Quality Competition in Supply Chain Networks with Applications to Information Asymmetry, Product Differentiation, Outsourcing, and Supplier Selection

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Supply Chain Networks

Supply chains are networks consisting of multiple decision-makers, such as manufacturers, transporters/distributors, and retailers, that participate in the processes of the production, delivery, and sales of goods as well as services so as to satisfy consumers at the demand markets (cf. Nagurney (2006)).

Supply Chain Networks - Quality

The quality of the products being produced and delivered in supply chain networks has been recognized as "the single most important force leading to the economic growth of companies in international markets" (Feigenbaum (1982)), and the most important factor affecting a business unit's performance and competitiveness (Buzzell and Gale (1987)).

Poor quality products, whether inferior durable goods such as automobiles, or consumables such as pharmaceuticals and food, may negatively affect the safety and the well-being of consumers with, possibly, associated fatal consequences.

Overview

In this dissertation, I contribute to the modeling and analysis of quality competition in supply chain networks under scenarios of information asymmetry, product differentiation, outsourcing, and under supplier selection.

Overview

Specifically, the dissertation addresses the following fundamental questions:

- What are the equilibrium product quality levels of competing firms and how to compute their values?
- How do these quality levels evolve over time until the equilibrium is achieved?
- How stable are the equilibria?
- What are the impacts on product quality, costs, and profits, of minimum quality regulations?

Definitions of Quality

- Excellence (e.g., Tuchman (1980), Garvin (1984), and Pirsig (1992)).
- Value (Feigenbaum (1951), Abbott (1955), and Cronin and Taylor (1992)).
- The extent to which a product or service meets and/or exceeds a customer's expectations (Feigenbaum (1983), Parasuraman, Zeithaml, and Berry (1985), Buzzell and Gale (1987), and Grönroos (1990)).
- The degree to which a specific product conforms to a design or specification (Shewhart (1931), Juran (1951), Levitt (1972), Gilmore (1974), Crosby (1979), Deming (1986), and Chase and Aquilano (1992)).

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Quality Cost

- "Cost incurred in ensuring and assuring quality as well as the loss incurred when quality is not achieved" (ASQC (1971) and BS (1990)).
- Four categories of quality-related costs: the prevention cost, the appraisal cost, the internal failure cost, and the external failure cost.
- Quality cost is understood as the sum of the four categories of quality-related costs, and they are convex functions of quality conformance level.
- The external failure cost, which is the compensation cost incurred when customers are dissatisfied with the quality of the products, is utilized to measure the disrepute of the firm in addition to the cost of quality in this dissertation.

Methodologies

- Variational Inequality Theory
- Projected Dynamical Systems
- Game Theory
- Multicriteria Decision-Making
- Algorithm - Euler Method & Modified Projection Method

Variational Inequality

Definition 2.1

The finite-dimensional variational inequality problem, $\mathrm{VI}(F, \mathcal{K})$*, is to determine a vector* $X^* \in \mathcal{K} \subset \mathcal{R}^n$*, such that*

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{2.1}$$

where F *is a given continuous function from* $\mathcal{K}$ *to* $\mathcal{R}^n$*,* $\mathcal{K}$ *is a given closed convex set, and* $\langle \cdot, \cdot \rangle$ *denotes the inner product in n-dimensional Euclidean space.*

Projected Dynamical Systems

The class of ordinary differential equations that this dissertation focuses on takes on the following form:

$$\dot{X} = \Pi_{\mathcal{K}}(X, -F(X)), \quad X(0) = X_0 \in \mathcal{K}, \tag{2.19}$$

where $\dot{X}$ *denotes the rate of change of vector* X, $\mathcal{K}$ *is closed convex set, corresponding to the constraint set in a particular application, and* $F(X)$ *is a vector field defined on* $\mathcal{K}$.

Theorem 2.12

Assume that $\mathcal{K}$ *is a convex polyhedron. Then the equilibrium points of the* $\mathrm{PDS}(\mathrm{F}, \mathcal{K})$ *coincide with the solutions of* $\mathrm{VI}(\mathrm{F}, \mathcal{K})$. *Hence, for* $X^* \in \mathcal{K}$ *and satisfying*

$$\dot{X} = 0 = \Pi_{\mathcal{K}}(X^*, -F(X^*)) \tag{2.22}$$

also satisfies

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{2.23}$$

This section corresponds to Chapter 3 of the dissertation, and is based on the paper:

Nagurney, A., Li, D., 2014a. Equilibria and dynamics of supply chain network competition with information asymmetry in quality and minimum quality standards, *Computational Management Science* 11(3), 285-315.

where a full list of references can be found.

Background and Motivation

Given the distances that may be involved as well as the types of products that are consumed in supply chain networks, there may be information asymmetry associated with knowledge about the quality of the products. Specifically, when there is no differentiation by brands or labels, products from different firms are viewed as being homogeneous for consumers.

Literature Review

- Akerlof, G. A., 1970. The market for 'lemons': Quality uncertainty and the market mechanism. *Quarterly Journal of Economics* 84(3), 488-500.
- Leland, H. E., 1979. Quacks, lemons, and licensing: A theory of minimum quality standards. *Journal of Political Economy* 87(6), 1328-1346.
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- Baltzer, K., 2012. Standards vs. labels with imperfect competition and asymmetric information. *Economics Letters* 114(1), 61-63.

Overview

I develop both static and dynamic competitive supply chain network models with information asymmetry in quality.

- The information asymmetry in quality occurs between the firms, producing the product, and the consumers.
- I consider multiple profit-maximizing firms, which may have multiple plants at their disposal.
- The firms compete in multiple demand markets in product shipments and product quality levels.
- Quality is associated with the manufacturing plants, and is also tracked through the transportation process
- I demonstrate how minimum quality standards can be incorporated into the framework, which has wide relevance for policy-making and regulation.

Supply Chain Network Topology

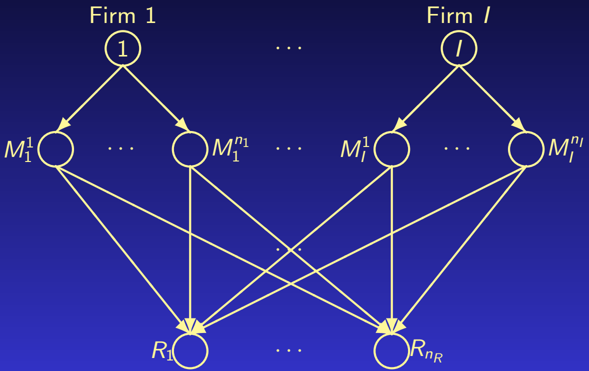


Figure: Supply Chain Network Topology

The Equilibrium Model

Conservation of flow equations

$$s_{ij} = \sum_{k=1}^{n_R} Q_{ijk}, \quad i = 1, \ldots, I; j = 1, \ldots, n_i, \tag{3.1}$$

$$d_k = \sum_{i=1}^{I} \sum_{j=1}^{n_i} Q_{ijk}, \quad k = 1, \ldots, n_R, \tag{3.2}$$

$$Q_{ijk} \geq 0, \quad i = 1, \ldots, I; j = 1, \ldots, n_i; k = 1, \ldots, n_R. \tag{3.3}$$

For each firm i, I group its Q_{ijk}s into the vector $Q_i \in R_+^{n_i n_R}$, and then group all such vectors for all firms into the vector $Q \in R_+^{\sum_{i=1}^{I} n_i n_R}$.

I also group all s_{ij}s into the vector $s \in R_+^{\sum_{i=1}^{I} n_i}$ and all d_ks into the vector $d \in R_+^{n_R}$.

The quality levels cannot be lower than 0% conformance level; thus,

Nonnegative quality level of firm i's manufacturing plant M_i^j

$$q_{ij} \geq 0, \quad i = 1, \ldots, I; j = 1, \ldots, n_i. \tag{3.4}$$

For each firm i, I group its own plant quality levels into the vector $q_i \in R_+^{n_i}$ and then group all such vectors for all firms into the vector $q \in R_+^{\sum_{i=1}^{I} n_i}$.

Production cost function at firm i's manufacturing plant M_i^j

$$f_{ij} = f_{ij}(s, q), \quad i = 1, \ldots, I; j = 1, \ldots, n_i. \tag{3.5a}$$

In view of (3.1),

$$\hat{f}_{ij} = \hat{f}_{ij}(Q, q) \equiv f_{ij}(s, q), \quad i = 1, \ldots, I; j = 1, \ldots, n_i. \tag{3.5b}$$

Transportation cost function associated with shipping the product produced at firm i's manufacturing plant M_i^j to demand market R_k

$$\hat{c}_{ijk} = \hat{c}_{ijk}(Q, q), \quad i = 1, \ldots, I; j = 1, \ldots, n_i; k = 1, \ldots, n_R. \tag{3.6}$$

The transportation cost is such that the quality of the product is not degraded as it undergoes the shipment process.

The production cost functions and the transportation functions are assumed to be convex, continuous, and twice continuously differentiable.

Consumers' perception of the quality of all such product, which may come from different firms, is for the average quality level.

Consumers' perception of the quality of the product at demand market R_k

$$\hat{q}_k = \frac{\sum_{i=1}^{I} \sum_{j=1}^{n_i} Q_{ijk} q_{ij}}{d_k}, \quad k = 1, \ldots, n_R \tag{3.7}$$

with the average (perceived) quality levels grouped into the vector $\hat{q} \in R_+^{n_R}$.

The demand price at demand market R_k

$$\rho_k = \rho_k(d, \hat{q}), \quad k = 1, \ldots, n_R. \tag{3.8a}$$

In light of (3.2) and (3.7),

$$\hat{\rho}_k = \hat{\rho}_k(Q, q) \equiv \rho_k(d, \hat{q}), \quad k = 1, \ldots, n_R. \tag{3.8b}$$

Each demand price function is, typically, assumed to be monotonically decreasing in product quantity but increasing in terms of the average product quality.

The demand price functions are continuous and twice continuously differentiable.

The strategic variables of firm i are its product shipments $\{Q_i\}$ and its quality levels q_i.

The profit/utility U_i of firm i; $i = 1, \ldots, I$

$$U_i = \sum_{k=1}^{n_R} \rho_k(d, \hat{q}) \sum_{j=1}^{n_i} Q_{ijk} - \sum_{j=1}^{n_i} f_{ij}(s, q) - \sum_{k=1}^{n_R} \sum_{j=1}^{n_i} \hat{c}_{ijk}(Q, q), \quad (3.9a)$$

which is equivalent to

$$U_i = \sum_{k=1}^{n_R} \hat{\rho}_k(Q, q) \sum_{j=1}^{n_i} Q_{ijk} - \sum_{j=1}^{n_i} \hat{f}_{ij}(Q, q) - \sum_{k=1}^{n_R} \sum_{j=1}^{n_i} \hat{c}_{ijk}(Q, q). \quad (3.9b)$$

Assume that for each firm i the profit function $U_i(Q, q)$ is concave with respect to the variables in Q_i and q_i, and is continuous and twice continuously differentiable.

Definition: A Cournot-Nash Equilibrium

Let K^i denote the feasible set corresponding to firm i, where $K^i \equiv \{(Q_i, q_i) | Q_i \geq 0, \text{ and } q_i \geq 0\}$ and define $K \equiv \prod_{i=1}^{I} K^i$.

A product shipment and quality level pattern $(Q^*, q^*) \in K$ is said to constitute a supply chain network Cournot-Nash equilibrium with information asymmetry in quality if for each firm i; $i = 1, \ldots, I$,

$$U_i(Q_i^*, q_i^*, \hat{Q}_i^*, \hat{q}_i^*) \geq U_i(Q_i, q_i, \hat{Q}_i^*, \hat{q}_i^*), \quad \forall (Q_i, q_i) \in K^i, \tag{3.11}$$

where

$$\hat{Q}_i^* \equiv (Q_1^*, \ldots, Q_{i-1}^*, Q_{i+1}^*, \ldots, Q_I^*) \quad \textit{and} \quad \hat{q}_i^* \equiv (q_1^*, \ldots, q_{i-1}^*, q_{i+1}^*, \ldots, q_I^*).$$

Variational Inequality Formulation

Theorem 3.1

Then the product shipment and quality pattern $(Q^*, q^*) \in K$ is a supply chain network Cournot-Nash equilibrium with quality information asymmetry if and only if it satisfies the variational inequality

$$-\sum_{i=1}^{I}\sum_{j=1}^{n_i}\sum_{k=1}^{n_R}\frac{\partial U_i(Q^*, q^*)}{\partial Q_{ijk}}\times(Q_{ijk}-Q^*_{ijk})-\sum_{i=1}^{I}\sum_{j=1}^{n_i}\frac{\partial U_i(Q^*, q^*)}{\partial q_{ij}}\times(q_{ij}-q^*_{ij})\geq 0,$$

$$\forall (Q, q) \in K, \tag{3.12}$$

$$
\begin{aligned}
&\sum_{i=1}^{I}\sum_{j=1}^{n_i}\sum_{k=1}^{n_R}\left[-\hat{\rho}_k(Q^*,q^*)-\sum_{l=1}^{n_R}\frac{\partial\hat{\rho}_l(Q^*,q^*)}{\partial Q_{ijk}}\sum_{h=1}^{n_i}Q^*_{ihl}+\sum_{h=1}^{n_i}\frac{\partial\hat{f}_{ih}(Q^*,q^*)}{\partial Q_{ijk}}\right.\\
&\left.+\sum_{h=1}^{n_i}\sum_{l=1}^{n_R}\frac{\partial\hat{c}_{ihl}(Q^*,q^*)}{\partial Q_{ijk}}\right]\times(Q_{ijk}-Q^*_{ijk})\\
&+\sum_{i=1}^{I}\sum_{j=1}^{n_i}\left[-\sum_{k=1}^{n_R}\frac{\partial\hat{\rho}_k(Q^*,q^*)}{\partial q_{ij}}\sum_{h=1}^{n_i}Q^*_{ihk}+\sum_{h=1}^{n_i}\frac{\partial\hat{f}_{ih}(Q^*,q^*)}{\partial q_{ij}}\right.\\
&\left.+\sum_{h=1}^{n_i}\sum_{k=1}^{n_R}\frac{\partial\hat{c}_{ihk}(Q^*,q^*)}{\partial q_{ij}}\right]\times(q_{ij}-q^*_{ij})\geq 0,\quad\forall(Q,q)\in K; \qquad (3.13)
\end{aligned}
$$

$(d^*, s^*, Q^*, q^*) \in K^1$ *is an equilibrium production, shipment, and quality level pattern if and only if it satisfies the variational inequality*

$$\sum_{k=1}^{n_R}[-\rho_k(d^*, \hat{q}^*)] \times (d_k - d_k^*) + \sum_{i=1}^{I}\sum_{j=1}^{n_i}\left[\sum_{h=1}^{n_i}\frac{\partial f_{ih}(s^*, q^*)}{\partial s_{ij}}\right] \times (s_{ij} - s_{ij}^*)$$

$$+\sum_{i=1}^{I}\sum_{j=1}^{n_i}\sum_{k=1}^{n_R}\left[-\sum_{l=1}^{n_R}\frac{\partial \rho_l(d^*, \hat{q}^*)}{\partial Q_{ijk}}\sum_{h=1}^{n_i}Q_{ihl}^* + \sum_{h=1}^{n_i}\sum_{l=1}^{n_R}\frac{\partial \hat{c}_{ihl}(Q^*, q^*)}{\partial Q_{ijk}}\right] \times (Q_{ijk} - Q_{ijk}^*)$$

$$+\sum_{i=1}^{I}\sum_{j=1}^{n_i}\left[-\sum_{k=1}^{n_R}\frac{\partial \rho_k(Q^*, \hat{q}^*)}{\partial q_{ij}}\sum_{h=1}^{n_i}Q_{ihk}^* + \sum_{h=1}^{n_i}\frac{\partial f_{ih}(s^*, q^*)}{\partial q_{ij}}\right.$$

$$\left.+\sum_{h=1}^{n_i}\sum_{k=1}^{n_R}\frac{\partial \hat{c}_{ihk}(Q^*, q^*)}{\partial q_{ij}}\right] \times (q_{ij} - q_{ij}^*) \geq 0, \qquad \forall (d, s, Q, q) \in K^1, \qquad (3.14)$$

where $K^1 \equiv \{(d, s, Q, q) | \, Q \geq 0, \; q \geq 0,$ *and (3.1), (3.2), and (3.7) hold*$\}$.

The Equilibrium Model - With Minimum Quality Standards

I now describe an extension of the above framework that incorporates minimum quality standards.

Nonnegative lower bounds on the quality levels at the manufacturing plants

$$q_{ij} \geq \underline{q}_{ij} \quad i = 1, \ldots, I; j = 1, \ldots, n_i \tag{3.17}$$

with the understanding that, if the lower bounds are all identically equal to zero, then (3.17) collapses to (3.4) and, if the lower bounds are positive, then they represent minimum quality standards.

Integrated Variational Inequality Formulation

I define a new feasible set $K^2 \equiv \{(Q, q) | Q \geq 0 \text{ and } (3.17) \text{ holds}\}$.

Corollary 3.1

The product shipment and quality pattern $(Q^*, q^*) \in K^2$ is a supply chain network Cournot-Nash equilibrium with quality information asymmetry in the presence of minimum quality standards if and only if it satisfies the variational inequality

$$-\sum_{i=1}^{I} \sum_{j=1}^{n_i} \sum_{k=1}^{n_R} \frac{\partial U_i(Q^*, q^*)}{\partial Q_{ijk}} \times (Q_{ijk} - Q^*_{ijk}) - \sum_{i=1}^{I} \sum_{j=1}^{n_i} \frac{\partial U_i(Q^*, q^*)}{\partial q_{ij}} \times (q_{ij} - q^*_{ij}) \geq 0,$$

$$\forall (Q, q) \in K^2, \tag{3.18}$$

$$\sum_{i=1}^{I}\sum_{j=1}^{n_i}\sum_{k=1}^{n_R}\left[-\hat{\rho}_k(Q^*,q^*)-\sum_{l=1}^{n_R}\frac{\partial\hat{\rho}_l(Q^*,q^*)}{\partial Q_{ijk}}\sum_{h=1}^{n_i}Q^*_{ihl}+\sum_{h=1}^{n_i}\frac{\partial\hat{f}_{ih}(Q^*,q^*)}{\partial Q_{ijk}}\right.$$
$$\left.+\sum_{h=1}^{n_i}\sum_{l=1}^{n_R}\frac{\partial\hat{c}_{ihl}(Q^*,q^*)}{\partial Q_{ijk}}\right]\times(Q_{ijk}-Q^*_{ijk})$$
$$+\sum_{i=1}^{I}\sum_{j=1}^{n_i}\left[-\sum_{k=1}^{n_R}\frac{\partial\hat{\rho}_k(Q^*,q^*)}{\partial q_{ij}}\sum_{h=1}^{n_i}Q^*_{ihk}+\sum_{h=1}^{n_i}\frac{\partial\hat{f}_{ih}(Q^*,q^*)}{\partial q_{ij}}\right.$$
$$\left.+\sum_{h=1}^{n_i}\sum_{k=1}^{n_R}\frac{\partial\hat{c}_{ihk}(Q^*,q^*)}{\partial q_{ij}}\right]\times(q_{ij}-q^*_{ij})\geq 0,\qquad\forall(Q,q)\in K^2. \tag{3.19}$$

Variational inequality (3.19) contains variational inequality (3.13) as a special case when the minimum quality standards are all zero.

Standard form VI

Determine $X^* \in \mathcal{K}$ where X is a vector in R^N, $F(X)$ is a continuous function such that $F(X) : X \mapsto \mathcal{K} \subset R^N$, and

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{3.20}$$

I define the vector $X \equiv (Q, q)$ and the vector $F(X) \equiv (F^1(X), F^2(X))$.

$N = \sum_{i=1}^{I} n_i n_R + \sum_{i=1}^{I} n_i$.

$F^1(X)$ consists of $F^1_{ijk} = -\frac{\partial U_i(Q,q)}{\partial Q_{ijk}}$; $i = 1, \ldots, I; j = 1, \ldots, n_i; k = 1, \ldots, n_R$, and $F^2(X)$ consist of $F^2_{ij} = -\frac{\partial U_i(Q,q)}{\partial q_{ij}}$; $i = 1, \ldots, I; j = 1, \ldots, n_i$.

I define the feasible set $\mathcal{K} \equiv K^2$.

The Dynamic Model

A dynamic adjustment process for product shipments and quality levels

$$\dot{Q}_{ijk} = \begin{cases} \frac{\partial U_i(Q,q)}{\partial Q_{ijk}}, & \text{if} \quad Q_{ijk} > 0 \\ \max\{0, \frac{\partial U_i(Q,q)}{\partial Q_{ijk}}\}, & \text{if} \quad Q_{ijk} = 0. \end{cases} \tag{3.23}$$

$$\dot{q}_{ij} = \begin{cases} \frac{\partial U_i(Q,q)}{\partial q_{ij}}, & \text{if} \quad q_{ij} > \underline{q}_{ij} \\ \max\{\underline{q}_{ij}, \frac{\partial U_i(Q,q)}{\partial q_{ij}}\}, & \text{if} \quad q_{ij} = \underline{q}_{ij}. \end{cases} \tag{3.26}$$

Qualitative Properties - Existence and Uniqueness Results

Assumption 3.1

Suppose that in the supply chain network model with information asymmetry in quality there exists a sufficiently large M, such that for any (i, j, k),

$$\frac{\partial U_i(Q, q)}{\partial Q_{ijk}} < 0, \tag{3.28}$$

for all shipment patterns Q with $Q_{ijk} \geq M$ and that there exists a sufficiently large $\bar{M}$, such that for any (i, j),

$$\frac{\partial U_i(Q, q)}{\partial q_{ij}} < 0, \tag{3.29}$$

for all quality level patterns q with $q_{ij} \geq \bar{M} \geq \underline{q}_{ij}$.

Proposition 3.1

Any supply chain network problem with information asymmetry in quality that satisfies Assumption 3.1 possesses at least one equilibrium shipment and quality level pattern satisfying variational inequality (3.19) (or (3.20)).

Proposition 3.2

Suppose that F is strictly monotone at any equilibrium point of the variational inequality problem defined in (3.20). Then it has at most one equilibrium point.

Theorem 3.2

Suppose that F is strongly monotone. Then there exists a unique solution to variational inequality (3.20); equivalently, to variational inequality (3.19).

Qualitative Properties - Stability Properties

Theorem 3.3

(i). *If $F(X)$ is monotone, then every supply chain network equilibrium with information asymmetry, X^*, provided its existence, is a global monotone attractor for the projected dynamical system. If $F(X)$ is locally monotone at X^*, then it is a monotone attractor for the projected dynamical system.*

(ii). *If $F(X)$ is strictly monotone, the unique equilibrium X^*, given existence, is a strictly global monotone attractor for the projected dynamical system. If $F(X)$ is locally strictly monotone at X^*, then it is a strictly monotone attractor for the projected dynamical system.*

(iii). *If $F(X)$ is strongly monotone, then the unique supply chain network equilibrium with information asymmetry in quality, which is guaranteed to exist, is also globally exponentially stable for the projected dynamical system. If $F(X)$ is locally strongly monotone at X^*, then it is exponentially stable.*

The Algorithm - The Euler Method

Iteration τ of the Euler method

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - a_{\tau} F(X^{\tau})),$$

where $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$ and F is the function that enters the variational inequality problem (3.19).

For convergence of the general iterative scheme, which induces the Euler method, the sequence $\{a_{\tau}\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_{\tau} = \infty$, $a_{\tau} > 0$, $a_{\tau} \to 0$, as $\tau \to \infty$.

