# Supply Chain Networks: Challenges and Opportunities from Analysis to Design

Anna Nagurney

Isenberg School of Management University of Massachusetts Amherst, Massachusetts 01003

International Conference on Computational Management Science Vienna, Austria July 28-30, 2010

### Acknowledgments

I would like to thank Professor Georg Pflug and the organizing committee for the opportunity to deliver this keynote talk at this CMS 2010 conference.

Special acknowledgments and thanks to my doctoral students and collaborators who have made research always stimulating and rewarding.

#### Outline

- Background and Motivation
- ▶ Why User Behavior Must be Captured in Network Design
- Methodology for Formulation, Analysis, and Computations and an Empirical Application to Electric Power Supply Chains
- ► Network Design Through Mergers and Acquisitons
- ▶ A Challenging Network Design Problem and Model for Critical Needs with Outsourcing
- Applications to Vaccine Production and Emergency Preparedness and Humanitarian Logistics
- ► The Algorithm and Explicit Formulae
- Numerical Examples
- What About Sustainability and Supply Chain Network Design?
- ► Summary, Conclusions, and Suggestions for Future Research

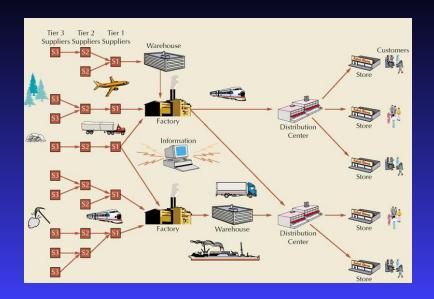
### Background and Motivation

Supply chains are the *critical infrastructure and backbones* for the production, distribution, and consumption of goods as well as services in our globalized *Network Economy*.

Supply chains, in their most fundamental realization, consist of manufacturers and suppliers, distributors, retailers, and consumers at the demand markets.

Today, supply chains may span thousands of miles across the globe, involve numerous suppliers, retailers, and consumers, and be underpinned by multimodal transportation and telecommunication networks.

### A General Supply Chain,



### **Examples of Supply Chains**

- ► food and food products
- ► high tech products
- automotive
- energy (oil, electric power, etc.)
- clothing and toys
- ▶ humanitarian relief
- ► healthcare supply chains.

# Food Supply Chains







## High Tech Products







# Automotive Supply Chains



# **Energy Supply Chains**



# Clothing and Toys



# Healthcare Supply Chains



### Humanitarian Relief



Supply chains may be characterized by *decentralized decision-making* associated with the different economic agents or by *centralized* decision-making.

Supply chains are are, in fact, Complex Network Systems.

Hence, any formalism that seeks to model supply chains and to provide quantifiable insights and measures must be a system-wide one and network-based.

Indeed, such crucial issues as the stability and resiliency of supply chains, as well as their adaptability and responsiveness to events in a global environment of increasing risk and uncertainty can only be rigorously examined from the view of supply chains as network systems.

## Characteristics of Supply Chains and Networks Today

- ► *large-scale nature* and complexity of network topology;
- congestion, which leads to nonlinearities;
- alternative behavior of users of the networks, which may lead to paradoxical phenomena;
- possibly conflicting criteria associated with optimization;
- ➤ interactions among the underlying networks themselves, such as the Internet with electric power networks, financial networks, and transportation and logistical networks;
- ► recognition of *their fragility and vulnerability*;
- policies surrounding networks today may have major impacts not only economically, but also socially, politically, and security-wise.

Changes in the availability of supplies, price shocks, as well as disruptions to transportation modes or telecommunications may have negative effects and consequences that propagate throughout the supply chain.

On the other hand, increases in demand for a product, entirely new demand markets, decreases in transaction costs, new suppliers, and even new modes of transaction, may provide new opportunities for profit maximization for manufacturers, distributors, as well as retailers, and new linkages that were not previously possible.



# Supply Chain Network Design Must Capture the Behavior of Users

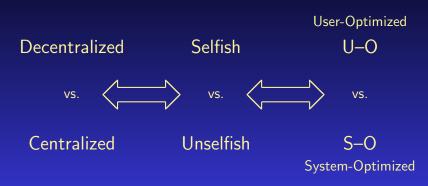


The importance of behavior will now be illustrated through a famous example known as the Braess paradox which demonstrates what can happen under *U-O* as opposed to *S-O* behavior.

Although the paradox was presented in the context of transportation networks, it is relevant to other network systems in which decision-makers act in a noncooperative (competitive) manner.

### Behavior on Congested Networks

Decision-makers select their cost-minimizing routes.



Flows are routed so as to minimize the total cost to society.

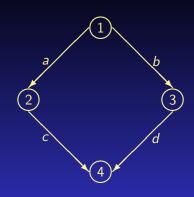
## The Braess (1968) Paradox

Assume a network with a single O/D pair (1,4). There are 2 paths available to travelers:  $p_1 = (a, c)$  and  $p_2 = (b, d)$ .

For a travel demand of **6**, the equilibrium path flows are  $x_{p_1}^* = x_{p_2}^* = 3$  and

The equilibrium path travel cost is

$$C_{p_1} = C_{p_2} = 83$$



$$c_a(f_a) = 10f_a, \quad c_b(f_b) = f_b + 50,$$
  
 $c_c(f_c) = f_c + 50, \quad c_d(f_d) = 10f_d.$ 

### Adding a Link Increases Travel Cost for All!

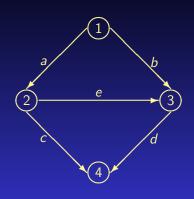
Adding a new link creates a new path  $p_3 = (a, e, d)$ .

The original flow distribution pattern is no longer an equilibrium pattern, since at this level of flow the cost on path  $p_3$ ,  $C_{p_3} = 70$ .

The new equilibrium flow pattern network is

$$x_{p_1}^* = x_{p_2}^* = x_{p_3}^* = 2.$$

The equilibrium path travel cost:  $C_{p_1} = C_{p_2} = C_{p_3} = 92$ .



$$c_e(f_e) = f_e + 10$$

The 1968 Braess article has been translated from German to English and appears as:

"On a Paradox of Traffic Planning,"

D. Braess, A. Nagurney, and T. Wakolbinger (2005) Transportation Science **39**, 446-450.







#### The Braess Paradox Around the World



1969 - Stuttgart, Germany - The traffic worsened until a newly built road was closed.

