The Application of Supernetworks to Critical Infrastructure from Transportation to Electric Power

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Acknowledgments

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Outline

- What Are Supernetworks?
- Why Behavior Matters and Paradoxes
- Methodologies for Formulation, Analysis, and Computations
- An Empirical Application to Electric Power Supply Chains
- Which Nodes and Links Really Matter?
- Some Applications of the N-Q Measure
- From Transportation Networks to the Internet
- What About Transportation Network Robustness?
- Which Nodes and Links Matter Environmentally?
- What About Disaster Relief?
- Summary, Conclusions, and Suggestions for Future Research

What Are Supernetworks?

We Are in a New Era of Decision-Making Characterized by:

- *complex interactions* among decision-makers in organizations;
- *alternative and, at times, conflicting criteria* used in decision-making;
- *constraints on resources*: human, financial, natural, time, etc.;
- *global reach* of many decisions;
- *high impact* of many decisions;
- *increasing risk and uncertainty*;
- the *importance of dynamics* and realizing a timely response to evolving events.

The Era of Supernetworks

Supernetworks are *Networks of Networks*, and their prevalence in the world around us is illustrated by:

- *multimodal transportation networks*;
- *complex supply chain networks consisting of manufacturers, shippers and carriers, distributors, and retailers*;
- *electric power generation and distribution networks*,
- *multitiered financial networks*, and
- *social network platforms such as Facebook and Twitter*, along with the Internet itself.

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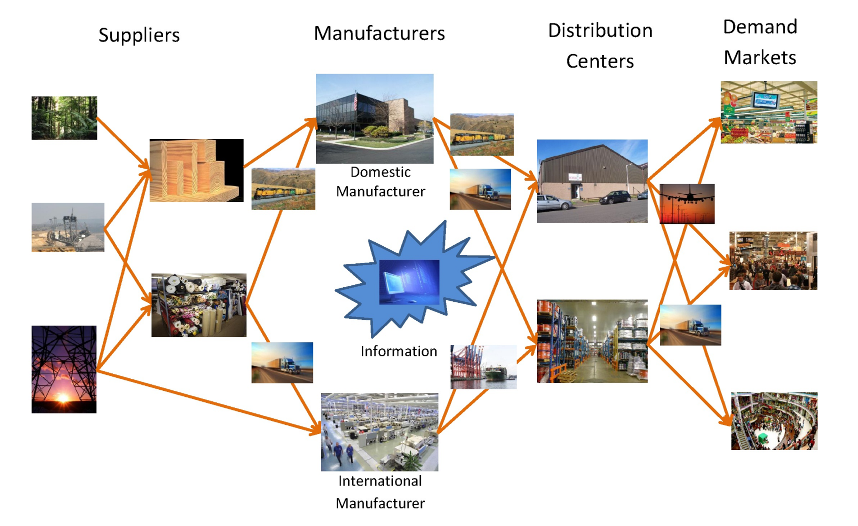
Multimodal Transportation

Complex Logistical Networks

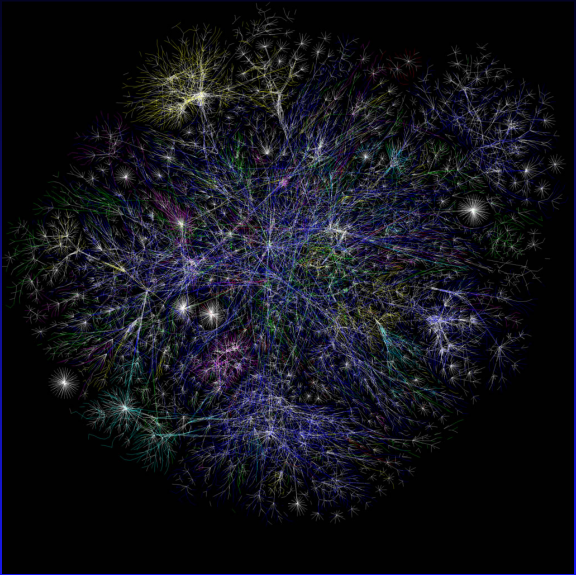


Electric Power Generation and Distribution Networks

Financial Networks

Social Networks

Visual Image of the Internet (opte.org)



Characteristics of Networks Today

- *large-scale nature* and complexity of network topology;
- *congestion*, which leads to nonlinearities;
- *alternative behavior of users of the networks*, which may lead to paradoxical phenomena;
- *possibly conflicting criteria associated with optimization*;
- *interactions among the underlying networks themselves*, such as the Internet with electric power, financial, and transportation and logistical networks;
- recognition of *their fragility and vulnerability*;
- policies surrounding networks today may have major impacts not only economically, but also *socially, politically, and security-wise*.

Many of today's networks are characterized by both a large-scale nature and complexity of the underlying network topology, so we are in an era of Supernetworks.

In Chicago's Regional Transportation Network, there are *12,982 nodes, 39,018 links, and 2,297,945 origin/destination (O/D) pairs*, whereas in the Southern California Association of Governments model there are *3,217 origins and/or destinations, 25,428 nodes, and 99,240 links, plus 6 distinct classes of users*.

In the case of the Internet, there are now over *3 billion users*, 40% of the world's population.

The Most Congested US Metropolitan Areas in 2013

According to INRIX Inc. which publishes a National Traffic Scorecard, the 2013 Top Congested Metropolitan Areas Were:

Rank	City	Hours Wasted in Traffic	Percent Change in Congestion
1	Los Angeles	64	+8.5%
2	Honolulu	60	+18%
3	San Francisco	56	+13%
4	Austin, TX	41	+9%
5	New York	53	+5%
6	Bridgeport, CT	42	+9%
7	San Jose, CA	35	+10%
8	Seattle	37	+7%
9	Boston	38	+22%
10	Washington, D.C.	40	-1%

The Costs of Congestion

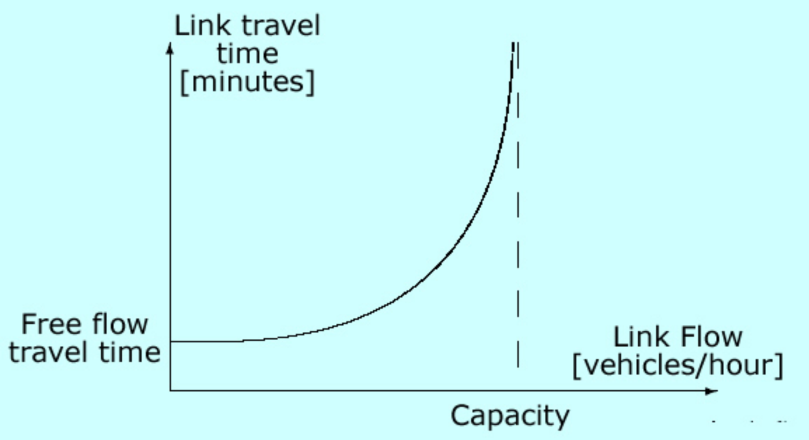
According to the 2015 Urban Mobility report, prepared by the Texas A&M Transportation Institute and INRIX Inc., Americans spend 6.9 billion hours battling traffic and burn 3.1 billion gallons of fuel while nudging inch by inch down the roadway.

The total nationwide price tag: $160 billion or $960 per commuter.

There is also a **freight capacity crisis in parts of the US especially in the Northeast**.

The United States economy depends on trucks to deliver nearly 70 percent of all freight transported annually in the U.S., accounting for $671 billion worth of manufactured and retail goods transported by truck in the U.S. alone.

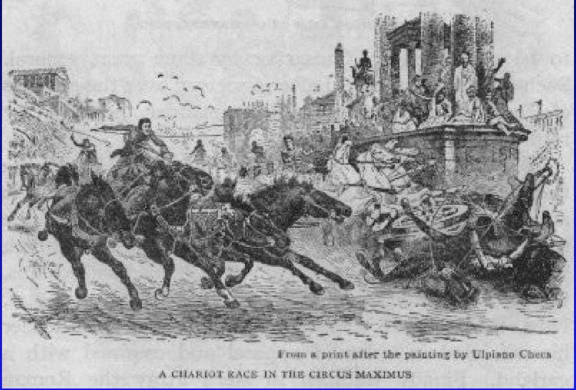
Capturing Link Congestion

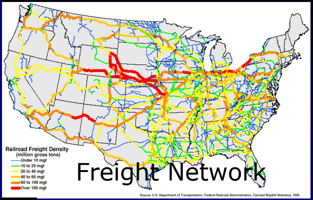


In a typical user link travel time (or cost) function, the free flow travel time referes to the travel time to traverse the link when there is zero flow or traffic on the link (zero vehicles).

Congestion is Not a New Phenomenon

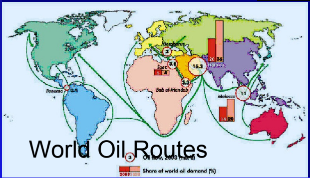
The study of the efficient operation of transportation networks dates to ancient Rome with a classical example being the publicly provided Roman road network and the time of day chariot policy, whereby chariots were banned from the ancient city of Rome at particular times of day.

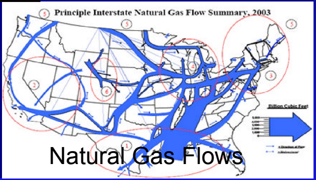




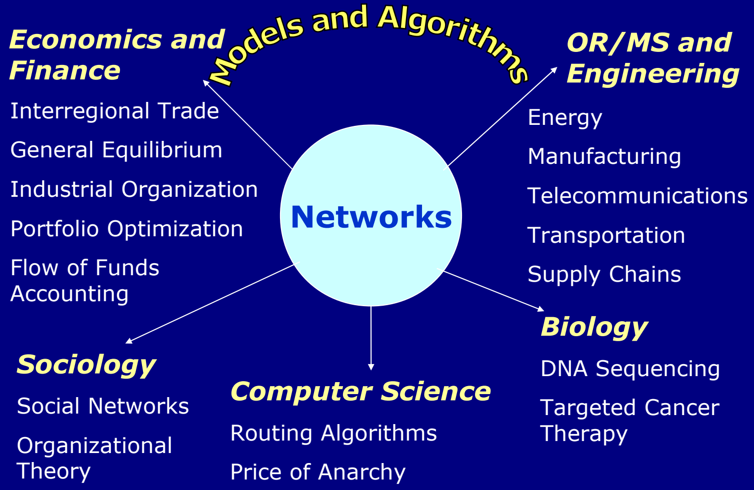
Network

Systems

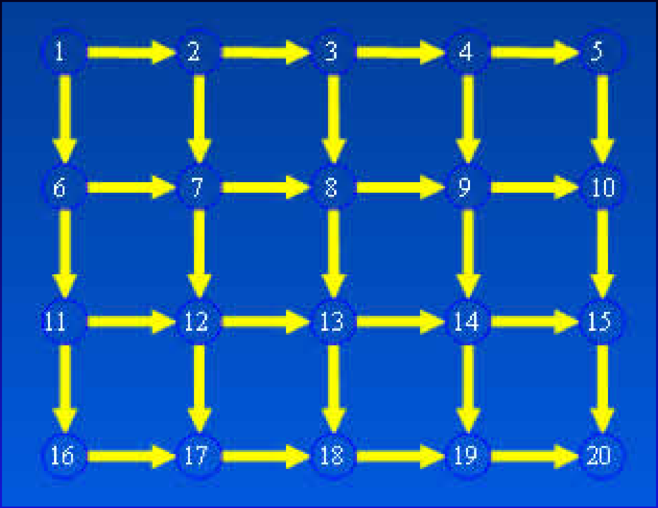




Interdisciplinary Impact of Networks



Network Components



The components of networks as a theoretical (modeling, analysis, and solution) construct include: nodes, links, and flows.

