Equilibria and Dynamics of Supply Chain Network Competition with Information Asymmetry in Quality and Minimum Quality Standards

Anna Nagurney
John F. Smith Memorial Professor
and
Dong (Michelle) Li
Doctoral Student

Department of Operations & Information Management
Isenberg School of Management
University of Massachusetts
Amherst, Massachusetts 01003

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where a full list of references can be found.

- Background and Motivation
- Supply Chain Network Competition with Information Asymmetry in Quality
- Qualitative Properties
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- Summary and Conclusions

Poor quality products, whether inferior durable goods such as automobiles, or consumables such as pharmaceuticals and food, may negatively affect the safety and the well-being of consumers with, possibly, associated fatal consequences.

- In 2008, fake heparin made by a Chinese manufacturer not only led to recalls of drugs in over ten European countries (Payne (2008)), but also resulted in the deaths of 81 Americans (Harris (2011)).
- In 2009, more than 400 peanut butter products were recalled after 8 people died and more than 500 people (half of them children) were sickened by salmonella poisoning (Harris (2009)).
- In 2010, four Japanese car-makers, including Toyota and Nissan, recalled 3.4 million vehicles sold around the globe, because the airbags were at risk of catching fire (Kubota and Klayman (2013)).
- In 2013, Taylor Farms, a large vegetable supplier, was under investigation in connection with an illness outbreak affecting hundreds of people in the US (Strom (2013)).

Background and Motivation - Information Asymmetry

Given the distances that may be involved as well as the types of products that are consumed in supply chain networks, there may be information asymmetry associated with knowledge about the quality of the products. Specifically, when there is no differentiation by brands or labels, products from different firms are viewed as being homogeneous for consumers.

Related literature

- Akerlof, G. A., 1970. The market for 'lemons': Quality uncertainty and the market mechanism. *Quarterly Journal of Economics* 84(3), 488-500.
- Leland, H. E., 1979. Quacks, lemons, and licensing: A theory of minimum quality standards. *Journal of Political Economy* 87(6), 1328-1346.
- Shapiro, C., 1983. Premiums for high quality products as returns to reputations. *Quarterly Journal of Economics* 98(4), 659-679.
- Ronnen, U., 1991. Minimum quality standards, fixed costs, and competition. *RAND Journal of Economics* 22(4), 490-504.
- Baltzer, K., 2012. Standards vs. labels with imperfect competition and asymmetric information. *Economics Letters* 114(1), 61-63.

Related literature

- Nagurney, A., Li, D., 2013. A dynamic network oligopoly model with transportation costs, product differentiation, and quality competition. *Computational Economics* in press.
- Nagurney, A., Li, D., Nagurney, L. S., 2013. Pharmaceutical supply chain networks with outsourcing under price and quality competition. *International Transactions in Operational Research* 20(6), 859-888.
- Nagurney, A., Li, D., Wolf, T., Saberi, S., 2013. A network economic game theory model of a service-oriented Internet with choices and quality competition. *Netnomics* 14(1-2), 1-25.
- Nagurney, A., Wolf, T., 2013. A Cournot-Nash-Bertrand game theory model of a service-oriented Internet with price and quality competition among network transport providers. *Computational Management Science* in press.

Overview

We develop both static and dynamic competitive supply chain network models with information asymmetry in quality.

- The information asymmetry in quality occurs between the firms, producing the product, and the consumers.
- We consider multiple profit-maximizing firms, which may have multiple plants at their disposal.
- The firms compete in multiple demand markets in product shipments and product quality levels.
- Quality is associated with the manufacturing plants, and is also tracked through the transportation process
- We demonstrate how minimum quality standards can be incorporated into the framework, which has wide relevance for policy-making and regulation.

Network Topology

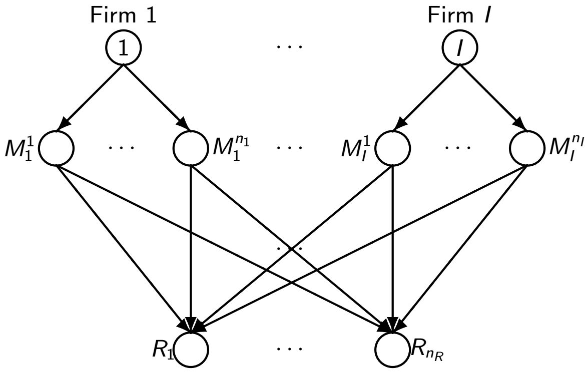

Figure: The Supply Chain Network Topology

Conservation of flow equations

$$s_{ij} = \sum_{k=1}^{n_R} Q_{ijk}, \quad i = 1, \ldots, I; j = 1, \ldots, n_i, \tag{1}$$

$$d_k = \sum_{i=1}^{I} \sum_{j=1}^{n_i} Q_{ijk}, \quad k = 1, \ldots, n_R, \tag{2}$$

$$Q_{ijk} \geq 0, \quad i = 1, \ldots, I; j = 1, \ldots, n_i; k = 1, \ldots, n_R. \tag{3}$$

For each firm i, we group its Q_{ijk}s into the vector $Q_i \in R_+^{n_i n_R}$, and then group all such vectors for all firms into the vector $Q \in R_+^{\sum_{i=1}^{I} n_i n_R}$.

We also group all s_{ij}s into the vector $s \in R_+^{\sum_{i=1}^{I} n_i}$ and all d_ks into the vector $d \in R_+^{n_R}$.

We define and quantify quality as the quality conformance level, that is, the degree to which a specific product conforms to a design or specification (Gilmore (1974), Juran and Gryna (1988)).

The quality levels cannot be lower than 0% defect-free level; thus,

Nonnegative quality level of firm i's manufacturing plant M_i^j

$$q_{ij} \geq 0, \quad i = 1, \ldots, I; j = 1, \ldots, n_i. \tag{4}$$

For each firm i, we group its own plant quality levels into the vector $q_i \in R_+^{n_i}$ and then group all such vectors for all firms into the vector $q \in R_+^{\sum_{i=1}^{I} n_i}$.

Production cost function at firm i's manufacturing plant M_i^j

$$f_{ij} = f_{ij}(s, q), \quad i = 1, \ldots, I; j = 1, \ldots, n_i. \tag{5a}$$

In view of (1),

$$\hat{f}_{ij} = \hat{f}_{ij}(Q, q) \equiv f_{ij}(s, q), \quad i = 1, \ldots, I; j = 1, \ldots, n_i. \tag{5b}$$

Transportation cost function associated with shipping the product produced at firm i's manufacturing plant M_i^j to demand market R_k

$$\hat{c}_{ijk} = \hat{c}_{ijk}(Q, q), \quad i = 1, \ldots, I; j = 1, \ldots, n_i; k = 1, \ldots, n_R. \tag{6}$$

Note that, according to (6), the transportation cost is such that the quality of the product is not degraded as it undergoes the shipment process.