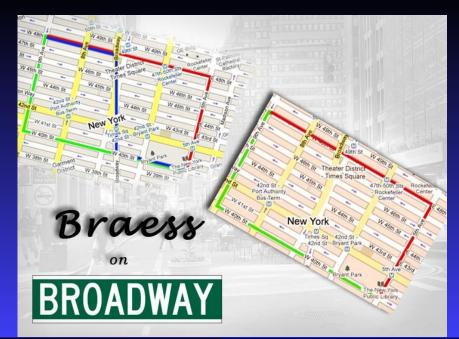
1990 - Earth Day - New York City - 42<sup>nd</sup> Street was closed and traffic flow improved.





2002 - Seoul, Korea - A 6 lane road built over the Cheonggyecheon River that carried 160,000 cars per day and was perpetually jammed was torn down to improve traffic flow.





Under S-O behavior, the total cost in the network is minimized, and the new route  $p_3$ , under the same demand, would not be used.

The Braess paradox never occurs in S-O networks.

### Other Networks that Behave like Traffic Networks



The Internet and electric power networks

Methodology for Formulation, Analysis, and Computations and

an Empirical Application to Electric Power Supply Chains

### The Variational Inequality Problem

We utilize the theory of variational inequalities for the formulation, analysis, and solution of both centralized and decentralized supply chain network problems.

#### **Definition: The Variational Inequality Problem**

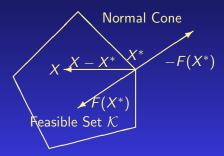
The finite-dimensional variational inequality problem, VI(F, K), is to determine a vector  $X^* \in K$ , such that:

$$\langle F(X^*)^T, X - X^* \rangle \ge 0, \quad \forall X \in \mathcal{K},$$

where F is a given continuous function from K to  $R^N$ , K is a given closed convex set, and  $\langle \cdot, \cdot \rangle$  denotes the inner product in  $R^N$ .

## Geometric Interpretation of VI(F, K)

In particular,  $F(X^*)$  is "orthogonal" to the feasible set K at the point  $X^*$ .



The variational inequality problem contains, as special cases, such mathematical programming problems as:

- systems of equations,
- optimization problems,
- complementarity problems,
- and is related to the fixed point problem.

Hence, it is a natural methodology for a spectrum of supply chain network problems from centralized to decentralized ones as well as to design problems.

### **Electric Power Supply Chains**

We have developed an empirical, large-scale electric supply chain network equilibrium model, formulated it as a VI problem, and were able to solve it by exploiting the connection between electric power supply chain networks and transportation networks using our proof of a hypothesis posed in the classic book, Studies in the Economics of Transportation, by Beckmann, McGuire, and Winsten (1956).

The paper, "An Integrated Electric Power Supply Chain and Fuel Market Network Framework: Theoretical Modeling with Empirical Analysis for New England," by Z. Liu and A. Nagurney was published in *Naval Research Logistics* (2009), **56**, 600-624.

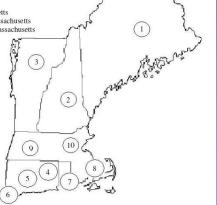
# An Empirical Example of an Electric Power Supply Chain for New England

There are 82 generating companies who own and operate 573 generating units. We considered 5 types of fuels: natural gas, residual fuel oil, distillate fuel oil, jet fuel, and coal. The whole area was divided into 10 regions:

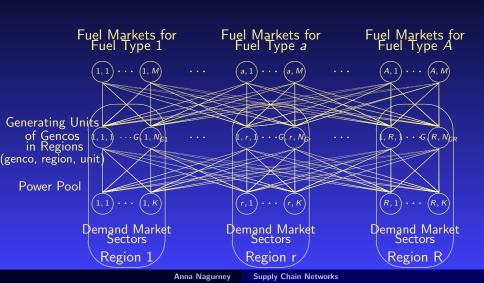
- 1. Maine,
- 2. New Hampshire,
- 3. Vermont,
- 4. Connecticut (excluding Southwest Connecticut),
- 5. Southwestern Connecticut (excluding the Norwalk-Stamford area),
- 6. Norwalk-Stamford area,
- 7. Rhode Island,
- 8. Southeastern Massachusetts,
- 9. Western and Central Massachusetts,
- 10. Boston/Northeast Massachusetts.

### Graphic of New England

- 1. Maine
- 2. New Hampshire
- 3. Vermont
- Connecticut (excluding Southwestern Connecticut)
- Southwestern Connecticut (excluding the Norwalk-Stamford area)
- 6. Norwalk-Stamford area
- 7. Rhode Island
- 8. Southeastern Massachusetts
- 9. Western and Central Massachusetts
- 10. Boston/Northeastern Massachusetts



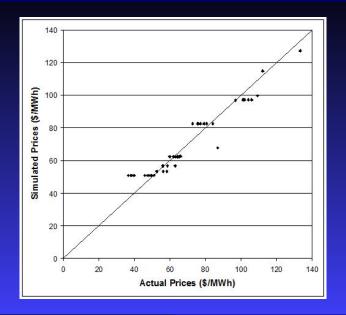
# The Electric Power Supply Chain Network with Fuel Supply Markets



We tested the model on the data of July 2006 which included  $24 \times 31 = 744$  hourly demand/price scenarios. We sorted the scenarios based on the total hourly demand, and constructed the load duration curve. We divided the duration curve into 6 blocks ( $L_1 = 94$  hours, and  $L_w = 130$  hours; w = 2, ..., 6) and calculated the average regional demands and the average weighted regional prices for each block.

The empirical model had on the order of 20,000 variables.

# Actual Prices Vs. Simulated Prices (\$/Mwh)



# Supply Chain Network Design Through Mergers and Acquisitions

Supply chain network design (and redesign) can be accomplished through link and node additions (as well as their removals).

It can be accomplished by modifying the link capacities (expanding certain ones and, if applicable, reducing or selling off others).

It can also be accomplished through the integration of networks as in mergers and acquisitions

and, as we will show later, through the design of the network from scratch.

# Mergers and Acquisitions and Network Synergies

Recently, we introduced a system-optimization perspective for supply chains in which firms are engaged in multiple activities of production, storage, and distribution to the demand markets and proposed a cost synergy measure associated with evaluating proposed mergers:

♦ A. Nagurney (2009) "A System-Optimization Perspective for Supply Chain Network Integration: The Horizontal Merger Case," Transportation Research E 45, 1-15.