Explicit Formulae for the Computation of the Product Shipments and Quality Levels

$$Q_{ijk}^{\tau+1} = \max\{0, Q_{ijk}^{\tau} + a_\tau(\hat{\rho}_k(Q^\tau, q^\tau) + \sum_{l=1}^{n_R} \frac{\partial \hat{\rho}_l(Q^\tau, q^\tau)}{\partial Q_{ijk}} \sum_{h=1}^{n_i} Q_{ihl}^{\tau} - \sum_{h=1}^{n_i} \frac{\partial \hat{f}_{ih}(Q^\tau, q^\tau)}{\partial Q_{ijk}} - \sum_{h=1}^{n_i} \sum_{l=1}^{n_R} \frac{\partial \hat{c}_{ihl}(Q^\tau, q^\tau)}{\partial Q_{ijk}})\} \tag{3.30}$$

$$q_{ij}^{\tau+1} = \max\{\underline{q}_{ij}, q_{ij}^{\tau} + a_\tau(\sum_{k=1}^{n_R} \frac{\partial \hat{\rho}_k(Q^\tau, q^\tau)}{\partial q_{ij}} \sum_{h=1}^{n_i} Q_{ihk}^{\tau} - \sum_{h=1}^{n_i} \frac{\partial \hat{f}_{ih}(Q^\tau, q^\tau)}{\partial q_{ij}} - \sum_{h=1}^{n_i} \sum_{k=1}^{n_R} \frac{\partial \hat{c}_{ihk}(Q^\tau, q^\tau)}{\partial q_{ij}})\}. \tag{3.31}$$

Numerical Examples - Example 3.1

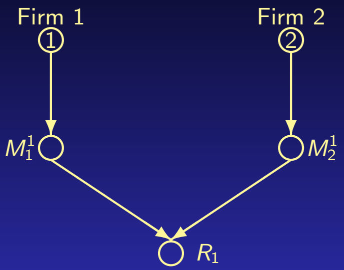


Figure: Supply Chain Network Topology for Example 3.1

The production cost functions are:

$$\hat{f}_{11}(Q_{111}, q_{11}) = 0.8Q_{111}^2 + 0.5Q_{111} + 0.25Q_{111}q_{11} + 0.5q_{11}^2,$$

$$\hat{f}_{21}(Q_{211}, q_{21}) = Q_{211}^2 + 0.8Q_{211} + 0.3Q_{211}q_{21} + 0.65q_{21}^2.$$

The total transportation cost functions are:

$$\hat{c}_{111}(Q_{111}, q_{11}) = 1.2Q_{111}^2 + Q_{111} + 0.25Q_{211} + 0.25q_{11}^2,$$

$$\hat{c}_{211}(Q_{211}, q_{21}) = Q_{211}^2 + Q_{211} + 0.35Q_{111} + 0.3q_{21}^2.$$

The demand price function at the demand market is:

$$\hat{\rho}_1(Q, \hat{q}) = 2250 - (Q_{111} + Q_{211}) + 0.8\hat{q}_1,$$

with the average quality expression given by:

$$\hat{q}_1 = \frac{Q_{111}q_{11} + Q_{211}q_{21}}{Q_{111} + Q_{211}}.$$

Also, there are no positive imposed minimum quality standards, so that:

$$\underline{q}_{11} = \underline{q}_{21} = 0.$$

The Euler method yields the following equilibrium solution.

$$Q^*_{111} = 323.42, \quad Q^*_{211} = 322.72,$$

$$q^*_{11} = 32.43, \quad q^*_{21} = 16.91,$$

with the equilibrium demand at the demand market being $d^*_1 = 646.14$, and the average quality level at R_1, $\hat{q}_1$, being 24.68.

The incurred demand market price at the equilibrium is:

$$\hat{\rho}_1 = 1623.60.$$

The profits of the firms are, respectively, 311,926.68 and 313,070.55.

The Jacobian matrix of $F(X) = -\nabla U(Q, q)$ for this problem and evaluated at the equilibrium point is:

$$J(Q_{111}, Q_{211}, q_{11}, q_{21}) = \begin{pmatrix} 5.99 & 1.01 & -0.35 & -0.20 \\ 0.99 & 6.01 & -0.20 & -0.30 \\ -0.35 & 2.00 & 1.50 & 0 \\ 0.20 & -0.30 & 0 & 1.90 \end{pmatrix}.$$

The eigenvalues of $\frac{1}{2}(J + J^T)$ are: 1.47, 1.88, 5.03, and 7.02, and are all positive.

Thus, the equilibrium solution is unique, and the conditions for convergence of the algorithm are also satisfied.

Moreover, according to Theorem 3.3, the equilibrium solution X^* to this example is a strictly monotone attractor and it is also exponentially stable.

Example 3.1 - Sensitivity Analysis

I conducted sensitivity analysis by varying $\underline{q}_{11}$ and $\underline{q}_{21}$ beginning with their values set at 0 and increasing them to reflect the imposition of minimum quality standards set to 200, 400, 600, 800, and 1000.

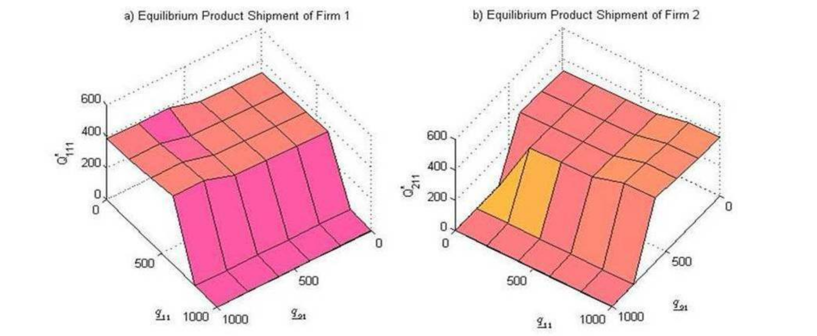


Figure: Equilibrium Product Shipments as $\underline{q}_{11}$ and $\underline{q}_{21}$ Vary in Example 3.1

A Supply Chain Network Model with Information Asymmetry in Quality, Minimum Quality Standards, and Quality Competition

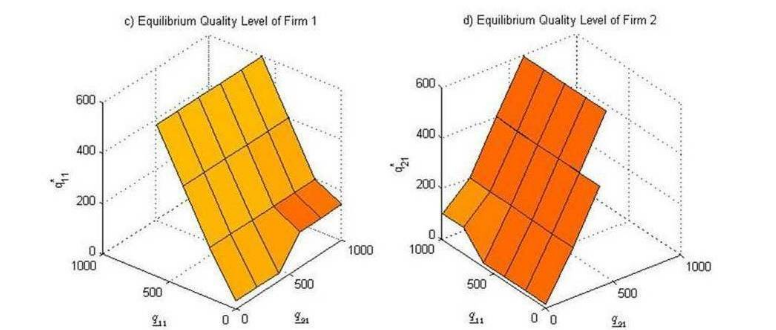


Figure: Equilibrium Quality Levels as $\underline{q}_{11}$ and $\underline{q}_{21}$ Vary in Example 3.1

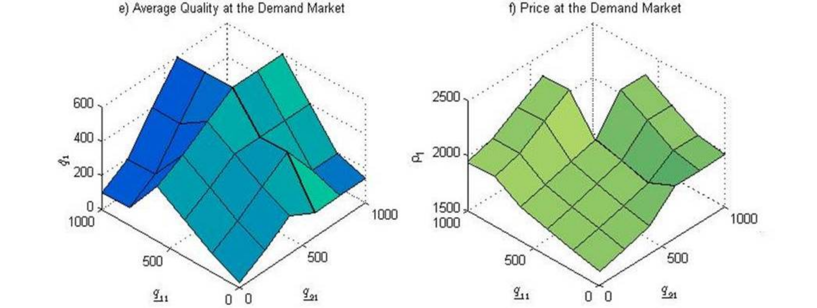


Figure: Average Quality and The Price at R_1 as $\underline{q}_{11}$ and $\underline{q}_{21}$ Vary in Example 3.1

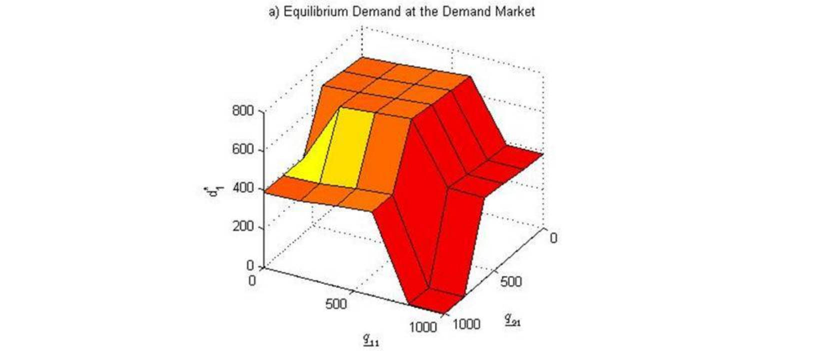


Figure: Equilibrium Demand at R_1 as $\underline{q}_{11}$ and $\underline{q}_{21}$ Vary in Example 3.1

A Supply Chain Network Model with Information Asymmetry in Quality, Minimum Quality Standards, and Quality Competition

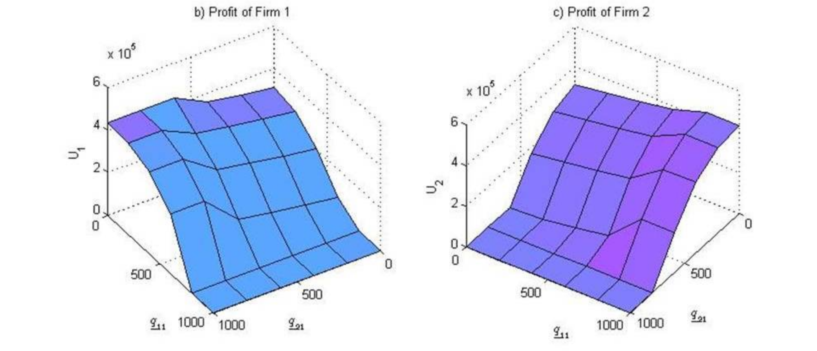


Figure: The Profits of the Firms as $\underline{q}_{11}$ and $\underline{q}_{21}$ Vary in Example 3.1

- Ronnen (1991): "low-quality sellers can be better off ... and high-quality sellers are worse off."
- Akerlof (1970): "good cars may be driven out of the market by lemons."
- The lower the competitor's quality level, the more harmful the competitor is to the firm with the high minimum quality standard.

Numerical Examples - Example 3.2

Example 3.2 is built from Example 3.1. I assume that the new plant for each firm has the same associated data as its original one. This would represent a scenario in which each firm builds an identical plant in proximity to its original one.

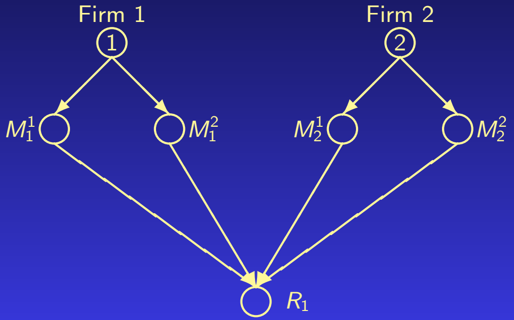


Figure: Supply Chain Network Topology for Example 3.2

The production cost functions at the new manufacturing plants are:

$$\hat{f}_{12}(Q_{121}, q_{12}) = 0.8Q_{121}^2 + 0.5Q_{121} + 0.25Q_{121}q_{12} + 0.5q_{12}^2,$$

$$\hat{f}_{22}(Q_{221}, q_{22}) = Q_{221}^2 + 0.8Q_{221} + 0.3Q_{221}q_{22} + 0.65q_{22}^2.$$

The total transportation cost functions on the new links are:

$$\hat{c}_{121}(Q_{121}, q_{12}) = 1.2Q_{121}^2 + Q_{121} + 0.25Q_{221} + 0.25q_{12}^2,$$

$$\hat{c}_{221}(Q_{221}, q_{22}) = Q_{221}^2 + Q_{221} + 0.35Q_{121} + 0.3q_{22}^2.$$

The demand price function retains its functional form, but with the new potential shipments added so that:

$$\hat{\rho}_1 = 2250 - (Q_{111} + Q_{211} + Q_{121} + Q_{221}) + 0.8\hat{q}_1,$$

with the average quality at R_1 expressed as:

$$\hat{q}_1 = \frac{Q_{111}q_{11} + Q_{211}q_{21} + Q_{121}q_{12} + Q_{221}q_{22}}{Q_{111} + Q_{211} + Q_{121} + Q_{211}}.$$

Also, at the new manufacturing plants, as in the original ones:

$$\underline{q}_{12} = \underline{q}_{22} = 0.$$

The Euler method converges to the following equilibrium solution.

$$Q_{111}^* = 225.96, \quad Q_{121}^* = 225.96, \quad Q_{211}^* = 225.54, \quad Q_{221}^* = 225.54.$$

$$q_{11}^* = 22.65, \quad q_{12}^* = 22.65, \quad q_{21}^* = 11.83, \quad q_{22}^* = 11.83,$$

The equilibrium demand at R_1 is, hence, $d_1^* = 903$. The average quality level, $\hat{q}_1$, now equal to 17.24.

Note that the average quality level has dropped precipitously from its value of 24.68 in Example 3.1.

The incurred demand market price at R_1 is:

$$\hat{\rho}_1 = 1,360.78.$$

The profits of the firms are, respectively, 406,615.47 and 407,514.97.

The strategy of building an identical plant at the same location as the original one appears to be cost-wise and profitable for the firms; however, at the expense of a decrease in the average quality level at the demand market, as reflected in the results for Example 3.2.

The Jacobian matrix of $F(X) = -\nabla U(Q,q)$ evaluated at X^* for Example 3.2, is

$$J(Q_{111}, Q_{121}, Q_{211}, Q_{221}, q_{11}, q_{12}, q_{21}, q_{22})$$
$$= \begin{pmatrix} 5.99 & 1.99 & 1.00 & 1.00 & -0.25 & -0.10 & -0.10 & -0.10 \\ 1.00 & 6.00 & 1.00 & 1.00 & -0.10 & -0.25 & -0.10 & -0.10 \\ 1.00 & 1.00 & 6.00 & 2.01 & -0.10 & -0.10 & -0.20 & -0.10 \\ 1.00 & 1.00 & 2.00 & 6.00 & -0.10 & -0.10 & -0.10 & -0.20 \\ -0.25 & -0.10 & 0.10 & 0.10 & 1.50 & 0 & 0 & 0 \\ -0.10 & -0.25 & 0.10 & 0.10 & 0 & 1.50 & 0 & 0 \\ 0.10 & 0.10 & -0.20 & -0.10 & 0 & 0 & 1.90 & 0 \\ 0.10 & 0.10 & -0.10 & -0.20 & 0 & 0 & 0 & 1.90 \end{pmatrix}.$$

The Jacobian matrix for this example is strictly diagonally dominant, which guarantees its positive-definiteness.

Thus, the equilibrium solution X^* is unique, the conditions for convergence of the algorithm are also satisfied, and the equilibrium solution is a strictly monotone attractor. Moreover, X^* is exponentially stable.

Numerical Examples - Example 3.4

In Example 3.4, there is a new demand market, R_2, added to Example 3.3, which is located closer to both firms' manufacturing plants than the original demand market R_1.

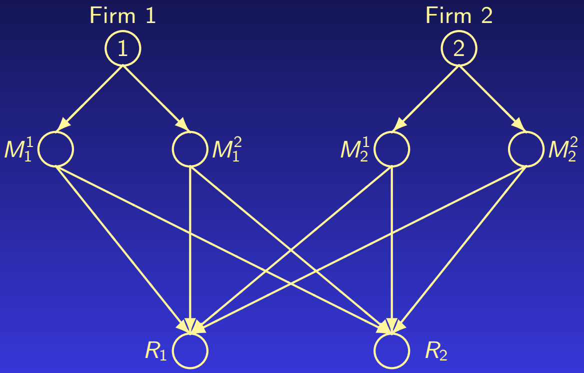


Figure: Supply Chain Network Topology for Example 3.4

The total transportation cost functions for transporting the product to R_2 for both firms, respectively, are:

$$\hat{c}_{112}(Q_{112}, q_{11}) = 0.8Q_{112}^2 + Q_{112} + 0.2Q_{212} + 0.05q_{11}^2,$$

$$\hat{c}_{122}(Q_{122}, q_{12}) = 0.75Q_{122}^2 + Q_{122} + 0.25Q_{222} + 0.03q_{12}^2,$$

$$\hat{c}_{212}(Q_{212}, q_{21}) = 0.6Q_{212}^2 + Q_{212} + 0.3Q_{112} + 0.02q_{21}^2,$$

$$\hat{c}_{222}(Q_{222}, q_{22}) = 0.5Q_{222}^2 + 0.8Q_{222} + 0.25Q_{122} + 0.05q_{22}^2.$$

The production cost functions at the manufacturing plants have the same functional forms as in Example 3.3, but now they include the additional shipments to the new demand market, R_2, that is:

$$\hat{f}_{12}(Q_{121}, Q_{122}, q_{12}) = 0.3(Q_{121}+Q_{122})^2+0.1(Q_{121}+Q_{122})+0.3(Q_{121}+Q_{122})q_{12}+0.4q_{12}^2,$$

$$\hat{f}_{22}(Q_{221}, Q_{222}, q_{22}) = 1.2(Q_{221}+Q_{222})^2+0.5(Q_{221}+Q_{222})+0.3(Q_{221}+Q_{222})q_{22}+0.5q_{22}^2.$$

$$\hat{f}_{11}(Q_{111}, Q_{112}, q_{11}) = 0.8(Q_{111}+Q_{112})^2+0.5(Q_{111}+Q_{112})+0.25(Q_{111}+Q_{112})q_{11}+0.5q_{11}^2,$$

$$\hat{f}_{21}(Q_{211}, Q_{212}, q_{21}) = (Q_{211}+Q_{212})^2+0.8(Q_{211}+Q_{212})+0.3(Q_{211}+Q_{212})q_{21}+0.65q_{21}^2.$$

In this example, consumers at the new demand market R_2 are more sensitive to the quality of the product than consumers at the original demand market R_1. The demand price functions for both the demand markets are, respectively:

$$\hat{\rho}_1 = 2250 - (Q_{111} + Q_{211} + Q_{121} + Q_{221}) + 0.8\hat{q}_1,$$

$$\hat{\rho}_2 = 2250 - (Q_{112} + Q_{122} + Q_{212} + Q_{222}) + 0.9\hat{q}_2,$$

where

$$\hat{q}_1 = \frac{Q_{111}q_{11} + Q_{211}q_{21} + Q_{121}q_{12} + Q_{221}q_{22}}{Q_{111} + Q_{211} + Q_{121} + Q_{211}},$$

and

$$\hat{q}_2 = \frac{Q_{112}q_{11} + Q_{212}q_{21} + Q_{122}q_{12} + Q_{222}q_{22}}{Q_{112} + Q_{212} + Q_{122} + Q_{222}}.$$

The equilibrium solution is as below.

$$Q^*_{111} = 208.70, \quad Q^*_{121} = 211.82, \quad Q^*_{211} = 203.90, \quad Q^*_{221} = 129.79,$$

$$Q^*_{112} = 165.39, \quad Q^*_{122} = 352.11, \quad Q^*_{212} = 182.30, \quad Q^*_{222} = 200.05.$$

$$q^*_{11} = 53.23, \quad q^*_{12} = 79.08, \quad q^*_{21} = 13.41, \quad q^*_{22} = 13.82.$$

The equilibrium demand at the two demand markets is now $d^*_1 = 754.21$ and $d^*_2 = 899.85$. The value of $\hat{q}_1$ is 42.94 and that of $\hat{q}_2$ is 46.52.

The incurred demand market prices are:

$$\hat{\rho}_1 = 1,530.15, \quad \hat{\rho}_2 = 1,392.03.$$

The profits of the firms are, respectively, 882,342.15 and 651,715.83.

- Due to the addition of R_2, which has associated lower transportation costs, each firm ships more product to demand market R_2 than to R_1. The total demand $d_1 + d_2$ is now 88.76% larger than the total demand d_1 in Example 3.2.
- The average quality levels increase, which leads to the increase in the prices and both firms' profits.

The Jacobian matrix of $-\nabla U(Q, q)$, for Example 3.4, evaluated at the equilibrium is

$$J(Q_{111}, Q_{121}, Q_{211}, Q_{221}, Q_{112}, Q_{122}, Q_{212}, Q_{222}, q_{11}, q_{12}, q_{21}, q_{22})$$

$$= \begin{pmatrix}
5.99 & 1.98 & 1.02 & 1.02 & 1.60 & 0 & 0 & 0 & -0.29 & -0.10 & -0.10 & -0.06 \\
1.98 & 6.17 & 1.04 & 1.04 & 0 & 0.60 & 0 & 0 & -0.10 & -0.25 & -0.10 & -0.06 \\
0.98 & 0.96 & 6.03 & 2.03 & 0 & 0 & 2.00 & 0 & -0.12 & -0.13 & -0.17 & -0.08 \\
0.98 & 0.96 & 2.03 & 7.43 & 0 & 0 & 0 & 2.40 & -0.12 & -0.13 & -0.12 & -0.13 \\
1.60 & 0 & 0 & 0 & 5.19 & 1.98 & 1.02 & 1.02 & -0.34 & -0.15 & -0.08 & -0.09 \\
0 & 0.60 & 0 & 0 & 1.98 & 4.07 & 1.03 & 1.03 & -0.07 & -0.37 & -0.08 & -0.09 \\
0 & 0 & 2.00 & 0 & 0.98 & 0.97 & 5.24 & 2.04 & -0.10 & -0.20 & -0.19 & -0.12 \\
0 & 0 & 0 & 2.40 & 0.98 & 0.97 & 2.04 & 5.44 & -0.10 & -0.20 & -0.10 & -0.20 \\
-0.29 & -0.10 & 0.12 & 0.12 & -0.34 & -0.07 & 0.10 & 0.10 & 1.60 & 0 & 0 & 0 \\
-0.10 & -0.25 & 0.13 & 0.13 & -0.15 & -0.37 & 0.20 & 0.20 & 0 & 1.36 & 0 & 0 \\
0.10 & 0.10 & -0.17 & -0.12 & 0.08 & 0.08 & -0.19 & -0.10 & 0 & 0 & 1.94 & 0 \\
0.06 & 0.06 & -0.08 & -0.13 & 0.09 & 0.09 & -0.12 & -0.20 & 0 & 0 & 0 & 1.70
\end{pmatrix}.$$

The eigenvalues of $\frac{1}{2}(J + J^T)$ are all positive and are: 1.29, 1.55, 1.66, 1.71, 1.93, 2.04, 3.76, 4.73, 6.14, 7.55, 8.01, and 11.78.

Therefore, both the uniqueness of the equilibrium solution and the conditions for convergence of the algorithm are guaranteed.

The equilibrium solution to Example 3.4 is a strictly monotone attractor and is exponentially stable.

Example 3.4 - Sensitivity Analysis

I multiply the coefficient of the second Q_{ijk} term, that is, the linear one, in each of the transportation cost functions $\hat{c}_{ijk}$ by a positive factor β, but retain the other transportation cost functions as in Example 3.4. I vary β from 0 to 50, 100, 150, 200, 250, 300, and 350.

