Management of Knowledge Intensive Systems As Supernetworks: Modeling, Analysis, Computations, and Applications

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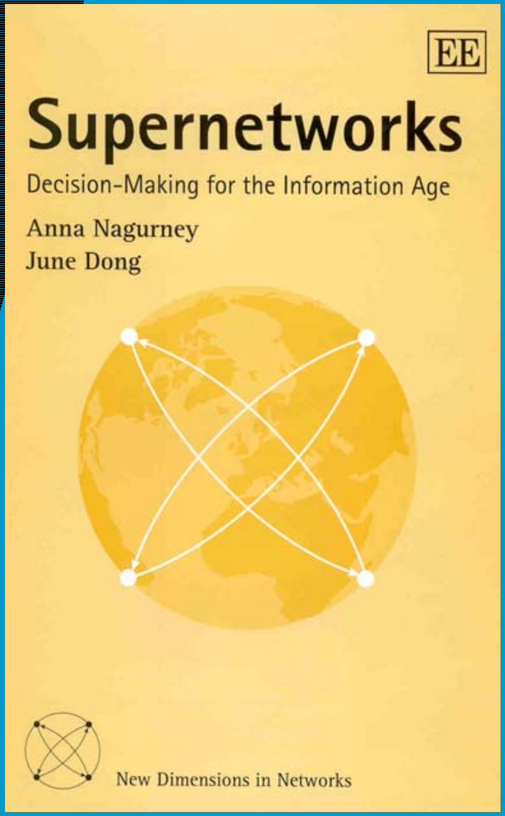
Supernetworks



- Supply chain networks with electronic commerce
- Financial networks with intermediation
- Telecommunicating versus commuting decision-making
- Teleshopping versus shopping decision-making,
- Transportation and location decisions
- Supply chain networks with environment issues

Supernetworks

- Provide tools to study interrelated networks
- Allow to apply efficient algorithms for computation
- Provide visual aids to see the dynamic changes

Introduction

- Knowledge intensive organizations
  - News organizations
  - Intelligence agencies
  - Global financial institutions

Introduction

- Challenges
  - How to respond in a timely manner to new events
  - How to best manage the scale and scope of their coverage and ultimate reach globally
  - How to best manage various production allocation processes efficiently
  - Respond to their customers' needs in an environment of increasing uncertainty and risk

Challenges to the Decision Makers

- Large scale
- Complex
- Multicriteria
- Time

Information management tools are needed

Knowledge Supernetworks

- To capture graphical format
- To provide alternatives
- To determine the optimal allocation of resources
- To schedule the activities

The Knowledge Supernetwork

- Network $G = [N, L]$
  - *N* notes
  - *L* directed links
    - Physical links
    - Abstract links
      - Associated with a factor of production or activity required for knowledge production
        - *Information,Transportation, Financial transaction, Communication link, Interface link*
    - A path corresponds to a production process
  - *W J* O/D pairs
    - *An O/D pair corresponds to the beginning and the end of knowledge production*

