Pharmaceutical Supply Chain Networks with Outsourcing Under Price and Quality Competition

Anna Nagurney[1], Dong (Michelle) Li[1], and Ladimer S. Nagurney[2]

[1]Isenberg School of Management
University of Massachusetts, Amherst, Massachusetts 01003

[2]Department of Electrical and Computer Engineering
University of Hartford, West Hartford, CT 06117

INFORMS Annual Meeting, Minneapolis, MN
Oct 6-9, 2013

Acknowledgments

This research was supported, in part, by the National Science Foundation (NSF) grant CISE #1111276, for the NeTS: Large: Collaborative Research: Network Innovation Through Choice project awarded to the University of Massachusetts Amherst. This support is gratefully acknowledged.

This presentation is based on the paper:

Nagurney, A., Li, D., and Nagurney, L., 2013. Pharmaceutical Supply Chain Networks with Outsourcing Under Price and Quality Competition, in press in the *International Transactions in Operational Research*,

where a full list of references can be found.

Outline

- Background and Motivation
- The Pharmaceutical Supply Chain Network Model with Outsourcing Under Price and Quality Competition
- The Underlying Dynamics
- The Algorithm
- Numerical Examples
- Summary and Conclusions

Background and Motivation

Outsourcing is defined as the behavior of moving some of a firm's responsibilities and/or internal processes, such as product design or manufacturing, to a third party company (Chase, Jacobs, and Aquilano (2004)).


Background and Motivation

There is, currently, a tremendous shift from in-house manufacturing towards outsourcing in US pharmaceutical companies.

- The US market for outsourced pharmaceutical manufacturing is expanding at the rate of 10% to 12% annually (Olson and Wu (2011)).
- Up to 40% of the drugs that Americans take are imported, and more than 80% of the active ingredients for drugs sold in the United States are outsourced (Economy In Crisis (2010)).
- Pharmaceutical companies are increasingly farming out activities other than production, such as research and development activities (Higgins and Rodriguez (2006)).

Background and Motivation

There is, currently, a tremendous shift from in-house manufacturing towards outsourcing in US pharmaceutical companies.

- The US market for outsourced pharmaceutical manufacturing is expanding at the rate of 10% to 12% annually (Olson and Wu (2011)).
- Up to 40% of the drugs that Americans take are imported, and more than 80% of the active ingredients for drugs sold in the United States are outsourced (Economy In Crisis (2010)).
- Pharmaceutical companies are increasingly farming out activities other than production, such as research and development activities (Higgins and Rodriguez (2006)).

Background and Motivation

There is, currently, a tremendous shift from in-house manufacturing towards outsourcing in US pharmaceutical companies.

- The US market for outsourced pharmaceutical manufacturing is expanding at the rate of 10% to 12% annually (Olson and Wu (2011)).
- Up to 40% of the drugs that Americans take are imported, and more than 80% of the active ingredients for drugs sold in the United States are outsourced (Economy In Crisis (2010)).
- Pharmaceutical companies are increasingly farming out activities other than production, such as research and development activities (Higgins and Rodriguez (2006)).

Background and Motivation

Quality issues in outsourced products must be of paramount concern.

- In 2006, outsourced cold medicine contained a toxic substance used in antifreeze, which can cause death (Bogdanich and Hooker (2007)).
- In 2008, fake heparin made by one Chinese manufacturer not only led to recalls of drugs in over ten European countries (Payne (2008)), but also caused the deaths of 81 American citizens (The New York Times (2011)).

Quality issues in outsourced products must be of paramount concern.

- In 2006, outsourced cold medicine contained a toxic substance used in antifreeze, which can cause death (Bogdanich and Hooker (2007)).
- In 2008, fake heparin made by one Chinese manufacturer not only led to recalls of drugs in over ten European countries (Payne (2008)), but also caused the deaths of 81 American citizens (The New York Times (2011)).

Background and Motivation

A more comprehensive supply chain network model that captures

- contractor selection,
- the minimization of the disrepute of the pharmaceutical firm, as well as
- the competition among contractors,

is an imperative.

Literature Review

Related literature

- Balakrishnan, K., Mohan, U., Seshadri, S., 2008. Outsourcing of front-end business processes: Quality, information, and customer contact. *Journal of Operations Management* 26 (6), 288-302.
- Kaya, M., Özer, Ö., 2009. Quality risk in outsourcing: Noncontractible product quality and private quality cost information. *Naval Research Logistics* 56, 669-685.
- Kaya, O., 2011. Outsourcing vs. in-house production: A comparison of supply chain contracts with effort dependent demand. *Omega* 39, 168-178.

Background and Motivation

We develop a pharmaceutical supply chain network model which takes into account the quality concerns in the context of global outsourcing.

- This model captures the behaviors of the pharmaceutical firm and its potential contractors.
- The contractors compete by determining the prices that they charge the pharmaceutical firm for manufacturing and delivering the product to the demand markets and the quality levels of the products to maximize its total profit.
- The pharmaceutical firm seeks to minimize its total cost, which includes its weighted disrepute cost, which is influenced by the quality of the product produced by its contractors and the amount of product that is outsourced.

Background and Motivation

We develop a pharmaceutical supply chain network model which takes into account the quality concerns in the context of global outsourcing.

- This model captures the behaviors of the pharmaceutical firm and its potential contractors.
- The contractors compete by determining the prices that they charge the pharmaceutical firm for manufacturing and delivering the product to the demand markets and the quality levels of the products to maximize its total profit.
- The pharmaceutical firm seeks to minimize its total cost, which includes its weighted disrepute cost, which is influenced by the quality of the product produced by its contractors and the amount of product that is outsourced.

Background and Motivation

We develop a pharmaceutical supply chain network model which takes into account the quality concerns in the context of global outsourcing.

- This model captures the behaviors of the pharmaceutical firm and its potential contractors.
- The contractors compete by determining the prices that they charge the pharmaceutical firm for manufacturing and delivering the product to the demand markets and the quality levels of the products to maximize its total profit.
- The pharmaceutical firm seeks to minimize its total cost, which includes its weighted disrepute cost, which is influenced by the quality of the product produced by its contractors and the amount of product that is outsourced.