The production cost functions and the transportation functions are assumed to be convex, continuous, and twice continuously differentiable.

Since firms do not differentiate the products as well as their quality levels, consumers' perception of the quality of all such product, which may come from different firms, is for the average quality level.

Consumers' perception of the quality of the product at demand market R_k

$$\hat{q}_k = \frac{\sum_{i=1}^{I} \sum_{j=1}^{n_i} Q_{ijk} q_{ij}}{d_k}, \quad k = 1, \ldots, n_R \tag{7}$$

with the average (perceived) quality levels grouped into the vector $\hat{q} \in R_+^{n_R}$.

The demand price at demand market R_k

$$\rho_k = \rho_k(d, \hat{q}), \quad k = 1, \ldots, n_R. \tag{8a}$$

In light of (2) and (7),

$$\hat{\rho}_k = \hat{\rho}_k(Q, q) \equiv \rho_k(d, \hat{q}), \quad k = 1, \ldots, n_R. \tag{8b}$$

Each demand price function is, typically, assumed to be monotonically decreasing in product quantity but increasing in terms of the average product quality.

We assume that the demand price functions are continuous and twice continuously differentiable.

The strategic variables of firm i are its product shipments $\{Q_i\}$ and its quality levels q_i.

The profit/utility U_i of firm i; $i = 1, \ldots, I$

$$U_i = \sum_{k=1}^{n_R} \rho_k(d, \hat{q}) \sum_{j=1}^{n_i} Q_{ijk} - \sum_{j=1}^{n_i} f_{ij}(s, q) - \sum_{k=1}^{n_R} \sum_{j=1}^{n_i} \hat{c}_{ijk}(Q, q), \qquad (9a)$$

which is equivalent to

$$U_i = \sum_{k=1}^{n_R} \hat{\rho}_k(Q, q) \sum_{j=1}^{n_i} Q_{ijk} - \sum_{j=1}^{n_i} \hat{f}_{ij}(Q, q) - \sum_{k=1}^{n_R} \sum_{j=1}^{n_i} \hat{c}_{ijk}(Q, q). \qquad (9b)$$

Assume that for each firm i the profit function $U_i(Q, q)$ is concave with respect to the variables in Q_i and q_i, and is continuous and twice continuously differentiable.

Let K^i denote the feasible set corresponding to firm i, where $K^i \equiv \{(Q_i, q_i) | Q_i \geq 0, \text{ and } q_i \geq 0\}$ and define $K \equiv \prod_{i=1}^{I} K^i$.

Definition 1

A product shipment and quality level pattern $(Q^*, q^*) \in K$ is said to constitute a supply chain network Cournot-Nash equilibrium with information asymmetry in quality if for each firm i; $i = 1, \dots, I$,

$$U_i(Q_i^*, q_i^*, \hat{Q}_i^*, \hat{q}_i^*) \geq U_i(Q_i, q_i, \hat{Q}_i^*, \hat{q}_i^*), \quad \forall (Q_i, q_i) \in K^i, \tag{11}$$

where

$$\hat{Q}_i^* \equiv (Q_1^*, \dots, Q_{i-1}^*, Q_{i+1}^*, \dots, Q_I^*) \quad \textit{and} \quad \hat{q}_i^* \equiv (q_1^*, \dots, q_{i-1}^*, q_{i+1}^*, \dots, q_I^*).$$

The Equilibrium Model - Variational Inequality Formulation

Theorem 2

Then the product shipment and quality pattern $(Q^*, q^*) \in K$ *is a supply chain network Cournot-Nash equilibrium with quality information asymmetry according to Definition 1 if and only if it satisfies the variational inequality*

$$-\sum_{i=1}^{I}\sum_{j=1}^{n_i}\sum_{k=1}^{n_R}\frac{\partial U_i(Q^*, q^*)}{\partial Q_{ijk}}\times(Q_{ijk}-Q^*_{ijk})-\sum_{i=1}^{I}\sum_{j=1}^{n_i}\frac{\partial U_i(Q^*, q^*)}{\partial q_{ij}}\times(q_{ij}-q^*_{ij})\geq 0,$$

$$\forall (Q, q) \in K, \tag{12}$$

that is,

$$\sum_{i=1}^{I}\sum_{j=1}^{n_i}\sum_{k=1}^{n_R}\Bigg[-\hat{\rho}_k(Q^*,q^*)-\sum_{l=1}^{n_R}\frac{\partial\hat{\rho}_l(Q^*,q^*)}{\partial Q_{ijk}}\sum_{h=1}^{n_i}Q^*_{ihl}+\sum_{h=1}^{n_i}\frac{\partial\hat{f}_{ih}(Q^*,q^*)}{\partial Q_{ijk}}$$

$$+\sum_{h=1}^{n_i}\sum_{l=1}^{n_R}\frac{\partial\hat{c}_{ihl}(Q^*,q^*)}{\partial Q_{ijk}}\Bigg]\times(Q_{ijk}-Q^*_{ijk})$$

$$+\sum_{i=1}^{I}\sum_{j=1}^{n_i}\Bigg[-\sum_{k=1}^{n_R}\frac{\partial\hat{\rho}_k(Q^*,q^*)}{\partial q_{ij}}\sum_{h=1}^{n_i}Q^*_{ihk}+\sum_{h=1}^{n_i}\frac{\partial\hat{f}_{ih}(Q^*,q^*)}{\partial q_{ij}}$$

$$+\sum_{h=1}^{n_i}\sum_{k=1}^{n_R}\frac{\partial\hat{c}_{ihk}(Q^*,q^*)}{\partial q_{ij}}\Bigg]\times(q_{ij}-q^*_{ij})\geq 0,\quad\forall(Q,q)\in K;\qquad(13)$$

Equivalently,

$(d^*, s^*, Q^*, q^*) \in K^1$ *is an equilibrium production, shipment, and quality level pattern if and only if it satisfies the variational inequality*

$$\sum_{k=1}^{n_R}[-\rho_k(d^*,\hat{q}^*)] \times (d_k - d_k^*) + \sum_{i=1}^{I}\sum_{j=1}^{n_i}\left[\sum_{h=1}^{n_i}\frac{\partial f_{ih}(s^*,q^*)}{\partial s_{ij}}\right] \times (s_{ij}-s_{ij}^*)$$

$$+\sum_{i=1}^{I}\sum_{j=1}^{n_i}\sum_{k=1}^{n_R}\left[-\sum_{l=1}^{n_R}\frac{\partial \rho_l(d^*,\hat{q}^*)}{\partial Q_{ijk}}\sum_{h=1}^{n_i}Q_{ihl}^* + \sum_{h=1}^{n_i}\sum_{l=1}^{n_R}\frac{\partial \hat{c}_{ihl}(Q^*,q^*)}{\partial Q_{ijk}}\right]\times(Q_{ijk}-Q_{ijk}^*)$$

$$+\sum_{i=1}^{I}\sum_{j=1}^{n_i}\left[-\sum_{k=1}^{n_R}\frac{\partial \rho_k(Q^*,\hat{q}^*)}{\partial q_{ij}}\sum_{h=1}^{n_i}Q_{ihk}^* + \sum_{h=1}^{n_i}\frac{\partial f_{ih}(s^*,q^*)}{\partial q_{ij}}\right.$$

$$\left.+\sum_{h=1}^{n_i}\sum_{k=1}^{n_R}\frac{\partial \hat{c}_{ihk}(Q^*,q^*)}{\partial q_{ij}}\right] \times (q_{ij}-q_{ij}^*) \geq 0, \qquad \forall (d,s,Q,q) \in K^1, \qquad (14)$$

where $K^1 \equiv \{(d,s,Q,q)|\, Q \geq 0,\ q \geq 0,$ *and (1), (2), and (7) hold*$\}$.