Conservation of Flow Equations

Consider *k* knowledge productions, a typical one as *i*

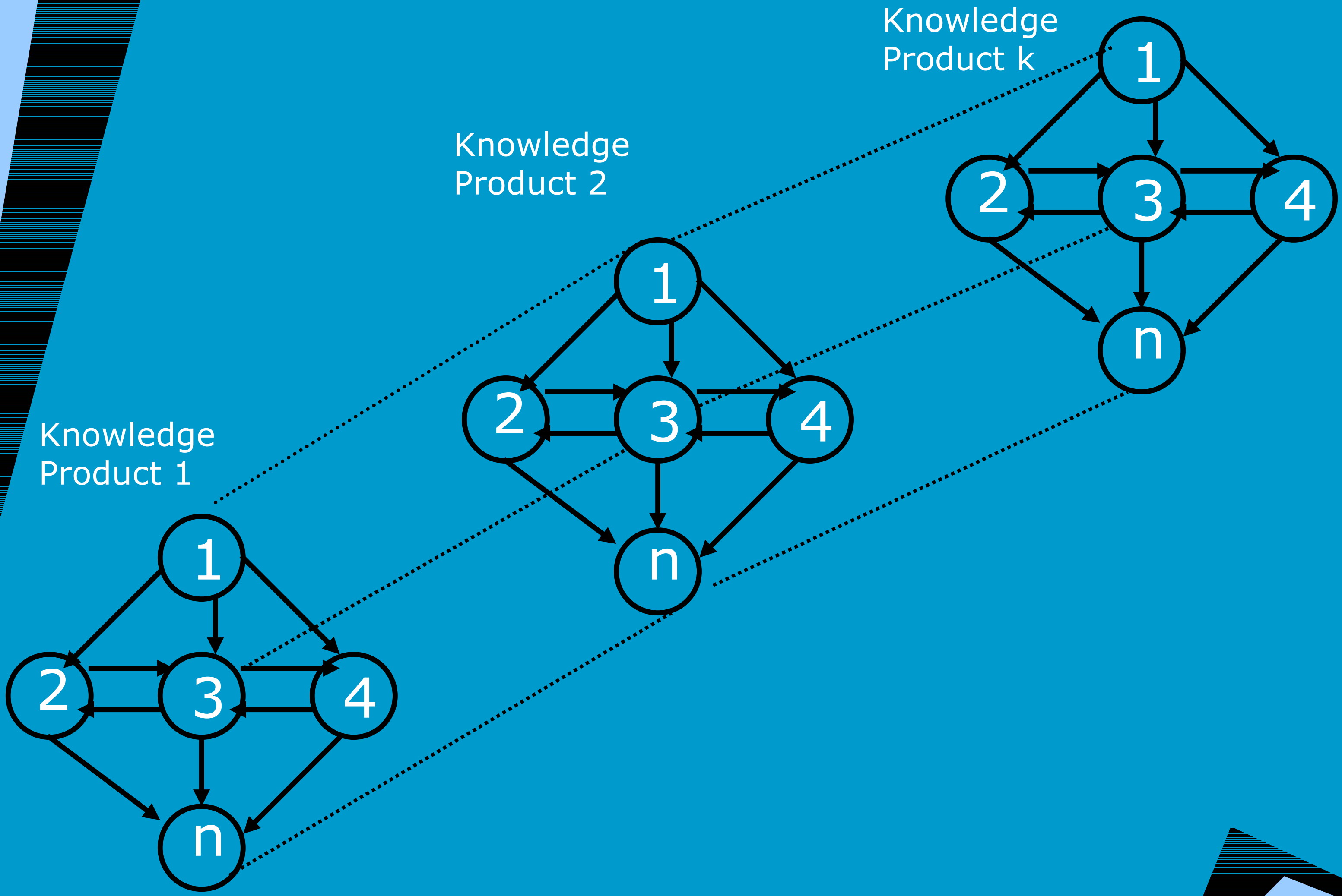
f_a^i : The flow of knowledge product *i* on link *a*

x_p^i : Path flow of knowledge product *i* on path *p*

$$f_a^i = \sum_{p \in P} x_p^i \delta_{ap}, \forall i, a$$

$$d_w^i = \sum_{p \in P} x_p^i, \forall i, w$$

The Knowledge Supernetwork



The Multicriteria Decision-Making Problem

Assume there are *H* criteria on each link *a* for each product *i*. A unit function associated with criterion *h*, link *a*, and product *i* is

$$c^i_{ha} = c^i_{ha}(f), \forall i, h.a,$$

The total function is

$$\hat{c}^i_{ha} = c^i_{ha}(f) \times f^i_a, \forall i, h, a.$$

The Multicriteria Decision-Making Problem

Assume there are H objective functions for the decision-maker:

$$\phi_h(f), h = 1, \ldots, H$$

where,

$$\phi_h(f) = \sum_{i,a} \hat{c}^i_{ha}(f)$$

Criteria Examples

Total (Production) Cost: $\sum_{i,a} \pi_a^i(f) \times f_a^i$

Total (Production) Time: $\sum_{i,a} \tau_a^i(f) \times f_a^i$

Total Risk: $\sum_{i,a} \rho_a^i(f) \times f_a^i$

The Multicriteria Decision-Making Problem

The decision-maker seeks to

$$Minimize \;\; Z(\phi_1(f), \phi_2(f), \ldots, \phi_H(f))$$

$$subject \;\; to:$$

$$f \in K.$$

The Multicriteria Decision-Making Problem

Assume the decision-maker weights the various objectives with weight w_h associated with objective function h.

Hence we have the following optimization problem

$$Minimize_{f \in K} \quad Z(f) = \sum_{h=1}^{H} w_h \phi_h(f)$$

A Special Multicriteria Decision-Making Problem

With three criteria, *cost*, *time*, and *risk*, the optimization problem can be expressed as:

$$Minimize_{f \in K}\ \mathrm{Z(f)}$$

$$= w_1 \sum_{i,a} \pi_a^i(f) \times f_a^i + w_2 \sum_{i,a} \tau_a^i(f) \times f_a^i + w_3 \sum_{i,a} \rho_a^i(f) \times f_a^i$$

Variational Inequality Formulation (Fixed Demand)

The optimal solution f^* to problem (1) is equivalent to the solution of the following variational inequality problem:

$$\langle \nabla Z(f^*), f - f^* \rangle \geq 0, \forall f \in K, \quad (VI1)$$

Where ∇Z is the gradient of the function Z.

The Knowledge Supernetwork Optimality Conditions

$$\frac{\partial Z(f^*)}{\partial x_p^i} \equiv \hat{C}_p^{'i}(f^*) \begin{cases} = \lambda_w^i, \text{ if } x_p^{i*} > 0 \\ \geq \lambda_w^i, \text{ if } x_p^{i*} = 0 \end{cases}$$

At optimality, all the utilized paths (i.e. those with positive flow) for a product connecting an O/D pair have equal and minimal *generalized marginal total costs*.

Solution Properties

- A solution f^* to (VI1) exists, given that the feasible set K is compact and the marginal total costs on the links are continuous for all the products.
- The solution to (VI1) is unique under the assumption that the ∇Z is strictly monotone.

Modeling Extensions

- Elastic Demand Model with Known Price Functions
- Elastic Demand Model with Known Demand Functions

Elastic Demand Model with Known Price Functions

Equilibrium Conditions in the Case of Known Price Functions

We can immediately extend optimality conditions (17), in the fixed demand case, as follows: for all products i; $i = 1, \ldots, k$; all O/D pairs $\omega \in W$, and all paths $p \in P_\omega$, a link flow and demand pattern (f^*, d^*) is said to be in equilibrium if the following holds:

$$\hat{C}'^{\,i}_p(f^*) \begin{cases} = \lambda^i_\omega(d^*), & \text{if} \quad x^{i*}_p > 0 \\ \geq \lambda^i_\omega(d^*), & \text{if} \quad x^{i*}_p = 0. \end{cases} \tag{20}$$

Theorem 2: Variational Inequality Formulation (Price Functions Known)

The link flow and demand pattern $(f^*, d^*) \in \mathcal{K}$ satisfying equilibrium conditions (20) also satisfies the variational inequality problem:

$$\sum_{i,a} \sum_{j=1}^{k} \sum_{h=1}^{H} \sum_{b \in \mathcal{L}} w_h \frac{\partial \hat{c}^j_{hb}(f)}{\partial f^i_{ha}} \times (f^i_a - f^{i*}_a) - \sum_{i,\omega} \lambda^i_\omega(d^*) \times (d^i_\omega - d^{i*}_\omega) \geq 0, \quad \forall (f, d) \in \mathcal{K}$$