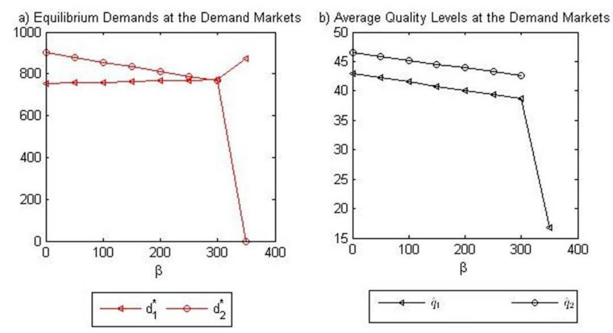


Figure: The Equilibrium Demands and Average Quality Levels as β Varies in Example 3.4

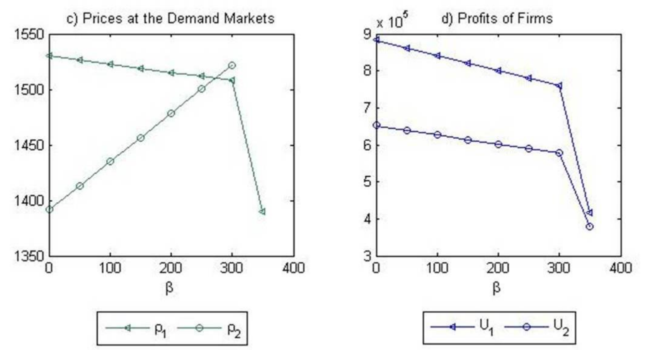


Figure: Prices at the Demand Markets and the Profits of the Firms as β Varies in Example 3.4

Summary and Conclusions

- I developed a rigorous framework for the modeling, analysis, and computation of solutions to competitive supply chain network problems in static and dynamic settings in which there is information asymmetry in quality.
- I also demonstrated how this framework can capture the inclusion of policy interventions in the form of minimum quality standards.
- It contributes to the literature on supply chains with quality competition and reveals the spectrum of insights that can be obtained through computations, supported by theoretical analysis.
- Finally, it contributes to the integration of economics with operations research and the management sciences.

This section corresponds to Chapter 4 of the dissertation, and is based on the paper:

Nagurney, A., Li, D., 2014b. A dynamic network oligopoly model with transportation costs, product differentiation, and quality competition. *Computational Economics* 44(2), 201-229.

Overview

In this section, I develop a supply chain network model with differentiated products and quality levels. I present both the static version, in an equilibrium context, which I formulate as a finite-dimensional variational inequality problem, and develop its dynamic counterpart, using projected dynamical systems theory.

Literature Review

- Hotelling, H., 1929. Stability in competition. *The Economic Journal* 39, 41-57.
- Abbott, L., 1953. Vertical equilibrium under pure quality competition. *The American Economic Review* 43(5), 826-845.
- Dixit, A., 1979. Quality and quantity competition. *Review of Economic Studies* 46(4), 587-599.
- Banker, R. D., Khosla, I., and Sinha, K. J., 1998. *Quality and competition. Management Science* 44(9), 1179-1192.
- Brekke, K. R., Siciliani, L., Straume, O. R., 2010. Price and quality in spatial competition. *Regional Science and Urban Economics* 40(6), 471-480.

Supply Chain Network Topology

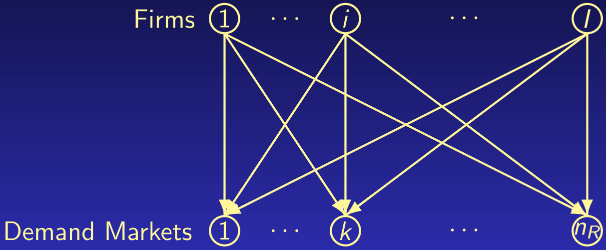


Figure: Supply Chain Network Topology

The Equilibrium Model

Conservation of flow equations

$$s_i = \sum_{k=1}^{n_R} Q_{ik}, \quad i = 1, \ldots, I, \tag{4.1}$$

$$d_{ik} = Q_{ik}, \quad i = 1, \ldots, I; k = 1, \ldots, n_R, \tag{4.2}$$

$$Q_{ik} \geq 0, \quad i = 1, \ldots, I; k = 1, \ldots, n_R. \tag{4.3}$$

I group the production outputs into the vector $s \in R^I_+$, the demands into the vector $d \in R^{In_R}_+$, and the product shipments into the vector $Q \in R^{In_R}_+$.

The Equilibrium Model

Production cost function for firm i

$$\hat{f}_i = \hat{f}_i(s, q_i), \quad i = 1, \ldots, I. \tag{4.5}$$

As a special case, the above functions can take on the form

$$\hat{f}_i(s, q_i) = f_i(s, q_i) + g_i(q_i), \quad i = 1, \ldots, I. \tag{4.6}$$

The production cost functions (4.4) (and (4.5)) are assumed to be convex and twice continuously differentiable.

Nonnegative quality level for firm i's product

$$q_i \geq 0, \quad i = 1, \ldots, I. \tag{4.4}$$

I group the quality levels of all firms into the vector $q \in R^I_+$.

Demand price function for firm i's product at demand market k

$$\rho_{ik} = \rho_{ik}(d, q), \quad i = 1, \ldots, I; k = 1, \ldots, n_R. \tag{4.7}$$

The demand price for a product at a demand market is allowed to depend, in general, upon the entire consumption pattern, as well as on all the levels of quality of all the products.

The demand price functions are, typically, assumed to be monotonically decreasing in product quantity but increasing in terms of product quality.

Transportation cost function

$$\hat{c}_{ik} = \hat{c}_{ik}(Q_{ik}), \quad i = 1, \ldots, I; k = 1, \ldots, n_R. \tag{4.8}$$

The demand price functions (4.7) and the total transportation cost functions (4.8) are assumed to be continuous and twice continuously differentiable.

The strategic variables of firm i are its product shipments $\{Q_i\}$ where $Q_i = (Q_{i1}, \ldots, Q_{in_R})$ and its quality level q_i.

Utility function

$$U_i = \sum_{k=1}^{n_R} \rho_{ik} d_{ik} - f_i - g_i - \sum_{k=1}^{n_R} \hat{c}_{ik}. \tag{4.9}$$

Variational Inequality Formulation

Theorem 4.1

$(Q^*, q^*) \in K$ *is a network Cournot-Nash equilibrium if and only if it satisfies the variational inequality*

$$-\sum_{i=1}^{I}\sum_{k=1}^{n_R} \frac{\partial U_i(Q^*, q^*)}{\partial Q_{ik}} \times (Q_{ik} - Q_{ik}^*) - \sum_{i=1}^{I} \frac{\partial U_i(Q^*, q^*)}{\partial q_i} \times (q_i - q_i^*) \geq 0,$$

$$\forall (Q, q) \in K, \quad (4.13)$$

$(s^*, Q^*, d^*, q^*) \in K^1$ is an equilibrium production, shipment, consumption, and quality level pattern if and only if it satisfies

$$\sum_{i=1}^{I} \frac{\partial \hat{f}_i(s^*, q_i^*)}{\partial s_i} \times (s_i - s_i^*)$$

$$+ \sum_{i=1}^{I} \sum_{k=1}^{n_R} \left[\frac{\partial \hat{c}_{ik}(Q_{ik}^*)}{\partial Q_{ik}} - \sum_{l=1}^{n_R} \frac{\partial \rho_{il}(d^*, q^*)}{\partial d_{ik}} \times d_{il}^* \right] \times (Q_{ik} - Q_{ik}^*)$$

$$- \sum_{i=1}^{I} \sum_{k=1}^{n_R} \rho_{ik}(d^*, q^*) \times (d_{ik} - d_{ik}^*)$$

$$+ \sum_{i=1}^{I} \left[\frac{\partial \hat{f}_i(s^*, q_i^*)}{\partial q_i} - \sum_{l=1}^{n_R} \frac{\partial \rho_{il}(d^*, q^*)}{\partial q_i} \times d_{il}^* \right] \times (q_i - q_i^*) \geq 0,$$

$$(s, Q, d, q) \in K^1, \tag{4.14}$$

where $K^1 \equiv \{(s, Q, d, q) |\, Q \geq 0,\ q \geq 0,$ *and* (4.1) *and* (4.2) *hold*$\}$.

Standard form VI

Determine $X^* \in \mathcal{K} \subset R^N$, such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{4.19}$$

where F is a given continuous function from $\mathcal{K}$ to R^N, and $\mathcal{K}$ is a closed and convex set.

I define the $(In_R + I)$ - dimensional vector $X \equiv (Q, q)$ and the $(In_R + I)$ - dimensional row vector $F(X) = (F^1(X), F^2(X))$.

$F^1(X)$ consists of $F^1_{ik}(X) \equiv -\frac{\partial U_i(Q,q)}{\partial Q_{ik}}$, and $F^2(X)$ consists of $F^2_i(X) \equiv -\frac{\partial U_i(Q,q)}{\partial q_i}$.

The feasible set $\mathcal{K} \equiv K$.

The Dynamic Model

A dynamic adjustment process for quantity and quality levels

$$\dot{Q}_{ik} = \begin{cases} \frac{\partial U_i(Q,q)}{\partial Q_{ik}}, & \text{if} \quad Q_{ik} > 0 \\ \max\{0, \frac{\partial U_i(Q,q)}{\partial Q_{ik}}\}, & \text{if} \quad Q_{ik} = 0. \end{cases} \tag{4.24}$$

$$\dot{q}_{i} = \begin{cases} \frac{\partial U_i(Q,q)}{\partial q_{i}}, & \text{if} \quad q_{i} > 0 \\ \max\{0, \frac{\partial U_i(Q,q)}{\partial q_{i}}\}, & \text{if} \quad q_{i} = 0. \end{cases} \tag{4.27}$$

Qualitative Properties - Existence and Uniqueness Results

Assumption 4.1

Suppose that in a network oligopoly model there exists a sufficiently large M, such that for any (i, k),

$$\frac{\partial U_i(Q, q)}{\partial Q_{ik}} < 0, \tag{4.29}$$

for all shipment patterns Q with $Q_{ik} \geq M$ and that there exists a sufficiently large $\bar{M}$, such that for any i,

$$\frac{\partial U_i(Q, q)}{\partial q_i} < 0, \tag{4.30}$$

for all quality level patterns q with $q_i \geq \bar{M}$.

Proposition 4.1

Any network oligopoly problem that satisfies Assumption 4.1 possesses at least one equilibrium shipment and quality level pattern.

Proposition 4.2

Suppose that F is strictly monotone at any equilibrium point of the variational inequality problem defined in (4.13). Then it has at most one equilibrium point.

Theorem 4.2

Suppose that F is strongly monotone. Then there exists a unique solution to variational inequality (4.19); equivalently, to variational inequality (4.14).

Qualitative Properties - Stability Properties

Theorem 4.3

(i). *If $-\nabla U(Q, q)$ is monotone, then every supply chain network Cournot-Nash equilibrium, provided its existence, is a global monotone attractor for the utility gradient process.*

(ii). *If $-\nabla U(Q, q)$ is strictly monotone, then there exists at most one supply chain network Cournot-Nash equilibrium. Furthermore, provided existence, the unique spatial Cournot-Nash equilibrium is a strictly global monotone attractor for the utility gradient process.*

(iii). *If $-\nabla U(Q, q)$ is strongly monotone, then there exists a unique supply chain network Cournot-Nash equilibrium, which is globally exponentially stable for the utility gradient process.*

Explicit Formulae for the Computation of the Product Shipments and Quality Levels

$$Q_{ik}^{\tau+1} = \max\{0, Q_{ik}^{\tau} + a_\tau(\rho_{ik}(d^\tau, q^\tau) + \sum_{k=1}^{n_R} \frac{\partial \rho_{il}(d^\tau, q^\tau)}{\partial d_{ik}} d_{il}^\tau - \frac{\partial \hat{f}_i(s^\tau, q_i^\tau)}{\partial s_i} - \frac{\partial \hat{c}_{ik}(Q_{ik}^\tau)}{\partial Q_{ik}})\}, \quad (4.31)$$

$$q_i^{\tau+1} = \max\{0, q_i^\tau + a_\tau(\sum_{k=1}^{n_R} \frac{\partial \rho_{il}(d^\tau, q^\tau)}{\partial q_i} d_{il}^\tau - \frac{\partial \hat{f}_i(s^\tau, q_i^\tau)}{\partial q_i})\}, \quad (4.32)$$

$$d_{ik}^{\tau+1} = Q_{ik}^{\tau+1}, \quad (4.33)$$

$$s_i^{\tau+1} = \sum_{k=1}^{n_R} Q_{ik}^{\tau+1}. \quad (4.34)$$

Numerical Examples - Example 4.3

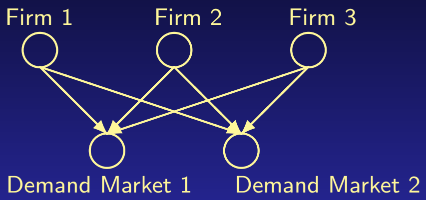


Figure: Supply Chain Network Topology for Example 4.3

The production cost functions are:

$$\hat{f}_1(s, q_1) = s_1^2 + s_1 s_2 + s_1 s_3 + 2q_1^2 + 39,$$

$$\hat{f}_2(s, q_2) = 2s_2^2 + 2s_1 s_2 + 2s_3 s_2 + q_2^2 + 37,$$

$$\hat{f}_3(s, q_3) = s_3^2 + s_1 s_3 + s_3 s_2 + 8q_3^2 + 60.$$

The total transportation cost functions are:

$$\hat{c}_{11}(Q_{11}) = Q_{11}^2 + 10, \quad \hat{c}_{12}(Q_{12}) = 5Q_{12}^2 + 7,$$

$$\hat{c}_{21}(Q_{21}) = 7Q_{21}^2 + 10, \quad \hat{c}_{22}(Q_{22}) = 2Q_{22}^2 + 5,$$

$$\hat{c}_{31}(Q_{31}) = 2Q_{31}^2 + 9, \quad \hat{c}_{32}(Q_{32}) = 3Q_{32}^2 + 8,$$

The demand price functions are:

$$\rho_{11}(d,q) = 100 - d_{11} - 0.4d_{21} - 0.1d_{31} + 0.3q_1 + 0.05q_2 + 0.05q_3,$$

$$\rho_{12}(d,q) = 100 - 2d_{12} - d_{22} - 0.1d_{32} + 0.4q_1 + 0.2q_2 + 0.2q_3,$$

$$\rho_{21}(d,q) = 100 - 0.6d_{11} - 1.5d_{21} - 0.1d_{31} + 0.1q_1 + 0.5q_2 + 0.1q_3,$$

$$\rho_{22}(d,q) = 100 - 0.7d_{12} - 1.7d_{22} - 0.1d_{32} + 0.01q_1 + 0.6q_2 + 0.01q_3,$$

$$\rho_{31}(d,q) = 100 - 0.2d_{11} - 0.4d_{21} - 1.8d_{31} + 0.2q_1 + 0.2q_2 + 0.7q_3,$$

$$\rho_{32}(d,q) = 100 - 0.1d_{12} - 0.3d_{22} - 2d_{32} + 0.2q_1 + 0.1q_2 + 0.4q_3.$$

The Jacobian of $-\nabla U(Q,q)$ is

$$J_(Q_{11}, Q_{12}, Q_{21}, Q_{22}, Q_{31}, Q_{32}, q_1, q_2, q_3)$$

$$= \begin{pmatrix} 6 & 2 & 1.4 & 1 & 1.1 & 1 & -0.3 & -0.05 & -0.05 \\ 2 & 16 & 1 & 2 & 1 & 1.1 & -0.4 & -0.2 & -0.2 \\ 2.6 & 2 & 21 & 4 & 2.1 & 2 & -0.1 & -0.5 & -0.5 \\ 2 & 2.7 & 4 & 7.4 & 2 & 2.1 & -0.01 & -0.6 & -0.01 \\ 1.2 & 1 & 1.4 & 1 & 9.6 & 2 & -0.2 & -0.2 & -0.7 \\ 1 & 1.1 & 1 & 1.3 & 2 & 12 & -0.2 & -0.1 & -0.4 \\ -0.3 & -0.4 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & -0.5 & -0.6 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & -0.7 & -0.4 & 0 & 0 & 16 \end{pmatrix}.$$

The Euler method converges to the equilibrium solution: $Q_{11}^* = 12.63$, $Q_{12}^* = 3.45$, $Q_{21}^* = 1.09$, $Q_{22}^* = 3.21$, $Q_{31}^* = 6.94$, $Q_{32}^* = 5.42$, $q_1^* = 1.29$, $q_2^* = 1.23$, $q_3^* = 0.44$ in 42 iterations.

The profits of the firms are $U_1 = 601.67$, $U_2 = 31.48$, and $U_3 = 403.97$.

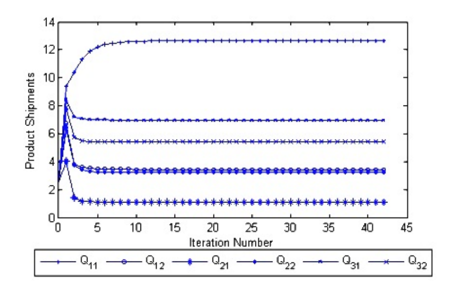


Figure: Product shipments for Example 4.3

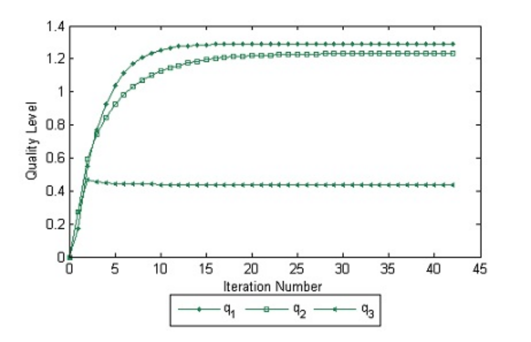


Figure: Quality levels for Example 4.3

Numerical Examples - Example 4.4

The new demand price functions associated with demand market 2 are now:

$$\rho_{12}(d,q) = 100 - 2d_{12} - d_{22} - 0.1d_{32} + 0.49q_1 + 0.2q_2 + 0.2q_2,$$

$$\rho_{22}(d,q) = 100 - 0.7d_{12} - 1.7d_{22} - 0.1d_{32} + 0.01q_1 + 0.87q_2 + 0.01q_3,$$

and

$$\rho_{32}(d,q) = 100 - 0.1d_{12} - 0.3d_{22} - 2d_{32} + 0.2q_1 + 0.1q_2 + 1.2q_3.$$

The Jacobian of $-\nabla U(Q,q)$ is now:

$$J(Q_{11},Q_{12},Q_{21},Q_{22},Q_{31},Q_{32},q_1,q_2,q_3)$$

$$= \begin{pmatrix} 6 & 2 & 1.4 & 1 & 1.1 & 1 & -0.3 & -0.05 & -0.05 \\ 2 & 16 & 1 & 2 & 1 & 1.1 & -0.49 & -0.2 & -0.2 \\ 2.6 & 2 & 21 & 4 & 2.1 & 2 & -0.1 & -0.5 & -0.5 \\ 2 & 2.7 & 4 & 7.4 & 2 & 2.1 & -0.01 & -0.87 & -0.01 \\ 1.2 & 1 & 1.4 & 1 & 9.6 & 2 & -0.2 & -0.2 & -0.7 \\ 1 & 1.1 & 1 & 1.3 & 2 & 12 & -0.2 & -0.1 & -1.2 \\ -0.3 & -0.49 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & -0.5 & -0.87 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & -0.7 & -1.2 & 0 & 0 & 16 \end{pmatrix}.$$

The computed equilibrium solution is now: $Q_{11}^* = 13.41$, $Q_{12}^* = 3.63$, $Q_{21}^* = 1.41$, $Q_{22}^* = 4.08$, $Q_{31}^* = 3.55$, $Q_{32}^* = 2.86$, $q_1^* = 1.45$, $q_2^* = 2.12$, $q_3^* = 0.37$. The profits of the firms are now: $U_1 = 682.44$, $U_2 = 82.10$, and $U_3 = 93.19$.

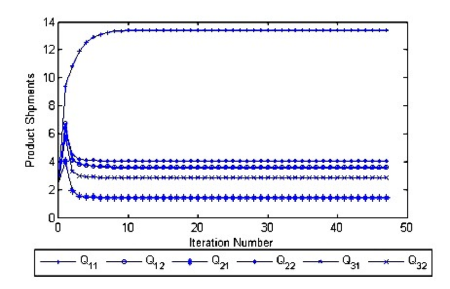


Figure: Product shipments for Example 4.4

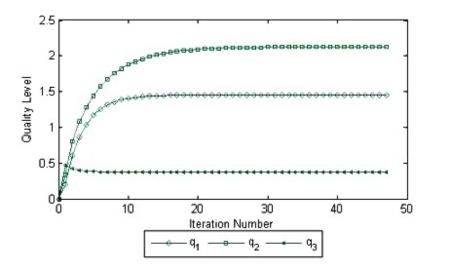


Figure: Quality levels for Example 4.4

The equilibrium quality levels of the three firms changes, with those of firm 1 and firm 2, increasing, relative to their values in Example 4.3.

The profit of firm 3 decreases by 76.9%, while the profits of the firms 1 and 2 increase 13.4% and 160.8%, respectively.

Summary and Conclusions

- I developed a new network oligopoly model with product differentiation and quality levels, in a network framework.
- I provided alternative variational inequality formulations and proposed a continuous-time adjustment process.
- I provided qualitative properties of existence and uniqueness of the dynamic trajectories and also gave conditions for stability analysis and associated results.
- I, through several numerical examples, illustrated the model and theoretical results, in order to demonstrate how the contributions in this paper could be applied in practice.

This section corresponds to Chapter 5 of the dissertation, and is based on the paper:

Nagurney, A., Li, D., and Nagurney, L., 2013. Pharmaceutical supply chain networks with outsourcing under price and quality competition. *International Transactions in Operational Research* 20(6), 859-888.

Background and Motivation

Outsourcing of manufacturing/production has long been noted in supply chain management and it has become prevalent in numerous manufacturing industries.

Quality issues in outsourced products must be of paramount concern.

- The suspension of the license of Pan Pharmaceuticals, the world's fifth largest contract manufacturer of health supplements, due to quality failure, caused costly consequences in terms of product recalls and credibility losses (Allen (2003)).
- Fake heparin made by a Chinese manufacturer led to recalls of drugs in over ten European countries (Payne (2008)), and resulted in the deaths of 81 Americans (Harris (2011)).
- More than 400 peanut butter products were recalled after 8 people died and more than 500 people in 43 states, half of them children, were sickened by salmonella poisoning, the source of which was a peanut butter plant in Georgia (Harris (2009)).

A more comprehensive supply chain network model that captures

- contractor selection,
- the minimization of the disrepute of the firm, as well as
- the competition among contractors,

is an imperative.

Literature Review

- Sridhar, S. S., Balachandran, B. V., 1997. Incomplete information, task assignment, and managerial control systems. *Management Science* 43(6), 764-778.
- Riordan, M. H., Sappington, D. E., 1987. Information, incentives, and organizational mode. *The Quarterly Journal of Economics* 102(2), 243-263.
- Kaya, O., 2011. Outsourcing vs. in-house production: A comparison of supply chain contracts with effort dependent demand. *Omega* 39, 168-178.
- Gray, J. V., Roth, A. V., Leiblein, M. V., 2011. Quality risk in offshore manufacturing: Evidence from the pharmaceutical industry. *Journal of Operations Management* 29(7-8), 737-752.

Overview

I develop a supply chain network model which takes into account the quality concerns in the context of outsourcing.

- This model captures the behaviors of the firm and its potential contractors.
- The contractors compete by determining the prices that they charge the firm for manufacturing and delivering the product to the demand markets and the quality levels of the products to maximize its total profit.
- The firm seeks to minimize its total cost, which includes its weighted disrepute cost, which is influenced by the quality of the product produced by its contractors and the amount of product that is outsourced.
- I develop both a static version of the model (at the equilibrium state) and also a dynamic one using the theory of projected dynamical systems.

Assumptions

The game theory supply chain network model developed in this section is based on the following assumptions:

- The original firm pays the transaction cost, which includes the costs of evaluating suppliers and the negotiation costs. The

  production/distribution costs and the quality cost information of the contractors are known by the firm through the transaction cost.