Examples of Minimum Quality Standards

- Corporate Average Fuel Economy (CAFE) Regulations
- CE (Conformité Européenne) Marking
- China Compulsory Certificate (CCC) Marking
- FCC (Federal Communications Commission) Declaration of Conformity

We now describe an extension of the above framework that incorporates minimum quality standards.

Nonnegative lower bounds on the quality levels at the manufacturing plants

$$q_{ij} \geq \underline{q}_{ij} \quad i = 1, \ldots, I; j = 1, \ldots, n_i \tag{15}$$

with the understanding that, if the lower bounds are all identically equal to zero, then (15) collapses to (4) and, if the lower bounds are positive, then they represent minimum quality standards.

We define a new feasible set $K^2 \equiv \{(Q, q) | Q \geq 0 \text{ and } (15) \text{ holds}\}$.

Corollary 1

The product shipment and quality pattern $(Q^*, q^*) \in K^2$ is a supply chain network Cournot-Nash equilibrium with quality information asymmetry in the presence of minimum quality standards if and only if it satisfies the variational inequality

$$-\sum_{i=1}^{I}\sum_{j=1}^{n_i}\sum_{k=1}^{n_R}\frac{\partial U_i(Q^*, q^*)}{\partial Q_{ijk}} \times (Q_{ijk} - Q^*_{ijk}) - \sum_{i=1}^{I}\sum_{j=1}^{n_i}\frac{\partial U_i(Q^*, q^*)}{\partial q_{ij}} \times (q_{ij} - q^*_{ij}) \geq 0,$$

$$\forall (Q, q) \in K^2, \tag{16}$$

that is,

$$\sum_{i=1}^{I}\sum_{j=1}^{n_i}\sum_{k=1}^{n_R}\left[-\hat{\rho}_k(Q^*,q^*)-\sum_{l=1}^{n_R}\frac{\partial\hat{\rho}_l(Q^*,q^*)}{\partial Q_{ijk}}\sum_{h=1}^{n_i}Q^*_{ihl}+\sum_{h=1}^{n_i}\frac{\partial\hat{f}_{ih}(Q^*,q^*)}{\partial Q_{ijk}}\right.$$
$$\left.+\sum_{h=1}^{n_i}\sum_{l=1}^{n_R}\frac{\partial\hat{c}_{ihl}(Q^*,q^*)}{\partial Q_{ijk}}\right]\times(Q_{ijk}-Q^*_{ijk})$$
$$+\sum_{i=1}^{I}\sum_{j=1}^{n_i}\left[-\sum_{k=1}^{n_R}\frac{\partial\hat{\rho}_k(Q^*,q^*)}{\partial q_{ij}}\sum_{h=1}^{n_i}Q^*_{ihk}+\sum_{h=1}^{n_i}\frac{\partial\hat{f}_{ih}(Q^*,q^*)}{\partial q_{ij}}\right.$$
$$\left.+\sum_{h=1}^{n_i}\sum_{k=1}^{n_R}\frac{\partial\hat{c}_{ihk}(Q^*,q^*)}{\partial q_{ij}}\right]\times(q_{ij}-q^*_{ij})\geq 0,\qquad \forall(Q,q)\in K^2. \qquad (17)$$

Variational inequality (17) contains variational inequality (13) as a special case when the minimum quality standards are all zero.

Standard Form VI

Determine $X^* \in \mathcal{K}$ *where* X *is a vector in* R^N, $F(X)$ *is a continuous function such that* $F(X) : X \mapsto \mathcal{K} \subset R^N$, *and*

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{18}$$

We define the vector $X \equiv (Q, q)$ and the vector $F(X) \equiv (F^1(X), F^2(X))$.

$N = \sum_{i=1}^{I} n_i n_R + \sum_{i=1}^{I} n_i$.

$F^1(X)$ consists of $F^1_{ijk} = -\frac{\partial U_i(Q,q)}{\partial Q_{ijk}}$; $i = 1, \ldots, I; j = 1, \ldots, n_i; k = 1, \ldots, n_R$,
and $F^2(X)$ consist of $F^2_{ij} = -\frac{\partial U_i(Q,q)}{\partial q_{ij}}$; $i = 1, \ldots, I; j = 1, \ldots, n_i$.

We define the feasible set $\mathcal{K} \equiv K^2$.

We now describe the underlying dynamics for the evolution of product shipments and quality levels under information asymmetry in quality until the equilibrium satisfying variational inequality (17) is achieved.

A dynamic adjustment process for product shipments and quality levels

$$\dot{Q}_{ijk} = \begin{cases} \frac{\partial U_i(Q,q)}{\partial Q_{ijk}}, & \text{if} \quad Q_{ijk} > 0 \\ \max\{0, \frac{\partial U_i(Q,q)}{\partial Q_{ijk}}\}, & \text{if} \quad Q_{ijk} = 0. \end{cases} \tag{19}$$

$$\dot{q}_{ij} = \begin{cases} \frac{\partial U_i(Q,q)}{\partial q_{ij}}, & \text{if} \quad q_{ij} > \underline{q}_{ij} \\ \max\{\underline{q}_{ij}, \frac{\partial U_i(Q,q)}{\partial q_{ij}}\}, & \text{if} \quad q_{ij} = \underline{q}_{ij}. \end{cases} \tag{20}$$

The pertinent ordinary differential equation (ODE) for the adjustment processes of the product shipments and quality levels:

$$\dot{X} = \Pi_{\mathcal{K}}(X, -F(X)), \tag{21}$$

where, since $\mathcal{K}$ is a convex polyhedron, according to Dupuis and Nagurney (1993), $\Pi_{\mathcal{K}}(X, -F(X))$ is the projection, with respect to $\mathcal{K}$, of the vector $-F(X)$ at X defined as

$$\Pi_{\mathcal{K}}(X, -F(X)) = \lim_{\delta \to 0} \frac{P_{\mathcal{K}}(X - \delta F(X)) - X}{\delta} \tag{22}$$

with $P_{\mathcal{K}}$ denoting the projection map:

$$P(X) = \text{argmin}_{z \in \mathcal{K}} \|X - z\|, \tag{23}$$

where $\|\cdot\| = \langle x, x \rangle$, and $F(X) = -\nabla U(Q, q)$.