Elastic Demand Model with Known Demand Functions

Equilibrium Conditions in the Case of Known Demand Functions

We now give the equilibrium conditions governing the knowledge supernetwork model in which the demand functions are known. In particular, we have that (cf. (17) and (20)): for all products i; $i = 1, \ldots, m$; all O/D pairs $\omega \in W$, and all paths $p \in P_\omega$, a pattern of link flows, prices, and demands is in equilibrium if it satisfies the conditions:

$$\hat{C}_p^{i'}(f^*) \begin{cases} = \lambda_\omega^{i*}, & \text{if} \quad x_p^{i*} > 0 \\ \geq \lambda_\omega^{i*}, & \text{if} \quad x_p^{i*} = 0 \end{cases} \tag{23}$$

and

$$d_\omega^i(\lambda^*) \begin{cases} = \sum_{p \in P_\omega} x_p^{i*}, & \text{if} \quad \lambda_\omega^{i*} > 0 \\ \leq \sum_{p \in P_\omega} x_p^{i*}, & \text{if} \quad \lambda_\omega^{i*} = 0. \end{cases} \tag{24}$$

Elastic Demand Model with Known Demand Functions

Theorem 3: Variational Inequality Formulation (Demand Functions Known)

Let $\mathcal{K}$ now denote the feasible set defined by $\mathcal{K} \equiv \{(f,d,\lambda)|\lambda \geq 0, \exists x \geq 0$ such that $(1), (2)$ hold$\}$. The vector $(f^*, d^*, \lambda^*) \in \mathcal{K}$ is an equilibrium according to (23) and (24) if and only if it satisfies the variational inequality problem:

$$\sum_{i,a}\sum_{j=1}^{k}\sum_{h=1}^{H}\sum_{b\in\mathcal{L}} w_h \frac{\partial \hat{c}^j_{hb}(f^*)}{\partial f^i_a} \times (f^i_a - f^{i*}_a) - \sum_{i,\omega} \lambda^{i*}_\omega \times (d^i_\omega - d^{i*}_\omega)$$

$$+\sum_{i,\omega}(d^{i*}_\omega - d^i_\omega(\lambda^*)) \times (\lambda^i_\omega - \lambda^{i*}_\omega) \geq 0, \quad \forall (f,d,\lambda) \in \mathcal{K}. \tag{25}$$

Computational Procedure

We use Euler method to solve the Variational Inequality problem, in standard form is as following:

$$\left\langle F(X^*), X - X^* \right\rangle \geq 0, \forall X \in \mathrm{K}$$

The Euler Method

Step 0: Initialization

Set $X^0 \in \mathcal{K}$ and set $T = 0$. T is an iteration counter which may also be interpreted as a time period.

Step 1: Computation

Compute X^{T+1} by solving the variational inequality problem:

$$X^{T+1} = P_{\mathcal{K}}(X^T - a_T F(X^T)),$$

where $\{a_T\}$ is a sequence of positive scalars satisfying: $\sum_{T=0}^{\infty} a_T = \infty$, $a_T \rightarrow 0$, as $T \rightarrow \infty$

and $P_{\mathcal{K}}$ is the projection of X on the set $\mathcal{K}$ defined as:

$$y = P_{\mathcal{K}} X = \arg\min_{z \in \mathcal{K}} \|X - z\|.$$

Step 2: Convergence Verification

If $\|X^{T+1} - X^T\| \leq \epsilon$, for some $\epsilon > 0$, a prespecified tolerance, then stop; else, set $T = T + 1$, and go to Step 1,

Why Euler Method?

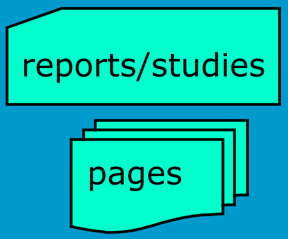
- In the context of the knowledge supernetwork models, the induced subproblems can be solved exactly and in closed form.
- The Euler method suggests a natural underlying dynamics to the problems.
- The proposed scheme further lays the foundations for the ultimate development of dynamic versions of the models.

