Supply Chain Network Operations Management and Design of A Sustainable Blood Banking System

Amir Masoumi

SCHMGT 597LG - Humanitarian Logistics and Healthcare

Isenberg School of Management
University of Massachusetts
Amherst, Massachusetts 01003

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Overview

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For each of the two problems:

- mathematical formulation is derived;
- an algorithm is proposed;
- solutions to several numerical cases are computed; and
- case-based analysis is conducted.

1. Introduction
2. Operations Management of Blood Banking Systems
3. Blood Banking Systems Network Design

What is a perishable product?

A perishable product has a limited lifetime during which it can be used, after which it should be discarded (Federgruen, Prastacos, and Zipkin (1986)).

Examples of perishable goods include dairy products, baked goods, fruits and vegetables, medicines and vaccines, cut flowers, etc.

Examples of Perishable Products

Supply Chain Management

Supply chains are the essential infrastructure for the production, distribution, and consumption of goods as well as services in today's globalized network economy, and, in their most basic realization, consist of manufacturers and suppliers, distributors, retailers, and consumers at the demand markets (Nagurney (2006)).

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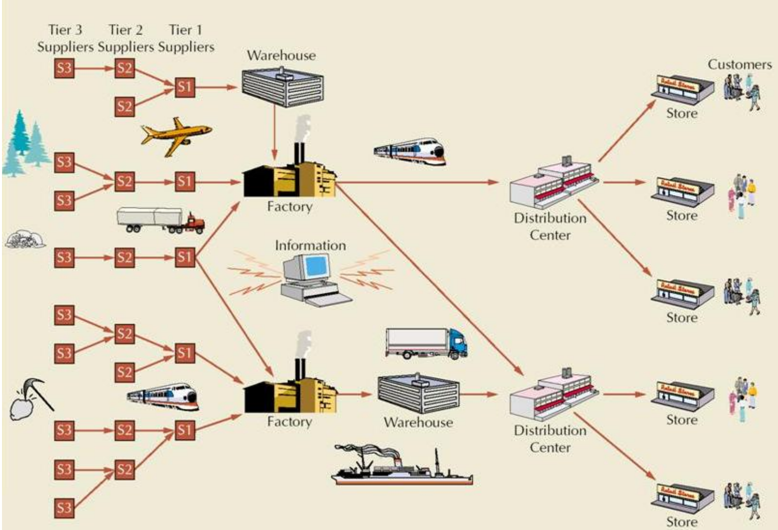
Accordingly, supply chain management (SCM) is defined as the management of a network of interconnected businesses involved in the ultimate provision of product and service packages required by end customers (Harland (1996) and Christopher (2005)).

Supply Chain Network



Background and Motivation

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An event where donors come to give blood is called a blood drive. This can occur at a blood bank but they are often set up at a location in the community such as a shopping center, workplace, school, or house of worship.

Background and Motivation: Types of Donation

- Allogeneic (Homologous): a donor gives blood for storage at a blood bank for transfusion to an unknown recipient.
- Directed: a person, often a family member, donates blood for transfusion to a specific individual.
- Autologous: a person has blood stored that will be transfused back to the donor at a later date, usually after surgery.

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In the developed world, most blood donors are unpaid volunteers who give blood for an established community supply. In poorer countries, donors usually give blood when family or friends need a transfusion.

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The donor must answer questions about medical history and take a short physical examination to make sure the donation is not hazardous to his or her health.

If a potential donor does not meet these criteria, they are deferred. This term is used because many donors who are ineligible may be allowed to donate later.

More on Blood Donation

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The US does not have a centralized blood donation service. The American Red Cross collects a little less than half of the blood used, the other half is collected by independent agencies, most of which are members of America's Blood Centers. The US military collects blood from service members for its own use, but also draws blood from the civilian supply.

Apheresis

Apheresis is a blood donation method where the blood is passed through an apparatus that separates out one particular constituent and returns the remainder to the donor.

Apheresis is used to collect:

- Platelets: must be pooled from multiple donations.
- Plasma: can be combined with Plateletpheresis.
- White Blood Cells.
- Double Red Cells.

Background and Motivation

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Hospitals with as many days of surgical delays as 50 or even 120 have been observed (Whitaker et al. (2007)).

Background and Motivation

The hospital cost of a unit of red blood cells in the US had a 6.4% increase from 2005 to 2007.

In the US, this criticality has become more of an issue in the Northeastern and Southwestern states since this cost is meaningfully higher compared to that of the Southeastern and Central states.

Life Cycles of Blood Products

The collected blood is usually stored as separate components.

- Platelets: the longest shelf life is 7 days.
- Red Blood Cells: a shelf life of 35-42 days at refrigerated temperatures
- Plasma: can be stored frozen for up to one year.

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It is difficult to have a stockpile of blood to prepare for a disaster.

After the 9/11 terrorist attacks, it became clear that collecting during a disaster was impractical and that efforts should be focused on maintaining an adequate supply at all times.

Background and Motivation

In 2006, the national estimate for the number of units of blood components outdated by blood centers and hospitals was 1,276,000 out of 15,688,000 units.

Hospitals were responsible for approximately 90% of the outdates, where this volume of medical waste imposes discarding costs to the already financially-stressed hospitals (The New York Times (2010)).

Background and Motivation

While many hospitals have their waste burned to avoid polluting the soil through landfills, the incinerators themselves are one of the nations leading sources of toxic pollutants such as dioxins and mercury (Giusti (2009)).

Healthcare facilities in the United States are second only to the food industry in producing waste, generating more than 6,600 tons per day, and more than 4 billion pounds annually (Fox News (2011)).

Methodologies

The methodologies utilized in this research include:

- optimization theory,
- network theory,
- game theory,
- multicriteria decision-making,
- risk analysis, and,
- variational inequality theory.

This section is based on the following paper:

Nagurney, A., Masoumi, A., Yu, M. (2012) Supply chain network operations management of a blood banking system with cost and risk minimization. *Computational Management Science*, in press.

The paper has already been cited in:

Rossetti, M., Buyurgan, N., Pohl, E. (2012) Medical supply logistics. In: *Handbook of healthcare system scheduling*. International Series in Operations Research & Management Science, 168, 245-280.


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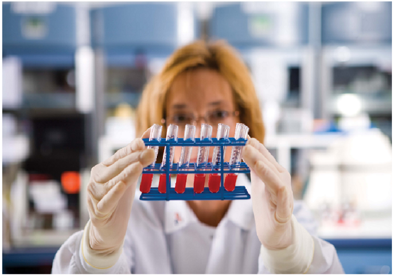

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Regionalized Blood Supply Chain Network Model

A generalized network optimization model was developed for the complex supply chain of human blood, which is a life-saving, perishable product.

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More specifically, the framework was of multicriteria centralized (system-optimization) type for a regionalized blood supply chain network.

Regionalized Blood Supply Chain Network Model

A network topology was assumed where the top level (origin) corresponds to the organization; .i.e., the regional division management of the American Red Cross. The bottom level (destination) nodes correspond to the demand sites - typically the hospitals and the other surgical medical centers.

The paths joining the origin node to the destination nodes represent sequences of supply chain network activities that ensure that the blood is collected, tested, processed, and, ultimately, delivered to the demand sites.