Definitions

Quality levels and quality cost are quantified and incorporated.

- Quality level is defined as the "quality conformance level", the degree to which a specific product conforms to a design or specification (Juran and Gryna (1988)), and it should be within 0 and 100 percent of defects levels.
- Quality costs are defined as "costs incurred in ensuring and assuring quality as well as the loss incurred when quality is not achieved" (ASQC (1971) and BS (1990)), which is widely believed to be a convex function of the quality conformance level (see, e.g., Juran and Gryna (1988), Campanella (1990), Feigenhaum (1983), Porter and Rayner (1992), and Shank and Govindarajan (1994)).

Definitions

Quality levels and quality cost are quantified and incorporated.

- Quality level is defined as the "quality conformance level", the degree to which a specific product conforms to a design or specification (Juran and Gryna (1988)), and it should be within 0 and 100 percent of defects levels.
- Quality costs are defined as "costs incurred in ensuring and assuring quality as well as the loss incurred when quality is not achieved" (ASQC (1971) and BS (1990)), which is widely believed to be a convex function of the quality conformance level (see, e.g., Juran and Gryna (1988), Campanella (1990), Feigenhaum (1983), Porter and Rayner (1992), and Shank and Govindarajan (1994)).

Definitions

The external failure cost, which is described as following, is defined and incorporated as the cost of disrepute for the original firm in this work.

- External failure cost, which is one category of quality related cost, is the compensation cost incurred when customers are unsatisfied with the quality of the products.(see, e.g., Crosby (1979), Harrington (1987), Plunkett and Dale (1988), Juran and Gryna (1993), and Rapley, Prickett, and Elliot (1999)).

Assumptions

The game theory supply chain network model developed in this paper is based on the following assumptions:

- The original pharmaceutical firm pays the transaction cost, which includes the costs of evaluating suppliers, negotiation costs, the monitoring and the enforcement of the contract in order to ensure the quality (see Picot (1991), Franceschini (2003), Heshmati (2003), and Liu and Nagurney (2011a)).

  The production/distribution costs and the quality cost information of the contractors are known by the firm through the transaction cost.

- In-house supply chain activities can ensure a 100% perfect quality conformance level (see Schneiderman (1986) and Kaya (2011)).

Assumptions

The game theory supply chain network model developed in this paper is based on the following assumptions:

- The original pharmaceutical firm pays the transaction cost, which includes the costs of evaluating suppliers, negotiation costs, the monitoring and the enforcement of the contract in order to ensure the quality (see Picot (1991), Franceschini (2003), Heshmati (2003), and Liu and Nagurney (2011a)).

  The production/distribution costs and the quality cost information of the contractors are known by the firm through the transaction cost.

- In-house supply chain activities can ensure a 100% perfect quality conformance level (see Schneiderman (1986) and Kaya (2011)).

We develop both a static version of the model (at the equilibrium state) and also a dynamic one using the theory of projected dynamical systems (cf. Nagurney and Zhang (1996)).

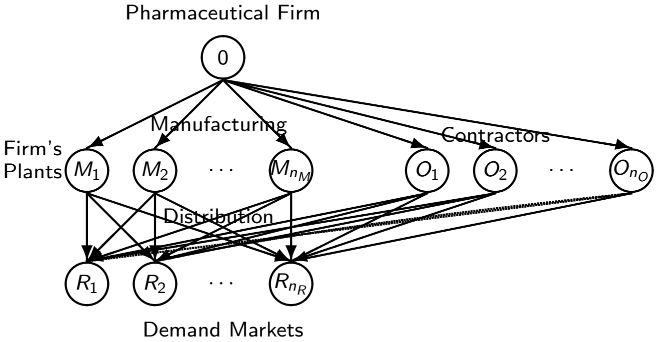

Figure: The Pharmaceutical Supply Chain Network Topology with Outsourcing

n: $n = n_M + n_O$, the number of manufacturing plants, whether in-house or belonging to the contractors'.

Q_{jk}: the nonnegative amount of pharmaceutical product produced at manufacturing plant j and delivered to demand market k. We group the $\{Q_{jk}\}$ elements into the vector $Q \in R_{+}^{nn_R}$.

d_k: the demand for the product at demand market k, assumed known and fixed.

π_{jk}: the price charged by contractor j for producing and delivering a unit of the product to k. We group the $\{\pi_{jk}\}$ elements for contractor j into the vector $\pi_j \in R_{+}^{n_R}$ and then group all such vectors for all the contractors into the vector $\pi \in R^{n_O n_R}$.

$dc(q\prime)$: the cost of disrepute, which corresponds to the external failure quality cost.

The Behavior of the Pharmaceutical Firm

We assume that in-house activities can ensure a 100% perfect quality conformance level.

Quality level of contractor j

$$0 \leq q_j \leq q^U, \quad j = 1, \ldots, n_O, \tag{1}$$

where q^U is the value representing perfect quality achieved by the pharmaceutical firm in its in-house manufacturing.

Average quality level of the pharmaceutical firm

$$q\prime = \frac{\sum_{j=n_M+1}^{n} \sum_{k=1}^{n_R} Q_{jk} q_{j-n_M} + (\sum_{j=1}^{n_M} \sum_{k=1}^{n_R} Q_{jk}) q^U}{\sum_{k=1}^{n_R} d_k}. \tag{2}$$

The Behavior of the Pharmaceutical Firm

The total utility maximization objective of the pharmaceutical firm

$$\text{Maximize}_Q \quad U_0(Q,q,\pi) = -\sum_{j=1}^{n_M} f_j(\sum_{k=1}^{n_R} Q_{jk}) - \sum_{j=1}^{n_M}\sum_{k=1}^{n_R} \hat{c}_{jk}(Q_{jk})$$

$$-\sum_{j=1}^{n_O}\sum_{k=1}^{n_R} \pi^*_{jk} Q_{n_M+j,k} - \sum_{j=1}^{n_O} tc_j(\sum_{k=1}^{n_R} Q_{n_M+j,k}) - \omega dc(q\prime). \quad (3)$$

subject to:

$$\sum_{j=1}^{n} Q_{jk} = d_k, \quad k = 1,\dots,n_R, \quad (4)$$

$$Q_{jk} \geq 0, \quad j = 1,\dots,n; k = 1,\dots,n_R, \quad (5)$$

with $q\prime$ in (3) as in (2).