Components of Common Physical Networks

Network System	Nodes	Links	Flows
Transportation	Intersections, Homes, Workplaces, Airports, Railyards	Roads, Airline Routes, Railroad Track	Automobiles, Trains, and Planes,
Manufacturing and logistics	Workstations, Distribution Points	Processing, Shipment	Components, Finished Goods
Communication	Computers, Satellites, Telephone Exchanges	Fiber Optic Cables Radio Links	Voice, Data, Video
Energy	Pumping Stations, Plants	Pipelines, Transmission Lines	Water, Gas, Oil, Electricity

Network Components

It is important to realize that there may be systems that, at first glance, do not seem to be networks, but, after further thought and creativity, one may be able to represent the system as a network!

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The advantages of doing so are many:

• One can then see similarities and differences in structure across different problem domains.

• One can avail oneself of powerful network-based analytical tools.

• One can represent what may be an extremely complex problem graphically through a network, which can suggest further insights and extensions.

Network Problems and Methodologies

Networks consist of nodes, links, flows, and *behavior* associated with their operation, usage, and management.

Network problems are a distinct class of problems and they come in various forms and formulations, i.e., as optimization (linear or nonlinear) problems or as equilibrium problems and even dynamic network problems.

Methodologies that we have been using and extending to formulate, analyze, and compute solutions to large-scale network problems have included: network optimization, network theory, game theory, multicriteria decision-making, risk management, variational inequality theory, and projected dynamical systems theory.

Why Behavior Matters and Paradoxes

Network Models from Analysis to Design Must Capture the Behavior of Users

Behavior on Congested Networks

Decision-makers select their cost-minimizing routes.

				User-Optimized or Equilibrium
Decentralized		Selfish		U–O
vs.	⟺	vs.	⟺	vs.
Centralized		Unselfish		S–O
				System-Optimized

Flows are routed so as to minimize the total cost to society.

Two fundamental principles of flow (travel) behavior, due to Wardrop (1952), with terms coined by Dafermos and Sparrow (1969).

User-optimized (U-O) (network equilibrium) Problem – each user determines his/her cost minimizing route of travel between an origin/destination, until an equilibrium is reached, in which no user can decrease his/her cost of travel by unilateral action (in the sense of Nash).

System-optimized (S-O) Problem – users are allocated among the routes so as to minimize the total cost in the system, where the total cost is equal to the sum over all the links of the link's user cost times its flow.

The U-O problems, under certain simplifying assumptions, possesses optimization reformulations. But now we can handle cost asymmetries, multiple modes of transport, and different classes of travelers, without such assumptions.

We Can State These Conditions Mathematically!

The U-O and S-O Conditions

Definition: U-O or Network Equilibrium – Fixed Demands

A path flow pattern x^*, with nonnegative path flows and O/D pair demand satisfaction, is said to be U-O or in equilibrium, if the following condition holds for each O/D pair $w \in W$ and each path $p \in P_w$:

$$C_p(x^*) \begin{cases} = \lambda_w, & \textit{if} \quad x_p^* > 0, \\ \geq \lambda_w, & \textit{if} \quad x_p^* = 0. \end{cases}$$

Definition: S-O Conditions

A path flow pattern x with nonnegative path flows and O/D pair demand satisfaction, is said to be S-O, if for each O/D pair $w \in W$ and each path $p \in P_w$:

$$\hat{C}'_p(x) \begin{cases} = \mu_w, & \text{if} \quad x_p > 0, \\ \geq \mu_w, & \text{if} \quad x_p = 0, \end{cases}$$

where $\hat{C}'_p(x) = \sum_{a \in \mathcal{L}} \frac{\partial \hat{c}_a(f_a)}{\partial f_a} \delta_{ap}$, and μ_w is a Lagrange multiplier.

The Braess Paradox Illustrates

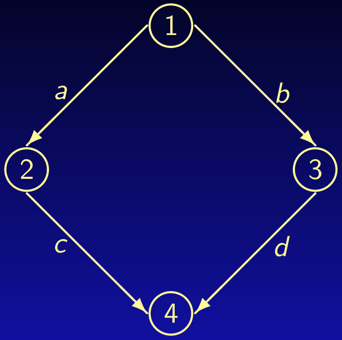
Why Behavior on Networks is Important

The Braess (1968) Paradox

Assume a network with a single O/D pair (1,4). There are 2 paths available to travelers: $p_1 = (a, c)$ and $p_2 = (b, d)$.

For a travel demand of **6,** the equilibrium path flows are $x^*_{p_1} = x^*_{p_2} = 3$ and

The equilibrium path travel cost is $C_{p_1} = C_{p_2} = 83.$



$$c_a(f_a) = 10f_a, \quad c_b(f_b) = f_b + 50,$$

$$c_c(f_c) = f_c + 50, \quad c_d(f_d) = 10f_d.$$

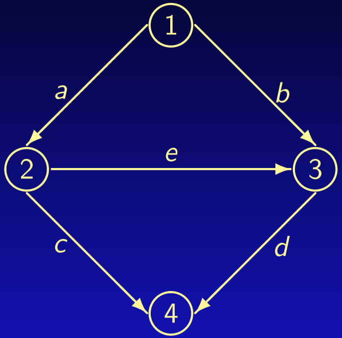
Adding a Link Increases Travel Cost for All!

Adding a new link creates a new path $p_3 = (a, e, d)$.

The original flow distribution pattern is no longer an equilibrium pattern, since at this level of flow the cost on path p_3, $C_{p_3} = 70$.

The new equilibrium flow pattern network is
$x^*_{p_1} = x^*_{p_2} = x^*_{p_3} = 2$.

The equilibrium path travel cost:
$C_{p_1} = C_{p_2} = C_{p_3} = 92$.



$c_e(f_e) = f_e + 10$

Under S-O behavior, the total cost in the network is minimized, and the new route p_3, under the same demand of 6, would not be used.

The Braess paradox never occurs in S-O networks.

The 1968 Braess article has been translated from German to English and appears as:

"On a Paradox of Traffic Planning,"

D. Braess, A. Nagurney, and T. Wakolbinger (2005) *Transportation Science* **39**, 446-450.

Über ein Paradoxon aus der Verkehrsplanung

Von D. Braess, Münster [1])

Eingegangen am 28. März 1968

Zusammenfassung: Für die Straßenverkehrsplanung möchte man den Verkehrsfluß auf den einzelnen Straßen des Netzes abschätzen, wenn die Zahl der Fahrzeuge bekannt ist, die zwischen den einzelnen Punkten des Straßennetzes verkehren. Welche Wege am günstigsten sind, hängt nun nicht nur von der Beschaffenheit der Straße ab, sondern auch von der Verkehrsdichte. Es ergeben sich nicht immer optimale Fahrzeiten, wenn jeder Fahrer nur für sich den günstigsten Weg heraussucht. In einigen Fällen kann sich durch Erweiterung des Netzes der Verkehrsfluß sogar so umlagern, daß größere Fahrzeiten erforderlich werden.

Summary: For each point of a road network let be given the number of cars starting from it, and the destination of the cars. Under these conditions one wishes to estimate the distribution of the traffic flow. Whether a street is preferable to another one depends not only upon the quality of the road but also upon the density of the flow. If every driver takes that path which looks most favorable to him, the resultant running times need not be minimal. Furthermore it is indicated by an example that an extension of the road network may cause a redistribution of the traffic which results in longer individual running times.

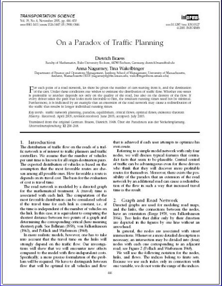
1. Einleitung

Für die Verkehrsplanung und Verkehrssteuerung interessiert, wie sich der Fahrzeugstrom auf die einzelnen Straßen des Verkehrsnetzes verteilt. Bekannt sei dabei die Anzahl der Fahrzeuge für alle Ausgangs- und Zielpunkte. Bei der Berechnung wird davon ausgegangen, daß von den möglichen Wegen jeweils der günstigste gewählt wird. Wie günstig ein Weg ist, richtet sich nach dem Aufwand, der zum Durchfahren nötig ist. Die Grundlage für die Bewertung des Aufwandes bildet die Fahrzeit.

Für die mathematische Behandlung wird das Straßennetz durch einen gerichteten Graphen beschrieben. Zur Charakterisierung der Bögen gehört die Angabe des Zeitaufwandes. Die Bestimmung der günstigen Stromverteilungen kann als gelöst betrachtet werden, wenn die Bewertung konstant ist, d. h., wenn die Fahrzeiten unabhängig von der Größe des Verkehrsflusses sind. Sie ist dann äquivalent mit der bekannten Aufgabe, den kürzesten Abstand zweier Punkte eines Graphen und den zugehörigen kritischen Pfad zu bestimmen [1], [5], [7].

Will man das Modell aber realistischer gestalten, ist zu berücksichtigen, daß die benötigte Zeit stark von der Stärke des Verkehrs abhängt. Wie die folgenden Untersuchungen zeigen, ergeben sich dann gegenüber dem Modell mit konstanter (belastungsunabhängiger) Bewertung z. T. völlig neue Aspekte. Dabei erweist sich schon eine Präzisierung der Problemstellung als notwendig; denn es ist zwischen dem Strom zu unterscheiden, der für alle am günstigsten ist,, und dem, der sich einstellt, wenn jeder Fahrer nur seinen eigenen Weg optimalisiert.

[1]) Priv.-Doz. Dr. Dietrich Braess, Institut für numerische und instrumentelle Mathematik, 44 Münster, Hüfferstr. 3 a.



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On a Paradox of Traffic Planning

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For each point of a road network, let there be given the number of cars starting from it, and the destination of the cars. Under these conditions one wishes to estimate the distribution of traffic flow. Whether one street is preferable to another depends not only on the quality of the road, but also on the density of the flow. If every driver takes the path that looks most favorable to him, the resultant running times need not be minimal. Furthermore, it is indicated by an example that an extension of the road network may cause a redistribution of the traffic that results in longer individual running times.

Key words: traffic network planning; paradox; equilibrium; critical flows; optimal flows; existence theorem
History: Received: April 2005; revision received: June 2005; accepted: July 2005.