Components of a Regionalized Blood Banking System

- ARC Regional Division Management (Top Tier),

Blood Collection

- Blood Collection Sites (Tier 2), denoted by: $CS_1, CS_2, \ldots, CS_{n_{CS}}$,

Shipment of Collected Blood

- Blood Centers (Tier 3), denoted by: $BC_1, BC_2, \ldots, BC_{n_{BC}}$,

Testing and Processing

- Component Labs (Tier 4), denoted by: $CL_1, CL_2, \ldots, CL_{n_{CL}}$,

Storage

- Storage Facilities (Tier 5), denoted by: $SF_1, SF_2, \ldots, SF_{n_{SF}}$,

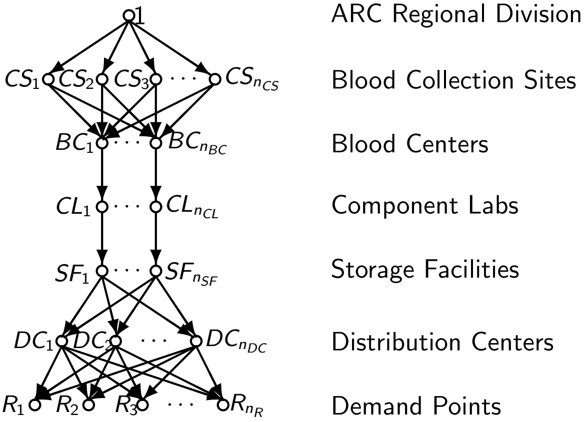
Shipment

- Distribution Centers (Tier 6), denoted by: $DC_1, DC_2, \ldots, DC_{n_DC}$,

Distribution

- Demand Points (Tier 7), denoted by: $R_1, R_2, \ldots, R_{n_R}$

Figure 3.1: Supply Chain Network Topology for a Regionalized Blood Bank



Graph $G = [N, L]$, where N denotes the set of nodes and L the set of links.

Regionalized Blood Supply Chain Network Model

The formalism is that of multicriteria optimization, where the organization seeks to determine the optimal levels of blood processed on each supply chain network link subject to the minimization of the total cost associated with its various activities of blood collection, shipment, processing and testing, storage, and distribution, in addition to the total discarding cost as well as the minimization of the total supply risk, subject to the uncertain demand being satisfied as closely as possible at the demand sites.

Notation

- c_a: the unit operational cost on link a.
- $\hat{c}_a$: the total operational cost on link a.
- f_a: the flow of whole blood/red blood cell on link a.
- p: a path in the network joining the origin node to a destination node representing the activities and their sequence.
- $\mathcal{P}_k$: the set of paths, which represent alternative associated possible supply chain network processes joining $(1, R_k)$.
- $\mathcal{P}$: the set of all paths joining node 1 to the demand nodes.
- n_p: the number of paths from the origin to the demand markets.
- x_p: the nonnegative flow of the blood on path p.
- d_k: the uncertain demand for blood at demand location k.
- v_k: the projected demand for blood at demand location k.

Formulation

Total Operational Cost on Link a

$$\hat{c}_a(f_a) = f_a \times c_a(f_a), \qquad \forall a \in L, \tag{3.1}$$

assumed to be convex and continuously differentiable.

Let P_k be the probability distribution function of d_k, that is, $P_k(D_k) = Prob(d_k \leq D_k) = \int_0^{D_k} \mathcal{F}_k(t)d(t)$. Therefore,

Shortage and Surplus of Blood at Demand Point R_k

$$\Delta_k^- \equiv \max\{0, d_k - v_k\}, \qquad k = 1, \ldots, n_R, \tag{3.2}$$

$$\Delta_k^+ \equiv \max\{0, v_k - d_k\}, \qquad k = 1, \ldots, n_R, \tag{3.3}$$

Expected Values of Shortage and Surplus

$$E(\Delta_k^-) = \int_{v_k}^{\infty} (t - v_k)\mathcal{F}_k(t)d(t), \qquad k = 1, \ldots, n_R, \tag{3.4}$$

$$E(\Delta_k^+) = \int_{0}^{v_k} (v_k - t)\mathcal{F}_k(t)d(t), \qquad k = 1, \ldots, n_R. \tag{3.5}$$

Expected Total Penalty at Demand Point k

$$E(\lambda_k^- \Delta_k^- + \lambda_k^+ \Delta_k^+) = \lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+), \tag{3.6}$$

where λ_k^- is a large penalty associated with the shortage of a unit of blood, and λ_k^+ is the incurred cost of a unit of surplus blood.

Formulation

Arc Multiplier, and Waste/Loss on a link

Let α_a correspond to the percentage of loss over link a, and f'_a denote the final flow on that link. Thus,

$$f'_a = \alpha_a f_a, \qquad \forall a \in L. \tag{3.7}$$

Therefore, the waste/loss on link a, denoted by w_a, is equal to:

$$w_a = f_a - f'_a = (1 - \alpha_a) f_a, \qquad \forall a \in L. \tag{3.8}$$

Total Discarding Cost function

$$\hat{z}_a = \hat{z}_a(f_a), \qquad \forall a \in L. \tag{3.9b}$$

Non-negativity of Flows

$$x_p \geq 0, \qquad \forall p \in \mathcal{P}, \tag{3.10}$$

Path Multiplier, and Projected Demand

$$\mu_p \equiv \prod_{a \in p} \alpha_a, \qquad \forall p \in \mathcal{P}, \tag{3.11}$$

where μ_p is the throughput factor on path p. Thus, the projected demand at R_k is equal to:

$$v_k \equiv \sum_{p \in \mathcal{P}_k} x_p \mu_p, \qquad k = 1, \ldots, n_R. \tag{3.12}$$

Relation between Link and Path Flows

$$\alpha_{ap} \equiv \begin{cases} \delta_{ap} \prod_{a'<a} \alpha_{a'}, & \text{if } \{a' < a\} \neq \emptyset, \\ \\ \delta_{ap}, & \text{if } \{a' < a\} = \emptyset, \end{cases} \tag{3.13}$$

where $\{a' < a\}$ denotes the set of the links preceding link a in path p. Also, δ_{ap} is defined as equal to 1 if link a is contained in path p; otherwise, it is equal to zero. Therefore,

$$f_a = \sum_{p \in \mathcal{P}} x_p \, \alpha_{ap}, \qquad \forall a \in L. \tag{3.14}$$

Cost Objective Function

Minimization of Total Costs

$$\text{Minimize} \quad \sum_{a \in L} \hat{c}_a(f_a) + \sum_{a \in L} \hat{z}_a(f_a) + \sum_{k=1}^{n_R} (\lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+)) \tag{3.15}$$

subject to: constraints (3.10), (3.12), and (3.14).

Supply Side Risk

One of the most significant challenges for the ARC is to capture the risk associated with different activities in the blood supply chain network. Unlike the demand which can be projected, albeit with some uncertainty involved, the amount of donated blood at the collection sites has been observed to be highly stochastic.

Risk Objective Function

$$\text{Minimize} \quad \sum_{a \in L_1} \hat{r}_a(f_a), \tag{3.16}$$

where $\hat{r}_a = \hat{r}_a(f_a)$ is the total risk function on link a, and L_1 is the set of blood collection links.

The Multicriteria Optimization Formulation

θ: the weight associated with the risk objective function, assigned by the decision maker.

Multicriteria Optimization Formulation in Terms of Link Flows

$$\text{Minimize} \quad \sum_{a\in L} \hat{c}_a(f_a) + \sum_{a\in L} \hat{z}_a(f_a) + \sum_{k=1}^{n_R} (\lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+))$$

$$+ \theta \sum_{a\in L_1} \hat{r}_a(f_a) \tag{3.17}$$

subject to: constraints (3.10), (3.12), and (3.14).

The Multicriteria Optimization Formulation

Multicriteria Optimization Formulation in Terms of Path Flows

$$\text{Minimize} \quad \sum_{p\in\mathcal{P}}\left(\hat{C}_p(x)+\hat{Z}_p(x)\right)+\sum_{k=1}^{n_R}\left(\lambda_k^- E(\Delta_k^-)+\lambda_k^+ E(\Delta_k^+)\right)$$

$$+\,\theta\sum_{p\in\mathcal{P}}\hat{R}_p(x) \tag{3.18}$$

subject to: constraints (3.10) and (3.12).