- In-house supply chain activities can ensure a 100% perfect quality conformance level (see Schneiderman (1986) and Kaya (2011)).

The Supply Chain Network Topology

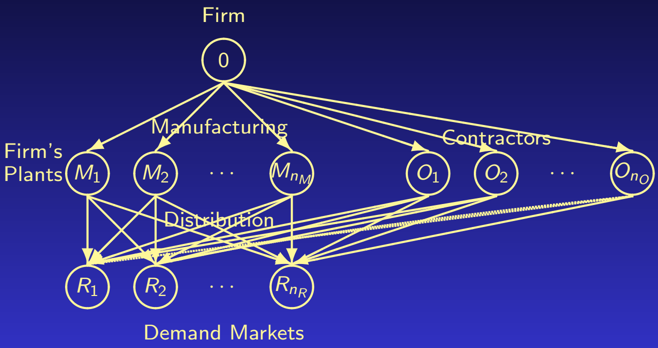


Figure: Supply Chain Network Topology with Outsourcing

The Equilibrium Model

Quality level of contractor j

$$0 \leq q_j \leq q^U, \quad j = 1, \ldots, n_O, \tag{5.1}$$

where q^U is the value representing perfect quality achieved by the firm in its in-house manufacturing.

Average quality level of the firm

$$q\prime = \frac{\sum_{j=n_M+1}^{n} \sum_{k=1}^{n_R} Q_{jk} q_{j-n_M} + (\sum_{j=1}^{n_M} \sum_{k=1}^{n_R} Q_{jk}) q^U}{\sum_{k=1}^{n_R} d_k}. \tag{5.2}$$

The Equilibrium Model - The Behavior of the Firm

The total utility maximization objective of firm i

$$\text{Maximize}_Q \quad U_0(Q, q^*, \pi^*) = -\sum_{j=1}^{n_M} f_j(\sum_{k=1}^{n_R} Q_{jk}) - \sum_{j=1}^{n_M}\sum_{k=1}^{n_R} \hat{c}_{jk}(Q_{jk})$$

$$-\sum_{j=1}^{n_O}\sum_{k=1}^{n_R} \pi^*_{jk} Q_{n_M+j,k} - \sum_{j=1}^{n_O} tc_j(\sum_{k=1}^{n_R} Q_{n_M+j,k}) - \omega dc(q\prime). \qquad (5.3)$$

subject to:

$$\sum_{j=1}^{n} Q_{jk} = d_k, \quad k = 1, \dots, n_R, \qquad (5.4)$$

$$Q_{jk} \geq 0, \quad j = 1, \dots, n; k = 1, \dots, n_R, \qquad (5.5)$$

with $q\prime$ in (5.3) as in (5.2).

Variational Inequality Formulation

Theorem 5.1

Determine $Q^* \in K^0$, such that:

$$-\sum_{h=1}^{n}\sum_{l=1}^{n_R} \frac{\partial U_0(Q^*, q^*, \pi^*)}{\partial Q_{hl}} \times (Q_{hl} - Q^*_{hl}) \geq 0, \quad \forall Q \in K^0, \tag{5.6}$$

where $K^0 \equiv \{Q | Q \in R_+^{nn_R} \text{ with } (5.4) \text{ satisfied}\}$, *for* $h = 1, \ldots, n_M$; $l = 1, \ldots, n_R$:

$$-\frac{\partial U_o}{\partial Q_{hl}} = \left[\frac{\partial f_h(\sum_{k=1}^{n_R} Q_{hk})}{\partial Q_{hl}} + \frac{\partial \hat{c}_{hl}(Q_{hl})}{\partial Q_{hl}} + \omega \frac{\partial dc(q')}{\partial Q_{hl}}\right]$$

$$= \left[\frac{\partial f_h(\sum_{k=1}^{n_R} Q_{hk})}{\partial Q_{hl}} + \frac{\partial \hat{c}_{hl}(Q_{hl})}{\partial Q_{hl}} + \omega \frac{\partial dc(q')}{\partial q'} \frac{q^U}{\sum_{k=1}^{n_R} d_k}\right],$$

and for $h = n_M + 1, \ldots, n$; $l = 1, \ldots, n_R$:

$$-\frac{\partial U_o}{\partial Q_{hl}} = \left[\pi^*_{h-n_M,l} + \frac{\partial tc_{h-n_M}(\sum_{k=1}^{n_R} Q_{hk})}{\partial Q_{hl}} + \omega \frac{\partial dc(q')}{\partial Q_{hl}}\right]$$

$$= \left[\pi^*_{h-n_M,l} + \frac{\partial tc_{h-n_M}(\sum_{k=1}^{n_R} Q_{hk})}{\partial Q_{hl}} + \omega \frac{\partial dc(q')}{\partial q'} \frac{q_h}{\sum_{k=1}^{n_R} d_k}\right].$$

The Equilibrium Model - The Behavior of the Contractors

The total utility maximization objective of contractor j

$$\text{Maximize}_{q_j,\pi_j} \quad U_j(Q^*, q, \pi) = \sum_{k=1}^{n_R} \pi_{jk} Q^*_{n_M+j,k} - \sum_{k=1}^{n_R} sc_{jk}(Q^*, q) - \hat{q}c_j(q) - \sum_{k=1}^{n_R} oc_{jk}(\pi_{jk}) \tag{5.7}$$

subject to:

$$\pi_{jk} \geq 0, \quad k = 1, \ldots, n_R, \tag{5.8}$$

and (5.1) for each j.

The cost functions in each contractor's utility function are continuous, twice continuously differentiable, and convex.

Variational Inequality Formulation

Theorem 5.2

$(q^*, \pi^*) \in \mathcal{K}^1$ is a Bertrand-Nash equilibrium if and only if it satisfies the variational inequality:

$$-\sum_{j=1}^{n_O} \frac{\partial U_j(Q^*, q^*, \pi^*)}{\partial q_j} \times (q_j - q_j^*) - \sum_{j=1}^{n_O} \sum_{k=1}^{n_R} \frac{\partial U_j(Q^*, q^*, \pi^*)}{\partial \pi_{jk}} \times (\pi_{jk} - \pi_{jk}^*) \geq 0,$$

$$\forall (q, \pi) \in \mathcal{K}^1, \tag{5.12}$$

with notice that: for $j = 1, \ldots, n_O$:

$$-\frac{\partial U_j}{\partial q_j} = \sum_{k=1}^{n_R} \frac{\partial sc_{jk}(Q^*, q)}{\partial q_j} + \frac{\partial \hat{c}_j(q)}{\partial q_j}, \tag{5.13}$$

and for $j = 1, \ldots, n_O$; $k = 1, \ldots, n_R$:

$$-\frac{\partial U_J}{\partial \pi_{jk}} = \sum_{r=1}^{n_R} \frac{\partial oc_{jk}(\pi_{jk})}{\partial \pi_{jk}} - Q^*_{n_M+j,k}. \tag{5.14}$$

The Equilibrium Conditions for the Supply Chain Network with Outsourcing

Definition 5.2

The equilibrium state of the supply chain network with outsourcing is one where both variational inequalities (5.6) and (5.12) hold simultaneously.

Theorem 5.3

Determine $(Q^*, q^*, \pi^*) \in \mathcal{K}$, such that:

$$-\sum_{h=1}^{n}\sum_{l=1}^{n_R}\frac{\partial U_0(Q^*,q^*,\pi^*)}{\partial Q_{hl}}\times(Q_{hl}-Q^*_{hl})-\sum_{j=1}^{n_O}\frac{\partial U_j(Q^*,q^*,\pi^*)}{\partial q_j}\times(q_j-q^*_j)$$

$$-\sum_{j=1}^{n_O}\sum_{k=1}^{n_R}\frac{\partial U_j(Q^*,q^*,\pi^*)}{\partial \pi_{jk}}\times(\pi_{jk}-\pi^*_{jk})\geq 0,\quad \forall (Q,q,\pi)\in\mathcal{K}, \tag{5.15}$$

where $\mathcal{K} \equiv K^0 \times \mathcal{K}^1$.

Standard form VI

Determine $X^* \in \mathcal{K}$ where X is a vector in R^N, $F(X)$ is a continuous function such that $F(X) : X \mapsto \mathcal{K} \subset R^N$, and

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{5.16}$$

where $\mathcal{K}$ is closed and convex.

I define the vector $X \equiv (Q, q, \pi)$. Also, $N = nn_R + n_O + n_o n_R$.

The first nn_R components of F are given by:
$-\frac{\partial U_0(Q,q,\pi)}{\partial Q_{hl}}$,for $h = 1, \ldots, n$; $l = 1, \ldots, n_R$;

the next n_O components of F are given by:
$-\frac{\partial U_j(Q,q,\pi)}{\partial q_j}$,for $j = 1, \ldots, n_O$;

and the subsequent $n_O n_R$ components of F are given by:
$-\frac{\partial U_j(Q,q,\pi)}{\partial \pi_{jk}}$, for $j = 1, \ldots, n_O$; $k = 1, \ldots, n_R$.

The Dynamic Model

The pertinent ordinary differential equation (ODE):

$$\dot{X} = \Pi_{\mathcal{K}}(X, -F(X)), \tag{5.18}$$

where $F(X) = -\nabla U(Q, q, \pi)$.

X^* solves the variational inequality problem (5.15) equivalently, (5.16), if and only if it is a stationary point of the ODE (5.18), that is,

$$\dot{X} = 0 = \Pi_{\mathcal{K}}(X^*, -F(X^*)).$$

Qualitative Results

Theorem 5.4

Suppose that F is strongly monotone. Then there exists a unique solution to variational inequality (5.16); equivalently, to variational inequality (5.15).

Theorem 5.5

(i). *If $F(X)$ is monotone, then every supply chain network equilibrium, as defined in Definition 5.2, provided its existence, is a global monotone attractor for the utility gradient processes.*

(ii). *If $F(X)$ is strictly monotone, then there exists at most one supply chain network equilibrium. Furthermore, given existence, the unique equilibrium is a strictly global monotone attractor for the utility gradient processes.*

(iii). *If $F(X)$ is strongly monotone, then the unique supply chain network equilibrium, which is guaranteed to exist, is also globally exponentially stable for the utility gradient processes.*

Explicit Formulae for Quality Levels and Contractor Prices

$$q_j^{\tau+1} = \min\{q^U, \max\{0, q_j^\tau + a_\tau(-\sum_{k=1}^{n_R} \frac{\partial sc_{jk}(Q^\tau, q^\tau)}{\partial q_j} - \frac{\partial \hat{q}c_j(q^\tau)}{\partial q_j})\}\},$$

$$j = 1, \ldots, n_O, \quad (5.20)$$

$$\pi_{jk}^{\tau+1} = \max\{0, \pi_{jk}^\tau + a_\tau(-\sum_{r=1}^{n_R} \frac{\partial oc_{jk}(\pi_{jk}^\tau)}{\partial \pi_{jk}} + Q_{n_M+j,k}^\tau)\},$$

$$j = 1, \ldots, n_O; k = 1, \ldots, n_R. \quad (5.21)$$

The Algorithm - The Euler Method

The strictly convexquadratic programming problem

$$X^{\tau+1} = \text{Minimize}_{X \in \mathcal{K}} \quad \frac{1}{2}\langle X, X\rangle - \langle X^{\tau} - a_{\tau} F(X^{\tau}), X\rangle. \tag{5.19}$$

In order to obtain the values of the product flows at each iteration, I apply the exact equilibration algorithm, originated by Dafermos and Sparrow (1969) and applied to many different applications of networks with special structure (cf. Nagurney (1999) and Nagurney and Zhang (1996)).

An Illustrative Example

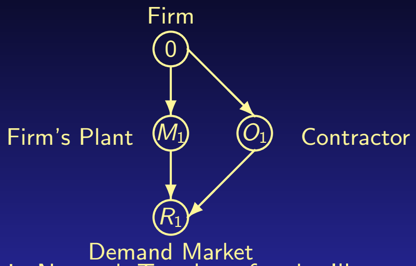


Figure: Supply Chain Network Topology for the Illustrative Example

The demand at demand market R_1: 1,000, q^U=100, and ω=1.

The firm's production cost:

$$f_1(Q_{11}) = Q_{11}^2 + Q_{11}.$$

Total transportation cost:

$$\hat{c}_{11}(Q_{11}) = .5Q_{11}^2 + Q_{11}.$$

Transaction cost:

$$tc_1(Q_{21}) = .05Q_{21}^2 + Q_{21}.$$

The contractor's total cost of production and distribution:

$$sc_{11}(Q_{21}, q_1) = Q_{21}q_1.$$

Total quality cost:

$$\hat{q}c_1(q_1) = 10(q_1 - 100)^2.$$

Opportunity cost:

$$oc_{11}(\pi_{11}) = .5(\pi_{11} - 10)^2.$$

Cost of disrepute:

$$dc(q\prime) = 100 - q\prime,$$

where $q\prime = \frac{Q_{21}q_1 + Q_{11}100}{1000}$.

The Euler method yields the following product flow, quality level, and price pattern:

$$Q_{11}^* = 270.50, \quad Q_{21}^* = 729.50, \quad q_1^* = 63.52, \quad \pi_{11}^* = 739.50.$$

The total cost incurred by the firm is 677,128.65 with the contractor earning a profit of 213,786.67. The value of $q\prime$ is 73.39.

The Jacobian matrix of $F(X) = -\nabla U(Q, q, \pi)$, for this example, is

$$J(Q_{11}, Q_{21}, q_1, \pi_{11}) = \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & .1 & -.001 & 1 \\ 0 & 1 & 20 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}.$$

Both the existence and the uniqueness of the solution to variational inequality (5.15) with respect to this example are guaranteed.

A Variant

The in-house transportation cost function is reduced by a factor of 10:

$$\hat{c}_{11}(Q_{11}) = .05Q_{11}^2 + .1Q_{11}.$$

The Euler method again yields the following equilibrium solution:

$$Q_{11}^* = 346.86,\ Q_{21}^* = 653.14, \quad q_1^* = 67.34, \quad \pi_{11}^* = 663.15.$$

The firm's total cost is now 581,840.07 and the contractor's profit is now 165,230.62. The value of ql is now 78.67.

The Jacobian of F for the variant is also strongly positive-definite.

An Illustrative Example - Sensitivity Analysis

I returned to the original example and increase the demand for the product at R_1 in increments of 1,000.

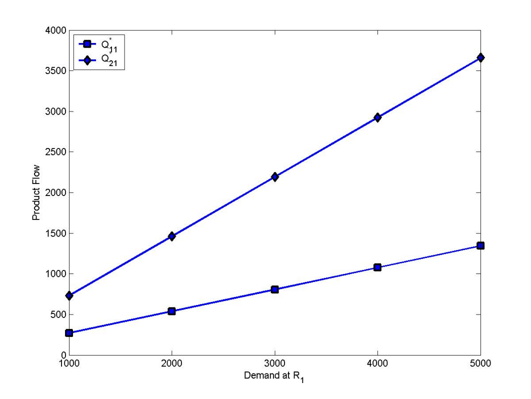


Figure: Equilibrium Product Flows as the Demand Increases for the Illustrative Example

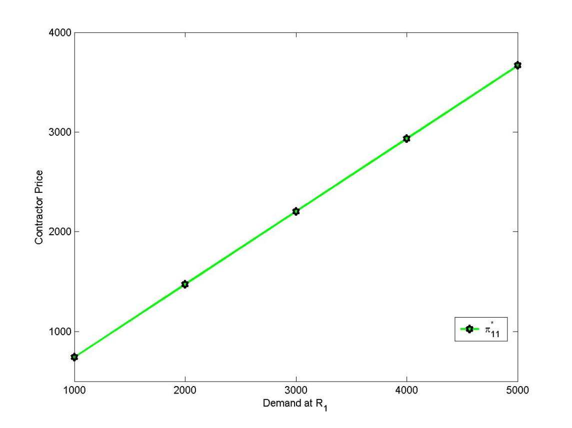


Figure: Equilibrium Contractor Prices as the Demand Increases for the Illustrative Example

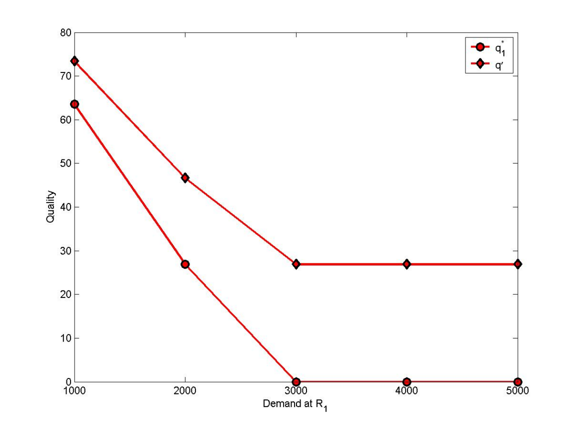


Figure: Equilibrium Contractor Quality Level and the Average Quality as the Demand Increases for the Illustrative Example

Additional Numerical Examples - Example 5.1

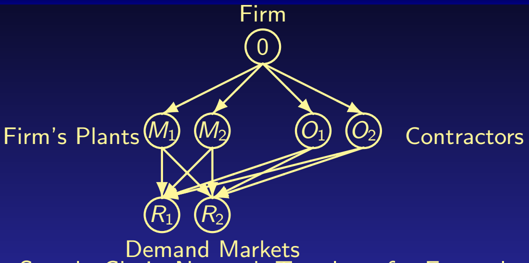


Figure: Supply Chain Network Topology for Example 5.1

The demand at R_1 is 1,000 and it is 500 at R_2. q^U=100, and ω=1.

The production cost functions:

$$f_1(\sum_{k=1}^{2} Q_{1k}) = (Q_{11} + Q_{12})^2 + 2(Q_{11} + Q_{12}),$$

$$f_2(\sum_{k=1}^{2} Q_{2k}) = 1.5(Q_{21} + Q_{22})^2 + 2(Q_{21} + Q_{22}).$$

The transportation cost functions:

$$\hat{c}_{11}(Q_{11}) = 1.5Q_{11}^2 + 10Q_{11}, \quad \hat{c}_{12}(Q_{12}) = 1Q_{12}^2 + 25Q_{12},$$

$$\hat{c}_{21}(Q_{21}) = 1Q_{21}^2 + 5Q_{21}, \quad \hat{c}_{22}(Q_{22}) = 2.5Q_{22}^2 + 40Q_{22}.$$

Transaction cost functions:

$$tc_1(Q_{31} + Q_{32}) = .5(Q_{31} + Q_{32})^2 + .1(Q_{31} + Q_{32}),$$

$$tc_2(Q_{41} + Q_{42}) = .25(Q_{41} + Q_{42})^2 + .2(Q_{41} + Q_{42}).$$

The contractors' total cost of production and distribution:

$$sc_{11}(Q_{31}, q_1) = Q_{31}q_1, \quad \hat{s}c_{12}(Q_{32}, q_1) = Q_{32}q_1,$$

$$sc_{21}(Q_{41}, q_2) = 2Q_{41}q_2, \quad \hat{s}c_{22}(Q_{42}, q_2) = 2Q_{42}q_2.$$

Total quality cost functions:

$$\hat{q}c_1(q_1) = 5(q_1 - 100)^2, \quad \hat{q}c_2(q_2) = 10(q_1 - 100)^2.$$

Opportunity cost functions:

$$oc_{11}(\pi_{11}) = .5(\pi_{11} - 10)^2, \quad oc_{12}(\pi_{12}) = (\pi_{12} - 10)^2,$$

$$oc_{21}(\pi_{21}) = (\pi_{21} - 5)^2, \quad oc_{22}(\pi_{22}) = .5(\pi_{22} - 20)^2.$$

Cost of disrepute:

$$dc(q\prime) = 100 - q\prime$$

where $q\prime = \frac{Q_{31}q_1 + Q_{32}q_1 + Q_{41}q_2 + Q_{42}q_2 + Q_{11}100 + Q_{12}100 + Q_{21}100 + Q_{22}100}{1500}$.

The Euler method yields the following equilibrium solution.

$$Q_{11}^* = 95.77, \quad Q_{12}^* = 85.51, \quad Q_{21}^* = 118.82, \quad Q_{22}^* = 20.27,$$

$$Q_{31}^* = 213.59, \quad Q_{32}^* = 224.59, \quad Q_{41}^* = 571.83, \quad Q_{42}^* = 169.63.$$

$$q_1^* = 56.18, \quad q_2^* = 25.85,$$

$$\pi_{11}^* = 223.57, \quad \pi_{12}^* = 122.30, \quad \pi_{21}^* = 290.92, \quad \pi_{22}^* = 189.61.$$

The total cost of the firm is 610,643.26 and the profits of the contractors' are 5,733.83 and 9,294.44. The value of $q_{\prime}$ is 50.55.
The Jacobian matrix of $F(X) = -\nabla U(Q, q, \pi)$ is

$$J(Q_{11}, Q_{12}, Q_{21}, Q_{22}, Q_{31}, Q_{32}, Q_{41}, Q_{42}, q_1, q_2, \pi_{11}, \pi_{12}, \pi_{21}, \pi_{22})$$

$$= \begin{pmatrix}
5 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 5 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 3 & 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & -6.67 \times 10^{-4} & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & -6.67 \times 10^{-4} & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0.5 & 0 & -6.67 \times 10^{-4} & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0.5 & 0 & -6.67 \times 10^{-4} & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 20 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}.$$

Example 5.1 - Sensitivity Analysis

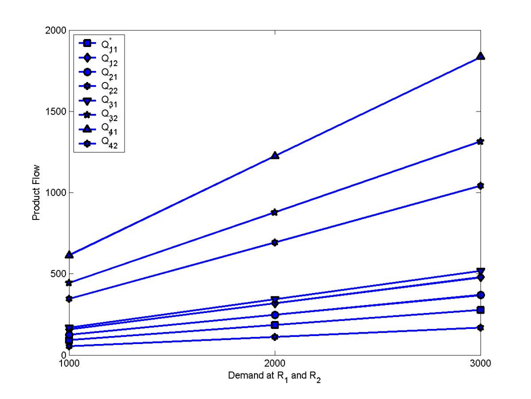


Figure: Equilibrium Product Flows as the Demand Increases for Example 5.1

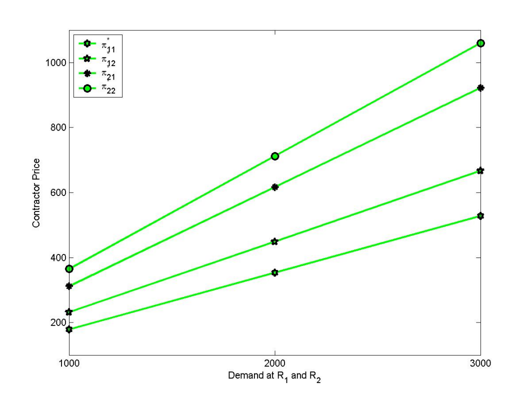


Figure: Equilibrium Contractor Prices as the Demand Increases for Example 5.1

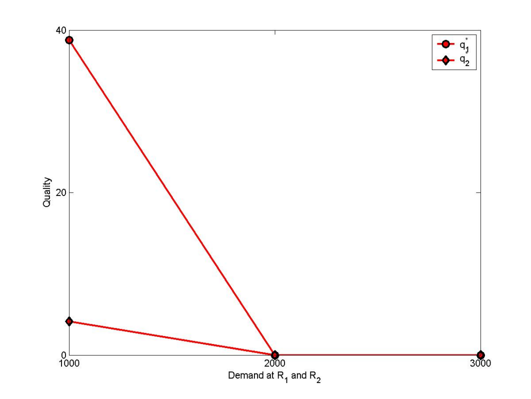


Figure: Equilibrium Quality Levels as the Demand Increases for Example 5.1

Additional Numerical Examples - Example 5.2

I consider the following disruption. The data are as in Example 5.1 but contractor O_2 is not able to provide any production and distribution services. This could arise due to a natural disaster, adulteration in its production process, and/or an inability to procure an ingredient.

