# Lecture 8
# Tolls

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## TOLLS

Let the S-O flow pattern be $f$.

Let U-O flow pattern be $f'$.

What do we know?

The total cost on the network with the flow pattern $f$ is **always**

$$\leq$$

the total cost on the network with the flow pattern $f'$.

# Tolls

**Remedy**

Modify the travel costs as perceived by users by charging them a toll. This procedure does not change the travel cost spent as perceived by society (tolls aren't lost).

Then try to determine the **toll pattern** in such a way that the S-O becomes at the same time U-O.

Conventional Toll Collection

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## Two Types of Toll Policies

**Link Toll collection policy** - a toll $r_a$ is associated with each link $a$ of the network.

Then we set $R_L = r_a;\ a \in L$ - a link toll policy.

**Path Toll collection policy** - a toll $r_p$ is associated with each path $p$ of the network.

Then we set $R_p = r_p;\ p \in P$ - a path toll policy.

**Before imposition of tolls**

Travel cost incurred by users as perceived by society & by users themselves is the same.

1. on a link $a -$ $c_a(f_a)$
2. on a path $p -$ $C_p = \sum_a c_a(f_a)\delta_{ap}$

**After imposition of tolls**

Travel costs incurred by users as perceived by society is **still** given by 1 and 2. The tolls aren't lost.

However, the travel cost as perceived by **users** changes as follows:

1. Link Toll Policy

$$1.1 \quad C_p^r = C_p^r(f) = \sum_a [c_a(f_a) + r_a] \times \delta_{ap} = C_p(f) + \sum_a r_a \delta_{ap}$$

2. Path Toll Policy

$$2.1 \quad C_p^r = C_p^r(f) = C_p(f) + r_p$$

After the imposition of tolls, the corresponding S-O pattern is the same as before the imposition of tolls.

However, the U-O pattern changes.

**1. Link toll collection problem**

Determine a link toll pattern $R_L = r_a;\ a \in L$ so that the S-O pattern for a given network is **at the same time** U-O.

**2. Path toll collection problem**

Determine a path toll pattern $R_p = r_p;\ p \in P$ so that the S-O pattern for a given network is **at the same time** U-O.

# Tolls

We know that $f$ is the **unique** S-O pattern if and only if it satisfies, for every O/D pair $w$:

$$\hat{C}'_{p_1}(f) = ... = \hat{C}'_{p_{s'}}(f) = \lambda_w \leq \hat{C}'_{p_{s'+1}}(f) \leq ... \leq \hat{C}'_{p_{m'}}(f)$$

$$F_{p_r} > 0; \; r = 1, ..., s' \qquad \textbf{(II)}$$
$$F_{p_r} = 0; \; r = s'+1, ..., m' \; ,$$

where

$$\hat{C}'_p(f) = \sum_a \frac{\partial \hat{c}_a(f_a)}{\partial f_a} \delta_{ap}$$

# Tolls

**After the imposition of tolls,** the above S-O pattern is at the same time U-O if and only if it satisfies the equilibrium conditions (U-O), that is

$$C_{p_1}(f) = \ldots = C_{p_s}(f) = v_w \leq C_{p_{s+1}}(f) \leq \ldots \leq C_{p_m}(f)$$

$F_{p_r} > 0;\ r = 1, \ldots, s.$ **(I)**
$F_{p_r} = 0;\ r = s+1, \ldots, m.$

where $C_p(f)$ is given by (1.1) for a link toll policy and (2.1) for a path toll policy.

∗ When do (I) and (II) have the same solution?

# Tolls

If for a $f$ that satisfies II:

$\hat{C}'_p(f) = C^r_p(f)$ then all $p \in P, \ w \in \Omega$ **(1)**

then (I) and (II) coincide; hence, we will have identical S-O and U-O solutions.

From (1) we have:

(3.1) $\hat{C}'_p = C_p(f) + r_p$

(case of path toll policy)

(3.2) $\hat{C}'_p = \sum_a (c_a(f_a) + r_a)\delta_{ap} = C_p(f) + \sum_a r_a$

(case of link toll policy)

# Tolls

(3.1) suggests the following **path toll policy:**

$r_p = \hat{C}'_p - C_p(f)$, for all paths $p$, where $f$ = S-O pattern.

Hence, we have a path toll policy:

$R_p = r_p; \; p \in P$

**Link toll policy**

We can replace (3.2) by:

$$\hat{C}'_p = C_p(f) + \sum_a r_a \delta_{ap} = \sum_a (c_a(f_a) + r_a)\delta_{ap}$$

or

$$\sum_a \hat{c}'_a(f_a)\delta_{ap} = \sum_a (c_a(f_a) + r_a)\delta_{ap}$$

or

$$\hat{c}'_a(f_a) = c_a(f_a) + r_a$$

or

$$r_a = \hat{c}'_a(f_a) - c_a(f_a).$$

# Tolls

$f$: is a S-O solution.