The total costs on path p are expressed as:

$$\hat{C}_p(x) = x_p \times C_p(x), \quad \forall p \in \mathcal{P}, \tag{3.19a}$$

$$\hat{Z}_p(x) = x_p \times Z_p(x), \quad \forall p \in \mathcal{P}, \tag{3.19b}$$

$$\hat{R}_p(x) = x_p \times R_p(x), \quad \forall p \in \mathcal{P}, \tag{3.19c}$$

The unit cost functions on path p are, in turn, defined as below:

$$C_p(x) \equiv \sum_{a \in L} c_a(f_a)\alpha_{ap}, \quad \forall p \in \mathcal{P}, \tag{3.20a}$$

$$Z_p(x) \equiv \sum_{a \in L} z_a(f_a)\alpha_{ap}, \quad \forall p \in \mathcal{P}, \tag{3.20b}$$

$$R_p(x) \equiv \sum_{a \in L_1} r_a(f_a)\alpha_{ap}, \quad \forall p \in \mathcal{P}. \tag{3.20c}$$

Variational Inequality Formulation: Feasible Set and Decision Variables

Let K denote the feasible set such that:

$$K \equiv \{x | x \in R_+^{n_p}\}. \tag{3.28}$$

The multicriteria optimization problem is characterized, under the assumptions, by a convex objective function and a convex feasible set.

The path flows are grouped into the vector x. Also, the link flows, and the projected demands are grouped into the respective vectors f and v.

Variational Inequality Formulation

Theorem 3.1

The vector x^* is an optimal solution to the multicriteria optimization problem (3.18), subject to (3.10) and (3.12), if and only if it is a solution to the variational inequality problem: determine the vector of optimal path flows $x^* \in K$, such that:

$$\sum_{k=1}^{n_R} \sum_{p \in \mathcal{P}_k} \left[\frac{\partial(\sum_{q \in \mathcal{P}} \hat{C}_q(x^*))}{\partial x_p} + \frac{\partial(\sum_{q \in \mathcal{P}} \hat{Z}_q(x^*))}{\partial x_p} + \lambda_k^+ \mu_p P_k \left(\sum_{p \in \mathcal{P}_k} x_p^* \mu_p \right) \right.$$

$$\left. -\lambda_k^- \mu_p \left(1 - P_k \left(\sum_{p \in \mathcal{P}_k} x_p^* \mu_p \right) \right) + \theta \frac{\partial(\sum_{q \in \mathcal{P}} \hat{R}_q(x^*))}{\partial x_p} \right] \times [x_p - x_p^*] \geq 0, \qquad \forall x \in K. \tag{3.39}$$

Variational Inequality Formulation

Theorem 3.1 (cont'd)

The variational inequality mentioned, in turn, can be rewritten in terms of link flows as: determine the vector of optimal link flows, and the vector of optimal projected demands $(f^*, v^*) \in K^1$, such that:

$$\sum_{a\in L_1}\left[\frac{\partial \hat{c}_a(f_a^*)}{\partial f_a}+\frac{\partial \hat{z}_a(f_a^*)}{\partial f_a}+\theta\,\frac{\partial \hat{r}_a(f_a^*)}{\partial f_a}\right]\times[f_a-f_a^*]+\sum_{a\in L_1^C}\left[\frac{\partial \hat{c}_a(f_a^*)}{\partial f_a}+\frac{\partial \hat{z}_a(f_a^*)}{\partial f_a}\right]\times[f_a-f_a^*]$$

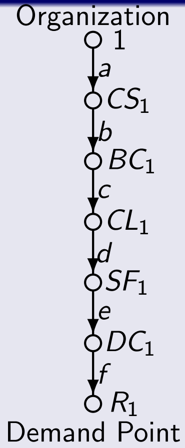
$$+\sum_{k=1}^{n_R}\left[\lambda_k^+P_k(v_k^*)-\lambda_k^-(1-P_k(v_k^*))\right]\times[v_k-v_k^*]\geq 0,\qquad \forall (f,v)\in K^1, \qquad (3.40)$$

where K^1 denotes the feasible set as defined below:

$$K^1 \equiv \{(f,v)|\exists x\geq 0,\ (3.12),\ \text{and (3.14) hold}\}.$$

Illustrative Numerical Examples

Example 3.1



Supply Chain Network Topology for Numerical Examples 3.1 and 3.2

Example 3.1 (cont'd)

The total cost functions on the links were:

$$\hat{c}_a(f_a) = f_a^2 + 6f_a, \quad \hat{c}_b(f_b) = 2f_b^2 + 7f_b, \quad \hat{c}_c(f_c) = f_c^2 + 11f_c,$$

$$\hat{c}_d(f_d) = 3f_d^2 + 11f_d, \quad \hat{c}_e(f_e) = f_e^2 + 2f_e, \quad \hat{c}_f(f_f) = f_f^2 + f_f.$$

No waste was assumed so that $\alpha_a = 1$ for all links. Hence, all the functions $\hat{z}_a$ were set equal to 0.

The total risk cost function on the blood collection link a was: $\hat{r}_a = 2f_a^2$, and the weight associated with the risk objective, θ, was assumed to be 1.

Example 3.1 (cont'd)

There is only a single path p_1 which was defined as:
$p_1 = (a, b, c, d, e, f)$ with $\mu_{p_1} = 1$.

Demand for blood followed a uniform distribution on $[0, 5]$ so that:
$P_1(x_{p_1}) = \frac{x_{p_1}}{5}$.

The penalties were: $\lambda_1^- = 100, \quad \lambda_1^+ = 0$.

Example 3.1: Solution

Substitution of the numerical values, and solving the variational inequality (3.39) yields:

$$x_{p_1}^* = 1.48,$$

and the corresponding optimal link flow pattern:

$$f_a^* = f_b^* = f_c^* = f_d^* = f_e^* = f_f^* = 1.48.$$

Also, the projected demand is equal to:

$$v_1^* = x_{p_1}^* = 1.48.$$

Example 3.2

Example 3.2 had the same data as Example 3.1 except that now there was a loss associated with the testing and processing link with $\alpha_c = .8$, and $\hat{z}_c = .5f_c^2$.

Solving the variational inequality (3.39) yields:

$$x_{p_1}^* = 1.39,$$

which, in turn, yields:

$$f_a^* = f_b^* = f_c^* = 1.39,$$

$$f_d^* = f_e^* = f_f^* = 1.11.$$

The projected demand was:

$$v_1^* = x_{p_1}^* \mu_{p_1} = 1.11.$$

Summary of the Numerical Examples 3.1 and 3.2

An Interesting Paradox

Comparing the results of Examples 3.1 and 3.2 reveals the fact that when perishability is taken into consideration, with $\alpha_c = .8$ and the data above, the organization chooses to produce/ship slightly smaller quantities so as to minimize the discarding cost of the waste, despite the shortage penalty of λ_1^-.

Sensitivity Analysis Results

Table 3.1: Computed Optimal Path Flows $x^*_{p_1}$ and Optimal Values of the Objective Function as α_c and λ^-_1 Vary

λ^-_1 \ α_c		.2	.4	.6	.8	1
100	$x^*_{p_1}$	0.00	0.58	1.16	1.39	1.48
	OF	250.00	246.96	234.00	218.83	204.24
200	$x^*_{p_1}$	0.88	2.40	2.83	2.77	2.61
	OF	494.19	439.52	376.23	326.94	288.35
300	$x^*_{p_1}$	2.10	3.74	3.86	3.54	3.20
	OF	715.12	581.15	464.85	387.17	331.44
400	$x^*_{p_1}$	3.20	4.76	4.57	4.03	3.55
	OF	914.75	689.71	525.36	425.56	357.63
500	$x^*_{p_1}$	4.20	5.57	5.09	4.37	3.79
	OF	1096.03	775.55	569.30	452.16	375.23

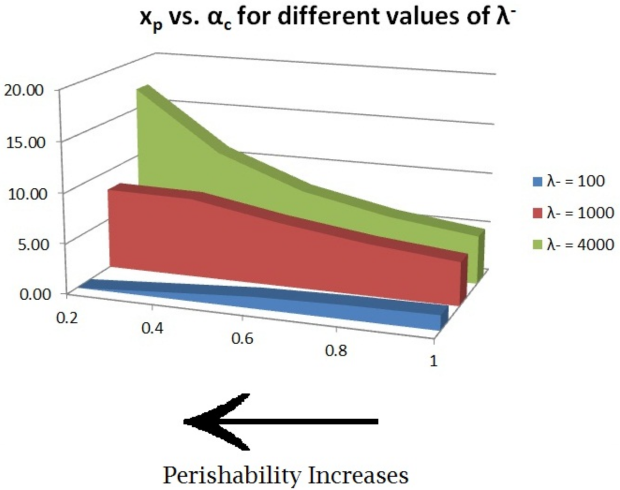
Note: Numbers in red are the paradoxical results.