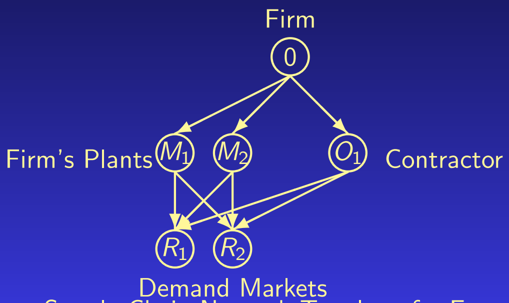


Figure: Supply Chain Network Topology for Example 5.2

The new equilibrium solution is:

$$Q_{11}^* = 218.06, \quad Q_{12}^* = 141.79, \quad Q_{21}^* = 260.20, \quad Q_{22}^* = 25.96,$$

$$Q_{31}^* = 521.74, \quad Q_{32}^* = 332.25.$$

$$q_1^* = 14.60,$$

$$\pi_{11}^* = 531.74, \quad \pi_{12}^* = 176.12.$$

The new average quality level is $q\prime$=51.38. The total cost of the firm is now 1,123,226.62 whereas the profit of the first contractor was now 123,460.67.

Summary and Conclusions

I developed a supply chain network game theory model, in both equilibrium and dynamic versions, to capture contractor selection, based on the competition among the contractors in the prices that they charge as well as the quality levels of the products that they produce.

- I introduced a disrepute cost associated with the average quality at the demand markets.
- The numerical studies included sensitivity analysis results as well as a disruption to the supply chain network in that a contractor is no longer available for production and distribution.

1. Introduction
2. Quality & Information Asymmetry
   - A Supply Chain Network Model with Information Asymmetry in Quality, Minimum Quality Standards, and Quality Competition
3. Quality & Product Differentiation
   - A Supply Chain Network Model with Transportation Costs, Product Differentiation, and Quality Competition
4. Quality & Outsourcing
   - A Supply Chain Network Model with Outsourcing and Quality and Price Competition
   - A Supply Chain Network Model with Product Differentiation, Outsourcing, and Quality and Price Competition
5. Quality & Supplier Selection
   - A Supply Chain Network Model with Supplier Selection and Quality and Price Competition
6. Future Research

This section corresponds to Chapter 6 of the dissertation, and is based on the paper:

Nagurney, A., Li, D., 2015. A supply chain network game theory model with product differentiation, outsourcing of production and distribution, and quality and price competition. *Annals of Operations Research* 228(1), 479-503.

Background and Motivation

In addition to the increasing volume of outsourcing, the supply chain networks weaving the original manufacturers and the contractors are becoming increasingly complex.

If quality issues in outsourcing are to be considered in complex supply chain networks with multiple firms and contractors, product differentiation cannot be ignored.

- When consumers observe a brand of a product, they consider the quality, function, and reputation of that particular brand name.
- With outsourcing, chances are that the product was manufactured by a completely different company than the brand indicates, but the level of quality and the reputation associated with the outsourced product still remain with the "branded" original firm.
- If a product is recalled for a faulty part and that part was outsourced, the original firm is the one that carries the burden of correcting its damaged reputation.

Overview

In this section, I develop a model that extends the one in Chapter 5. It is a supply chain network game theory model with product differentiation, possible outsourcing of production and distribution, quantity and quality competition among the firms, and quality and price competition among the contractors.

Moreover, the in-house quality levels are no longer assumed to be perfect, but, rather, are strategic variables of the firms, since in-house quality failures may also occur (cf. Beamish and Bapuji (2008)).

Supply Chain Network Topology

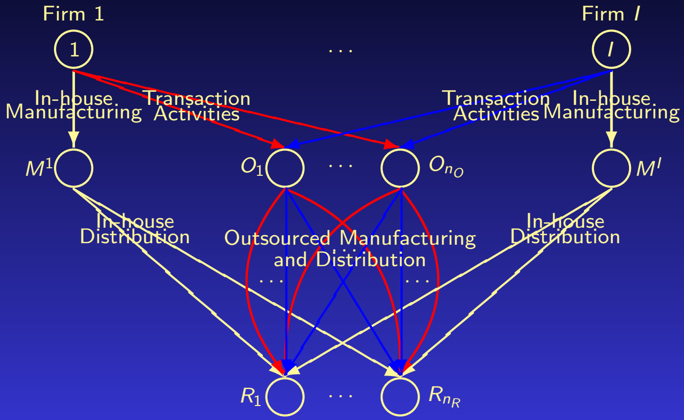


Figure: Supply Chain Network Topology with Outsourcing

The Model - The Behavior of the Firms

Quality levels

$$0 \leq q_{ij} \leq q^U, \quad i = 1, \ldots, I; j = 1, \ldots, n_O, \tag{6.1}$$

$$0 \leq q_i \leq q^U, \quad i = 1, \ldots, I, \tag{6.2}$$

where q^U is the value representing perfect quality achieved by the firm in its in-house manufacturing.

Average quality levels of the firms

$$q_i'(Q_i, q_i, q_i^2) = \frac{\sum_{k=1}^{n_R} \sum_{j=2}^{n} Q_{ijk} q_{i,j-1} + \sum_{k=1}^{n_R} Q_{i1k} q_i}{\sum_{k=1}^{n_R} d_{ik}}, \quad i = 1, \ldots, I. \tag{6.3}$$

The total utility maximization objective of firm i

$$\text{Maximize}_{Q_i,q_i} \quad U_i^1 = -f_i(Q^1,q^1) - c_i(q^1) - \sum_{k=1}^{n_R} \hat{c}_{ik}(Q^1,q^1) - \sum_{j=1}^{n_O}\sum_{k=1}^{n_R} \pi^*_{ijk} Q_{i,1+j,k}$$

$$- \sum_{j=1}^{n_O} tc_{ij}\Big(\sum_{k=1}^{n_R} Q_{i,1+j,k}\Big) - \omega_i dc_i(q_i'(Q_i,q_i,q_i^{2^*})) \tag{6.4}$$

subject to:

$$\sum_{j=1}^{n} Q_{ijk} = d_{ik}, \quad i=1,\ldots,I;\ k=1,\ldots,n_R, \tag{6.5}$$

$$Q_{ijk} \geq 0, \quad i=1,\ldots,I;\ j=1,\ldots,n;\ k=1,\ldots,n_R, \tag{6.6}$$

and (6.2).

All the cost functions in (6.4) are continuous, twice continuously differentiable, and convex.

Variational Inequality Formulation

Theorem 6.1

$(Q^*, q^{1^*}) \in \mathcal{K}^1$ is a Counot-Nash equilibrium if and only if it satisfies the variational inequality:

$$-\sum_{i=1}^{I}\sum_{h=1}^{n}\sum_{m=1}^{n_R} \frac{\partial U_i^1(Q^*, q^{1^*}, q^{2^*}, \pi_i^*)}{\partial Q_{ihm}} \times (Q_{ihm} - Q_{ihm}^*)$$

$$-\sum_{i=1}^{I} \frac{\partial U_i^1(Q^*, q^{1^*} q^{2^*}, \pi_i^*)}{\partial q_i} \times (q_i - q_i^*) \geq 0,$$

$$\forall (Q, q^1) \in \mathcal{K}^1, \tag{6.8}$$

with notice that: for $h=1$; $i=1,\ldots,I$; $m=1,\ldots,n_R$:

$$-\frac{\partial U_i^1}{\partial Q_{ihm}} = \left[\frac{\partial f_i}{\partial Q_{ihm}} + \sum_{k=1}^{n_R} \frac{\partial \hat{c}_{ik}}{\partial Q_{ihm}} + \omega_i \frac{\partial dc_i}{\partial q_i'} \frac{\partial q_i'}{\partial Q_{ihm}}\right],$$

for $h=2,\ldots,n$; $i=1,\ldots,I$; $m=1,\ldots,n_R$:

$$-\frac{\partial U_i^1}{\partial Q_{ihm}} = \left[\pi^*_{i,h-1,m} + \frac{\partial tc_{i,h-1}}{\partial Q_{ihm}} + \omega_i \frac{\partial dc_i}{\partial q_i'} \frac{\partial q_i'}{\partial Q_{ihm}}\right],$$

for $i=1,\ldots,I$:

$$-\frac{\partial U_i^1}{\partial q_i} = \left[\frac{\partial f_i}{\partial q_i} + \frac{\partial c_i}{\partial q_i} + \sum_{k=1}^{n_R} \frac{\partial \hat{c}_{ik}}{\partial q_i} + \omega_i \frac{\partial dc_i}{\partial q_i'} \frac{\partial q_i'}{\partial q_i}\right].$$

The Model - The Behavior of the Contractors

The total utility maximization objective of contractor j

$$\text{Maximize}_{q_j,\pi_j} \quad U_j^2 = \sum_{k=1}^{n_R}\sum_{i=1}^{I}\pi_{ijk}Q^*_{i,1+j,k} - \sum_{k=1}^{n_R}\sum_{i=1}^{I}sc_{ijk}(Q^{2*},q^2) - \hat{c}_j(q^2)$$

$$-\sum_{k=1}^{n_R}\sum_{i=1}^{I}oc_{ijk}(\pi) \tag{6.9}$$

subject to:

$$\pi_{ijk} \geq 0, \quad j = 1,\ldots,n_O; k = 1,\ldots,n_R, \tag{6.10}$$

and (6.1) for each j. are at their equilibrium values.

The cost functions in each contractor's utility function are continuous, twice continuously differentiable, and convex.

Variational Inequality Formulation

Theorem 6.2

$(q^{2^*}, \pi^*) \in \mathcal{K}^2$ is a Bertrand-Nash equilibrium if and only if it satisfies the variational inequality:

$$-\sum_{l=1}^{I}\sum_{j=1}^{n_O}\frac{\partial U_j^2(Q^{2^*}, q^{2^*}, \pi^*)}{\partial q_{lj}} \times (q_{lj} - q_{lj}^*)$$

$$-\sum_{l=1}^{I}\sum_{j=1}^{n_O}\sum_{k=1}^{n_R}\frac{\partial U_j^2(Q^{2^*}, q^{2^*}, \pi^*)}{\partial \pi_{ljk}} \times (\pi_{ljk} - \pi_{ljk}^*) \geq 0,$$

$$\forall (q^2, \pi) \in \mathcal{K}^2. \tag{6.12}$$

with notice that: for $j = 1, \ldots, n_O$; $l = 1, \ldots, I$:

$$-\frac{\partial U_j^2}{\partial q_{lj}} = \sum_{i=1}^{I} \sum_{k=1}^{n_R} \frac{\partial sc_{ijk}}{\partial q_{lj}} + \frac{\partial \hat{c}_j}{\partial q_{lj}},$$

and for $j = 1, \ldots, n_O$; $l = 1, \ldots, I$; $k = 1, \ldots, n_R$:

$$-\frac{\partial U_j^2}{\partial \pi_{ljk}} = \sum_{i=1}^{I} \sum_{r=1}^{n_R} \frac{\partial oc_{ijr}}{\partial \pi_{ljk}} - Q^*_{l,1+j,k}.$$

The Equilibrium Conditions for the Supply Chain Network with Outsourcing

Definition 6.3

The equilibrium state of the supply chain network with product differentiation, outsourcing, and quality and price competition is one where both variational inequalities (6.8) and (6.12) hold simultaneously.

The Equilibrium Conditions for the Supply Chain Network with Outsourcing

Theorem 6.3

The equilibrium conditions governing the supply chain network model with product differentiation, outsourcing, and quality competition are equivalent to the solution of the variational inequality problem: determine $(Q^*, q^{1^*}, q^{2^*}, \pi^*) \in \mathcal{K}$, *such that:*

$$-\sum_{i=1}^{I}\sum_{h=1}^{n}\sum_{m=1}^{n_R} \frac{\partial U_i^1(Q^*, q^{1^*}, q^{2^*}, \pi^*)}{\partial Q_{ihm}} \times (Q_{ihm} - Q^*_{ihm}) - \sum_{i=1}^{I} \frac{\partial U_i^1(Q^*, q^{1^*}, q^{2^*}, \pi^*)}{\partial q_i}$$

$$\times (q_i - q_i^*) - \sum_{l=1}^{I}\sum_{j=1}^{n_O} \frac{\partial U_j^2(Q^*, q^{1^*}, q^{2^*}, \pi^*)}{\partial q_{lj}} \times (q_{lj} - q^*_{lj})$$

$$-\sum_{l=1}^{I}\sum_{j=1}^{n_O}\sum_{k=1}^{n_R} \frac{\partial U_j^2(Q^*, q^{1*}, q^{2*}, \pi^*)}{\partial \pi_{ljk}} \times (\pi_{ljk} - \pi^*_{ljk}) \geq 0,$$

$$\forall (Q, q^1, q^2, \pi) \in \mathcal{K}. \tag{6.13}$$

Explicit Formulae for Quality Levels and Contractor Prices

$$q_i^{\tau+1} = \min\{q^U, \max\{0, q_i^\tau + a_\tau(\frac{\partial f_i(Q^{1^\tau}, q^{1^\tau})}{\partial q_i} + \frac{\partial c_i(q^{1^\tau})}{\partial q_i} + \sum_{k=1}^{n_R} \frac{\partial \hat{c}_{ik}(Q^{1^\tau}, q^{1^\tau})}{\partial q_i}$$

$$+\omega_i \frac{\partial dc_i(q_i'^\tau)}{\partial q_i'} \frac{\partial q_i'(Q_i^\tau, q_i^\tau, q_i^{2^\tau})}{\partial q_i})\}\}; \quad (6.16)$$

$$q_{lj}^{\tau+1} = \min\{q^U, \max\{0, q_{lj}^\tau + a_\tau(\sum_{i=1}^{I}\sum_{k=1}^{n_R} \frac{\partial sc_{ijk}(Q^{2^\tau}, q^{2^\tau})}{\partial q_{lj}} + \frac{\partial \hat{c}_j(q^{2^\tau})}{\partial q_{lj}})\}\}. \quad (6.17)$$

$$\pi_{ljk}^{\tau+1} = \max\{0, \pi_{ljk}^\tau + a_\tau(\sum_{i=1}^{I}\sum_{r=1}^{n_R} \frac{\partial oc_{ijk}(\pi_{ljk}^\tau)}{\partial \pi_{ljk}} - Q_{l,1+j,k}^\tau)\}. \quad (6.18)$$

Similar to the discussion in Chapter 5 (cf. (5.3)), at each iteration τ, the values of the product flows can be determined by the exact equilibration algorithm of Dafermos and Sparrow (1969).

Numerical Examples

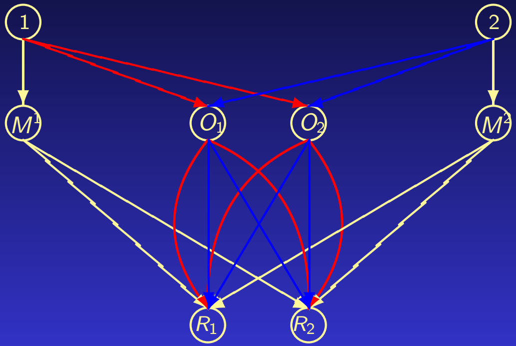


Figure: The Supply Chain Network Topology for the Numerical Examples

Numerical Examples - Example 6.1

The production cost functions at the in-house manufacturing plants are:

$$f_1(Q^1, q^1) = (Q_{111}+Q_{112})^2+1.5(Q_{111}+Q_{112})+2(Q_{211}+Q_{212})+.2q_1(Q_{111}+Q_{112}),$$

$$f_2(Q^1, q^1) = 2(Q_{211}+Q_{212})^2+.5(Q_{211}+Q_{212})+(Q_{111}+Q_{112})+.1q_2(Q_{211}+Q_{212}).$$

The total transportation cost functions for the in-house manufactured products are:

$$\hat{c}_{11}(Q_{111}) = Q_{111}^2 + 5Q_{111}, \quad \hat{c}_{12}(Q_{112}) = 2.5Q_{112}^2 + 10Q_{112},$$
$$\hat{c}_{21}(Q_{211}) = .5Q_{211}^2 + 3Q_{211}, \quad \hat{c}_{22}(Q_{212}) = 2Q_{212}^2 + 5Q_{212}.$$

The in-house total quality cost functions for the two original firms are given by:

$$c_1(q_1) = (q_1 - 80)^2 + 10, \quad c_2(q_2) = (q_2 - 85)^2 + 20.$$

The transaction cost functions are:

$$tc_{11}(Q_{121}+Q_{122}) = .5(Q_{121}+Q_{122})^2 + 2(Q_{121}+Q_{122}) + 100,$$

$$tc_{12}(Q_{131}+Q_{132}) = .7(Q_{131}+Q_{132})^2 + .5(Q_{131}+Q_{132}) + 150,$$

$$tc_{21}(Q_{221}+Q_{222}) = .5(Q_{221}+Q_{222})^2 + 3(Q_{221}+Q_{222}) + 75,$$

$$tc_{22}(Q_{221}+Q_{222}) = .75(Q_{231}+Q_{232})^2 + .5(Q_{231}+Q_{232}) + 100.$$

The contractors' total cost functions of production and distribution are:

$$sc_{111}(Q_{121}, q_{11}) = .5Q_{121}q_{11}, \quad sc_{112}(Q_{122}, q_{11}) = .5Q_{122}q_{11},$$

$$sc_{121}(Q_{131}, q_{12}) = .5Q_{131}q_{12}, \quad sc_{122}(Q_{132}, q_{12}) = .5Q_{132}q_{12},$$

$$sc_{211}(Q_{221}, q_{21}) = .3Q_{221}q_{21}, \quad sc_{212}(Q_{222}, q_{21}) = .3Q_{222}q_{21},$$

$$sc_{221}(Q_{231}, q_{22}) = .25Q_{231}q_{22}, \quad sc_{222}(Q_{232}, q_{22}) = .25Q_{232}q_{22}.$$

The total quality cost functions of the contractors are:

$$\hat{c}_1(q_{11}, q_{21}) = (q_{11}-75)^2 + (q_{21}-75)^2 + 15,$$

$$\hat{c}_2(q_{12}, q_{22}) = 1.5(q_{12}-75)^2 + 1.5(q_{22}-75)^2 + 20.$$

The contractors' opportunity cost functions are:

$$oc_{111}(\pi_{111}) = (\pi_{111} - 10)^2, \quad oc_{121}(\pi_{121}) = .5(\pi_{121} - 5)^2,$$
$$oc_{112}(\pi_{112}) = .5(\pi_{112} - 5)^2, \quad oc_{122}(\pi_{122}) = (\pi_{122} - 15)^2,$$
$$oc_{211}(\pi_{211}) = 2(\pi_{211} - 20)^2, \quad oc_{221}(\pi_{221}) = .5(\pi_{221} - 5)^2,$$
$$oc_{212}(\pi_{212}) = .5(\pi_{212} - 5)^2, \quad oc_{222}(\pi_{222}) = (\pi_{222} - 15)^2.$$

The original firms' disrepute cost functions are:

$$dc_1(q_1') = 100 - q_1', \quad dc_2(q_2') = 100 - q_2',$$

where

$$q_1' = \frac{Q_{121}q_{11} + Q_{131}q_{12} + Q_{111}q_1 + Q_{122}q_{11} + Q_{132}q_{12} + Q_{112}q_1}{d_{11} + d_{12}},$$

and

$$q_2' = \frac{Q_{221}q_{21} + Q_{231}q_{22} + Q_{211}q_2 + Q_{222}q_{21} + Q_{232}q_{22} + Q_{212}q_2}{d_{21} + d_{22}}.$$

ω_1 and ω_2 are 1. q^U is 100.

The Euler method converges in 255 iterations and yields the following equilibrium solution.
The computed product flows are:

$$Q^*_{111} = 13.64, \quad Q^*_{121} = 26.87, \quad Q^*_{131} = 9.49, \quad Q^*_{112} = 9.34, \quad Q^*_{122} = 42.85,$$

$$Q^*_{132} = 47.81, \quad Q^*_{211} = 16.54, \quad Q^*_{221} = 47.31, \quad Q^*_{231} = 11.16, \quad Q^*_{212} = 12.65,$$

$$Q^*_{222} = 62.90, \quad Q^*_{232} = 74.45.$$

The computed quality levels of the original firms and the contractors are:

$$q^*_1 = 77.78, \quad q^*_2 = 83.61, \quad q^*_{11} = 57.57, \quad q^*_{12} = 65.45,$$

$$q^*_{21} = 58.47, \quad q^*_{22} = 67.87.$$

The equilibrium prices are:

$$\pi^*_{111} = 23.44, \quad \pi^*_{112} = 47.85, \quad \pi^*_{121} = 14.49, \quad \pi^*_{122} = 38.91,$$
$$\pi^*_{211} = 31.83, \quad \pi^*_{212} = 67.90, \quad \pi^*_{221} = 16.16, \quad \pi^*_{222} = 52.23.$$

The total costs of the original firms' are, respectively, 11,419.90 and 24,573.94, with their incurred disrepute costs being 36.32 and 34.69. The profits of the contractors are 567.84 and 440.92. The values of q'_1 and q'_2 are, respectively, 63.68 and 65.31.

Example 6.1 - Sensitivity Analysis

I conduct sensitivity analysis by varying the weights that the firms impose on the disrepute, ω, which is the vector of ω_i; $i = 1, 2$, with $\omega = (0,0), (1000,1000), (2000,2000), (3000,3000), (4000,4000), (5000,5000)$.