Theorem 2

X^* solves the variational inequality problem (17) if and only if it is a stationary point of the ODE (21), that is,

$$\dot{X} = 0 = \Pi_{\mathcal{K}}(X^*, -F(X^*)). \tag{24}$$

Assumption 1

Suppose that in the supply chain network model with information asymmetry in quality there exists a sufficiently large M, such that for any (i, j, k),

$$\frac{\partial U_i(Q, q)}{\partial Q_{ijk}} < 0, \tag{25}$$

for all shipment patterns Q with $Q_{ijk} \geq M$ and that there exists a sufficiently large $\bar{M}$, such that for any (i, j),

$$\frac{\partial U_i(Q, q)}{\partial q_{ij}} < 0, \tag{26}$$

for all quality level patterns q with $q_{ij} \geq \bar{M} \geq \underline{q}_{ij}$.

Proposition 1

Any supply chain network problem with information asymmetry in quality that satisfies Assumption 1 possesses at least one equilibrium shipment and quality level pattern satisfying variational inequality (17) (or (18)).

Proposition 2

Suppose that F is strictly monotone at any equilibrium point of the variational inequality problem defined in (18). Then it has at most one equilibrium point.

Theorem 3

Suppose that F is strongly monotone. Then there exists a unique solution to variational inequality (18); equivalently, to variational inequality (17).

Theorem 4

(i). *If* $F(X)$ *is monotone, then every supply chain network equilibrium with information asymmetry,* X^**, provided its existence, is a global monotone attractor for the projected dynamical system. If* $F(X)$ *is locally monotone at* X^**, then it is a monotone attractor for the projected dynamical system.*

(ii). *If* $F(X)$ *is strictly monotone, the unique equilibrium* X^**, given existence, is a strictly global monotone attractor for the projected dynamical system. If* $F(X)$ *is locally strictly monotone at* X^**, then it is a strictly monotone attractor for the projected dynamical system.*

(iii). *If* $F(X)$ *is strongly monotone, then the unique supply chain network equilibrium with information asymmetry in quality, which is guaranteed to exist, is also globally exponentially stable for the projected dynamical system. If* $F(X)$ *is locally strongly monotone at* X^**, then it is exponentially stable.*

Iteration τ of the Euler method

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - a_{\tau}F(X^{\tau})), \tag{27}$$

where $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$ and F is the function that enters the variational inequality problem (17).

For convergence of the general iterative scheme, which induces the Euler method, the sequence $\{a_{\tau}\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_{\tau} = \infty$, $a_{\tau} > 0$, $a_{\tau} \to 0$, as $\tau \to \infty$.

$$Q_{ijk}^{\tau+1} = \max\{0, Q_{ijk}^{\tau} + a_\tau(\hat{\rho}_k(Q^\tau, q^\tau) + \sum_{l=1}^{n_R} \frac{\partial \hat{\rho}_l(Q^\tau, q^\tau)}{\partial Q_{ijk}} \sum_{h=1}^{n_i} Q_{ihl}^{\tau} - \sum_{h=1}^{n_i} \frac{\partial \hat{f}_{ih}(Q^\tau, q^\tau)}{\partial Q_{ijk}}$$

$$- \sum_{h=1}^{n_i} \sum_{l=1}^{n_R} \frac{\partial \hat{c}_{ihl}(Q^\tau, q^\tau)}{\partial Q_{ijk}})\} \quad (28)$$

$$q_{ij}^{\tau+1} = \max\{\underline{q}_{ij}, q_{ij}^{\tau} + a_\tau(\sum_{k=1}^{n_R} \frac{\partial \hat{\rho}_k(Q^\tau, q^\tau)}{\partial q_{ij}} \sum_{h=1}^{n_i} Q_{ihk}^{\tau} - \sum_{h=1}^{n_i} \frac{\partial \hat{f}_{ih}(Q^\tau, q^\tau)}{\partial q_{ij}}$$

$$- \sum_{h=1}^{n_i} \sum_{k=1}^{n_R} \frac{\partial \hat{c}_{ihk}(Q^\tau, q^\tau)}{\partial q_{ij}})\}. \quad (29)$$

Theorem 5

In the supply chain network model with information asymmetry in quality, let $F(X) = -\nabla U(Q, q)$, where we group all U_i; $i = 1, \ldots, I$, into the vector $U(Q, q)$, be strictly monotone at any equilibrium shipment pattern and quality levels and assume that Assumption 1 is satisfied. Furthermore, assume that F is uniformly Lipschitz continuous. Then there exists a unique equilibrium product shipment and quality level pattern $(Q^*, q^*) \in \mathcal{K}^2$, and any sequence generated by the Euler method as given by (27) above, with explicit formulae at each iteration given by (28) and (29), where $\{a_\tau\}$ satisfies $\sum_{\tau=0}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \to 0$, as $\tau \to \infty$ converges to (Q^*, q^*).

We implemented the Euler method using Matlab on a Lenovo E46A. The convergence tolerance is 10^{-6}, so that the algorithm is deemed to have converged when the absolute value of the difference between each successive product shipment and quality level is less than or equal to 10^{-6}.

The sequence $\{a_\tau\}$ is set to: $.3\{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \ldots\}$. We initialized the algorithm by setting the product shipments equal to 20 and the quality levels equal to 0.

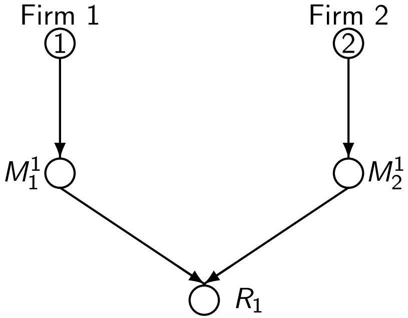

Figure: The Supply Chain Network Topology for Example 1

The production cost functions are:

$$\hat{f}_{11}(Q_{111}, q_{11}) = 0.8Q_{111}^2 + 0.5Q_{111} + 0.25Q_{111}q_{11} + 0.5q_{11}^2,$$

$$\hat{f}_{21}(Q_{211}, q_{21}) = Q_{211}^2 + 0.8Q_{211} + 0.3Q_{211}q_{21} + 0.65q_{21}^2.$$

The total transportation cost functions are:

$$\hat{c}_{111}(Q_{111}, q_{11}) = 1.2Q_{111}^2 + Q_{111} + 0.25Q_{211} + 0.25q_{11}^2,$$

$$\hat{c}_{211}(Q_{211}, q_{21}) = Q_{211}^2 + Q_{211} + 0.35Q_{111} + 0.3q_{21}^2.$$

The demand price function at the demand market is:

$$\hat{\rho}_1(Q, \hat{q}) = 2250 - (Q_{111} + Q_{211}) + 0.8\hat{q}_1,$$

with the average quality expression given by:

$$\hat{q}_1 = \frac{Q_{111}q_{11} + Q_{211}q_{21}}{Q_{111} + Q_{211}}.$$

Also, we have that there are no positive imposed minimum quality standards, so that:

$$\underline{q}_{11} = \underline{q}_{21} = 0.$$

The Euler method converges in 437 iterations and yields the following equilibrium solution.

$$Q^*_{111} = 323.42, \quad Q^*_{211} = 322.72,$$

$$q^*_{11} = 32.43, \quad q^*_{21} = 16.91,$$

with the equilibrium demand at the demand market being $d^*_1 = 646.14$, and the average quality level at R_1, $\hat{q}_1$, being 24.68.