Sensitivity Analysis Results

Table 3.1 (cont'd)

λ_1^- \ α_c		.2	.4	.6	.8	1
1000	$x_{p_1}^*$	8.09	7.95	6.41	5.19	4.33
	OF	1799.11	1027.94	681.89	515.88	415.67
2000	$x_{p_1}^*$	12.69	9.80	7.27	5.68	4.65
	OF	2631.32	1224.45	755.64	554.47	439.05
3000	$x_{p_1}^*$	15.33	10.58	7.60	5.86	4.76
	OF	3107.51	1307.25	783.73	568.57	447.39
4000	$x_{p_1}^*$	17.03	11.01	7.77	5.96	4.82
	OF	3415.88	1352.89	798.54	575.88	451.68

Sensitivity Analysis Results



The Solution Algorithm

The realization of Euler Method for the solution of the blood bank supply chain management problem governed by variational inequality (3.39) induces subproblems that can be solved explicitly and in closed form.

At iteration τ of the Euler method one computes:

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - a_{\tau}F(X^{\tau})), \tag{3.59}$$

where $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$, and F is the function that enters the standard form variational inequality problem (3.44).

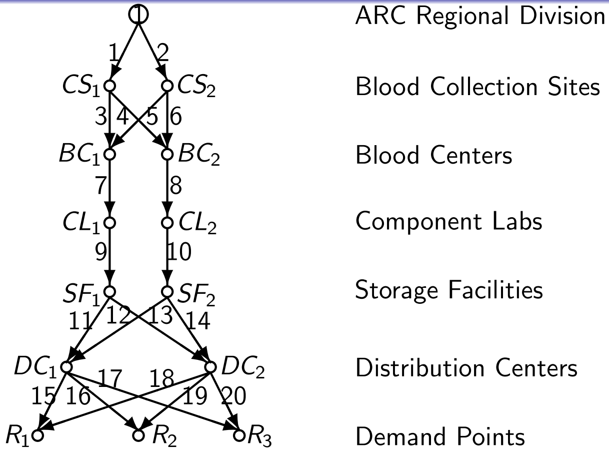
Explicit Formulae for the Euler Method Applied to Variational Inequality Formulation

$$x_p^{\tau+1} = \max\{0, x_p^\tau + a_\tau(\lambda_k^- \mu_p(1 - P_k(\sum_{p\in\mathcal{P}_k} x_p^\tau \mu_p)) - \lambda_k^+ \mu_p P_k(\sum_{p\in\mathcal{P}_k} x_p^\tau \mu_p)$$

$$-\frac{\partial(\sum_{q\in\mathcal{P}} \hat{C}_q(x^\tau))}{\partial x_p} - \frac{\partial(\sum_{q\in\mathcal{P}} \hat{Z}_q(x^\tau))}{\partial x_p} - \theta \frac{\partial(\sum_{q\in\mathcal{P}} \hat{R}_q(x^\tau))}{\partial x_p})\}, \forall p \in \mathcal{P}. \quad (3.60)$$

The above was applied to calculate the updated product flow during the steps of the Euler Method for the blood banking optimization problem.

Example 3.3



Supply Chain Network Topology for Numerical Example 3.3

The demands at these demand points followed uniform probability distribution on the intervals [5,10], [40,50], and [25,40], respectively:

$$P_1\Big(\sum_{p\in\mathcal{P}_1}\mu_p x_p\Big)=\frac{\sum_{p\in\mathcal{P}_1}\mu_p x_p-5}{5},\quad P_2\Big(\sum_{p\in\mathcal{P}_2}\mu_p x_p\Big)=\frac{\sum_{p\in\mathcal{P}_2}\mu_p x_p-40}{10},$$

$$P_3\Big(\sum_{p\in\mathcal{P}_3}\mu_p x_p\Big)=\frac{\sum_{p\in\mathcal{P}_3}\mu_p x_p-25}{15}.$$

$$\lambda_1^-=2200,\quad \lambda_1^+=50,$$
$$\lambda_2^-=3000,\quad \lambda_2^+=60,$$
$$\lambda_3^-=3000,\quad \lambda_3^+=50.$$

$$\hat{r}_1(f_1)=2f_1^2,\quad \text{and}\quad \hat{r}_2(f_2)=1.5f_2^2,$$

and $\theta=0.7$.

Solution Procedure

The Euler method (cf. (3.60)) for the solution of variational inequality (3.39) was implemented in Matlab. A Microsoft Windows System with a Dell PC at the University of Massachusetts Amherst was used for all the computations. The sequence is set as $a_{\tau} = .1(1, \frac{1}{2}, \frac{1}{2}, \cdots)$, and the convergence tolerance was $\epsilon = 10^{-6}$.

Example 3.3 Results

Table 3.2: Total Cost and Total Discarding Cost Functions and Solution for Numerical Example 3.3

Link a	α_a	$\hat{c}_a(f_a)$	$\hat{z}_a(f_a)$	f_a^*
1	.97	$6f_1^2 + 15f_1$	$.8f_1^2$	54.72
2	.99	$9f_2^2 + 11f_2$	$.7f_2^2$	43.90
3	1.00	$.7f_3^2 + f_3$	$.6f_3^2$	30.13
4	.99	$1.2f_4^2 + f_4$	$.8f_4^2$	22.42
5	1.00	$f_5^2 + 3f_5$	$.6f_5^2$	19.57
6	1.00	$.8f_6^2 + 2f_6$	$.8f_6^2$	23.46
7	.92	$2.5f_7^2 + 2f_7$	$.5f_7^2$	49.39
8	.96	$3f_8^2 + 5f_8$	$.8f_8^2$	42.00
9	.98	$.8f_9^2 + 6f_9$	$.4f_9^2$	43.63
10	1.00	$.5f_{10}^2 + 3f_{10}$	$.7f_{10}^2$	39.51

Example 3.3 Results

Table 3.2 (cont'd)

Link a	α_a	$\hat{c}_a(f_a)$	$\hat{z}_a(f_a)$	f_a^*
11	1.00	$.3f_{11}^2 + f_{11}$	$.3f_{11}^2$	29.68
12	1.00	$.5f_{12}^2 + 2f_{12}$	$.4f_{12}^2$	13.08
13	1.00	$.4f_{13}^2 + 2f_{13}$	$.3f_{13}^2$	26.20
14	1.00	$.6f_{14}^2 + f_{14}$	$.4f_{14}^2$	13.31
15	1.00	$1.3f_{15}^2 + 3f_{15}$	$.7f_{15}^2$	5.78
16	1.00	$.8f_{16}^2 + 2f_{16}$	$.4f_{16}^2$	25.78
17	.98	$.5f_{17}^2 + 3f_{17}$	$.5f_{17}^2$	24.32
18	1.00	$.7f_{18}^2 + 2f_{18}$	$.7f_{18}^2$	.29
19	1.00	$.6f_{19}^2 + 4f_{19}$	$.4f_{19}^2$	18.28
20	.98	$1.1f_{20}^2 + 5f_{20}$	$.5f_{20}^2$	7.29

The computed amounts of projected demand for each of the three demand points were:

$$v_1^* = 6.06, \quad v_2^* = 44.05, \quad \text{and} \quad v_3^* = 30.99.$$

This section is based on the following paper:

Nagurney, A., Masoumi, A. (2012) Supply chain network design of a sustainable blood banking system. In: Boone, T., Jayaraman, V., and Ganeshan, R., Editors, *Sustainable supply chains: Models, methods and public policy implications.* Springer, London, England, in press.

Sustainable Blood Banking System Supply Chain Network Design Model

A generalized network model for design/redesign of the complex supply chain of human blood was developed. The framework is that of multicriteria (centralized) system-optimization for a regionalized blood supply chain network.