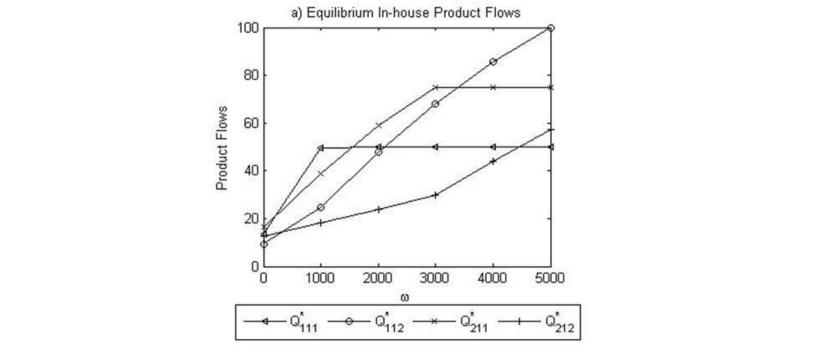


Figure: Equilibrium In-house Product Flows as ω Increases for Example 6.1

A Supply Chain Network Model with Product Differentiation, Outsourcing, and Quality and Price Competition

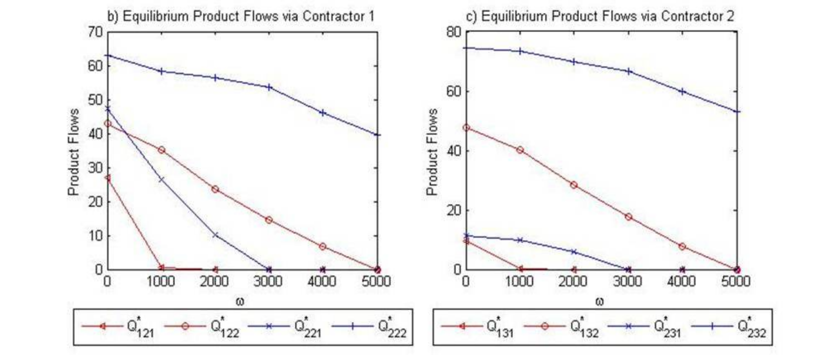


Figure: Equilibrium Outsourced Product Flows as ω Increases for Example 6.1

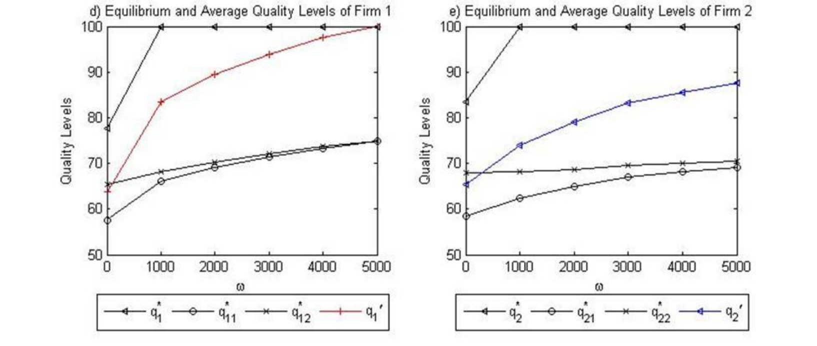


Figure: Equilibrium and Average Quality Levels as ω Increases for Example 6.1

A Supply Chain Network Model with Product Differentiation, Outsourcing, and Quality and Price Competition

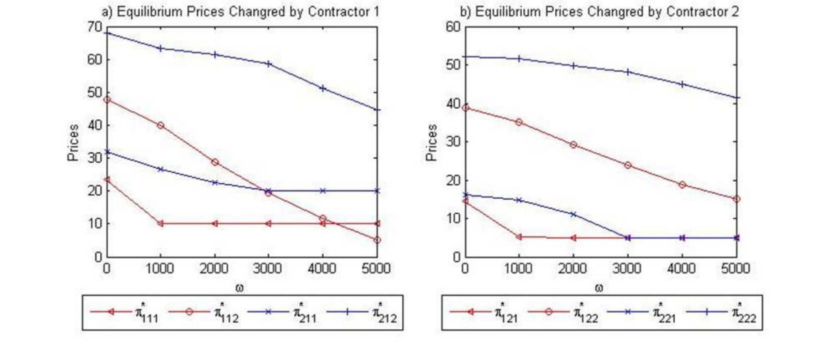


Figure: Equilibrium Prices as ω Increases for Example 6.1

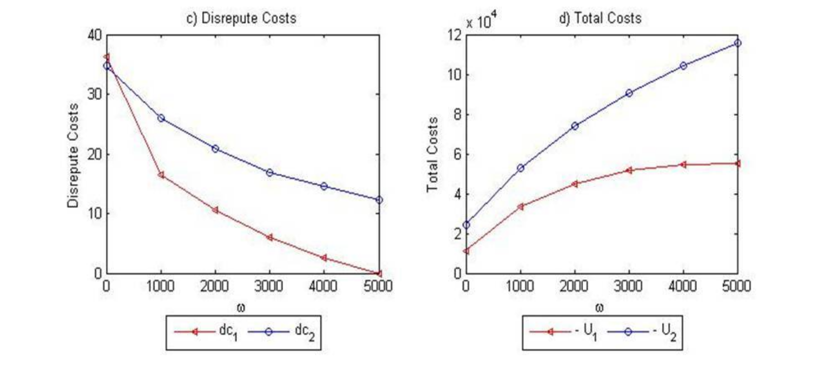


Figure: The Disrepute Costs and Total Costs of the Firms as ω Increases for Example 6.1

Numerical Examples - Example 6.3

In Example 6.3, I consider the scenario that the in-house transportation from the two firms to each demand market gets much more congested than before, and each firm's in-house quantities also affect the other firm's in-house transportation costs.

The total in-house transportation cost functions of the two firms now become:

$$\hat{c}_{11}(Q_{111}, Q_{211}, q_1) = Q_{111}^2 + 1.5Q_{111}q_1 + 7Q_{211},$$

$$\hat{c}_{12}(Q_{112}, Q_{212}.q_1) = 2.5Q_{112}^2 + 2Q_{112}q_1 + 10Q_{212},$$

$$\hat{c}_{21}(Q_{211}, Q_{111}, q_2) = .5Q_{211}^2 + 3Q_{211}q_2 + 8Q_{111},$$

$$\hat{c}_{22}(Q_{212}, Q_{112}, q_2) = 2Q_{212}^2 + 2Q_{212}q_2 + 10Q_{112}.$$

The remaining data are identical to those in Example 6.1.

The total costs of firm 1 and firm 2 associated with different ω values are displayed. The total cost of firm 1 increases monotonically, whether ω_1 or ω_2 increases.

Table: Total Costs of Firm 1 with Different Sets of ω_1 and ω_2

ω	$\omega_1 = 0$	$\omega_1 = 1000$	$\omega_1 = 2000$	$\omega_1 = 3000$	$\omega_1 = 4000$	$\omega_1 = 5000$
$\omega_2 = 0$	12,999.09	45,135.09	61,322.22	71,463.36	77,437.89	80,462.63
$\omega_2 = 1000$	13,218.71	45,348.05	61,535.18	71,676.32	77,650.85	80,675.60
$\omega_2 = 2000$	13,425.67	45,571.40	61,758.53	71,899.67	77,874.20	80,898.94
$\omega_2 = 3000$	13,666.29	45,812.52	61,999.65	72,140.79	78,115.32	81,114.01
$\omega_2 = 4000$	14,091.85	46,034.08	62,221.20	72,362.34	78,336.88	81,361.62
$\omega_2 = 5000$	14,091.85	46,239.00	62,426.12	72,567.26	78,541.80	81,566.54

Table: Total Costs of Firm 2 with Different Sets of ω_1 and ω_2

ω	$\omega_1 = 0$	$\omega_1 = 1000$	$\omega_1 = 2000$	$\omega_1 = 3000$	$\omega_1 = 4000$	$\omega_1 = 5000$
$\omega_2 = 0$	27,585.65	28,203.96	28,561.15	28,798.10	29,005.24	29,187.92
$\omega_2 = 1000$	62,896.33	63,626.00	63,983.19	64,220.14	64,427.28	64,609.96
$\omega_2 = 2000$	92,753.88	93,312.11	93,669.30	93,906.25	94,113.39	94,296.07
$\omega_2 = 3000$	116,378.40	116,981.94	117,339.13	117,576.08	117,783.22	117,965.90
$\omega_2 = 4000$	135,237.43	135,872.91	136,230.10	136,467.05	136,674.19	136,856.87
$\omega_2 = 5000$	150,231.01	150,886.51	151,243.69	151,480.65	151,687.79	151,870.47

Summary and Conclusions

- I developed a supply chain network game theory model with product differentiation, outsourcing, quantity and quality competition among multiple firms, and price and quality competition among multiple contractors.
- This model provides the optimal make-or-buy as well as contractor selection decisions for each original firm.
- I modeled the impact of quality on in-house and outsourced production and transportation and on the reputation of each firm.
- I provided solutions to a series of numerical examples, accompanied by sensitivity analysis.

1. Introduction
2. Quality & Information Asymmetry
   - A Supply Chain Network Model with Information Asymmetry in Quality, Minimum Quality Standards, and Quality Competition
3. Quality & Product Differentiation
   - A Supply Chain Network Model with Transportation Costs, Product Differentiation, and Quality Competition
4. Quality & Outsourcing
   - A Supply Chain Network Model with Outsourcing and Quality and Price Competition
   - A Supply Chain Network Model with Product Differentiation, Outsourcing, and Quality and Price Competition
5. Quality & Supplier Selection
   - A Supply Chain Network Model with Supplier Selection and Quality and Price Competition
6. Future Research

This section corresponds to Chapter 7 of the dissertation, and is based on the paper:

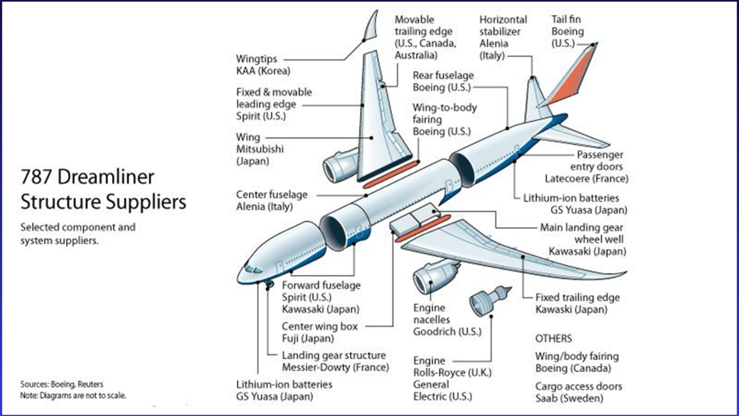
Li, D., Nagurney, A., 2015. A general multitiered supply chain network model of quality competition with suppliers.

Background and Motivation

Indeed, products are made of materials and components, and the components and materials are produced and supplied not by the firms that process them into products but by suppliers in globalized supply chain networks.

The quality of a product depends not only on the quality of the firm that produces and delivers it, but also on the quality of the components provided by the firm's suppliers (Robinson and Malhotra (2005) and Foster (2008)).

The number of components comprising a finished product may be small or immense as in aircraft manufacturing and other complex high-tech products.



In recent years, a series of recalls caused by suppliers' poor quality components/materials has received intensive attention.

- In 2007, the toy giant Mattel recalled 19 million toy cars because of a supplier's lead paint and small, poorly designed magnets, which could harm children if ingested (Story and Barboza (2007)).
- In 2010, four Japanese car-makers, including Toyota and Nissan, recalled 3.6 million vehicles sold around the globe, because the airbags supplied by Takata Corp., were at risk of catching fire (Kubota and Klayman (2013)). The recalls are still ongoing and have expanded to other companies as well (Tabuchi and Jensen (2014)).
- In 2013, in the food industry, Taylor Farms, a large vegetable supplier, was under investigation in connection with an illness outbreak affecting hundreds of people in the US (Strom (2013)).

Furthermore, since suppliers, which may be located on-shore or off-shore, supply chain networks of firms may be more vulnerable to disruptions than ever before.

Literature Review

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Overview

- The firms are responsible for assembling the products under their brand names using the components from their suppliers, and delivering the products to multiple demand markets.
- Firms also have the option of producing their own components, if necessary.
- The firms compete in product quantity, the preservation level of its assembly process, the contracted component quantities produced by the suppliers, and in in-house component quantities and quality level.
- The potential suppliers may either provide distinct components to the firms, or provide the same component in which case they compete non-cooperatively with one another in terms of quality and prices.
- A formula to quantify the quality of the finished product based on the quality of the individual components.

The Multitiered Supply Chain Network Topology with Suppliers

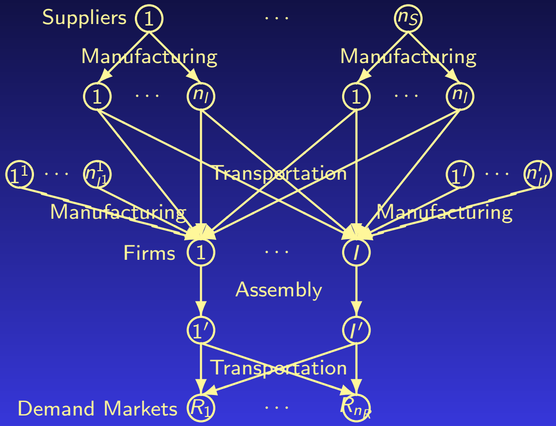

Figure: The Multitiered Supply Chain Network Topology with Suppliers

The Model

Conservation of flow equation

$$Q_{ik} = d_{ik}, \quad i = 1, \ldots, I; k = 1, \ldots, n_R. \tag{7.1}$$

Nonnegative shipment volumes

$$Q_{ik} \geq 0, \quad i = 1, \ldots, I; k = 1, \ldots, n_R. \tag{7.2}$$

Quality levels

$$q^U_{il} \geq q^S_{jil} \geq 0, \quad j = 1, \ldots, n_S; i = 1, \ldots, I; l = 1, \ldots, n_{l^i}, \tag{7.3}$$

$$q^U_{il} \geq q^F_{il} \geq 0, \quad i = 1, \ldots, I; l = 1, \ldots, n_{l^i}. \tag{7.4}$$

The average quality level of product i's component l

$$q_{il} = \frac{q^F_{il} Q^F_{il} + \sum_{j=1}^{n_S} Q^S_{jil} q^S_{jil}}{Q^F_{il} + \sum_{j=1}^{n_S} Q^S_{jil}}, \quad i = 1, \ldots, I; l = 1, \ldots, n_{l^i}. \tag{7.5}$$

The Model

The quality level of a finished product

$$q_i = \alpha_i^F (\sum_{l=1}^{n_{l_i}} \omega_{il} q_{il}), \quad i = 1, \ldots, I; l = 1. \tag{7.6}$$

α_i^F captures the percentage of the quality preservation of product i in the assembly process.

$$0 \le \alpha_i^F \le 1, \quad i = 1, \ldots, I. \tag{7.7}$$

ω_{il} is the ratio of the importance of the quality of firm i's component l in one unit product i to the quality associated with one unit product i (i.e., q_i).

$$\sum_{l=1}^{n_{l^i}} \omega_{il} = 1, \quad i = 1, \ldots, I. \tag{7.8}$$

The Model - The Behavior of the Firms

The total utility maximization objective of firm i

$$\text{Maximize}_{Q_i,Q_i^F,Q_i^S,q_i^F,\alpha_i^F} \quad U_i^F=\sum_{k=1}^{n_R}\hat{\rho}_{ik}(Q,Q^F,Q^S,q^F,q^{S^*},\alpha^F)d_{ik}-f_i(Q,\alpha^F)$$

$$-\sum_{l=1}^{n_{l^i}} f_{il}^F(Q^F,q^F)-\sum_{k=1}^{n_R}\hat{c}_{ik}^F(Q,Q^F,Q^S,q^F,q^{S^*},\alpha^F)-\sum_{j=1}^{n_S}\sum_{l=1}^{n_{l^i}} tc_{ijl}(Q^S)-\sum_{j=1}^{n_S}\sum_{l=1}^{n_{l^i}}\pi_{jil}^*Q_{jil}^S \tag{7.11}$$

subject to:

$$\sum_{k=1}^{n_R}Q_{ik}\theta_{il}\leq\sum_{j=1}^{n_S}Q_{jil}^S+Q_{il}^F, \quad i=1,\dots,I;\, l=1,\dots,n_{l^i}, \tag{7.12}$$

$$CAP_{jil}^S\geq Q_{jil}^S\geq 0, \quad j=1,\dots,n_S;\, i=1,\dots,I;\, l=1,\dots,n_{l^i}, \tag{7.13}$$

$$CAP_{il}^F\geq Q_{il}^F\geq 0, \quad i=1,\dots,I;\, l=1,\dots,n_{l^i}, \tag{7.14}$$

and (7.1), (7.2), (7.4), and (7.7).

Variational Inequality Formulation

Theorem 7.1

$(Q^*, Q^{F*}, Q^{S*}, q^{F*}, \alpha^{F*}) \in \overline{\mathcal{K}}^F$ is a Counot-Nash equilibrium if and only if it satisfies the variational inequality:

$$-\sum_{i=1}^{I}\sum_{k=1}^{n_R} \frac{\partial U_i^F(Q^*, Q^{F*}, Q^{S*}, q^{F*}, \alpha^{F*}, \pi_i^*, q^{S*})}{\partial Q_{ik}} \times (Q_{ik} - Q_{ik}^*)$$

$$-\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}} \frac{\partial U_i^F(Q^*, Q^{F*}, Q^{S*}, q^{F*}, \alpha^{F*}, \pi_i^*, q^{S*})}{\partial Q_{il}^F} \times (Q_{il}^F - Q_{il}^{F*})$$

$$-\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}} \frac{\partial U_i^F(Q^*, Q^{F*}, Q^{S*}, q^{F*}, \alpha^{F*}, \pi_i^*, q^{S*})}{\partial Q_{jil}^S} \times (Q_{jil}^S - Q_{jil}^{S*})$$

$$-\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}} \frac{\partial U_i^F(Q^*, Q^{F*}, Q^{S*}, q^{F*}, \bar{q}^{F*}, \pi_i^*, q^{S*})}{\partial q_{il}^F} \times (q_{il}^F - q_{il}^{F*})$$

$$-\sum_{i=1}^{I} \frac{\partial U_i^F(Q^*, Q^{F*}, Q^{S*}, q^{F*}, \alpha^{F*}, \pi_i^*, q^{S*})}{\partial \alpha_i^F} \times (\alpha_i^F - \alpha_i^{F*}) \geq 0, \quad \forall (Q, Q^F, Q^S, q^F, \alpha^F) \in \overline{\mathcal{K}}^F, \tag{7.16}$$

A Supply Chain Network Model with Supplier Selection and Quality and Price Competition

equivalently, $(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \lambda^*) \in \mathcal{K}^F$ *is a Counot-Nash equilibrium if and only if it satisfies the variational inequality:*

$$\sum_{i=1}^{I}\sum_{k=1}^{n_R}\left[\sum_{h=1}^{n_R}\frac{\partial \hat{c}^F_{ih}(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, q^{S^*}, \alpha^{F^*})}{\partial Q_{ik}} - \sum_{h=1}^{n_R}\frac{\partial \hat{\rho}_{ih}(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, q^{S^*}, \alpha^{F^*})}{\partial Q_{ik}}\right.$$

$$\left.\times Q^*_{ih} + \frac{\partial f_i(Q^*, \alpha^{F^*})}{\partial Q_{ik}} - \hat{\rho}_{ik}(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, q^{S^*}, \alpha^{F^*}) + \sum_{l=1}^{n_{l^i}}\lambda^*_{il}\theta_{il}\right] \times (Q_{ik} - Q^*_{ik})$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\left[\sum_{m=1}^{n_{l^i}}\frac{\partial f^F_{im}(Q^{F^*}, q^{F^*})}{\partial Q^F_{il}} + \sum_{h=1}^{n_R}\frac{\partial \hat{c}^F_{ih}(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, q^{S^*}, \alpha^{F^*})}{\partial Q^F_{il}}\right.$$

$$\left.-\sum_{h=1}^{n_R}\frac{\partial \hat{\rho}_{ih}(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, q^{S^*}, \alpha^{F^*})}{\partial Q^F_{il}} \times Q^*_{ih} - \lambda^*_{il}\right] \times (Q^F_{il} - Q^{F^*}_{il})$$

$$+\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\left[\pi^*_{jil} + \sum_{h=1}^{n_S}\sum_{m=1}^{n_{l^i}}\frac{\partial tc_{ihm}(Q^{S^*})}{\partial Q^S_{jil}} + \sum_{h=1}^{n_R}\frac{\partial \hat{c}^F_{ih}(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, q^{S^*}, \alpha^{F^*})}{\partial Q^S_{jil}}\right.$$

$$\left.-\sum_{h=1}^{n_R}\frac{\partial \hat{\rho}_{ih}(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, q^{S^*}, \alpha^{F^*})}{\partial Q^S_{jil}} \times Q^*_{ih} - \lambda^*_{il}\right] \times (Q^S_{jil} - Q^{S^*}_{jil})$$

A Supply Chain Network Model with Supplier Selection and Quality and Price Competition

$$
\begin{aligned}
&+\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\left[\sum_{m=1}^{n_{l^i}}\frac{\partial f^F_{im}(Q^{F^*},q^{F^*})}{\partial q^F_{il}}+\sum_{h=1}^{n_R}\frac{\partial \hat{c}^F_{ih}(Q^*,Q^{F^*},Q^{S^*},q^{F^*},q^{S^*},\alpha^{F^*})}{\partial q^F_{il}}\right.\\
&\qquad\left.-\sum_{h=1}^{n_R}\frac{\partial \hat{\rho}_{ih}(Q^*,Q^{F^*},Q^{S^*},q^{F^*},q^{S^*},\alpha^{F^*})}{\partial q^F_{il}}\times Q^*_{ih}\right]\times(q^F_{il}-q^{F^*}_{il})\\
&+\sum_{i=1}^{I}\left[\frac{\partial f_i(Q^{F^*},\alpha^{F^*})}{\partial \alpha^F_i}+\sum_{h=1}^{n_R}\frac{\partial \hat{c}^F_{ih}(Q^*,Q^{F^*},Q^{S^*},q^{F^*},q^{S^*},\alpha^{F^*})}{\partial \alpha^F_i}\right.\\
&\qquad\left.-\sum_{h=1}^{n_R}\frac{\partial \hat{\rho}_{ih}(Q^*,Q^{F^*},Q^{S^*},q^{F^*},q^{S^*},\alpha^{F^*})}{\partial \alpha^F_i}\times Q^*_{ih}\right]\times(\alpha^F_i-\alpha^{F^*}_i)\\
&+\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\left[\sum_{j=1}^{n_S}Q^{S^*}_{jil}+Q^{F^*}_{il}-\sum_{k=1}^{n_R}Q^*_{ik}\theta_{il}\right]\times(\lambda_{il}-\lambda^*_{il})\geq 0,\quad \forall(Q,Q^F,Q^S,q^F,\alpha^F,\lambda)\in\mathcal{K}^F. \qquad (7.17)
\end{aligned}
$$

The Model - The Behavior of the Suppliers

The total utility maximization objective of supplier j

$$\text{Maximize}_{\pi_j, q_j^S} \quad U_j^S = \sum_{i=1}^{I}\sum_{l=1}^{n_{li}} \pi_{jil} Q_{jil}^{S^*} - \sum_{l=1}^{n_l} f_{jl}^S(Q^{S^*}, q^S) - \sum_{i=1}^{I}\sum_{l=1}^{n_{li}} \hat{c}_{jil}^S(Q^{S^*}, q^S)$$

$$- \sum_{i=1}^{I}\sum_{l=1}^{n_{li}} oc_{jil}(\pi) \tag{7.19}$$

subject to:

$$\pi_{jil} \geq 0, \quad j = 1, \ldots, n_S; i = 1, \ldots, I; l = 1, \ldots, n_{li}, \tag{7.20}$$

and (7.3).

Variational Inequality Formulation

Theorem 7.2

$(\pi^*, q^{S^*}) \in \mathcal{K}^S$ is a Bertrand-Nash equilibrium if and only if it satisfies the variational inequality:

$$-\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}} \frac{\partial U_j^S(Q^{S^*}, \pi^*, q^{S^*})}{\partial \pi_{jil}} \times (\pi_{jil} - \pi_{jil}^*)$$

$$-\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}} \frac{\partial U_j^S(Q^{S^*}, \pi^*, q^{S^*})}{\partial q_{jil}^S} \times (q_{jil}^S - q_{jil}^{S^*}) \geq 0, \forall (\pi, q^S) \in \mathcal{K}^S. \quad (7.22)$$

The Equilibrium Conditions for the Supply Chain Network with Supplier Selection and Quality and Price Competition

Definition 7.3

The equilibrium state of the multitiered supply chain network with suppliers is one where both variational inequalities (7.16), or, equivalently, (7.17), and (7.22) hold simultaneously.