We assume that all the cost functions in (3) are continuous, continuously differentiable, and convex.

Variational Inequality Formulation

Theorem 1

Determine $Q^* \in K^0$, *such that:*

$$-\sum_{h=1}^{n}\sum_{l=1}^{n_R}\frac{\partial U_0(Q^*, q^*, \pi^*)}{\partial Q_{hl}} \times (Q_{hl} - Q^*_{hl}) \geq 0, \quad \forall Q \in K^0, \qquad (6)$$

where $K^0 \equiv \{Q | Q \in R_+^{nn_R} \text{ with } (4) \text{ satisfied}\}$,

Variational Inequality Formulation

Theorem 1 (cont'd)

for $h = 1, \ldots, n_M$; $l = 1, \ldots, n_R$:

$$-\frac{\partial U_o}{\partial Q_{hl}} = \left[\frac{\partial f_h(\sum_{k=1}^{n_R} Q_{hk})}{\partial Q_{hl}} + \frac{\partial \hat{c}_{hl}(Q_{hl})}{\partial Q_{hl}} + \omega \frac{\partial dc(q\prime)}{\partial Q_{hl}}\right]$$

$$= \left[\frac{\partial f_h(\sum_{k=1}^{n_R} Q_{hk})}{\partial Q_{hl}} + \frac{\partial \hat{c}_{hl}(Q_{hl})}{\partial Q_{hl}} + \omega \frac{\partial dc(q\prime)}{\partial q\prime} \frac{q^U}{\sum_{k=1}^{n_R} d_k}\right],$$

and for $h = n_M + 1, \ldots, n$; $l = 1, \ldots, n_R$:

$$-\frac{\partial U_o}{\partial Q_{hl}} = \left[\pi_{h-n_M,l} + \frac{\partial tc_{h-n_M}(\sum_{k=1}^{n_R} Q_{hk})}{\partial Q_{hl}} + \omega \frac{\partial dc(q\prime)}{\partial Q_{hl}}\right]$$

$$= \left[\pi_{h-n_M,l} + \frac{\partial tc_{h-n_M}(\sum_{k=1}^{n_R} Q_{hk})}{\partial Q_{hl}} + \omega \frac{\partial dc(q\prime)}{\partial q\prime} \frac{q_h}{\sum_{k=1}^{n_R} d_k}\right].$$

The total utility maximization objective of contractor j

$$\text{Maximize}_{q_j,\pi_j} \quad U_j(Q,q,\pi)=\sum_{k=1}^{n_R}\pi_{jk}Q^*_{n_M+j,k}-\sum_{k=1}^{n_R}\hat{sc}_{jk}(Q^*,q)-\hat{qc}_j(q)$$

$$-\sum_{k=1}^{n_R}oc_{jk}(\pi_{jk}) \quad (7)$$

subject to:

$$\pi_{jk}\geq 0, \quad k=1,\ldots,n_R, \quad (8)$$

and (1) for each j.

We assume that the cost functions in each contractor's utility function are continuous, continuously differentiable, and convex.

Opportunity Cost

- "the loss of potential gain from other alternatives when one alternative is chosen" (New Oxford American Dictionary (2010)).
- Opportunity cost functions include both explicit and implicit costs (Mankiw (2011)).
- Nobel laureate Akerlof (1970): "The Market for Lemons: Quality Uncertainty and the Market Mechanism".
- It has been emphasized in pharmaceutical firm competition by Grabowski and Vernon (1990), Palmer and Raftery (1999), and Cockburn (2004).
- Gan and Litvinov (2003) also constructed opportunity cost functions that are functions of prices but in an energy application.

Opportunity Cost

- "the loss of potential gain from other alternatives when one alternative is chosen" (New Oxford American Dictionary (2010)).
- Opportunity cost functions include both explicit and implicit costs (Mankiw (2011)).
- Nobel laureate Akerlof (1970): "The Market for Lemons: Quality Uncertainty and the Market Mechanism".
- It has been emphasized in pharmaceutical firm competition by Grabowski and Vernon (1990), Palmer and Raftery (1999), and Cockburn (2004).
- Gan and Litvinov (2003) also constructed opportunity cost functions that are functions of prices but in an energy application.

Opportunity Cost

- "the loss of potential gain from other alternatives when one alternative is chosen" (New Oxford American Dictionary (2010)).
- Opportunity cost functions include both explicit and implicit costs (Mankiw (2011)).
- Nobel laureate Akerlof (1970): "The Market for Lemons: Quality Uncertainty and the Market Mechanism".
- It has been emphasized in pharmaceutical firm competition by Grabowski and Vernon (1990), Palmer and Raftery (1999), and Cockburn (2004).
- Gan and Litvinov (2003) also constructed opportunity cost functions that are functions of prices but in an energy application.

Opportunity Cost

- "the loss of potential gain from other alternatives when one alternative is chosen" (New Oxford American Dictionary (2010)).
- Opportunity cost functions include both explicit and implicit costs (Mankiw (2011)).
- Nobel laureate Akerlof (1970): "The Market for Lemons: Quality Uncertainty and the Market Mechanism".
- It has been emphasized in pharmaceutical firm competition by Grabowski and Vernon (1990), Palmer and Raftery (1999), and Cockburn (2004).
- Gan and Litvinov (2003) also constructed opportunity cost functions that are functions of prices but in an energy application.