Translated from the original German: Braess, Dietrich. 1968. Über ein Paradoxon aus der Verkehrsplanung. *Unternehmensforschung* **12** 258–268.

1. Introduction

The distribution of traffic flow on the roads of a traffic network is of interest to traffic planners and traffic controllers. We assume that the number of vehicles per unit time is known for all origin-destination pairs. The expected distribution of vehicles is based on the assumption that the most favorable routes are chosen among all possible ones. How favorable a route is depends on its travel cost. The basis for the evaluation of cost is travel time.

The road network is modeled by a directed graph for the mathematical treatment. A (travel) time is associated with each link. The computation of the most favorable distribution can be considered solved if the travel time for each link is constant, i.e., if the time is independent of the number of vehicles on the link. In this case, it is equivalent to computing the shortest distance between two points of a graph and determining the corresponding critical (here meaning shortest) path. See Bellman (1958), von Falkenhausen (1963), and Pollack and Wiebenson (1960).

In more realistic models, however, one has to take into account that the travel time on the links will strongly depend on the traffic flow. Our investigations will show that we will encounter new effects compared to the model with flow-independent costs. Specifically, a more precise formulation of the problem will be required. We have to distinguish between flow that will be optimal for all vehicles and flow that is achieved if each user attempts to optimize his own route.

Referring to a simple model network with only four nodes, we will discuss typical features that contradict facts that seem to be plausible. Central control of traffic can be advantageous even for those drivers who think that they will discover more profitable routes for themselves. Moreover, there exists the possibility of the paradox that an extension of the road network by an additional road can cause a redistribution of the flow in such a way that increased travel time is the result.

2. Graph and Road Network

Directed graphs are used for modeling road maps, and the links, the connections between the nodes, have an orientation (Berge 1958, von Falkenhausen 1966). Two links that differ only by their direction are depicted in the figures by one line without an arrowhead.

In general, the nodes are associated with street intersections. Whenever a more detailed description is necessary, an intersection may be divided into (four) nodes with each one corresponding to an adjacent road; see Figure 2 (Pollack and Wiebenson 1960).

We will use the following notation for the nodes, links, and flows. The indices belong to finite sets. Because we use each index only in connection with one variable, we do not write the range of the indices.

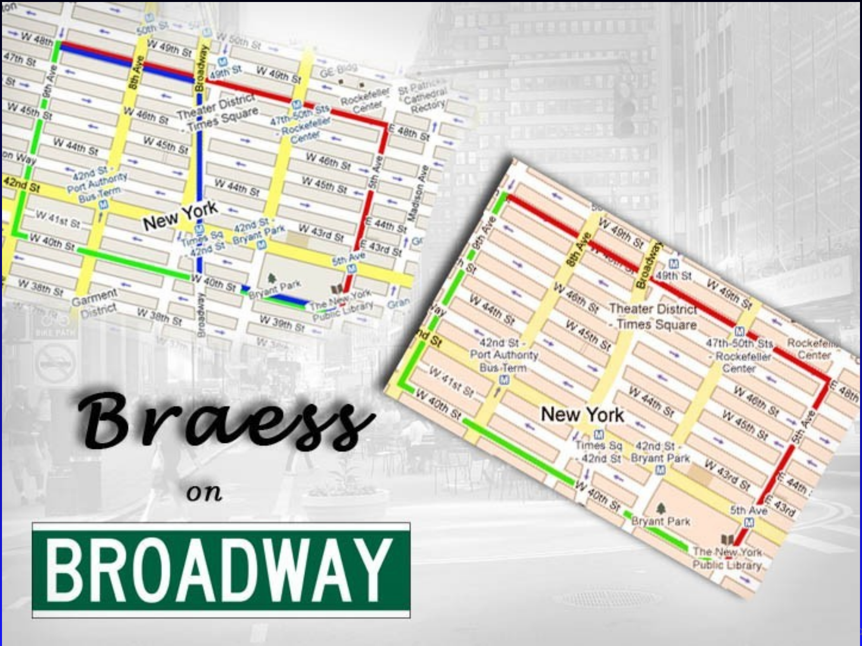
446

The Braess Paradox Around the World

1969 - Stuttgart, Germany - The traffic worsened until a newly built road was closed.

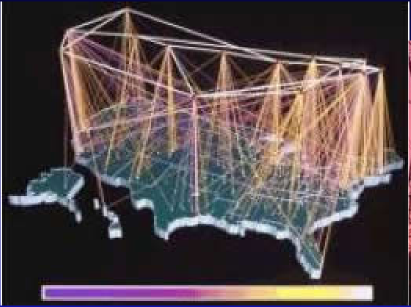
1990 - Earth Day - New York City - 42^{nd} Street was closed and traffic flow improved.

2002 - Seoul, Korea - A 6 lane road built over the Cheonggyecheon River that carried 160,000 cars per day and was perpetually jammed was torn down to improve traffic flow.


Braess
on
BROADWAY

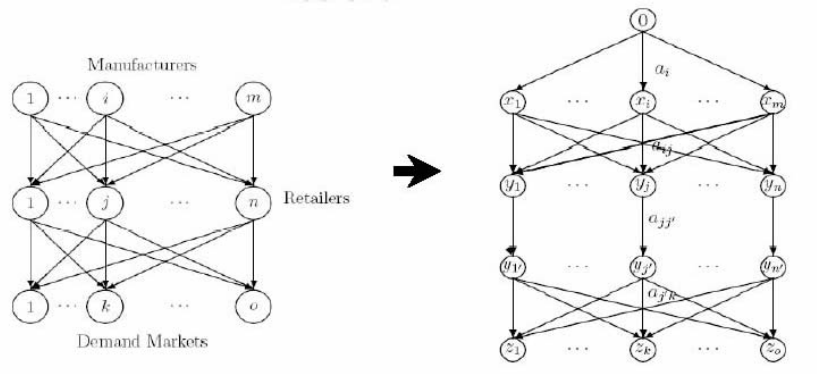
Interview on Broadway for *America Revealed* on March 15, 2011

Other Networks that Behave like Traffic Networks



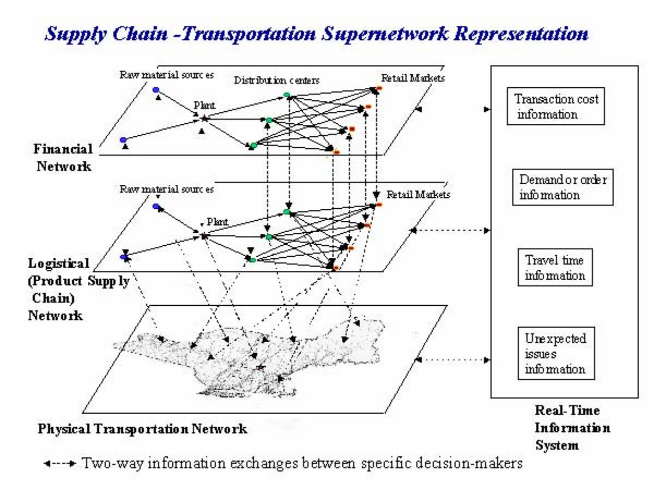
The Internet, electric power networks, and even multitiered financial networks!

Representation of Supply Chains as Transportation Networks



The equivalence between supply chains and transportation networks established in Nagurney, *Transportation Research E* **42** (2006), 293-316.

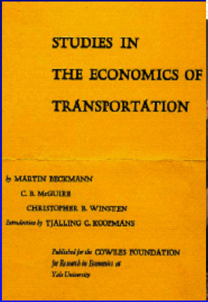
Representation of Supply Chains as Networks



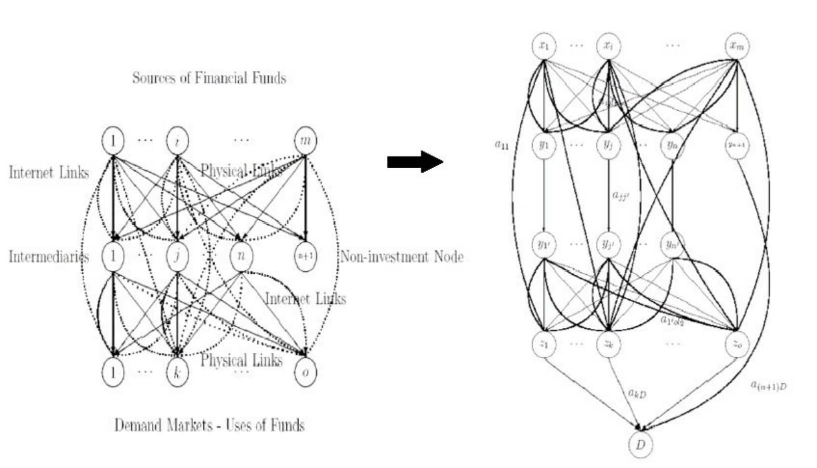
Multilevel supply chain established by Nagurney, Ke, Cruz, Hancock, and Southworth in *Environment & Planning B* **29** (2002), 795-818.

In 1952, Copeland in his book, *A Study of Moneyflows in the United States*, NBER, NY, asked whether money flows lie water or electricity?

In 1956, Beckmann, McGuire, and Winsten in their classic book, *Studies in the Economics of Transportation*, Yale University Press, hypothesized that electric power generation and distribution networks could be transformed into transportation network equilibrium problems.

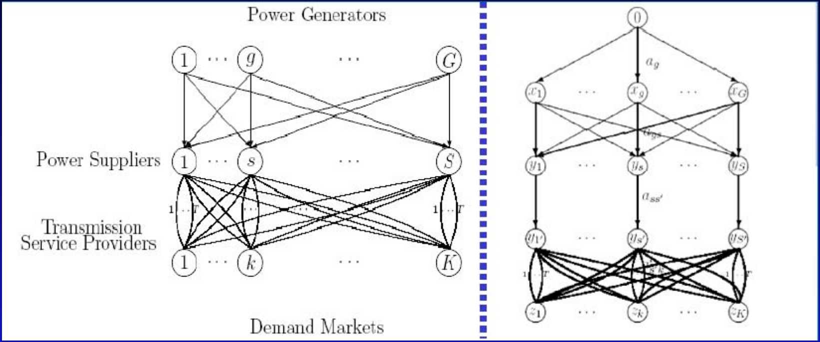


Transportation Network Equilibrium Reformulation of the Financial Network Equilibrium Model with Intermediation



Liu and Nagurney, *Computational Management Science* (2007).

Representation of Electric Power Networks as Transportation Networks



The transportation network equilibrium reformulation of electric power supply chain networks by Nagurney, Liu, Cojocaru, and Daniele, *Transportation Research E* **43** (2007), 624-646.

Hence, we have shown that both electricity as well as money flow like transportation flows.

Methodologies for Formulation, Analysis, and Computations

The Variational Inequality Problem

We utilize the theory of variational inequalities for the formulation, analysis, and solution of both centralized and decentralized network problems.

