Design of Sustainable Supply Chains for Sustainable Cities

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Acknowledgments

I would like to thank Dr. Peter Nijkamp and Dr. Emmanouil Tranos for the invitation to speak at this workshop.

Special thanks to my students and other collaborators who have made research so rewarding.

This presentation is based on the paper with the same title.

Outline

- Background and Motivation
- Supply Chains
- The Sustainable Supply Chain Network Model with Frequency of Activities
- Numerical Examples
- Summary, Conclusions, and Suggestions for Future Research

Background and Motivation

What is Sustainability?

The general definition of sustainability is development that meets the needs of the present without compromising the ability of future generations to meet their own needs (World Commission Environment and Development (WCED) (1987)).

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- What *resources* will future generations require?
- What *level of emissions* can be released without negatively affecting future generations?
- What *policies* are required to achieve sustainability?
- What are the effects of *market forces*, etc.?

Pollution and Environmental Impacts

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The average surface temperature of the earth, expressed as a global average, has increased by about 0.74C over the past hundred years (between 1906 and 2005) with 11 of the 12 warmest years occurring between 1995 and 2006 (IPCC (2007)).

Pollution and Environmental Impacts

In the US alone, greenhouse gas emissions are projected to rise 35% between 2005 and 2030 due to fewer forests and agricultural land to absorb the carbon, an increasing population, expansion of the economy, and an increased use of fossil fuel powered power plants to generate energy (Creyts et al. (2007)).

Cities and Sustainability

Cities, *as dynamic complex networks, are the systems in which more people now live than don't* and which represent the economic engines for commerce, research and development, education, health care, and even culture.

Cities have evolved over space and time on built infrastructure from transportation networks to telecommunication and electric power networks.

At the same time, cities are *the centers of resource usage* from electricity and other forms of energy and fuel, to food, water, and a plethora of other products.

Some Examples of Cities

Cities and Sustainability

Cities are also *the repositories and generators of waste output and other environmental pollutants, such as carbon and other emissions, sewage, noise, etc.*

Some Negative Externalities Associated with Cities

The term *Sustainable Cities* has come into increasing use in the past two decades, with a focus of making cities more livable, with an eye not only on the present generation but towards future ones, as well (cf. Nijkamp and Perrels (1994), Capello, Nijkamp, and Pepping (1999), Knickerbocker (2007), Grant Thornton (2011)).

A recent World Bank report (see Suzuki et al. (2009)) noted that the world is shrinking with cheaper air travel, large-scale commercial shipping, and expanding road networks. Today, only 10% of the globe's land area is considered to be remote, that is, more than 48 hours from a large city.

Our world is becoming a network of interconnected cities or a supernetwork of cities.

Cities and Sustainability

According to Alusi et al. (2011), *urbanization is one of the most pressing and complex challenges of the 21st century*, with the citizenry characterized by a growing awareness of a threat to the sustainability of the earth's natural environment, coupled with the increase in the number of people moving into and living in cities.

Cities and Supply Chains

Cities are supplied by a complex array of supply chains servicing an immense spectrum of economic activities from food stores and restaurants, office supplies and high tech equipment, apparel, construction materials, as well as raw materials, to name just a few.

Supply Chains

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Supply chains are the *critical infrastructure and backbones* for the production, distribution, and consumption of goods as well as services in our globalized *Network Economy*.

Supply chains, in their most fundamental realization, *consist of manufacturers and suppliers, distributors, retailers, and consumers at the demand markets*.

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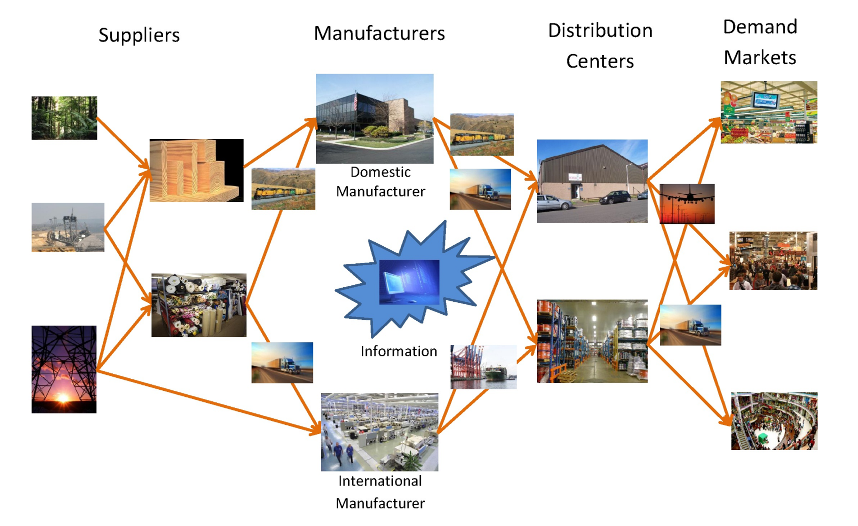
Today, supply chains may span thousands of miles across the globe, involve numerous suppliers, retailers, and consumers, and be underpinned by multimodal transportation and telecommunication networks.