$*$ determines a link toll policy $R_L = r_a; \; a \in L$

$*$ **There is (typically) only 1 way to determine (assign) a link toll policy.**

**Remark**

To each link toll policy $R_L = r_a;\; a \in L$ can construct a corresponding path toll policy $R_p$ which is induced if we set:

$$r_p = \sum_a r_a \delta_{ap}$$

But, converse is **not** true.

Given a path toll policy $R_p = r_p;\; p \in P$ we cannot usually find a link toll policy $R_L$ that satisfies the above equation for each path.

# Tolls

We have a system of equations with the $r_a$'s as unknowns.

**# of equations = # of paths**
**# of unknowns = # of links**

Usually $\#$ of paths $>$ $\#$ of links.

Hence, system does not always have a solution.

However, we have shown that system has always one solution given by:

$$r_p = \hat{C}'_p(f) - C_p(f)$$

**The General Case**

A toll policy renders a S-O flow pattern, user-optimized, if and only if for every O/D pair $w$:

$$\left.\begin{array}{c} C^r_{p_1}(f) = C_{p_1}(f) + r_{p_1} = v^r_w \\ C^r_{p_2}(f) = C_{p_2}(f) + r_{p_2} = v^r_w \\ \vdots \\ C^r_{p_s}(f) = C_{p_s}(f) + r_{p_s} = v^r_w \end{array}\right\} F_{p_i} > 0; \qquad i = 1, ..., s$$

$$\left.\begin{array}{c} C^r_{p_{s+1}}(f) = C_{p_{s+1}}(f) + r_{p_{s+1}} = v^r_w \\ \vdots \\ C^r_{p_m}(f) = C_{p_m}(f) + r_{p_m} = v^r_w \end{array}\right\} F_{p_i} = 0; \qquad i = s+1, ..., m$$

or, equivalently,

$$
\begin{aligned}
&r_{p_1} = v_w^r - C_{p_1}(f)\\
&\vdots\\
&r_{p_s} = v_w^r - C_{p_s}(f)\\
&r_{p_{s+1}} \geq v_w^r - C_{p_{s+1}}(f)\\
&\vdots\\
&r_{p_m} \geq v_w^r - C_{p_m}(f)
\end{aligned}
$$

In theory, free to choose $v_w^r$ any way you want.

**Tolls around the world**

Toll roads are found in many countries. The way they are funded and operated may differ from country to country. Some of these toll roads are privately owned and operated. Others are owned by the government. Some of the government-owned toll roads are privately operated.

Under such BOT (Build-Operate-Transfer) systems , private companies build the roads and are given a limited franchise. Ownership is transferred to the government when the franchise expires.

# Tolls

Throughout the world, this type of arrangement is prevalent in Australia, South Korea, Japan, Philippines, and Canada.

The (BOT) system is a fairly new concept that is gaining ground in the United States, with Arkansas, California, Delaware, Florida, Illinois, Indiana, Mississippi, Texas, and Virginia already building and operating toll roads under this scheme.

Pennsylvania, Massachusetts, New Jersey, and Tennessee are also considering the BOT methodology for future highway projects.


Electronic (Cash-Free) Toll Collection

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**Solution of Path Toll Problem**

Clear from the previous pages that we can construct an infinite # of solutions $R_p = r_p;\ p \in P$ to our path toll problem.

**Method:** We select **a priori** for each O/D pair the level of the user's travel cost $v_w^r$. Then determine: $r_{p_1}, ..., r_{p_s}$, that satisfy preceding relationships.

$*$ Freedom in choosing $v_w^r$ - User's Cost.

Hence, possibility of imposing additional requirements.

**Solution of Link Toll Problem**

We need: $C_p^r(f) = \hat{C}_p^r$

$$C_p^r(f) = C_p(f) + \sum r_a \delta_{ap} = \sum_a \hat{c}_a'(f_a)\delta_{ap} =$$

$$\sum_a [c_a(f_a) + r_a]\delta_{ap} = \sum_a \hat{c}_a'(f_a)\delta_{ap}$$

and $r_a = \hat{c}_a'(f_a) - c_a(f_a)$, for all links $a$.

This is in general the **only** solution to the link toll policy problem that is nonnegative.

**Summary**

**LTP & PTP**

**Step 1:** First determine the S-O flow pattern $f$ for a given network. Before proceeding to Step 3, distinguish between LTP & PTP.

**Link Toll Policy**

**Step 2:** For each link $a$ compute tolls $r_a, \;\; a \in L : r_a = \hat{c}_a'(f_a) - c_a(f_a)$ using $f_a$'s calculated in Step 1. The link toll policy $R_L$ is a solution to the link toll problem.

## Path Toll Policy

**Step 2:** For each path $p$ in the network compute user's travel cost $C_p(f) = \sum_a c_a(f_a)\delta_{ap}$ using $f_a$'s calculated in Step 1

**Step 3:** Select the level of user's cost $v_w^r$ after the imposition of tolls for every flow so that a certain objective is met.

**Step 4:** Compute now for each $w$ the tolls $r_p = v_w^r - C_p^r(f)$

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