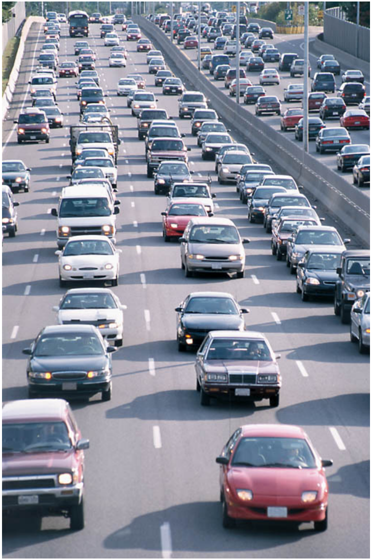
Lecture 9
Modeling Extensions

Dr. Anna Nagurney

John F. Smith Memorial Professor
Isenberg School of Management
University of Massachusetts
Amherst, Massachusetts 01003



Highway Traffic

www.env-ne.org

Modeling Extensions

Standard Model $c_a = c_a(f_a)$

The more general model to handle 2-way traffic & intersections:

Extended Model $c_a = c_a(f)$

Multimodal Model to handle multimodal networks.

$$c_a^1 = c_a^1(f_a^1, f_a^2, \cdots, f_a^k)$$

$$\vdots$$

$$c_a^k = c_a^k(f_a^1, f_a^2, \cdots, f_a^k)$$

Modeling Extensions

Extended Model

$c_a = c_a(f): \qquad f = f_a: \; a \in L$

Travel cost on path p:

$$C_p = C_p(f) = \sum_{a \in L} c_a(f)\delta_{ap}$$

Modeling Extensions

Simplest Example

User's cost:

$$c_a(f) = \sum_{b \in L} g_{ab} f_b + h_a$$

(expect main effect to be from $g_{aa} f_a$)

Total link cost:

$$\hat{c}_a(f) = \sum_{b \in L} g_{ab} f_b \times f_a + h_a f_a$$

In particular, c_a may be independent of some of the flows.

Modeling Extensions

User Equilibrium with Elastic Demand

Until now all the models that we have considered assumed a **fixed demand** associated with the O/D pairs.

In many situations, however, the demands may be influenced by the level of service on the network.

Modeling Extensions

For example, as congestion increases, motorists may decide to use a different mode of travel (e.g., subway) or forego some trips altogether.

To handle this situation, we introduce an **elastic demand** function.

Increasing Travel Demand, India

blogs.taz.de

The Elastic Demand Model

For each O/D pair w, we have now a demand function:

$$d_w = d_w(\lambda_w),$$

where λ_w denotes the disutility of traveling between O/D pair w.

We also assume that d_w admits an inverse

$$\lambda_w = \lambda_w(d_w).$$

Modeling Extensions

Then the conservation of flow equations are given by:

$$d_w = \sum_{p \in P_w} F_p, \text{ for all } w$$

and

$$f_a = \sum_p F_p \delta_{ap} \text{ for all } a$$

We also assume (for the time being) that $c_a = c_a(f_a)$, for all a.

U-O (Equilibrium) Conditions

A feasible path flow and demand pattern (F^*, d^*) is said to be U-O if and only if for each O/D pair w, and each $p \in P_w$:

$$C_p(f^*) \begin{cases} = \lambda_w(d_w^*), & \text{if } F_p^* > 0 \\ \geq \lambda_w(d_w^*), & \text{if } F_p^* = 0 \end{cases}$$

Equivalent Minimization Formulation

Minimize

$$\sum_a \int_a^{f_a} c_a(x) d_x - \sum_w \int_0^{d_w} \lambda_w(y) d_y$$

subject to:

$$f_a = \sum_p F_p \delta_{ap}, \text{ for all } a$$

$$d_w = \sum_{p \in P_w} F_p, \text{ for all } w$$

$$F_p \geq 0, \text{ for all } p.$$

Modeling Extensions

Although there exist algorithms to compute the equilibrium in the elastic demand problem, the elastic demand problem can be transformed into a **fixed demand** problem, for which efficient algorithm exists.

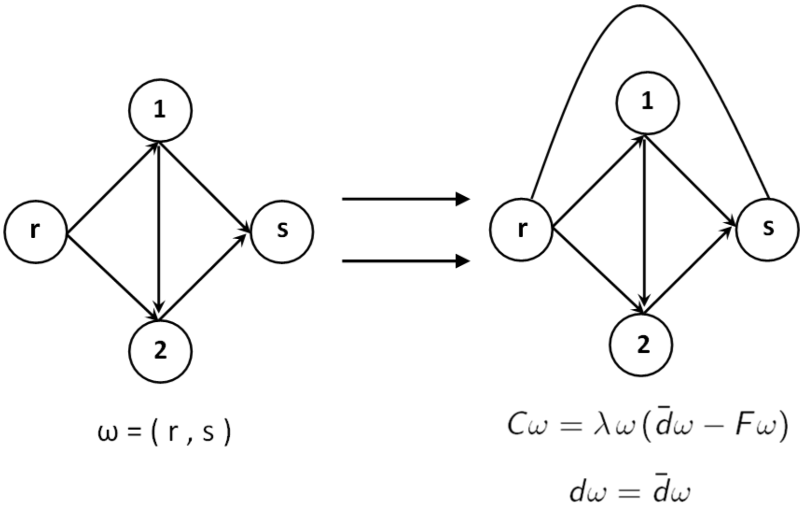
Solution by Network Representation
Excess Demand Reformulation

For each O/D pair w, we construct a path connecting O/D pair w consisting of a single link which we denote by w. Given an upper bound $\bar{d}_w > d_w(\lambda_w)$, for every w, we construct associated functions on the links thus:

$$c_w = \lambda_w(\bar{d}_w - f_w)$$

F_w denotes the flow on a new disjoint path w.

Example



References

⇒ Gartner N. (1980), Optimal Traffic Assignment with Elastic Demands: A Review; Part II: Algorithmic Approaches, *Transportation Science*, 14: pp. 192-208.

⇒ Nagurney A. and Dong J. (2002), *Supernetworks: Decision-Making for the Information Age*, Edward Elgar Publishing

http://supernet.som.umass.edu/bookser/supbook.html