A General Supply Chain

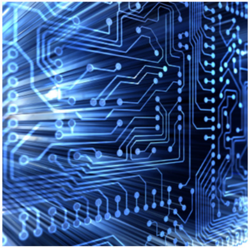


Examples of Supply Chains

- food and food products
- high tech products
- automotive
- energy (oil, electric power, etc.)
- clothing and toys
- humanitarian relief
- healthcare supply chains.

Food Supply Chains

High Tech Products



Automotive Supply Chains

Energy Supply Chains

Clothing and Toys

Healthcare Supply Chains

Sustainability of supply chains is, hence, a precursor to the sustainability of our cities. According to a Business for Social Responsibility (2009) paper, it is now widely acknowledged that making significant progress on mitigating the impact of climate change depends on reducing the negative environmental impacts of supply chains through their redesign and enhanced management (see also McKinsey Quarterly (2008)).

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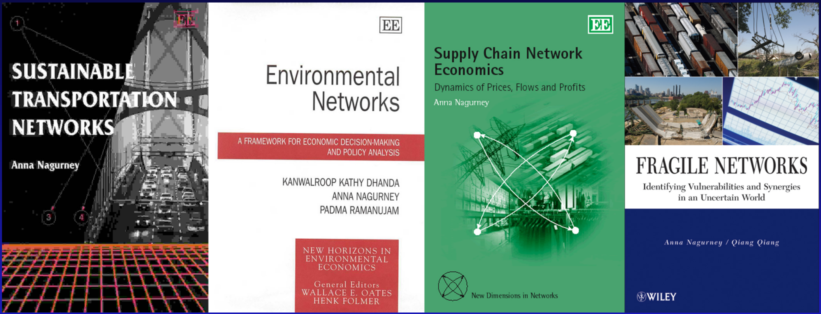
As noted by Capgemini in its 2008 report: *2016: Future Supply Chain*, *"Preserving energy and raw materials and other resources like water will become a crucial aspect in future supply chains, as costs will likely remain volatile and supplies will continue to dwindle."* These conditions may well create substantial pressure on current supply chain models.

Our Approach to Sustainability



Why More Research is Needed

Although the importance of sustainable supply chains to the sustainability of cities is being increasingly recognized (cf. Grant Thornton (2011)), in terms of not only the enhancement of business processes in terms of efficiency and cost reduction but also the reduction of negative environmental externalities as well as waste *there have been only limited modeling efforts that capture supply chains within a cities framework*.

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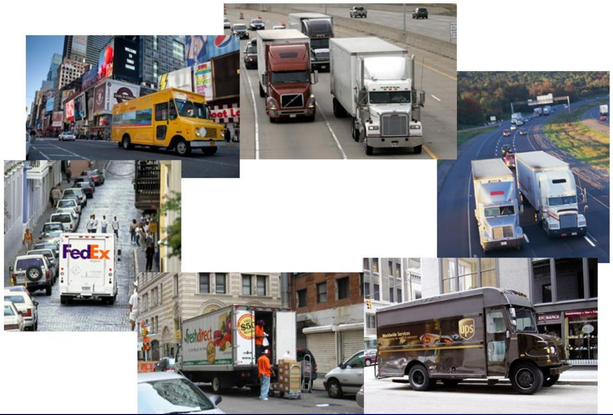
Models of sustainable supply chains are important since *they enable the evaluation (before expensive investments are actually made) as to alternative network designs, technologies, as well as sensitivities to cost and demand structures*.

Why More Research is Needed

The edited volume of Taniguchi and Thompson (2004), which focuses on logistics systems for sustainable cities, *emphasized the unique features of urban logistical systems, which may include more frequent freight shipments and deliveries, with the concomitant negative externalities*.



Why More Research is Needed

Geroliminis and Daganzo (2005) further emphasize that the environmental impacts of logistical activities are most severe where population densities are highest, that is, in cities.

They have identified *innovative practices of cities around the globe in terms of their logistics systems and sustainability*, including the use of alternative modes of transportation, such as, for example, even bicycles for deliveries in Amsterdam.

The Sustainable Supply Chain Network Model
with
Frequency of Activities

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The Sustainable Supply Chain Network Model with Frequency of Activities

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• Logistics in cities are often characterized by more frequent shipments, especially using primarily freight vehicles such as trucks.

• However, the scope of our model is broader and we also capture the optimal frequencies of the other activities, that is, those of manufacturing, storage, etc.

The Sustainable Supply Chain Network Model with Frequency of Activities

This paper is inspired, in part, by a recent paper presentation by Martin J. Beckmann. Vehicle and passenger flows in mass transportation: Optimal routing of buses and planes. Presented at the Symposium on Transportation Network Design and Economics, Northwestern University, Evanston, Illinois, January 29, 2010.

The Sustainable Supply Chain Network Model with Frequencies

We consider the supply chain network topology depicted in Figure 1 but note that this network is simply representative and more disaggregation can be included, depending on the application.

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The top level (origin) node 0 corresponds to the firm and the bottom level (destination) nodes correspond to the demand sites, which can denote, for example, retailers or consumers, that the firm wishes to supply.

