 2's Supply Chain	Ranking
Supplier 1	0.6897		0.6598	
Component 1	0.5121	2	0.6590	1
Component 2	0.6721	1	−0.0505	3
Component 3	−0.0438	3	0.4710	2

	Importance for the Whole Supply Chain	Importance for Firm 1's Supply Chain	Importance for Firm 2's Supply Chain
Supplier 1	0.6721	0.6897	0.6598
Ranking	2	1	3
Component 1	0.5984	0.5121	0.6590
Ranking	2	3	1
Component 2	0.2476	0.6721	−0.0505
Ranking	2	1	3
Component 3	0.2586	−0.0438	0.4710
Ranking	2	3	1

Example 3 is the same as Example 2, except that two more suppliers are now available to the firms in addition to supplier 1.

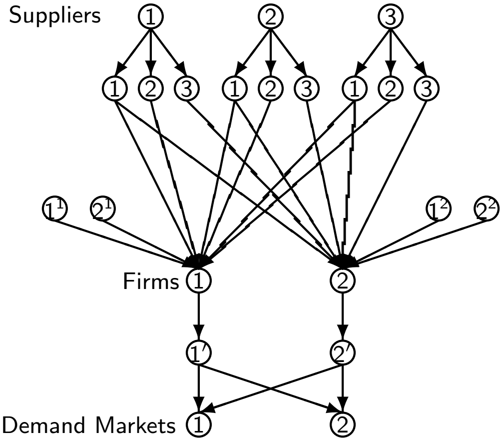


Figure: Example 3

Numerical Examples - Example 3

The data associated with suppliers 2 and 3 are following.

The capacities of suppliers 2 and 3 are:

$$CAP^S_{211} = 60, \quad CAP^S_{212} = 70, \quad CAP^S_{221} = 50, \quad CAP^S_{222} = 60,$$

$$CAP^S_{311} = 50, \quad CAP^S_{312} = 80, \quad CAP^S_{321} = 80, \quad CAP^S_{322} = 60.$$

The production costs of the suppliers are:

$$f^S_{21}(Q^S_{211}, Q^S_{221}) = Q^S_{211} + Q^S_{221}, \quad f^S_{22}(Q^S_{212}) = 3Q^S_{212}, \quad f^S_{23}(Q^S_{222}) = 2Q^S_{222},$$

$$f^S_{31}(Q^S_{311}, Q^S_{321}) = 10(Q^S_{311} + Q^S_{321}), \quad f^S_{32}(Q^S_{312}) = Q^S_{312}, \quad f^S_{33}(Q^S_{322}) = 2.5Q^S_{322}.$$

The transportation costs are:

$$tc^S_{211}(Q^S_{211}, Q^S_{212}) = 0.5Q^S_{211} + 0.2Q^S_{212}, \quad tc^S_{212}(Q^S_{212}, Q^S_{211}) = 0.3Q^S_{212} + 0.1Q^S_{211},$$

$$tc^S_{221}(Q^S_{221}, Q^S_{222}) = 0.8Q^S_{221} + 0.2Q^S_{222}, \quad tc^S_{222}(Q^S_{222}, Q^S_{221}) = 0.75Q^S_{222} + 0.1Q^S_{221},$$

$$tc^S_{311}(Q^S_{311}, Q^S_{312}) = 0.4Q^S_{311} + 0.05Q^S_{312}, \quad tc^S_{312}(Q^S_{312}, Q^S_{311}) = 0.4Q^S_{312} + 0.2Q^S_{311},$$

$$tc^S_{321}(Q^S_{321}, Q^S_{322}) = 0.7Q^S_{321} + 0.1Q^S_{322}, \quad tc^S_{322}(Q^S_{322}, Q^S_{321}) = 0.6Q^S_{322} + 0.1Q^S_{321}.$$

Numerical Examples - Example 3

The opportunity costs are:

$$oc_2(\pi_{211}, \pi_{212}, \pi_{221}, \pi_{222}) = (\pi_{211}-6)^2+0.75(\pi_{212}-5)^2+0.3(\pi_{221}-8)^2+0.5(\pi_{222}-4)^2,$$

$$oc_3(\pi_{311}, \pi_{312}, \pi_{321}, \pi_{322}) = 0.5(\pi_{311}-5)^2+1.5(\pi_{312}-5)^2+0.5(\pi_{321}-3)^2+0.5(\pi_{322}-4)^2.$$

The transaction costs of the firms now become:

$$c_{121}(Q^S_{211}) = 0.5Q^{S^2}_{211} + Q^S_{211}, \quad c_{122}(Q^S_{212}) = 0.25Q^{S^2}_{212} + 0.3Q^S_{212},$$

$$c_{221}(Q^S_{221}) = Q^{S^2}_{221} + 0.1Q^S_{221}, \quad c_{222}(Q^S_{222}) = Q^{S^2}_{222} + 0.5Q^S_{222},$$

$$c_{131}(Q^S_{311}) = 0.2Q^{S^2}_{311} + 0.3Q^S_{311}, \quad c_{132}(Q^S_{312}) = 0.5Q^{S^2}_{312} + 0.2Q^S_{312},$$

$$c_{231}(Q^S_{321}) = 0.1Q^{S^2}_{321} + 0.1Q^S_{321}, \quad c_{232}(Q^S_{322}) = 0.5Q^{S^2}_{322} + 0.1Q^S_{322}.$$

Numerical Examples - Example 3

The Euler method converges in 563 iterations.