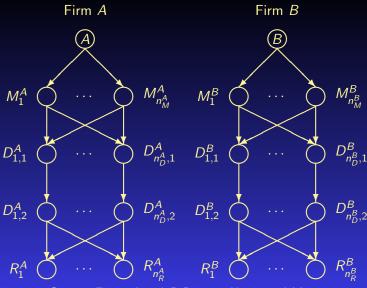


Figure 1: Case 0: Firms A and B Prior to Horizontal Merger

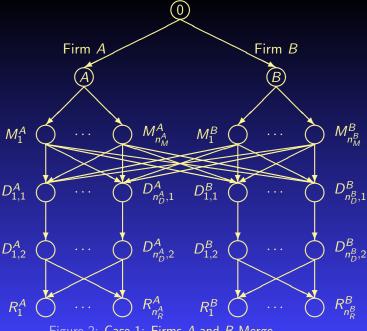


Figure 2: Case 1: Firms A and B Merge

**Anna Nagurney** 

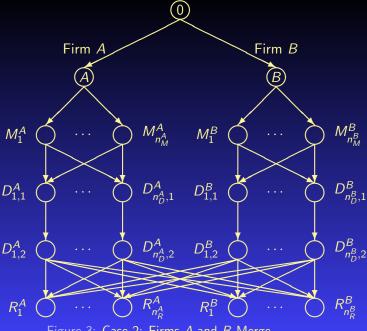


Figure 3: Case 2: Firms A and B Merge

**Anna Nagurney** 

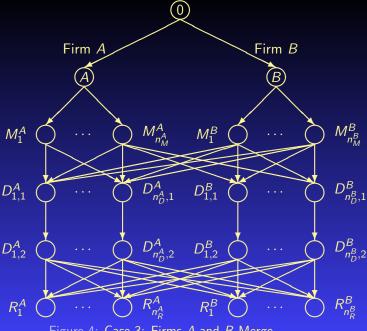


Figure 4: Case 3: Firms A and B Merge

**Anna Nagurney** 

#### Synergy Measure

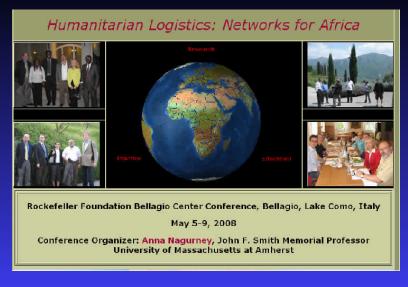
The measure that we utilized in Nagurney (2009) to capture the gains, if any, associated with a horizontal merger Case i; i = 1, 2, 3 is as follows:

$$S^{i} = \left[\frac{TC^{0} - TC^{i}}{TC^{0}}\right] \times 100\%,$$

where  $TC^i$  is the total cost associated with the value of the objective function  $\sum_{a\in L^i} \hat{c}_a(f_a)$  for i=0,1,2,3 evaluated at the optimal solution for Case i. Note that  $\mathcal{S}^i$ ; i=1,2,3 may also be interpreted as *synergy*.

This model can also be applied to the teaming of organizations in the case of humanitarian operations.

#### Bellagio Conference on Humanitarian Logistics



See: http://hlogistics.som.umass.edu/

# The Supply Chain Network Oligopoly Model (Nagurney, Computational Management Science (2010), in press.)

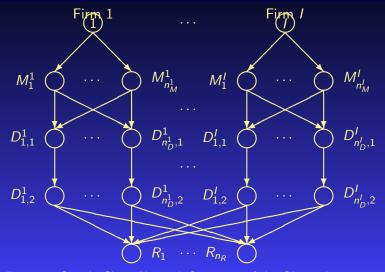


Figure 5: Supply Chain Network Structure of the Oligopoly

#### Mergers Through Coalition Formation

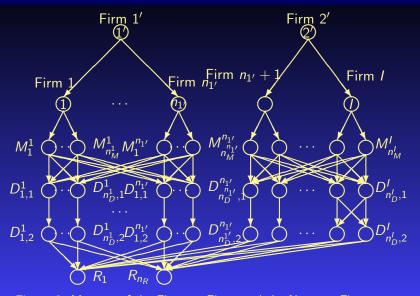


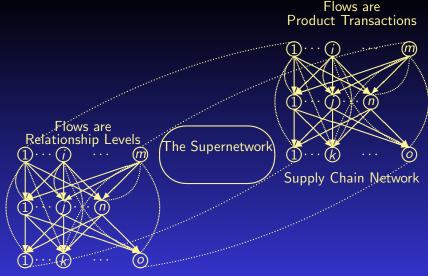
Figure 6: Mergers of the First  $n_{1'}$  Firms and the Next  $n_{2'}$  Firms

In addition, supply chain network design can be accomplished through the evolution and integration of disparate network systems, including social networks.

#### **Two References:**

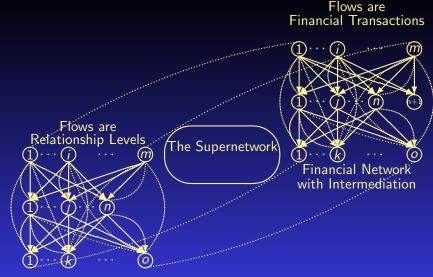
T. Wakolbinger and A. Nagurney (2004) "Dynamic Supernetworks for the Integration of Social Networks and Supply Chains with Electronic Commerce: Modeling and Analysis of Buyer-Seller Relationships with Computations," *Netnomics* **6**, 153-185.

A. Nagurney, T. Wakolbinger, and L. Zhao (2006) "The Evolution and Emergence of Integrated Social and Financial Networks with Electronic Transactions: A Dynamic Supernetwork Theory for the Modeling, Analysis, and Computation of Financial Flows and Relationship Levels," *Computational Economics* 27, 353-393.



Social Network

Figure 7: The Multilevel Supernetwork Structure of the Integrated Supply Chain / Social Network System



Social Network

Figure 8: The Multilevel Supernetwork Structure of the Integrated Financial Network / Social Network System

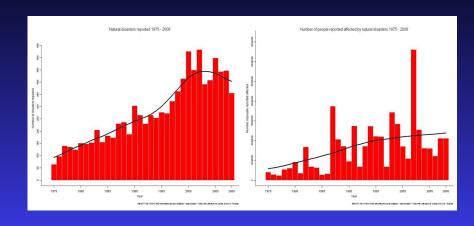
# A Challenging Network Design Problem and

Model for Critical Needs with Outsourcing

The number of disasters is increasing globally, as is the number of people affected by disasters. At the same time, with the advent of increasing globalization, viruses are spreading more quickly and creating new challenges for medical and health professionals, researchers, and government officials.

Between 2000 and 2004, the average annual number of disasters was 55% higher than in the period 1994 through 1999, with 33% more humans affected in the former period than in the latter (cf. Balcik and Beamon (2008) and Nagurney and Qiang (2009)).

# Natural Disasters (1975–2008)



However, although the average number of disasters has been increasing annually over the past decade the average percentage of needs met by different sectors in the period 2000 through 2005 identifies significant shortfalls.