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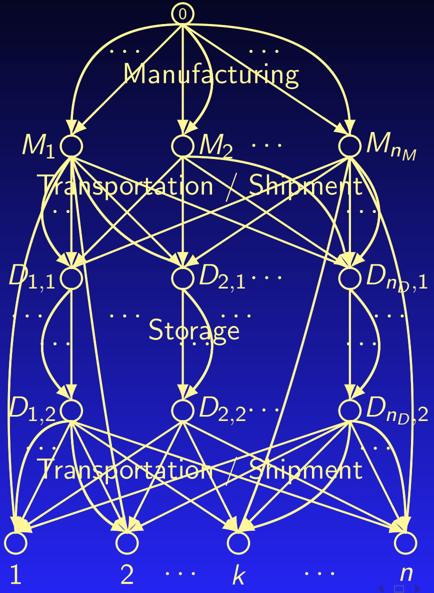
The top level (origin) node 0 corresponds to the firm and the bottom level (destination) nodes correspond to the demand sites, which can denote, for example, retailers or consumers, that the firm wishes to supply.

The paths joining the origin node to the destination nodes depict sequences of supply chain network activities that guarantee that the product is produced and is delivered to the demand sites.

The Supply Chain Network Topology



The Sustainable Supply Chain Network Model with Frequencies

The firm is considering n_M manufacturing facilities/plants; n_D distribution centers, and is to serve the n demand locations with the respective demands given by: d_1, d_2, ..., d_n. The links from the top-tiered node 0 are connected to the manufacturing nodes of the firm, which are denoted, respectively, by: $M_1, \ldots, M_{n_M}$ These links represent the manufacturing links.

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There may be multiple alternative links joining node 0 to each of the manufacturing nodes in order to depict different possible technologies associated with a given manufacturing plant, which can be associated with different levels of environmental impacts and associated costs as well as waste production.

The Sustainable Supply Chain Network Model with Frequencies

The links from the manufacturing nodes are connected to the distribution center nodes of the firm, and are denoted by $D_{1,1}, \ldots, D_{n_D,1}$. These links correspond to the possible transportation/shipment links between the manufacturing plants and the distribution centers where the product will be stored.

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In addition, we allow for the possibility, if relevant and feasible, of direct links from the manufacturing nodes to the demand points.

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In addition, we allow for the possibility, if relevant and feasible, of direct links from the manufacturing nodes to the demand points.

A firm may decide that, rather than having the product shipped and stored and then distributed to the retailers and consumers that it may be beneficial (cost-wise and/or environmentally) to ship the product directly. There may be multiple such links joining a manufacturing node to a demand node to denote alternatives.

The Sustainable Supply Chain Network Model with Frequencies

The model can allow for outsourcing of production, transportation, etc., with appropriate changes in the cost functions (see below) so that contracts can also be captured. In that case, the firm may agree on a fixed unit price or cost for a particular link activity.

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The links joining nodes $D_{1,1}, \ldots, D_{n_D,1}$ with nodes $D_{1,2}, \ldots, D_{n_D,2}$ represent the possible storage links, and here, for flexibility, and an eye towards sustainability, we allow for multiple possible storage links to represent different levels of environmental impacts.

The Sustainable Supply Chain Network Model with Frequencies

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There are multiple possible transportation/shipment links joining the nodes $D_{1,2}, \ldots, D_{n_D,2}$ with the demand nodes: $1, \ldots, n$ since there may exist multiple modes of transportation for distribution purposes and the firm may wish to select one with its degree of desired environmental impact. Note that in Figure 1 such alternatives are depicted as distinct links joining a pair of nodes.

The Sustainable Supply Chain Network Model with Frequencies

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The supply chain network consisting of the graph $G = [N, L]$, with N denoting the set of nodes and L the set of directed links.

The firm seeks to minimize the total costs associated with its production, storage, and transportation/distribution activities, along with the total cost of link operation frequencies, plus the total cost of environmental impact and waste, which we elaborate upon below, subject to the demand being satisfied at the demand sites.

The Sustainable Supply Chain Network Model with Frequencies

We denote the links by a, b, etc., and the total cost on a link a by $\hat{c}_a$. For the sake of generality, we note that the total costs are generalized costs and may include, for example, risk, time, etc. (see also Nagurney (2010)).

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A path p in the network (see, e.g., Figure 1) joining node 0, which is the origin node, to a demand node, which is a destination node, represents the activities and their sequence associated with producing the product and having it, ultimately, delivered. Let w_k denote the pair of origin/destination (O/D) nodes $(0, k)$ and let P_{w_k} denote the set of paths, which represent alternative associated possible supply chain network processes, joining $(0, k)$. P is the set of all paths joining node 0 to the demand nodes. n_P denotes the number of paths from the organization to the demand markets.

The Sustainable Supply Chain Network Model with Frequencies

Let x_p represent the nonnegative flow of the product on path p joining (origin) node 0 with a (destination) demand node. Let d_k denote the demand, which is assumed to be known and fixed, for the product at demand location k.

The Sustainable Supply Chain Network Model with Frequencies

Let x_p represent the nonnegative flow of the product on path p joining (origin) node 0 with a (destination) demand node. Let d_k denote the demand, which is assumed to be known and fixed, for the product at demand location k.