Sustainable Blood Banking System Supply Chain Network Design Model

The organization seeks to determine the optimal levels of blood processed on each supply chain network link coupled with the optimal levels of capacity escalation/reduction in its blood banking supply chain network activities,

subject to:

the minimization of the total cost associated with its various activities of blood collection, shipment, processing and testing, storage, and distribution, in addition to the total discarding cost as well as the minimization of the total supply risk, subject to the uncertain demand being satisfied as closely as possible at the demand sites.

Total Investment Cost of Capacity Enhancement/Reduction on Links

$$\hat{\pi}_a = \hat{\pi}_a(u_a), \qquad \forall a \in L, \tag{4.16}$$

where u_a denotes the change in capacity on link a, and $\hat{\pi}_a$ is the total investment cost of such change.

Total Investment Cost of Capacity Enhancement/Reduction on Links

$$\hat{\pi}_a = \hat{\pi}_a(u_a), \qquad \forall a \in L, \tag{4.16}$$

where u_a denotes the change in capacity on link a, and $\hat{\pi}_a$ is the total investment cost of such change.

Capacity Adjustments Constraints

$$f_a \leq \bar{u}_a + u_a, \qquad \forall a \in L, \tag{4.18}$$

and

$$-\bar{u}_a \leq u_a, \qquad \forall a \in L, \tag{4.19}$$

where $\bar{u}_a$ denotes the nonnegative existing capacity on link a.

The Multicriteria Optimization Formulation

θ: the weight associated with the risk objective function, assigned by the decision maker.

Multicriteria Optimization Formulation in Terms of Link Flows

$$\text{Minimize} \quad \sum_{a \in L} \hat{c}_a(f_a) + \sum_{a \in L} \hat{z}_a(f_a) + \sum_{a \in L} \hat{\pi}_a(u_a)$$

$$+ \sum_{k=1}^{n_R} (\lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+)) + \theta \sum_{a \in L_1} \hat{r}_a(f_a), \tag{4.21}$$

subject to: constraints (4.11), (4.13), (4.15), (4.18), and (4.19).

The Multicriteria Optimization Formulation

Multicriteria Optimization Formulation in Terms of Path Flows

$$\text{Minimize} \quad \sum_{p \in \mathcal{P}} (\hat{C}_p(x) + \hat{Z}_p(x)) + \sum_{a \in L} \hat{\pi}_a(u_a)$$

$$+ \sum_{k=1}^{n_R} (\lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+)) + \theta \sum_{p \in \mathcal{P}} \hat{R}_p(x), \tag{4.22}$$

subject to: constraints (4.11), (4.13), (4.15), (4.18), and (4.19).

Variational Inequality Formulation: Feasible Set and Decision Variables

Let K denote the feasible set such that:

$$K \equiv \{(x, u, \gamma) | x \in R_+^{n_P}, \ (4.19) \text{ holds, and } \ \gamma \in R_+^{n_L}\}. \tag{4.27}$$

The multicriteria optimization problem is characterized, under my assumptions, by a convex objective function and a convex feasible set.

Variational Inequality Formulation: Feasible Set and Decision Variables

Let K denote the feasible set such that:

$$K \equiv \{(x, u, \gamma) | x \in R_+^{n_p}, \text{ (4.19) holds, and } \gamma \in R_+^{n_L}\}. \tag{4.27}$$

The multicriteria optimization problem is characterized, under my assumptions, by a convex objective function and a convex feasible set.

The path flows, the link flows, and the projected demands are grouped into the respective vectors x, f, and v. Also, the link capacity changes are grouped into the vector u. Lastly, the Lagrange multipliers corresponding to the links capacity adjustment constraints are grouped into the vector γ.

Variational Inequality Formulation (Path Flows)

Theorem 4.1

The multicriteria optimization problem, subject to its constraints, is equivalent to the variational inequality problem: determine the vector of optimal path flows, the vector of optimal capacity adjustments, and the vector of optimal Lagrange multipliers $(x^*, u^*, \gamma^*) \in K$, such that:

$$\sum_{k=1}^{n_R} \sum_{p\in\mathcal{P}_k} \left[\frac{\partial(\sum_{q\in\mathcal{P}} \hat{C}_q(x^*))}{\partial x_p} + \frac{\partial(\sum_{q\in\mathcal{P}} \hat{Z}_q(x^*))}{\partial x_p} + \lambda_k^+ \mu_p P_k \left(\sum_{p\in\mathcal{P}_k} x_p^* \mu_p \right) \right.$$

$$\left. -\lambda_k^- \mu_p \left(1 - P_k \left(\sum_{p\in\mathcal{P}_k} x_p^* \mu_p \right) \right) + \sum_{a\in L} \gamma_a^* \delta_{ap} + \theta \frac{\partial(\sum_{q\in\mathcal{P}} \hat{R}_q(x^*))}{\partial x_p} \right] \times [x_p - x_p^*]$$

$$+ \sum_{a\in L} \left[\frac{\partial \hat{\pi}_a(u_a^*)}{\partial u_a} - \gamma_a^* \right] \times [u_a - u_a^*] + \sum_{a\in L} \left[\bar{u}_a + u_a^* - \sum_{p\in\mathcal{P}} x_p^* \alpha_{ap} \right] \times [\gamma_a - \gamma_a^*] \geq 0,$$

$$\forall (x, u, \gamma) \in K. \qquad (4.29)$$

Variational Inequality Formulation (Link Flows)

Theorem 4.1 (cont'd)

The variational inequality mentioned, in turn, can be rewritten in terms of link flows as: determine the vector of optimal link flows, the vectors of optimal projected demands and the link capacity adjustments, and the vector of optimal Lagrange multipliers $(f^*, v^*, u^*, \gamma^*) \in K^1$, such that:

$$\sum_{a\in L}\left[\frac{\partial \hat{c}_a(f_a^*)}{\partial f_a}+\frac{\partial \hat{z}_a(f_a^*)}{\partial f_a}+\gamma_a^*+\theta\,\frac{\partial \hat{r}_a(f_a^*)}{\partial f_a}\right]\times[f_a-f_a^*]$$

$$+\sum_{a\in L}\left[\frac{\partial \hat{\pi}_a(u_a^*)}{\partial u_a}-\gamma_a^*\right]\times[u_a-u_a^*]+\sum_{k=1}^{n_R}\left[\lambda_k^+P_k(v_k^*)-\lambda_k^-(1-P_k(v_k^*))\right]\times[v_k-v_k^*]$$

$$+\sum_{a\in L}[\bar{u}_a+u_a^*-f_a^*]\times[\gamma_a-\gamma_a^*]\geq 0, \qquad \forall (f,v,u,\gamma)\in K^1, \tag{4.30}$$

where K^1 denotes the feasible set as defined below:

$$K^1\equiv\{(f,v,u,\gamma)|\exists x\geq 0,\ (4.13),(4.15),\ \text{and}\ (4.19)\ \text{hold},\ \text{and}\ \gamma\geq 0\}. \tag{4.31}$$

Explicit Formulae for the Euler Method Applied to Variational Inequality Formulation

$$x_p^{\tau+1} = \max\{0, x_p^\tau + a_\tau(\lambda_k^- \mu_p(1 - P_k(\sum_{p\in\mathcal{P}_k} x_p^\tau \mu_p)) - \lambda_k^+ \mu_p P_k(\sum_{p\in\mathcal{P}_k} x_p^\tau \mu_p)$$

$$-\frac{\partial(\sum_{q\in\mathcal{P}} \hat{C}_q(x^\tau))}{\partial x_p} - \frac{\partial(\sum_{q\in\mathcal{P}} \hat{Z}_q(x^\tau))}{\partial x_p} - \sum_{a\in L} \gamma_a^\tau \delta_{ap} - \theta \frac{\partial(\sum_{q\in\mathcal{P}} \hat{R}_q(x^\tau))}{\partial x_p})\},$$