According to Development Initiatives (2006), based on data in the Financial Tracking System of the Office for the Coordination of Humanitarian Affairs, from 2000-2005, the average needs met by different sectors in the case of disasters were:

- ▶ 79% by the food sector;
- ▶ 37% of the health needs;
- ▶ 35% of the water and sanitation needs;
- ▶ 28% of the shelter and non-food items, and
- ▶ 24% of the economic recovery and infrastructure needs.

#### Hurricane Katrina in 2005



Hurricane Katrina has been called an "American tragedy," in which essential services failed completely (Guidotti (2006)).

#### Haiti Earthquake in 2010

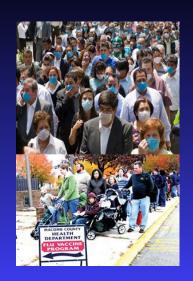


Delivering the humanitarian relief supplies (water, food, medicines, etc.) to the victims was a major logistical challenge.

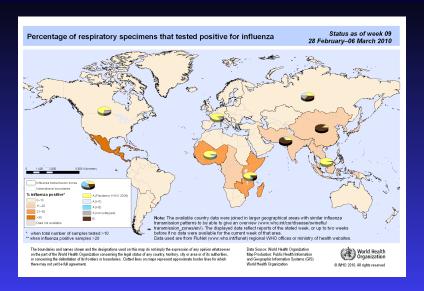
# H1N1 (Swine) Flu

As of May 2, 2010, worldwide, more than 214 countries and overseas territories or communities have reported laboratory confirmed cases of pandemic influenza H1N1 2009, including over 18,001 deaths (www.who.int).

Parts of the globe experienced serious flu vaccine shortages, both seasonal and H1N1 (swine) ones, in late 2009.

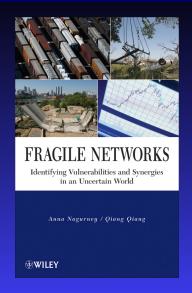


#### Map of Influenza Activity and Virus Subtypes



#### Source: World Health Organization

### Fragile Networks



We are living in a world of Fragile Networks.

Anna Nagurney

### Background and Motivation

Underlying the delivery of goods and services in times of crises, such as in the case of disasters, pandemics, and life-threatening major disruptions, are supply chains, without which essential products do not get delivered in a timely manner, with possible increased disease, injuries, and casualties.

It is clear that better-designed supply chain networks would have facilitated and enhanced various emergency preparedness and relief efforts and would have resulted in less suffering and lives lost.

Supply chain networks provide the logistical backbones for the provision of products as well as services both in corporate as well as in emergency and humanitarian operations.

Here we focus on supply chains in the case of

# Critical Needs Products.

#### Critical Needs Products

Critical needs products are those that are essential to the survival of the population, and can include, for example, vaccines, medicine, food, water, etc., depending upon the particular application.

The demand for the product should be met as nearly as possible since otherwise there may be additional loss of life.

In times of crises, a system-optimization approach is mandated since the demands for critical supplies should be met (as nearly as possible) at minimal total cost.

#### An Overview of Some of the Relevant Literature

- ➤ M. J. Beckmann, C. B. McGuire, and C. B. Winsten (1956) Studies in the Economics of Transportation, Yale University Press, New Haven, Connecticut.
- ► S. C. Dafermos and F. T. Sparrow (1969) "The Traffic Assignment Problem for a General Network," *Journal of Research of the National Bureau of Standards* **73B**, 91-118.
- ▶ D. E. Boyce, H. S. Mahmassani, and A. Nagurney (2005) "A Retrospective on Beckmann, McGuire, and Winsten's Studies in the Economics of Transportation," Papers in Regional Science 84, 85-103.
- ► A. Nagurney (2009), "A System-Optimization Perspective for Supply Chain Network Integration: The Horizontal Merger Case," *Transportation Research E* **45**, 1-15.

- ▶ A. Nagurney, T. Woolley, and Q. Qiang (2010) "Multiproduct Supply Chain Horizontal Network Integration: Models, Theory, and Computational Results," *International Journal of Operational Research* 17, 333-349.
- ► A. Nagurney (2010) "Formulation and Analysis of Horizontal Mergers Among Oligopolistic Firms with Insights into the Merger Paradox: A Supply Chain Network Perspective," Computational Management Science, in press.
- ► A. Nagurney (2010) "Supply Chain Network Design Under Profit Maximization and Oligopolistic Competition," Transportation Research E 46, 281-294.

This part of the presentation is based on the paper:

"Supply Chain Network Design for Critical Needs with Outsourcing,"

A. Nagurney, M. Yu, and Q. Qiang, to appear in *Papers in Regional Science*,

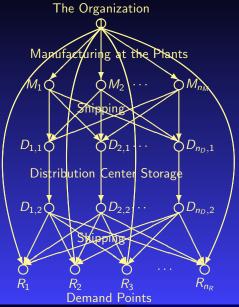
where additional background as well as references can be found.

We assume that the organization (government, humanitarian one, socially responsible firm, etc.) is considering  $n_M$  manufacturing facilities/plants;  $n_D$  distribution centers, but must serve the  $n_R$  demand points.

The supply chain network is modeled as a network G = [N, L], consisting of the set of nodes N and the set of links L. Let  $L^1$  and  $L^2$  denote the links associated with "in house" supply chain activities and the outsourcing activities, respectively. The paths joining the origin node to the destination nodes represent sequences of supply chain network activities that ensure that the product is produced and, ultimately, delivered to those in need at the demand points.

The optimization model can handle both design (from scratch) and redesign scenarios.

### Supply Chain Network Topology with Outsourcing



#### The Links

The possible manufacturing links from the top-tiered node 1 are connected to the possible manufacturing nodes of the organization, which are denoted, respectively, by:  $M_1, \ldots, M_{n_M}$ .

The possible shipment links from the manufacturing nodes, are connected to the possible distribution center nodes of the organization, denoted by  $D_{1,1}, \ldots, D_{n_D,1}$ .

The links joining nodes  $D_{1,1}, \ldots, D_{n_D,1}$  with nodes  $D_{1,2}, \ldots, D_{n_D,2}$  correspond to the possible storage links.

There are possible shipment links joining the nodes  $D_{1,2}, \ldots, D_{n_D,2}$  with the demand nodes:  $R_1, \ldots, R_{n_D}$ .

There are also outsourcing links, which may join the top node to each bottom node (or the relevant nodes for which the outsourcing activity is feasible, as in production, storage, or distribution, or a combination thereof). The organization does not control the capacities on these links since they have been established by the particular firm that corresponds to the outsource link.

The ability to outsource supply chain network activities for critical needs products provides alternative pathways for the production and delivery of products during times of crises such as disasters.