The following conservation of flow equation must hold:

$$d_k \equiv \sum_{p \in P_{w_k}} x_p, \quad k = 1, \ldots, n, \tag{1}$$

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Let f_a denote the flow of the product on link a. The following conservation of flow equations satisfied:

$$f_a = \sum_{p \in P} x_p \delta_{ap}, \quad \forall a \in L, \tag{2}$$

where $\delta_{ap} = 1$, if link a is contained in path p, and $\delta_{ap} = 0$,

The Sustainable Supply Chain Network Model with Frequencies

The path flows must be nonnegative, that is,

$$x_p \geq 0, \quad \forall p \in P, \tag{3}$$

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The total operational cost on a link, be it a manufacturing / production link, a transportation / shipment link, or a storage link is assumed to be a function of the flow of the product on the link; see, for example, Nagurney and Nagurney (2010) and Nagurney (2006) and the references therein. We have, thus, that

$$\hat{c}_a = \hat{c}_a(f_a), \quad \forall a \in L. \tag{4}$$

The Sustainable Supply Chain Network Model with Frequencies

We assume that the total cost on each link is convex and is continuously differentiable.

We denote the total cost of operating link a at a frequency γ_a by $\hat{\pi}_a$, $\forall a \in L$, and assume that

$$\hat{\pi}_a = \hat{\pi}_a(\gamma_a), \quad \forall a \in L. \tag{5}$$

These frequency operational cost functions are assumed to be convex and continuously differentiable and to have bounded second order partial derivatives.

The Sustainable Supply Chain Network Model with Frequencies

The firm seeks to determine the optimal levels of product processed on each supply chain network link coupled with the optimal levels of frequency link operation subject to the minimization of the total cost.

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The firm is faced with the following objective function:

$$\text{Minimize} \quad \sum_{a \in L} \hat{c}_a(f_a) + \hat{\pi}_a(\gamma_a). \tag{6}$$

The Sustainable Supply Chain Network Model with Frequencies

The firm is concerned with the environmental impact of its activities, which can include not only the emissions generated but also noise pollution, as well as other types of pollution and infrastructure deterioration.

Let $\hat{e}_a(f_a, \gamma_a)$, $\forall a \in L$, denote the environmental impact function associated with link a. Let $\hat{z}_a(f_a)$, $a \in L$, denote the waste management cost associated with link a, $a \in L$. They are assumed to be convex and continuously differentiable and to have bounded second order partial derivatives, as are the ones above.

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The second objective of the firm is then given by:

$$\text{Minimize} \quad \sum_{a \in L} \hat{e}_a(f_a, \gamma_a) + \hat{z}_a(f_a). \tag{7}$$

The Multicriteria Optimization Problem for Sustainable Supply Chain Network Design with Frequency of Activities

A nonnegative constant ω is now assigned to the environmental criterion (7). The constant ω is a weight that the firm assigns. Of course, ω can also be interpreted as a "tax" imposed by the governmental/environmental authority (see, e.g., Wu et al. (2006)).

The Sustainable Supply Chain Network Model with Frequencies

We assume, as given, a parameter $\bar{u}_a$, $\forall a \in L$. These parameters denote the existing capacities of the links.

For example, for a manufacturing link $\bar{u}_a$ would denote the capacity of production, that is, the volume of the product that could be produced on the link; for a storage link a, the capacity would denote how much of the product could be stored there, and, similarly, for a transportation/shipment link, $\bar{u}_a$ would represent the amount that could be shipped (could denote a truckload, for example).

The Sustainable Supply Chain Network Model with Frequencies

Using results from multicriteria optimization (see, e.g., Nagurney and Dong (2002)), one can then construct the following:

$$\text{Minimize} \quad \sum_{a \in L} \hat{c}_a(f_a) + \hat{\pi}_a(\gamma_a) + \omega\left(\sum_{a \in L} \hat{e}_a(f_a, \gamma_a) + \hat{z}_a(f_a)\right). \tag{8}$$

The firm seeks to solve (8), subject to the constraints: (1), (2), (3), and

$$f_a \leq \bar{u}_a \gamma_a, \quad \forall a \in L. \tag{9}$$

$$0 \leq \gamma_a, \quad \forall a \in L. \tag{10}$$

Constraint (9) guarantees that the product flow on a link does not exceed that link's capacity times the frequency of replenishment.

The Sustainable Supply Chain Network Model with Frequencies

We associate the Lagrange multiplier μ_a with constraint (9) for each link $a \in L$ and denote the associated optimal Lagrange multiplier by μ_a^*. These terms may also be interpreted as the price or value of an additional unit of "capacity" on link a. We group these Lagrange multipliers into the respective vectors μ and μ^*.

The Variational Inequality Formulation

Theorem 1

The optimization problem (8), subject to the constraints (1) – (3), (9), and (10), is equivalent to the variational inequality problem: determine the vectors of link flows, link operation frequencies, and Lagrange multipliers $(f^*, \gamma^*, \mu^*) \in K$, *such that:*

$$\sum_{a\in L}\left[\frac{\partial \hat{c}_a(f_a^*)}{\partial f_a} + \omega\frac{\partial \hat{e}_a(f_a^*, \gamma_a^*)}{\partial f_a} + \omega\frac{\partial \hat{z}_a(f_a^*)}{\partial f_a} + \mu_a^*\right] \times [f_a - f_a^*]$$

$$+\sum_{a\in L}\left[\frac{\partial \hat{\pi}_a(\gamma_a^*)}{\partial \gamma_a} + \omega\frac{\partial \hat{e}_a(f_a^*, \gamma_a^*)}{\partial \gamma_a} - \bar{u}_a\mu_a^*\right] \times [\gamma_a - \gamma_a^*]$$

$$+\sum_{a\in L}[\bar{u}_a\gamma_a^* - f_a^*] \times [\mu_a - \mu_a^*] \geq 0, \quad \forall (f, \gamma, \mu) \in K, \qquad (11)$$