Specific Applications
-- A News Organization (Eg. CNN)

- A knowledge organization
- Produces knowledge
- Disseminates knowledge
- Its product is in the form of processed information or knowledge
    - An O/D pair — News Programs
    - Products — News Segments
    - Demand for each product  minutes
    - Links  activities that are needed to produce the news segment

Specific Applications
-- An Intelligence Agency

- O/D pairs
- Demand
- Links
  - information processing
  - Transformation
  - Acquisition
  - synthesizing



Transportation and communication links may be involved

Specific Applications
-- A Global Financial Institute

- O/D pairs: clients
- Demands: financial products
- Links: transaction links (physical or electronic)

Numerical Examples

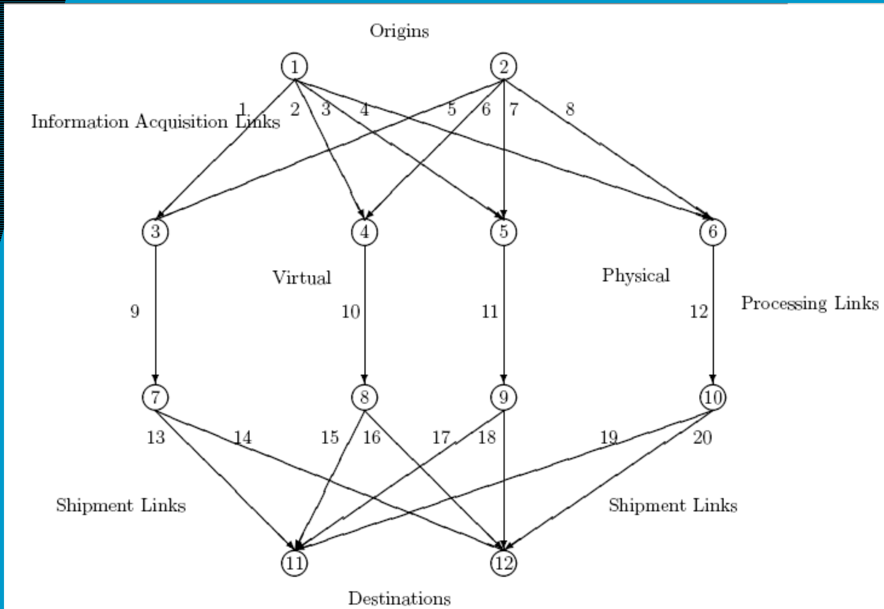


Figure 2: The Knowledge Supernetwork for the Numerical Examples

- Two O/D Pairs

  $w_1 = (1,11)$

  $w_2 = (2,12)$
- Demands

  $d_{w_1} = 80$

  $d_{w_2} = 160$
- Three Criteria

  Cost, Time, Risk

The marginal Cost Functions

Table 1: The Marginal Total Link Criterion Functions Representing: Cost, Time, and Risk for the Numerical Examples