Definition: The Variational Inequality Problem
The finite-dimensional variational inequality problem, $\mathrm{VI}(F, \mathcal{K})$*, is to determine a vector* $X^* \in \mathcal{K}$*, such that:*

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K},$$

where F *is a given continuous function from* $\mathcal{K}$ *to* R^N*,* $\mathcal{K}$ *is a given closed convex set, and* $\langle \cdot, \cdot \rangle$ *denotes the inner product in* R^N*.*

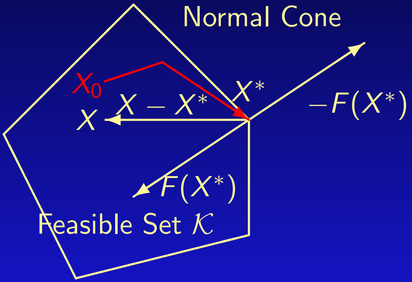
The variational inequality problem contains, as special cases, such mathematical programming problems as:

- systems of equations,
- optimization problems,
- complementarity problems,
- and is related to the fixed point problem.

Hence, it is a natural methodology for a spectrum of congested network problems from centralized to decentralized ones as well as to design problems.

Geometric Interpretation of VI($F, \mathcal{K}$) and a Projected Dynamical System (Dupuis and Nagurney, Nagurney and Zhang)

In particular, $F(X^*)$ is "orthogonal" to the feasible set $\mathcal{K}$ at the point X^*.



Associated with a VI is a Projected Dynamical System, which provides a natural underlying dynamics associated with travel (and other) behavior to the equilibrium.

To model the *dynamic behavior of supernetworks*, including transportation networks and supply chains, we utilize *projected dynamical systems* (PDSs) advanced by Dupuis and Nagurney (1993) in *Annals of Operations Research* and by Nagurney and Zhang (1996) in our book *Projected Dynamical Systems and Variational Inequalities with Applications*.

Such nonclassical dynamical systems are now being used in

evolutionary games (Sandholm (2005, 2011)),

ecological predator-prey networks (Nagurney and Nagurney (2011a, b)), and

even *neuroscience* (Girard et al. (2008)).

Some of My Books

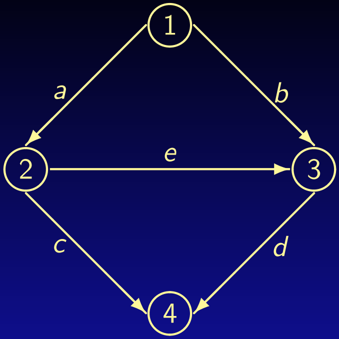
Question: When does the U-O solution coincide with the S-O solution?

Answer: In a general network, when the user link cost functions are given by:

$$c_a(f_a) = c_a^0 f_a^{\beta},$$

for all links, with $c_a^0 \geq 0$, and $\beta \geq 0$.

In particular, if $c_a(f_a) = c_a^0$, that is, in the case of *uncongested networks*, this result always holds.



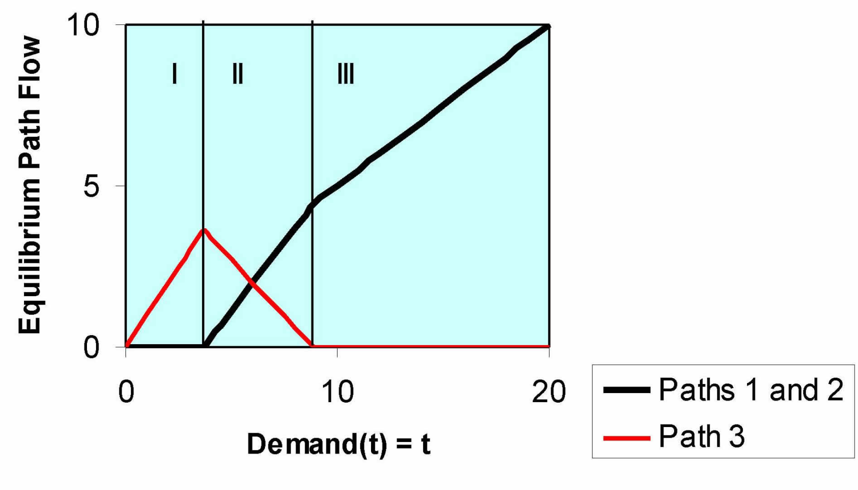
Recall the Braess network with the added link e.

What happens as the demand increases?

For Networks with Time-Dependent Demands

We Use Evolutionary Variational Inequalities

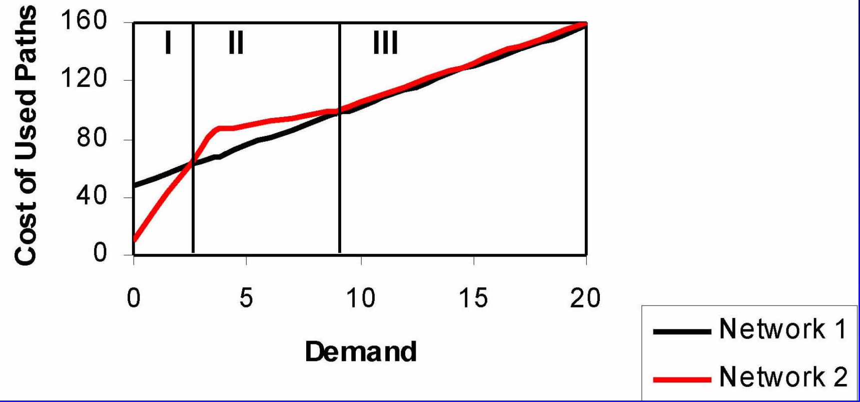
The U-O Solution of the Braess Network with Added Link (Path) and Time-Varying Demands Solved as an *Evolutionary Variational Inequality* (Nagurney, Daniele, and Parkes (2007)).



In Demand Regime I, Only the New Path is Used.

In Demand Regime II, the travel demand lies in the range [2.58, 8.89], and *the Addition of a New Link (Path) Makes Everyone Worse Off*!

In Demand Regime III, when the travel demand exceeds 8.89, *Only the Original Paths are Used!*



The new path is never used, under U-O behavior, when the demand exceeds 8.89, even out to infinity!

An Empirical Application to Electric Power Supply Chains

Electric Power Supply Chains

We developed **an empirical, large-scale electric supply chain network equilibrium model**, formulated it as a VI problem, and were able to solve it by **exploiting the connection between electric power supply chain networks and transportation networks** using our proof of a hypothesis posed in the classic book, *Studies in the Economics of Transportation*, by Beckmann, McGuire, and Winsten (1956).

The paper, "An Integrated Electric Power Supply Chain and Fuel Market Network Framework: Theoretical Modeling with Empirical Analysis for New England," by Z. Liu and A. Nagurney was published in *Naval Research Logistics* **56** (2009) pp 600-624;

http://supernet.isenberg.umass.edu/articles/LiuNagurneyNRL.pdf

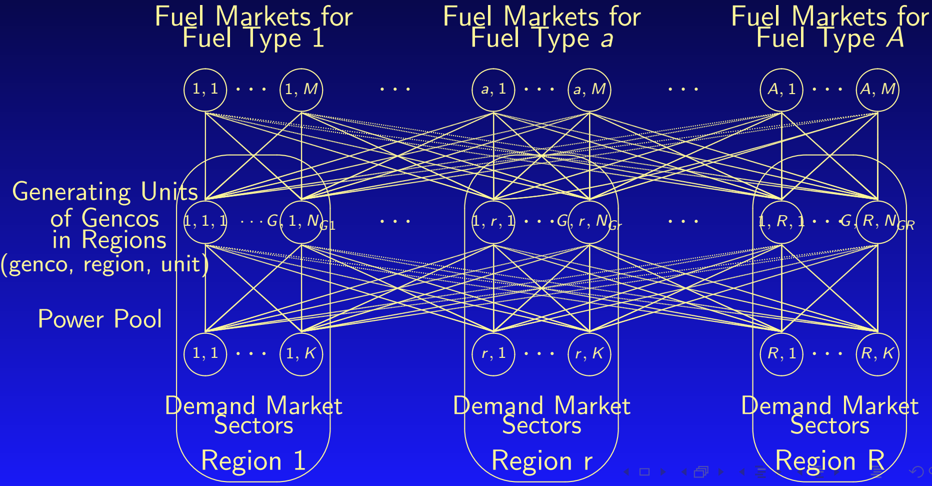
An Empirical Example of an Electric Power Supply Chain for New England

There are 82 generating companies who own and operate 573 generating units. We considered 5 types of fuels: natural gas, residual fuel oil, distillate fuel oil, jet fuel, and coal. The whole area was divided into 10 regions:

1. Maine,
2. New Hampshire,
3. Vermont,
4. Connecticut (excluding Southwest Connecticut),
5. Southwestern Connecticut (excluding the Norwalk-Stamford area),
6. Norwalk-Stamford area,
7. Rhode Island,
8. Southeastern Massachusetts,
9. Western and Central Massachusetts,
10. Boston/Northeast Massachusetts.

Graphic of New England

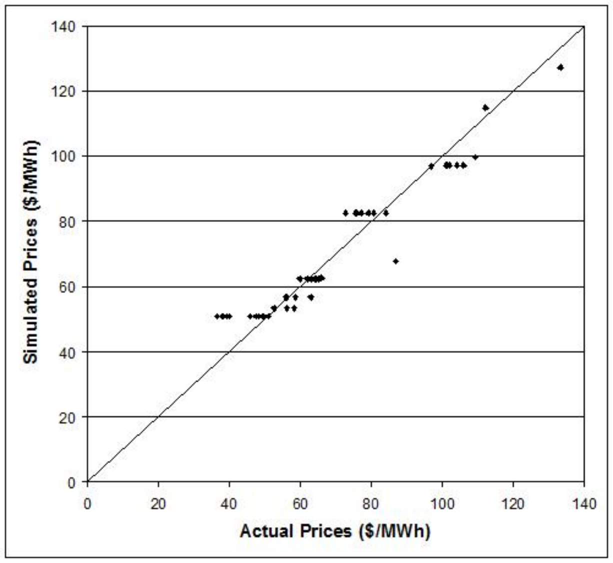
The Electric Power Supply Chain Network with Fuel Supply Markets



We tested the model on the data of July 2006 which included $24 \times 31 = 744$ hourly demand/price scenarios. We sorted the scenarios based on the total hourly demand, and constructed the load duration curve. We divided the duration curve into 6 blocks ($L_1 = 94$ hours, and $L_w = 130$ hours; $w = 2, ..., 6$) and calculated the average regional demands and the average weighted regional prices for each block.