$K \equiv \{(f, \gamma, \mu) | \exists x \geq 0,$ *and* $(1), (2),$ *and* (10) *hold, and* $\mu \geq 0\}$, *where* f *is the vector of link flows,* γ *is the vector of link operation frequencies, and* μ *is the vector of Lagrange multipliers.*

The Variational Inequality Formulation

Variational inequality (11) can be put into standard form (see Nagurney (1999)): determine $X^* \in \mathcal{K}$ such that:

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{12}$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in $\mathcal{N}$-dimensional Euclidean space. If we define the column vectors: $X \equiv (f, \gamma, \mu)$ and $F(X) \equiv (F_1(X), F_2(X), F_3(X))$, such that

$$F_1(X) \equiv [\frac{\partial \hat{c}_a(f_a)}{\partial f_a} + \omega \frac{\partial \hat{e}_a(f_a, \gamma_a)}{\partial f_a} + \omega \frac{\partial \hat{z}_a(f_a)}{\partial f_a} + \mu_a; \quad a \in L], \tag{13}$$

$$F_2(X) \equiv \left[\frac{\partial \hat{\pi}_a(\gamma_a)}{\partial \gamma_a} + \omega \frac{\partial \hat{e}_a(f_a, \gamma_a)}{\partial \gamma_a} - \bar{u}_a \mu_a; a \in L \right], \tag{14}$$

$$F_3(X) \equiv [\bar{u}_a \gamma_a - f_a; \quad a \in L], \tag{15}$$

and define $\mathcal{K} \equiv K$, then (11) can be re-expressed as (12).

Special Case of the Model

Let $\bar{u}_a = 1$ and let $\hat{\pi}_a$ now denote the total cost associated with investment to a level of operation γ_a on link a, for $a \in L$.

Corollary 1

Under the preceding assumptions, the optimality conditions for the sustainable supply chain network model take on the following variational inequality form: determine the vectors of link flows, link capacity investments, and Lagrange multipliers $(f^*, \gamma^*, \mu^*) \in K^1$*:*

$$\sum_{a \in L} \left[\frac{\partial \hat{c}_a(f_a^*)}{\partial f_a} + \omega \frac{\partial \hat{e}_a(f_a^*, \gamma_a^*)}{\partial f_a} + \omega \frac{\partial \hat{z}_a(f_a^*)}{\partial f_a} + \mu_a^* \right] \times [f_a - f_a^*]$$

$$+ \sum_{a \in L} \left[\frac{\partial \hat{\pi}_a(\gamma_a^*)}{\partial \gamma_a} + \omega \frac{\partial \hat{e}_a(f_a^*, \gamma_a^*)}{\partial \gamma_a} - \mu_a^* \right] \times [\gamma_a - \gamma_a^*]$$

$$+ \sum_{a \in L} [\gamma_a^* - f_a^*] \times [\mu_a - \mu_a^*] \geq 0, \quad \forall (f, \gamma, \mu) \in K^1, \tag{16}$$

$K^1 \equiv \{(f, \gamma, \mu) | \exists x \geq 0, (1), (2),$ *and* (10) *hold with* $\bar{u}_a = 1, \forall a, \mu \geq 0\}$.

Theorem 2

The function $F(X)$ as defined following (12) (see (13) – (15)), under the assumptions above, is monotone, that is,

$$\langle (F(X^1) - F(X^2))^T, X^1 - X^2 \rangle \geq 0, \quad \forall X^1, X^2 \in \mathcal{K}. \qquad (17)$$

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Theorem 3

The function $F(X)$ as defined following (12) is Lipschitz continuous, that is,

$$\|F(X^1) - F(X^2)\| \leq \|X^1 - X^2\|, \quad \forall X^1, X^2 \in \mathcal{K}. \tag{18}$$

It is important to realize that linear functions are convex and continuously differentiable. Hence, our model can be applied (and solved) under many different not unreasonable cost settings.

Theorem 4: Convergence

Assume that the function that enters the variational inequality (11) (or (12)) has at least one solution and satisfies the conditions in Theorem 2 and in Theorem 3. Then the modified projection method converges to the solution of variational inequality (11) (or (12)) and, similarly, due to Corollary 1, to the solution of (16).

Numerical Examples

Numerical Examples

The modified projected method was implemented in FORTRAN and a Unix system at the University of Massachusetts Amherst was used for all the computations.

We initialized the algorithm by equally distributing the demand at each demand site among all the paths joining the firm node 0 to the demand node.

All other variables (the link frequencies and the Lagrange multipliers) were initialized to zero. We used the equilibration algorithm (cf. Dafermos and Sparrow (1969) and Nagurney (1999)) for the solution of the embedded quadratic programming network optimization problems.

The numerical examples were solved to a high degree of accuracy since the imposed convergence criterion guaranteed that the absolute value of successive iterates differed by no more than 10^{-5}.