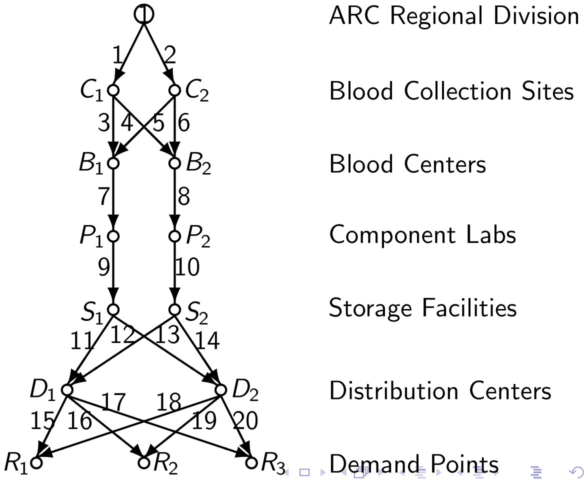
$$\forall p \in \mathcal{P}; \tag{4.37}$$

$$u_a^{\tau+1} = \max\{-\bar{u}_a, u_a^\tau + a_\tau(\gamma_a^\tau - \frac{\partial \hat{\pi}_a(u_a^\tau)}{\partial u_a})\}, \quad \forall a \in L; \tag{4.38}$$

$$\gamma_a^{\tau+1} = \max\{0, \gamma_a^\tau + a_\tau(\sum_{p\in\mathcal{P}} x_p^\tau \alpha_{ap} - \bar{u}_a - u_a^\tau)\}, \quad \forall a \in L. \tag{4.39}$$

Illustrative Numerical Examples

The Supply Chain Network Topology for the Numerical Examples 4.1-4.5



Example 4.1: Design from Scratch

The demands at these demand points followed uniform probability distribution on the intervals [5,10], [40,50], and [25,40], respectively:

$$P_1\Big(\sum_{p\in\mathcal{P}_1}\mu_p x_p\Big) = \frac{\displaystyle\sum_{p\in\mathcal{P}_1}\mu_p x_p - 5}{5}, \quad P_2\Big(\sum_{p\in\mathcal{P}_2}\mu_p x_p\Big) = \frac{\displaystyle\sum_{p\in\mathcal{P}_2}\mu_p x_p - 40}{10},$$

$$P_3\Big(\sum_{p\in\mathcal{P}_3}\mu_p x_p\Big) = \frac{\displaystyle\sum_{p\in\mathcal{P}_3}\mu_p x_p - 25}{15}.$$

$$\begin{aligned}\lambda_1^- &= 2800, \quad \lambda_1^+ = 50,\\ \lambda_2^- &= 3000, \quad \lambda_2^+ = 60,\\ \lambda_3^- &= 3100, \quad \lambda_3^+ = 50.\end{aligned}$$

$$\hat{r}_1(f_1) = 2f_1^2, \ \hat{r}_2(f_2) = 1.5f_2^2, \ \text{and } \theta = 0.7$$

Link a	α_a	$\hat{c}_a(f_a)$	$\hat{z}_a(f_a)$	$\hat{\pi}_a(u_a)$	f_a^*	u_a^*	γ_a^*
1	.97	$6f_1^2 + 15f_1$	$.8f_1^2$	$.8u_1^2 + u_1$	47.18	47.18	76.49
2	.99	$9f_2^2 + 11f_2$	$.7f_2^2$	$.6u_2^2 + u_2$	39.78	39.78	48.73
3	1.00	$.7f_3^2 + f_3$	$.6f_3^2$	$u_3^2 + 2u_3$	25.93	25.93	53.86
4	.99	$1.2f_4^2 + f_4$	$.8f_4^2$	$2u_4^2 + u_4$	19.38	19.38	78.51
5	1.00	$f_5^2 + 3f_5$	$.6f_5^2$	$u_5^2 + u_5$	18.25	18.25	37.50
6	1.00	$.8f_6^2 + 2f_6$	$.8f_6^2$	$1.5u_6^2 + 3u_6$	20.74	20.74	65.22
7	.92	$2.5f_7^2 + 2f_7$	$.5f_7^2$	$7u_7^2 + 12u_7$	43.92	43.92	626.73
8	.96	$3f_8^2 + 5f_8$	$.8f_8^2$	$6u_8^2 + 20u_8$	36.73	36.73	460.69
9	.98	$.8f_9^2 + 6f_9$	$.4f_9^2$	$3u_9^2 + 2u_9$	38.79	38.79	234.74
10	1.00	$.5f_{10}^2 + 3f_{10}$	$.7f_{10}^2$	$5.4u_{10}^2 + 2u_{10}$	34.56	34.56	375.18
11	1.00	$.3f_{11}^2 + f_{11}$	$.3f_{11}^2$	$u_{11}^2 + u_{11}$	25.90	25.90	52.80
12	1.00	$.5f_{12}^2 + 2f_{12}$	$.4f_{12}^2$	$1.5u_{12}^2 + u_{12}$	12.11	12.11	37.34
13	1.00	$.4f_{13}^2 + 2f_{13}$	$.3f_{13}^2$	$1.8u_{13}^2 + 1.5u_{13}$	17.62	17.62	64.92
14	1.00	$.6f_{14}^2 + f_{14}$	$.4f_{14}^2$	$u_{14}^2 + 2u_{14}$	16.94	16.94	35.88
15	1.00	$.4f_{15}^2 + f_{15}$	$.7f_{15}^2$	$.5u_{15}^2 + 1.1u_{15}$	5.06	5.06	6.16
16	1.00	$.8f_{16}^2 + 2f_{16}$	$.4f_{16}^2$	$.7u_{16}^2 + 3u_{16}$	24.54	24.54	37.36
17	.98	$.5f_{17}^2 + 3f_{17}$	$.5f_{17}^2$	$2u_{17}^2 + u_{17}$	13.92	13.92	56.66
18	1.00	$.7f_{18}^2 + f_{18}$	$.7f_{18}^2$	$u_{18}^2 + u_{18}$	0.00	0.00	1.00
19	1.00	$.6f_{19}^2 + 4f_{19}$	$.4f_{19}^2$	$u_{19}^2 + 2u_{19}$	15.93	15.93	33.86
20	.98	$1.1f_{20}^2 + 5f_{20}$	$.5f_{20}^2$	$.8u_{20}^2 + u_{20}$	12.54	12.54	21.06

Table 4.1: Total Cost Functions and Solution for Example 4.1

Example 4.1 Results (cont'd)

The values of the total investment cost and the cost objective criterion were 42, 375.96 and 135, 486.43, respectively.

The computed amounts of projected demand for each of the three demand points were:

$$v_1^* = 5.06, \quad v_2^* = 40.48, \text{ and } v_3^* = 25.93.$$

Note that the values of the projected demand were closer to the lower bounds of their uniform probability distributions due to the relatively high cost of setting up a new blood supply chain network from scratch.

Example 4.2: Increased Penalties

Example 4.2 had the exact same data as Example 4.1 with the exception of the penalties per unit shortage which were ten times larger.