Theorem 7.3

Determine $(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \lambda^*, \pi^*, q^{S^*}) \in \mathcal{K}$, *such that:*

$$\sum_{i=1}^{I}\sum_{k=1}^{n_R}\left[\frac{\partial f_i(Q^*, \alpha^{F^*})}{\partial Q_{ik}} + \sum_{h=1}^{n_R}\frac{\partial \hat{c}^F_{ih}(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, q^{S^*}, \alpha^{F^*})}{\partial Q_{ik}}\right.$$

$$\left.-\sum_{h=1}^{n_R}\frac{\partial \hat{\rho}_{ih}(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, q^{S^*}, \alpha^{F^*})}{\partial Q_{ik}} \times Q^*_{ih} - \hat{\rho}_{ik}(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, q^{S^*}, \alpha^{F^*}) + \sum_{l=1}^{n_{l^i}}\lambda^*_{il}\theta_{il}\right]$$

$$\times(Q_{ik} - Q^*_{ik}) + \sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\left[\sum_{m=1}^{n_{l^i}}\frac{\partial f^F_{im}(Q^{F^*}, q^{F^*})}{\partial Q^F_{il}} + \sum_{h=1}^{n_R}\frac{\partial \hat{c}^F_{ih}(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, q^{S^*}, \alpha^{F^*})}{\partial Q^F_{il}}\right.$$

$$\left.-\sum_{h=1}^{n_R}\frac{\partial \hat{\rho}_{ih}(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, q^{S^*}, \alpha^{F^*})}{\partial Q^F_{il}} \times Q^*_{ih} - \lambda^*_{il}\right] \times (Q^F_{il} - Q^{F^*}_{il})$$

$$+\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\left[\pi^*_{jil} + \sum_{h=1}^{n_S}\sum_{m=1}^{n_{l^i}}\frac{\partial tc_{ihm}(Q^{S^*})}{\partial Q^S_{jil}} + \sum_{h=1}^{n_R}\frac{\partial \hat{c}^F_{ih}(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, q^{S^*}, \alpha^{F^*})}{\partial Q^S_{jil}}\right.$$

$$\left.-\sum_{h=1}^{n_R}\frac{\partial \hat{\rho}_{ih}(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, q^{S^*}, \alpha^{F^*})}{\partial Q^S_{jil}} \times Q^*_{ih} - \lambda^*_{il}\right] \times (Q^S_{jil} - Q^{S^*}_{jil})$$

A Supply Chain Network Model with Supplier Selection and Quality and Price Competition

$$+\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\left[\sum_{m=1}^{n_{l^i}}\frac{\partial f^F_{im}(Q^{F^*},q^{F^*})}{\partial q^F_{il}}+\sum_{h=1}^{n_R}\frac{\partial \hat{c}^F_{ih}(Q^*,Q^{F^*},Q^{S^*},q^{F^*},q^{S^*},\alpha^{F^*})}{\partial q^F_{il}}\right.$$

$$\left.-\sum_{h=1}^{n_R}\frac{\partial \hat{\rho}_{ih}(Q^*,Q^{F^*},Q^{S^*},q^{F^*},q^{S^*},\alpha^{F^*})}{\partial q^F_{il}}\times Q^*_{ih}\right]\times(q^F_{il}-q^{F^*}_{il})$$

$$+\sum_{i=1}^{I}\left[\frac{\partial f_i(Q^{F^*},\alpha^{F^*})}{\partial \alpha^F_i}+\sum_{h=1}^{n_R}\frac{\partial \hat{c}^F_{ih}(Q^*,Q^{F^*},Q^{S^*},q^{F^*},q^{S^*},\alpha^{F^*})}{\partial \alpha^F_i}\right.$$

$$\left.-\sum_{h=1}^{n_R}\frac{\partial \hat{\rho}_{ih}(Q^*,Q^{F^*},Q^{S^*},q^{F^*},q^{S^*},\alpha^{F^*})}{\partial \alpha^F_i}\times Q^*_{ih}\right]\times(\alpha^F_i-\alpha^{F^*}_i)$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\left[\sum_{j=1}^{n_S}Q^{S^*}_{jil}+Q^{F^*}_{il}-\sum_{k=1}^{n_R}Q^*_{ik}\theta_{il}\right]\times(\lambda_{il}-\lambda^*_{il})+\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\left[\sum_{g=1}^{I}\sum_{m=1}^{n_{l^i}}\frac{\partial oc_{jgm}(\pi^*)}{\partial \pi_{jil}}-Q^{S^*}_{jil}\right]$$

$$\times(\pi_{jil}-\pi^*_{jil})+\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\left[\sum_{m=1}^{n_l}\frac{\partial f^S_{jm}(Q^{S^*},q^{S^*})}{\partial q^S_{jil}}+\sum_{g=1}^{I}\sum_{m=1}^{n_{l^i}}\frac{\partial \hat{c}^S_{jgm}(Q^{S^*},q^{S^*})}{\partial q^S_{jil}}\right]\times(q^S_{jil}-q^{S^*}_{jil})\geq 0,$$

$$\forall (Q,Q^F,Q^S,q^F,\alpha^F,\lambda,\pi,q^S)\in\mathcal{K}, \tag{7.24}$$

Standard form VI

We now put variational inequality (7.24) into standard form (cf. (2.1)): Determine $X^* \in \mathcal{K}$ where X is a vector in R^N, $F(X)$ is a continuous function such that $F(X): X \mapsto \mathcal{K} \subset R^N$, and

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{7.25}$$

where $\langle \cdot, \cdot \rangle$ is the inner product in the N-dimensional Euclidean space, $N = In_R + 3\sum_{i=1}^{I} n_{I^i} + 3n_S \sum_{i=1}^{I} n_{I^i} + I$, and $\mathcal{K}$ is closed and convex. Define the vector $X \equiv (Q, Q^F, Q^S, q^F, \alpha^F, \lambda, \pi, q^S)$.

Standard form VI

We also put variational inequality (7.23) into standard form: Determine $Y^* \in \overline{\mathcal{K}}$ where Y is a vector in R^M, $G(Y)$ is a continuous function such that $G(Y): Y \mapsto \overline{\mathcal{K}} \subset R^M$, and

$$\langle G(Y^*), Y - Y^* \rangle \geq 0, \quad \forall Y \in \overline{\mathcal{K}}, \tag{7.27}$$

where $M = In_R + 2\sum_{i=1}^{I} n_{I^i} + 3n_S \sum_{i=1}^{I} n_{I^i} + I$, and $\overline{\mathcal{K}}$ is closed and convex.

Qualitative Properties

Assumption 7.1

Suppose that in our multitiered supply chain network model with suppliers and quality competition, there exist a sufficiently large Π, such that,

$$\pi_{jil} \leq \Pi, \quad j = 1, \ldots, n_S; i = 1, \ldots, I; l = 1, \ldots, n_{l^i}. \tag{7.28}$$

Theorem 7.4

With Assumption 7.1 satisfied, there exists at least one solution to variational inequality (7.25) and (7.27), equivalently, (7.24) and (7.23).

Theorem 7.5

Under the assumptions in Theorems 7.1 and 7.2, the $F(X)$ that enters variational inequality (7.25), is monotone, that is,

$$\langle F(X') - F(X''), X' - X'' \rangle \geq 0, \quad \forall X', X'' \in \mathcal{K}, \tag{7.29}$$

and the $G(Y)$ that enters variational inequality (7.27) is also monotone,

$$\langle G(Y') - G(Y''), Y' - Y'' \rangle \geq 0, \quad \forall Y', Y'' \in \overline{\mathcal{K}}. \tag{7.30}$$

Theorem 7.6

Assume that the function $G(Y)$ in variational inequality (7.27) is strictly monotone on $\bar{\mathcal{K}}$. Then, if variational inequality (7.27) admits a solution, $(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi^*, q^{S^*})$, that is the only solution.

The Algorithm - The Modified Projection Method

The modified projection method

Step 0: Initialization
Start with $X^0 \in \mathcal{K}$. Set $\mathcal{T} := 1$ and select a, such that $0 < a \leq \frac{1}{L}$, where L is the Lipschitz continuity constant for $F(X)$.

Step 1: Construction and Computation
Compute $\overline{X}^{\mathcal{T}-1}$ by solving the variational inequality subproblem:

$$\langle \overline{X}^{\mathcal{T}-1} + (aF(X^{\mathcal{T}-1}) - X^{\mathcal{T}-1})^T, X - \overline{X}^{\mathcal{T}-1} \rangle \geq 0, \quad \forall X \in \mathcal{K}.$$

Step 2: Adaptation
Compute $X^{\mathcal{T}}$ by solving the variational inequality subproblem:

$$\langle X^{\mathcal{T}} + (aF(\overline{X}^{\mathcal{T}-1}) - X^{\mathcal{T}-1})^T, X - X^{\mathcal{T}} \rangle \geq 0, \quad \forall X \in \mathcal{K}.$$

Step 3: Convergence Verification
If $| X_l^{\mathcal{T}} - X_l^{\mathcal{T}-1} | \leq \epsilon$, for all l, with $\epsilon > 0$, a prespecified tolerance, then stop; else set $\mathcal{T} := \mathcal{T} + 1$, and go to step 1.

Numerical Examples and Sensitivity Analysis - Example 7.1

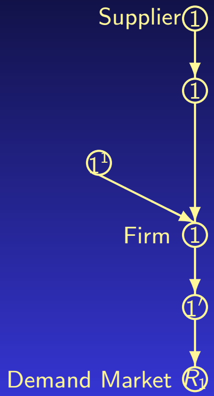

Figure: Supply Chain Network Topology for Example 7.1

The product of firm 1 requires only one component 1^1. 2 units of 1^1 are needed for producing one unit of firm 1's product. Thus,

$$\theta_{11} = 2.$$

The capacity of the supplier is:

$$CAP^S_{111} = 120.$$

The firm's capacity for producing its component is:

$$CAP^F_{11} = 80.$$

The value that represents the perfect component quality is:

$$q^U_{11} = 75.$$

The supplier's production cost is:

$$f^S_{11}(Q^S_{111}, q^S_{111}) = 5Q^S_{111} + 0.8(q^S_{111} - 62.5)^2.$$

The supplier's transportation cost is:

$$\hat{c}^S_{111}(Q^S_{111}, q^S_{111}) = 0.5Q^S_{111} + 0.2(q^S_{111} - 125)^2 + 0.3Q^S_{111}q^S_{111},$$

and its opportunity cost is:

$$oc_{111}(\pi_{111}) = 0.7(\pi_{111} - 100)^2.$$

The firm's assembly cost is:

$$f_1(Q_{11}, \alpha_1^F) = 0.75Q_{11}^2 + 200\alpha_1^{F^2} + 200\alpha_1^F + 25Q_{11}\alpha_1^F.$$

The firm's production cost for producing its component is:

$$f_{11}^F(Q_{11}^F, q_{11}^F) = 2.5Q_{11}^{F^2} + 0.5(q_{11}^F - 60)^2 + 0.1Q_{11}^F q_{11}^F,$$

and its transaction cost is:

$$tc_{111}(Q_{111}^S) = 0.5Q_{111}^{S^2} + Q_{111}^S + 100.$$

The firm's transportation cost for shipping its product to the demand market is:

$$c_{11}^F(Q_{11}, q_1) = 0.5Q_{11}^2 + 0.02q_1^2 + 0.1Q_{11}q_1,$$

and the demand price function at demand market R_1 is:

$$\rho_{11}(d_{11}, q_1) = -d_{11} + 0.7q_1 + 1000,$$

where $q_1 = \alpha_1^F \omega_{11} \frac{Q_{11}^F q_{11}^F + Q_{111}^S q_{111}^S}{Q_{11}^F + Q_{111}^S}$ and $\omega_{11} = 1$.

The equilibrium solution obtained using the modified projection method is:

$$Q_{11}^{*} = 89.26, \quad Q_{11}^{F^*} = 60.16, \quad Q_{111}^{S^*} = 118.38, \quad q_{11}^{F^*} = 71.17,$$

$$q_{111}^{S^*} = 57.25, \quad \pi_{11}^{*} = 184.53, \quad \alpha_{1}^{F^*} = 1.00, \quad \lambda_{11}^{*} = 305.25.$$

with the induced demand, demand price, and product quality being

$$d_{11} = 89.26, \quad \rho_{11} = 954.10, \quad q_1 = 61.94.$$

The profit of the firm is 33,331.69, and the profit of the supplier is 13,218.67.

For this example, the eigenvalues of the symmetric part of the Jacobian matrix of $G(Y)$ are all positive. Therefore, $\nabla G(Y)$ is positive-definite, and $G(Y)$ is strictly monotone. The uniqueness of the solution $(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi^*, q^{S^*})$ and the convergence of the modified projection method are then guaranteed.

Example 1 - Sensitivity Analysis

I maintain the capacity of the firm at 80, and vary the capacity of the supplier from 0 to 20, 40, 60, 80, 100, and 120.

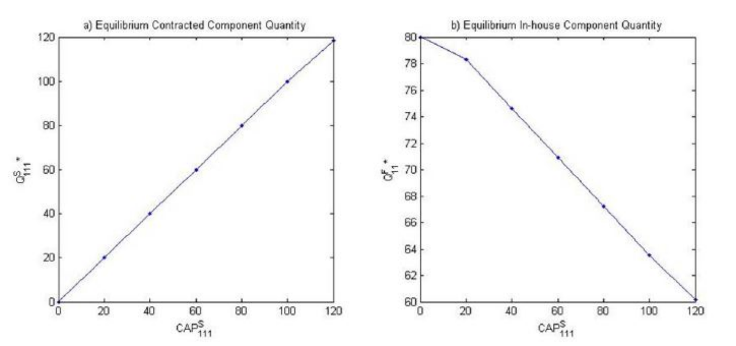

Figure: Equilibrium Component Quantities as the Capacity of the Supplier Varies

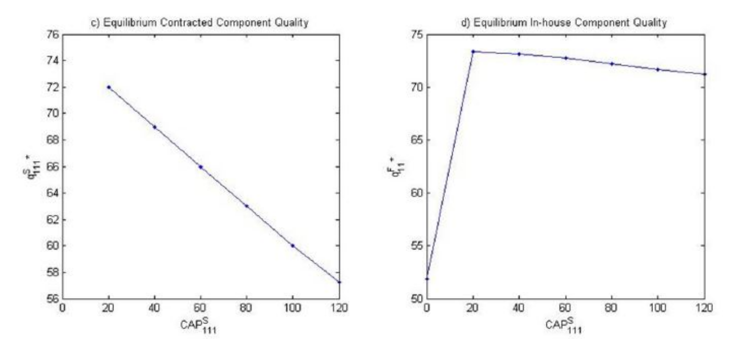

Figure: Equilibrium Component Quality Levels as the Capacity of the Supplier Varies

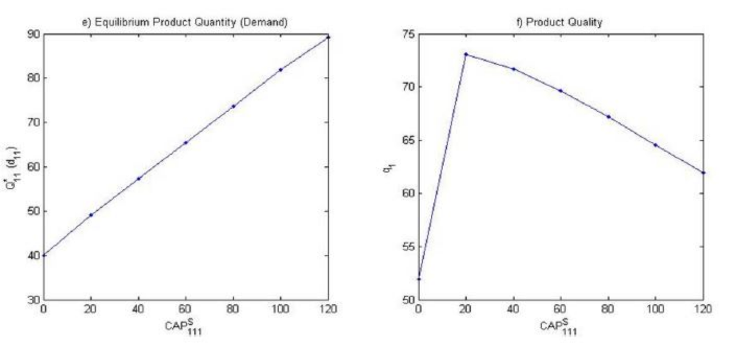

Figure: Equilibrium Product Quantity (Demand) and Product Quality as the Capacity of the Supplier Varies

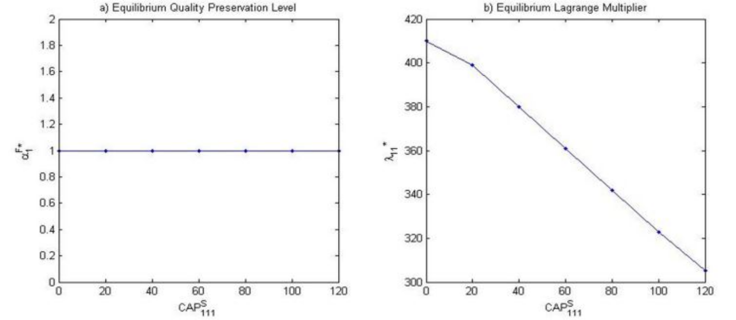

Figure: Equilibrium Quality Preservation Level and Equilibrium Lagrange Multiplier as the Capacity of the Supplier Varies

A Supply Chain Network Model with Supplier Selection and Quality and Price Competition

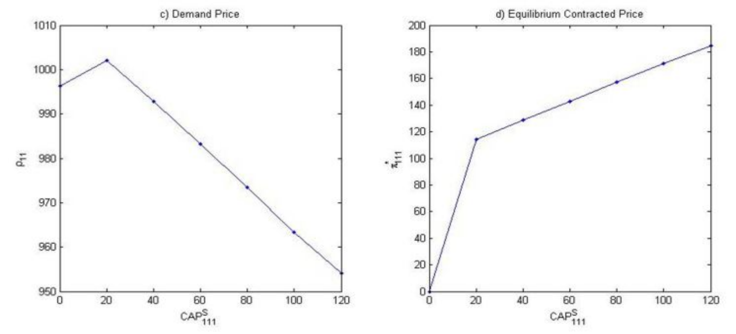

Figure: Demand Price and Equilibrium Contracted Price as the Capacity of the Supplier Varies

A Supply Chain Network Model with Supplier Selection and Quality and Price Competition

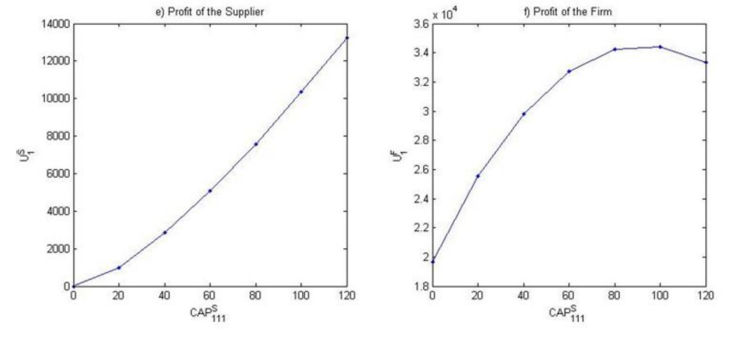

Figure: The Supplier's Profit and the Firm's Profit as the Capacity of the Supplier Varies

I then maintain the capacity of the supplier at 120, and vary the capacity of the firm from 0 to 20, 40, 60, and 80.

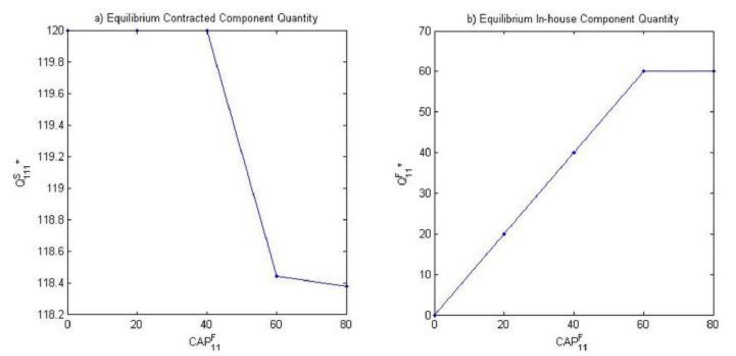

Figure: Equilibrium Component Quantities as the Capacity of the Firm Varies

A Supply Chain Network Model with Supplier Selection and Quality and Price Competition

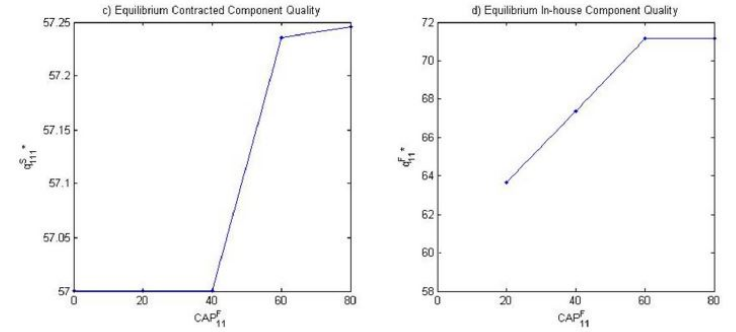

Figure: Equilibrium Component Quality Levels as the Capacity of the Firm Varies

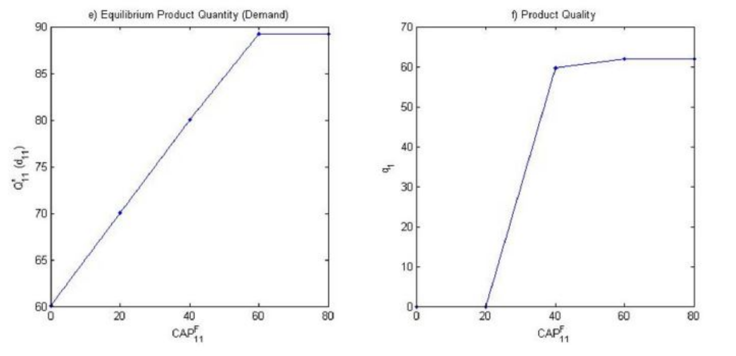

Figure: Equilibrium Product Quantity (Demand) and Product Quality as the Capacity of the Firm Varies

A Supply Chain Network Model with Supplier Selection and Quality and Price Competition

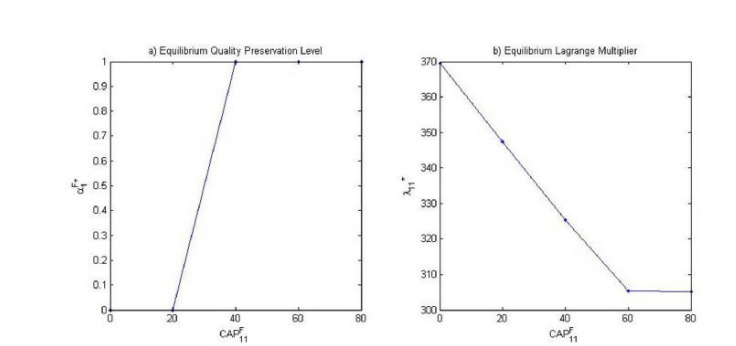

Figure: Equilibrium Quality Preservation Level and Equilibrium Lagrange Multiplier as the Capacity of the Firm Varies

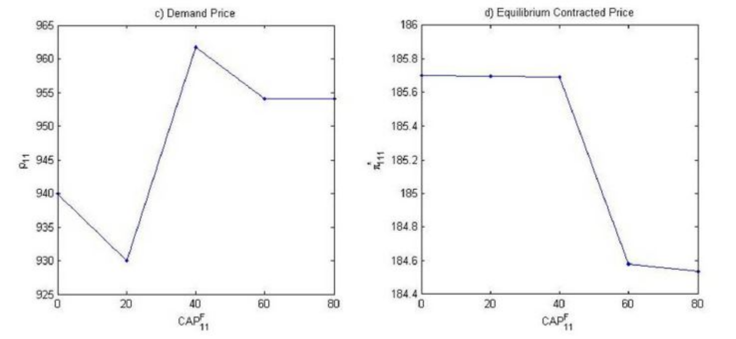

Figure: Demand Price and Equilibrium Contracted Price as the Capacity of the Firm Varies

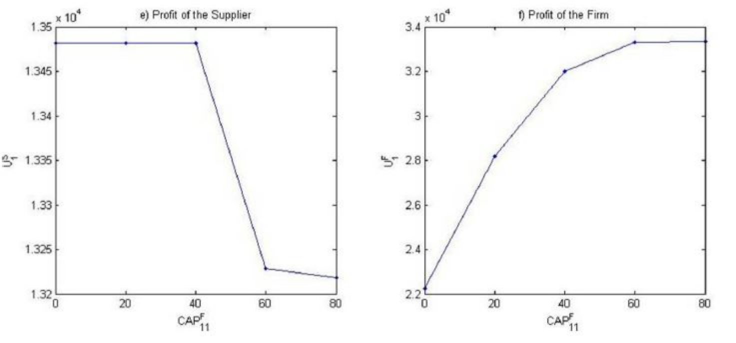

Figure: The Supplier's Profit and the Firm's Profit as the Capacity of the Firm Varies