#### Demands, Path Flows, and Link Flows

Let  $d_k$  denote the demand at demand point k;  $k=1,\ldots,n_R$ , which is a random variable with probability density function given by  $\mathcal{F}_k(t)$ . Let  $x_p$  represent the nonnegative flow of the product on path p;  $f_a$  denote the flow of the product on link a.

#### Conservation of Flow Between Path Flows and Link Flows

$$f_a = \sum_{p \in P} x_p \delta_{ap}, \quad \forall a \in L, \tag{1}$$

that is, the total amount of a product on a link is equal to the sum of the flows of the product on all paths that utilize that link.  $\delta_{ap}=1$  if link a is contained in path p, and  $\delta_{ap}=0$ , otherwise.

#### Supply Shortage and Surplus

Let

$$v_k \equiv \sum_{p \in P_{w_k}} x_p, \quad k = 1, \dots, n_R, \tag{2}$$

where  $v_k$  can be interpreted as the *projected demand* at demand market k;  $k = 1, ..., n_R$ . Then,

$$\Delta_k^- \equiv \max\{0, d_k - v_k\}, \quad k = 1, \dots, n_R, \tag{3}$$

$$\Delta_k^+ \equiv \max\{0, v_k - d_k\}, \quad k = 1, \dots, n_R, \tag{4}$$

where  $\Delta_k^-$  and  $\Delta_k^+$  represent the supply shortage and surplus at demand point k, respectively. The expected values of  $\Delta_k^-$  and  $\Delta_k^+$  are given by:

$$E(\Delta_k^-) = \int_{v_0}^{\infty} (t - v_k) \mathcal{F}_k(t) d(t), \quad k = 1, \dots, n_R,$$
 (5)

$$E(\Delta_k^+) = \int_0^{\nu_k} (\nu_k - t) \mathcal{F}_k(t) d(t), \quad k = 1, \dots, n_R.$$
 (6)

#### The Operation Costs, Investment Costs and Penalty Costs

The total cost on a link is assumed to be a function of the flow of the product on the link. We have, thus, that

$$\hat{c}_a = \hat{c}_a(f_a), \quad \forall a \in L.$$
 (7)

We denote the nonnegative existing capacity on a link a by  $\bar{u}_a$ ,  $\forall a \in L$ . Note that the organization can add capacity to the "in house" link a;  $\forall a \in L^1$ . We assume that

$$\hat{\pi}_a = \hat{\pi}_a(u_a), \quad \forall a \in L^1. \tag{8}$$

The expected total penalty at demand point k;  $k = 1, ..., n_R$ , is,

$$E(\lambda_k^- \Delta_k^- + \lambda_k^+ \Delta_k^+) = \lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+), \tag{9}$$

where  $\lambda_k^-$  is the unit penalty of supply shortage at demand point k and  $\lambda_k^+$  is that of supply surplus. Note that  $\lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+)$  is a function of the path flow vector x.

## The Supply Chain Network Design Optimization Problem

The organization seeks to determine the optimal levels of product processed on each supply chain network link (including the outsourcing links) coupled with the optimal levels of capacity investments in its supply chain network activities subject to the minimization of the total cost.

The total cost includes the total cost of operating the various links, the total cost of capacity investments, and the expected total supply shortage/surplus penalty.

## The Supply Chain Network Design Optimization Problem

Minimize 
$$\sum_{a \in L} \hat{c}_a(f_a) + \sum_{a \in L^1} \hat{\pi}_a(u_a) + \sum_{k=1}^{n_R} (\lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+))$$
(10)

subject to: constraints (1), (2) and

$$f_a \le \bar{u}_a + u_a, \quad \forall a \in L^1,$$
 (11)

$$f_a \leq \bar{u}_a, \quad \forall a \in L^2,$$
 (12)

$$u_a \ge 0, \quad \forall a \in L^1,$$
 (13)

$$x_p \ge 0, \quad \forall p \in P.$$
 (14)

#### The Feasible Set

We associate the Lagrange multiplier  $\omega_a$  with constraint (11) for link  $a \in L^1$  and we denote the associated optimal Lagrange multiplier by  $\omega_a^*$ . Similarly, Lagrange multiplier  $\gamma_a$  is associated with constraint (12) for link  $a \in L^2$  with the optimal multiplier denoted by  $\gamma_a^*$ . These two terms may also be interpreted as the price or value of an additional unit of capacity on link a. We group these Lagrange multipliers into the vectors  $\omega$  and  $\gamma$ , respectively. Let K denote the feasible set such that

$$K \equiv \{(x, u, \omega, \gamma) | x \in R_+^{n_P}, u \in R_+^{n_{L^1}}, \omega \in R_+^{n_{L^1}}, \text{ and } \gamma \in R_+^{n_{L^2}}\}.$$

#### Theorem

The optimization problem is equivalent to the variational inequality problem: determine the vector of optimal path flows, the vector of optimal link capacity enhancements, and the vectors of optimal Lagrange multipliers  $(x^*, u^*, \omega^*, \gamma^*) \in K$ , such that:

$$\sum_{k=1}^{n_R} \sum_{p \in P_{w_k}} \left[ \frac{\partial \hat{C}_p(x^*)}{\partial x_p} + \sum_{a \in L^1} \omega_a^* \delta_{ap} + \sum_{a \in L^2} \gamma_a^* \delta_{ap} + \lambda_k^+ P_k \left( \sum_{p \in P_{w_k}} x_p^* \right) \right]$$

$$-\lambda_k^- \left( 1 - P_k \left( \sum_{p \in P_{w_k}} x_p^* \right) \right) \times [x_p - x_p^*]$$

$$+ \sum_{a \in L^1} \left[ \frac{\partial \hat{\pi}_a(u_a^*)}{\partial u_a} - \omega_a^* \right] \times [u_a - u_a^*] + \sum_{a \in L^1} [\bar{u}_a + u_a^* - \sum_{p \in P} x_p^* \delta_{ap}] \times [\omega_a - \omega_a^*]$$

$$+ \sum_{a \in L^2} [\bar{u}_a - \sum_{a \in P} x_p^* \delta_{ap}] \times [\gamma_a - \gamma_a^*] \ge 0, \quad \forall (x, u, \omega, \gamma) \in K.$$

$$(15)$$

## Theorem (cont'd.)

In addition, (15) can be reexpressed in terms of links flows as: determine the vector of optimal link flows, the vectors of optimal projected demands and link capacity enhancements, and the vectors of optimal Lagrange multipliers  $(f^*, v^*, u^*, \omega^*, \gamma^*) \in K^1$ , such that:

$$\sum_{a \in L^{1}} \left[ \frac{\partial \hat{c}_{a}(f_{a}^{*})}{\partial f_{a}} + \omega_{a}^{*} \right] \times \left[ f_{a} - f_{a}^{*} \right] + \sum_{a \in L^{2}} \left[ \frac{\partial \hat{c}_{a}(f_{a}^{*})}{\partial f_{a}} + \gamma_{a}^{*} \right] \times \left[ f_{a} - f_{a}^{*} \right]$$

$$+ \sum_{a \in L^{1}} \left[ \frac{\partial \hat{\pi}_{a}(u_{a}^{*})}{\partial u_{a}} - \omega_{a}^{*} \right] \times \left[ u_{a} - u_{a}^{*} \right]$$

$$\frac{1}{n_{R}}$$

$$+ \sum_{k=1} \left[ \lambda_{k}^{+} P_{k}(v_{k}^{*}) - \lambda_{k}^{-} (1 - P_{k}(v_{k}^{*})) \right] \times \left[ v_{k} - v_{k}^{*} \right] + \sum_{a \in L^{1}} \left[ \bar{u}_{a} + u_{a}^{*} - f_{a}^{*} \right] \times \left[ \omega_{a} - \omega_{a}^{*} \right]$$

$$+ \sum_{a \in L^{2}} \left[ \bar{u}_{a} - f_{a}^{*} \right] \times \left[ \gamma_{a} - \gamma_{a}^{*} \right] \ge 0, \quad \forall (f, v, u, \omega, \gamma) \in K^{1},$$
(16)

where  $K^1 \equiv \{(f, v, u, \omega, \gamma) | \exists x \geq 0, \text{ and } (1), (2), (13), \text{ and } (14) \text{ hold,}$ and  $\omega \geq 0, \ \gamma \geq 0\}.$ 

# Applications to Vaccine Production and

Emergency Preparedness and Humanitarian Logistics

## Applications to Vaccine Production

Consider a vaccine manufacturer who is gearing up for next year's production of H1N1 (swine) flu vaccine. Governments around the world are beginning to contract with this company for next year's flu vaccine.

By applying the general theoretical model to the company's data, the firm can determine whether it needs to expand its facilities (or not), how much of the vaccine to produce where, how much to store where, and how much to have shipped to the various demand points. Also, it can determine whether it should outsource any of its vaccine production and at what level.

The firm by solving the model with its company-relevant data can then ensure that the price that it receives for its vaccine production and delivery is appropriate and that it recovers its incurred costs and obtains, if negotiated correctly, an equitable profit.

## Applications to Emergency Preparedness and Humanitarian Logistics

A company can, using the model, prepare and plan for an emergency such as a natural disaster in the form of a hurricane and identify where to store a necessary product (such as food packets, for example) so that the items can be delivered to the demand points in a timely manner and at minimal total cost.

In August 2005 Hurricane Katrina hit the US and this natural disaster cost immense damage with repercussions that continue to this day. While US state and federal officials came under severe criticism for their handling of the storm's aftermath, Wal-Mart had prepared in advance and through its logistical efficiencies had dozens of trucks loaded with supplies for delivery before the hurricane even hit landfall.

### The Algorithm

At an iteration  $\tau$  of the Euler method (see Dupuis and Nagurney (1993) and Nagurney and Zhang (1996)) one computes:

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - \mathsf{a}_{\tau} \mathsf{F}(X^{\tau})), \tag{17}$$

where  $P_{\mathcal{K}}$  is the projection on the feasible set  $\mathcal{K}$  and F is the function that enters the variational inequality problem: determine  $X^* \in \mathcal{K}$  such that

$$\langle F(X^*)^T, X - X^* \rangle \ge 0, \quad \forall X \in \mathcal{K},$$
 (18)

where  $\langle \cdot, \cdot \rangle$  is the inner product in *n*-dimensional Euclidean space,  $X \in \mathbb{R}^n$ , and F(X) is an *n*-dimensional function from  $\mathcal{K}$  to  $\mathbb{R}^n$ , with F(X) being continuous.

The sequence  $\{a_{\tau}\}$  must satisfy:  $\sum_{\tau=0}^{\infty} a_{\tau} = \infty$ ,  $a_{\tau} > 0$ ,  $a_{\tau} \to 0$ , as  $\tau \to \infty$ .

## Explicit Formulae for (17) to the Supply Chain Network Design Variational Inequality (15)

$$x_p^{\tau+1} = \max\{0, x_p^{\tau} + a_{\tau}(\lambda_k^-(1 - P_k(\sum_{p \in P_{w_k}} x_p^{\tau})) - \lambda_k^+ P_k(\sum_{p \in P_{w_k}} x_p^{\tau})$$

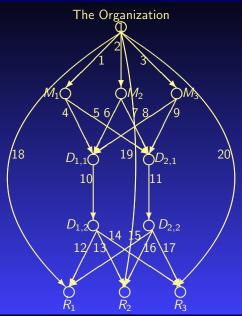
$$-\frac{\partial \hat{C}_{p}(x^{\tau})}{\partial x_{p}} - \sum_{a \in L^{1}} \omega_{a}^{\tau} \delta_{ap} - \sum_{a \in L^{2}} \gamma_{a}^{\tau} \delta_{ap})\}, \forall p \in P;$$
 (19)

$$u_a^{\tau+1} = \max\{0, u_a^{\tau} + a_{\tau}(\omega_a^{\tau} - \frac{\partial \hat{\pi}_a(u_a^{\tau})}{\partial u_a})\}, \quad \forall a \in L^1;$$
 (20)

$$\omega_a^{\tau+1} = \max\{0, \omega_a^{\tau} + a_{\tau}(\sum_{p \in P} \chi_p^{\tau} \delta_{ap} - \bar{u}_a - u_a^{\tau})\}, \quad \forall a \in L^1;$$
 (21)

$$\gamma_a^{\tau+1} = \max\{0, \gamma_a^{\tau} + a_{\tau}(\sum x_p^{\tau} \delta_{ap} - \bar{u}_a)\}, \quad \forall a \in L^2.$$
 (22)

## Numerical Examples



Anna Nagurney

Supply Chain Networks

#### Example 1

The demands at the three demand points followed a uniform probability distribution on the intervals [0, 10], [0, 20], and [0, 30], respectively:

$$P_1(\sum_{p \in P_{w_1}} x_p) = \frac{\sum_{p \in P_{w_1}} x_p}{10}, \quad P_2(\sum_{p \in P_{w_2}} x_p) = \frac{\sum_{p \in P_{w_2}} x_p}{20},$$
$$P_3(\sum_{p \in P_{w_3}} x_p) = \frac{\sum_{p \in P_{w_3}} x_p}{30},$$

where  $w_1 = (1, R_1)$ ,  $w_2 = (1, R_2)$ , and  $w_3 = (1, R_3)$ . The penalties were:

$$\lambda_1^- = 50, \quad \lambda_1^+ = 0; \quad \lambda_2^- = 50, \quad \lambda_2^+ = 0; \quad \lambda_3^- = 50, \quad \lambda_3^+ = 0.$$

The capacities associated with the three outsourcing links were:

$$\bar{u}_{18} = 5$$
,  $\bar{u}_{19} = 10$ ,  $\bar{u}_{20} = 5$ .