The incurred demand market price at the equilibrium is:

$$\hat{\rho}_1 = 1623.60.$$

The profits of the firms are, respectively, 311,926.68 and 313,070.55.

The Jacobian matrix of $F(X) = -\nabla U(Q, q)$ for this problem and evaluated at the equilibrium point is:

$$J(Q_{111}, Q_{211}, q_{11}, q_{21}) = \begin{pmatrix} 5.99 & 1.01 & -0.35 & -0.20 \\ 0.99 & 6.01 & -0.20 & -0.30 \\ -0.35 & 2.00 & 1.50 & 0 \\ 0.20 & -0.30 & 0 & 1.90 \end{pmatrix}.$$

The eigenvalues of $\frac{1}{2}(J + J^T)$ are: 1.47, 1.88, 5.03, and 7.02, and are all positive.

Thus, $F(X^*)$ is strongly monotone, the equilibrium solution is unique, and the conditions for convergence of the algorithm are also satisfied (cf. Theorem 5).

Moreover, according to Theorem 4, the equilibrium solution X^* to this example is exponentially stable.

We conducted sensitivity analysis by varying $\underline{q}_{11}$ and $\underline{q}_{21}$ beginning with their values set at 0 and increasing them to reflect the imposition of minimum quality standards set to 200, 400, 600, 800, and 1000.

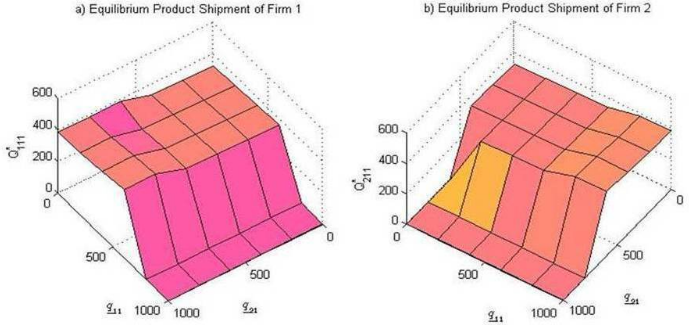

Figure: Equilibrium Product Shipments as $\underline{q}_{11}$ and $\underline{q}_{21}$ Vary in Example 1

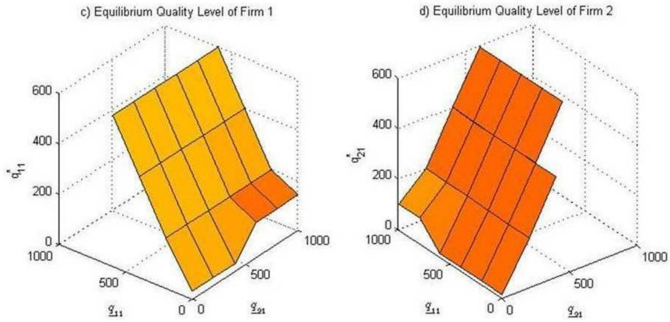

Figure: Equilibrium Quality Levels as $\underline{q}_{11}$ and $\underline{q}_{21}$ Vary in Example 1

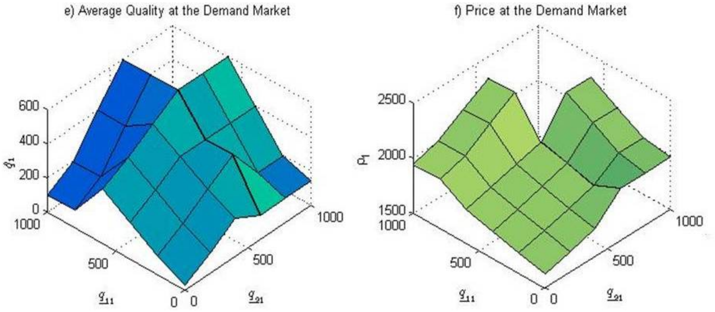

Figure: Average Quality at the Demand Market and Price at the Demand Market as $\underline{q}_{11}$ and $\underline{q}_{21}$ Vary in Example 1

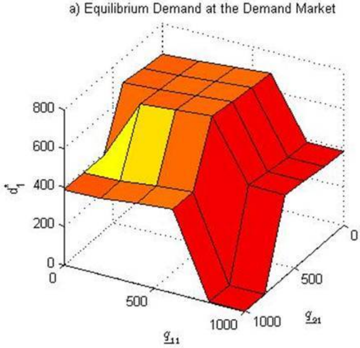

Figure: Equilibrium Demand at R_1 as $\underline{q}_{11}$ and $\underline{q}_{21}$ Vary in Example 1

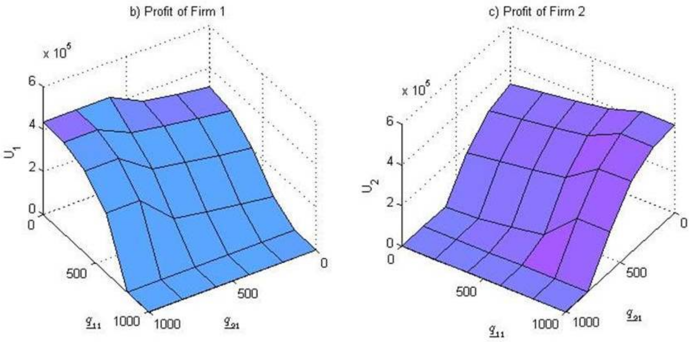

Figure: The Profits of the Firms as $\underline{q}_{11}$ and $\underline{q}_{21}$ Vary in Example 1

- As the minimum quality standard of a firm increases, its equilibrium quality level increases, and its equilibrium shipment quantity decreases as does its profit.
- A firm prefers a free ride, that is, it prefers that the other firm improve its product quality and, hence, the price, rather than have it increase its own quality.
- When there is an enforced higher minimum quality standard imposed on a firm's plant(s), the firm is forced to achieve a higher quality level, which may bring its own profit down but raise the competitor's profit.
- Ronnen (1991): "low-quality sellers can be better off ... and high-quality sellers are worse off."