Numerical Examples

The numerical examples consisted of a firm faced with 3 possible manufacturing plants, each of which had 2 possible technologies, 2 distribution centers, each of which also had 2 distinct technologies, and the firm had to supply the 3 demand points.

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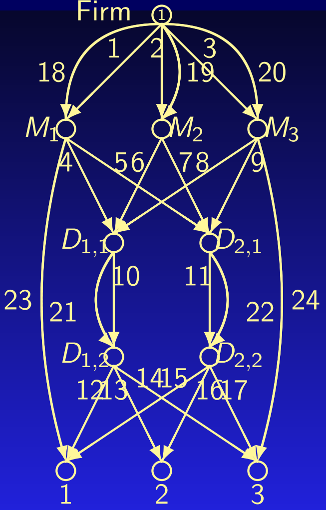
The first two examples had link capacities as reported in Table 1. The third numerical example (since it was a supply chain network design example) had $\bar{u}_a = 1$ for all links a, with the interpretation that the optimal values for the γ_a^*, for all links $a \in L$, would reflect the effective optimal capacities of the corresponding links (see, e.g., Nagurney (2010)).

The Supply Chain Network Topology $G = [N, L]$ for the Examples



The Input Data

Table 1: Total Operating and Frequency Cost Functions, Environmental Impact Cost and Waste Cost Functions, and Link Capacities for Numerical Examples 1 and 2

Link	$\hat{c}_a(f_a)$	$\hat{\pi}_a(\gamma_a)$	$\hat{e}_a(f_a, \gamma_a)$	$\hat{z}_a(f_a)$	$\bar{u}_a$
1	$.5f_1^2 + 2f_1$	$.5\gamma_1^2 + \gamma_1$	$.05f_1^2 + f_1 + 1.5\gamma_1^2 + 2\gamma_1$	$.05f_1^2 + f_1$	100.
2	$.5f_2^2 + f_2$	$2.5\gamma_2^2 + \gamma_2$	$.1f_2^2 + f_2 + 2\gamma_2^2 + 2\gamma_2$	$.1f_2^2 + 2f_2$	100.
3	$.5f_3^2 + f_3$	$\gamma_3^2 + 2\gamma_3$	$.15f_3^2 + 2f_3 + 2.5\gamma_3^2 + \gamma_3$	$.25f_3^2 + 5f_3$	200.
4	$1.5f_4^2 + 2f_4$	$\gamma_4^2 + \gamma_4$	$.05f_4^2 + .1f_4 + .1\gamma_4^2 + .2\gamma_4$	$.05f_4^2 + 2f_4$	20.
5	$f_5^2 + 3f_5$	$2.5\gamma_5^2 + 2\gamma_5$	$.05f_5^2 + .1f_5 + .05\gamma_5^2 + .1\gamma_5$	$.1f_5^2 + 3f_5$	20.
6	$f_6^2 + 2f_6$	$.5\gamma_6^2 + \gamma_6$	$.1f_6^2 + .1f_6 + .05\gamma_6^2 + .1\gamma_6$	$.05f_6^2 + f_6$	20.
7	$.5f_7^2 + 2f_7$	$.5\gamma_7^2 + \gamma_7$	$.05f_7^2 + .2f_7 + .1\gamma_7^2 + .2\gamma_7$	$.25f_7^2 + f_7$	20.
8	$.5f_8^2 + 2f_8$	$1.5\gamma_8^2 + \gamma_8$	$.05f_8^2 + .1f_8 + .1\gamma_8^2 + .3\gamma_8$	$.2f_8^2 + 2f_8$	10.
9	$f_9^2 + 5f_9$	$2\gamma_9^2 + 3\gamma_9$	$.05f_9^2 + .1f_9 + .1\gamma_9^2 + .2\gamma_9$	$.1f_9^2 + 5f_9$	10.
10	$.5f_{10}^2 + 2f_{10}$	$\gamma_{10}^2 + 5\gamma_{10}$	$.2f_{10}^2 + f_{10} + 1.5\gamma_{10}^2 + 3\gamma_{10}$	$.05f_{10}^2 + 5f_{10}$	50.
11	$f_{11}^2 + f_{11}$	$.5\gamma_{11}^2 + 3\gamma_{11}$	$.25f_{11}^2 + 3f_{11} + 2\gamma_{11}^2 + 3\gamma_{11}$	$.1f_{11}^2 + 2f_{11}$	50.
12	$.5f_{12}^2 + 2f_{12}$	$.5\gamma_{12}^2 + \gamma_{12}$	$.05f_{12}^2 + .1f_{12} + .1_{12}^2 + .2\gamma_{12}$	$.05f_{12}^2 + 3f_{12}$	15.
13	$.5f_{13}^2 + 5f_{13}$	$.5\gamma_{13}^2 + \gamma_{13}$	$.1f_{13}^2 + .1f_{13} + .05\gamma_{13}^2 + .1\gamma_{13}$	$.05f_{13}^2 + 5f_{13}$	15.
14	$f_{14}^2 + 7f_{14}$	$2\gamma_{14}^2 + 5\gamma_{14}$	$.15f_{14}^2 + .2f_{14} + .1\gamma_{14}^2 + .1\gamma_{14}$	$.05f_{14}^2 + 3f_{14}$	15.
15	$f_{15}^2 + 2f_{15}$	$.5\gamma_{15}^2 + \gamma_{15}$	$.05f_{15}^2 + .3f_{15} + .1\gamma_{15}^2 + .2\gamma_{15}$	$.1f_{15}^2 + 5f_{15}$	20.