We set 
$$\bar{u}_a = 0$$
 for all links  $a \in L^1$ .

Anna Nagurney

Table 1: Total Cost Functions and Solution for Example 1

Link a	$\hat{c}_a(f_a)$	$\hat{\pi}_{a}(u_{a})$	$f_a^*$	$U_a^*$	$\omega_a^*$	$\gamma_{\sf a}^*$
1	$f_1^2 + 2f_1$	$.5u_1^2 + u_1$	1.34	1.34	2.34	_
2	$.5f_2^2 + f_2$	$.5u_2^2 + u_2$	2.47	2.47	3.47	_
3	$.5f_3^2 + f_3$	$.5u_3^2 + u_3$	2.05	2.05	3.05	_
4	$1.5f_4^2 + 2f_4$	$.5u_4^2 + u_4$	0.61	0.61	1.61	_
5	$f_5^2 + 3f_5$	$.5u_5^2 + u_5$	0.73	0.73	1.73	_
6	$f_6^2 + 2f_6$	$.5u_6^2 + u_6$	0.83	0.83	1.83	_
7	$.5f_7^2 + 2f_7$	$.5u_7^2 + u_7$	1.64	1.64	2.64	_
8	$.5f_8^2 + 2f_8$	$.5u_8^2 + u_8$	1.67	1.67	2.67	_
9	$f_9^2 + 5f_9$	$.5u_9^2 + u_9$	0.37	0.37	1.37	_
10	$.5f_{10}^2 + 2f_{10}$	$.5u_{10}^2 + u_{10}$	3.11	3.11	4.11	_
11	$f_{11}^2 + f_{11}$	$.5u_{11}^2 + u_{11}$	2.75	2.75	3.75	_
12	$.5f_{12}^2 + 2f_{12}$	$.5u_{12}^2 + u_{12}$	0.04	0.04	1.04	_
13	$.5f_{13}^2 + 5f_{13}$	$.5u_{13}^2 + u_{13}$	0.00	0.00	0.45	_

Table 2: Total Cost Functions and Solution for Example 1 (continued)

Link a	$\hat{c}_a(f_a)$	$\hat{\pi}_{a}(u_{a})$	$f_a^*$	u <sub>a</sub> *	$\omega_a^*$	$\gamma_a^*$
14	$f_{14}^2$	$.5u_{14}^2 + u_{14}$	3.07	3.07	4.07	-
15	$f_{15}^2 + 2f_{15}$	$.5u_{15}^2 + u_{15}$	0.00	0.00	0.45	-
16	$.5f_{16}^2 + 3f_{16}$	$.5u_{16}^2 + u_{16}$	0.00	0.00	0.45	-
17	$.5f_{17}^2 + 2f_{17}$	$.5u_{17}^2 + u_{17}$	2.75	2.75	3.75	_
18	10 <i>f</i> <sub>18</sub>	_	5.00	-	-	14.77
19	12f <sub>19</sub>	_	10.00	-	-	13.00
20	15f <sub>20</sub>	_	5.00	-	-	16.96

Note that the optimal supply chain network design for Example 1 is, hence, as the initial topology but with links 13, 15, and 16 removed since those links have zero capacities and associated flows. Note that the organization took advantage of outsourcing to the full capacity available.

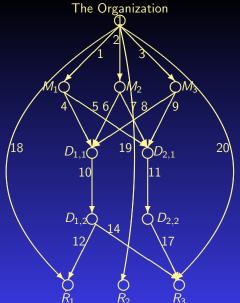


Figure 9: The Optimal Supply Chain Network Design for Example 1

### Example 2

Example 2 had the identical data to that in Example 1 except that we now assumed that the organization had capacities on its supply chain network activities where  $\bar{u}_a = 10$ , for all  $a \in L^1$ .

Table 3: Total Cost Functions and Solution for Example 2

Link a	$\hat{c}_a(f_a)$	$\hat{\pi}_a(u_a)$	$f_a^*$	$u_a^*$	$\omega_a^*$	$\gamma_a^*$
1	$f_1^2 + 2f_1$	$.5u_1^2 + u_1$	1.84	0.00	0.00	_
2	$.5f_2^2 + f_2$	$.5u_2^2 + u_2$	4.51	0.00	0.00	-
3	$.5f_3^2 + f_3$	$.5u_3^2 + u_3$	3.85	0.00	0.00	_
4	$1.5f_4^2 + 2f_4$	$.5u_4^2 + u_4$	0.88	0.00	0.00	_
5	$f_5^2 + 3f_5$	$.5u_5^2 + u_5$	0.97	0.00	0.00	_
6	$f_6^2 + 2f_6$	$.5u_6^2 + u_6$	1.40	0.00	0.00	_
7	$.5f_7^2 + 2f_7$	$.5u_7^2 + u_7$	3.11	0.00	0.00	_
8	$.5f_8^2 + 2f_8$	$.5u_8^2 + u_8$	3.47	0.00	0.00	_
9	$f_9^2 + 5f_9$	$.5u_9^2 + u_9$	0.38	0.00	0.00	_

Table 4: Total Cost Functions and Solution for Example 2 (continued)

Link a	$\hat{c}_a(f_a)$	$\hat{\pi}_{a}(u_{a})$	$f_a^*$	u <sub>a</sub> *	$\omega_a^*$	$\gamma_a^*$
10	$.5f_{10}^2 + 2f_{10}$	$.5u_{10}^2 + u_{10}$	5.75	0.00	0.00	-
11	$f_{11}^2 + f_{11}$	$.5u_{11}^2 + u_{11}$	4.46	0.00	0.00	-
12	$.5f_{12}^2 + 2f_{12}$	$.5u_{12}^2 + u_{12}$	0.82	0.00	0.00	-
13	$1.5f_{13}^2 + 5f_{13}$	$.5u_{13}^2 + u_{13}$	0.52	0.00	0.00	-
14	$f_{14}^2$	$.5u_{14}^2 + u_{14}$	4.41	0.00	0.00	_
15	$f_{15}^2 + 2f_{15}$	$.5u_{15}^2 + u_{15}$	0.00	0.00	0.00	_
16	$1.5f_{16}^2 + 3f_{16}$	$.5u_{16}^2 + u_{16}$	0.05	0.00	0.00	_
17	$.5f_{17}^2 + 2f_{17}$	$.5u_{17}^2 + u_{17}$	4.41	0.00	0.00	_
18	10 <i>f</i> <sub>18</sub>	_	5.00	_	-	10.89
19	$12f_{19}$	_	10.00	_	-	11.59
20	$15f_{20}$	_	5.00	_	_	11.96

Note that links 13 and 16 now have positive associated flows although at very low levels.