$$\lambda_1^- = 28000, \qquad \lambda_2^- = 30000, \qquad \lambda_3^- = 31000.$$

Link a	α_a	$\hat{c}_a(f_a)$	$\hat{z}_a(f_a)$	$\hat{\pi}_a(u_a)$	f_a^*	u_a^*	γ_a^*
1	.97	$6f_1^2+15f_1$	$.8f_1^2$	$.8u_1^2+u_1$	63.53	63.53	102.65
2	.99	$9f_2^2+11f_2$	$.7f_2^2$	$.6u_2^2+u_2$	53.53	53.53	65.23
3	1.00	$.7f_3^2+f_3$	$.6f_3^2$	$u_3^2+2u_3$	34.93	34.93	71.85
4	.99	$1.2f_4^2+f_4$	$.8f_4^2$	$2u_4^2+u_4$	26.08	26.08	105.34
5	1.00	$f_5^2+3f_5$	$.6f_5^2$	$u_5^2+u_5$	24.50	24.50	50.00
6	1.00	$.8f_6^2+2f_6$	$.8f_6^2$	$1.5u_6^2+3u_6$	27.96	27.96	86.89
7	.92	$2.5f_7^2+2f_7$	$.5f_7^2$	$7u_7^2+12u_7$	59.08	59.08	839.28
8	.96	$3f_8^2+5f_8$	$.8f_8^2$	$6u_8^2+20u_8$	49.48	49.48	613.92
9	.98	$.8f_9^2+6f_9$	$.4f_9^2$	$3u_9^2+2u_9$	52.18	52.18	315.05
10	1.00	$.5f_{10}^2+3f_{10}$	$.7f_{10}^2$	$5.4u_{10}^2+2u_{10}$	46.55	46.55	504.85
11	1.00	$.3f_{11}^2+f_{11}$	$.3f_{11}^2$	$u_{11}^2+u_{11}$	35.01	35.01	71.03
12	1.00	$.5f_{12}^2+2f_{12}$	$.4f_{12}^2$	$1.5u_{12}^2+u_{12}$	16.12	16.12	49.36
13	1.00	$.4f_{13}^2+2f_{13}$	$.3f_{13}^2$	$1.8u_{13}^2+1.5u_{13}$	23.93	23.93	87.64
14	1.00	$.6f_{14}^2+f_{14}$	$.4f_{14}^2$	$u_{14}^2+2u_{14}$	22.63	22.63	47.25
15	1.00	$.4f_{15}^2+f_{15}$	$.7f_{15}^2$	$.5u_{15}^2+1.1u_{15}$	9.33	9.33	10.43
16	1.00	$.8f_{16}^2+2f_{16}$	$.4f_{16}^2$	$.7u_{16}^2+3u_{16}$	29.73	29.73	44.62
17	.98	$.5f_{17}^2+3f_{17}$	$.5f_{17}^2$	$2u_{17}^2+u_{17}$	19.89	19.89	80.55
18	1.00	$.7f_{18}^2+f_{18}$	$.7f_{18}^2$	$u_{18}^2+u_{18}$	0.00	0.00	1.00
19	1.00	$.6f_{19}^2+4f_{19}$	$.4f_{19}^2$	$u_{19}^2+2u_{19}$	18.99	18.99	39.97
20	.98	$1.1f_{20}^2+5f_{20}$	$.5f_{20}^2$	$.8u_{20}^2+u_{20}$	18.98	18.98	31.37

Table 4.2: Total Cost Functions and Solution for Example 4.2

Example 4.2 Results (cont'd)

Raising the shortage penalties increased the level of activities in almost all the network links. The new projected demand values were:

$$v_1^* = 9.33, \quad v_2^* = 48.71, \text{ and } v_3^* = 38.09.$$

Here the projected demand values were closer to the upper bounds of their uniform probability distributions.

Thus, the values of the total investment cost and the cost objective criterion, were $75,814.03$ and $177,327.31$, respectively, which were significantly higher than Example 4.1.

Example 4.3: Redesign Problem

The existing capacities for links were chosen close to the optimal solution for corresponding capacities in Example 4.1.

All other parameters were the same as in Example 4.1.

Link a	α_a	$\hat{c}_a(f_a)$	$\hat{z}_a(f_a)$	$\hat{\pi}_a(u_a)$	$\bar{u}_a$	f_a^*	u_a^*	γ_a^*
1	.97	$6f_1^2 + 15f_1$	$.8f_1^2$	$.8u_1^2 + u_1$	48	54.14	6.14	10.83
2	.99	$9f_2^2 + 11f_2$	$.7f_2^2$	$.6u_2^2 + u_2$	40	43.85	3.85	5.62
3	1.00	$.7f_3^2 + f_3$	$.6f_3^2$	$u_3^2 + 2u_3$	26	29.64	3.64	9.29
4	.99	$1.2f_4^2 + f_4$	$.8f_4^2$	$2u_4^2 + u_4$	20	22.35	2.35	10.39
5	1.00	$f_5^2 + 3f_5$	$.6f_5^2$	$u_5^2 + u_5$	19	20.10	1.10	3.20
6	1.00	$.8f_6^2 + 2f_6$	$.8f_6^2$	$1.5u_6^2 + 3u_6$	21	22.88	1.88	8.63
7	.92	$2.5f_7^2 + 2f_7$	$.5f_7^2$	$7u_7^2 + 12u_7$	44	49.45	5.45	88.41
8	.96	$3f_8^2 + 5f_8$	$.8f_8^2$	$6u_8^2 + 20u_8$	37	41.40	4.40	72.88
9	.98	$.8f_9^2 + 6f_9$	$.4f_9^2$	$3u_9^2 + 2u_9$	39	43.67	4.67	30.04
10	1.00	$.5f_{10}^2 + 3f_{10}$	$.7f_{10}^2$	$5.4u_{10}^2 + 2u_{10}$	35	38.95	3.95	44.70
11	1.00	$.3f_{11}^2 + f_{11}$	$.3f_{11}^2$	$u_{11}^2 + u_{11}$	26	29.23	3.23	7.45
12	1.00	$.5f_{12}^2 + 2f_{12}$	$.4f_{12}^2$	$1.5u_{12}^2 + u_{12}$	13	13.57	0.57	2.72
13	1.00	$.4f_{13}^2 + 2f_{13}$	$.3f_{13}^2$	$1.8u_{13}^2 + 1.5u_{13}$	18	22.05	4.05	16.07
14	1.00	$.6f_{14}^2 + f_{14}$	$.4f_{14}^2$	$u_{14}^2 + 2u_{14}$	17	16.90	-0.10	1.81
15	1.00	$.4f_{15}^2 + f_{15}$	$.7f_{15}^2$	$.5u_{15}^2 + 1.1u_{15}$	6	6.62	0.62	1.72
16	1.00	$.8f_{16}^2 + 2f_{16}$	$.4f_{16}^2$	$.7u_{16}^2 + 3u_{16}$	25	25.73	0.73	4.03
17	.98	$.5f_{17}^2 + 3f_{17}$	$.5f_{17}^2$	$2u_{17}^2 + u_{17}$	14	18.92	4.92	20.69
18	1.00	$.7f_{18}^2 + f_{18}$	$.7f_{18}^2$	$u_{18}^2 + u_{18}$	0	0.00	0.00	1.00
19	1.00	$.6f_{19}^2 + 4f_{19}$	$.4f_{19}^2$	$u_{19}^2 + 2u_{19}$	16	17.77	1.77	5.53
20	.98	$1.1f_{20}^2 + 5f_{20}$	$.5f_{20}^2$	$.8u_{20}^2 + u_{20}$	13	12.10	-0.62	0.00

Table 4.3: Total Cost Functions and Solution for Example 4.3

Example 4.3 Results (cont'd)

The optimal Lagrangian multipliers, γ_a^*, i.e., the shadow prices of the capacity adjustment constraints, were considerably smaller than their counterparts in Example 4.1.

So, the respective values of the capacity investment cost and the cost criterion were 856.36 and $85,738.13$.

The computed projected demand values:

$$v_1^* = 6.62, \quad v_2^* = 43.50, \text{ and } v_3^* = 30.40.$$

Example 4.4: Increased Demands

The existing capacities, the shortage penalties, and the cost functions were the same as in Example 4.3.

However, the demands at the three hospitals were escalated, following uniform probability distributions on the intervals [10,17], [50,70], and [30,60], respectively.