Opportunity Cost

- "the loss of potential gain from other alternatives when one alternative is chosen" (New Oxford American Dictionary (2010)).
- Opportunity cost functions include both explicit and implicit costs (Mankiw (2011)).
- Nobel laureate Akerlof (1970): "The Market for Lemons: Quality Uncertainty and the Market Mechanism".
- It has been emphasized in pharmaceutical firm competition by Grabowski and Vernon (1990), Palmer and Raftery (1999), and Cockburn (2004).
- Gan and Litvinov (2003) also constructed opportunity cost functions that are functions of prices but in an energy application.

Definition: A Nash-Bertrand Equilibrium

Let K^j denote the feasible set according to contractor j, where $K^j \equiv \{(q_j, \pi_j) | \pi_j \text{ satisfies } (8) \text{ and } q_j \text{ satisfies } (1) \text{ for } j\}$, and $\mathcal{K}^1 \equiv \prod_{j=1}^{n_O} K^j$. They are closed and convex.

Definition 1

A quality level and price pattern $(q^*, \pi^*) \in \mathcal{K}^1$ is said to constitute a Bertrand-Nash equilibrium if for each contractor j; $j = 1, \ldots, n_O$

$$U_j(Q^*, q_j^*, \hat{q_j^*}, \pi_j^*, \hat{\pi_j^*}) \geq U_j(Q^*, q_j, \hat{q_j^*}, \pi_j, \hat{\pi_j^*}), \quad \forall (q_j, \pi_j) \in K^j, \tag{9}$$

where

$$\hat{q_j^*} \equiv (q_1^*, \ldots, q_{j-1}^*, q_{j+1}^*, \ldots, q_{n_O}^*), \tag{10}$$

$$\hat{\pi_j^*} \equiv (\pi_1^*, \ldots, \pi_{j-1}^*, \pi_{j+1}^*, \ldots, \pi_{n_O}^*). \tag{11}$$

Variational Inequality Formulation

Theorem 2

$(q^*, \pi^*) \in \mathcal{K}^1$ *is a Bertrand-Nash equilibrium according to Definition 1 if and only if it satisfies the variational inequality:*

$$-\sum_{j=1}^{n_O} \frac{\partial U_j(Q^*, q^*, \pi^*)}{\partial q_j} \times (q_j - q_j^*) - \sum_{j=1}^{n_O} \sum_{k=1}^{n_R} \frac{\partial U_j(Q^*, q^*, \pi^*)}{\partial \pi_{jk}} \times (\pi_{jk} - \pi_{jk}^*) \geq 0,$$

$$\forall (q, \pi) \in \mathcal{K}^1, \tag{12}$$

Variational Inequality Formulation

Theorem 2 (cont'd)

with notice that: for $j = 1, \ldots, n_O$:

$$-\frac{\partial U_j}{\partial q_j} = \sum_{k=1}^{n_R} \frac{\partial \hat{sc}_{jk}(Q,q)}{\partial q_j} + \frac{\partial \hat{qc}_j(q)}{\partial q_j}, \tag{13}$$

and for $j = 1, \ldots, n_O$; $k = 1, \ldots, n_R$:

$$-\frac{\partial U_J}{\partial \pi_{jk}} = \frac{\partial oc_{jk}(\pi_{jk})}{\partial \pi_{jk}} - Q_{n_M+j,k}. \tag{14}$$

The Equilibrium Conditions for the Supply Chain Network with Outsourcing and with Price and Quality Competition

Definition 2

The equilibrium state of the pharmaceutical supply chain network with outsourcing is one where both variational inequalities (6) and (12) hold simultaneously.

Variational Inequality Formulation

Theorem 3

Determine $(Q^*, q^*, \pi^*) \in \mathcal{K}$, *such that:*

$$-\sum_{h=1}^{n}\sum_{l=1}^{n_R}\frac{\partial U_0(Q^*,q^*,\pi^*)}{\partial Q_{hl}}\times(Q_{hl}-Q^*_{hl})-\sum_{j=1}^{n_O}\frac{\partial U_j(Q^*,q^*,\pi^*)}{\partial q_j}\times(q_j-q^*_j)$$

$$-\sum_{j=1}^{n_O}\sum_{k=1}^{n_R}\frac{\partial U_j(Q^*,q^*,\pi^*)}{\partial \pi_{jk}}\times(\pi_{jk}-\pi^*_{jk})\geq 0, \quad \forall (Q,q,\pi)\in\mathcal{K}, \qquad (15)$$

where $\mathcal{K} \equiv K^0 \times \mathcal{K}^1$.

Standard Form of Variational Inequality Formulation

Standard Form

Determine $X^* \in \mathcal{K}$ *where* X *is a vector in* R^N, $F(X)$ *is a continuous function such that* $F(X): X \mapsto \mathcal{K} \subset R^N$, *and*

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{16}$$

where $\langle \cdot, \cdot \rangle$ *is the inner product in the N-dimensional Euclidean space, and* $\mathcal{K}$ *is closed and convex.*

Standard Form of Variational Inequality Formulation

Standard Form

We define the vector $X \equiv (Q, q, \pi)$*. Also,* $N = nn_R + n_O + n_o n_R$*.*
Note that (16) may be rewritten as:

$$\sum_{i=1}^{N} F_i(X^*) \times (X_i - X_i^*) \geq 0, \quad \forall X \in \mathcal{K}, \tag{17}$$

where first nn_R *components of* F *are given by:*
$-\frac{\partial U_0(Q,q,\pi)}{\partial Q_{hl}}$*,for* $h = 1, \ldots, n$*;* $l = 1, \ldots, n_R$*;*
the next n_O *components of* F *are given by:*
$-\frac{\partial U_j(Q,q,\pi)}{\partial q_j}$*,for* $j = 1, \ldots, n_O$*;*
and the subsequent $n_O n_R$ *components of* F *are given by:*
$-\frac{\partial U_j(Q,q,\pi)}{\partial \pi_{jk}}$*, for* $j = 1, \ldots, n_O$*;* $k = 1, \ldots, n_R$*.*

The Underlying Dynamics

Recall the pertinent ordinary differential equation (ODE):

$$\dot{X} = \Pi_{\mathcal{K}}(X, -F(X)), \tag{18}$$

where, since $\mathcal{K}$ is a convex polyhedron, according to Dupuis and Nagurney (1993), $\Pi_{\mathcal{K}}(X, -F(X))$ is the projection, with respect to $\mathcal{K}$, of the vector $-F(X)$ at X defined as

$$\Pi_{\mathcal{K}}(X, -F(X)) = \lim_{\delta \to 0} \frac{P_{\mathcal{K}}(X - \delta F(X)) - X}{\delta} \tag{19}$$

with $P_{\mathcal{K}}$ denoting the projection map:

$$P(X) = \text{argmin}_{x \in \mathcal{K}} \|X - z\|, \tag{20}$$

where $\|\cdot\| = \langle x^T, x \rangle$, and $F(X) = -\nabla U(Q, q, \pi)$.