The empirical model had on the order of $20,000$ *variables.*

Actual Prices Vs. Simulated Prices ($/Mwh)



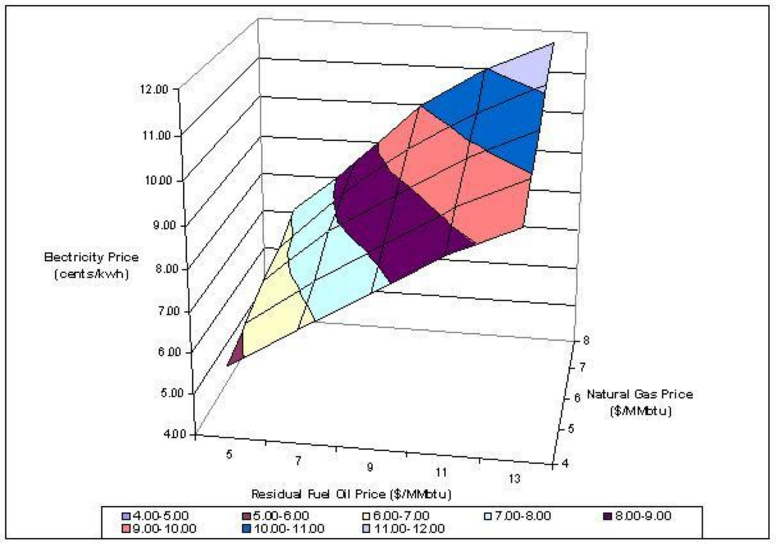
Sensitivity Analysis

We used the same demand data, and then varied the prices of natural gas and residual fuel oil. We assumed that the percentage change of distillate fuel oil and jet fuel prices were the same as that of the residual fuel oil price.

The next figure presents the average electricity price for the two peak blocks under oil/gas price variations.

The surface in the figure represents the average peak electricity prices under different natural gas and oil price combinations.

Sensitivity Analysis



Supernetworks with Social Networks In addition, supernetworks arise through the evolution and integration of disparate network systems, including social networks.

Two References:

A. Nagurney, T. Wakolbinger, and L. Zhao (2006) "The Evolution and Emergence of Integrated Social and Financial Networks with Electronic Transactions: A Dynamic Supernetwork Theory for the Modeling, Analysis, and Computation of Financial Flows and Relationship Levels," *Computational Economics* **27**, 353-393.

J. M. Cruz, A. Nagurney, and T. Wakolbinger (2006) "Financial Engineering of the Integration of Global Supply Chain Networks and Social Networks with Risk Management," *Naval Research Logistics* **53**, 674-696.

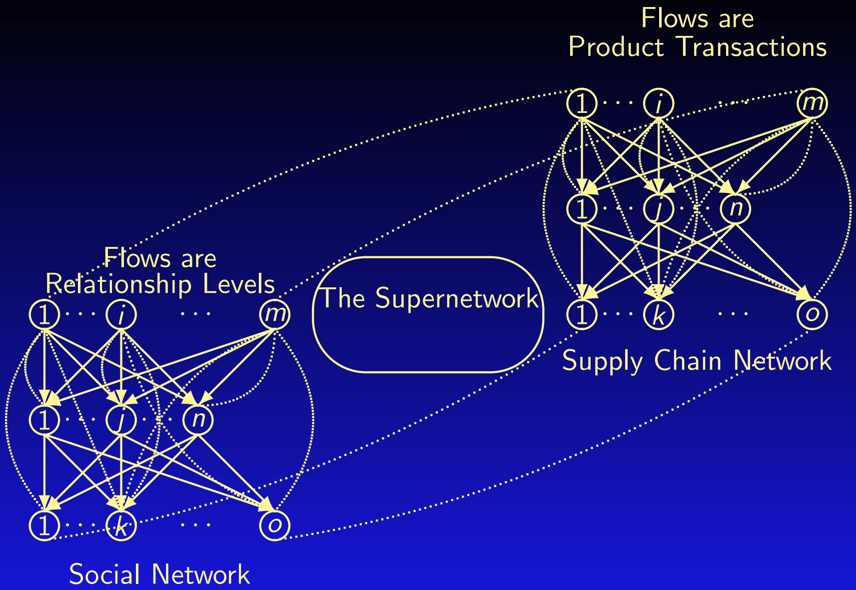


Figure 1: The Multilevel Supernetwork Structure of the Integrated Supply Chain / Social Network System

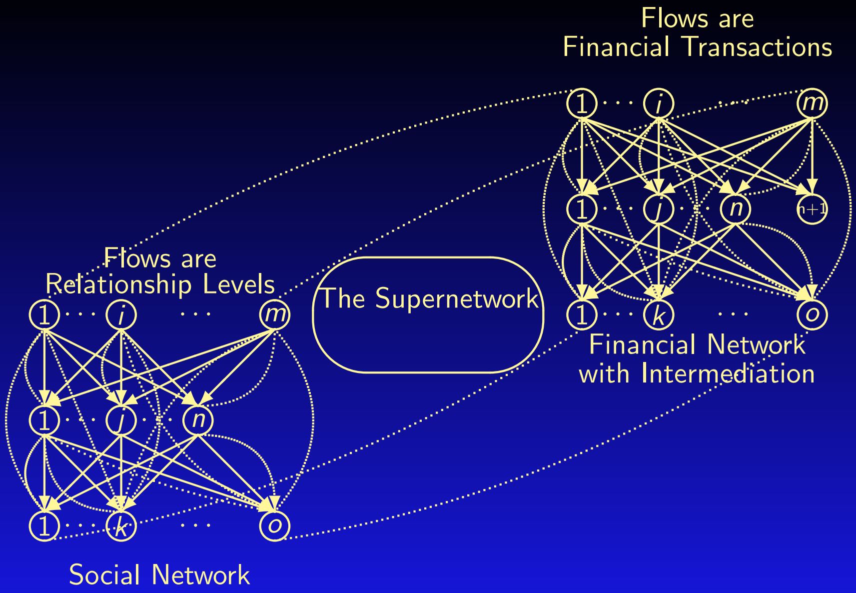


Figure 2: The Multilevel Supernetwork Structure of the Integrated Financial Network / Social Network System

Transportation Networks Needed Also in Disasters

Networks are the **fundamental critical infrastructure** for the movement of people and goods in our globalized **Network Economy**.

Transportation networks also serve as the primary conduit for **rescue, recovery, and reconstruction in disasters.**

Recent disasters have vividly demonstrated the importance and vulnerability of transportation

- The biggest blackout in North America, August 14, 2003;
- Indonesian tsunami and earthquake, December 26, 2004;
- Hurricane Katrina, August 23, 2005;
- Minneapolis I35 Bridge collapse, August 1, 2007;
- The Sichuan earthquake on May 12, 2008;
- The Haiti earthquake that struck on January 12, 2010 and the Chilean one on February 27, 2010;
- The triple disaster in Japan on March 11, 2011;
- Superstorm Sandy, October 29, 2012, second costliest hurricane in US history.

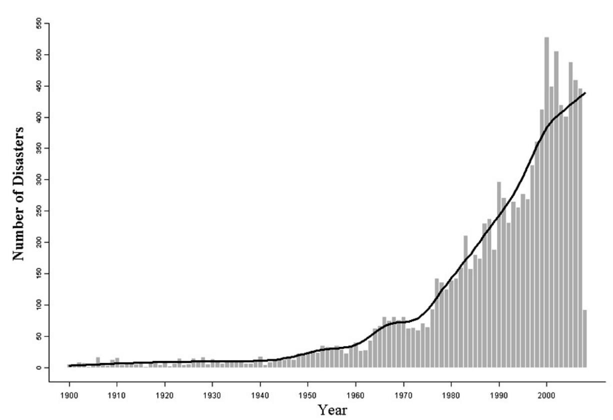
Hurricane Katrina in 2005

Hurricane Katrina has been called an *"American tragedy,"* in which essential services failed completely.

The Haitian and Chilean Earthquakes

The Triple Disaster in Japan on March 11, 2011

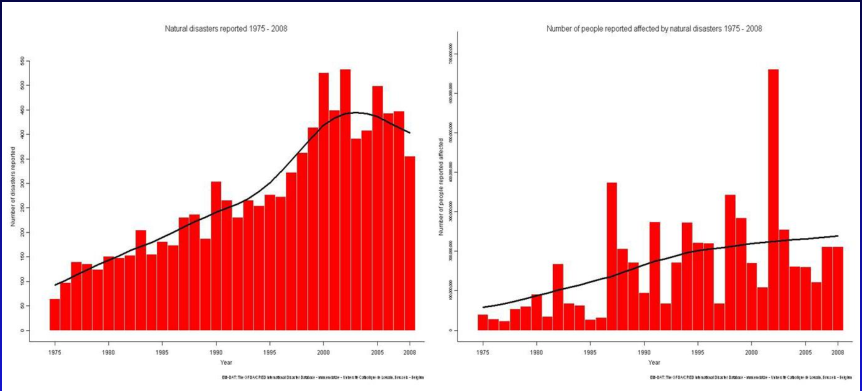
Disasters have brought an unprecedented impact on human lives in the 21st century and the number of disasters is growing. From January to October 2005, *an estimated 97,490 people were killed in disasters globally; 88,117 of them because of natural disasters*.



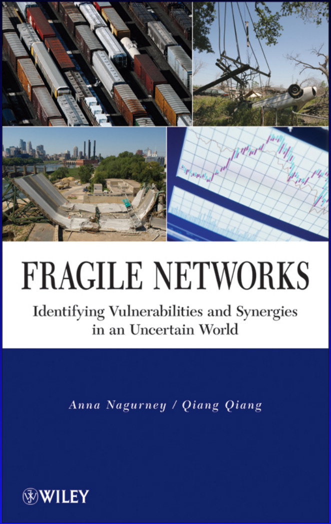
Frequency of disasters [Source: Emergency Events Database (2008)]

Disasters have a catastrophic effect on human lives and a region's or even a nation's resources.

Natural Disasters (1975–2008)



Fragile Networks



We are living in a world of *Fragile Networks*.

Some of the Recent Literature on Network Vulnerability

- Latora and Marchiori (2001, 2002, 2004)
- Holme, Kim, Yoon and Han (2002)
- Taylor and Deste (2004)
- Murray-Tuite and Mahmassani (2004)
- Chassin and Posse (2005)
- Barrat, Barthlemy and Vespignani (2005)
- Sheffi (2005)
- DallAsta, Barrat, Barthlemy and Vespignani (2006)
- Jenelius, Petersen and Mattson (2006, 2012)
- Taylor and DEste (2007)
- Nagurney and Qiang (2007, 2008, 2009)
- Qiang and Nagurney (2012)
- Qiang, Nagurney, and Dong (2009)
- Barker, Nicholson, Ramirez-Marquez (2015)

Network Centrality Measures

- Barrat et al. (2004, pp. 3748), The identification of the most central nodes in the system is a major issue in network characterization.
- Centrality Measures for Non-Weighted Networks
  - Degree, betweenness (node and edge), closeness (Freeman (1979), Girvan and Newman (2002))
  - Eigenvector centrality (Bonacich (1972))
  - Flow centrality (Freeman, Borgatti and White (1991))
  - Betweenness centrality using flow (Izquierdo and Hanneman (2006))
  - Random-work betweenness, Current-flow betweenness (Newman and Girvan (2004))
- Centrality Measures for Weighted Networks (Very Few)
  - Weighted betweenness centrality (Dall'Asta et al. (2006))
  - Network efficiency measure (Latora-Marchiori (2001))

Which Nodes and Links Really Matter?