The Input Data

Table 2: Total Operating and Frequency Cost Functions, Environmental Impact Cost and Waste Cost Functions, and Link Capacities for Numerical Examples 1 and 2

16	$.5f_{16}^2 + 3f_{16}$	$\gamma_{16}^2 + \gamma_{16}$	$.05f_{16}^2 + .1f_{16} + .1\gamma_{16}^2 + .1\gamma_{16}$	$.15f_{16}^2 + 3f_{16}$	20.
17	$.5f_{17}^2 + 2f_{17}$	$.5\gamma_{17}^2 + \gamma_{17}$	$.15f_{17}^2 + .3f_{17} + .05\gamma_{17}^2 + .1\gamma_{17}$	$.1f_{17}^2 + 5f_{17}$	20.
18	$.5f_{18}^2 + f_{18}$	$\gamma_{18}^2 + 2\gamma_{18}$	$.2f_{18}^2 + 2f_{18} + 2\gamma_{18}^2 + 3\gamma_{18}$	$.05f_{18}^2 + f_{18}$	100.
19	$.5f_{19}^2 + 2f_{19}$	$\gamma_{19}^2 + \gamma_{19}$	$.25f_{19}^2 + 3f_{19} + 3\gamma_{19}^2 + 4\gamma_{19}$	$.1f_{19}^2 + 2f_{19}$	200.
20	$1.5f_{20}^2 + f_{20}$	$\gamma_{20}^2 + \gamma_{20}$	$.3f_{20}^2 + 3f_{20} + 2.5\gamma_{20}^2 + 5\gamma_{20}$	$.15f_{20}^2 + f_{20}$	100.
21	$.5f_{21}^2 + 2f_{21}$	$\gamma_{21}^2 + 3\gamma_{21}$	$.1f_{21}^2 + 3f_{21} + 1.5\gamma_{21}^2 + 4\gamma_{21}$	$.15f_{21}^2 + 3f_{21}$	100.
22	$f_{22}^2 + 3f_{22}$	$.5\gamma_{22}^2 + 2\gamma_{22}$	$.05f_{22}^2 + 4f_{22} + 2.5\gamma_{22}^2 + 4\gamma_{22}$	$.25f_{22}^2 + 5f_{22}$	100.
23	$.5f_{23}^2 + f_{23}$	$.25\gamma_{23}^2 + \gamma_{23}$	$.2f_{23}^2 + f_{23} + \gamma_{23}^2 + 2\gamma_{23}$	$.2f_{23}^2 + 4f_{23}$	150.
24	$f_{24}^2 + f_{24}$	$.25\gamma_{24}^2 + \gamma_{24}$	$.1f_{24}^2 + 3f_{24} + .05\gamma_{24}^2 + 2\gamma_{24}$	$.1f_{24}^2 + 2f_{24}$	150.

Example 1

In Example 1 the demands were:

$$d_1 = 100, \quad d_2 = 200, \quad d_3 = 100.$$

The total operating and frequency cost, the environmental impact, and the waste cost functions were as reported in Table 1.

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The total operating and frequency cost, the environmental impact, and the waste cost functions were as reported in Table 1.

In Example 1 we assumed that the firm did not care about the environmental impact and the waste generated generated in its supply chain and, hence, $\omega = 0$. The computed solution is reported in Table 2.

Example 1 Solution

Table 3: Example 1 Optimal Solution

Link	f_a^*	γ_a^*	μ_a^*
1	74.61	.7461	.0175
2	58.08	.5808	.0390
3	100.71	.5035	.0150
4	25.30	1.2651	.1765
5	24.89	1.2443	.4111
6	46.75	2.3373	.1669
7	68.45	3.4228	.2211
8	49.52	4.9520	1.5856
9	11.20	1.1202	.7481
10	60.73	1.2146	.1486
11	52.76	1.0551	.0811
12	0.00	.0000	.0000
13	108.47	7.2307	.5481
14	13.10	.83733	.5662
15	0.00	.0000	.0000

Example 1 Solution

Table 4: Example 1 Optimal Solution

Link a	f_a^*	γ_a^*	μ_a^*
16	91.53	4.5766	.5076
17	13.01	.6506	.0826
18	75.58	.7558	.0351
19	57.12	.2856	.0079
20	33.90	.3390	.0168
21	60.84	.6084	.0422
22	51.79	.5179	.0252
23	100.00	.6667	.0089
24	73.89	.4926	.0083

Example 1 Solution

The total cost (see objective function (6)) was: 55,920.97. The total environmental impact cost (see objective function (7)) was: 11,966.57, and the total waste costs were: 15,551.25. The value of the objective function (8) was, hence, 55.920.97.

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All the demand for demand market 1 is fulfilled through link 23 since links 12 and 15 have zero product flow on them. Of course, the corresponding frequencies of operating these links is also zero.

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All the demand for demand market 1 is fulfilled through link 23 since links 12 and 15 have zero product flow on them. Of course, the corresponding frequencies of operating these links is also zero.