Theorem: Equilibrium Condition

Theorem 4

X^* solves the variational inequality problem (15) equicalently, (16), if and only if it is a stationary point of the ODE (18), that is,

$$\dot{X} = 0 = \Pi_{\mathcal{K}}(X^*, -F(X^*)). \tag{21}$$

Theorem 5

Suppose that F is strongly monotone. Then there exists a unique solution to variational inequality (16); equivalently, to variational inequality (15).

Theorem 6

- *If $F(X)$ is monotone, then every supply chain network equilibrium, as defined in Definition 2, provided its existence, is a global monotone attractor for the utility gradient processes.*
- *If $F(X)$ is strictly monotone, then there exists at most one supply chain network equilibrium. Furthermore, given existence, the unique equilibrium is a strictly global monotone attractor for the utility gradient processes.*
- *If $F(X)$ is strongly monotone, then the unique supply chain network equilibrium, which is guaranteed to exist, is also globally exponentially stable for the utility gradient processes.*

Stability Under Monotonicity

The monotonicity of a function F is closely related to the positive-definiteness of its Jacobian ∇F (cf. Nagurney (1999)).

Particularly,

- if ∇F is positive-semidefinite, F is monotone;
- if ∇F is positive-definite, F is strictly monotone;
- and, if ∇F is strongly positive definite, in the sense that the symmetric part of $\nabla F, (\nabla F^T + \nabla F)/2$, has only positive eigenvalues, then F is strongly monotone.

The Algorithm - The Euler Method

Iteration τ of the Euler method (see also Nagurney and Zhang (1996))

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - a_{\tau} F(X^{\tau})), \tag{29}$$

where $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$ and F is the function that enters the variational inequality problem (16).

For convergence of the general iterative scheme, which induces the Euler method, the sequence $\{a_{\tau}\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_{\tau} = \infty$, $a_{\tau} > 0$, $a_{\tau} \to 0$, as $\tau \to \infty$.

The Algorithm - The Euler Method

We emphasize that this is the first time that this algorithm is being adapted and applied for the solution of supply chain network game theory problems under Nash-Bertrand competition and with outsourcing.

The Algorithm - The Euler Method

The strictly convexquadratic programming problem

$$X^{\tau+1} = \text{Minimize}_{X \in \mathcal{K}} \quad \frac{1}{2}\langle X, X \rangle - \langle X^{\tau} - a_{\tau} F(X^{\tau}), X \rangle. \tag{30}$$

In order to obtain the values of the product flows at each iteration, we can apply the exact equilibration algorithm, originated by Dafermos and Sparrow (1969) and applied to many different applications of networks with special structure (cf. Nagurney (1999) and Nagurney and Zhang (1996)).

Explicit Formulae for Quality Levels and Contractor Prices

$$q_j^{\tau+1} = \min\{q^U, \max\{0, q_j^\tau + a_\tau(-\sum_{k=1}^{n_R} \frac{\partial \hat{sc}_{jk}(Q^\tau, q^\tau)}{\partial q_j} - \frac{\partial \hat{qc}_j(q^\tau)}{\partial q_j})\}\},$$

$$j = 1, \ldots, n_O, \tag{31}$$

$$\pi_{jk}^{\tau+1} = \max\{0, \pi_{jk}^\tau + a_\tau(-\frac{\partial oc_{jk}(\pi_{jk}^\tau)}{\partial \pi_{jk}} + Q_{n_M+j,k}^\tau)\},$$

$$j = 1, \ldots, n_O; k = 1, \ldots, n_R. \tag{32}$$

The Convergence Result

Theorem 7

In the supply chain network model with outsourcing, let $F(X) = -\nabla U(Q, q, \pi)$ be strongly monotone. Also, assume that F is uniformly Lipschitz continuous. Then there exists a unique equilibrium product flow, quality level, and price pattern $(Q^*, q^*, \pi^*) \in \mathcal{K}$ and any sequence generated by the Euler method as given by (29) above, where $\{a_\tau\}$ satisfies $\sum_{\tau=0}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \to 0$, as $\tau \to \infty$ converges to (Q^*, q^*, π^*).

Note that convergence also holds if $F(X)$ is monotone (cf. Theorem 8.6 in Nagurney and Zhang (1996)) provided that the price iterates are bounded.

An Illustrative Example

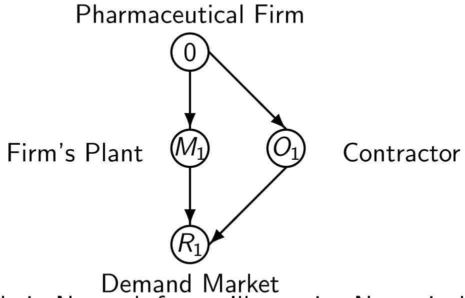

Figure: Supply Chain Network for an Illustrative Numerical Example

An Illustrative Example

The demand at demand market R_1: 1,000, q^U=100, and ω=1.