Link a	$\sum_{b\in\mathcal{L}} \frac{\partial \hat{\delta}_{1b}(f)}{\partial f_a}$	$\sum_{b\in\mathcal{L}} \frac{\partial \hat{\delta}_{2b}(f)}{\partial f_a}$	$\sum_{b\in\mathcal{L}} \frac{\partial \hat{\delta}_{3b}(f)}{\partial f_a}$
1	$.00005f_1^4 + f_1 + f_2 + 2$	$.00005f_1^4 + 2f_1 + f_2 + 2$	$2f_1 + 4$
2	$.00003f_2^4 + f_2 + .5f_5 + 1$	$.00003f_2^4 + 2f_2 + f_1 + 1$	$3f_2 + 2$
3	$.00005f_3^4 + 4f_3 + f_4 + 1$	$.00005f_3^4 + 3f_3 + .5f_4 + 3$	$f_3 + 1$
4	$.00003f_4^4 + 6f_4 + 2f_5 + 4$	$.00003f_4^4 + 7f_4 + 3f_3 + 1$	$f_4 + 1$
5	$f_5 + 1$	$f_5 + 2$	$2f_5 + 5$
6	$.00007f_6^4 + f_6 + .5f_2 + 1$	$.00007f_6^4 + 2f_6 + f_5 + 1$	$3f_6 + 6$
7	$8f_7 + 7$	$4f_7 + 6$	$f_7 + 1$
8	$.00001f_8^4 + 7f_8 + 3f_5 + 6$	$.00001f_8^4 + 4f_8 + 2f_7 + 1$	$f_8 + 1$
9	$2f_9 + 1$	$2f_9 + 1$	$5f_9 + 12$
10	$.00003f_{10}^4 + 2f_{10} + f_9 + 1$	$.00003f_{10}^4 + 2f_{10} + f_9 + 1$	$11f_{10} + 11$
11	$.00004f_{11}^4 + 2f_{11} + f_{10} + 4$	$.00004f_{11}^4 + 4f_{11} + 2f_{12} + 2$	$f_{11} + 1$
12	$.00002f_{12}f^4 + 2f_{12} + f_{11} + 2$	$.00002f_{12}^4 + 4f_{12} + 2f_{11} + 1$	$f_{12} + 1$
13	$.00003f_{13}^4 + 9f_{13} + 3f_{14} + 3$	$.00003f_{13}^4 + 3f_{13} + f_{14} + 2$	$f_{13} + 11$
14	$5f_{14} + 3$	$4f_{14} + 2$	$6f_{14} + 21$
15	$6f_{15} + 4$	$4f_{15} + 1$	$7f_{15} + 14$
16	$10f_{16} + 10$	$2f_{16} + 10$	$5f_{16} + 10$
17	$5f_{17} + 10$	$5f_{17} + 10$	$f_{17} + 2$
18	$f_{18} + 20$	$6f_{18} + 20$	$2f_{18} + 1$
19	$6f_{19} + 20$	$5f_{19} + 10$	$f_{19} + 1$
20	$10f_{20} + 15$	$4f_{20} + 10$	$f_{20} + 1$

Equilibrium Link and Path Flows and the Generalized Path Marginal Costs

- Fixed demands and Criteria weighted equally

Link a	f_a^*
1	23.31
2	16.21
3	21.05
4	19.43
5	65.03
6	30.11
7	31.16
8	33.70
9	88.34
10	46.32
11	52.21
12	53.13
13	23.31
14	65.03
15	16.21
16	30.11
17	21.05
18	31.16
19	19.43
20	33.70

Path p	x_p^*
p_1	23.31
p_2	16.21
p_3	21.05
p_4	19.43
p_5	65.03
p_6	30.11
p_7	31.16
p_8	33.70

The incurred generalized path marginal total costs (cf. (18)) were:

O/D pair ω_1:

$$\hat{C}'_{p_1} = 1595.36, \quad \hat{C}'_{p_2} = 1595.41, \quad \hat{C}'_{p_3} = 1595.42, \quad \hat{C}'_{p_4} = 1595.42,$$

O/D pair ω_2:

$$\hat{C}'_{p_5} = 2078.50, \quad \hat{C}'_{p_6} = 2078.42, \quad \hat{C}'_{p_7} = 2078.42, \quad \hat{C}'_{p_8} = 2078.43.$$

What Do We Gain From the Model?

- Incorporate various related elements (networks) into the supernetwork structure
- View problems in a systematic way to see the whole picture
- Incorporate multicriteria into the decision-making process to capture the sometimes conflict issues
- The model provides us the optimal allocation of activities
- and the allocation of resources
- It also will allow us to see the dynamic of the changes in reaching the optimal solution

Future Directions

- Incorporate competition
  - Several knowledge organizations may share a subset of links (information resources, transformation processor, etc.)
- Introduce uncertainty into the framework
- Conduct empirical tests to validate the models