Example 1 - Sensitivity Analysis - Investing in Capacity Changing

From \ To	CAP^S_{111}=0	20	40	60	80	100	120
CAP^S_{111}=0	–	0.97, 5.89	2.86, 10.17	5.08, 13.09	7.57, 14.62	10.37, 14.77	13.22, 13.69
20	-0.97, -5.89	–	1.90, 4.28	4.09, 7.20	6.60, 8.73	9.40, 8.88	12.25, 7.80
40	-2.86, -10.17	-1.90, -4.28	–	2.20, 2.92	4.70, 4.45	7.51, 4.60	10.36, 3.52
60	-5.06, -13.09	-4.09, -7.20	-2.20, -2.92	–	2.50, 1.53	5.31, 1.68	8.16, 0.60
80	-7.57, -14.62	-6.60, -8.73	-4.70, -4.45	-2.50, -1.53	–	2.81, 0.15	*5.66, -0.93*
100	-10.37, -14.77	-9.40, -8.88	-7.51, -4.60	-5.31, -1.68	-2.81, -0.15	–	*2.85, -1.08*
120	-13.22, -13.69	-12.25, -7.80	-10.36, -3.52	-8.16, -0.60	*-5.65, 0.93*	*-2.85, 1.08*	–

Figure: Maximum Acceptable Investments ($\times 10^3$) for Capacity Changing when the Capacity of the Firm Maintains 80 but that of the Supplier Varies

From \ To	CAP^F_{11}=0	20	40	60	80
CAP^F_{11}=0	–	0.00, 5.94	0.00, 9.77	*-0.25, 11.10*	*-0.26, 11.10*
20	0.00, -5.94	–	0.00, 3.83	*-0.25, 5.16*	*-0.26, 5.16*
40	0.00, -9.77	0.00, -3.83	–	*-0.25, 1.33*	*-0.26, 1.33*
60	*0.25, -11.10*	*0.25, -5.16*	*0.25, -1.33*	–	*-0.01, 0.004*
80	*0.26, -11.10*	*0.26, -5.16*	*0.26, -1.33*	*0.01, -0.004*	–

Figure: Maximum Acceptable Investments ($\times 10^3$) for Capacity Changing when the Capacity of the Supplier Maintains 120 but that of the Firm Varies

Numerical Examples and Sensitivity Analysis - Example 7.2

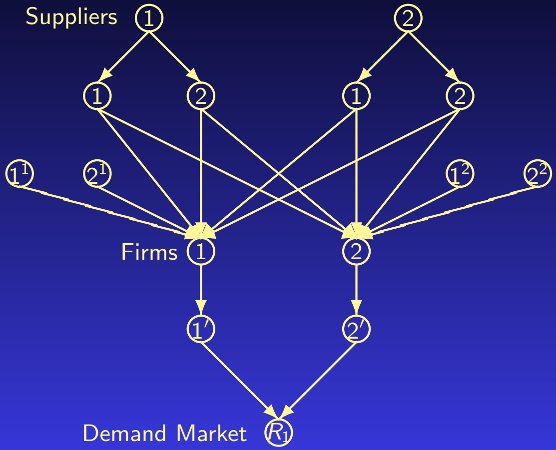

Figure: Supply Chain Network Topology for Example 7.2

$$\theta_{11} = 1, \quad \theta_{12} = 2, \quad \theta_{21} = 2, \quad \theta_{22} = 1.$$

The ratio of the importance of the quality of the components to the quality of one unit product is:

$$\omega_{11} = 0.2, \quad \omega_{12} = 0.8, \quad \omega_{21} = 0.4, \quad \omega_{22} = 0.6.$$

The capacities of the suppliers are:

$$CAP^S_{111} = 80, \quad CAP^S_{112} = 100, \quad CAP^S_{121} = 100, \quad CAP^S_{122} = 60,$$
$$CAP^S_{211} = 60, \quad CAP^S_{212} = 100, \quad CAP^S_{221} = 100, \quad CAP^S_{222} = 50.$$

The firms' capacities for in-house component production are:

$$CAP^F_{11} = 30, \quad CAP^F_{12} = 30, \quad CAP^F_{21} = 30, \quad CAP^F_{22} = 30.$$

The values representing the perfect component quality are:

$$q^U_{11} = 60, \quad q^U_{12} = 75, \quad q^U_{21} = 60, \quad q^U_{22} = 75.$$

The suppliers' production costs are:

$$f_{11}^S = 0.4(Q_{111}^S + Q_{121}^S) + 1.5(q_{111}^S - 50)^2 + 1.5(q_{121}^S - 50)^2 + q_{211}^S + q_{221}^S,$$

$$f_{12}^S = 0.4(Q_{112}^S + Q_{122}^S) + 2(q_{112}^S - 45)^2 + 2(q_{122}^S - 45)^2 + q_{212}^S + q_{222}^S,$$

$$f_{21}^S = Q_{211}^S + Q_{221}^S + 2(q_{211}^S - 31.25)^2 + 2(q_{221}^S - 31.25)^2 + q_{111}^S + q_{121}^S,$$

$$f_{12}^S = Q_{212}^S + Q_{222}^S + (q_{212}^S - 85)^2 + (q_{222}^S - 85)^2 + q_{112}^S + q_{122}^S.$$

Their transportation costs are:

$$\hat{c}_{111}^S = 0.2Q_{111}^S + 1.2(q_{111}^S - 41.67)^2, \quad \hat{c}_{112}^S = 0.1Q_{112}^S + 1.2(q_{112}^S - 37.5)^2,$$

$$\hat{c}_{121}^S = 0.2Q_{121}^S + 1.4(q_{121}^S - 39.29)^2, \quad \hat{c}_{122}^S = 0.1Q_{122}^S + 1.1(q_{122}^S - 36.36)^2,$$

$$\hat{c}_{211}^S = 0.3Q_{211}^S + 1.3(q_{211}^S - 30.77)^2, \quad \hat{c}_{212}^S = 0.4Q_{212}^S + 1.7(q_{212}^S - 32.35)^2,$$

$$\hat{c}_{221}^S = 0.2Q_{221}^S + 1.3(q_{221}^S - 30.77)^2, \quad \hat{c}_{222}^S = 0.1Q_{222}^S + 1.5(q_{222}^S - 30)^2.$$

The opportunity costs of the suppliers are:

$$oc_{111} = 5(\pi_{111} - 80)^2 + 0.5\pi_{211}, \quad oc_{112} = 9(\pi_{112} - 80)^2 + \pi_{212},$$

$$oc_{121} = 5(\pi_{121} - 100)^2 + \pi_{221}, \quad oc_{122} = 7.5(\pi_{122} - 50)^2 + 0.1\pi_{222},$$

$$oc_{211} = 5(\pi_{211} - 50)^2 + 2\pi_{111}, \quad oc_{212} = 8(\pi_{212} - 70)^2 + 0.5\pi_{112},$$

$$oc_{221} = 9(\pi_{221} - 60)^2 + \pi_{121}, \quad oc_{222} = 8(\pi_{222} - 60)^2 + 0.5\pi_{122}.$$

The firms' assembly costs are:

$$f_1(Q_{11},\alpha_1^F)=3Q_{11}^2+0.5Q_{11}\alpha_1^F+100\alpha_1^{F^2}+50\alpha_1^F,$$

$$f_2(Q_{21},\alpha_2^F)=2.75Q_{21}^2+0.6Q_{21}\alpha_2^F+100\alpha_2^{F^2}+50\alpha_2^F.$$

Their production costs for producing components are:

$$f_{11}^F(Q_{11}^F,q_{11}^F)=Q_{11}^{F^2}+0.0001Q_{11}^Fq_{11}^F+1.1(q_{11}^F-36.36)^2,$$

$$f_{12}^F(Q_{12}^F,q_{12}^F)=1.25Q_{12}^{F^2}+0.0001Q_{12}^Fq_{12}^F+1.2(q_{12}^F-41.67)^2,$$

$$f_{21}^F(Q_{21}^F,q_{21}^F)=Q_{21}^{F^2}+0.0001Q_{21}^Fq_{21}^F+1.5(q_{21}^F-33.33)^2,$$

$$f_{22}^F(Q_{22}^F,q_{22}^F)=0.75Q_{22}^{F^2}+0.0001Q_{22}^Fq_{22}^F+1.25(q_{22}^F-36)^2.$$

The transaction costs are:

$$tc_{111}(Q_{111}^S)=0.5Q_{111}^{S^2}+Q_{111}^S+100,\quad tc_{112}(Q_{112}^S)=0.5Q_{112}^{S^2}+0.5Q_{112}^S+150,$$

$$tc_{121}(Q_{211}^S)=0.75Q_{211}^{S^2}+0.75Q_{211}^S+150,\quad tc_{122}(Q_{212}^S)=Q_{212}^{S^2}+Q_{212}^S+100,$$

$$tc_{211}(Q_{121}^S)=0.75Q_{121}^{S^2}+Q_{121}^S+150,\quad tc_{212}(Q_{122}^S)=0.5Q_{122}^{S^2}+0.75Q_{122}^S+100,$$

$$tc_{221}(Q_{221}^S)=0.8Q_{221}^{S^2}+0.25Q_{221}^S+100,\quad tc_{222}(Q_{222}^S)=0.5Q_{222}^{S^2}+Q_{222}^S+175.$$

The firms' transportation costs are:

$$c^F_{11}(Q_{11}, q_1) = 3Q_{11}^2 + 0.3Q_{11}q_1 + 0.25q_1,$$

$$c^F_{21}(Q_{21}, q_2) = 3Q_{21}^2 + 0.3Q_{21}q_2 + 0.1q_2,$$

and the demand price functions are:

$$\rho_{11}(d_{11}, d_{21}, q_1, q_2) = -3d_{11} - 1.3d_{21} + q_1 + 0.74q_2 + 2200,$$

$$\rho_{21}(d_{21}, d_{11}, q_2, q_1) = -3.5d_{21} - 1.4d_{11} + 1.1q_2 + 0.9q_1 + 1800,$$

where $q_1 = \alpha^F_1(\omega_{11}\frac{Q^F_{11}q^F_{11}+Q^S_{111}q^S_{111}+Q^S_{211}q^S_{211}}{Q^F_{11}+Q^S_{111}+Q^S_{211}} + \omega_{12}\frac{Q^F_{12}q^F_{12}+Q^S_{112}q^S_{112}+Q^S_{212}q^S_{212}}{Q^F_{12}+Q^S_{112}+Q^S_{212}})$ and $q_2 = \alpha^F_2(\omega_{21}\frac{Q^F_{21}q^F_{21}+Q^S_{121}q^S_{121}+Q^S_{221}q^S_{221}}{Q^F_{21}+Q^S_{121}+Q^S_{221}} + \omega_{22}\frac{Q^F_{22}q^F_{22}+Q^S_{122}q^S_{122}+Q^S_{222}q^S_{222}}{Q^F_{22}+Q^S_{122}+Q^S_{222}})$.

The modified projection method converges to the following equilibrium solution:

$$Q_{11}^* = 93.56, \quad Q_{21}^* = 71.34,$$

$$Q_{11}^{F*} = 30.00, \quad Q_{12}^{F*} = 30.00, \quad Q_{21}^{F*} = 30.00, \quad Q_{22}^{F*} = 30.00,$$
$$Q_{111}^{S*} = 27.37, \quad Q_{112}^{S*} = 100.00, \quad Q_{121}^{S*} = 45.44, \quad Q_{122}^{S*} = 23.35,$$
$$Q_{211}^{S*} = 36.19, \quad Q_{212}^{S*} = 57.12, \quad Q_{221}^{S*} = 67.24, \quad Q_{222}^{S*} = 17.99,$$
$$q_{11}^{F*} = 38.26, \quad q_{12}^{F*} = 45.15, \quad q_{21}^{F*} = 34.93, \quad q_{22}^{F*} = 41.71,$$
$$q_{111}^{S*} = 46.30, \quad q_{112}^{S*} = 42.19, \quad q_{121}^{S*} = 44.83, \quad q_{122}^{S*} = 41.94,$$
$$q_{211}^{S*} = 31.06, \quad q_{212}^{S*} = 51.85, \quad q_{221}^{S*} = 31.06, \quad q_{222}^{S*} = 52.00,$$
$$\pi_{111}^* = 82.74, \quad \pi_{112}^* = 85.56, \quad \pi_{121}^* = 104.54, \quad \pi_{122}^* = 51.56,$$
$$\pi_{211}^* = 53.62, \quad \pi_{212}^* = 73.57, \quad \pi_{221}^* = 63.74, \quad \pi_{222}^* = 61.12,$$
$$\alpha_1^{F*} = 1.00, \quad \alpha_2^{F*} = 1.00,$$
$$\lambda_{11}^* = 109.83, \quad \lambda_{12}^* = 187.06, \quad \lambda_{21}^* = 172.34, \quad \lambda_{22}^* = 76.58,$$

and the induced demands, demand prices, and product quality levels are:

$$d_{11} = 93.56, \quad d_{21} = 71.34, \quad \rho_{11} = 1,901.07, \quad \rho_{21} = 1,504.22,$$

$$q_1 = 44.06, \quad q_2 = 41.13.$$

The firms' profits are 94,610.69 and 57,787.69, respectively, and those of the suppliers are 15,671.13 and 6923.20.

The eigenvalues of the symmetric part of the Jacobian matrix of $G(Y)$ (cf. (7.27)) are all positive. Therefore, theuniqueness of the solution $(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi^*, q^{S^*})$ and the convergence of the modified projection method are guaranteed.

Example 7.2 - Supplier Disruption Analysis and the Values of Suppliers

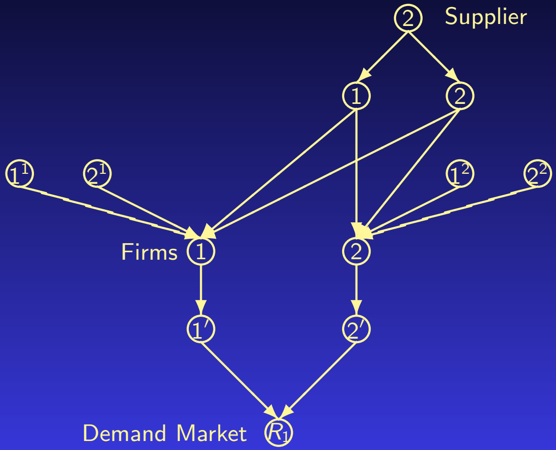

Figure: Supply Chain Network Topology With Disruption to Supplier 1

The equilibrium solution achieved by the modified projection method is:

$$Q_{11}^* = 65.00, \quad Q_{21}^* = 65.00,$$
$$Q_{11}^{F^*} = 30.00, \quad Q_{12}^{F^*} = 30.00, \quad Q_{21}^{F^*} = 30.00, \quad Q_{22}^{F^*} = 30.00,$$
$$Q_{111}^{S^*} = 0.00, \quad Q_{112}^{S^*} = 0.00, \quad Q_{121}^{S^*} = 0.00, \quad Q_{122}^{S^*} = 0.00,$$
$$Q_{211}^{S^*} = 35.00, \quad Q_{212}^{S^*} = 100.00, \quad Q_{221}^{S^*} = 100.00, \quad Q_{222}^{S^*} = 35.00,$$
$$q_{11}^{F^*} = 38.26, \quad q_{12}^{F^*} = 45.16, \quad q_{21}^{F^*} = 34.93, \quad q_{22}^{F^*} = 41.75,$$
$$q_{211}^{S^*} = 31.06, \quad q_{212}^{S^*} = 51.85, \quad q_{221}^{S^*} = 31.06, \quad q_{222}^{S^*} = 52.00,$$
$$\pi_{211}^* = 53.50, \quad \pi_{212}^* = 76.25, \quad \pi_{221}^* = 65.56, \quad \pi_{222}^* = 62.19,$$
$$\alpha_1^{F^*} = 1.00, \quad \alpha_2^{F^*} = 1.00,$$
$$\lambda_{11}^* = 107.53, \quad \lambda_{12}^* = 448.93, \quad \lambda_{21}^* = 242.02, \quad \lambda_{22}^* = 95.98,$$

and the induced demands, demand prices, and product quality levels are:

$$d_{11} = 65.00, \quad d_{21} = 65.00 \quad \rho_{11} = 1,998.07, \quad \rho_{21} = 1,569.17,$$
$$q_1 = 47.12, \quad q_2 = 41.14.$$

The firms' profits are 80,574.83 and 57,406.47, respectively, and supplier 2's profit is 13,635.49. The uniqueness of the solution $(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi^*, q^{S^*})$ is guaranteed.

A Supply Chain Network Model with Supplier Selection and Quality and Price Competition

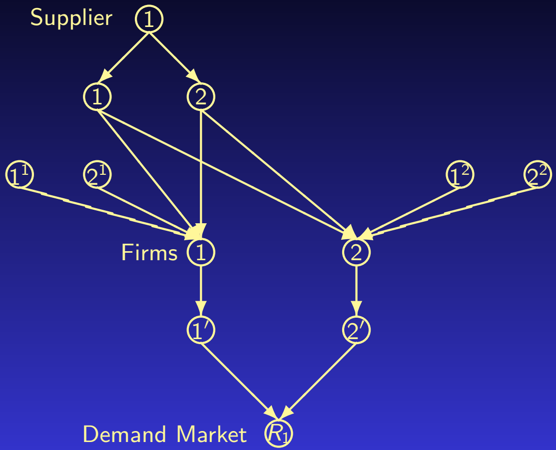

Figure: Supply Chain Network Topology With Disruption to Supplier 2

The modified projection method converges to the following equilibrium solution:

$$Q_{11}^* = 65.00, \quad Q_{21}^* = 63.79,$$

$$Q_{11}^{F^*} = 30.00, \quad Q_{12}^{F^*} = 30.00, \quad Q_{21}^{F^*} = 30.00, \quad Q_{22}^{F^*} = 30.00,$$
$$Q_{111}^{S^*} = 35.00, \quad Q_{112}^{S^*} = 100.00, \quad Q_{121}^{S^*} = 97.58, \quad Q_{122}^{S^*} = 33.79,$$
$$Q_{211}^{S^*} = 0.00, \quad Q_{212}^{S^*} = 0.00, \quad Q_{221}^{S^*} = 0.00, \quad Q_{222}^{S^*} = 0.00,$$
$$q_{11}^{F^*} = 38.26, \quad q_{12}^{F^*} = 45.16, \quad q_{21}^{F^*} = 34.93, \quad q_{22}^{F^*} = 41.75,$$
$$q_{111}^{S^*} = 46.30, \quad q_{112}^{S^*} = 42.19, \quad q_{121}^{S^*} = 44.83, \quad q_{122}^{S^*} = 41.94,$$
$$\pi_{111}^* = 83.50, \quad \pi_{112}^* = 85.56, \quad \pi_{121}^* = 109.76, \quad \pi_{122}^* = 52.25,$$
$$\alpha_1^{F^*} = 1.00, \quad \alpha_2^{F^*} = 1.00,$$
$$\lambda_{11}^* = 119.17, \quad \lambda_{12}^* = 442.79, \quad \lambda_{21}^* = 256.75, \quad \lambda_{22}^* = 86.75.$$

The induced demands, demand prices, and product quality levels are:

$$d_{11} = 65.00, \quad d_{21} = 63.79, \quad \rho_{11} = 1,996.05, \quad \rho_{21} = 1,570.59,$$

$$q_1 = 42.82, \quad q_2 = 42.11.$$

The firms' profits are 83,895.42 and 53,610.96, respectively, and supplier 1's profit is 22,729.18. The uniqueness of the solution $(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi^*, q^{S^*})$ is guaranteed.

- Without supplier 1, the profit of firm 1 decreases by 14.84%, and that of firm 2 decreases by 0.66%. Therefore, from this perspective, supplier 1 is more important to firm 1 than to firm 2. The value of supplier 1 to firm 1 is 14,035.86, and that to firm 2 is 381.22, which are measured by the associated profit declines.

- Without supplier 2, firm 1's profit declines by 11.33%, and that of firm 2 reduces by 7.23%. Thus, supplier 2 is more important to firm 1 than to firm 2 under this disruption. The value of supplier 2 to firm 1 is 10,715.27, and that to firm 2 is 4,176.73.

- In addition, according to the above results, supplier 1 is more important than supplier 2 to firm 1, and to firm 2, supplier 2 is more important.

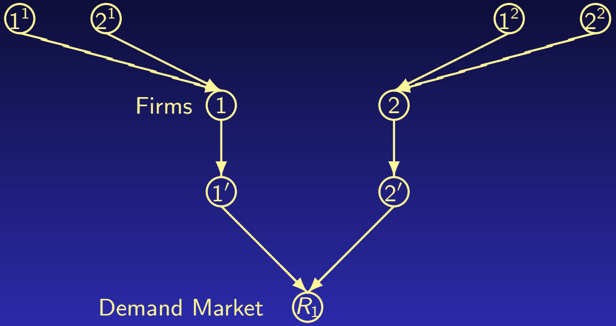

Figure: Supply Chain Network Topology With Disruption to Suppliers 1 and 2

The equilibrium solution obtained using the modified projection method is:

$$Q_{11}^* = 15.00, \quad Q_{21}^* = 15.00,$$

$$Q_{11}^{F^*} = 15.00, \quad Q_{12}^{F^*} = 30.00, \quad Q_{21}^{F^*} = 30.00, \quad Q_{22}^{F^*} = 30.00,$$
$$Q_{111}^{S^*} = 0.00, \quad Q_{112}^{S^*} = 0.00, \quad Q_{121}^{S^*} = 0.00, \quad Q_{122}^{S^*} = 0.00,$$
$$Q_{211}^{S^*} = 0.00, \quad Q_{212}^{S^*} = 0.00, \quad Q_{221}^{S^*} = 0.00, \quad Q_{222}^{S^*} = 0.00,$$
$$q_{11}^{F^*} = 37.29, \quad q_{12}^{F^*} = 45.08, \quad q_{21}^{F^*} = 35.71., \quad q_{22}^{F^*} = 37.90,$$
$$\alpha_1^{F^*} = 1.00, \quad \alpha_2^{F^*} = 1.00,$$
$$\lambda_{11}^* = 30.46, \quad \lambda_{12}^* = 967.28, \quad \lambda_{21}^* = 772.88, \quad \lambda_{22}^* = 22.63.$$

The induced demands, demand prices, and the product quality levels are:

$$d_{11} = 15.00, \quad d_{21} = 15.00, \quad \rho_{11} = 2,206.42, \quad \rho_{21} = 1,806.40,$$

$$q_1 = 43.52, \quad q_2 = 37.02.$$

The firms' profits are 30,016.91 and 24,391.32, respectively. The uniqueness of the solution $(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi^*, q^{S^*})$ is guaranteed.

Compared to Example 7.2, without the suppliers, the demands at the demand market decrease, the firms' product quality levels decrease, and the prices at the demand market increase. Firm 1's profit deceases by 68.27%, firm 2's reduces by 57.79%. The value of the suppliers to firm 1 is 64,593.78, and that to firm 2 is 33,396.37.

Summary and Conclusions

- The novelty of this framework lies in its generality and its computability.
- It is illustrated with numerical examples, accompanied by sensitivity analysis that explores such critical issues as the impacts of capacity disruptions and the potential investments in capacity enhancements.
- I also conduct sensitivity analysis to reveal the impacts of specific supplier unavailability along with their values as reflected in the profits of the firms and in the quality of the finished products.
- With knowledge of the values of the suppliers to the firms, the firms can make more specific, targeted efforts in their supplier management strategies and in their contingency plans in the case of supplier disruptions.

Future Research

- Models in Chapter 3 can be extended to capture the quality information asymmetry caused by the time delay of consumer's learning process of quality in spatial price networks for new products.
- I would also like to extend Chapters 5 and 6 to incorporate the quality information asymmetry that lies between firms and their potential contractors and the transaction costs and risk caused by it.
- I plan to formulate quality degradation in competitive transportation networks for perishable products.
- The dynamics of quality competition in multitiered supply chain networks with suppliers, on which Chapter 8 focused, can be developed using projected dynamical systems theory with stability results presented.
- I would like to conduct empirical research to apply the models and results in this dissertation to real-life quality competition problems in supply chain networks under the four scenarios discussed and modeled.

Thank You!!