  Akerlof (1970): "good cars may be driven out of the market by lemons."

  The lower the competitor's quality level, the more harmful the competitor is to the firm with the high minimum quality standard.

The implications of the sensitivity analysis for policy-makers are clear – the imposition of a one-sided quality standard can have a negative impact on the firm in one's region (or country).

Moreover, policy-makers should prevent firms located in regions with very low minimum quality standards from entering the market; otherwise, they may not only bring the average quality level at the demand market(s) down and hurt the consumers, but such products may also harm the profits of the other firms with much higher quality levels and even drive them out of the market.

Example 2 is built from Example 1. We assume that the new plant for each firm has the same associated data as its original one. This would represent a scenario in which each firm builds an identical plant in proximity to its original one.

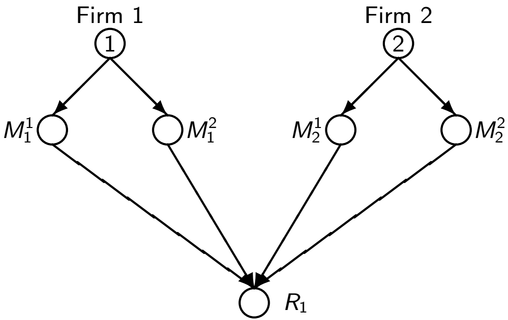

Figure: The Supply Chain Network Topology for Examples 2 and 3

The production cost functions at the new manufacturing plants are:

$$\hat{f}_{12}(Q_{121}, q_{12}) = 0.8Q_{121}^2 + 0.5Q_{121} + 0.25Q_{121}q_{12} + 0.5q_{12}^2,$$

$$\hat{f}_{22}(Q_{221}, q_{22}) = Q_{221}^2 + 0.8Q_{221} + 0.3Q_{221}q_{22} + 0.65q_{22}^2.$$

The total transportation cost functions on the new links are:

$$\hat{c}_{121}(Q_{121}, q_{12}) = 1.2Q_{121}^2 + Q_{121} + 0.25Q_{221} + 0.25q_{12}^2,$$

$$\hat{c}_{221}(Q_{221}, q_{22}) = Q_{221}^2 + Q_{221} + 0.35Q_{121} + 0.3q_{22}^2.$$

The demand price function retains its functional form, but with the new potential shipments added so that:

$$\hat{\rho}_1 = 2250 - (Q_{111} + Q_{211} + Q_{121} + Q_{221}) + 0.8\hat{q}_1,$$

with the average quality at R_1 expressed as:

$$\hat{q}_1 = \frac{Q_{111}q_{11} + Q_{211}q_{21} + Q_{121}q_{12} + Q_{221}q_{22}}{Q_{111} + Q_{211} + Q_{121} + Q_{211}}.$$

Also, at the new manufacturing plants we have that, as in the original ones:

$$\underline{q}_{12} = \underline{q}_{22} = 0.$$

The Euler method converges in 408 iterations to the following equilibrium solution.

$$Q^*_{111} = 225.96, \quad Q^*_{121} = 225.96, \quad Q^*_{211} = 225.54, \quad Q^*_{221} = 225.54.$$

$$q^*_{11} = 22.65, \quad q^*_{12} = 22.65, \quad q^*_{21} = 11.83, \quad q^*_{22} = 11.83,$$

The equilibrium demand at R_1 is, hence, $d^*_1 = 903$. The average quality level, $\hat{q}_1$, now equal to 17.24.

Note that the average quality level has dropped precipitously from its value of 24.68 in Example 1.

The incurred demand market price at R_1 is:

$$\hat{\rho}_1 = 1,360.78.$$

The profits of the firms are, respectively, 406,615.47 and 407,514.97.

The strategy of building an identical plant at the same location as the original one appears to be cost-wise and profitable for the firms; however, at the expense of a decrease in the average quality level at the demand market, as reflected in the results for Example 2.

The Jacobian matrix of $F(X) = -\nabla U(Q, q)$ evaluated at X^* for Example 2, is

$$J(Q_{111}, Q_{121}, Q_{211}, Q_{221}, q_{11}, q_{12}, q_{21}, q_{22})$$

$$= \begin{pmatrix} 5.99 & 1.99 & 1.00 & 1.00 & -0.25 & -0.10 & -0.10 & -0.10 \\ 1.00 & 6.00 & 1.00 & 1.00 & -0.10 & -0.25 & -0.10 & -0.10 \\ 1.00 & 1.00 & 6.00 & 2.01 & -0.10 & -0.10 & -0.20 & -0.10 \\ 1.00 & 1.00 & 2.00 & 6.00 & -0.10 & -0.10 & -0.10 & -0.20 \\ -0.25 & -0.10 & 0.10 & 0.10 & 1.50 & 0 & 0 & 0 \\ -0.10 & -0.25 & 0.10 & 0.10 & 0 & 1.50 & 0 & 0 \\ 0.10 & 0.10 & -0.20 & -0.10 & 0 & 0 & 1.90 & 0 \\ 0.10 & 0.10 & -0.10 & -0.20 & 0 & 0 & 0 & 1.90 \end{pmatrix}.$$

We note that the Jacobian matrix for this example is strictly diagonally dominant, which guarantees its positive-definiteness.

Thus, $F(X^*)$ is strongly monotone, the equilibrium solution X^* is unique, the conditions for convergence of the algorithm are also satisfied, and the equilibrium solution is exponentially stable.

Example 3 is constructed from Example 2, but now the new plant for Firm 1 is located in a country where the production cost is much lower but the total transportation cost to the demand market R_1 is higher.

The location of the second plant of Firm 2 also changes, resulting in both a higher production cost and a higher transportation cost to R_1.

The production cost functions of the new plants are:

$$\hat{f}_{12}(Q_{121}, q_{12}) = 0.3Q_{121}^2 + 0.1Q_{121} + 0.3Q_{121}q_{12} + 0.4q_{12}^2,$$

$$\hat{f}_{22}(Q_{221}, q_{22}) = 1.2Q_{221}^2 + 0.5Q_{221} + 0.3Q_{221}q_{22} + 0.5q_{22}^2.$$

The total transportation cost functions on the new links are now:

$$\hat{c}_{121}(Q_{121}, q_{12}) = 1.8Q_{121}^2 + Q_{121} + 0.25Q_{221} + 0.25q_{12}^2,$$

$$\hat{c}_{221}(Q_{221}, q_{22}) = 1.5Q_{221}^2 + 0.8Q_{221} + 0.3Q_{121} + 0.3q_{22}^2.$$

The Euler method converges in 498 iterations, yielding the equilibrium solution:

$$Q^*_{111} = 232.86, \quad Q^*_{121} = 221.39, \quad Q^*_{211} = 240.82, \quad Q^*_{221} = 178.45,$$

$$q^*_{11} = 25.77, \quad q^*_{12} = 19.76, \quad q^*_{21} = 10.64, \quad q^*_{22} = 9.37,$$

with an equilibrium demand $d^*_1 = 873.52$, and the average quality level at R_1, $\hat{q}_1$, equal to 16.73.