The firm's production cost:

$$f_1(Q_{11}) = Q_{11}^2 + Q_{11}.$$

Total transportation cost:

$$\hat{c}_{11}(Q_{11}) = .5Q_{11}^2 + Q_{11}.$$

Transaction cost:

$$tc_1(Q_{21}) = .05Q_{21}^2 + Q_{21}.$$

An Illustrative Example

The contractor's total cost of production and distribution:

$$\hat{s}c_{11}(Q_{21}, q_1) = Q_{21}q_1.$$

Total quality cost:

$$\hat{q}c_1(q_1) = 10(q_1 - 100)^2.$$

Opportunity cost:

$$oc_{11}(\pi_{11}) = .5(\pi_{11} - 10)^2.$$

Cost of disrepute:

$$dc(q\prime) = 100 - q\prime,$$

where $q\prime = \frac{Q_{21}q_1 + Q_{11}100}{1000}$.

An Illustrative Example

We set the convergence tolerance to 10^{-3}. The Euler method was initialized with $Q_{11}^0 = Q_{21}^0 = 500$, $q_1^0 = 1$, and $\pi_{11}^0 = 0$. The sequence $\{a_\tau\}$ was set to: $\{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \ldots\}$.

The Euler method converged in 87 iterations and yielded the following product flow, quality level, and price pattern:

$$Q_{11}^* = 270.50, \quad Q_{21}^* = 729.50, \quad q_1^* = 63.52, \quad \pi_{11}^* = 739.50.$$

The total cost incurred by the pharmaceutical firm was 677,128.65 with the contractor earning a profit of 213,786.67. The value of $q\prime$ was 73.39.

An Illustrative Example

The Jacobian matrix of $F(X) = -\nabla U(Q, q, \pi)$, for this example, is

$$J(Q_{11}, Q_{21}, q_1, \pi_{11}) = \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & .1 & -.001 & 1 \\ 0 & 1 & 20 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}.$$

Both the existence and the uniqueness of the solution to variational inequality (15) with respect to this example are guaranteed.

A Variant

The transportation cost function was reduced by a factor of 10:

$$\hat{c}_{11}(Q_{11}) = .05Q_{11}^2 + .1Q_{11}.$$

The Euler method again required 87 iterations for convergence and yielded the following equilibrium solution:

$$Q_{11}^* = 346.86,\ Q_{21}^* = 653.14, \quad q_1^* = 67.34, \quad \pi_{11}^* = 663.15.$$

The pharmaceutical firm's total costs were now 581,840.07 and the contractor's profits were now 165,230.62. The value of $q\prime$ was now 78.67.

The Jacobian of F for the variant is also strongly positive-definite with the only change in the Jacobian matrix above being that the 3 is replaced by 2.1.

Sensitivity Analysis

We returned to the original example and increased the demand for the pharmaceutical product at R_1 in increments of 1,000.

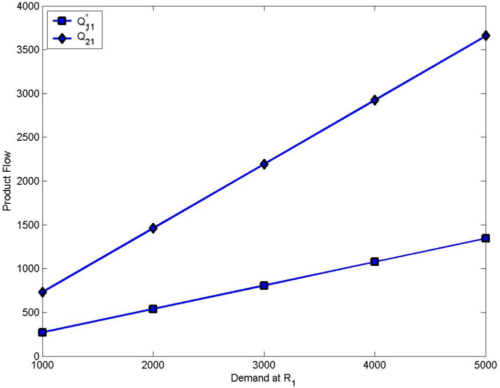

Figure: Equilibrium Product Flows as the Demand Increases for the Illustrative Example

Sensitivity Analysis

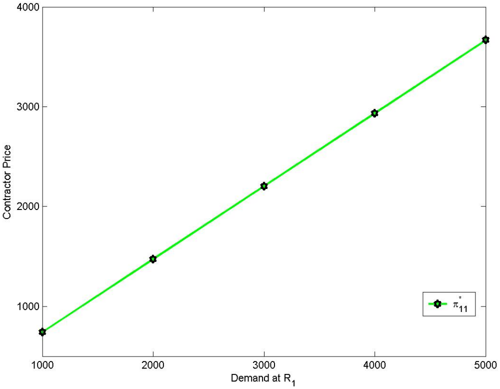

Figure: Equilibrium Contractor Prices as the Demand Increases for the Illustrative Example

Sensitivity Analysis

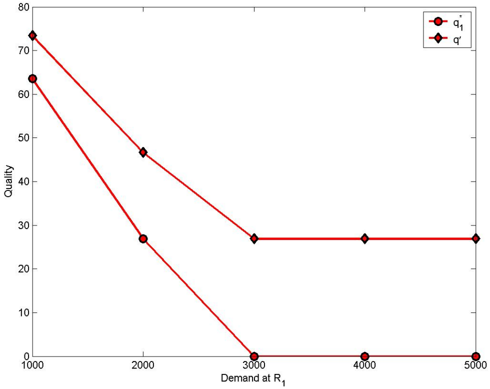

Figure: Equilibrium Contractor Quality Level and the Average Quality as the Demand Increases for the Illustrative Example

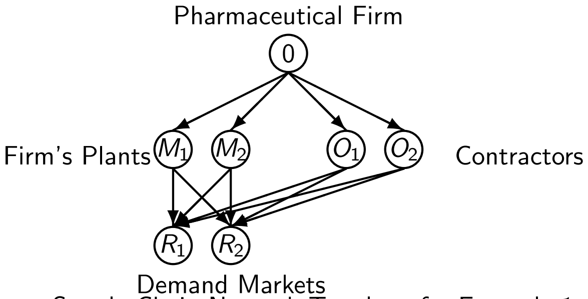

Figure: Supply Chain Network Topology for Example 1

Additional Numerical Examples - Example 1

The demand at R_1 was 1,000 and it was 500 at R_2. q^U=100, and ω=1.