Also, since, in this example, the firm is not at all concerned about its environmental impact and wastes generated, the value of the objective function corresponds to the total operational and frequency costs.

Example 2

Example 2 had the identical data as in Example 1 except that the firm was now concerned about the environment with $\omega = 1$. The new computed solution is given in Table 3.

Example 2 Solution

Table 5: Example 2 Optimal Solution

Link a	f_a^*	γ_a^*	μ_a^*
1	90.32	.9032	.0661
2	62.87	.6287	.0866
3	84.87	.4223	.0298
4	31.83	1.5913	.2351
5	30.93	1.5471	.4994
6	53.64	2.6821	.2026
7	59.35	2.9677	.2381
8	35.89	3.5892	1.2784
9	8.70	.8703	.6857
10	60.29	1.2057	.2806
11	52.30	1.0461	.2246
12	0.00	.0000	.0000
13	109.41	7.2947	.6089
14	11.95	.7966	.5630
15	0.00	.0000	.0000

Example 2 Solution

Table 6: Example 2 Optimal Solution

Link a	f_a^*	γ_a^*	μ_a^*
16	90.59	4.5294	.5533
17	8.41	.4204	.0781
18	72.45	.7245	.0935
19	50.13	.2506	.0350
20	39.77	.3977	.0878
21	61.07	.6107	.1005
22	46.69	.4669	.0880
23	100.00	.6667	.0311
24	79.64	.5310	.0085

Example 2 Solution

The total cost (see objective function (6)) was now: 56,632.07. The environmental impact cost (see objective function (7)) was now: 11,468.64. The waste cost was: 14,326.37. The value of the objective function (8) was, hence, 82,427.09. Due to the higher weight on the environmental and waste costs, the impact on the environment was reduced. However, as a consequence, the total cost is now higher than in Example 1 although not substantially so.

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Links 12 and 15, which are transportation/shipment links, are not used/operated, as was also the case in Example 1.

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Links 12 and 15, which are transportation/shipment links, are not used/operated, as was also the case in Example 1.

As expected, there is a transfer of production to the more environmentally-friendly manufacturing plants, with the associated technologies of production.

Example 3

Example 3 had the same data as Example 2 except that the $\bar{u}_1 = 1$ for all links $a = 1, \ldots, 24$. hence, the firm, in Example 1, was interested in designing a sustainable supply chain network for the product, with concern for the environment.

Example 3

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We kept the cost data for Example 3 as in Example 2 for comparison purposes. For actual design purposes one would need to increase the values of the $\hat{\pi}_a$ functions for all links $a \in L$, since these would then reflect actual construction/investment costs in the links (cf. Nagurney (2010)).

Here our goal was to demonstrate the flexibility of the modeling and computational framework.

The computed solution is reported in Table 4.

Example 3 Solution

Table 7: Example 3 Optimal Solution

Link a	f_a^*	γ_a^*	μ_a^*
1	97.25	97.25	391.9421
2	52.05	52.05	471.3385
3	70.95	70.95	499.5281
4	39.78	39.78	88.7147
5	24.65	24.65	127.7666
6	53.33	53.33	59.7681
7	54.01	4.01	66.0133
8	19.80	19.80	64.6667
9	8.43	8.43	38.5899
10	56.30	56.30	289.4470
11	47.05	47.05	241.1960
12	0.00	.0000	.0000
13	112.92	112.92	125.3066
14	0.00	.0000	5.1009
15	0.00	.0000	.3203

Example 3 Solution

Table 8: Example 3 Optimal Solution

Link a	f_a^*	γ_a^*	μ_a^*
16	87.08	87.08	192.6962
17	0.00	.0000	.0000
18	67.18	67.18	407.9846
19	55.29	55.29	447.2140
20	57.28	57.28	406.8561
21	56.61	56.61	289.9844
22	40.04	40.04	246.2007
23	100.00	100.00	252.9854
24	100.00	100.00	51.0052

Example 3 Solution

The total cost was: 122,625.56. The environmental impact was now: 102,133.26. The waste cost was: 13,464.07. The value of the objective function (8) was 238,222.89.

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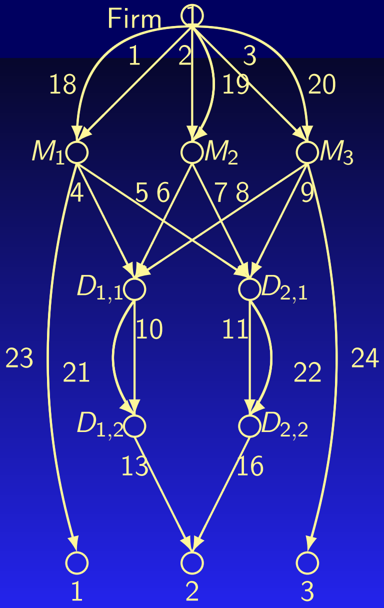
Demand points 1 and 3 are now served exclusively through direct shipments following the manufacture of the product.

The Optimal Supply Chain Network Topology for Example 3



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The above examples, although stylized, illustrate the practicality and flexibility of the sustainable supply chain network modeling approach and algorithm.

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- Numerical examples illustrated the relevance and scope of the theoretical and computational approach.
- Future research may include the inclusion of competition as well as perishable products.

Thank You!

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