The incurred demand market price is

$$\hat{\rho}_1 = 1,389.86.$$

The profits of the firms are, respectively, 415,706.05 and 378,496.95,

- Because of the high transportation cost to the demand market, the quantity produced at and shipped from M_1^2 decreases, in comparison to the value in Example 2.
- Because of the higher manufacturing cost at Firm 2's foreign plant, M_2^2, the total supply of the product from Firm 2 now decreases.
- The demand at demand market R_1 decreases and the average quality there decreases slightly.

The Jacobian matrix of $F(X) = -\nabla U(Q, q)$ at equilibrium is

$$J(Q_{111}, Q_{121}, Q_{211}, Q_{221}, q_{11}, q_{12}, q_{21}, q_{22})$$

$$= \begin{pmatrix} 5.99 & 1.99 & 1.01 & 1.01 & -0.27 & -0.10 & -0.11 & -0.08 \\ 1.99 & 6.20 & 1.00 & 1.00 & -0.10 & -0.21 & -0.11 & -0.08 \\ 0.99 & 1.00 & 6.01 & 2.01 & -0.11 & -0.11 & -0.20 & -0.08 \\ 0.99 & 1.00 & 2.01 & 7.41 & -0.11 & -0.11 & -0.11 & -0.17 \\ -0.27 & -0.10 & 0.11 & 0.11 & 1.50 & 0 & 0 & 0 \\ -0.10 & -0.21 & 0.11 & 0.11 & 0 & 1.30 & 0 & 0 \\ 0.11 & 0.11 & -0.20 & -0.11 & 0 & 0 & 1.90 & 0 \\ 0.08 & 0.08 & -0.08 & -0.17 & 0 & 0 & 0 & 1.60 \end{pmatrix}.$$

This Jacobian matrix is strictly diagonally dominant, and, hence, it is positive-definite.

Thus, the uniqueness of the computed equilibrium is guaranteed. Also, the conditions for convergence of the algorithm are satisfied. The equilibrium solution is exponentially stable.

There is a new demand market, $R_2$is located closer to both firms' manufacturing plants than demand market R_1.

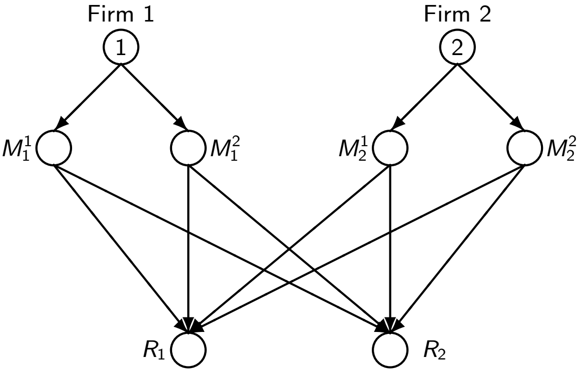

Figure: The Supply Chain Network Topology for Example 4

The total transportation cost functions for transporting the product to R_2 for both firms, respectively, are:

$$\hat{c}_{112}(Q_{112}, q_{11}) = 0.8Q_{112}^2 + Q_{112} + 0.2Q_{212} + 0.05q_{11}^2,$$

$$\hat{c}_{122}(Q_{122}, q_{12}) = 0.75Q_{122}^2 + Q_{122} + 0.25Q_{222} + 0.03q_{12}^2,$$

$$\hat{c}_{212}(Q_{212}, q_{21}) = 0.6Q_{212}^2 + Q_{212} + 0.3Q_{112} + 0.02q_{21}^2,$$

$$\hat{c}_{222}(Q_{222}, q_{22}) = 0.5Q_{222}^2 + 0.8Q_{222} + 0.25Q_{122} + 0.05q_{22}^2.$$

The production cost functions at the manufacturing plants have the same functional forms as in Example 3, but now they include the additional shipments to the new demand market, R_2, that is:

$$\hat{f}_{12}(Q_{121}, Q_{122}, q_{12}) = 0.3(Q_{121}+Q_{122})^2+0.1(Q_{121}+Q_{122})+0.3(Q_{121}+Q_{122})q_{12}+0.4q_{12}^2,$$

$$\hat{f}_{22}(Q_{221}, Q_{222}, q_{22}) = 1.2(Q_{221}+Q_{222})^2+0.5(Q_{221}+Q_{222})+0.3(Q_{221}+Q_{222})q_{22}+0.5q_{22}^2.$$

$$\hat{f}_{11}(Q_{111}, Q_{112}, q_{11}) = 0.8(Q_{111}+Q_{112})^2+0.5(Q_{111}+Q_{112})+0.25(Q_{111}+Q_{112})q_{11}+0.5q_{11}^2,$$

$$\hat{f}_{21}(Q_{211}, Q_{212}, q_{21}) = (Q_{211}+Q_{212})^2+0.8(Q_{211}+Q_{212})+0.3(Q_{211}+Q_{212})q_{21}+0.65q_{21}^2.$$

In this example, consumers at the new demand market R_2 are more sensitive to the quality of the product than consumers at the original demand market R_1. The demand price functions for both the demand markets are, respectively:

$$\hat{\rho}_1 = 2250 - (Q_{111} + Q_{211} + Q_{121} + Q_{221}) + 0.8\hat{q}_1,$$

$$\hat{\rho}_2 = 2250 - (Q_{112} + Q_{122} + Q_{212} + Q_{222}) + 0.9\hat{q}_2,$$

where

$$\hat{q}_1 = \frac{Q_{111}q_{11} + Q_{211}q_{21} + Q_{121}q_{12} + Q_{221}q_{22}}{Q_{111} + Q_{211} + Q_{121} + Q_{211}},$$

and

$$\hat{q}_2 = \frac{Q_{112}q_{11} + Q_{212}q_{21} + Q_{122}q_{12} + Q_{222}q_{22}}{Q_{112} + Q_{212} + Q_{122} + Q_{222}}.$$

The Euler method converges in 597 iterations, and the equilibrium solution is as below.

$$Q^*_{111} = 208.70, \quad Q^*_{121} = 211.82, \quad Q^*_{211} = 203.90, \quad Q^*_{221} = 129.79,$$

$$Q^*_{112} = 165.39, \quad Q^*_{122} = 352.11, \quad Q^*_{212} = 182.30, \quad Q^*_{222} = 200.05.$$

$$q^*_{11} = 53.23, \quad q^*_{12} = 79.08, \quad q^*_{21} = 13.41, \quad q^*_{22} = 13.82.$$

The equilibrium demand at the two demand markets is now $d^*_1 = 754.21$ and $d^*_2 = 899.85$. The value of $\hat{q}_1$ is 42.94 and that of $\hat{q}_2$ is 46.52.