The production cost functions:

$$f_1(\sum_{k=1}^{2} Q_{1k}) = (Q_{11} + Q_{12})^2 + 2(Q_{11} + Q_{12}),$$

$$f_2(\sum_{k=1}^{2} Q_{2k}) = 1.5(Q_{21} + Q_{22})^2 + 2(Q_{21} + Q_{22}).$$

The transportation cost functions:

$$\hat{c}_{11}(Q_{11}) = 1.5Q_{11}^2 + 10Q_{11}, \quad \hat{c}_{12}(Q_{12}) = 1Q_{12}^2 + 25Q_{12},$$

$$\hat{c}_{21}(Q_{21}) = 1Q_{21}^2 + 5Q_{21}, \quad \hat{c}_{22}(Q_{22}) = 2.5Q_{22}^2 + 40Q_{22}.$$

Transaction cost functions:

$$tc_1(Q_{31} + Q_{32}) = .5(Q_{31} + Q_{32})^2 + .1(Q_{31} + Q_{32}),$$

$$tc_2(Q_{41} + Q_{42}) = .25(Q_{41} + Q_{42})^2 + .2(Q_{41} + Q_{42}).$$

Additional Numerical Examples - Example 1

The contractors' total cost of production and distribution:

$$\hat{s}c_{11}(Q_{31}, q_1) = Q_{31}q_1, \quad \hat{s}c_{12}(Q_{32}, q_1) = Q_{32}q_1,$$

$$\hat{s}c_{21}(Q_{41}, q_2) = 2Q_{41}q_2, \quad \hat{s}c_{22}(Q_{42}, q_2) = 2Q_{42}q_2.$$

Total quality cost functions:

$$\hat{q}c_1(q_1) = 5(q_1 - 100)^2, \quad \hat{q}c_2(q_2) = 10(q_1 - 100)^2.$$

Opportunity cost functions:

$$oc_{11}(\pi_{11}) = .5(\pi_{11} - 10)^2, \quad oc_{12}(\pi_{12}) = (\pi_{12} - 10)^2,$$

$$oc_{21}(\pi_{21}) = (\pi_{21} - 5)^2, \quad oc_{22}(\pi_{22}) = .5(\pi_{22} - 20)^2.$$

Cost of disrepute:

$$dc(q\prime) = 100 - q\prime$$

where $q\prime = \frac{Q_{31}q_1 + Q_{32}q_1 + Q_{41}q_2 + Q_{42}q_2 + Q_{11}100 + Q_{12}100 + Q_{21}100 + Q_{22}100}{1500}$.

Additional Numerical Examples - Example 1

The Euler method converged in 153 iterations and yielded the following equilibrium solution.

$$Q_{11}^* = 95.77, \quad Q_{12}^* = 85.51, \quad Q_{21}^* = 118.82, \quad Q_{22}^* = 20.27,$$

$$Q_{31}^* = 213.59, \quad Q_{32}^* = 224.59, \quad Q_{41}^* = 571.83, \quad Q_{42}^* = 169.63.$$

$$q_1^* = 56.18, \quad q_2^* = 25.85,$$

$$\pi_{11}^* = 223.57, \quad \pi_{12}^* = 122.30, \quad \pi_{21}^* = 290.92, \quad \pi_{22}^* = 189.61.$$

The total cost of the pharmaceutical firm was 610,643.26 and the profits of the contractors' were: 5,733.83 and 9,294.44. The value of $q\prime$ was 50.55.

Additional Numerical Examples - Example 1

The Jacobian matrix of $F(X) = -\nabla U(Q, q, \pi)$ is

$$J(Q_{11}, Q_{12}, Q_{21}, Q_{22}, Q_{31}, Q_{32}, Q_{41}, Q_{42}, q_1, q_2, \pi_{11}, \pi_{12}, \pi_{21}, \pi_{22})$$

$$= \begin{pmatrix}
5 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 5 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 3 & 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & -6.67 \times 10^{-4} & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & -6.67 \times 10^{-4} & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0.5 & 0 & -6.67 \times 10^{-4} & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0.5 & 0 & -6.67 \times 10^{-4} & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 20 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}.$$

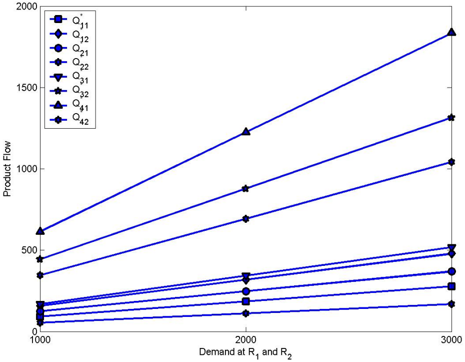

Figure: Equilibrium Product Flows as the Demand Increases for Example 1

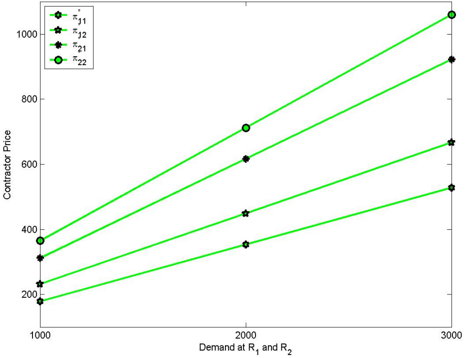

Figure: Equilibrium Contractor Prices as the Demand Increases for Example 1

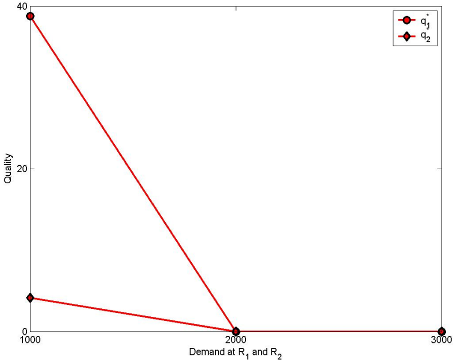

Figure: Equilibrium Quality Levels as the Demand Increases for Example 1

Additional Numerical Examples - Example 2

We considered the following disruption. The data were as in Example 1 but contractor O_2 was not able to provide any production and distribution services. This could arise due to a natural disaster, adulteration in its production process, and/or an inability to procure an ingredient.