The Nagurney and Qiang (N-Q) Network Efficiency / Performance Measure

Definition: A Unified Network Performance Measure

The network performance/efficiency measure, $\mathcal{E}(\mathcal{G}, d)$, for a given network topology $\mathcal{G}$ and the equilibrium (or fixed) demand vector d, is:

$$\mathcal{E} = \mathcal{E}(\mathcal{G}, d) = \frac{\sum_{w \in W} \frac{d_w}{\lambda_w}}{n_W},$$

where recall that n_W is the number of O/D pairs in the network, and d_w and λ_w denote, for simplicity, the equilibrium (or fixed) demand and the equilibrium disutility for O/D pair w, respectively.

The Importance of Nodes and Links

Definition: Importance of a Network Component

The importance of a network component $g \in \mathcal{G}$, $I(g)$, *is measured by the relative network efficiency drop after* g *is removed from the network:*

$$I(g) = \frac{\triangle \mathcal{E}}{\mathcal{E}} = \frac{\mathcal{E}(\mathcal{G}, d) - \mathcal{E}(\mathcal{G} - g, d)}{\mathcal{E}(\mathcal{G}, d)}$$

where $\mathcal{G} - g$ *is the resulting network after component* g *is removed from network* $\mathcal{G}$.

The Approach to Identifying the Importance of Network Components

The elimination of a link is treated in the N-Q network efficiency measure by removing that link while the removal of a node is managed by removing the links entering and exiting that node.

In the case that the removal results in no path connecting an O/D pair, we simply assign the demand for that O/D pair to an abstract path with a cost of infinity.

The N-Q measure is well-defined even in the case of disconnected networks.

The Advantages of the N-Q Network Efficiency Measure

- The measure captures *demands, flows, costs, and behavior of users*, in addition to network topology.
- The resulting importance definition of network components is applicable and *well-defined even in the case of disconnected networks*.
- It can be used to identify the *importance (and ranking) of either nodes, or links, or both*.
- It can be applied to *assess the efficiency/performance of a wide range of network systems, including financial systems and supply chains under risk and uncertainty*.
- It is applicable also to *elastic demand networks*.
- It is *applicable to dynamic networks*, *including the Internet*.

Some Applications of the N-Q Measure

The Sioux Falls Network

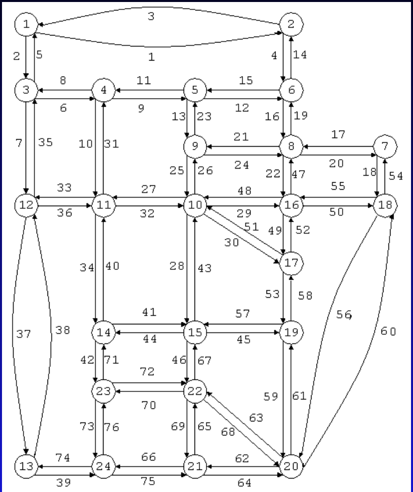


Figure 3: The Sioux Falls network with 24 nodes, 76 links, and 528 O/D pairs of nodes.

Importance of Links in the Sioux Falls Network

The computed network efficiency measure $\mathcal{E}$ for the Sioux Falls network is $\mathcal{E} = 47.6092$. Links 27, 26, 1, and 2 are the most important links, and hence special attention should be paid to protect these links accordingly, while the removal of links 13, 14, 15, and 17 would cause the least efficiency loss.

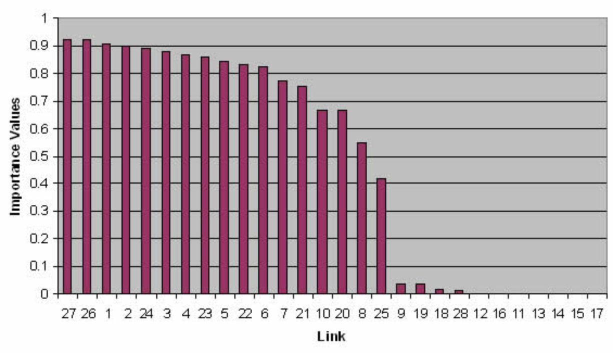


Figure 4: The Sioux Falls network link importance rankings

According to the European Environment Agency (2004), *since 1990, the annual number of extreme weather and climate related events has doubled, in comparison to the previous decade*. These events account for approximately 80% of all economic losses caused by catastrophic events. In the course of climate change, catastrophic events are projected to occur more frequently (see Schulz (2007)).

Schulz (2007) applied *N-Q network efficiency measure to a German highway system in order to identify the critical road elements* and found that this measure provided more reasonable results than the measure of Taylor and DEste (2007).

The N-Q measure can also be used to assess which links should be added to improve efficiency. *This measure was used for the evaluation of the proposed North Dublin (Ireland) Metro system* (October 2009 Issue of *ERCIM News*).

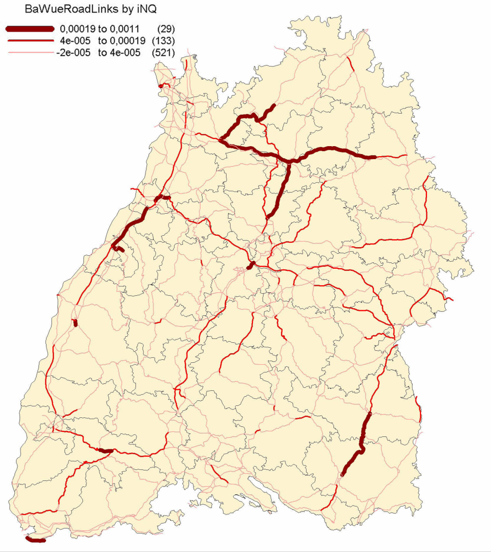
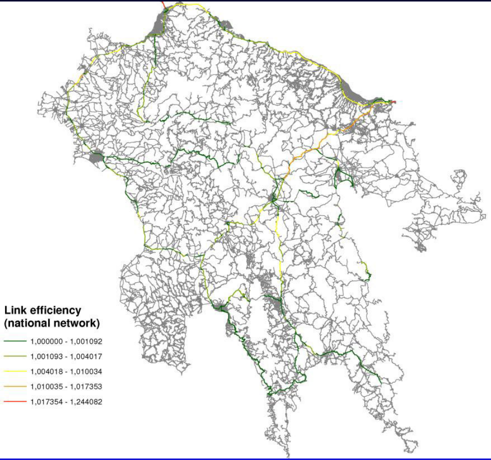


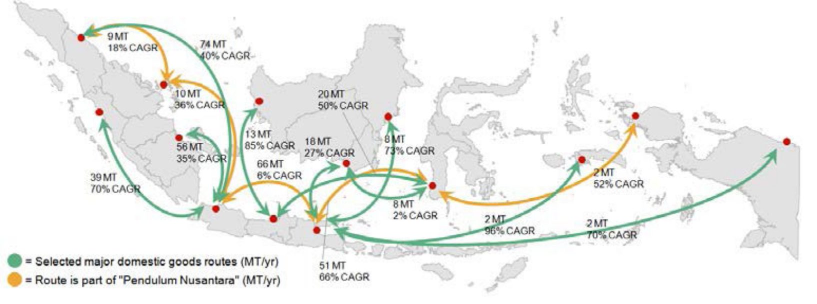
Figure 5: Comparative Importance of the links for the Baden - Wurttemberg Network – Modelling and analysis of transportation networks in earthquake prone areas via the N-Q measure, Tyagunov et al.

Mitsakis et al. (2014) applied the N-Q measure to identify the importance of links in Peloponessus, Greece. The work was inspired by the immense fires that hit this region in 2007.



The N-Q measure is noted in the "Guidebook for Enhancing Resilience of European Road Transport in Extreme Weather Events," 2014.

The N-Q measure has also been used to assess new shipping routes in Indonesia in a report, "State of Logistics - Indonesia 2015."



From Transportation Networks to the Internet

Network Efficiency Measure for Dynamic Networks - Continuous Time

The network efficiency for the network $\mathcal{G}$ with time-varying demand d for $t \in [0, T]$, denoted by $\mathcal{E}(\mathcal{G}, d, T)$, is defined as follows:

$$\mathcal{E}(\mathcal{G}, d, T) = \frac{\int_0^T [\sum_{w \in W} \frac{d_w(t)}{\lambda_w(t)}]/n_W \, dt}{T}.$$

Note that the above measure is the average network performance over time of the dynamic network.

Network Efficiency Measure for Dynamic Networks - Discrete Time

Let $d_w^1,\ d_w^2, ..., d_w^H$ denote demands for O/D pair w in H discrete time intervals, given, respectively, by: $[t_0, t_1], (t_1, t_2], ..., (t_{H-1}, t_H]$, where $t_H \equiv T$. We assume that the demand is constant in each such time interval for each O/D pair. Moreover, we denote the corresponding minimal costs for each O/D pair w at the H different time intervals by: $\lambda_w^1,\ \lambda_w^2, ..., \lambda_w^H$. The demand vector d, in this special discrete case, is a vector in $R^{n_W \times H}$.

Dynamic Network Efficiency: Discrete Time Version

The network efficiency for the network $(\mathcal{G}, d)$ *over* H *discrete time intervals:*
$[t_0, t_1], (t_1, t_2], ..., (t_{H-1}, t_H]$, *where* $t_H \equiv T$, *and with the respective constant demands:*
$d_w^1,\ d_w^2, ..., d_w^H$ *for all* $w \in W$ *is defined as follows:*

$$\mathcal{E}(\mathcal{G}, d, t_H = T) = \frac{\sum_{i=1}^{H}[(\sum_{w \in W} \frac{d_w^i}{\lambda_w^i})(t_i - t_{i-1})/n_W]}{t_H}.$$

Special Case

Assume that $d_w(t) = d_w$*, for all O/D pairs* $w \in W$ *and for* $t \in [0, T]$*. Then, the dynamic network efficiency measure collapses to the Nagurney and Qiang measure:*

$$\mathcal{E} = \frac{1}{n_W} \sum_{w \in W} \frac{d_w}{\lambda_w}.$$

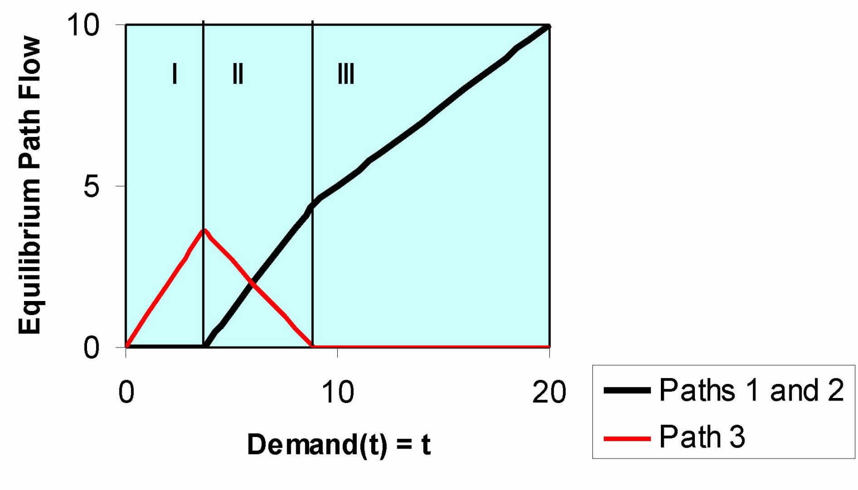
Importance of a Network Component

The importance of network component g of network $\mathcal{G}$ with demand d over time horizon T is defined as follows:

$$I(g,d,T)=\frac{\mathcal{E}(\mathcal{G},d,T)-\mathcal{E}(\mathcal{G}-g,d,T)}{\mathcal{E}(\mathcal{G},d,T)}$$

where $\mathcal{E}(\mathcal{G}-g,d,T)$ is the dynamic network efficiency after component g is removed.