The incurred demand market prices are:

$$\hat{\rho}_1 = 1,530.15, \quad \hat{\rho}_2 = 1,392.03.$$

The profits of the firms are, respectively, 882,342.15 and 651,715.83.

- Due to the addition of R_2, which has associated lower transportation costs, each firm ships more product to demand market R_2 than to R_1. The total demand $d_1 + d_2$ is now 88.76% larger than the total demand d_1 in Example 2.

- The average quality levels increase, which leads to the increase in the prices and both firms' profits.

The Jacobian matrix of $-\nabla U(Q, q)$, for Example 4, evaluated at the equilibrium is

$$J(Q_{111}, Q_{121}, Q_{211}, Q_{221}, Q_{112}, Q_{122}, Q_{212}, Q_{222}, q_{11}, q_{12}, q_{21}, q_{22})$$

$$= \begin{pmatrix}
5.99 & 1.98 & 1.02 & 1.02 & 1.60 & 0 & 0 & 0 & -0.29 & -0.10 & -0.10 & -0.06 \\
1.98 & 6.17 & 1.04 & 1.04 & 0 & 0.60 & 0 & 0 & -0.10 & -0.25 & -0.10 & -0.06 \\
0.98 & 0.96 & 6.03 & 2.03 & 0 & 0 & 2.00 & 0 & -0.12 & -0.13 & -0.17 & -0.08 \\
0.98 & 0.96 & 2.03 & 7.43 & 0 & 0 & 0 & 2.40 & -0.12 & -0.13 & -0.12 & -0.13 \\
1.60 & 0 & 0 & 0 & 5.19 & 1.98 & 1.02 & 1.02 & -0.34 & -0.15 & -0.08 & -0.09 \\
0 & 0.60 & 0 & 0 & 1.98 & 4.07 & 1.03 & 1.03 & -0.07 & -0.37 & -0.08 & -0.09 \\
0 & 0 & 2.00 & 0 & 0.98 & 0.97 & 5.24 & 2.04 & -0.10 & -0.20 & -0.19 & -0.12 \\
0 & 0 & 0 & 2.40 & 0.98 & 0.97 & 2.04 & 5.44 & -0.10 & -0.20 & -0.10 & -0.20 \\
-0.29 & -0.10 & 0.12 & 0.12 & -0.34 & -0.07 & 0.10 & 0.10 & 1.60 & 0 & 0 & 0 \\
-0.10 & -0.25 & 0.13 & 0.13 & -0.15 & -0.37 & 0.20 & 0.20 & 0 & 1.36 & 0 & 0 \\
0.10 & 0.10 & -0.17 & -0.12 & 0.08 & 0.08 & -0.19 & -0.10 & 0 & 0 & 1.94 & 0 \\
0.06 & 0.06 & -0.08 & -0.13 & 0.09 & 0.09 & -0.12 & -0.20 & 0 & 0 & 0 & 1.70
\end{pmatrix}.$$

The eigenvalues of $\frac{1}{2}(J + J^T)$ are all positive and are: 1.29, 1.55, 1.66, 1.71, 1.93, 2.04, 3.76, 4.73, 6.14, 7.55, 8.01, and 11.78.

Therefore, both the uniqueness of the equilibrium solution and the conditions for convergence of the algorithm are guaranteed.

The equilibrium solution to Example 4 is exponentially stable.

I multiply the coefficient of the second Q_{ijk} term, that is, the linear one, in each of the transportation cost functions $\hat{c}_{ijk}$ by a positive factor β, but retain the other transportation cost functions as in Example 4. I vary β from 0 to 50, 100, 150, 200, 250, 300, and 350.

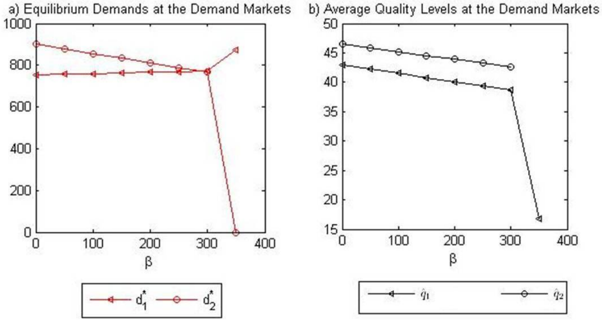

Figure: The Equilibrium Demands and Average Quality Levels as β Varies in Example 4

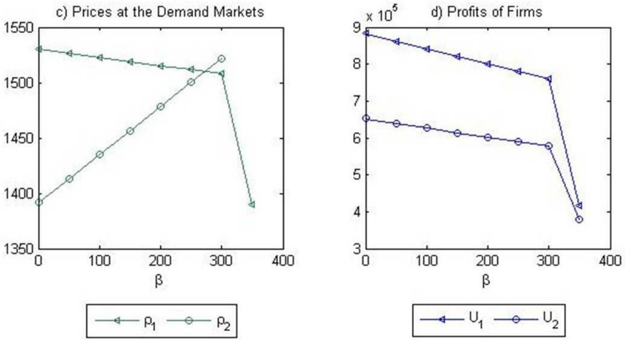

Figure: Prices at the Demand Markets and the Profits of the Firms as β Varies in Example 4

As β increases, that is, as R_2 is located farther, the transportation costs to R_2 increase.

- Firms ship less of the product to R_2 while their shipments to R_1 increase. At the same time, firms cannot afford higher quality as the total costs of both firms increase, so the average quality levels at both demand markets decrease.
- Due to the changes in the demands and the average quality levels, the price at R_1 decreases, but that at R_2 increases, and the profits of both firms decrease.
- When $\beta = 350$, demand market R_2 will be removed from the supply chain network, due to the demand there dropping to zero. Thus, when $\beta = 350$, the results of Example 4 are the same as those for Example 3.

- We developed a rigorous framework for the modeling, analysis, and computation of solutions to competitive supply chain network problems in static and dynamic settings in which there is information asymmetry in quality.
- We also demonstrated how our framework can capture the inclusion of policy interventions in the form of minimum quality standards.
- It contributes to the literature on supply chains with quality competition and reveals the spectrum of insights that can be obtained through computations, supported by theoretical analysis.
- Finally, it contributes to the integration of economics with operations research and the management sciences.

In future research, we plan on exploring issues and applications of information asymmetry in quality in various imperfectly competitive environments, including those arising in healthcare settings. We also intend to assess the value of product differentiation for both producers and consumers alike and the role that minimum quality standards can play in such settings.

Thank you!

- For more information, please visit http://supernet.isenberg.umass.edu.