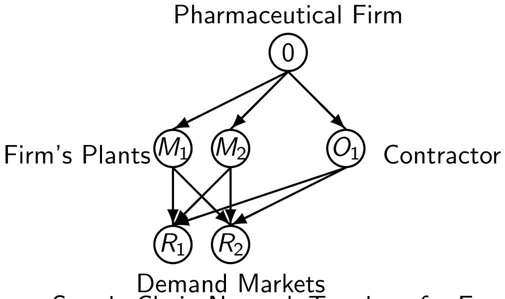

Figure: Supply Chain Network Topology for Example 2

Additional Numerical Examples - Example 2

The Euler method converged in 73 iterations and yielded the following new equilibrium solution.

$$Q_{11}^* = 218.06, \quad Q_{12}^* = 141.79, \quad Q_{21}^* = 260.20, \quad Q_{22}^* = 25.96,$$

$$Q_{31}^* = 521.74, \quad Q_{32}^* = 332.25.$$

$$q_1^* = 14.60,$$

$$\pi_{11}^* = 531.74, \quad \pi_{12}^* = 176.12.$$

The new average quality level was $q\prime$=51.38. The total cost of the pharmaceutical firm was now 1,123,226.62 whereas the profit of the first contractor was now 123,460.67.

Summary and Conclusions

We developed a supply chain network game theory model, in both equilibrium and dynamic versions, to capture contractor selection, based on the competition among the contractors in the prices that they charge as well as the quality levels of the pharmaceutical products that they produce.

Summary and Conclusions

- We introduced a disrepute cost associated with the average quality at the demand markets.
- We utilized variational inequality theory for the formulation of the equilibrium conditions.
- We provided stability analysis results as well as an algorithm that can be interpreted as a discretization of the continuous-time adjustment processes.
- Our numerical studies included sensitivity analysis results as well as a disruption to the supply chain network in that a contractor is no longer available for production and distribution.

Summary and Conclusions

- We introduced a disrepute cost associated with the average quality at the demand markets.
- We utilized variational inequality theory for the formulation of the equilibrium conditions.
- We provided stability analysis results as well as an algorithm that can be interpreted as a discretization of the continuous-time adjustment processes.
- Our numerical studies included sensitivity analysis results as well as a disruption to the supply chain network in that a contractor is no longer available for production and distribution.

Summary and Conclusions

- We introduced a disrepute cost associated with the average quality at the demand markets.
- We utilized variational inequality theory for the formulation of the equilibrium conditions.
- We provided stability analysis results as well as an algorithm that can be interpreted as a discretization of the continuous-time adjustment processes.
- Our numerical studies included sensitivity analysis results as well as a disruption to the supply chain network in that a contractor is no longer available for production and distribution.

Summary and Conclusions

- We introduced a disrepute cost associated with the average quality at the demand markets.
- We utilized variational inequality theory for the formulation of the equilibrium conditions.
- We provided stability analysis results as well as an algorithm that can be interpreted as a discretization of the continuous-time adjustment processes.
- Our numerical studies included sensitivity analysis results as well as a disruption to the supply chain network in that a contractor is no longer available for production and distribution.

Summary and Conclusions

This paper is a contribution to the literature on outsourcing with a focus on quality with an emphasis on the pharmaceutical industry which is a prime example of an industry where the quality of a product is paramount. It also is an interesting application of game theory and associated methodologies.

The ideas in this paper may be adapted, with appropriate modifications, to other industries.

Thank you!


The Virtual Center for Supernetworks
Supernetworks for Optimal Decision-Making and Improving the Global Quality of Life
Director's Welcome
About the Director
Projects
Supernetworks Laboratory
Center Associates
Media Coverage
What's New
Downloadable Articles
Visuals
Audio/Video
Books
Commentaries & OpEds
The Supernetwork Sentinel
Congratulations & Kudos
Professor Nagurney and her Doctoral Students May 7, 2013
The Virtual Center for Supernetworks is an interdisciplinary center at the Isenberg School of Management that advances knowledge on large-scale networks and integrates operations research and management science, engineering, and economics. Its Director is Dr. Anna Nagurney, the John F. Smith Memorial Professor of Operations Management.
Mission: The Virtual Center for Supernetworks fosters the study and application of supernetworks and serves as a resource on networks ranging from transportation and logistics, including supply chains, and the Internet, to a spectrum of economic networks.
The Applications of Supernetworks Include: decision-making, optimization, and game theory; supply chain management; critical infrastructure from transportation to electric power networks; financial networks; knowledge and social networks; energy, the environment, and sustainability; risk management; network vulnerability, resiliency, and performance metrics; humanitarian logistics and healthcare.
Announcements and Notes
Photos of Center Activities
Photos of Network Innovators
Friends of the Center
Course Lectures
Fulbright Lectures
UMass Amherst INFORMS Student Chapter
Professor Anna Nagurney's Blog
Network Classics
Doctoral Dissertations
Conferences
Journals
Societies
Archive
Announcements and Notes from the Center Director Professor Anna Nagurney
Updated: September 9, 2013
Follow
Professor Anna Nagurney's Blog
RENeW
Research, Education, Networks, and the World: A Female Professor Speaks
Sustaining the Supply Chain
Mathematical Moments Podcast
PBS VIDEO
America Revealed
New Book
Networks Against Time
Photos of Center Activities
The Braess Paradox
Translation
Information
Photos
Publications
You are visitor number
00083025
to the Virtual Center for Supernetworks.
WGBY Connecting Point
Humanitarian Logistics: Networks for Africa
The New York Times
ENERGY FOR TOMORROW CONFERENCE
Funding for the Center has been provided by:
The National Science Foundation
The AT&T Foundation
The Rockefeller Foundation
The John F. Smith Memorial Fund of the University of Massachusetts
The Isenberg School of Management - University of Massachusetts.
Contact the Center: supernet@isenberg.umass.edu
ISENBERG
UMASS AMHERST
Copyright 2001-2013
This is an official web page of the Eugene M. Isenberg School of Management University of Massachusetts Amherst
Maintained by supernet@isenberg.umass.edu