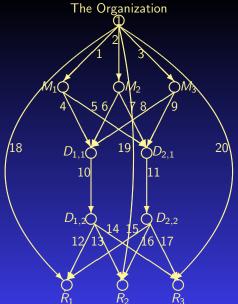


Figure 10: The Optimal Supply Chain Network Design for Example 2

### Example 3

Example 3 had the same data as Example 2 except that we changed the probability distributions so that we now had:

$$P_1(\sum_{p \in P_{w_1}} x_p) = \frac{\sum_{p \in P_{w_1}} x_p}{110},$$

$$P_2(\sum_{p \in P_{w_2}} x_p) = \frac{\sum_{p \in P_{w_2}} x_p}{120},$$

$$P_3(\sum_{p \in P_{w_3}} x_p) = \frac{\sum_{p \in P_{w_3}} x_p}{130}.$$

Table 5: Total Cost Functions and Solution for Example 3

Link a	$\hat{c}_a(f_a)$	$\hat{\pi}_{a}(u_{a})$	$f_a^*$	и*	$\omega_a^*$	$\gamma_a^*$
1	$f_1^2 + 2f_1$	$.5u_1^2 + u_1$	4.23	0.00	0.00	_
2	$.5f_2^2 + f_2$	$.5u_2^2 + u_2$	9.06	0.00	0.00	_
3	$.5f_3^2 + f_3$	$.5u_3^2 + u_3$	8.61	0.00	0.00	_
4	$1.5f_4^2 + 2f_4$	$.5u_4^2 + u_4$	2.05	0.00	0.00	_
5	$f_5^2 + 3f_5$	$.5u_5^2 + u_5$	2.18	0.00	0.00	_
6	$f_6^2 + 2f_6$	$.5u_6^2 + u_6$	3.28	0.00	0.00	_
7	$.5f_7^2 + 2f_7$	$.5u_7^2 + u_7$	5.77	0.00	0.00	_
8	$.5f_8^2 + 2f_8$	$.5u_8^2 + u_8$	7.01	0.00	0.00	_
9	$f_9^2 + 5f_9$	$.5u_9^2 + u_9$	1.61	0.00	0.00	_
10	$.5f_{10}^2 + 2f_{10}$	$.5u_{10}^2 + u_{10}$	12.34	2.34	3.34	_
11	$f_{11}^2 + f_{11}$	$.5u_{11}^2 + u_{11}$	9.56	0.00	0.00	_
12	$.5f_{12}^2 + 2f_{12}$	$.5u_{12}^2 + u_{12}$	5.82	0.00	0.00	_
13	$.5f_{13}^2 + 5f_{13}$	$.5u_{13}^2 + u_{13}$	2.38	0.00	0.00	_

Table 6: Total Cost Functions and Solution for Example 3 (continued)

Link a	$\hat{c}_a(f_a)$	$\hat{\pi}_{a}(u_{a})$	$f_a^*$	$u_a^*$	$\omega_a^*$	$\gamma_{\sf a}^*$
14	$f_{14}^2$	$.5u_{14}^2 + u_{14}$	4.14	0.00	0.00	_
15	$f_{15}^2 + 2f_{15}$	$.5u_{15}^2 + u_{15}$	2.09	0.00	0.00	-
16	$.5f_{16}^2 + 3f_{16}$	$.5u_{16}^2 + u_{16}$	2.75	0.00	0.00	-
17	$.5f_{17}^2 + 2f_{17}$	$.5u_{17}^2 + u_{17}$	4.72	0.00	0.00	-
18	10 <i>f</i> <sub>18</sub>	-	5.00	_	-	34.13
19	$12f_{19}$	-	10.00	_	-	31.70
20	15 <i>f</i> <sub>20</sub>	_	5.00	_	-	29.66

The optimal supply chain network design for Example 3 has the initial topology since there are now positive flows on all the links. It is also interesting to note that there is a significant increase in production volumes by the organization at its manufacturing plants.

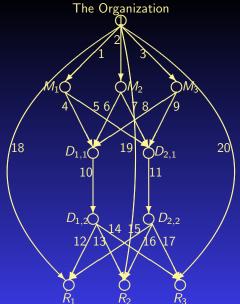


Figure 11: The Optimal Supply Chain Network Design for Example 3

What About Sustainability and

Supply Chain Network Design?

Sustainability of supply chains has also emerged as a major theme in both research and practice since the impacts of climate change have made both producers and consumers more cognizant of their decision-making and how their decisions affect the environment.











Photos of oil spill crisis in Gulf of Mexico, May 2010

Recent results in this dimension:

"Sustainable Supply Chain Network Design: A Multicriteria Perspective,"

A. Nagurney, and L. S. Nagurney, *International Journal of Sustainable Engineering* (2010), in press.



## Summary, Conclusions, and Suggestions for Future Research

- ► We emphasized the importance of capturing behavior in supply chain modeling, analysis, and design.
- ► We discussed a variety of network design approaches: the addition of links; the integration of networks as in mergers and acquisitions; and the design from scratch (and redesign).
- ► We developed an integrated framework for the design of supply chain networks for critical products with outsourcing.
- ► The model utilizes cost minimization within a system-optimization perspective as the primary objective and captures rigorously the uncertainty associated with the demand for critical products at the various demand points.

- ➤ The supply chain network design model allows for the investment of enhanced link capacities and the investigation of whether the product should be outsourced or not.
- ➤ The framework can be applied in numerous situations in which the goal is to produce and deliver a critical product at minimal cost so as to satisfy the demand at various demand points, as closely as possible, given associated penalties for under- and over-supply.
- ▶ We noted the importance of sustainable supply chain network design but presented only highlights, due to time limitations.

- ► We expect that future research will include design for robustness and resiliency.
- ► Some recent research that we have begun in this direction: "Modeling of Supply Chain Risk Under Disruptions with Performance Measurement and Robustness Analysis," Q. Qiang, A. Nagurney, and J. Dong (2009), in *Managing Supply Chain Risk and Vulnerability: Tools and Methods for Supply Chain Decision Makers*, T. Wu and J. Blackhurst, s, Springer, London, England, 91-111.

#### THANK YOU!



For more information, see: http://supernet.som.umass.edu