Table: Equilibrium Solution and Incurred Demand Prices for Example 3

Q^*	$Q^*_{11} = 21.82$	$Q^*_{12} = 9.61$	$Q^*_{21} = 24.23$	$Q^*_{22} = 12.41$
Q^{F^*}	$Q^{F^*}_{11} = 5.57$	$Q^{F^*}_{12} = 9.11$	$Q^{F^*}_{21} = 6.48$	$Q^{F^*}_{22} = 12.94$
Q^{S^*}	$Q^{S^*}_{111} = 13.71$	$Q^{S^*}_{112} = 32.64$	$Q^{S^*}_{121} = 21.77$	$Q^{S^*}_{122} = 30.68$
	$Q^{S^*}_{211} = 20.45$	$Q^{S^*}_{212} = 27.98$	$Q^{S^*}_{221} = 10.07$	$Q^{S^*}_{222} = 11.78$
	$Q^{S^*}_{311} = 23.13$	$Q^{S^*}_{312} = 24.56$	$Q^{S^*}_{321} = 34.94$	$Q^{S^*}_{322} = 17.86$
λ^*	$\lambda^*_{11} = 37.68$	$\lambda^*_{12} = 37.94$	$\lambda^*_{21} = 45.03$	$\lambda^*_{22} = 39.83$
π^*	$\pi^*_{111} = 23.71$	$\pi^*_{112} = 21.32$	$\pi^*_{121} = 31.77$	$\pi^*_{122} = 27.45$
	$\pi^*_{211} = 16.23$	$\pi^*_{212} = 23.65$	$\pi^*_{221} = 24.79$	$\pi^*_{222} = 15.78$
	$\pi^*_{311} = 28.13$	$\pi^*_{312} = 13.19$	$\pi^*_{321} = 37.94$	$\pi^*_{322} = 21.86$
d^*	$d^*_{11} = 21.82$	$d^*_{12} = 9.61$	$d^*_{21} = 24.23$	$d^*_{22} = 12.41$
ρ	$\rho_{11} = 443.04$	$\rho_{12} = 418.38$	$\rho_{21} = 440.64$	$\rho_{22} = 365.58$

The profits of the firms are now 4,968.67 and 5,758.13, and the profits of the suppliers are 1,375.22, 725.17, and 837.44, respectively.

Numerical Examples - Example 3 - Performance Measures

Table: Supply Chain Network Performance Measure Values for Example 3

	$\mathcal{E}(G)$	$\mathcal{E}(G-1)$	$\mathcal{E}(G-2)$	$\mathcal{E}(G-3)$	$\mathcal{E}(G-\sum_{j=1}^{nS} j)$
Whole Supply Chain	0.0403	0.0334	0.0361	0.0332	0.0086

	$\mathcal{E}_i(G_i)$	$\mathcal{E}_i(G_i-1)$	$\mathcal{E}_i(G_i-2)$	$\mathcal{E}_i(G_i-3)$	$\mathcal{E}_i(G_i-\sum_{j=1}^{nS} j)$
Firm 1's Supply Chain	0.0361	0.0309	0.0303	0.0309	0.0067
Firm 2's Supply Chain	0.0445	0.0358	0.0419	0.0355	0.0105

Table: Importance and Rankings of Suppliers for Example 3

	Importance for the Whole Supply Chain	Ranking
Supplier 1	0.1717	2
Supplier 2	0.1035	3
Supplier 3	0.1760	1
All Suppliers	0.7864	

	Importance for Firm 1's Supply Chain	Ranking	Importance for Firm 2's Supply Chain	Ranking
Supplier 1	0.1443	2	0.1939	2
Supplier 2	0.1612	1	0.0566	3
Supplier 3	0.1438	3	0.2021	1
All Suppliers	0.8139		0.7641	

	Importance for the Whole Supply Chain	Importance for Firm 1's Supply Chain	Importance for Firm 2's Supply Chain
Supplier 1	0.1717	0.1443	0.1939
Ranking	2	3	1
Supplier 2	0.1035	0.1612	0.0566
Ranking	2	1	3
Supplier 3	0.1760	0.1438	0.2021
Ranking	2	3	1
All Suppliers	0.7864	0.8139	0.7641
Ranking	2	1	3

Summary and Conclusions

- The behaviors of both suppliers and firms are captured in order to be able to assess both supply chain network performance as well as vulnerabilities.
- The firms have the option of producing the components needed in-house.
- A unified variational inequality is constructed, whose solution yields the equilibrium quantities of the components, produced in-house and/or contracted for, the quantities of the final products, the prices charged by the suppliers, as well as the Lagrange multipliers.
- The model is used for the introduction of supply chain network performance measures for the entire supply chain network economy consisting of all the firms as well as for that of an individual firm.
- Importance indicators are constructed that allow for the ranking of suppliers for the whole supply chain or that of an individual firm, as well as for the supplier components.

Thank you!

- For more information, please visit http://supernet.isenberg.umass.edu.