The U-O Solution of the Braess Network with Added Link (Path) and Time-Varying Demands Solved as an *Evolutionary Variational Inequality* In a Model of the Internet (Nagurney, Daniele, and Parkes (2007)).



Importance of Nodes and Links in the Dynamic Braess Network Using the N-Q Measure When $T = 10$

Link	Importance Value	Importance Ranking
a	0.2604	1
b	0.1784	2
c	0.1784	2
d	0.2604	1
e	-0.1341	3

Node	Importance Value	Importance Ranking
1	1.0000	1
2	0.2604	2
3	0.2604	2
4	1.0000	1

Link e is never used after $t = 8.89$ and in the range $t \in [2.58, 8.89]$, it increases the cost, so the fact that link e has a negative importance value makes sense; over time, its removal would, on the average, improve the network efficiency!

What About Transportation Network Robustness?

The concept of *system robustness* has been studied in engineering and computer science. IEEE (1990) defined robustness as *"the degree to which a system or component can function correctly in the presence of invalid inputs or stressful environmental conditions."*

Gribble (2001) defined system robustness as *"the ability of a system to continue to operate correctly across a wide range of operational conditions, and to fail gracefully outside of that range."*

Ali et al. (2003) considered an allocation mapping to be robust if it *"guarantees the maintenance of certain desired system characteristics despite fluctuations in the behavior of its component parts or its environment."*

Schillo et al. (2001) argued that robustness has to be studied *"in relation to some definition of performance measure."*

Holmgren (2007) stated: *"Robustness signifies that the system will retain its system structure (function) intact (remain unchanged or nearly unchanged) when exposed to perturbations."*

Definition: Network Robustness Measure Under User-Optimizing Decision-Making Behavior

The robustness measure $\mathcal{R}^\gamma$ for a network $\mathcal{G}$ with the vector of user link cost functions c, the vector of link capacities u, the vector of demands d (either fixed or elastic) is defined as the relative performance retained under a given uniform capacity retention ratio γ with $\gamma \in (0, 1]$ so that the new capacities are given by γu. Its mathematical definition is

$$\mathcal{R}^\gamma = \mathcal{R}(\mathcal{G}, c, \gamma, u) = \frac{\mathcal{E}^\gamma}{\mathcal{E}} \times 100\%$$

where $\mathcal{E}$ and $\mathcal{E}^\gamma$ are the network performance measures with the original capacities and the remaining capacities, respectively.

For example, if $\gamma = .8$, this means that the user link cost functions now have the link capacities given by $.8u_a$ for all links $a \in \mathcal{L}$; if $\gamma = .4$, then the link capacities become $.4u_a$ for all links $a \in \mathcal{L}$, and so on.

According to this Definition, *a network under a given level of capacity retention or deterioration is considered to be robust if the network performance stays close to the original level*.

We can also study network robustness from *the perspective of network capacity enhancement*.

Such an analysis *provides insights into link investments*. In this case $\gamma \geq 1$ and, for definiteness (and as suggested in Nagurney and Qiang (2009)), we refer to the network robustness measure in this context as the *"capacity increment ration."*

An Application to the Anaheim Network

Each link of the Anaheim network has a link travel cost functional form of the BPR form. There are 461 nodes, 914 links, and 1,406 O/D pairs in the Anaheim network.

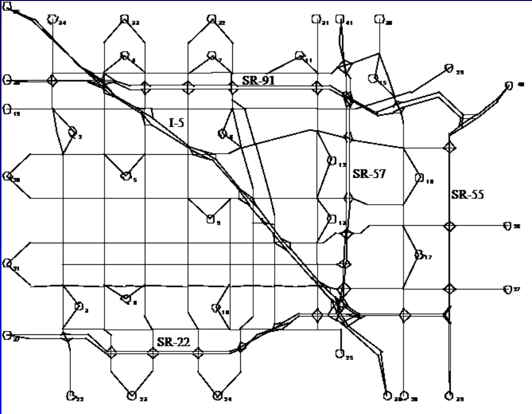


Figure 6: The Anaheim network

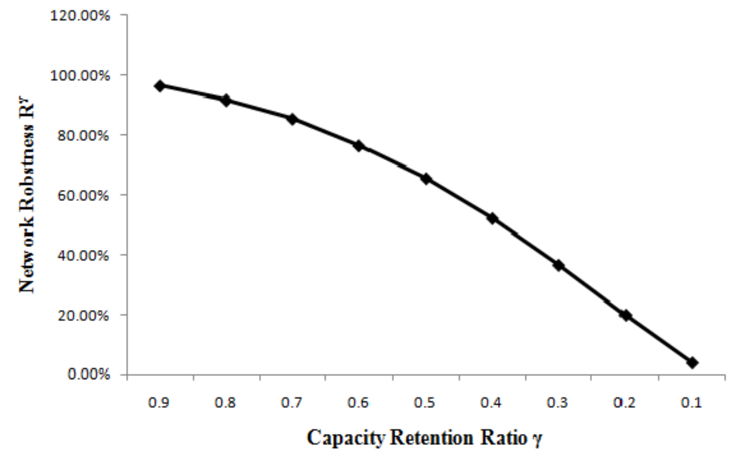


Figure 7: Robustness vs. Capacity Retention Ratio for the Anaheim Network

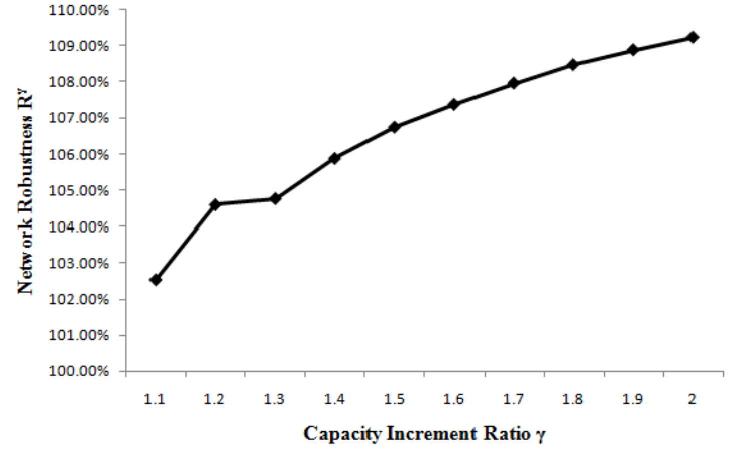


Figure 8: Robustness vs. Capacity Increment Ratio for the Anaheim Network

Different Perspectives on Transportation Network Robustness

Relative Total Cost Indices

The definition of the index under the user-optimizing flow pattern, denoted by $\mathcal{I}^{\gamma}_{U-O}$:

$$\mathcal{I}^{\gamma}_{U-O} = \mathcal{I}_{U-O}(\mathcal{G}, c, d, \gamma, u) = \frac{TC^{\gamma}_{U-O} - TC_{U-O}}{TC_{U-O}} \times 100\%,$$

where TC_{U-O} and TC^{γ}_{U-O} are the total network costs evaluated under the U-O flow pattern with the original capacities and the remaining capacities (i.e., γu), respectively.

The definition of the index under the system-optimizing flow pattern is:

$$\mathcal{I}^{\gamma}_{S-O} = \mathcal{I}_{S-O}(\mathcal{G}, c, d, \gamma, u) = \frac{TC^{\gamma}_{S-O} - TC_{S-O}}{TC_{S-O}} \times 100\%,$$

where TC_{S-O} and TC^{γ}_{S-O} are the total network costs evaluated at the S-O flow pattern with the capacities as above.

From these definitions, a network, under a given capacity retention/deterioration ratio γ (and either S-O or U-O behavior) is *considered to be robust if the index $\mathcal{I}^\gamma$ is low.*

This means that the relative total cost does not change much; hence the network may be viewed as being more robust than if the relative total cost were large.

We can also study the relative total cost improvement after capacity enhancement. In that case, because the relative total cost savings need to be computed, we reverse the order of subtraction in the previous expressions with $\gamma \geq 1$. Furthermore, γ is defined as the "capacity increment ratio."

Therefore, the larger the relative total cost index is, the greater the expected total cost savings for a capacity enhancement plan for a specific γ.

Relationship to the Price of Anarchy

The *price of anarchy*, $\mathcal{P}$, defined as

$$\mathcal{P} = \frac{TC_{U-O}}{TC_{S-O}},$$

captures the relationship between total costs *across* distinct behavioral principles, whereas the above indices are focused on the degradation of network performance *within* U-O or S-O behavior.

The relationship between the ratio of the two indices and the price of anarchy

$$\frac{I^{\gamma}_{S-O}}{I^{\gamma}_{U-O}} = \frac{[TC^{\gamma}_{S-O} - TC_{S-O}]}{[TC^{\gamma}_{U-O} - TC_{U-O}]} \times \mathcal{P}.$$

The term preceding the price of anarchy may be less than 1, greater than 1, or equal to 1, depending on the network and data.

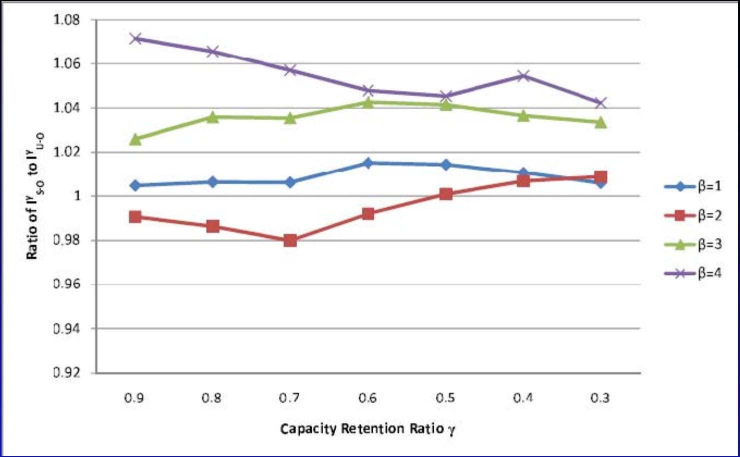


Figure 9: Example: The Sioux Falls network

This network is always more robust under U-O behavior except when β is equal to 2 (where β is the power to which the link flow is raised to into the BPR function) and $\gamma \in [0.5, 0.9]$.