Link a	α_a	$\hat{c}_a(f_a)$	$\hat{z}_a(f_a)$	$\hat{\pi}_a(u_a)$	$\bar{u}_a$	f_a^*	u_a^*	γ_a^*
1	.97	$6f_1^2 + 15f_1$	$.8f_1^2$	$.8u_1^2 + u_1$	48	65.45	17.45	28.92
2	.99	$9f_2^2 + 11f_2$	$.7f_2^2$	$.6u_2^2 + u_2$	40	53.36	13.36	17.03
3	1.00	$.7f_3^2 + f_3$	$.6f_3^2$	$u_3^2 + 2u_3$	26	35.87	9.87	21.74
4	.99	$1.2f_4^2 + f_4$	$.8f_4^2$	$2u_4^2 + u_4$	20	26.98	6.98	28.91
5	1.00	$f_5^2 + 3f_5$	$.6f_5^2$	$u_5^2 + u_5$	19	24.43	5.43	11.86
6	1.00	$.8f_6^2 + 2f_6$	$.8f_6^2$	$1.5u_6^2 + 3u_6$	21	27.87	6.87	23.60
7	.92	$2.5f_7^2 + 2f_7$	$.5f_7^2$	$7u_7^2 + 12u_7$	44	59.94	15.94	234.92
8	.96	$3f_8^2 + 5f_8$	$.8f_8^2$	$6u_8^2 + 20u_8$	37	50.21	13.21	178.39
9	.98	$.8f_9^2 + 6f_9$	$.4f_9^2$	$3u_9^2 + 2u_9$	39	52.94	13.94	85.77
10	1.00	$.5f_{10}^2 + 3f_{10}$	$.7f_{10}^2$	$5.4u_{10}^2 + 2u_{10}$	35	47.24	12.24	134.64
11	1.00	$.3f_{11}^2 + f_{11}$	$.3f_{11}^2$	$u_{11}^2 + u_{11}$	26	35.68	9.68	20.35
12	1.00	$.5f_{12}^2 + 2f_{12}$	$.4f_{12}^2$	$1.5u_{12}^2 + u_{12}$	13	16.20	3.20	10.61
13	1.00	$.4f_{13}^2 + 2f_{13}$	$.3f_{13}^2$	$1.8u_{13}^2 + 1.5u_{13}$	18	26.54	8.54	32.23
14	1.00	$.6f_{14}^2 + f_{14}$	$.4f_{14}^2$	$u_{14}^2 + 2u_{14}$	17	20.70	3.70	9.40
15	1.00	$.4f_{15}^2 + f_{15}$	$.7f_{15}^2$	$.5u_{15}^2 + 1.1u_{15}$	6	10.30	4.30	5.40
16	1.00	$.8f_{16}^2 + 2f_{16}$	$.4f_{16}^2$	$.7u_{16}^2 + 3u_{16}$	25	30.96	5.96	11.34
17	.98	$.5f_{17}^2 + 3f_{17}$	$.5f_{17}^2$	$2u_{17}^2 + u_{17}$	14	20.95	6.95	28.81
18	1.00	$.7f_{18}^2 + f_{18}$	$.7f_{18}^2$	$u_{18}^2 + u_{18}$	0	0.35	0.35	1.69
19	1.00	$.6f_{19}^2 + 4f_{19}$	$.4f_{19}^2$	$u_{19}^2 + 2u_{19}$	16	21.68	5.68	13.36
20	.98	$1.1f_{20}^2 + 5f_{20}$	$.5f_{20}^2$	$.8u_{20}^2 + u_{20}$	13	14.14	1.14	2.83

Table 4.4: Total Cost Functions and Solution for Example 4.4

Example 4.4 Results (cont'd)

A 50% increase in demand resulted in significant positive capacity changes as well as positive flows on all 20 links in the network.

The values of the total investment function and the cost criterion were $5,949.18$ and $166,445.73$, respectively.

The projected demand values were now:

$$v_1^* = 10.65, \quad v_2^* = 52.64, \quad \text{and} \quad v_3^* = 34.39.$$

Example 4.5: Decreased Demands

Example 4.5 was similar to Example 4.4, but now the demand suffered a decrease from the original demand scenario.

The demand at demand points 1, 2, and 3 followed a uniform probability distribution on the intervals [4,7], [30,40], and [15,30], respectively.

Link a	α_a	$\hat{c}_a(f_a)$	$\hat{z}_a(f_a)$	$\hat{\pi}_a(u_a)$	$\bar{u}_a$	f_a^*	u_a^*	γ_a^*
1	.97	$6f_1^2 + 15f_1$	$.8f_1^2$	$.8u_1^2 + u_1$	48	43.02	-0.62	0.00
2	.99	$9f_2^2 + 11f_2$	$.7f_2^2$	$.6u_2^2 + u_2$	40	34.54	-0.83	0.00
3	1.00	$.7f_3^2 + f_3$	$.6f_3^2$	$u_3^2 + 2u_3$	26	23.77	-1.00	0.00
4	.99	$1.2f_4^2 + f_4$	$.8f_4^2$	$2u_4^2 + u_4$	20	17.54	-0.25	0.00
5	1.00	$f_5^2 + 3f_5$	$.6f_5^2$	$u_5^2 + u_5$	19	15.45	-0.50	0.00
6	1.00	$.8f_6^2 + 2f_6$	$.8f_6^2$	$1.5u_6^2 + 3u_6$	21	18.40	-1.00	0.00
7	.92	$2.5f_7^2 + 2f_7$	$.5f_7^2$	$7u_7^2 + 12u_7$	44	38.99	-0.86	0.00
8	.96	$3f_8^2 + 5f_8$	$.8f_8^2$	$6u_8^2 + 20u_8$	37	32.91	-1.67	0.00
9	.98	$.8f_9^2 + 6f_9$	$.4f_9^2$	$3u_9^2 + 2u_9$	39	34.43	-0.33	0.00
10	1.00	$.5f_{10}^2 + 3f_{10}$	$.7f_{10}^2$	$5.4u_{10}^2 + 2u_{10}$	35	30.96	-0.19	0.00
11	1.00	$.3f_{11}^2 + f_{11}$	$.3f_{11}^2$	$u_{11}^2 + u_{11}$	26	23.49	-0.50	0.00
12	1.00	$.5f_{12}^2 + 2f_{12}$	$.4f_{12}^2$	$1.5u_{12}^2 + u_{12}$	13	10.25	-0.33	0.00
13	1.00	$.4f_{13}^2 + 2f_{13}$	$.3f_{13}^2$	$1.8u_{13}^2 + 1.5u_{13}$	18	18.85	0.85	4.57
14	1.00	$.6f_{14}^2 + f_{14}$	$.4f_{14}^2$	$u_{14}^2 + 2u_{14}$	17	12.11	-1.00	0.00
15	1.00	$.4f_{15}^2 + f_{15}$	$.7f_{15}^2$	$.5u_{15}^2 + 1.1u_{15}$	6	5.52	-0.48	0.63
16	1.00	$.8f_{16}^2 + 2f_{16}$	$.4f_{16}^2$	$.7u_{16}^2 + 3u_{16}$	25	20.68	-2.14	0.00
17	.98	$.5f_{17}^2 + 3f_{17}$	$.5f_{17}^2$	$2u_{17}^2 + u_{17}$	14	16.15	2.15	9.59
18	1.00	$.7f_{18}^2 + f_{18}$	$.7f_{18}^2$	$u_{18}^2 + u_{18}$	0	0.00	0.00	1.00
19	1.00	$.6f_{19}^2 + 4f_{19}$	$.4f_{19}^2$	$u_{19}^2 + 2u_{19}$	16	14.58	-1.00	0.00
20	.98	$1.1f_{20}^2 + 5f_{20}$	$.5f_{20}^2$	$.8u_{20}^2 + u_{20}$	13	7.34	-0.62	0.00

Table 4.5: Total Cost Functions and Solution for Example 4.5

Example 4.5 Results (cont'd)

As expected, most of the computed capacity changes were negative as a result of the diminished demand for blood at demand points.

The projected demand values were as follows:

$$v_1^* = 5.52, \quad v_2^* = 35.25, \quad \text{and} \quad v_3^* = 23.02.$$

The value of the total cost criterion for this Example was $51,221.32$.

Summary and Conclusions

A sustainable supply chain network design model was developed for the highly perishable human blood. The model:

- captures the perishability of this life-saving product through the use of arc multipliers;
- contains discarding costs associated with waste/disposal;
- determines the optimal enhancement/reduction of capacities as well as the determination of the capacities from scratch;
- can capture the cost-related effects of shutting down specific modules of the supply chain due to an economic crisis;
- handles uncertainty associated with demand points along with shortage/surplus penalties;
- quantifies the supply-side risk associated with procurement.

Thank You!