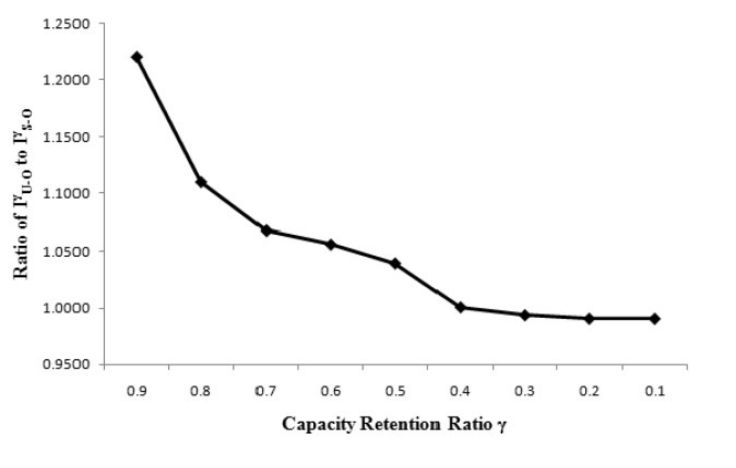


Figure 10: Example: The Anaheim network

This network is more robust under the S-O solution when the capacity retention ratio γ is above .3.

Which Nodes and Links Matter Environmentally?

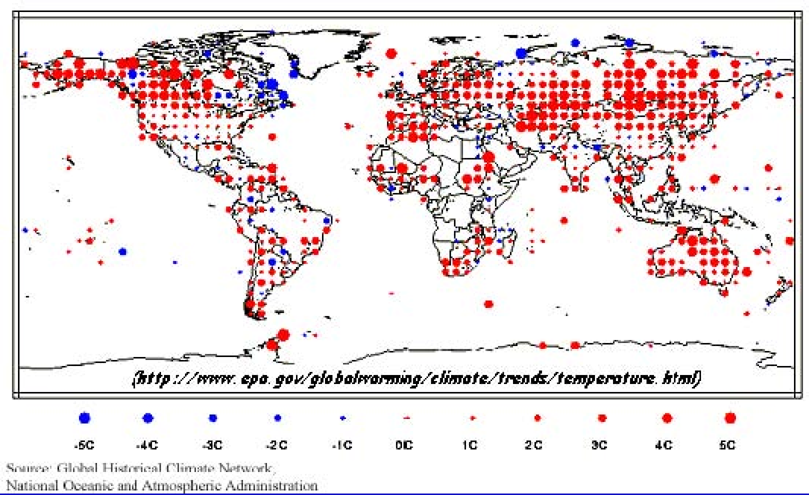


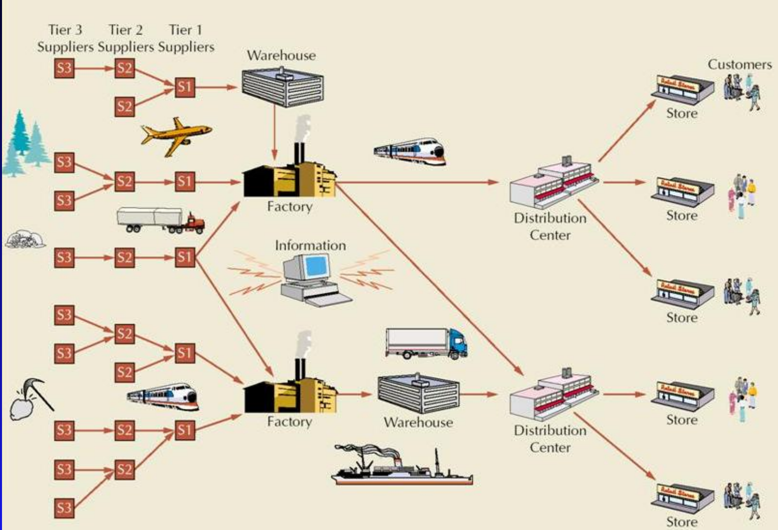
Figure 11: Global Annual Mean Temperature Trend 1950–1999

Figure 12: Impacts of climate change on transportation infrastructure

We have also extended our measures to construct environmental impact assessment indices and environmental link importance identifiers under either U-O or S-O behaviors.

What About Disaster Relief?

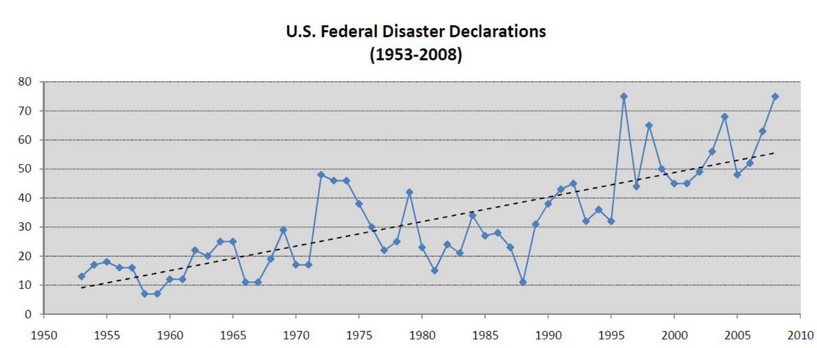
A General Supply Chain



Illustrations of Supply Chain Risk

Disaster Relief

In the US alone, the average number of disasters per year that exceeded a cost of 1 billion dollars in damages increased from 3.6 in the 2001-2005 period to 5.8 in 2006-2010 (NOAA).



Disaster Relief

Humanitarian Supply Chains

The supply chain is a critical component not only of corporations but also of humanitarian organizations and their logistical operations.

At least 50 cents of each dollar's worth of food aid is spent on transport, storage and administrative costs.

Vulnerability of Humanitarian Supply Chains

Extremely poor logistical infrastructures: Modes of transportation include trucks, barges, donkeys in Afghanistan, and elephants in Cambodia.

To ship the humanitarian goods to the affected area in the first 72 hours after disasters is crucial. The successful execution is not just a question of money but a difference between life and death.

Corporations expertise with logistics could help public response efforts for nonprofit organizations.

In the humanitarian sector, organizations are 15 to 20 years behind, as compared to the commercial arena, regarding supply chain network development.

It is clear that better-designed supply chain networks in which transportation plays a pivotal role would have facilitated and enhanced various emergency preparedness and relief efforts and would have resulted in less suffering and lives lost.

Risk Reduction Model of Liu and Nagurney (2011)

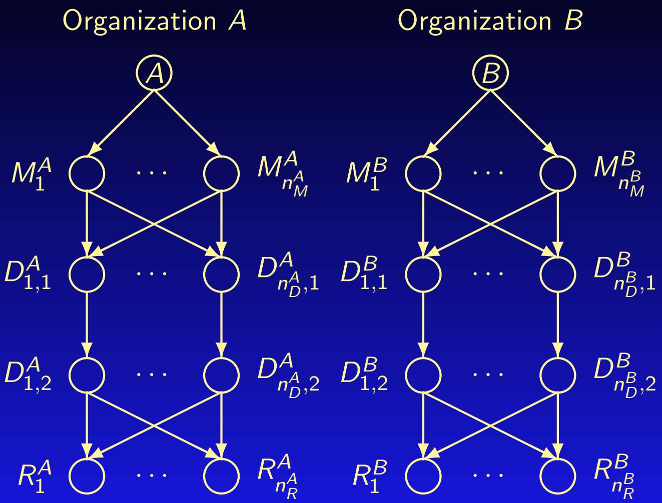


Figure 13: The Pre-Merger Supply Chain Network

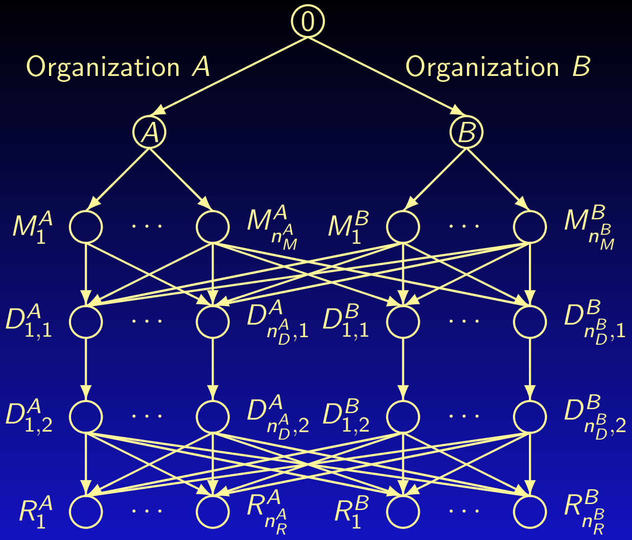


Figure 14: Organizations A and B Merge or “Team Up”

Integrated Disaster Relief Model of Nagurney, Masoumi, and Yu (2015)

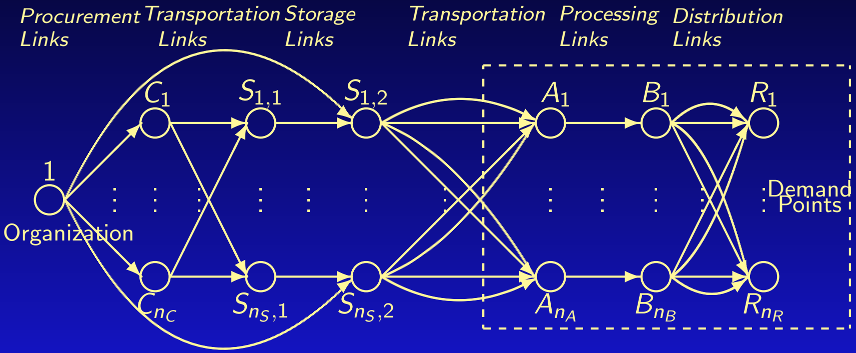


Figure 15: Network Topology of the Integrated Disaster Relief Supply Chain

Summary, Conclusions, and Suggestions for Future Research

- We discussed the *new era of supernetworks*.
- We emphasized the *importance of capturing behavior* in network modeling, analysis, and design and various paradoxes.
- We presented the *Nagurney-Qiang network performance / efficiency measure* and how it has been applied to identify the importance and rankings of nodes and links along with various applications.
- We discussed definitions of network robustness and measures for the evaluation from both S-O and U-O perspectives.
- We detailed network models for disaster relief and humanitarian operations.

- We have done extensive research on supply chains and disruption management and have also conducted research on Future Internet Architectures as part of a large NSF project.
- We expect that future research will include supernetwork design for robustness and resiliency.

THANK YOU!

For reference materials, see: http://supernet.som.umass.edu